# Chapter 1

1.1 (a) $x(t) = A \cos 2\pi f_0 t$ : Power signal

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} A^2 \cos^2 2\pi f_0 t\,dt$$

$$= \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \frac{A^2}{2}\,dt = \frac{A^2}{2}$$

(b)

$$x(t) = \begin{cases} A\cos 2\pi f_0 t & \text{for } -T_0/2 \le t \le \frac{T_0}{2} \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-T_0/2}^{T_0/2} A^2 \cos^2 2\pi f_0 t\,dt = \frac{A^2 T_0}{2}$$

(c) $$x(t) = \begin{cases} A\exp(-at) & \text{for } t > 0,\ a > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Energy signal

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_0^{\infty} A^2 \exp(-2at)\,dt$$

$$= \left[\frac{A^2 \exp(-2at)}{-2a}\right]_0^{\infty} = \frac{A^2}{2a}$$

(d) $x(t) = \cos t + 5\cos 2t \quad \text{for } -\infty < t < \infty$

Power signal

$$P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} \cos^2 t + 25\cos^2 2t \, dt\,; \quad \begin{array}{l} 2\pi f_0 = 1 \\ T_0 = 2\pi \end{array}$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}\left(\frac{1}{2} + \frac{25}{2}\right) dt = \frac{1}{2\pi}\left(26\pi\right) = 13$$

**1.2** $x(t) = \text{rect}(t/T)$

$$= \begin{cases} 1 & \text{for } -T/2 \le t \le T/2 \\ 0 & \text{elsewhere} \end{cases}$$

ESD $\Psi(f) = |X(f)|^2 = T^2 \,\text{sinc}^2(fT)$

$$E_x = \int_{-\infty}^{\infty} x^2(t)\,dt = \int_{-T/2}^{T/2} dt = T$$

**1.3** Using Equations (1.18) and (1.19)

$$G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2 \,\delta(f - nf_0)$$

$$P_x = \int_{-\infty}^{\infty} G_x(f)\,df = \int_{-\infty}^{\infty} \sum_{n=-\infty}^{\infty} |c_n|^2 \delta(f - nf_0)\,df$$

$$P_x = \sum_{n=-\infty}^{\infty} |c_n|^2$$

$\underline{\underline{1.4}}$ $\quad P_x = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x^2(t)\,dt\,; \qquad 2\pi f_0 = 10$

$$f_0 = 5/\pi$$

$$T_0 = \pi/5$$

$$P_x = \frac{5}{\pi}\int_{-\pi/10}^{\pi/10} 100\cos^2 10t + 400\cos^2 20t\;dt$$

$$= \frac{5}{2\pi}\int_{-\pi/10}^{\pi/10} 100\left(1+\cos 20t\right) + 400\left(1+\cos 40t\right)dt$$

$$= \frac{5}{2\pi}\Big[100t + 400t\Big]_{-\pi/10}^{\pi/10} = 250\ \text{W}$$

$\underline{\underline{1.5}}$ $\quad G_x(f) = \sum_{n=-\infty}^{\infty} |c_n|^2\,\delta(f - nf_0)$

$$c_1 = c_{-1} = \frac{10}{2} = 5; \quad c_2 = c_{-2} = \frac{20}{2} = 10$$

$$c_n = 0 \text{ for } n = 0, \pm 3, \pm 4, \cdots$$

$$G_x(f) = (5)^2\,\delta\!\left(f - \frac{5}{\pi}\right) + (5)^2\,\delta\!\left(f + 5/\pi\right)$$

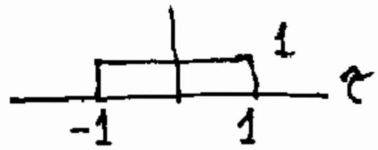
$$+ (10)^2\,\delta\!\left(f - 10/\pi\right) + (10)^2\,\delta\!\left(f + 10/\pi\right)$$

$$P_x = \int_{-\infty}^{\infty} G_x(f)\,df = 25 + 25 + 100 + 100$$

$$= 250\ \text{W}$$

**1.6** $\mathcal{F}\{R(\tau)\}$ must be a nonnegative function because $\mathcal{F}\{R(\tau)\} = G(f)$; and, the power spectral density, $G(f)$, must be a nonnegative function.

(a) $x(\tau) = \begin{cases} 1 & \text{for } -1 \le \tau \le 1 \\ 0 & \text{otherwise} \end{cases}$




**NO**
$$\begin{cases} 1.\ x(\tau) = x(-\tau) \ \checkmark \\ 2.\ x(0) \ge x(\tau) \ \checkmark \\ 3.\ \mathcal{F}\{x(\tau)\} \text{ is a positive \underline{and} \underline{negative} going function.} \end{cases}$$

(b) $x(\tau) = \delta(\tau) + \sin 2\pi f_0 \tau$

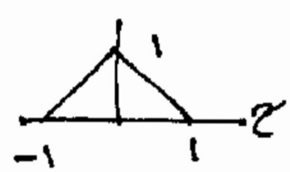
**NO** 1. $x(\tau) \ne x(-\tau)$ ×

(c) $x(t) = \exp(|\tau|)$

**NO**

1. $x(\tau) = x(-\tau)$ ✓
2. $x(0) \not\ge x(\tau)$ ×

(d) $x(\tau) = \begin{cases} -\tau + 1 & \text{for } 0 \le \tau \le 1 \\ \tau + 1 & \text{for } -1 \le \tau \le 0 \end{cases}$




**YES**
$$\begin{cases} 1.\ x(\tau) = x(-\tau) \ \checkmark \\ 2.\ x(0) \ge x(\tau) \ \checkmark \\ 3.\ \mathcal{F}\{x(\tau)\} = 2\,\mathrm{sinc}^2 f\tau \end{cases}$$

is a nonnegative function. ✓