Chapter 2

Fundamentals of Fluid Mechanics

2.1 Given, $\overline{U} = 25$ m/s, $n = 6$.

The turbulence number is given by

$$n = 100\,\frac{\sqrt{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}}{3\overline{U}}$$

But for isotropic turbulence,

$$\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$$

Therefore,

$$\begin{aligned} n &= 100\,\frac{\sqrt{3\overline{u'^2}}}{3\overline{U}} \\ 6 &= 100\,\frac{\sqrt{3\overline{u'^2}}}{3 \times 25} \\ \sqrt{3\overline{u'^2}} &= \frac{6 \times 3 \times 25}{100} \\ \overline{u'^2} &= \boxed{6.75\,\mathrm{m}^2/\mathrm{s}^2} \end{aligned}$$

2.2 Fluid acceleration is given by Equation (2.21), as

$$\frac{DV}{Dt} = \frac{\partial V}{\partial t} + V_x \frac{\partial V}{\partial x}$$

But $\partial V/\partial t = 0$ for steady flow. Therefore, we have

$$\frac{DV}{Dt} = V_x \frac{\partial V}{\partial x}$$

Thus, there is fluid acceleration.

The volumetric flow rate is given by

$$\begin{aligned}
\dot{Q} &= A\,V_x = e^{-x}\,V_x \\
V_x &= \frac{\dot{Q}}{e^{-x}} \\
\frac{\partial V_x}{\partial x} &= \dot{Q}\,e^{x}
\end{aligned}$$

since, $\dot{Q}$ is a constant. Therefore,

$$\frac{DV}{Dt} = \boxed{\left(\frac{\dot{Q}}{e^{-x}}\right)^2}$$

2.3 Given, $\frac{DT}{Dt} = 0.15°\mathrm{C/s}$ and $\frac{\partial T}{\partial x} = 0.9°\mathrm{C/m}$, $V_x = 0.72\,\mathrm{m/s}$

Using Euler's acceleration relation, Equation (2.20), we have

$$\begin{aligned}
\frac{\partial T}{\partial t} &= \frac{DT}{Dt} - V_x \frac{\partial T}{\partial x} \\
&= 0.15 - (0.72)(0.9) \\
&= \boxed{-\,0.498\,°\mathrm{C/s}}
\end{aligned}$$

Since $\partial T/\partial t$ is independent of x, at 3 m or any other location

$$\frac{\partial T}{\partial t} = -\,0.498°\mathrm{C/s}$$

2.4 Given that, $\left(\frac{DT}{Dt}\right)_{\max} = 0.006\,\mathrm{K/s} = 21.6\,\mathrm{K/hr}$

At $t = 2$ hr, $z = 0$;

$$\begin{aligned}
\frac{\partial T}{\partial t} &= 288\left(-e^{-0.02t}\right)(-0.02) \\
&= 5.534\,\mathrm{K/hr} \\
\frac{\partial T}{\partial z} &= -6.755\,\mathrm{K/km} \\
\frac{DT}{Dt} &= \frac{\partial T}{\partial t} + V_z\,\frac{\partial T}{\partial z}
\end{aligned}$$

The limiting condition on the ascent stipulates that

$$\left|\,5.534 - 6.755\,V_z\,\right| \le 21.6$$

Solving for V_{zmax}, we get

$$\begin{aligned}
6.755\,V_{\mathrm{zmax}} &= 21.6 + 5.534 \\
V_{\mathrm{zmax}} &= 4.017\,\mathrm{km/hr} \\
&= \frac{4.017}{3.6} \\
&= \boxed{1.12\,\mathrm{m/s}}
\end{aligned}$$

2.5 Let S to be the cross-sectional area of the tube and x coordinate is along the tube axis. Thus, the unit vector in the direction normal to the cross-section is $n = i$. The normal component of velocity V is $V \cdot n = u$. Since u varies only with r, the elemental area dA can be taken to be the annular strip $dA = 2\pi r dr$.

(a) The volume flow rate becomes

$$\begin{aligned}\dot{Q} &= \int_S u\,dA \\ &= \int_0^R u_{\max}\left(1-\frac{r^2}{R^2}\right)2\pi r dr \\ &= \boxed{\frac{1}{2}u_{\max}\,\pi R^2}\end{aligned}$$

The average velocity is

$$\begin{aligned}u_{\mathrm{av}} &= \frac{\dot{Q}}{A} \\ &= \frac{\frac{1}{2}u_{\max}\,\pi R^2}{\pi R^2} \\ &= \boxed{\frac{1}{2}u_{\max}}\end{aligned}$$

(b) For $R = 25$ mm and $u_{\max} = 10$ m/s, the volume flow rate is

$$\begin{aligned}\dot{Q} &= \frac{1}{2}(10)\pi(0.025)^2 \\ &= \boxed{0.00982\,\mathrm{m}^3/\mathrm{s}}\end{aligned}$$

