Section 1.2 Functions

1. domain, range, function

2. independent, dependent

3. No. The input element $x = 3$ cannot be assigned to more than exactly one output element.

4. To find $g(x+1)$ for $g(x) = 3x - 2$, substitute x with the quantity $x + 1$.

$$\begin{aligned} g(x+1) &= 3(x+1) - 2 \\ &= 3x + 3 - 2 \\ &= 3x + 1 \end{aligned}$$

5. No. The domain of the function $f(x) = \sqrt{1+x}$ is $[-1, \infty)$ which does not include $x = -2$.

6. The domain of a piece-wise function must be explicitly described, so that it can determine which equation is used to evaluate the function.

7. Yes. Each domain value is matched with only one range value.

8. No. The domain value of -1 is matched with two output values.

9. No. The National Football Conference, an element in the domain, is assigned to three elements in the range, the Giants, the Saints, and the Seahawks; The American Football Conference, an element in the domain, is also assigned to three elements in the range, the Patriots, the Ravens, and the Steelers.

10. Yes. Each element, or state, in the domain is assigned to exactly one element, or electoral votes, in the range.

11. Yes, the table represents y as a function of x. Each domain value is matched with only one range value.

12. No, the table does not represent a function. The input values of 0 and 1 are each matched with two different output values.

13. No, the graph does not represent a function. The input values 1, 2, and 3 are each matched with two outputs.

14. Yes, the graph represents a function. Each input value is matched with one output value.

15. (a) Each element of A is matched with exactly one element of B, so it does represent a function.
(b) The element 1 in A is matched with two elements, -2 and 1 of B, so it does not represent a function.
(c) Each element of A is matched with exactly one element of B, so it does represent a function.

16. (a) The element c in A is matched with two elements, 2 and 3 of B, so it is not a function.
(b) Each element of A is matched with exactly one element of B, so it does represent a function.
(c) This is not a function from A to B (it represents a function from B to A instead).

17. Both are functions. For each year there is exactly one and only one average price of a name brand prescription and average price of a generic prescription.

18. Since $b(t)$ represents the average price of a name brand prescription, $b(2009) \approx \$151$. Since $g(t)$ represents the average price of a generic prescription, $g(2006) \approx \$31$.

19. $x^2 + y^2 = 4 \Rightarrow y = \pm\sqrt{4 - x^2}$

Thus, y *is not* a function of x. For instance, the values $y = 2$ and $y = -2$ both correspond to $x = 0$.

20. $x = y^2 + 1$

$y = \pm\sqrt{x - 1}$

This *is not* a function of x. For example, the values $y = 2$ and $y = -2$ both correspond to $x = 5$.

21. $y = \sqrt{x^2 - 1}$

This *is* a function of x.

22. $y = \sqrt{x + 5}$

This *is* a function of x.

23. $2x + 3y = 4 \Rightarrow y = \frac{1}{3}(4 - 2x)$

Thus, y *is* a function of x.

24. $x = -y + 5 \Rightarrow y = -x + 5$

This *is* a function of x.

25. $y^2 = x^2 - 1 \Rightarrow y = \pm\sqrt{x^2 - 1}$

Thus, y *is not* a function of x. For instance, the values $y = \sqrt{3}$ and $y = -\sqrt{3}$ both correspond to $x = 2$.

26. $x + y^2 = 3 \Rightarrow y = \pm\sqrt{3 - x}$

Thus, y *is not* a function of x.

27. $y = |4 - x|$

This *is* a function of x.

28. $|y| = 3 - 2x \Rightarrow y = 3 - 2x$ or $y = -(3 - 2x)$

Thus, y *is not* a function of x.

29. $x = -7$ *does not* represent y as a function of x. All values of y correspond to $x = -7$.

30. $y = 8$ *is* a function of x, a constant function.

