Solutions to problems for
"Categorical Data Analysis for the Behavioral and Social Sciences"
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CHAPTER 2: PROBABILITY DISTRIBUTIONS

2.1 This probability can be determined using the hypergeometric distribution:

N = 25, n = 5, m = 12, k = 0

$$P(Y=0) = \frac{\binom{12}{0}\binom{25-12}{5-0}}{\binom{25}{5}} = \frac{\binom{3}{1}\binom{13}{5}}{\binom{25}{5}} = \frac{\left(\frac{12!}{0!(12!)}\right)\left(\frac{13!}{5!(8!)}\right)}{\left(\frac{25!}{5!(20)!}\right)} = \frac{1287}{53130} \approx 0.02\,.$$

2.2 Both of these probabilities can be determined using the hypergeometric distribution:

a. N = 20, n = 2, m = 15, k = 2

$$P(Y=2) = \frac{\binom{15}{2}\binom{20-15}{2-2}}{\binom{20}{2}} = \frac{\binom{15}{2}\binom{5}{0}}{\binom{20}{2}} = \frac{\left(\frac{15!}{2!(13!)}\right)\left(\frac{5!}{0!(5!)}\right)}{\left(\frac{20!}{2!(18)!}\right)} = \frac{105}{380} \approx 0.28$$

b. N = 20, n = 2, m = 15, k = 1

$$P(Y=1) = \frac{\binom{15}{1}\binom{20-15}{2-1}}{\binom{20}{2}} = \frac{\binom{15}{1}\binom{5}{1}}{\binom{20}{2}} = \frac{\left(\frac{15!}{1!(14!)}\right)\left(\frac{5!}{1!(4!)}\right)}{\left(\frac{20!}{2!(18)!}\right)} = \frac{75}{380} \approx 0.20$$

c. The expected number of students selected that are proficient in reading can be determined by finding the mean of the hypergeometric distribution:

$$\mu = \frac{nm}{N} = \frac{2(15)}{20} = 1.5$$

Since we cannot select “half” students, we would expect both students selected to be proficient in reading. This makes intuitive sense, since the majority of the students are proficient in reading.

2.3 This probability can be determined using the hypergeometric distribution:

N = 100, n = 6, m = 48, k = 3

$$P(Y=3) = \frac{\binom{48}{3}\binom{52}{3}}{\binom{100}{6}} = \frac{\left(\frac{48!}{3!(45!)}\right)\left(\frac{52!}{3!(49!)}\right)}{\left(\frac{100!}{6!(94)!}\right)} = \frac{382241600}{1192052400} \approx 0.32$$

2.4 This probability can be determined using the Bernoulli distribution. Two of the five applicants are female, so the proportion of female candidates is 2/5 = 0.4. Therefore, the probability of randomly selecting a female applicant is 0.4.

2.5 This probability can also be determined using the Bernoulli distribution. Fifteen of the twenty second grade students are proficient in reading. Therefore, the probability of randomly selecting one student that is proficient in reading is 15/20 = 0.75.

2.6 This probability can be determined using the Binomial distribution:

n = 10, k = 3, p = 0.67, and q = 0.33

$$P(Y = 3) = \binom{10}{3} 0.67^3 (0.33)^7 \approx 0.02$$

2.7 This probability can be determined using the Binomial distribution:

a. n = 20, k = 15, p = 0.25, and q = 0.75

$$P(Y = 15) = \binom{20}{15} 0.25^{15} (0.75)^5 < 0.001$$

b. The expected number of items the student will correctly answer can be determined by finding the mean of the distribution:

$\mu = np$ = 20(0.25) = 5.

2.8 This probability can be determined using the Binomial distribution and the laws of probability.

n = 5, k = 0, p = 0.4, and q = 0.6

$$P(Y = 0) = \binom{5}{0} 0.4^0 (0.6)^5 \approx 0.08$$

$P(Y \geq 1) = 1 - P(Y = 0) = 1 – 0.08 = .92$

2.9 This probability can be determined using the multinomial distribution:

$n = 5$ $y_1 = 2$ $p_1 = 0.45$ $y_2 = 1$ $p_1 = 0.15$ $y_3 = 3$ $p_1 = 0.40$

$$P(Y_1 = 2, Y_2 = 1, \text{and } Y_3 = 2) = \frac{5!}{2!(1!)(2!)} 0.45^2 (0.15^1)(0.40^2)$$

= 30(0.2025)(0.15)(0.16) = .1458

2.10 These probabilities can be determined using the poisson distribution:

a. $\lambda = 10$ k = 4

$$P(Y = 4) = \frac{e^{-10}10^4}{4!} = \frac{0.454}{24} \approx 0.019$$

b. $\lambda = 10$ k = 8

$$P(Y = 8) = \frac{e^{-10}10^8}{8!} \approx 0.113$$

2.11 This probability can be determined using the Poisson distribution:

$\lambda = 4$ k = 5

$$P(Y = 5) = \frac{e^{-4}4^5}{5!} \approx 0.156$$

2.12 Answers will vary