(c) For $\rho 1000$ kg/m^3, the mass flow rate through the tube is

$$\begin{aligned}\dot{m} &= \rho\,\dot{Q} \\ &= (1000)(0.00982) \\ &= \boxed{9.82\,\mathrm{kg/s}}\end{aligned}$$

2.6 The acceleration components are given by

$$\begin{aligned} a_x &= u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} \\ &= \left(x - y^2\right) + (xy + 2y)\,(-\,2y) \\ &= (2-1) + (2+2)(-2) \\ &= \boxed{-\,7\,\text{units}} \end{aligned}$$

$$\begin{aligned} a_y &= u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} \\ &= \left(x - y^2\right)(y) + (xy + 2y)\,(x+2) \\ &= (2-1) + (2+2)(4) \\ &= \boxed{17\,\text{units}} \end{aligned}$$

At $(x, y) = (2, 1)$,

$$\begin{aligned} V &= \boxed{1\,i + 4\,j} \\ a &= a_x\,i + a_y\,j \\ &= \boxed{-\,7\,i + 17\,j} \end{aligned}$$

The unit vector along 30° direction is

$$\overline{n} = \cos 30^\circ\,i + \sin 30^\circ\,j = \frac{\sqrt{3}}{2}\,i + \frac{1}{2}\,j$$

Therefore, the component of velocity along $\theta = 30^\circ$ is

$$\begin{aligned} V_{30^\circ} &= V \cdot n_{30^\circ} \\ &= 1\left(\frac{\sqrt{3}}{2}\right) + 4\left(\frac{1}{2}\right) \\ &= \boxed{2.87\,\text{units}} \end{aligned}$$

The maximum velocity is

$$V = \sqrt{1^2 + 4^2} = \boxed{4.123\,\text{units}} \qquad \text{at } 75.96^\circ \text{ from } x\text{-axis}$$

Maximum acceleration is

$$a = \sqrt{7^2 + 17^2} = \boxed{18.385\,\text{units}} \qquad \text{at } 292.38^\circ \text{ from } x\text{-axis}$$

The directions of the maximum velocity and accelerations are as shown in Figure S2.6.

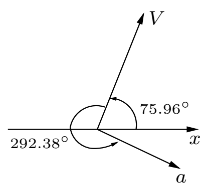

Figure S2.6

2.7 Let the control volume (CV) be as shown in Figure S2.7. The relative velocity of water at CV is $(V_1 + at)$.

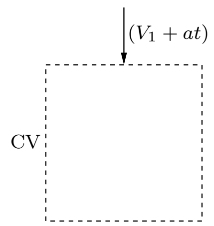

Figure S2.7

By continuity equation, we have

$$
\begin{aligned}
\frac{\partial}{\partial t}(\rho A_2 h) &= \rho (V_1 + at) A_1 \\
\frac{dh}{dt} &= (V_1 + at) \frac{A_1}{A_2} \\
\int_0^H dh &= \frac{A_1}{A_2} \int_0^t (V_1 + at)\, dt \\
H &= \frac{A_1}{A_2} \left(V_1 t + \frac{at^2}{2} \right) \\
t^2 + \frac{2V_1 t}{a} - \frac{2A_2}{A_1 a} H &= 0 \\
t &= \boxed{\frac{-V_1 \pm \sqrt{V_1^2 + 2\frac{A_2}{A_1}\, a\, H}}{a}}
\end{aligned}
$$

2.8 Considering the inflow and outflow along r, θ and z-directions, we have

For r-direction:

$$
\begin{aligned}
\int (\rho V \cdot dA)_{\text{in}} &= -\left[\rho - \left(\frac{\partial \rho}{\partial r}\right)\frac{dr}{2}\right]\left[V_r - \left(\frac{\partial V_r}{\partial r}\right)\frac{dr}{2}\right]\left(r - \frac{dr}{2}\right) d\theta dz \\
&= -\rho V_r r d\theta dz + \rho V_r \frac{dr}{2} d\theta dz + \rho \left(\frac{\partial V_r}{\partial r}\right) r \frac{dr}{2} d\theta dz \\
&\quad + V_r \left(\frac{\partial \rho}{\partial r}\right) r \frac{dr}{2} d\theta dz \\
\int (\rho V \cdot dA)_{\text{out}} &= \left[\rho + \left(\frac{\partial \rho}{\partial r}\right)\frac{dr}{2}\right]\left[V_r + \left(\frac{\partial V_r}{\partial r}\right)\frac{dr}{2}\right]\left(r + \frac{dr}{2}\right) d\theta dz \\
&= \rho V_r r d\theta dz + \rho V_r \frac{dr}{2} d\theta dz + \rho \left(\frac{\partial V_r}{\partial r}\right) r \frac{dr}{2} d\theta dz \\
&\quad + V_r \left(\frac{\partial \rho}{\partial r}\right) r \frac{dr}{2} d\theta dz
\end{aligned}
$$