31. $f(t) = 3t + 1$

(a) $f(2) = 3(2) + 1 = 7$
(b) $f(-4) = 3(-4) + 1 = -11$
(c) $f(t + 2) = 3(t + 2) + 1 = 3t + 7$

32. $g(y) = 7 - 3y$

(a) $g(0) = 7 - 3(0) = 7$

(b) $g\left(\frac{7}{3}\right) = 7 - 3\left(\frac{7}{3}\right) = 0$

(c) $g(s + 5) = 7 - 3(s + 5)$
$= 7 - 3s - 15 = -3s - 8$

33. $h(t) = t^2 - 2t$

(a) $h(2) = 2^2 - 2(2) = 0$

(b) $h(1.5) = (1.5)^2 - 2(1.5) = -0.75$

(c) $h(x - 4) = (x - 4)^2 - 2(x - 4)$
$= x^2 - 8x + 16 - 2x + 8$
$= x^2 - 10x + 24$

34. $V(r) = \frac{4}{3}\pi r^3$

(a) $V(3) = \frac{4}{3}\pi(3)^3 = 36\pi$

(b) $V\left(\frac{3}{2}\right) = \frac{4}{3}\pi\left(\frac{3}{2}\right)^3 = \frac{4}{3}\cdot\frac{27}{8}\pi = \frac{9\pi}{2}$

(c) $V(2r) = \frac{4}{3}\pi(2r)^3 = \frac{32\pi r^3}{3}$

35. $f(y) = 3 - \sqrt{y}$

(a) $f(4) = 3 - \sqrt{4} = 1$

(b) $f(0.25) = 3 - \sqrt{0.25} = 2.5$

(c) $f(4x^2) = 3 - \sqrt{4x^2} = 3 - 2|x|$

36. $f(x) = \sqrt{x + 8} + 2$

(a) $f(-4) = \sqrt{-4 + 8} + 2 = 4$

(b) $f(8) = \sqrt{8 + 8} + 2 = 6$

(c) $f(x - 8) = \sqrt{x - 8 + 8} + 2 = \sqrt{x} + 2$

37. $q(x) = \frac{1}{x^2 - 9}$

(a) $q(-3) = \frac{1}{(-3)^2 - 9} = \frac{1}{9 - 9} = \frac{1}{0}$ undefined

(b) $q(2) = \frac{1}{(2)^2 - 9} = \frac{1}{4 - 9} = -\frac{1}{5}$

(c) $q(y + 3) = \frac{1}{(y + 3)^2 - 9} = \frac{1}{y^2 + 6y + 9 - 9} = \frac{1}{y^2 + 6y}$

38. $q(t) = \frac{2t^2 + 3}{t^2}$

(a) $q(2) = \frac{2(2)^2 + 3}{(2)^2} = \frac{8 + 3}{4} = \frac{11}{4}$

(b) $q(0) = \frac{2(0)^2 + 3}{(0)^2}$ Division by zero is undefined.

(c) $q(-x) = \frac{2(-x)^2 + 3}{(-x)^2} = \frac{2x^2 + 3}{x^2}$

39. $f(x) = \frac{|x|}{x}$

(a) $f(9) = \frac{|9|}{9} = 1$

(b) $f(-9) = \frac{|-9|}{-9} = -1$

(c) $f(t) = \frac{|t|}{t} = \begin{cases} 1, & t > 0 \\ -1, & t < 0 \end{cases}$

$f(0)$ is undefined.

40. $f(x) = |x| + 4$

(a) $f(5) = |5| + 4 = 9$

(b) $f(-5) = |-5| + 4 = 9$

(c) $f(t) = |t| + 4$

41. $f(x) = \begin{cases} 2x + 1, & x < 0 \\ 2x + 2, & x \ge 0 \end{cases}$

(a) $f(-1) = 2(-1) + 1 = -1$

(b) $f(0) = 2(0) + 2 = 2$

(c) $f(2) = 2(2) + 2 = 6$

42. $f(x) = \begin{cases} 2x + 5, & x \le 0 \\ 2 - x, & x > 0 \end{cases}$

(a) $f(-2) = 2(-2) + 5 = 1$

(b) $f(0) = 2(0) + 5 = 5$

(c) $f(1) = 2 - 1 = 1$

43. $f(x) = \begin{cases} x^2 + 2, & x \le 1 \\ 2x^2 + 2, & x > 1 \end{cases}$

(a) $f(-2) = (-2)^2 + 2 = 6$

(b) $f(1) = (1)^2 + 2 = 3$

(c) $f(2) = 2(2)^2 + 2 = 10$

44. $f(x) = \begin{cases} x^2 - 4, & x \le 0 \\ 1 - 2x^2, & x > 0 \end{cases}$

(a) $f(-2) = (-2)^2 - 4 = 4 - 4 = 0$

(b) $f(0) = 0^2 - 4 = -4$

(c) $f(1) = 1 - 2(1^2) = 1 - 2 = -1$

45. $f(x) = \begin{cases} x + 2, & x < 0 \\ 4, & 0 \le x < 2 \\ x^2 + 1, & x \ge 2 \end{cases}$

