Chapter P
Fundamental Concepts of Algebra

Section P.1

Check Point Exercises

1. $8+6(x-3)^2 = 8+6(13-3)^2$
$= 8+6(10)^2$
$= 8+6(100)$
$= 8+600$
$= 608$

2. **a.** Since 2016 is 16 years after 2000, substitute 16 for x.
$T = -x^2+361x+3193$
$= -(16)^2+361(16)+3193$
$= 8713$
The average cost of tuition and fees at public U.S. colleges for the school year ending in 2016 was \$8713.

b. The formula underestimates the actual answer by \$65.

3. The elements common to $\{3, 4, 5, 6, 7\}$ and $\{3, 7, 8, 9\}$ are 3 and 7.
$\{3,4,5,6,7\} \cap \{3,7,8,9\} = \{3,7\}$

4. The union is the set containing all the elements of either set.
$\{3,4,5,6,7\} \cup \{3,7,8,9\} = \{3,4,5,6,7,8,9\}$

5. $\left\{-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}\right\}$

a. Natural numbers: $\sqrt{9}$ because $\sqrt{9}=3$

b. Whole numbers: $0,\ \sqrt{9}$

c. Integers: $-9,\ 0,\ \sqrt{9}$

d. Rational numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \sqrt{9}$

e. Irrational numbers: $\frac{\pi}{2},\ \sqrt{10}$

f. Real numbers: $-9,\ -1.3,\ 0,\ 0.\overline{3},\ \frac{\pi}{2},\ \sqrt{9},\ \sqrt{10}$

6. **a.** $\left|1-\sqrt{2}\right|$
Because $\sqrt{2} \approx 1.4,$ the number inside the absolute value bars is negative. The absolute value of x when $x < 0$ is $-x$. Thus,
$\left|1-\sqrt{2}\right| = -\left(1-\sqrt{2}\right) = \sqrt{2}-1$

b. $|\pi-3|$
Because $\pi \approx 3.14,$ the number inside the absolute value bars is positive. The absolute value of a positive number is the number itself. Thus,
$|\pi-3| = \pi-3.$

c. $\frac{|x|}{x}$
Because $x > 0,\ \ |x| = x.$
Thus, $\frac{|x|}{x} = \frac{x}{x} = 1$

7. $|-4-(5)| = |-9| = 9$
The distance between –4 and 5 is 9.

8. $7(4x^2+3x)+2(5x^2+x)$
$= 7(4x^2+3x)+2(5x^2+x)$
$= 28x^2+21x+10x^2+2x$
$= 38x^2+23x$

9. $6+4[7-(x-2)]$
$= 6+4[7-x+2)]$
$= 6+4[9-x]$
$= 6+36-4x$
$= 42-4x$

Concept and Vocabulary Check P.1

C1. expression

C2. b to the nth power; base; exponent

C3. formula; modeling; models

C4. intersection; $A \cap B$

C5. union; $A \cup B$

C6. natural

C7. whole

C8. integers

C9. rational

C10. irrational

C11. rational; irrational

C12. absolute value; x, $-x$

C13. $b+a$; ba

C14. $a+(b+c)$; $(ab)c$

C15. $ab+ac$

C16. 0; inverse; 0; identity

C17. inverse; 1; identity

C18. simplified

C19. a

Exercise Set P.1

1. $7+5(10)=7+50=57$

2. $8+6(5)=8+30=38$

3. $6(3)-8=18-8=10$

4. $8(3)-4=24-4=20$

5. $8^2+3(8)=64+24=88$

6. $6^2+5(6)=36+30=66$

7. $7^2-6(7)+3=49-42+3=7+3=10$

8. $8^2-7(8)+4=64-56+4=8+4=12$

9. $4+5(9-7)^3=4+5(2)^3$
$=4+5(8)=4+40=44$

10. $6+5(8-6)^3=6+5(2)^3$
$=6+5(8)$
$=6+40=46$

11. $8^2-3(8-2)=64-3(6)$
$=64-18=46$

12. $8^2-4(8-3)=64-4(5)=64-20=44$

13. $\dfrac{5(x+2)}{2x-14}=\dfrac{5(10+2)}{2(10)-14}$
$=\dfrac{5(12)}{6}$
$=5\cdot 2$
$=10$