For θ–direction:

$$
\begin{aligned}
\int(\rho V\cdot dA)_{\text{in}} &= -\left[\rho-\left(\frac{\partial\rho}{\partial\theta}\right)\frac{d\theta}{2}\right]\left[V_\theta-\left(\frac{\partial V_\theta}{\partial\theta}\right)\frac{d\theta}{2}\right]drdz \\
&= -\rho V_\theta drdz+\rho\left(\frac{\partial V_\theta}{\partial\theta}\right)\frac{d\theta}{2}drdz+V_\theta\left(\frac{\partial\rho}{\partial\theta}\right)\frac{d\theta}{2}drdz \\
\int(\rho V\cdot dA)_{\text{out}} &= \left[\rho+\left(\frac{\partial\rho}{\partial\theta}\right)\frac{d\theta}{2}\right]\left[V_\theta+\left(\frac{\partial V_\theta}{\partial\theta}\right)\frac{d\theta}{2}\right]drdz \\
&= \rho V_\theta drdz+\rho\frac{\partial V_\theta}{\partial\theta}\frac{d\theta}{2}drdz+V_\theta\left(\frac{\partial\rho}{\partial\theta}\right)\frac{d\theta}{2}drdz
\end{aligned}
$$

For z-direction:

$$
\begin{aligned}
\int(\rho V\cdot dA)_{\text{in}} &= -\left[\rho-\left(\frac{\partial\rho}{\partial z}\right)\frac{dz}{2}\right]\left[V_z-\left(\frac{\partial V_z}{\partial z}\right)\frac{dz}{2}\right]rd\theta dr \\
&= -\rho V_z rd\theta dr+\rho\left(\frac{\partial V_z}{\partial z}\right)\frac{dz}{2}rd\theta dr+V_z\left(\frac{\partial\rho}{\partial z}\right)\frac{dz}{2}rd\theta dr \\
\int(\rho V\cdot dA)_{\text{out}} &= \left[\rho+\left(\frac{\partial\rho}{\partial z}\right)\frac{dz}{2}\right]\left[V_z+\left(\frac{\partial V_z}{\partial z}\right)\frac{dz}{2}\right]r\,d\theta dr \\
&= \rho V_z rd\theta dr+\rho\left(\frac{\partial V_z}{\partial z}\right)\frac{dz}{2}rd\theta dr+V_z\left(\frac{\partial\rho}{\partial z}\right)\frac{dz}{2}rd\theta dz
\end{aligned}
$$

Therefore,

$$
\int_{cs}\rho V\cdot dA=\int(\rho V.dA)_r+\int(\rho V.dA)_\theta+\int(\rho V\cdot dA)_z
$$

$$
=[\rho\,V_r+r\,\left(\rho\,\left[\frac{\partial V_r}{\partial r}\right]+V_r\left[\frac{\partial\rho}{\partial r}\right]\right)+\left(\rho\left[\frac{\partial V_\theta}{\partial\theta}\right]+V_\theta\left[\frac{\partial\rho}{\partial\theta}\right]\right)
$$

$$
+\,r\,\left(\rho\left[\frac{\partial V_z}{\partial z}\right]+V_z\left[\frac{\partial\rho}{\partial z}\right]\right)]\,dr\,d\theta\,dz
$$

or

$$
\int_{cs}\rho V\cdot dA \;=\; \left[\rho\,V_r+r\frac{\partial(\rho V_r)}{\partial r}+\frac{\partial(\rho V_\theta)}{\partial\theta}+r\frac{\partial(\rho V_z)}{\partial z}\right]dr\,d\theta\,dz
$$

i.e. the net rate of mass flux exiting the control surface is given by

$$\left[\rho\, V_r + r\frac{\partial(\rho V_r)}{\partial r} + \frac{\partial(\rho V_\theta)}{\partial \theta} + r\frac{\partial(\rho V_z)}{\partial z}\right] dr\, d\theta\, dz$$

The mass inside the control volume at any instant of time is the product of the mass per unit volume, ρ and the volume $rdrd\theta dz$. Thus, the rate of change of mass inside the control volume is given by

$$\frac{\partial \rho}{\partial t}\, r\, d\theta\, dr\, dz$$

Thus, the differential form of mass conservation becomes

$$\left[\rho\, V_r + r\frac{\partial(\rho V_r)}{\partial r} + \frac{\partial(\rho V_\theta)}{\partial \theta} + r\frac{\partial(\rho V_z)}{\partial z}\right] dr\, d\theta\, dz + \frac{\partial \rho}{\partial t}\, r\, d\theta\, dr\, dz = 0$$

or

$$\boxed{\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial(\rho r V_r)}{\partial r} + \frac{1}{r}\frac{\partial(\rho V_\theta)}{\partial \theta} + \frac{\partial(\rho V_z)}{\partial z} = 0}$$

