CHAPTER TWO

PROB # 2.1

$I_g = \frac{1}{12} bh^3 = \left(\frac{1}{12}\right)(14)(21)^3 = 10{,}805 \text{ in.}^4$

f_r = modulus of rupture $= 7.5\sqrt{f'_c} = 7.5\sqrt{4000}$

$= 474$ psi

$M_{cr} = \dfrac{f_r I_g}{y_t} = \dfrac{(474)(10805)}{10.5} = 487{,}746$ in. lbs

$= \boxed{40.7 \text{ ft-k}}$

rgcm©

PROB # 2.2

$I_g = \left(\frac{1}{12}\right)(14)(21)^3 = 10{,}805 \text{ in.}^4$

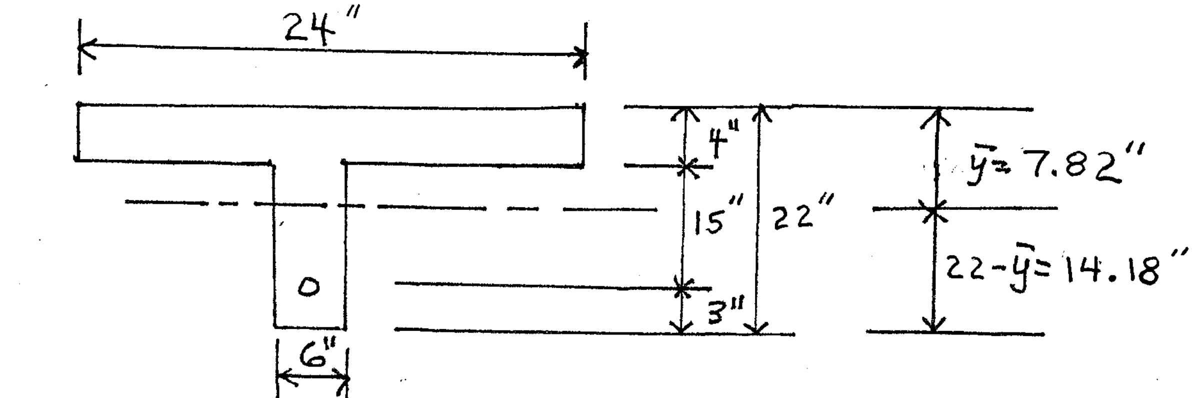
$f_r = 7.5\sqrt{4000} = 474$ psi

$M_{cr} = \dfrac{(474)(10805)}{10.5} = 487{,}746$ in. lbs

$= \boxed{40.7 \text{ ft-k}}$

rgcm©

PROB # 2.3



$A = (24)(4) + (6)(18) = 204 \text{ in.}^2$

$\bar{y} \text{ from top} = \dfrac{(24)(4)(2) + (6)(18)(13)}{204} = 7.82 \text{ in.}$

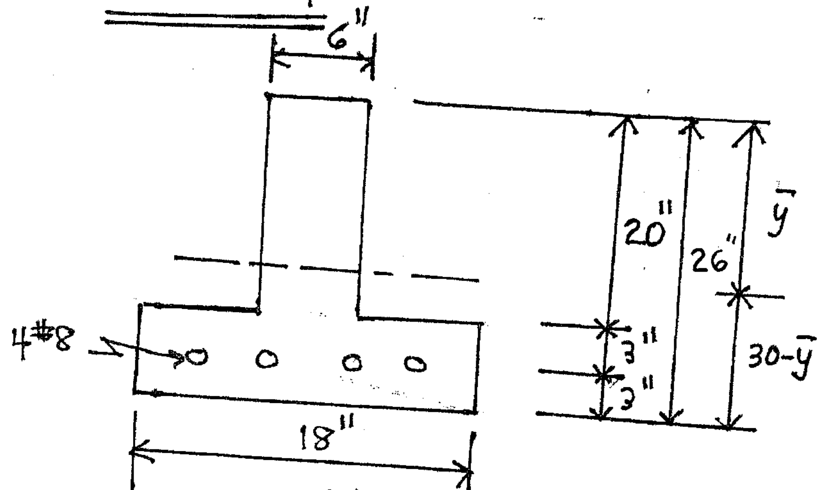
$I_g = \left(\frac{1}{3}\right)(24)(7.82^3 + 14.18^3) - \left(\frac{1}{3}\right)(18)(3.82^3 + 14.18^3) = 9194 \text{ in}^4$

$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$

$M_{cr} = \dfrac{(474)(9194)}{14.18} = 307{,}309 \text{ in. lbs.} = \boxed{25.6 \text{ ft-k}}$

✓gcm[c]

PROB # 2.4



$A = (6)(20) + (18)(6)$

$= 228 \text{ in.}^2$

$\bar{y} = \dfrac{6(20)10 + 18(23)6}{228}$

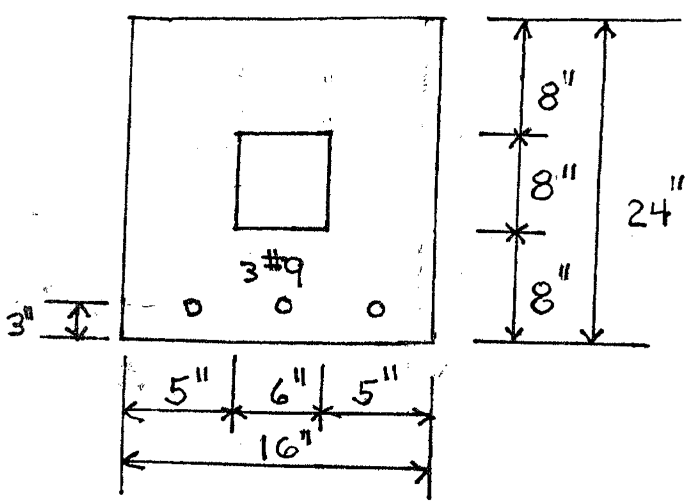
$= 16.16 \text{ in.}$

$I_g = \left(\frac{1}{3}\right)(6)(16.16)^3 + \left(\frac{1}{3}\right)(18)(9.84)^3 - \left(\frac{1}{3}\right)(12)(3.84)^3 = 13{,}930 \text{ in}^4$

$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$

$M_{cr} = \dfrac{(474)(13930)}{9.84} = 671{,}034 \text{ in.-lbs} = \boxed{55.9 \text{ ft-k}}$ ✓gcm[c]

PROB # 2.5



$$I_g = \left(\frac{1}{12}\right)(16)(24)^3 - \left(\frac{1}{12}\right)(6)(8)^3 = 18{,}176 \text{ in.}^4$$

$$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$$

$$M_{cr} = \frac{(474)(18176)}{12} = 717{,}950 \text{ in.-lbs.} = \boxed{59.8 \text{ ft-k}}$$

✓ gcm^c

PROB # 2.6

$$I_g = \left(\frac{1}{12}\right)(14)(24)^3 = 16{,}128 \text{ in.}^4$$

$$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$$

$$M_{cr} = \frac{(474)(16{,}128)}{12} = 637{,}056 \text{ in.-lbs}$$

$$= 53.1 \text{ ft-k}$$

$$\frac{w\ell^2}{8} = 53.1$$

$$w_{Total} = \frac{(8)(53.1)}{(28)^2} = 0.542 \text{ k/ft}$$

$$- \text{Beam wt} = -\frac{(16)(24)}{144}(0.150) = \underline{-0.400}$$

$$w \text{ not including beam wt} = \boxed{0.142 \text{ k/ft}}$$

✓ gcm^c

PROB #2.7

$$I_g = \left(\tfrac{1}{12}\right)(12)(30)^3 - \left(\tfrac{1}{12}\right)(8)(22)^3 = 19{,}901 \text{ in.}^4$$

$$f_r = 7.5\sqrt{4000} = 474 \text{ psi}$$

$$M_{cr} = \frac{(474)(19{,}901)}{15.00} = 628{,}872 \text{ in-lbs} = 52.41 \text{ ft-k}$$

$$\frac{w_T \ell^2}{8} = 52.41$$

$$w_T = \frac{(8)(52.41)}{(28)^2} = 0.535 \text{ k/ft}$$

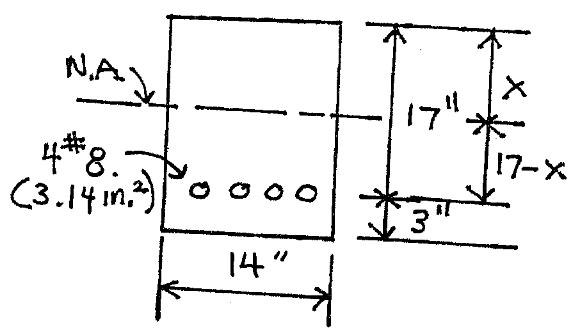
$$-\text{Bm wt} = \frac{(30)(12)-(8)(22)}{144}(0.150) = -0.192$$

w not including beam wt = $\boxed{0.343 \text{ k/ft}}$ ✓gcm©

PROB #2.8

Locate N.A.

$$(14x)\left(\tfrac{x}{2}\right) = (8)(3.14)(17-x)$$



$$7x^2 + 25.12x = 427$$

$$x = 6.22 \text{ in}$$

Moment of Inertia

$$I_x = \left(\tfrac{1}{3}\right)(14)(6.22)^3 + (8)(3.14)(10.78)^2 = 4042 \text{ in.}^4$$

Flexural stresses

$$f_c = \frac{(12)(60{,}000)(6.22)}{4042} = \boxed{1108 \text{ psi}}$$

$$f_s = \frac{(8)(12)(60{,}000)(10.78)}{4042} = \boxed{15{,}362 \text{ psi}}$$

✓gcm©

PROB# 2.9

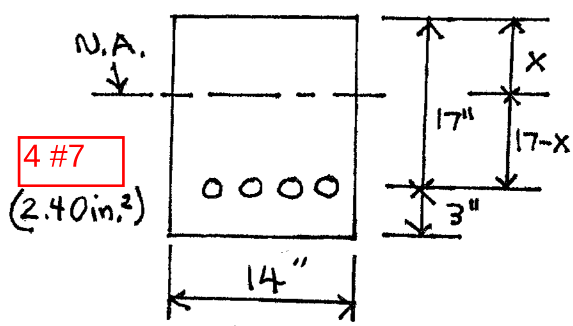
Locate N.A.

$$(14x)\left(\frac{x}{2}\right) = (8)(2.40)(17-x)$$

$$7x^2 = 326 - 19.2x$$

$$7x^2 + 19.2x - 326 = 0$$

$$x = 5.59 \text{ in}$$



Calculate moment of inertia

$$I_x = \left(\frac{1}{3}\right)(14)(5.59)^3 + (8)(2.40)(11.41)^2 = 3315 \text{ in}^4$$

Flexural stresses

$$f_c = \frac{(12)(60{,}000)(5.59)}{3315} = \boxed{1214 \text{ psi}}$$

$$f_s = \frac{(8)(12)(60{,}000)(11.41)}{3315} = \boxed{19{,}826 \text{ psi}}$$

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PROB# 2.10

Locate N.A.