(a) $f(-2) = (-2) + 2 = 0$

(b) $f(0) = 4$

(c) $f(2) = (2)^2 + 1 = 5$

46. $f(x) = \begin{cases} 5 - 2x, & x < 0 \\ 5, & 0 \le x < 1 \\ 4x + 1, & x \ge 1 \end{cases}$

(a) $f(-4) = 5 - 2(-4) = 13$

(b) $f(0) = 5$

(c) $f(1) = 4(1) + 1 = 5$

47. $f(x) = (x - 1)^2$

$\{(-2, 9), (-1, 4), (0, 1), (1, 0), (2, 1)\}$

48. $f(x) = x^2 - 3$

$\{(-2, 1), (-1, -2), (0, -3), (1, -2), (2, 1)\}$

49. $f(x) = |x| + 2$

$\{(-2, 4), (-1, 3), (0, 2), (1, 3), (2, 4)\}$

50. $f(x) = |x + 1|$

$\{(-2, 1), (-1, 0), (0, 1), (1, 2), (2, 3)\}$

51. $h(t) = \frac{1}{2}|t + 3|$

$h(-5) = \frac{1}{2}|-5 + 3| = \frac{1}{2}|-2| = \frac{1}{2}(2) = 1$

$h(-4) = \frac{1}{2}|-4 + 3| = \frac{1}{2}|-1| = \frac{1}{2}(1) = \frac{1}{2}$

$h(-3) = \frac{1}{2}|-3 + 3| = \frac{1}{2}|0| = 0$

$h(-2) = \frac{1}{2}|-2 + 3| = \frac{1}{2}|1| = \frac{1}{2}(1) = \frac{1}{2}$

$h(-1) = \frac{1}{2}|-1 + 3| = \frac{1}{2}|2| = \frac{1}{2}(2) = 1$

t	-5	-4	-3	-2	-1
$h(t)$	1	$\frac{1}{2}$	0	$\frac{1}{2}$	1

52. $f(s) = \frac{|s - 2|}{s - 2}$

$f(0) = \frac{|0 - 2|}{0 - 2} = \frac{2}{-2} = -1$

$f(1) = \frac{|1 - 2|}{1 - 2} = \frac{1}{-1} = -1$

$f\left(\frac{3}{2}\right) = \frac{\left|\frac{3}{2} - 2\right|}{\frac{3}{2} - 2} = \frac{\frac{1}{2}}{-\frac{1}{2}} = -1$

$f\left(\frac{5}{2}\right) = \frac{\left|\frac{5}{2} - 2\right|}{\frac{5}{2} - 2} = \frac{\frac{1}{2}}{\frac{1}{2}} = 1$

$f(4) = \frac{|4 - 2|}{4 - 2} = \frac{2}{2} = 1$

s	0	1	$\frac{3}{2}$	$\frac{5}{2}$	4
$f(s)$	-1	-1	-1	1	1

53.
$$\begin{aligned} f(x) = 15 - 3x &= 0 \\ 3x &= 15 \\ x &= 5 \end{aligned}$$

54.
$$\begin{aligned} f(x) = 5x + 1 &= 0 \\ 5x &= -1 \\ x &= -\frac{1}{5} \end{aligned}$$

55. $f(x) = \dfrac{9x - 4}{5} = 0$

$9x - 4 = 0$

$9x = 4$

$x = \dfrac{4}{9}$

56. $f(x) = \dfrac{2x-3}{7} = 0$

$2x - 3 = 0$

$2x = 3$

$x = \dfrac{3}{2}$

57. $f(x) = 5x^2 + 2x - 1$

Since $f(x)$ is a polynomial, the domain is all real numbers x.

58. $g(x) = 1 - 2x^2$

Because $g(x)$ is a polynomial, the domain is all real numbers x.

59. $h(t) = \dfrac{4}{t}$

Domain: All real numbers except $t = 0$

60. $s(y) = \dfrac{3y}{y+5}$

$y + 5 \neq 0$

$y \neq -5$

The domain is all real numbers $y \neq -5$.

61. $f(x) = \sqrt[3]{x-4}$

Domain: all real numbers x

62. $f(x) = \sqrt[4]{x^2 + 3x}$

$x^2 + 3x = x(x+3) \geq 0$

Domain: $x \leq -3$ or $x \geq 0$

63. $g(x) = \dfrac{1}{x} - \dfrac{3}{x+2}$

Domain: All real numbers except $x = 0,\ x = -2$

64. $h(x) = \dfrac{10}{x^2 - 2x}$

$x^2 - 2x \neq 0$

$x(x-2) \neq 0$

The domain is all real numbers except $x = 0, x = 2$.