14. $\dfrac{7(x-3)}{2x-16}=\dfrac{7(9-3)}{2(9)-16}=\dfrac{7(6)}{2}=7\cdot 3=21$

15. $\dfrac{2x+3y}{x+1}; x=-2, y=4$
$=\dfrac{2(-2)+3(4)}{-2+1}=\dfrac{-4+12}{-1}=\dfrac{8}{-1}=-8$

16. $\dfrac{2x+y}{xy-2x}$; $x=-2$ and $y=4$
$\dfrac{2(-2)+4}{(-2)(4)-2(-2)}=\dfrac{-4+4}{-8+4}=\dfrac{0}{4}=0$

17. $C=\dfrac{5}{9}(50-32)=\dfrac{5}{9}(18)=10$
50°F is equivalent to 10°C.

18. $C=\dfrac{5}{9}(F-32)=\dfrac{5}{9}(86-32)=\dfrac{5}{9}(54)=30$
86°F is equivalent to 30°C.

19. $h=4+60t-16t^2=4+60(2)-16(2)^2$
$=4+120-16(4)=4+120-64$
$=124-64=60$
Two seconds after it is kicked, the ball's height is 60 feet.

20. $h = 4 + 60t - 16t^2$

$= 4 + 60(3) - 16(3)^2$

$= 4 + 180 - 16(9)$

$= 4 + 180 - 144$

$= 184 - 144 = 40$

Three seconds after it is kicked, the ball's height is 40 feet.

21. $\{1,2,3,4\} \cap \{2,4,5\} = \{2,4\}$

22. $\{1,3,7\} \cap \{2,3,8\} = \{3\}$

23. $\{s,e,t\} \cap \{t,e,s\} = \{s,e,t\}$

24. $\{r,e,a,l\} \cap \{l,e,a,r\} = \{r,e,a,l\}$

25. $\{1,3,5,7\} \cap \{2,4,6,8,10\} = \{\ \}$

The empty set is also denoted by $\varnothing$.

26. $\{1,3,5,7\} \cap \{-5,-3,-1\} = \{\ \}$ or $\varnothing$

27. $\{a,b,c,d\} \cap \varnothing = \varnothing$

28. $\{w,y,z\} \cap \varnothing = \varnothing$

29. $\{1,2,3,4\} \cup \{2,4,5\} = \{1,2,3,4,5\}$

30. $\{1,3,7,8\} \cup \{2,3,8\} = \{1,2,3,7,8\}$

31. $\{1,3,5,7\} \cup \{2,4,6,8,10\}$

$= \{1,2,3,4,5,6,7,8,10\}$

32. $\{0,1,3,5\} \cup \{2,4,6\} = \{0,1,2,3,4,5,6\}$

33. $\{a,e,i,o,u\} \cup \varnothing = \{a,e,i,o,u\}$

34. $\{e,m,p,t,y\} \cup \varnothing = \{e,m,p,t,y\}$

35. **a.** $\sqrt{100}$

b. $0, \sqrt{100}$

c. $-9, 0, \sqrt{100}$

d. $-9, -\frac{4}{5}, 0, 0.25, 9.2, \sqrt{100}$

e. $\sqrt{3}$

f. $-9, -\frac{4}{5}, 0, 0.25, \sqrt{3}, 9.2, \sqrt{100}$

36. **a.** $\sqrt{49}$

b. $0, \sqrt{49}$

c. $-7, 0, \sqrt{49}$

d. $-7, -0.\overline{6}, 0, \sqrt{49}$

e. $\sqrt{50}$

f. $-7, -0.\overline{6}, 0, \sqrt{49}, \sqrt{50}$

37. **a.** $\sqrt{64}$

b. $0, \sqrt{64}$

c. $-11, 0, \sqrt{64}$

d. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{64}$

e. $\sqrt{5}, \pi$

f. $-11, -\frac{5}{6}, 0, 0.75, \sqrt{5}, \pi, \sqrt{64}$

38. **a.** $\sqrt{4}$

b. $0, \sqrt{4}$

c. $-5, 0, \sqrt{4}$

d. $-5, -0.\overline{3}, 0, \sqrt{4}$

e. $\sqrt{2}$

f. $-5, -0.\overline{3}, 0, \sqrt{2}, \sqrt{4}$

39. 0

40. Answers will vary. An example is $\frac{1}{2}$.

41. Answers will vary. An example is 2.

42. Answers will vary. An example is -2.

43. true; -13 is to the left of -2 on the number line.

