Chapter 2

Fundamentals of Flow in Closed Conduits

2.1. From the given data: $D_1 = 0.1$ m, $D_2 = 0.15\,\text{m}$, $V_1 = 2\,\text{m/s}$. Using these data, the following preliminary calculations are useful:

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.1)^2 = 0.007854\,\text{m}^2, \qquad A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.15)^2 = 0.01767\,\text{m}^2$$

Volumetric flow rate, Q, is given by

$$Q = A_1V_1 = (0.007854)(2) = \boxed{0.0157\,\text{m}^3/\text{s}}$$

According to the continuity equation,

$$A_1V_1 = A_2V_2 = Q \quad \rightarrow \quad V_2 = \frac{Q}{A_2} = \frac{0.0157}{0.01767} = \boxed{0.889\,\text{m/s}}$$

At 20°C, the density of water, ρ, is 998 kg/m^3, and the mass flow rate, $\dot{m}$, is given by

$$\dot{m} = \rho Q = (998)(0.0157) = \boxed{15.7\,\text{kg/s}}$$

2.xx From the given data: $D_1 = 0.2\,\text{m}$, $D_2 = 0.3\,\text{m}$, and $V_1 = 0.75\,\text{m/s}$. Using these data, the following preliminary calculations are useful:

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.03142\,\text{m}^2, \qquad A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.3)^2 = 0.07069\,\text{m}^2$$

(a) According to the continuity equation,

$$A_1V_1 = A_2V_2 \quad \rightarrow \quad V_2 = \frac{A_1V_1}{A_2} = \frac{(0.03142)(0.75)}{0.07069} = \boxed{0.333\,\text{m/s}}$$

(b) The volume flow rate, Q, is given by

$$Q = A_1V_1 = (0.03142)(0.75) = 0.02357\,\text{m}^3/\text{s} = \boxed{23.6\,\text{L/s}}$$

2.2. From the given data: $D_1 = 200$ mm, $D_2 = 100$ mm, $V_1 = 1$ m/s, and

$$A_1 = \frac{\pi}{4}D_1^2 = \frac{\pi}{4}(0.2)^2 = 0.0314\,\text{m}^2$$
$$A_2 = \frac{\pi}{4}D_2^2 = \frac{\pi}{4}(0.1)^2 = 0.00785\,\text{m}^2$$

The flow rate, Q_1, in the 200-mm pipe is given by

$$Q_1 = A_1V_1 = (0.0314)(1) = 0.0314\,\text{m}^3/\text{s}$$

and hence the flow rate, Q_2, in the 100-mm pipe is

$$Q_2 = \frac{Q_1}{2} = \frac{0.0314}{2} = \boxed{0.0157\,\text{m}^3/\text{s}}$$

The average velocity, V_2, in the 100-mm pipe is

$$V_2 = \frac{Q_2}{A_2} = \frac{0.0157}{0.00785} = \boxed{2\,\text{m/s}}$$

2.3. The velocity distribution in the pipe is

$$v(r) = V_0\left[1 - \left(\frac{r}{R}\right)^2\right] \tag{1}$$

and the average velocity, $\bar{V}$, is defined as

$$\bar{V} = \frac{1}{A}\int_A V\,dA \tag{2}$$

where

$$A = \pi R^2 \qquad \text{and} \qquad dA = 2\pi r dr \tag{3}$$

Combining Equations 1 to 3 yields

$$\bar{V} = \frac{1}{\pi R^2}\int_0^R V_0\left[1 - \left(\frac{r}{R}\right)^2\right] 2\pi r dr = \frac{2V_0}{R^2}\left[\int_0^R r dr - \int_0^R \frac{r^3}{R^2}dr\right] = \frac{2V_0}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{4R^2}\right]$$
$$= \frac{2V_0}{R^2}\frac{R^2}{4} = \boxed{\frac{V_0}{2}}$$

The flow rate, Q, is therefore given by

$$Q = A\bar{V} = \boxed{\frac{\pi R^2 V_0}{2}}$$

2.4.

$$\beta = \frac{1}{A\bar{V}^2}\int_A v^2\,dA = \frac{4}{\pi R^2 V_0^2}\int_0^R V_0^2\left[1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4}\right] 2\pi r dr$$
$$= \frac{8}{R^2}\left[\int_0^R r dr - \int_0^R \frac{2r^3}{R^2}dr + \int_0^R \frac{r^5}{R^4}dr\right] = \frac{8}{R^2}\left[\frac{R^2}{2} - \frac{R^4}{2R^2} + \frac{R^6}{6R^4}\right]$$
$$= \boxed{\frac{4}{3}}$$

2.5. $D = 0.2$ m, $Q = 0.06$ m^3/s, $L = 100$ m, $p_1 = 500$ kPa, $p_2 = 400$ kPa, $\gamma = 9.79$ kN/m^3.