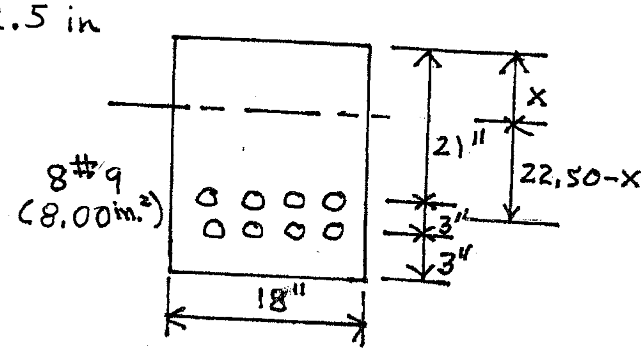
$d = 22.5$ in

$$(18x)\left(\frac{x}{2}\right) = (9)(8.00)(22.50-x)$$

$$9x^2 = 1620 - 72x$$

$$9x^2 + 72x = 1620$$

$$x = 10.0 \text{ in}$$



Moment of inertia

$$I_x = \left(\frac{1}{3}\right)(18)(10.0)^3 + (9)(8.00)(12.5)^2 = 17{,}250 \text{ in}^4$$

Flexural stresses

$$f_c = \frac{(12)(120{,}000)(10.0)}{17{,}250} = \boxed{835 \text{ psi}}$$

$$f_s = \frac{(9)(12)(120{,}000)(12.5)}{17{,}250} = \boxed{9391 \text{ psi}}$$

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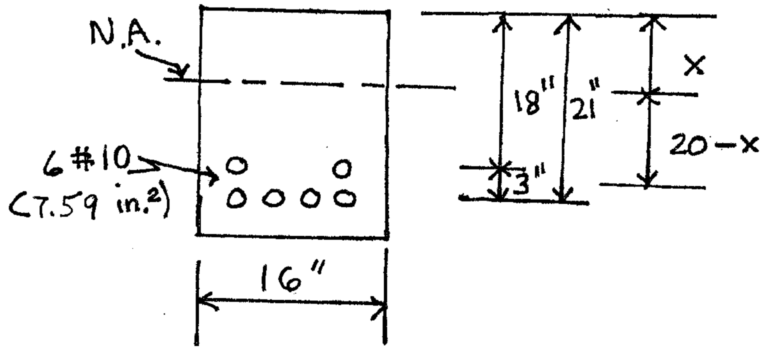
PROB #2.11

d = (18 x 2 x 1.27 + 21 x 4 x 1.27)/(6 x 1.27) = 20 in.



Locate N.A.

$$(16x)\left(\frac{x}{2}\right)=(10)(7.59)(20-x)$$

$$8x^2 = 1518 - 75.9x$$

$$8x^2 + 75.9x = 1518$$

$$x = 9.83 \text{ in}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(16)(9.83)^3 + (10)(7.59)(10.17)^2 = 12{,}916 \text{ in.}^4$$

Flexural stresses

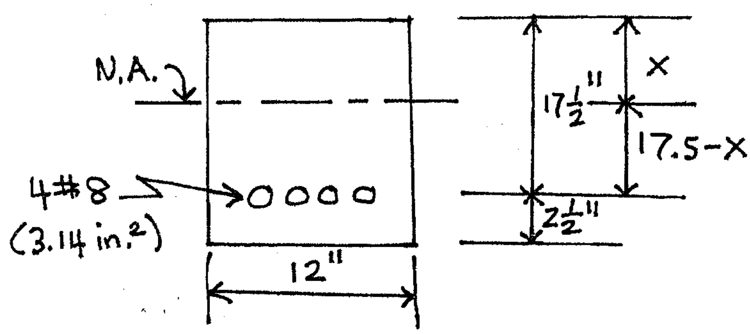
$$f_c = \frac{(12)(130{,}000)(9.83)}{12{,}916} = \boxed{1187 \text{ psi}}$$

rgcm

$$f_s = \frac{(10)(12)(130{,}000)(10.17)}{12{,}916} = \boxed{12{,}289 \text{ psi}}$$

The stress in the bottom layer of steel is determined using a distance from the neutral axis of 11.17 in. instead of 10.17 in as done above. This results in a value of steel stress, fs = 13,497 psi

PROB #2.12



Locate N.A.

$$(12x)\left(\frac{x}{2}\right) = (10)(3.14)(17.5-x)$$
$$6x^2 = 549.5 - 31.4x$$
$$6x^2 + 31.4x = 549.5$$
$$x = 7.30 \text{ in}^2$$

Moment of Inertia

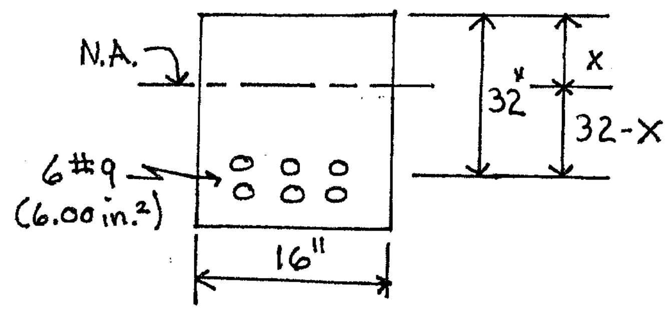
$$I_x = \left(\tfrac{1}{3}\right)(12)(7.30)^3 + (10)(3.14)(10.2)^2 = 4823 \text{ in.}^4$$

Moment and Flexural stresses

$$M = \frac{wl^2}{8} = \frac{(1.5)(24)^2}{8} = 108 \text{ ft-k}$$
$$f_c = \frac{(12)(108{,}000)(7.30)}{4823} = \boxed{1962 \text{ psi}}$$
$$f_s = \frac{(10)(12)(108{,}000)(10.2)}{4823} = \boxed{27{,}400 \text{ psi}}$$

✓JCM^c

PROB # 2-13



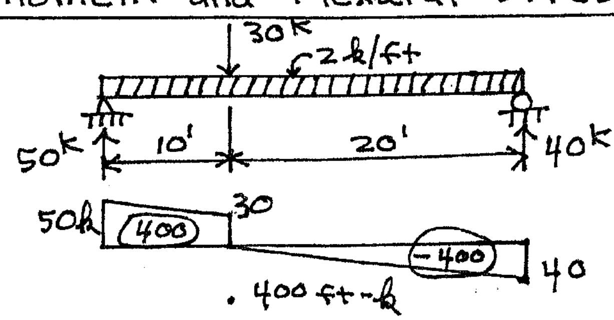
Locate N.A.

$$(16x)\left(\frac{x}{2}\right) = (10)(6.00)(32-x)$$
$$8x^2 = 1920 - 60x$$
$$8x^2 + 60x = 1920$$
$$x = 12.19 \text{ in}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(16)(12.19)^3 + (10)(6.00)(19.81)^2 = 33{,}207 \text{ in.}^4$$

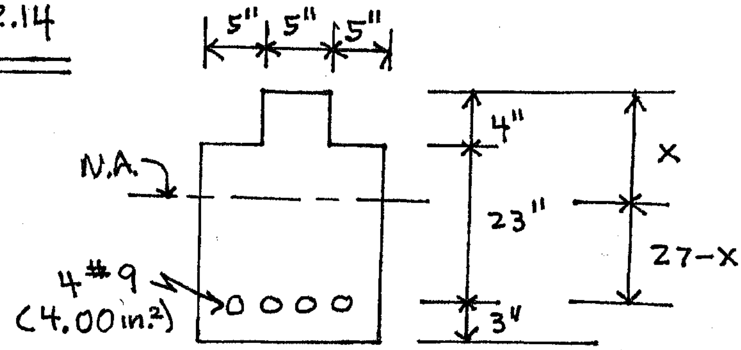
Moment and Flexural Stresses



$$f_c = \frac{(12)(400{,}000)(12.19)}{33{,}207} = \boxed{1762 \text{ psi}}$$ gcm

$$f_s = \frac{(10)(12)(400{,}000)(19.81)}{33{,}207} = \boxed{28{,}635 \text{ psi}}$$

PROB #2.14



Locate N.A. (assume $x > 4''$)

$$(5x)\left(\frac{x}{2}\right)+(2)(5)(x-4)\left(\frac{x-4}{2}\right)=(9)(4.00)(27-x)$$

$$2.5x^2 + 5x^2 - 40x + 80 = 972 - 36x$$

$$7.5x^2 - 4x = 892$$

$$x = 11.18 \text{ in} > 4 \text{ in. as assumed}$$

Moment of Inertia

$$I_x = \left(\tfrac{1}{3}\right)(5)(11.18)^3+(2)\left(\tfrac{1}{3}\right)(5)(7.18)^3+(9)(4.00)(15.82)^2$$

$$= 12{,}573 \text{ in.}^4$$

Flexural Stresses

$$f_c = \frac{(12)(70{,}000)(11.18)}{12{,}573} = \boxed{747 \text{ psi}}$$

$$f_s = \frac{(9)(12)(70{,}000)(15.82)}{12{,}573} = \boxed{9512 \text{ psi}}$$

✓ JCMc

PROB # 2.15

From solution of Prob. # 2.10

$x = 10.0$ in.

