we have

$$\mathcal{Z}\left[k^3\right] = \frac{z^{-1}(1 + 4z^{-1} + z^{-2})}{(1 - z^{-1})^4}$$

B-2-3.

Method 1: Noting that

$$\mathcal{Z}\left[te^{-at}\right] = \frac{Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}$$

we have

$$\mathcal{Z}\left[t^2e^{-at}\right] = \mathcal{Z}\left[-\frac{\partial}{\partial a}\, te^{-at}\right] = \frac{\partial}{\partial a}\left[\frac{-Te^{-aT}z^{-1}}{(1 - e^{-aT}z^{-1})^2}\right]$$

$$= \frac{T^2e^{-aT}(1 + e^{-aT}z^{-1})z^{-1}}{(1 - e^{-aT}z^{-1})^3}$$

Method 2:

$$\mathcal{Z}\left[t^2e^{-at}\right] = \sum_{k=0}^{\infty} (kT)^2e^{-akT}z^{-k}$$

$$= T^2\left[(e^{aT}z)^{-1} + 4(e^{aT}z)^{-2} + 9(e^{aT}z)^{-3} + 16(e^{aT}z)^{-4} + \cdots\right]$$

Referring to Problem A-2-2, we have

$$\mathcal{Z}\left[t^2e^{-at}\right] = \frac{T^2e^{-aT}z^{-1}(1 + e^{-aT}z^{-1})}{(1 - e^{-aT}z^{-1})^3}$$

B-2-4.

$$\mathcal{Z}\left[x(k)\right] = \mathcal{Z}\left[9k(2^{k-1})\right] - \mathcal{Z}\left[2^k\right] + \mathcal{Z}\left[3\right]$$

$$= \frac{9z^{-1}}{(1 - 2z^{-1})^2} - \frac{1}{1 - 2z^{-1}} + \frac{3}{1 - z^{-1}}$$

$$= \frac{2 + z^{-2}}{(1 - 2z^{-1})^2(1 - z^{-1})}$$

B-2-5. Referring to Problem A-2-4, we have

$$\mathcal{Z}\left[\sum_{h=0}^{k} a^h\right] = \frac{1}{1 - z^{-1}}\, X(z)$$

where

$$X(z) = \mathcal{Z}\left[a^h\right] = \frac{1}{1 - az^{-1}}$$

Hence

$$\mathcal{Z}\left[\sum_{h=0}^{k} a^h\right] = \frac{1}{1 - z^{-1}} \cdot \frac{1}{1 - az^{-1}}$$

B-2-6.

$$\mathcal{Z}\left[ka^{k-1}\right] = \mathcal{Z}\left[\frac{\partial}{\partial a} a^k\right] = \frac{\partial}{\partial a}\mathcal{Z}\left[a^k\right] = \frac{\partial}{\partial a}\left(\frac{1}{1 - az^{-1}}\right)$$

$$= \frac{z^{-1}}{(1 - az^{-1})^2} = \frac{z}{(z - a)^2}$$

$$\mathcal{Z}\left[k(k - 1)a^{k-2}\right] = \mathcal{Z}\left[\frac{\partial}{\partial a}(ka^{k-1})\right] = \frac{\partial}{\partial a}\left[\frac{z}{(z - a)^2}\right]$$

$$= \frac{2z}{(z - a)^3} = \frac{(2!)z}{(z - a)^3}$$

$$\mathcal{Z}\left[k(k - 1)\cdots(k - h + 1)a^{k-h}\right]$$

$$= \mathcal{Z}\left[\frac{\partial}{\partial a} k(k - 1)\cdots(k - h + 2)a^{k-h+1}\right]$$

$$= \frac{\partial}{\partial a} X(z, a)$$

where

$$X(z, a) = \frac{(h - 1)!\, z}{(z - a)^h}$$

Hence

$$\mathcal{Z}\left[k(k - 1)\cdots(k - h + 1)a^{k-h}\right] = \frac{\partial}{\partial a} X(z, a)$$

$$= (h - 1)!\, zh(z - a)^{-h-1} = \frac{h!\, z}{(z - a)^{h+1}}$$

B-2-7. From Figure 2-8 we have

$$x(0) = 0, \quad x(1) = 0, \quad x(2) = 0, \quad x(3) = \frac{1}{3}$$

$$x(4) = \frac{2}{3}, \quad x(k) = 1 \quad \text{for } k = 5, 6, 7, \ldots$$

Then

$$X(z) = \sum_{k=0}^{\infty} x(k)\, z^{-k}$$

$$= \frac{1}{3} z^{-3} + \frac{2}{3} z^{-4} + z^{-5} + z^{-6} + z^{-7} + \cdots$$

$$= \frac{1}{3} (z^{-3} + 2z^{-4}) + \frac{z^{-5}}{1 - z^{-1}}$$

$$= \frac{1}{3} \frac{z^{-3} + z^{-4} + z^{-5}}{1 - z^{-1}}$$

B-2-8. By dividing both numerator and denominator by z^4, we have

$$X(z) = 5 + 4z^{-1} + 3z^{-2} + 2z^{-3} + z^{-4}$$

This last equation is already in the form of a power series in z^{-1}. By inspection, we have

$$x(0) = 5$$
$$x(1) = 4$$
$$x(2) = 3$$
$$x(3) = 2$$
$$x(4) = 1$$
$$x(k) = 0 \qquad k \geqslant 5$$

Note that the given X(z) is the z transform of a signal of finite length.

B-2-9.

1. Partial-fraction-expansion method:

$$X(z) = \frac{z^{-1}(0.5 - z^{-1})}{(1 - 0.5z^{-1})(1 - 0.8z^{-1})^2} = \frac{z(0.5z - 1)}{(z - 0.5)(z - 0.8)^2}$$

Hence,

$$\frac{X(z)}{z} = -\frac{8.3333}{z - 0.5} + \frac{8.3333}{z - 0.8} - \frac{2}{(z - 0.8)^2}$$

or

$$X(z) = -\frac{8.3333}{1 - 0.5z^{-1}} + \frac{8.3333}{1 - 0.8z^{-1}} - \frac{2z^{-1}}{(1 - 0.8z^{-1})^2}$$

Thus,

$$x(k) = -8.3333(0.5)^k + 8.3333(0.8)^k - 2k(0.8)^{k-1}, \qquad k = 0, 1, 2, \ldots$$

2. Computational solution with MATLAB: