

1A.1 Estimation of dense-gas viscosity.

a. Table E.1 gives $T_c = 126.2$ K, $p_c = 33.5$ atm, and $\mu_c = 180 \times 10^{-6}$ g/cm·s for N₂. The reduced conditions for the viscosity estimation are then:

$$p_r = p/p_c = (1000 + 14.7)/33.5 \times 14.7 = 2.06$$

$$T_r = T/T_c = (273.15 + (68 - 32)/1.8)/126.2 = 2.32$$

At this reduced state, Fig. 1.3-1 gives $\mu_r = 1.15$. Hence, the predicted viscosity is $\mu = \mu_r/\mu_c = 1.15 \times 180 \times 10^{-6} = 2.07 \times 10^{-4}$ g/cm·s. This result is then converted into the requested units by use of Table F.3-4:

$$\mu = 2.07 \times 10^{-4} \times 6.7197 \times 10^{-2} = 1.4 \times 10^{-4} \text{ lb}_m/\text{ft}\cdot\text{s}$$

1A.2 Estimation of the viscosity of methyl fluoride.

a. CH_3F has $M = 16.04 - 1.008 + 19.00 = 34.03$ g/g-mole, $T_c = 4.55 + 273.15 = 277.70$ K, $p_c = 58.0$ atm, and $\tilde{V}_c = 34.03/0.300 = 113.4$ cm³/g-mole. The critical viscosity is then estimated as

$$\mu_c = 61.6(34.03 \times 277.70)^{1/2}(113.4)^{-2/3} = 255.6 \text{ micropoise}$$

from Eq. 1.3-1a, and

$$\mu_c = 7.70(34.03)^{1/2}(58.0)^{2/3}(277.7)^{-1/6} = 263.5 \text{ micropoise}$$

from Eq. 1.3-1b.

The reduced conditions for the viscosity estimate are $T_r = (370 + 273.15)/277.70 = 2.32$, $p_r = 120/58.0 = 2.07$, and the predicted μ_r from Fig. 1.3-1 is 1.1. The resulting predicted viscosity is

$$\begin{aligned} \mu = \mu_r \mu_c &= 1.1 \times 255.6 \times 10^{-6} = 2.8 \times 10^{-4} \text{ g/cm}\cdot\text{s via Eq.1.3-1a, or} \\ &1.1 \times 263.5 \times 10^{-6} = 2.9 \times 10^{-4} \text{ g/cm}\cdot\text{s via Eq.1.3-1b.} \end{aligned}$$

1A.3 Computation of the viscosities of gases at low density.

Equation 1.4-14, with molecular parameters from Table E.1 and collision integrals from Table E.2, gives the following results:

For O_2 : $M = 32.00$, $\sigma = 3.433 \text{ \AA}$, $\varepsilon/\text{K} = 113 \text{ K}$. Then at 20°C , $kT/\varepsilon = 293.15/113 = 2.594$ and $\Omega_\mu = 1.086$. Equation 1.4-14 then gives

$$\begin{aligned}\mu &= 2.6693 \times 10^{-5} \frac{\sqrt{32.00 \times 293.15}}{(3.433)^2 \times 1.086} \\ &= 2.02 \times 10^{-4} \text{ g/cm}\cdot\text{s} \\ &= 2.02 \times 10^{-5} \text{ Pa}\cdot\text{s} \\ &= 2.02 \times 10^{-2} \text{ mPa}\cdot\text{s}.\end{aligned}$$

The reported value in Table 1.1-3 is $2.04 \times 10^{-2} \text{ mPa}\cdot\text{s}$.

For N_2 : $M = 28.01$, $\sigma = 3.667 \text{ \AA}$, $\varepsilon/\text{K} = 99.8 \text{ K}$. Then at 20°C , $kT/\varepsilon = 293.15/99.8 = 2.937$ and $\Omega_\mu = 1.0447$. Equation 1.4-14 then gives

$$\begin{aligned}\mu &= 2.6693 \times 10^{-5} \frac{\sqrt{28.01 \times 293.15}}{(3.667)^2 \times 1.0447} \\ &= 1.72 \times 10^{-4} \text{ g/cm}\cdot\text{s} \\ &= 1.72 \times 10^{-5} \text{ Pa}\cdot\text{s} \\ &= 1.72 \times 10^{-2} \text{ mPa}\cdot\text{s}.\end{aligned}$$

The reported value in Table 1.1-3 is $1.75 \times 10^{-2} \text{ mPa}\cdot\text{s}$.

For CH_4 , $M = 16.04$, $\sigma = 3.780 \text{ \AA}$, $\varepsilon/\text{K} = 154 \text{ K}$. Then at 20°C , $kT/\varepsilon = 293.15/154 = 1.904$ and $\Omega_\mu = 1.197$. Equation 1.4-14 then gives

$$\begin{aligned}\mu &= 2.6693 \times 10^{-5} \frac{\sqrt{16.04 \times 293.15}}{(3.780)^2 \times 1.197} \\ &= 1.07 \times 10^{-4} \text{ g/cm}\cdot\text{s} \\ &= 1.07 \times 10^{-5} \text{ Pa}\cdot\text{s} \\ &= 1.07 \times 10^{-2} \text{ mPa}\cdot\text{s}.\end{aligned}$$

The reported value in Table 1.1-3 is $1.09 \times 10^{-2} \text{ mPa}\cdot\text{s}$.

1A.4 Gas-mixture viscosities at low density.

The data for this problem are as follows:

Component	M	μ , poise $\times 10^6$
1(H ₂)	2.016	88.4
2(CCl ₂ F ₂)	120.92	124.0

Insertion of these data into Eq. 1.4-16 gives the following coefficients for mixtures of H₂ and Freon-12 at this temperature:

$$\Phi_{11} = \Phi_{22} = 1.0$$

$$\begin{aligned} \Phi_{12} &= \frac{1}{\sqrt{8}} \left(1 + \frac{2.016}{120.92} \right)^{-1/2} \left[1 + \left(\frac{88.4}{124.0} \right)^{1/2} \left(\frac{120.92}{2.016} \right)^{1/4} \right]^2 \\ &= 3.934 \end{aligned}$$

$$\begin{aligned} \Phi_{21} &= \frac{1}{\sqrt{8}} \left(1 + \frac{120.92}{2.016} \right)^{-1/2} \left[1 + \left(\frac{124.0}{88.4} \right)^{1/2} \left(\frac{2.016}{120.92} \right)^{1/4} \right]^2 \\ &= 0.0920 \end{aligned}$$

Equation 1.4-15 then gives the predicted mixture viscosities:

$x_1 =$ $1 - x_2$	$\sum_1 =$ $\sum x_\beta \Phi_{1\beta}$	$\sum_2 =$ $\sum x_\beta \Phi_{2\beta}$	$A :=$ $x_1 \mu_1 / \sum_1$	$B :=$ $x_2 \mu_2 / \sum_2$	$A + B =$ $\mu_{\text{mix}} \times 10^6$	$\mu_{\text{obs,poise}} \times 10^6$
0.00	3.934	1.000	0.0	124.0	(124.0)	124.0
0.25	3.200	0.773	6.9	120.3	127.2	128.1
0.50	2.467	0.546	18.1	113.6	131.7	131.9
0.75	1.734	0.319	38.2	97.2	135.4	135.1
1.00	1.000	0.092	88.4	0.0	(88.4)	88.4

1A.5 Viscosities of chlorine-air mixtures at low density.

Equation 1.4-14 and Tables E.1, E.2 give the following viscosities at 75°F (= 273.15 + (75 - 32)/1.8 = 297.03 K) and 1 atm:

For component 1, (Cl₂), $M_1 = 70.91$, $\sigma_1 = 4.115 \text{ \AA}$, $\varepsilon_1/K = 357\text{K}$; hence, $\kappa T/\varepsilon_1 = 297.03/357 = 0.832$ and $\Omega_{\mu,1} = 1.754$, and

$$\mu_1 = 2.6693 \times 10^{-5} \frac{\sqrt{70.91 \times 297.03}}{(4.115)^2 \times 1.754} = 1.304 \times 10^{-4} \text{ g/cm}\cdot\text{s} = 0.01304 \text{ cp.}$$

For component 2, (air), $M_2 = 28.97$, $\sigma = 3.617 \text{ \AA}$, $\varepsilon_1/K = 97.0\text{K}$; hence, $\kappa T/\varepsilon_1 = 297.03/97.0 = 3.062$ and $\Omega_{\mu,1} = 1.033$, and

$$\mu_2 = 2.6693 \times 10^{-5} \frac{\sqrt{28.97 \times 297.03}}{(3.617)^2 \times 1.033} = 1.832 \times 10^{-4} \text{ g/cm}\cdot\text{s} = 0.01832 \text{ cp.}$$

Eq. 1.4-16 then gives the following coefficients for Eq. 1.4-15 at this temperature:

$$\Phi_{11} = \Phi_{22} = 1.0$$

$$\begin{aligned} \Phi_{12} &= \frac{1}{\sqrt{8}} \left(1 + \frac{70.91}{28.97} \right)^{-1/2} \left[1 + \left(\frac{0.01304}{0.01832} \right)^{1/2} \left(\frac{28.97}{70.91} \right)^{1/4} \right]^2 \\ &= 0.5339 \end{aligned}$$

$$\begin{aligned} \Phi_{21} &= \frac{1}{\sqrt{8}} \left(1 + \frac{28.97}{70.91} \right)^{-1/2} \left[1 + \left(\frac{0.01832}{0.01304} \right)^{1/2} \left(\frac{70.91}{28.97} \right)^{1/4} \right]^2 \\ &= 1.8360 \end{aligned}$$

Equation 1.4-15 then gives the predicted mixture viscosities:

$x_1 =$ $1 - x_2$	$\sum_1 =$ $\sum x_\beta \Phi_{1\beta}$	$\sum_2 =$ $\sum x_\beta \Phi_{2\beta}$	$A :=$ $x_1 \mu_1 / \sum_1$	$B :=$ $x_2 \mu_2 / \sum_2$	$A + B =$ $\mu_{\text{mix, cp.}} \times 10^6$
0.00	0.5339	1.000	0.0	0.01832	0.0183
0.25	0.6504	1.2090	0.005012	0.011365	0.0164
0.50	0.7670	1.4180	0.008501	0.006460	0.0150
0.75	0.8835	1.6270	0.011070	0.002815	0.0139
1.00	1.000	1.8360	0.01304	0.0	0.0130

1A.6 Estimation of liquid viscosity.

a. The calculated values for Eq. 1.5-9 at 0°C and 100°C are as follows:

T, K	273.15	373.15
$\rho, g/cm^3$	0.9998	0.9584
$\tilde{V} = M/\rho, cm^3/g\text{-mole}$	18.01	18.80
$\Delta\tilde{U}_{vap, T_b}, cal/g\text{-mole} = 897.5 \times 18.016 \times 252.16/453.59$	8989.	8989.
$\Delta\tilde{U}_{vap, T_b}/RT = 8989/1.98721/T$	16.560	12.120
$\exp 0.408\Delta\tilde{U}_{vap, T_b}/RT$	859.6	140.5
$\tilde{N}h/\tilde{V}, g/cm\cdot s$	2.22×10^{-4}	2.12×10^{-4}
Predicted liquid viscosity, g/cm·s	0.19	0.0298

b. The predicted values for Eq. 1.5-11 at 0°C and 100°C are:

T, K	273.15	373.15
$\tilde{N}h/\tilde{V}, g/cm\cdot s$	2.22×10^{-4}	2.12×10^{-4}
$\exp(3.8T_b/T)$	179.7	44.70
Predicted liquid viscosity, g/cm·s	0.0398	0.0095

Summary of results:

Temperature, °C	0	100
Observed viscosity, centipoise[=]g/cm·s×100	1.787	0.2821
Prediction of Eq. 1.5-9	19.	2.98
Prediction of Eq. 1.5-11	3.98	0.95

Both equations give poor predictions. This is not surprising, since the empirical formulas in Eqs. 1.5-8 *et seq.* are inaccurate for water and for other associated liquids.

1A.7 Molecular velocity and mean free path.

From eq. 1.4-1, the mean molecular velocity in O_2 at 273.2 K is

$$\bar{u} = \sqrt{\frac{8RT}{\pi M}} = \sqrt{\frac{8 \times 8.31451 \times 10^7 \times 273.2}{\pi \times 32.00}} = 4.25 \times 10^4 \text{ cm/s}$$

From eq. 1.4-3, the mean free path in O_2 at 1 atm and 273.2 K is

$$\lambda = \frac{RT}{\sqrt{2}\pi d^2 p \tilde{N}} = \frac{82.0578 \times 273.2}{\sqrt{2}\pi(3 \times 10^{-8})^2 \times 1 \times 6.02214 \times 10^{23}} = 9.3 \times 10^{-6} \text{ cm}$$

Hence, the ratio of the mean free path to the molecular diameter is $(9.3 \times 10^{-6} / 3 \times 10^{-8}) = 3.1 \times 10^4$ under these conditions. At liquid states, on the other hand, the corresponding ratio would be on the order of unity or even less.

1B.1 Velocity profiles and stress components

a. $\tau_{xy} = \tau_{yx} = -\mu b$, and all other τ_{ij} are zero.

$\rho v_x v_x = \rho b^2 y^2$, and all other $\rho v_i v_j$ are zero.

b. $\tau_{xy} = \tau_{yx} = -2\mu b$, and all other τ_{ij} are zero.

$\rho v_x v_x = \rho b^2 y^2$, $\rho v_x v_y = \rho v_y v_x = \rho b^2 xy$, $\rho v_y v_y = \rho b^2 x^2$, and all other $\rho v_i v_j$ are zero.

c. All τ_{ij} are zero

$\rho v_x v_x = \rho b^2 y^2$, $\rho v_x v_y = \rho v_y v_x = -\rho b^2 xy$, $\rho v_y v_y = \rho b^2 x^2$ and all other $\rho v_i v_j$ are zero.

d. $\tau_{xx} = \tau_{yy} = \mu b$, $\tau_{zz} = -2\mu b$, and all others are zero. the components of $\rho \mathbf{v} \mathbf{v}$ may be given in the matrix:

$$\rho \mathbf{v} \mathbf{v} = \begin{pmatrix} \rho v_x v_x = \frac{1}{4} \rho b^2 x^2 & \rho v_x v_y = \frac{1}{4} \rho b^2 xy & \rho v_x v_z = -\frac{1}{2} \rho b^2 xz \\ \rho v_y v_x = \frac{1}{4} \rho b^2 xy & \rho v_y v_y = \frac{1}{4} \rho b^2 y^2 & \rho v_y v_z = -\frac{1}{2} \rho b^2 yz \\ \rho v_z v_x = -\frac{1}{2} \rho b^2 xz & \rho v_z v_y = -\frac{1}{2} \rho b^2 yz & \rho v_z v_z = \rho b^2 z^2 \end{pmatrix}$$

1B.2 A fluid in a state of rigid rotation

a. A particle within a rigid body rotating with an angular velocity vector \mathbf{w} has a velocity given by $\mathbf{v} = [\mathbf{w} \times \mathbf{r}]$. If the angular velocity vector is in the $+z$ -direction, then there are two nonzero velocity components given by $v_x = -w_z y$ and $v_y = +w_z x$. Hence the magnitude of the angular velocity vector is b in Problem 1B.1(c).

b. For the velocity components of Problem 1B.1(c),

$$\frac{\partial v_y}{\partial x} + \frac{\partial v_x}{\partial y} = 0 \quad \text{and} \quad \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} = 2b$$

c. In Eq. 1.2-4, we selected only the linear *symmetric* combinations of derivatives of the velocity, so that in pure rotation there would be no viscous forces present. In (b) we see that the antisymmetric combination is nonzero in a purely rotational motion.

1B.3 Viscosity of suspensions

Expanding the Mooney expression, we get (with $\varepsilon = \phi/\phi_0$)

$$\begin{aligned}\frac{\mu_{\text{eff}}}{\mu_0} &= 1 + \left(\frac{\frac{5}{2}\phi}{1-\varepsilon}\right) + \frac{1}{2!}\left(\frac{\frac{5}{2}\phi}{1-\varepsilon}\right)^2 + \frac{1}{3!}\left(\frac{\frac{5}{2}\phi}{1-\varepsilon}\right)^3 + \dots \\ &= 1 + \frac{5}{2}\phi(1 + \varepsilon + \varepsilon^2 + \dots) + \frac{25}{8}\phi^2(1 + 2\varepsilon + \dots) + \frac{125}{48}\phi^3(1 + \dots) + \dots \\ &= 1 + \frac{5}{2}\phi + \phi^2\left(\frac{25}{8} + \frac{5}{2}\frac{1}{\phi_0}\right) + \phi^3\left(\frac{125}{48} + \frac{25}{4}\frac{1}{\phi_0} + \frac{5}{2}\frac{1}{\phi_0^2}\right) + \dots\end{aligned}$$

The first two terms match exactly with the first two terms in Eq. 1B.3-1. We can make the third term match exactly, by setting

$$\frac{25}{8} + \frac{5}{2}\frac{1}{\phi_0} = 7.17 \quad \text{whence } \phi_0 = 0.618$$

and the coefficient of ϕ^3 becomes

$$\frac{125}{48} + \frac{25}{4}\frac{1}{0.618} + \frac{5}{2}\frac{1}{0.382} = 20.26$$

If we try $\phi_0 = 0.70$, the coefficients of ϕ^2 and ϕ^3 become 6.70 and 17.6 respectively. This gives a somewhat better fit of Vand's data.

1C.1 Some consequences of the Maxwell-Boltzmann equation

a. The mean speed is

$$\bar{u} = \frac{\int_0^{\infty} u^3 e^{-mu^2/2kT} du}{\int_0^{\infty} u^2 e^{-mu^2/2kT} du} = \sqrt{\frac{2kT}{m}} \frac{\int_0^{\infty} \xi^3 e^{-\xi^2} d\xi}{\int_0^{\infty} \xi^2 e^{-\xi^2} d\xi} = \sqrt{\frac{2kT}{m}} \frac{\frac{1}{2}}{\frac{1}{4}\sqrt{\pi}} = \sqrt{\frac{8kT}{\pi m}}$$

b. First rewrite Eq. 1C.1-4 as

$$\bar{u}_x = \frac{\int_{-\infty}^{\infty} u_x e^{-mu_x^2/2kT} du_x}{\int_{-\infty}^{\infty} e^{-mu_x^2/2kT} du_x} \cdot \frac{\int_{-\infty}^{\infty} e^{-mu_y^2/2kT} du_y}{\int_{-\infty}^{\infty} e^{-mu_y^2/2kT} du_y} \cdot \frac{\int_{-\infty}^{\infty} e^{-mu_z^2/2kT} du_z}{\int_{-\infty}^{\infty} e^{-mu_z^2/2kT} du_z}$$

The integral over u_x in the numerator of the first factor is zero because the integrand is the product of a factor " u_x " (an odd function of the integration variable about $u_x = 0$) and an exponential function (an even function), and the range of integration extends equally far in the positive and negative directions.

c. The mean kinetic energy per molecule is

$$\frac{1}{2} m \overline{u^2} = \frac{1}{2} m \frac{\int_0^{\infty} u^4 e^{-mu^2/2kT} du}{\int_0^{\infty} u^2 e^{-mu^2/2kT} du} = \frac{1}{2} m \frac{2kT}{m} \frac{\int_0^{\infty} \xi^4 e^{-\xi^2} d\xi}{\int_0^{\infty} \xi^2 e^{-\xi^2} d\xi} = kT \frac{\frac{3}{8}\sqrt{\pi}}{\frac{1}{4}\sqrt{\pi}} = \frac{3}{2} kT$$

and is thus $\frac{1}{2} kT$ for each degree of freedom.

1C.2 The wall collision frequency

When we change to dimensionless variables in the second line of Eq. 1C.2-1, we get

$$\begin{aligned} Z &= n \left(\frac{m}{2\pi\kappa T} \right)^{3/2} \frac{2\kappa T}{m} \left(\int_0^\infty \xi e^{-\xi^2} d\xi \right) \sqrt{\frac{2\kappa T}{m}} \left(\int_0^\infty e^{-\xi^2} d\xi \right) \sqrt{\frac{2\kappa T}{m}} \left(\int_0^\infty e^{-\xi^2} d\xi \right) \\ &= n \left(\frac{m}{2\pi\kappa T} \right)^{3/2} \left(\frac{2\kappa T}{m} \cdot \frac{1}{2} \right) \left(\sqrt{\frac{2\kappa T}{m}} \cdot \sqrt{\pi} \right) \left(\sqrt{\frac{2\kappa T}{m}} \cdot \sqrt{\pi} \right) = n \sqrt{\frac{\kappa T}{2\pi m}} \end{aligned}$$

1C.3 The pressure in an ideal gas

a. The dimensions of the quantities in Eq. 1C.3-1 are

$$\begin{array}{lll} S & [=] & L^2 \\ u_x & [=] & L/t \\ \Delta t & [=] & t \\ m & [=] & M \\ f & [=] & (1/L^3)(L/t)^{-3} \\ du_x du_y du_z & [=] & (L/t)^3 \end{array}$$

Using these units, one finds that the expression on the right of Eq. 1C.3-1 has units of M/Lt^2 (which are the same as the units of force per area).

b. Combining Eqs. 1C.1-1 and 1C.3-1 we get

$$\begin{aligned} p &= 2nm \left(\frac{m}{2\pi kT} \right)^{3/2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} u_x^2 e^{-m(u_x^2 + u_y^2 + u_z^2)/2kT} du_x du_y du_z \\ &= 2nm \left(\frac{m}{2\pi kT} \right)^{3/2} \int_{-\infty}^{\infty} u_x^2 e^{-mu_x^2/2kT} du_x \cdot \int_{-\infty}^{\infty} e^{-mu_y^2/2kT} du_y \cdot \int_0^{\infty} e^{-mu_z^2/2kT} du_z \\ &= 2nm \left(\frac{2kT}{m} \right) \left(\frac{1}{\pi} \right)^{3/2} \int_0^{\infty} \xi^2 e^{-\xi^2} d\xi \cdot \int_{-\infty}^{\infty} \eta e^{-\eta^2} d\eta \cdot \int_{-\infty}^{\infty} \zeta^2 e^{-\zeta^2} d\zeta \\ &= 2nm \left(\frac{2kT}{m} \right) \left(\frac{1}{\pi} \right)^{3/2} \left(\frac{\sqrt{\pi}}{4} \right) (\sqrt{\pi}) (\sqrt{\pi}) = nkT \end{aligned}$$

1D.1 Uniform rotation of a fluid

a. For the special case that $\mathbf{w} = \delta_z w$, we get

$$\mathbf{v} = [\mathbf{w} \times \mathbf{r}] = \sum_i \sum_j \sum_k \varepsilon_{ijk} \delta_i w_j x_k = w(\delta_1 \varepsilon_{132} y + \delta_2 \varepsilon_{231} x) = w(-\delta_1 y + \delta_2 x)$$

Then using Eqs. A.6-1, 2, 13 and 14, we can get the velocity components in cylindrical coordinates

$$\begin{aligned} v_r &= (\mathbf{v} \cdot \delta_r) = w((- \delta_x y + \delta_y x) \cdot \delta_r) = w(-y \cos \theta + x \sin \theta) \\ &= w(-r \sin \theta \cos \theta + x \cos \theta \sin \theta) = 0 \end{aligned}$$

$$\begin{aligned} v_\theta &= (\mathbf{v} \cdot \delta_\theta) = w((- \delta_x y + \delta_y x) \cdot \delta_\theta) = w((-y)(-\sin \theta) + x \cos \theta) \\ &= w(r \sin \theta \sin \theta + r \cos \theta \cos \theta) = wr \end{aligned}$$

Therefore, the angular velocity of every point in the fluid is $v_\theta/r = w$, which is a constant, and there is no radial velocity. This is the way a rigid body rotates at constant angular velocity.

b. The vector operations are (using the abbreviated notation of §A.9 and the Einstein summation convention)

$$(\nabla \cdot \mathbf{v}) = \partial_i v_i = \partial_i \varepsilon_{imn} w_m x_n = \varepsilon_{imn} w_m \delta_{in} = \varepsilon_{nmn} w_m = 0$$

$$\begin{aligned} \{\nabla \mathbf{v}\}_{ij} &= \{\nabla[\mathbf{w} \times \mathbf{r}]\}_{ij} = \partial_i \varepsilon_{jmn} w_m x_n = \varepsilon_{jmn} w_m \delta_{in} = \varepsilon_{jmi} w_m \\ &= -\{\nabla \mathbf{v}\}_{ji} = -\{\nabla \mathbf{v}\}_{ij}^\dagger \end{aligned}$$

and from this last result we see that $\nabla \mathbf{v} + (\nabla \mathbf{v})^\dagger = 0$.

c. The results above indicate that for a fluid is a state of pure rotation, the tensor τ is identically zero. That is, there are no viscous stresses present in the fluid. This was the assertion made just before Eq. 1.2-4.

1D.2 Force on a surface of arbitrary orientation.

a. We can specify the surface area *and* the orientation of the surface of ΔOBC as $\mathbf{n}dS$. To project this surface onto the yz -plane, we take the dot product with δ_x , so that the area of ΔOBC is $(\mathbf{n} \cdot \delta_x)dS$.

b. The force per unit area on three triangles perpendicular to the three coordinate axes are

$$\text{Force on } \Delta OBC = \delta_x \pi_{xx} + \delta_y \pi_{xy} + \delta_z \pi_{xz}$$

$$\text{Force on } \Delta OCA = \delta_x \pi_{yx} + \delta_y \pi_{yy} + \delta_z \pi_{yz}$$

$$\text{Force on } \Delta OAB = \delta_x \pi_{zx} + \delta_y \pi_{zy} + \delta_z \pi_{zz}$$

c. Force balance on the volume OABC is then

$$\begin{aligned} \pi_n dS &= (\delta_x \pi_{xx} + \delta_y \pi_{xy} + \delta_z \pi_{xz})(\mathbf{n} \cdot \delta_x) dS \\ &\quad + (\delta_x \pi_{yx} + \delta_y \pi_{yy} + \delta_z \pi_{yz})(\mathbf{n} \cdot \delta_y) dS \\ &\quad + (\delta_x \pi_{zx} + \delta_y \pi_{zy} + \delta_z \pi_{zz})(\mathbf{n} \cdot \delta_z) dS \end{aligned}$$

or

$$\begin{aligned} \pi_n &= [\mathbf{n} \cdot \delta_x \delta_x \pi_{xx}] + [\mathbf{n} \cdot \delta_x \delta_y \pi_{xy}] + [\mathbf{n} \cdot \delta_x \delta_z \pi_{xz}] \\ &\quad + [\mathbf{n} \cdot \delta_y \delta_x \pi_{yx}] + [\mathbf{n} \cdot \delta_y \delta_y \pi_{yy}] + [\mathbf{n} \cdot \delta_y \delta_z \pi_{yz}] \\ &\quad + [\mathbf{n} \cdot \delta_z \delta_x \pi_{zx}] + [\mathbf{n} \cdot \delta_z \delta_y \pi_{zy}] + [\mathbf{n} \cdot \delta_z \delta_z \pi_{zz}] \\ &= \sum_i \sum_j [\mathbf{n} \cdot \delta_i \delta_j \pi_{ij}] = [\mathbf{n} \cdot \boldsymbol{\pi}] \end{aligned}$$