$d - x = 12.5$ in.

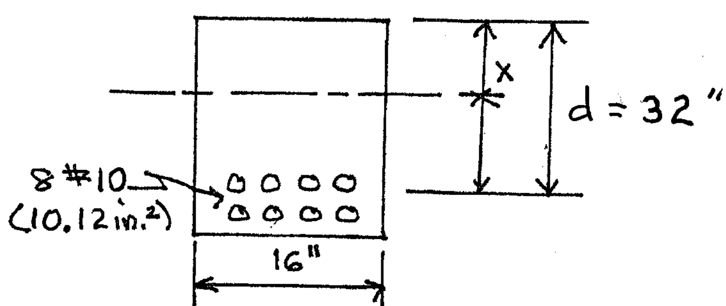
$I_x = 17{,}250 \text{ in.}^4$

✓ gcm

$$M_c = \frac{f_c I_x}{x} = \frac{(1800)(17{,}250)}{10} = 3{,}105{,}000 \text{ in.-lbs} = \boxed{258.8 \text{ ft-k}}$$

$$M_s = \frac{f_s I_x}{n(d-x)} = \frac{(24{,}000)(17{,}250)}{(9)(12.5)} = 3{,}680{,}000 \text{ in.-lbs} = \boxed{306.7 \text{ ft-k}}$$

PROB # 2.16



Locate N.A.

$$(16x)\left(\frac{x}{2}\right) = (10)(10.12)(32 - x)$$

$$8x^2 = 3238 - 101.2x$$

$$8x^2 + 101.2x = 3238$$

$$x = 14.76 \text{ in}$$

Moment of Inertia

$$I_x = \left(\tfrac{1}{3}\right)(16)(14.76)^3 + (10)(10.12)(32 - 14.76)^2 = 47{,}228 \text{ in}^4$$

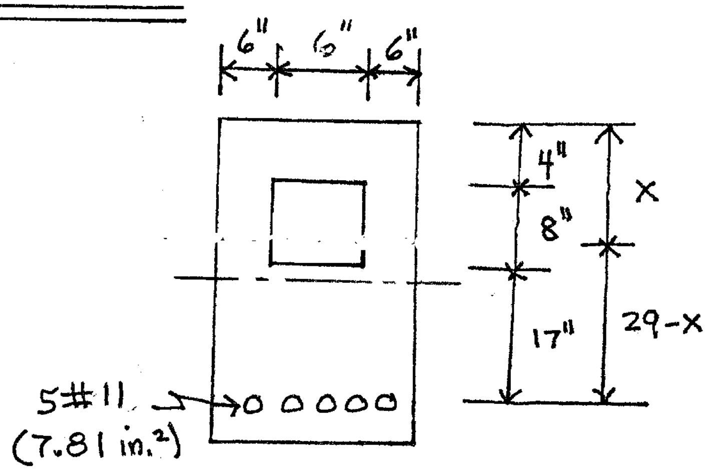
Resisting moment

✓ gcm

$$M_c = \frac{f_c I_x}{x} = \frac{(1125)(47{,}228)}{14.76} = 3{,}600{,}000 \text{ in.-lbs.} = \boxed{300 \text{ ft-k}}$$

$$M_s = \frac{f_s I_x}{n(d-x)} = \frac{(20{,}000)(47{,}228)}{(10)(17.24)} = 5{,}479{,}000 \text{ in.-lbs} = 456.6 \text{ ft-k}$$

PROB # 2.17



Assume N.A. passes below hole (i.e. $x > 12''$)

$$(18x)\left(\frac{x}{2}\right) - (6)(8)(x-8) = (9)(7.81)(29-x)$$

$$9x^2 - 48x + 384 = 2038 - 70.3x$$

$$9x^2 + 22.3x = 1654$$

$$x = 12.37 \text{ in.} > 12 \text{ in.}$$ assumption valid

Moment of Inertia

$$I_x = \left(\tfrac{1}{3}\right)(18)(12.37)^3 - \left(\tfrac{1}{12}\right)(6)(8)^3 - (6)(8)(4.37)^2$$
$$+ (9)(7.81)(29-12.37)^2 = 29,623 \text{ in}^4$$

Resisting Moment

$$M_c = \frac{f_c I_x}{x} = \frac{(1800)(29623)}{12.37} = 4310 \text{ in-k} = 359.2 \text{ ft-k} \leftarrow$$

$$M_s = \frac{f_s I_x}{n(d-x)} = \frac{(24,000)(29623)}{(9)(16.63)} = 4750 \text{ in-k} = 396 \text{ ft-k}$$

Allowable uniform load

$$\frac{w\ell^2}{8} = 359.2$$

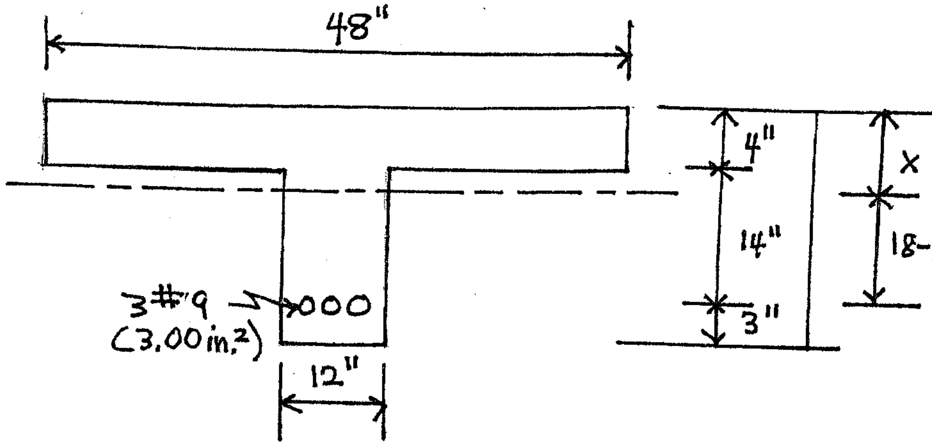
self-weight

$$w = \frac{(8)(359.2)}{(28)^2} = 3.665 - \frac{(18)(32)-(6)(8)}{144}(0.15) = \boxed{3.11 \text{ k/ft}}$$

JCMC

PROB # 2.18

Assume N.A. is in web



$$(48x)\left(\frac{x}{2}\right)-(36)(x-4)\left(\frac{x-4}{2}\right)=(10)(3.00)(18-x)$$

$$24x^2-18x^2+144x-288=540-30x$$

$$6x^2+174x=828$$

$$x=4.16 \text{ in}$$ (Neutral axis in web)

Moment of Inertia

$$I_x=\left(\frac{1}{3}\right)(48)(4.16)^3-\left(\frac{1}{3}\right)(36)(0.16)^3+10(3.00)(13.84)^2$$

$$=6898 \text{ in.}^4$$

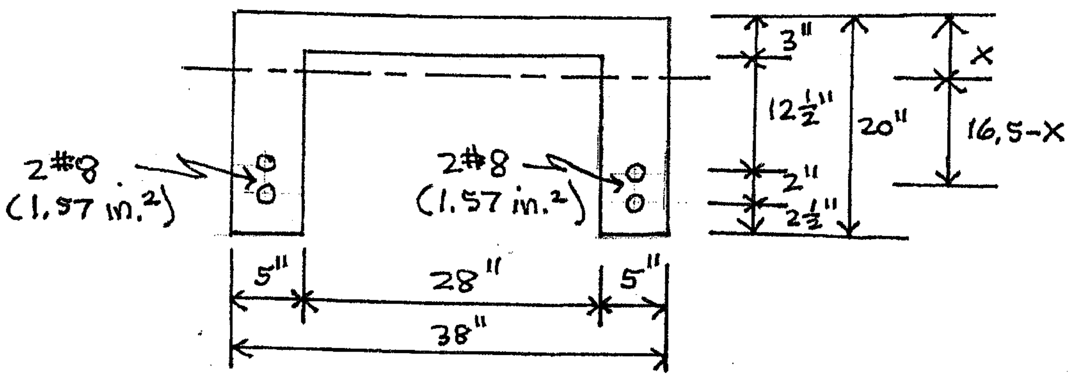
Flexural Stresses

$$f_c=\frac{(12)(100{,}000)(4.16)}{6898}=\boxed{724 \text{ psi}}$$

$$f_s=\frac{(10)(12)(100{,}000)(13.84)}{6898}=\boxed{24{,}077 \text{ psi}}$$

PROB # 2.19

Assume N.A. in stems



$$(38x)\left(\frac{x}{2}\right)-(28)(x-3)\left(\frac{x-3}{2}\right)=(9)(2\times1.57)(16.5-x)$$

$$19x^2-14x^2+84x-126=466.3-28.26x$$

$$5x^2+112.3x=592$$

$$x=4.41 \text{ in.} > \boxed{3} \text{ in}$$

Moment of Inertia

$$I_x=\left(\frac{1}{3}\right)(38)(4.41)^3-\left(\frac{1}{3}\right)(28)(1.41)^3+(9)(3.14)(12.09)^2$$

$$=5191 \text{ in.}^4$$

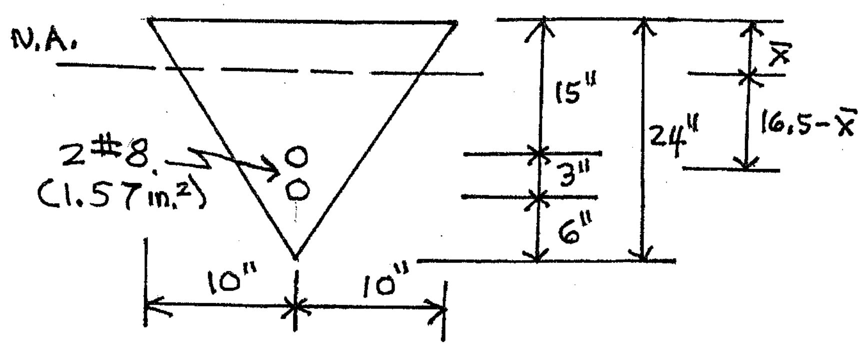
Flexural Stresses

$$f_c=\frac{(12)(120{,}000)(4.41)}{5191}=\boxed{1223 \text{ psi}}$$

$$f_s=\frac{(9)(12)(120{,}000)(12.09)}{5191}=\boxed{30{,}185 \text{ psi}}$$

rjcm

PROB #2.20



Locate N.A.