44. false; -6 is to the left of 2 on the number line.

45. true; 4 is to the right of -7 on the number line.

46. true; –13 is to the left of –5 on the number line.

47. true; $-\pi = -\pi$

48. true; –3 is to the right of –13 on the number line.

49. true; 0 is to the right of –6 on the number line.

50. true; 0 is to the right of –13 on the number line.

51. $|300| = 300$

52. $|-203| = 203$

53. $|12 - \pi| = 12 - \pi$

54. $|7 - \pi| = 7 - \pi$

55. $\left|\sqrt{2} - 5\right| = 5 - \sqrt{2}$

56. $\left|\sqrt{5} - 13\right| = 13 - \sqrt{5}$

57. $\dfrac{-3}{|-3|} = \dfrac{-3}{3} = -1$

58. $\dfrac{-7}{|-7|} = \dfrac{-7}{7} = -1$

59. $\big||-3| - |-7|\big| = |3 - 7| = |-4| = 4$

60. $\big||-5| - |-13|\big| = |5 - 13| = |-8| = 8$

61. $|x + y| = |2 + (-5)| = |-3| = 3$

62. $|x - y| = |2 - (-5)| = |7| = 7$

63. $|x| + |y| = |2| + |-5| = 2 + 5 = 7$

64. $|x| - |y| = |2| - |-5| = 2 - 5 = -3$

65. $\dfrac{y}{|y|} = \dfrac{-5}{|-5|} = \dfrac{-5}{5} = -1$

66. $\dfrac{|x|}{x} + \dfrac{|y|}{y} = \dfrac{|2|}{2} + \dfrac{|-5|}{-5} = \dfrac{2}{2} + \dfrac{5}{-5} = 1 + (-1) = 0$

67. The distance is $|2 - 17| = |-15| = 15$.

68. The distance is $|4 - 15| = |-11| = 11$.

69. The distance is $|-2 - 5| = |-7| = 7$.

70. The distance is $|-6 - 8| = |-14| = 14$.

71. The distance is $|-19 - (-4)| = |-19 + 4| = |-15| = 15$.

72. The distance is $|-26 - (-3)| = |-26 + 3| = |-23| = 23$.

73. The distance is
$|-3.6 - (-1.4)| = |-3.6 + 1.4| = |-2.2| = 2.2$.

74. The distance is
$|-5.4 - (-1.2)| = |-5.4 + 1.2| = |-4.2| = 4.2$.

75. $6 + (-4) = (-4) + 6$;
commutative property of addition

76. $11 \cdot (7 + 4) = 11 \cdot 7 + 11 \cdot 4$;
distributive property of multiplication over addition

77. $6 + (2 + 7) = (6 + 2) + 7$;
associative property of addition

78. $6 \cdot (2 \cdot 3) = 6 \cdot (3 \cdot 2)$;
commutative property of multiplication

79. $(2 + 3) + (4 + 5) = (4 + 5) + (2 + 3)$;
commutative property of addition

80. $7 \cdot (11 \cdot 8) = (11 \cdot 8) \cdot 7$;
commutative property of multiplication

81. $2(-8 + 6) = -16 + 12$;
distributive property of multiplication over addition

82. $-8(3 + 11) = -24 + (-88)$;
distributive property of multiplication over addition

83. $\dfrac{1}{x+3}(x+3) = 1; x \neq -3$,
inverse property of multiplication

84. $(x+4) + \big[-(x+4)\big] = 0$;
inverse property of addition

85. $\begin{aligned} 5(3x+4) - 4 &= 5 \cdot 3x + 5 \cdot 4 - 4 \\ &= 15x + 20 - 4 \\ &= 15x + 16 \end{aligned}$