$$R = \frac{D}{4} = \frac{0.2}{4} = 0.05\,\text{m}$$
$$\Delta h = \frac{p_1}{\gamma} - \frac{p_2}{\gamma} = \frac{500 - 400}{9.79} = 10.2\,\text{m}$$
$$\tau_0 = \frac{\gamma R \Delta h}{L} = \frac{(9.79 \times 10^3)(0.05)(10.2)}{100} = \boxed{49.9\ \text{N/m}^2}$$
$$A = \frac{\pi D^2}{4} = \frac{\pi(0.2)^2}{4} = 0.0314\,\text{m}^2$$
$$V = \frac{Q}{A} = \frac{0.06}{0.0314} = 1.91\,\text{m/s}$$
$$f = \frac{8\tau_0}{\rho V^2} = \frac{8(49.9)}{(998)(1.91)^2} = \boxed{0.11}$$

2.6. $T = 20^\circ$C, $V = 2$ m/s, $D = 0.25$ m, horizontal pipe, ductile iron. For ductile iron pipe, $k_s =$ 0.26 mm, and

$$\frac{k_s}{D} = \frac{0.26}{250} = 0.00104$$
$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(998.2)(2)(0.25)}{(1.002 \times 10^{-3})} = 4.981 \times 10^5$$

From the Moody diagram:

$$\boxed{f = 0.0202 \text{ (flow is not fully turbulent)}}$$

Using the Colebrook equation,

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{k_s/D}{3.7} + \frac{2.51}{\text{Re}\sqrt{f}}\right)$$

Substituting for k_s/D and Re gives

$$\frac{1}{\sqrt{f}} = -2\log\left(\frac{0.00104}{3.7} + \frac{2.51}{4.981 \times 10^5\sqrt{f}}\right)$$

By trial and error leads to

$$\boxed{f = 0.0204}$$

Using the Swamee-Jain equation,

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{k_s/D}{3.7} + \frac{5.74}{\text{Re}^{0.9}}\right] = -2\log\left[\frac{0.00104}{3.7} + \frac{5.74}{(4.981 \times 10^5)^{0.9}}\right]$$

which leads to

$$\boxed{f = 0.0205}$$

The head loss, h_f, over 100 m of pipeline is given by

$$h_f = f\frac{L}{D}\frac{V^2}{2g} = 0.0204\frac{100}{0.25}\frac{(2)^2}{2(9.81)} = 1.66\,\text{m}$$

Therefore the pressure drop, Δp, is given by

$$\Delta p = \gamma h_\mathrm{f} = (9.79)(1.66) = \boxed{16.3\,\mathrm{kPa}}$$

If the pipe is 1 m lower at the downstream end, f would not change, but the pressure drop, Δp, would then be given by

$$\Delta p = \gamma(h_\mathrm{f} - 1.0) = 9.79(1.66 - 1) = \boxed{6.46\,\mathrm{kPa}}$$

2.7. From the given data: $D = 25$ mm, $k_\mathrm{s} = 0.1$ mm, $\theta = 10°$, $p_1 = 550$ kPa, and $L = 100$ m. At 20°C, $\nu = 1.00 \times 10^{-6}$ m²/s, $\gamma = 9.79$ kN/m³, and

$$\frac{k_\mathrm{s}}{D} = \frac{0.1}{25} = 0.004$$

$$A = \frac{\pi}{4}D^2 = \frac{\pi}{4}(0.025)^2 = 4.909 \times 10^{-4}\,\mathrm{m}^2$$

$$h_\mathrm{f} = f\frac{L}{D}\frac{Q^2}{2gA^2} = f\frac{100}{0.025}\frac{Q^2}{2(9.81)(4.909 \times 10^{-4})^2} = 8.46 \times 10^8 fQ^2$$

The energy equation applied over 100 m of pipe is

$$\frac{p_1}{\gamma} + \frac{V^2}{2g} + z_1 = \frac{p_2}{\gamma} + \frac{V^2}{2g} + z_2 + h_\mathrm{f}$$

which simplifies to

$$p_2 = p_1 - \gamma(z_2 - z_1) - \gamma h_\mathrm{f}$$

$$p_2 = 550 - 9.79(100\sin 10°) - 9.79(8.46 \times 10^8 fQ^2)$$

$$p_2 = 380.0 - 8.28 \times 10^9 fQ^2$$

(a) For $Q = 2$ L/min $= 3.333 \times 10^{-5}$ m³/s,

$$V = \frac{Q}{A} = \frac{3.333 \times 10^{-5}}{4.909 \times 10^{-4}} = 0.06790\,\mathrm{m/s}$$

$$\mathrm{Re} = \frac{VD}{\nu} = \frac{(0.06790)(0.025)}{1 \times 10^{-6}} = 1698$$

Since Re < 2000, the flow is laminar when $Q = 2$ L/min. Hence,

$$f = \frac{64}{\mathrm{Re}} = \frac{64}{1698} = 0.03770$$

$$p_2 = 380.0 - 8.28 \times 10^9(0.03770)(3.333 \times 10^{-5})^2 = 380\,\mathrm{kPa}$$

Therefore, when the flow is 2 L/min, the pressure at the downstream section is $\boxed{380\text{ kPa}}$.
For $Q = 20$ L/min $= 3.333 \times 10^{-4}$ m³/s,

$$V = \frac{Q}{A} = \frac{3.333 \times 10^{-4}}{4.909 \times 10^{-4}} = 0.6790\,\mathrm{m/s}$$