$$(20x)\left(\frac{x}{2}\right)-\left(\frac{1}{2}\right)(x)\left(\frac{10}{24}x\right)(2)\left(\frac{1}{3}x\right)=(9)(1.57)(16.5-x)$$

$$10x^2 - 0.139x^3 = 233.5 - 14.13x$$

$$x^2 - 0.0139x^3 + 1.413x = 23.35$$

$x = 4.29''$ from equation solution

Moment of Inertia

$$I_x=\left(\frac{1}{3}\right)(20)(4.29)^3-\frac{(0.417\times4.29)(2)(4.29)^3}{12}$$

$$+9(1.57)(12.21)^2 = 2636\text{ in.}^4$$

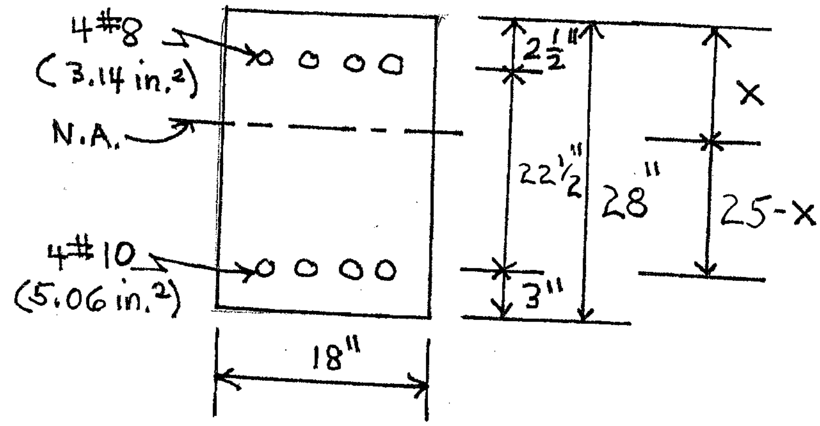
Flexural Stresses

$$f_c = \frac{(12)(90,000)(4.29)}{2636} = \boxed{1758\text{ psi}}$$

$$f_s = \frac{(9)(12)(90,000)(12.21)}{2636} = \boxed{45,023\text{ psi}}$$

✓ JCM

PROB #2.21



Locate N.A.

$$(18x)\left(\frac{x}{2}\right)+(\underbrace{17}_{2n-1})(3.14)(x-2.5)=(9)(5.06)(25-x)$$

$$9x^2+53.38x-133.45=1138.5-45.54x$$

$$9x^2+98.92x=1272$$

$$x=7.60 \text{ in.}$$

Moment of Inertia

$$I_x=\left(\tfrac{1}{3}\right)(18)(7.60)^3+(17)(3.14)(5.10)^2+(9)(5.06)(17.40)^2$$

$$=17{,}810 \text{ in.}^4$$

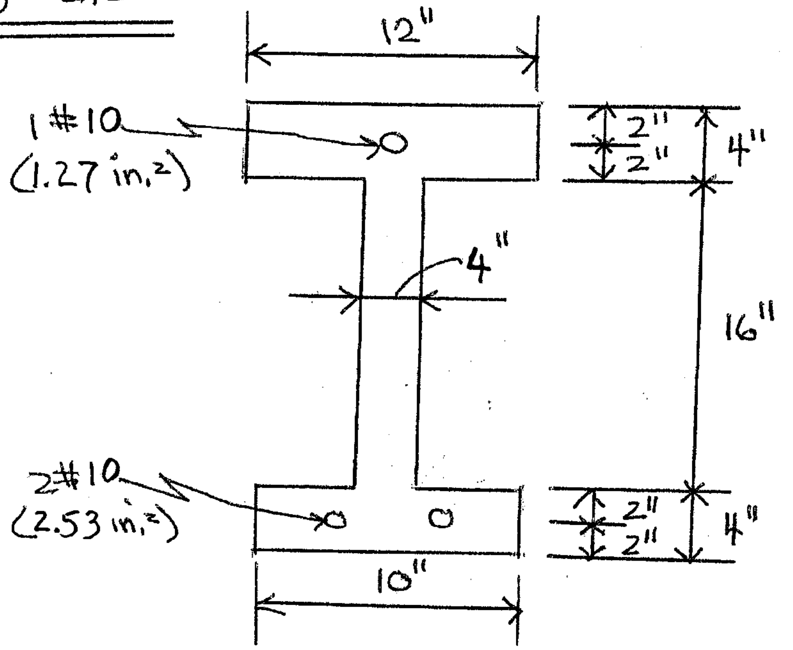
Flexural Stresses

$$f_c=\frac{(12)(250{,}000)(7.60)}{17{,}810}=\boxed{1280 \text{ psi}}$$ ✓ JCM

$$f_s'=\frac{2(9)(12)(250{,}000)(5.10)}{17{,}810}=\boxed{15{,}463 \text{ psi}}$$

$$f_s=\frac{(9)(12)(250{,}000)(17.40)}{17{,}810}=\boxed{26{,}378 \text{ psi}}$$

PROB # 2.22



Assume N.A. is in web

2n-1

$(12-4)(4)(x-2) + (4x)\left(\frac{x}{2}\right) + (15)(1.27)(x-2) = (8)(2.53)(22-x)$

$32x - 64 + 2x^2 + 19x - 38 = 445 - 20.24x$

$2x^2 + 71.24x = 547$

$x = 6.49$ in.

∴ N.A. in web as assumed

Moment of Inertia

$I_x = \left(\frac{1}{3}\right)(12)(6.49)^3 - \left(\frac{1}{3}\right)(8)(2.49)^3 + (15)(1.27)(4.49)^2 + (8)(2.53)(15.51)^2$

$= 6305$ in.^{4}

Allowable resisting moment

$M_c = \frac{(1800)(6305)}{6.49} = 1{,}748{,}690$ in.-lbs $= 145.7$ ft-k

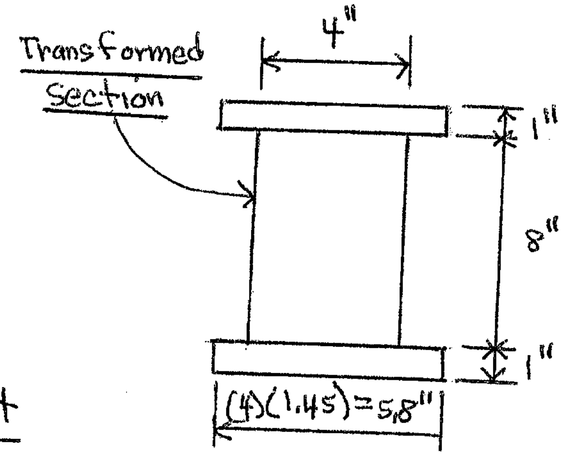
$M_s' = \frac{(24{,}000)(6305)}{(2)(8)(4.49)} = 2{,}106{,}347$ in.-lbs $= 175.5$ ft-k

$M_s = \frac{(24{,}000)(6305)}{(8)(15.51)} = 1{,}219{,}536$ in.-lbs $= \boxed{101.6 \text{ ft-k}}$

JMc

PROB# 2.23

$$n = \frac{29\times10^6}{20\times10^6} = 1.45$$



Moment of Inertia

$$I_x = \left(\tfrac{1}{12}\right)(5.8)(10)^3 - \left(\tfrac{1}{12}\right)(1.8)(8)^3$$
$$= 406.53 \text{ in.}^4$$

Allowable resisting moment

M_{res} for 29×10^6 psi steel

$$= \frac{(30{,}000)(406.53)}{(1.45)(5.00)} = 1{,}682{,}193 \text{ in.-lbs} = 140.18 \text{ ft lb} \leftarrow$$

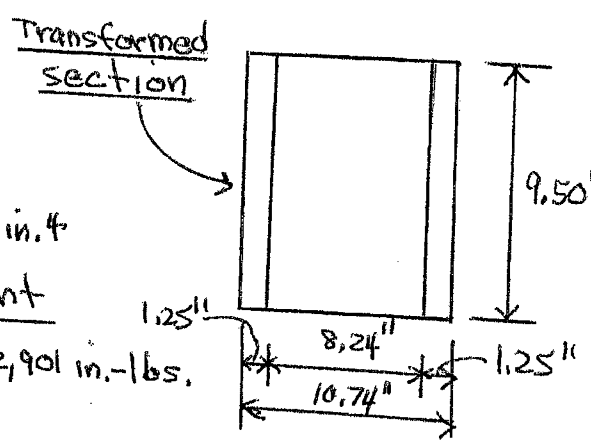
$$M_{res} \text{ for } 20\times10^6 \text{ psi steel} = \frac{(20{,}000)(406.53)}{4.00}$$
$$= 2{,}032{,}650 \text{ in. lbs} = 169.39 \text{ ft-k}$$

$$M_{res} = \boxed{140.18 \text{ ft-k}}$$

✓ gcm©

PROB #2.24

$$m = \frac{29\times10^6}{1.76\times10^6} = 16.48$$



Moment of inertia

$$I_x = \left(\tfrac{1}{12}\right)(10.74)(9.50)^3 = 767.35 \text{ in.}^4$$

Allowable resisting moment

$$M_{wood} = \frac{(1875)(767.35)}{4.75} = 302{,}901 \text{ in.-lbs.}$$
$$= 25.24 \text{ ft-k}$$