65. $g(y) = \dfrac{y+2}{\sqrt{y-10}}$

$y - 10 > 0$

$y > 10$

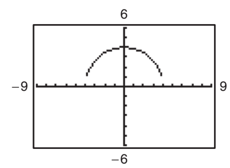
Domain: all $y > 10$

66. $f(x) = \dfrac{\sqrt{x+6}}{6+x}$

$x + 6 \geq 0$ for numerator and $x \neq -6$ for denominator.

Domain: all $x > -6$

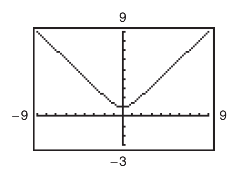
67. $f(x) = \sqrt{16 - x^2}$



Domain: $[-4, 4]$

Range: $[0, 4]$

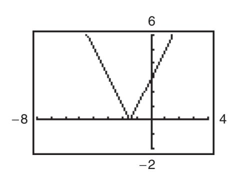
68. $f(x) = \sqrt{x^2 + 1}$



Domain: all real numbers

Range: $1 \leq y$

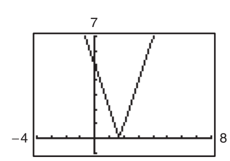
69. $g(x) = |2x + 3|$



Domain: $(-\infty, \infty)$

Range: $[0, \infty)$

70. $g(x) = |3x - 5|$



Domain: all real numbers

Range: $y \geq 0$

71. $A = \pi r^2,\ C = 2\pi r$

$$r = \frac{C}{2\pi}$$

$$A = \pi\left(\frac{C}{2\pi}\right)^2 = \frac{C^2}{4\pi}$$

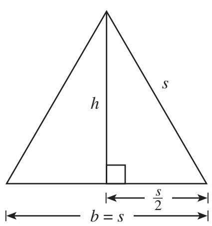
72. $A = \frac{1}{2}bh,$ in an equilateral triangle $b = s$ and:

$$s^2 = h^2 + \left(\frac{s}{2}\right)^2$$

$$h = \sqrt{s^2 - \left(\frac{s}{2}\right)^2}$$

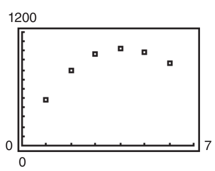
$$h = \sqrt{\frac{4s^2}{4} - \frac{s^2}{4}} = \frac{\sqrt{3}s}{2}$$

$$A = \frac{1}{2}s \cdot \frac{\sqrt{3}s}{2} = \frac{\sqrt{3}s^2}{4}$$



73. (a) From the table, the maximum volume seems to be 1024 cm^3, corresponding to $x = 4$.

(b)



Yes, V is a function of x.

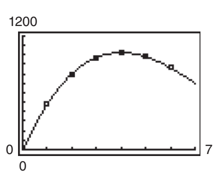
(c) $V = \text{length} \times \text{width} \times \text{height}$

$$= (24 - 2x)(24 - 2x)x$$

$$= x(24 - 2x)^2 = 4x(12 - x)^2$$

Domain: $0 < x < 12$

(d)



The function is a good fit. Answers will vary.

74. $A = \frac{1}{2}(\text{base})(\text{height}) = \frac{1}{2}xy.$

Since $(0, y)$, $(2, 1)$, and $(x, 0)$ all lie on the same line, the slopes between any pair of points are equal.

$$\frac{1-y}{2-0} = \frac{1-0}{2-x}$$

$$1 - y = \frac{2}{2-x}$$

$$y = 1 - \frac{2}{2-x} = \frac{x}{x-2}$$

Therefore, $A = \frac{1}{2}xy = \frac{1}{2}x\left(\frac{x}{x-2}\right) = \frac{x^2}{2x-4}.$

The domain is $x > 2$, since $A > 0$.

75. $A = l \cdot w = (2x)y = 2xy$

But $y = \sqrt{36 - x^2}$, so $A = 2x\sqrt{36 - x^2}$, $0 < x < 6$.

76. (a) $V = (\text{length})(\text{width})(\text{height}) = yx^2$

But, $y + 4x = 108$, or $y = 108 - 4x$.

Thus, $V = (108 - 4x)x^2$.