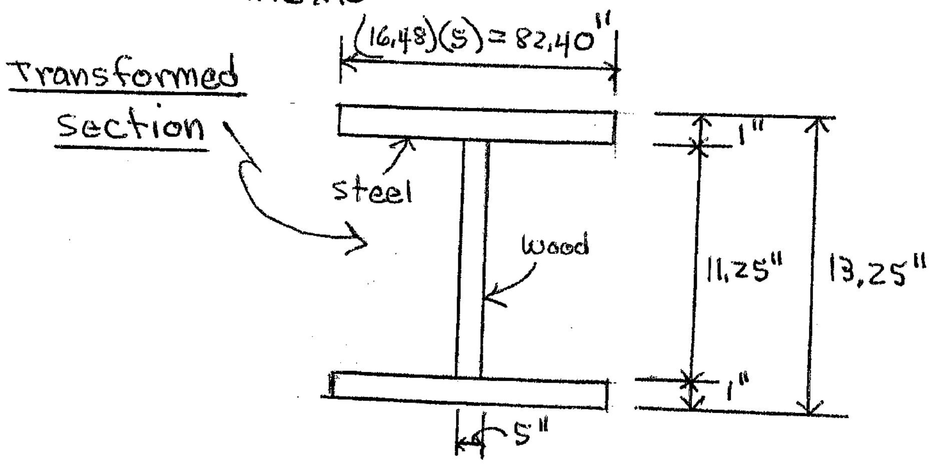
$$M_{steel} = \frac{(24{,}000)(767.35)}{(16.48)(4.75)} = 235{,}268 \text{ in.-lbs}$$

$$= \boxed{19.61 \text{ ft-k}} \leftarrow$$

✓ gcm©

PROB # 2.25

$$n = \frac{29 \times 10^6}{1.76 \times 10^6} = 16.48$$



Moment of Inertia

$$I_x = \left(\tfrac{1}{12}\right)(82.40)(13.25)^3 - \left(\tfrac{1}{12}\right)(77.40)(11.25)^3$$

$$= 6789.6 \text{ in.}^4$$

Allowable resisting moment

$$M_{wood} = \frac{(1800)(6789.6)}{5.625} = 2{,}172{,}662 \text{ in.-lbs}$$

$$= 181.06 \text{ ft-k}$$

$$M_{steel} = \frac{(24{,}000)(6789.6)}{(16.48)(6.625)}$$

$$= 1{,}492{,}493 \text{ in.-lbs} = \boxed{124.4 \text{ ft-k}} \leftarrow$$

r JCM

PROB # 2.26

Using 3 #8 bars (2.35 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(2.35)(60)}{(0.85)(4)(16)} = 2.96 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (2.35)(60)\left(21 - \frac{2.96}{2}\right)$$

$$= 2752 \text{ in.-k} = \boxed{229.3 \text{ ft-k}}$$ rgcm©

PROB # 2.27

Using 6 #10 bars (7.59 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(7.59)(60)}{(0.85)(4)(16)} = 8.37 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (7.59)(60)\left(26.25 - \frac{8.37}{2}\right)$$

$$= 10{,}048 \text{ in.-k} = \boxed{837.3 \text{ ft-k}}$$ rgcm©

PROB # 2.28

Using 4 #10 bars (5.06 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5.06)(60)}{(0.85)(4)(16)} = 5.58 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (5.06)(60)\left(25 - \frac{5.58}{2}\right)$$

$$= 6743 \text{ in.-k} = \boxed{561.9 \text{ ft-k}}$$ rgcm©

PROB # 2.29

Using 6 #9 bars (6.00 in.^2)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(6.00)(60)}{(0.85)(4)(16)} = 6.62 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (6.00)(60)\left(26 - \frac{6.62}{2}\right)$$

$$= 8169 \text{ in.-k} = \boxed{680.7 \text{ ft-k}}$$ rgcm^c

PROB # 2.30

Using 3 #9 bars (3.00 in.^2)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3.00)(60)}{(0.85)(4)(14)} = 3.78 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.00)(60)\left(21 - \frac{3.78}{2}\right)$$

$$= 3440 \text{ in.-k} = \boxed{286.6 \text{ ft-k}}$$ rgcm^c

PROB # 2.31

Using 8 #10 bars 10.12 in.^2)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(10.12)(60)}{(0.85)(4)(18)} = 9.92 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (10.12)(60)\left(28.5 - \frac{9.92}{2}\right)$$

$$= 14{,}293 \text{ in.-k} = \boxed{1191.1 \text{ ft-k}}$$ rgcm^c

PROB # 2.32

Using 4 #10 bars (5.06 in.^{2})

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(5.06)(60)}{(0.85)(5)(14)} = 5.10 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (5.06)(60)\left(20.5 - \frac{5.10}{2}\right)$$

$$= 5450 \text{ ft-k} = \boxed{454.1 \text{ ft-k}}$$ ✓ gcm

PROB # 2.33

Using 4 #11 bars (6.25 in.^{2})

$$a = \frac{A_s f_y}{0.8\ f'_c b} = \frac{(6.25)(75)}{(0.85)(5)(18)} = 6.13 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (6.25)(75)\left(25.5 - \frac{6.13}{2}\right)$$

$$= 10{,}517 \text{ in.-k} = \boxed{876.4 \text{ ft-k}}$$ ✓ gcm

PROB # 2.34

Using 6 #11 bars (9.37 in.^{2})

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(9.37)(60)}{(0.85)(3)(22)} = 10.02 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (9.37)(60)\left(36 - \frac{10.02}{2}\right)$$

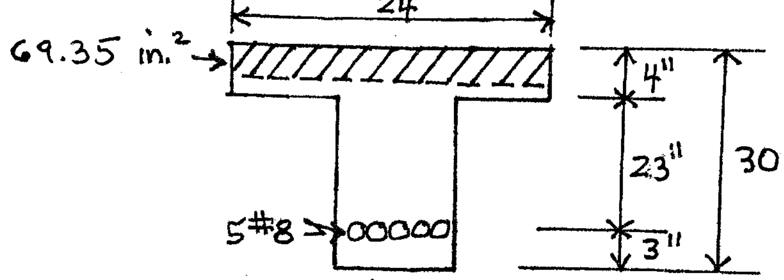
$$= 17{,}422 \text{ in.-k} = \boxed{1451.8 \text{ ft-k}}$$ ✓ gcm

PROB # 2.35

Using 5 #8 bars (3.93 in.^2)

$$0.85 f'_c A_c = A_s f_y$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(3.93)(60)}{(0.85)(4)} = 69.35 \text{ in.}^2$$



Noting that $69.35 \text{ in.}^2 < (4)(24) = 96 \text{ in.}^2$

$\therefore$ N.A. is in flange

$$a = \frac{69.35}{24} = 2.89 \text{ in.}$$

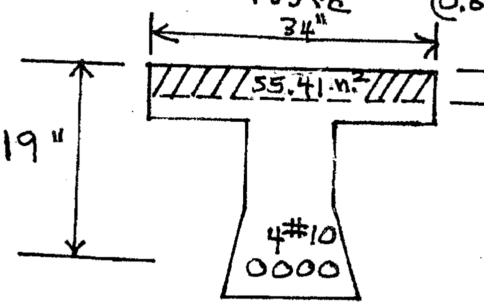
$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.93)(60)\left(27 - \frac{2.89}{2}\right)$$

$$= 6026 \text{ in.-k} = \boxed{502.2 \text{ ft-k}}$$ ✓ JCMc

PROB # 2.36

Using 4 #8 bars (3.14 in.^2)

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(3.14)(60)}{(0.85)(4)} = 55.41 \text{ in.}^2$$



$$a = \frac{55.41}{34} = 1.63 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.14)(60)\left(19 - \frac{1.63}{2}\right)$$

$$= 3426 \text{ in.-k} = \boxed{285.5 \text{ ft-k}}$$ ✓ JCMc

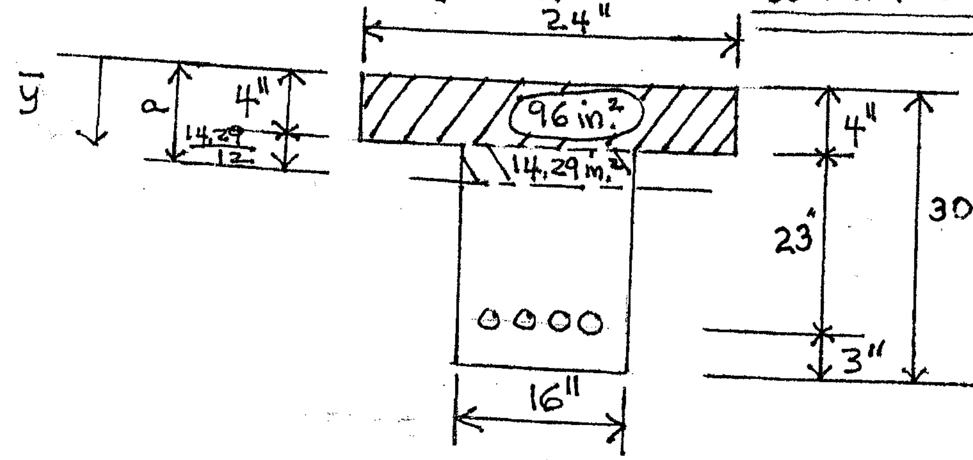
PROB #2.37

using 4 #11 bars (6.25 in.²)

$$0.85 f'_c A_c = A_s f_y$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(6.25)(60)}{(0.85)(4)} = 110.29 \text{ in.}^2$$

$$> (4)(24) = 96 \text{ in.}^2 \quad \therefore \text{ N.A. is in web}$$



$$a = 4 + \frac{14.29}{16} = 4 + 0.893 = 4.893 \text{ in.}$$

$$\bar{y} \text{ from top} = \frac{(96)(2) + (14.29)\left(4 + \frac{0.893}{2}\right)}{110.29} = 2.32 \text{ in.}$$

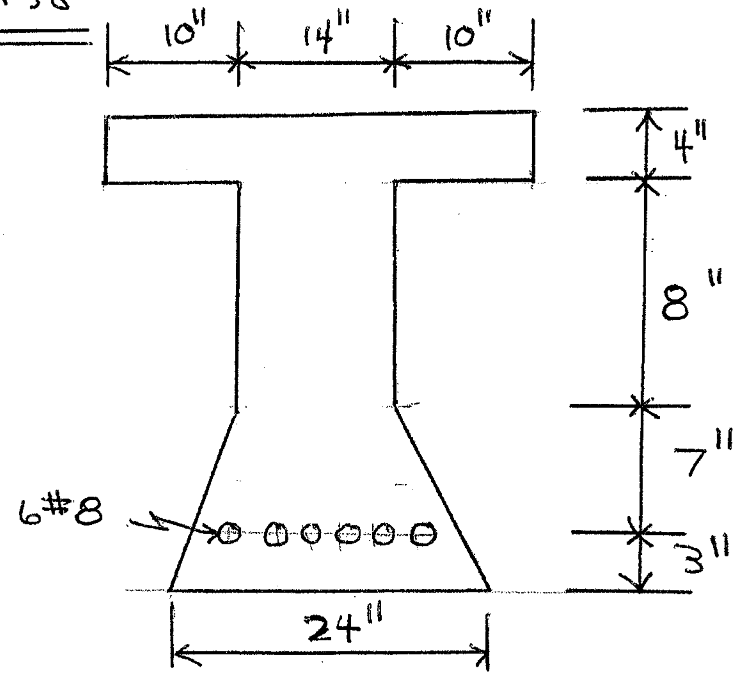
$$z = 27 - 2.32 = 24.68 \text{ in.}$$

$$M_n = A_s f_y z = (6.25)(60)(24.68)$$

$$= 9255 \text{ in.-k} = \boxed{771.3 \text{ ft-k}}$$

✓ J C McC

PROB #2.38



using 6#8 bars ($A_s = 4.71$ in.^{2})

$$0.85 f'_c A_c = A_s f_y$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(4.71)(60)}{(0.85)(4)} = 83.12 \text{ in.}^2$$

$$< (4)(34) = 136 \text{ in.}^2$$ ∴ N.A. is in flange

$$a = \frac{83.12}{34} = 2.44 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (4.71)(60)\left(19 - \frac{2.44}{2}\right)$$

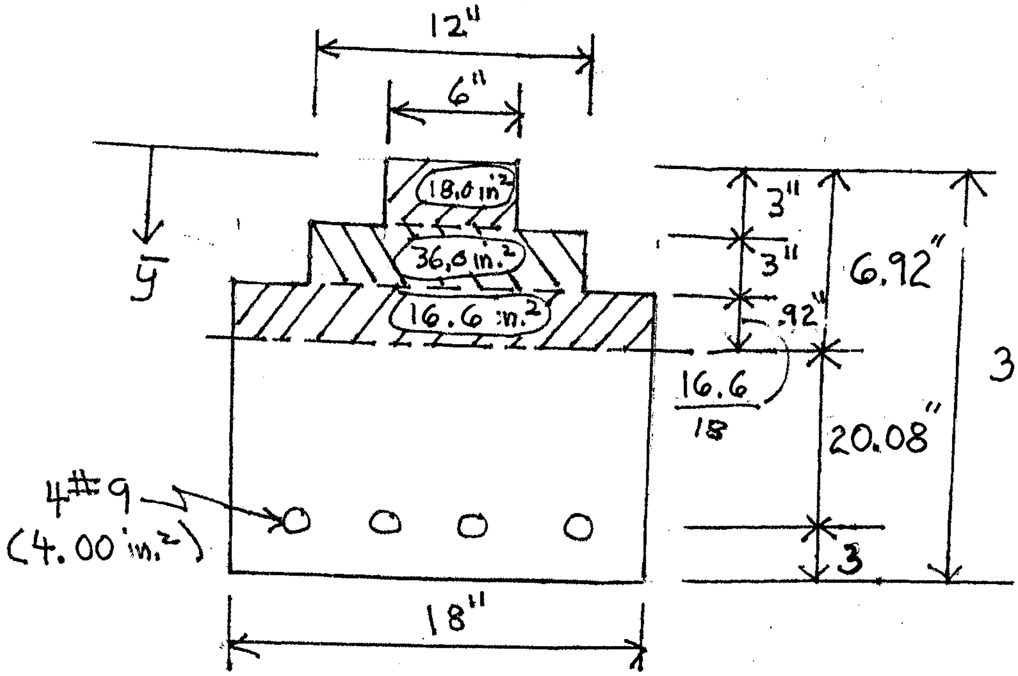
$$= 5024 \text{ in.-k} = \boxed{418.7 \text{ ft-k}}$$

JCM©

PROB #2.39

$$0.85 f'_c A_c = A_s f_y$$

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(4.00)(60)}{(0.85)(4)} = 70.6 \text{ in.}^2$$



$$\bar{y} \text{ from top} = \frac{(18)(1.50) + (36)(4.5) + (16.6)(6 + \frac{.92}{2})}{70.6}$$

$$= 4.20 \text{ in.}$$

$$z = 27 - 4.20 = 22.80 \text{ in.}$$

$$M_n = A_s f_y z = (4.00)(60)(22.80)$$

$$= 5472 \text{ in.-k} = \boxed{456 \text{ ft-k}}$$

✓gam

PROB #2.40

Using 3 #9 bars (3.00 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3.00)(60)}{(0.85)(4)(14)} = 3.78 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.00)(60)\left(23 - \frac{3.78}{2}\right)$$

$$= 3800 \text{ in.-k} = 316.6 \text{ ft-k}$$

$$\frac{w_n L^2}{8} = M_n = 316.6$$

$$w_n = \frac{(8)(316.6)}{(18)^2} = \boxed{7.82 \text{ k/ft}}$$

rgcmc

PROB #2.41

Using 4 #8 bars (3.14 in.²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3.14)(60)}{(0.85)(3)(16)} = 3.46 \text{ in.}$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3.14)(60)\left(20 - \frac{3.46}{2}\right)$$

$$= 3442 \text{ in.-k} = 286.8 \text{ ft-k}$$

$$\frac{w_n L^2}{8} = M_n = 286.8$$

$$w_n = \frac{(8)(286.8)}{(20)^2} = \boxed{5.74 \text{ k/ft}}$$

rgcmc

PROB # 2.42

$$I_g = \left(\frac{1}{12}\right)(350)(600)^3 = 6.3 \times 10^9 \ mm^4$$

$$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{28} = 3.704 \ MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(3.704)(6.3 \times 10^9)}{300}$$

$$= 7.78 \times 10^7 \ N\cdot mm = \boxed{77.8 \ kN\cdot m}$$ rgcm©

PROB # 2.43

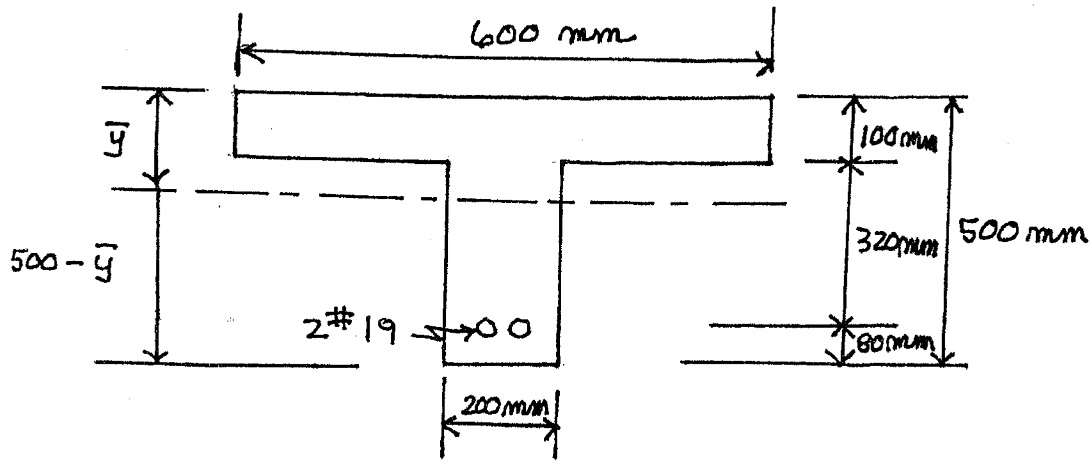
$$I_g = \left(\frac{1}{12}\right)(350)(500)^3 = 3.65 \times 10^9 \ mm^4$$

$$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{28} = 3.704 \ MPa$$

$$M_{cr} = \frac{f_r I_g}{y_t} = \frac{(3.704)(3.65 \times 10^9)}{250}$$

$$= 5.4 \times 10^7 \ N\cdot mm = \boxed{54.0 \ kN\cdot m}$$ rgcm©

PROB # 2.44



$$A = (600)(100) + (200)(400) = 140\ 000\ mm^2$$

$$\bar{y} = \frac{(60\ 000)(50) + (80\ 000)(300)}{140\ 000} = 192.86\ mm$$

$$I_x = \left(\frac{1}{3}\right)(200)(192.86^3 + 307.14^3) + \left(\frac{1}{12}\right)(400)(100)^3 + (400)(100)(142.86)^2 = 3.26 \times 10^9\ mm^4$$