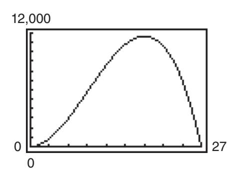
Since $y = 108 - 4x > 0$

$$4x < 108$$

$$x < 27.$$

Domain: $0 < x < 27$

(b)



(c) The highest point on the graph occurs at $x = 18$. The dimensions that maximize the volume are $18 \times 18 \times 36$ inches.

77. (a) Total cost = Variable costs + Fixed costs

$$C = 68.75x + 248{,}000$$

(b) Revenue = Selling price × Units sold

$$R = 99.99x$$

(c) Since $P = R - C$

$$P = 99.99x - (68.75x + 248{,}000)$$

$$P = 31.24x - 248{,}000.$$

78. (a) The independent variable is x and represents the month. The dependent variable is y and represents the monthly revenue.

(b) $f(x) = \begin{cases} -1.97x + 26.3, & 7 \le x \le 12 \\ 0.505x^2 - 1.47x + 6.3, & 1 \le x \le 6 \end{cases}$

Answers will vary.

(c) $f(5) = 11.575,$ and represents the revenue in May: $11,575.

(d) $f(11) = 4.63,$ and represents the revenue in November: $4630.

(e) The values obtained from the model are close approximations to the actual data.

79. (a) The independent variable is t and represents the year. The dependent variable is n and represents the numbers of miles traveled.

(b)

t	0	1	2	3	4	5
$n(t)$	3.95	3.96	3.98	3.99	4.00	4.02

t	6	7	8	9	10	11
$n(t)$	4.03	4.04	4.05	4.07	4.08	4.09

(c) The model fits the data well.

(d) Sample answer: No. The function may not accurately model other years

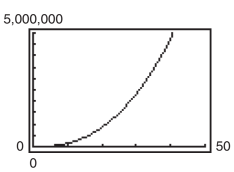
80. (a) $F(y) = 149.76\sqrt{10}y^{5/2}$

y	5	10	20	30	40
$F(y)$	26,474	149,760	847,170	2,334,527	4,792,320

(Answers will vary.)

F increases very rapidly as y increases.

(b)



$X_{min} = 0$
$X_{max} = 50$
$X_{scl} = 10$
$Y_{min} = 0$
$Y_{max} = 5{,}000{,}000$
$Y_{scl} = 500{,}000$

(c) From the table, $y \approx 22$ ft (slightly above 20). You could obtain a better approximation by completing the table for values of y between 20 and 30.

(d) By graphing $F(y)$ together with the horizontal line $y_2 = 1{,}000{,}000,$ you obtain $y \approx 21.37$ feet.

81. Yes. If $x = 30,$ $y = -0.01(30)^2 + 3(30) + 6$

$y = 6$ feet

Since the child trying to catch the throw is holding the glove at a height of 5 feet, the ball will fly over the glove.

82. (a) $\dfrac{f(2013) - f(2005)}{2013 - 2005} \approx \$525$ million/year

This represents the increase in sales per year from 2005 to 2013.

(b)

t	5	6	7	8	9
$S(t)$	217.3	136.9	237.4	518.8	981.1

t	10	11	12	13
$S(t)$	1624.2	2448.2	3453.1	4638.9

The model approximates the data well.

83. $f(x) = 2x$

$$\frac{f(x+c) - f(x)}{c} = \frac{2(x+c) - 2x}{c} = \frac{2c}{c} = 2, \; c \ne 0$$

84. $g(x) = 3x - 1$

$$g(x+h) = 3(x+h) - 1 = 3x + 3h - 1$$
$$g(x+h) - g(x) = (3x + 3h - 1) - (3x - 1) = 3h$$
$$\frac{g(x+h) - g(x)}{h} = \frac{3h}{h} = 3, \; h \ne 0$$

85. $f(x) = x^2 - x + 1,\; f(2) = 3$

$$\frac{f(2+h) - f(2)}{h} = \frac{(2+h)^2 - (2+h) + 1 - 3}{h} = \frac{4 + 4h + h^2 - 2 - h + 1 - 3}{h} = \frac{h^2 + 3h}{h} = h + 3, \; h \ne 0$$

86. $f(x) = x^3 + x$

$$f(x+h) = (x+h)^3 + (x+h) = x^3 + 3x^2h + 3xh^2 + h^3 + x + h$$
$$f(x+h) - f(x) = (x^3 + 3x^2h + 3xh^2 + h^3 + x + h) - (x^3 + x) = 3x^2h + 3xh^2 + h^3 + h = h(3x^2 + 3xh + h^2 + 1)$$
$$\frac{f(x+h) - f(x)}{h} = \frac{h(3x^2 + 3xh + h^2 + 1)}{h} = 3x^2 + 3xh + h^2 + 1, \; h \ne 0$$