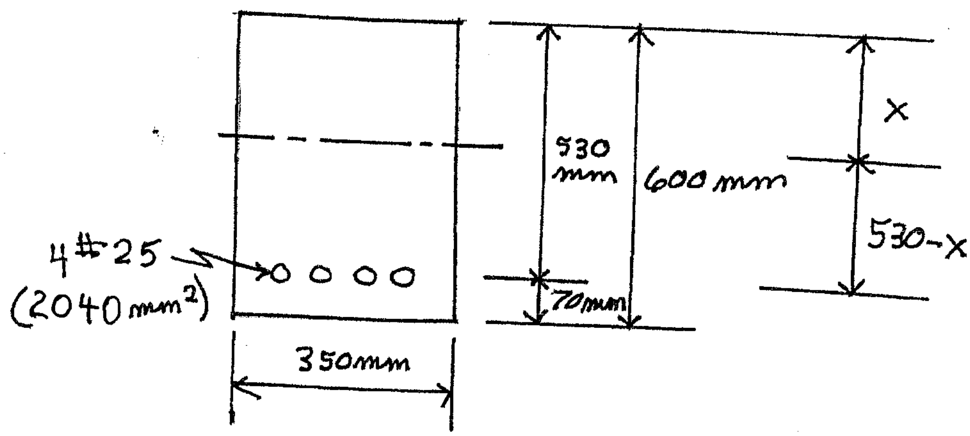
$$f_r = 0.7\sqrt{f'_c} = 0.7\sqrt{28} = 3.704\ MPa$$

$$M_{cr} = \frac{f_r I_x}{d - \bar{y}} = \frac{(3.704)(3.26 \times 10^9)}{307.14}$$

$$= 3.931 \times 10^7\ N \cdot mm = \boxed{39.31\ kN \cdot m}$$

rgcm©

PROB# 2.45



Locate N.A.

$$(350x)\left(\frac{x}{2}\right) = (8)(2040)(530-x)$$

$$175x^2 = 8.65\times10^6 - 16320x$$

$$175x^2 + 16{,}320x = 8.65\times10^6$$

$$x = 180.5 \text{ mm}$$

Moment of Inertia

$$I_x = \left(\tfrac{1}{3}\right)(350)(180.5)^3 + (8)(2040)(349.5)^2$$

$$= 2.69\times10^9 \text{ mm}^4$$

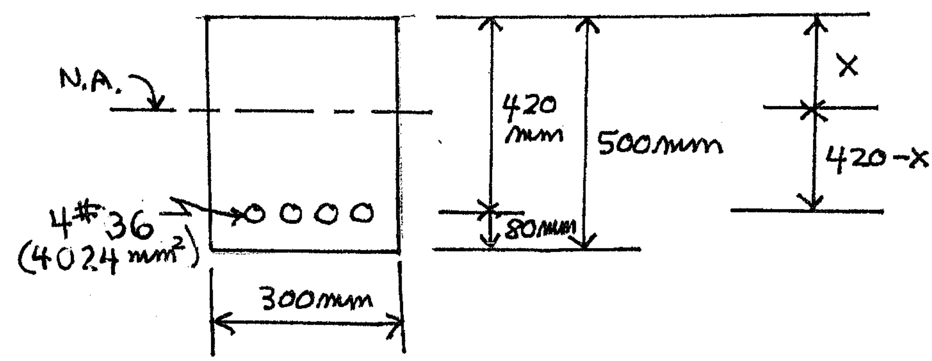
Flexural Stresses

$$f_c = \frac{(150)(10)^6(180.5)}{2.69\times10^9} = \boxed{10.07 \text{ MPa}}$$

$$f_s = \frac{(8)(150)(10)^6(349.5)}{2.69\times10^9} = \boxed{155.9 \text{ MPa}}$$

rgcm©

PROB # 2.46



Locate N.A.

$$(300x)\left(\frac{x}{2}\right) = (9)(4024)(420-x)$$

$$150x^2 = 15.21\times10^6 - 36{,}216x$$

$$150x^2 + 36{,}216x = 15.21\times10^6$$

$$x = 219.8 \text{ mm}$$

Moment of Inertia

$$I_x = \left(\frac{1}{3}\right)(300)(219.8)^3 + (9)(4024)(200.2)^2 = 2.51\times10^9 \text{ mm}^4$$

Flexural Stresses

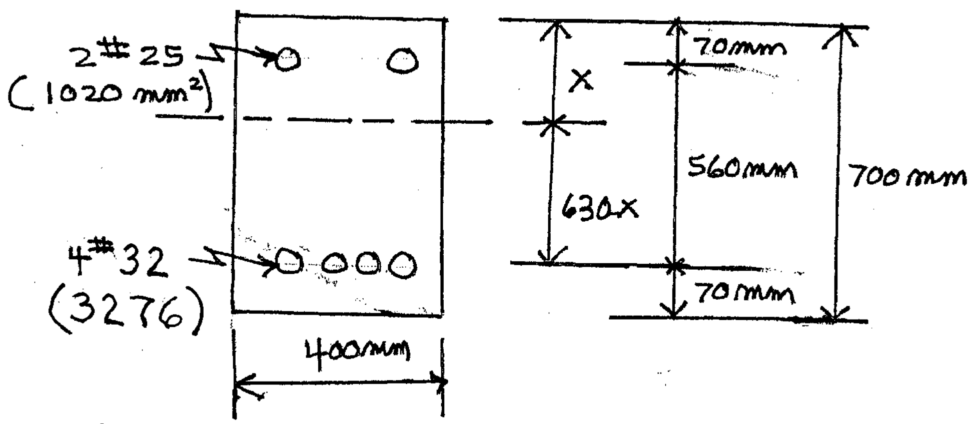
$$M = \frac{(20)(8)^2}{8} = 160 \text{ kN.m}$$

$$f_c = \frac{(160)(10)^6(219.8)}{2.51\times10^9} = \boxed{14.01 \text{ MPa}}$$

$$f_s = \frac{(9)(160)(10)^6(200.2)}{2.51\times10^9} = \boxed{114.9 \text{ MPa}}$$

rgcm©

PROB #2.47



Locate N.A.

$$(400x)\left(\frac{x}{2}\right) + (18-1)(1020)(x-70) = (9)(3276)(630-x)$$

$$200x^2 + 17340x - 1.213\times10^6 = 18.57\times10^6 - 29484x$$

$$200x^2 + 46{,}824x = 19.78\times10^6$$

$$x = 218.5 \text{ mm}$$

Moment of Inertia

$$I_x = \left(\tfrac{1}{3}\right)(400)(218.5)^3 + (18-1)(1020)(148.5)^2 + (9)(3276)(411.5)^2 = 6.77\times10^9 \text{ mm}^4$$

Flexural Stresses

$$f_c = \frac{(300)(10)^6(218.5)}{6.77\times10^9} = \boxed{9.68 \text{ MPa}}$$

$$f_s' = \frac{(18)(300)(10)^6(148.5)}{6.77\times10^9} = \boxed{118.4 \text{ MPa}}$$

$$f_s = \frac{(9)(300)(10)^6(411.5)}{6.77\times10^9} = \boxed{164.1 \text{ MPa}}$$

VJCMc

PROB#2.48

Using 3#36 bars ($3018\ mm^2$)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3018)(350)}{(0.85)(35)(300)} = 118.4\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3018)(350)\left(600 - \frac{118.4}{2}\right)$$

$$= 5.71 \times 10^8\ N\cdot mm = \boxed{571\ kN\cdot m}$$ ✓ JCM

PROB#2.49

Using 3#36 bars ($3018\ mm^2$)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3018)(350)}{(0.85)(28)(320)} = 138.69\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (3018)(350)\left(600 - \frac{138.69}{2}\right)$$

$$= 5.605 \times 10^8\ N\cdot mm = \boxed{560.5\ kN\cdot m}$$ ✓ JCM

PROB#2.50

Using 3#25 bars ($1530\ mm^2$)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(1530)(420)}{(0.85)(24)(350)} = 90.0\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (1530)(420)\left(530 - \frac{90}{2}\right)$$

$$= 3.117 \times 10^8\ N\cdot mm = \boxed{311.7\ kN\cdot m}$$ ✓ JCM

PROB#2.51

Using 3#32 bars ($2457\ mm^2$)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(2457)(420)}{(0.85)(42)(400)} = 72.26\ mm$$

$$M_n = A_s f_y \left(d - \frac{a}{2}\right) = (2457)(420)\left(660 - \frac{72.26}{2}\right)$$

$$= 6.44 \times 10^8\ N\cdot mm = \boxed{644\ kN\cdot m}$$ ✓ JCM

PROB # 2.52

Using 6 #25 bars (3060 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3060)(420)}{(0.85)(24)(350)} = 180 \text{ mm}$$

$$M_m = A_s f_y \left(d - \frac{a}{2}\right) = (3060)(420)\left(495 - \frac{180}{2}\right)$$

$$= 5.205 \times 10^8 \text{ N}\cdot\text{mm} = \boxed{520.5 \text{ kN}\cdot\text{m}}$$ ✓gcm

PROB # 2.53

Using 4 #36 bars (4024 mm²)

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(4024)(350)}{(0.85)(35)(300)} = 157.8 \text{ mm}$$

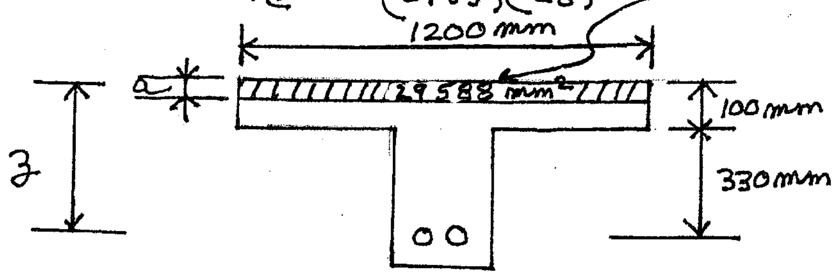
$$M_m = A_s f_y \left(d - \frac{a}{2}\right) = (4024)(420)\left(600 - \frac{157.8}{2}\right)$$

$$= 7.339 \times 10^8 \text{ N}\cdot\text{mm} = \boxed{734 \text{ kN}\cdot\text{m}}$$ ✓gcm

PROB # 2.54

Using 2 #36 bars (2012 mm²)

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(2012)(350)}{(0.85)(28)} = 29\,588 \text{ mm}^2$$



$$T = A_s f_y = (2012)(350) = 704\,200 \text{ N}$$

$$a = \frac{29{,}588}{1200} = 24.66 \text{ mm}$$

$$z = 430 - \frac{24.66}{2} = 417.7 \text{ mm}$$

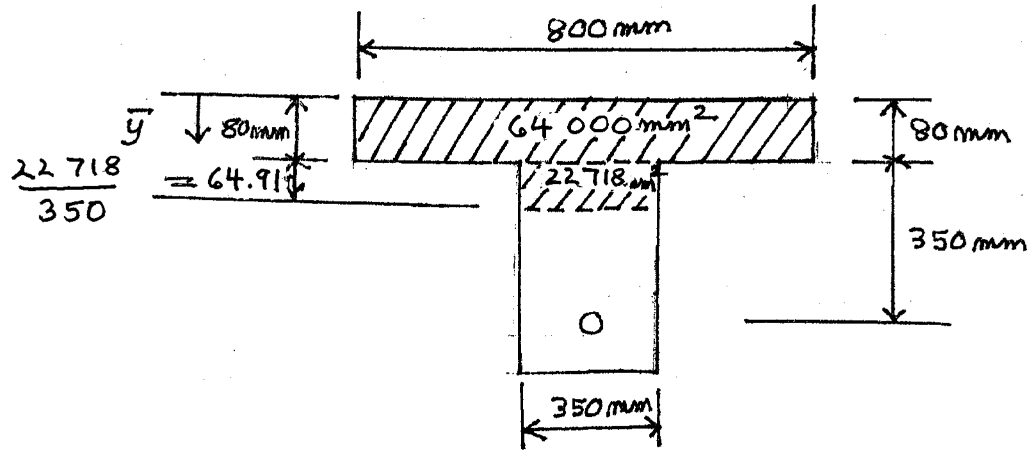
$$M_m = Tz = (704\,200)(417.7) = 2.941 \times 10^8 \text{ N}\cdot\text{mm}$$

$$= \boxed{294 \text{ kN}\cdot\text{m}}$$ ✓gcm

PROB #2.55

Using 6 #32 bars (4914 mm^2)

$$A_c = \frac{A_s f_y}{0.85 f'_c} = \frac{(4914)(420)}{(0.85)(28)} = 86718 \text{ mm}^2$$



$$\bar{y} = \frac{(64\,000)(40) + (22\,718)(80 + \frac{64.91}{2})}{86\,718} = 58.98 \text{ mm}$$

$$z = 430 - 58.98 = 371.02 \text{ mm}$$

$$T = A_s f_y = (4914)(420) = 2\,063\,880 \text{ N}$$

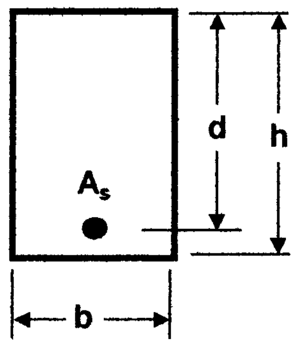
$$M_n = Tz = (2\,063\,880)(371) =$$

$$= 7.657 \times 10^8 \text{ N.mm}$$

$$= \boxed{766 \text{ kN.m}}$$ ✓ jcmc

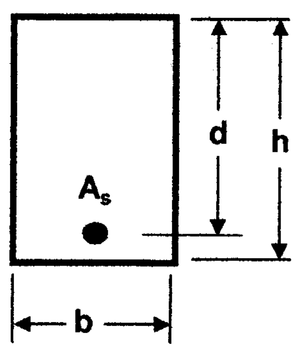
Prob. 2-56 **Repeat Prob. 2-27 using Chapter 2 Spreadsheet.**

		Units
f'_c =	4000	psi
b =	16	in.
d =	26.25	in.
A_s =	7.59	in^2
f_y =	60	ksi
a =	8.37	in.
M_n =	10048.1	in.-k
	837.3	kip-ft
ϕM_n =	753.6	kip-ft



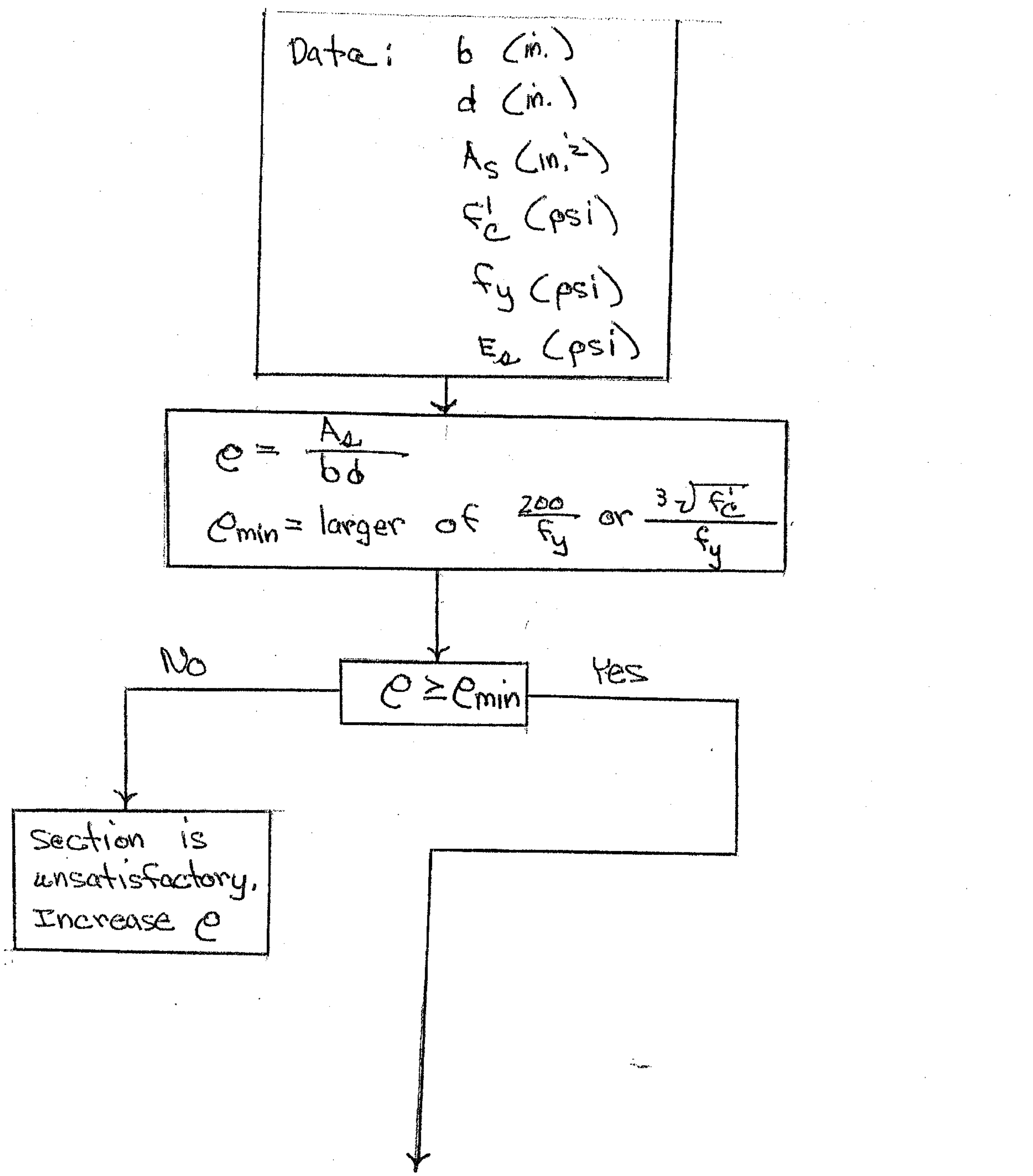
Prob. 2-57 **Repeat Prob. 2-28 using Chapter 2 Spreadsheet.**

		Units
f'_c =	4000	psi
b =	16	in.
d =	25	in.
A_s =	5.06	in^2
f_y =	60	ksi
a =	5.58	in.
M_n =	6742.8	in.-k
	561.9	kip-ft
ϕM_n =	505.7	kip-ft



PROB # 2.58 (1)

Flexural analysis of singly reinforced rectangular beams



PROB # 2.58 (2)

$\beta_1 = 0.85$ for $f'_c \leq 4000$ psi

$\beta_1 = 0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right)$ for $f'_c > 4000$ psi ≤ 8000 psi

$\beta_1 = 0.65$ for $f'_c > 8000$ psi

$\frac{c}{d} \leq 0.375$

No

section is unsatisfactory
Increase its size and/or
reduce steel area

Yes

$a = \frac{A_s f_y}{0.85 f'_c b}$

$M_n = A_s f_y \left(d - \frac{a}{2}\right)$

rgcm