87. False. The range of $f(x)$ is $(-1, \infty)$.

88. True. The first number in each ordered pair corresponds to exactly one second number.

89. $f(x) = \sqrt{x} + 2$

Domain: $[0, \infty)$ or $x \ge 0$

Range: $[2, \infty)$ or $y \ge 2$

90. $f(x) = \sqrt{x+3}$

Domain: $[-3, \infty)$ or $x \ge -3$

Range: $[0, \infty)$ or $y \ge 0$

91. No. f is not the independent variable. Because the value of f depends on the value of x, x is the independent variable and f is the dependent variable.

92. (a) The height h is a function of t because for each value of t there is exactly one corresponding value of h for $0 \le t \le 2.6$.

(b) The height after 0.5 second is about 20 feet. The height after 1.25 seconds is about 28 feet.

(c) From the graph, the domain is $0 \le t \le 2.6$.

(d) The time t is not a function of h because some values of h correspond to more than one value of t.

93. $12 - \dfrac{4}{x+2} = \dfrac{12(x+2)-4}{x+2} = \dfrac{12x+20}{x+2}$

94.
$$\begin{aligned}\frac{3}{x^2 + x - 20} + \frac{2x}{x^2 + 4x - 5} &= \frac{3}{(x+5)(x-4)} + \frac{2x}{(x+5)(x-1)}\\ &= \frac{3(x-1)}{(x+5)(x-4)(x-1)} + \frac{2x(x-4)}{(x+5)(x-1)(x-4)}\\ &= \frac{3x - 3 + 2x^2 - 8x}{(x+5)(x-4)(x-1)}\\ &= \frac{2x^2 - 5x - 3}{(x+5)(x-4)(x-1)}\end{aligned}$$

95.
$$\begin{aligned}\frac{2x^3+11x^2-6x}{5x}\cdot\frac{x+10}{2x^2+5x-3} &= \frac{x(2x^2+11x-6)(x+10)}{5x(2x-1)(x+3)}\\ &= \frac{(2x-1)(x+6)(x+10)}{5(2x-1)(x+3)}\\ &= \frac{(x+6)(x+10)}{5(x+3)},\ x \ne 0,\ \frac{1}{2}\end{aligned}$$

96. $\dfrac{x+7}{2(x-9)} \div \dfrac{x-7}{2(x-9)} = \dfrac{x+7}{2(x-9)}.\dfrac{2(x-9)}{x-7} = \dfrac{x+7}{x-7},\ x \ne 9$

Section 1.3 Graphs of Functions

1. decreasing

2. even

3. Domain: $1 \le x \le 4$ or $[1, 4]$

4. No. If a vertical line intersects the graph more than once, then it does not represent y as a function of x.

5. If $f(2) \ge f(2)$ for all x in $(0, 3)$, then $(2, f(2))$ is a relative maximum of f.

6. Since $f(x) = [\![x]\!] = n$, where n is an integer and $n \le x$, the input value of x needs to be greater than or equal to 5 but less than 6 in order to produce an output value of 5. So the interval $[5, 6)$ would yield a function value of 5.

7. Domain: all real numbers, $(-\infty, \infty)$

Range: $(-\infty, 1]$

$f(0) = 1$

8. Domain: all real numbers, $(-\infty, \infty)$

Range: all real numbers, $(-\infty, \infty)$

$f(0) = 2$

9. Domain: $[-4, 4]$

Range: $[0, 4]$

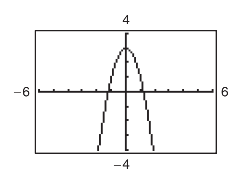
$f(0) = 4$

10. Domain: all real numbers, $(-\infty, \infty)$

Range: $[-3, \infty)$

$f(0) = -3$

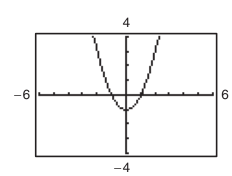
11. $f(x) = -2x^2 + 3$



Domain: $(-\infty, \infty)$

Range: $(-\infty, 3]$

12. $f(x) = x^2 - 1$



Domain: $(-\infty, \infty)$

Range: $[-1, \infty)$