Note: The elemental control surface areas are:

$(rd\theta dz)$ — normal to r-direction

$(drdz)$ — normal to θ-direction

$(d\theta dr)$ — normal to z-direction

For mass conservation,

$$\frac{\partial \rho}{\partial t} d(\text{volume}) + \sum(\text{net outflow of mass}) = 0$$

Considering the density at the centre of the CV as ρ and the velocity there as

$$V = i_r V_r + i_\theta V_\theta + i_z V_z$$

where i_r, i_θ, i_z are unit vectors in the r, θ and z directions, respectively, and V_r, V_θ, V_z are the velocity components in the r, θ and z directions, respectively. To evaluate $\int_{cs} \rho V \cdot dA$, we should account for the mass flux through each of the six faces of the control surface. The properties at each of the six faces of the control surface can be obtained from a Taylor's series expansion about the centre of the CV.

2.9 (a) Velocity at $(10, 6)$ and $t = 3$ s is

$$\boxed{V = (30\,i + 24\,j - 15\,k)\,\text{m/s}}$$

(b) At $t = 0$, the slope of the streamlines is

$$\boxed{\frac{dy}{dx} = \frac{4y}{3x}}$$

(c) The equation of the streamlines at $t = 0$ and passing through the point (10, 6), becomes

$$\begin{aligned}
\frac{dy}{dx} &= \frac{4y}{3x} \\
&= \frac{24}{30} \\
\frac{dx}{5} &= \frac{dy}{4}
\end{aligned}$$

Integrating this we get

$$\boxed{y = \frac{4}{5}\,x + c}$$

where c is an arbitrary constant.

(d) At $t = 0$, the streamlines are straight lines, at an angle of 38.66° to the x-axis.

2.10 By continuity equation, we have

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} = 0$$

Since the plates are infinitely long in z-direction, we have $z \to \infty$ and hence

$$\frac{\partial V_z}{\partial z} = 0$$

Also, for fully developed flow

$$\frac{\partial V_x}{\partial x} = 0$$

Thus, the continuity equation reduces to

$$\frac{\partial V_y}{\partial y} = 0$$

This implies that,

$$V_y = \text{constant}$$

Also, at the impervious plate surface, $V_y = 0$, therefore, $\boxed{V_y = 0}$ everywhere.

2.11 Assuming the fluid to be water, at the exit, the weight flow rate becomes

$$\begin{aligned}
\dot{W} &= \rho g A_2 u_2 = \rho g A_1 u_1 \\
u_1 &= \frac{200}{9.81 \times 10^3} \frac{1}{(\pi/4)\,(0.1)^2} \\
&= 2.6\,\mathrm{m/s} \\
u_2 &= \frac{200}{9.81 \times 10^3} \frac{1}{(\pi/4)\,(0.06)^2} \\
&= 7.21\,\mathrm{m/s}
\end{aligned}$$

Thus,

$$\begin{aligned}
u_{m1} &= \frac{2.6}{0.5} \\
&= \boxed{5.2\,\mathrm{m/s}} \\
u_{m2} &= \frac{7.21}{0.82} \\
&= \boxed{8.79\,\mathrm{m/s}}
\end{aligned}$$

2.12 From momentum theorem, the force F must be equal to the rate of change of momentum normal to the plate surface.

That is,

$$F = \boxed{\rho\, V_0\, \dot{Q}_0 \sin\alpha} \tag{1}$$

where $\dot{Q}_0 = V_0\, A_0$ and A_0 is the cross–sectional area of the jet.

By momentum theorem, the force balance parallel to the plate can be written as

$$\left(\rho\, \dot{Q}_1\, V_1 - \rho\, \dot{Q}_2\, V_2\right) - \rho\, V_0\, \dot{Q}_0 \cos\alpha = 0 \tag{2}$$

But $V_1 = V_2 = V_0$, therefore, Eq. (2) becomes

$$\dot{Q}_0 \cos\alpha = \dot{Q}_1 - \dot{Q}_2 \tag{3}$$

Also, by continuity,

$$\dot{Q}_0 = \dot{Q}_1 + \dot{Q}_2 \tag{4}$$

From Eqs. (3) and (4), we get

$$\dot{Q}_1 = \boxed{\frac{\dot{Q}_0}{2}\,(1 + \cos\alpha)}$$

$$\dot{Q}_2 = \boxed{\frac{\dot{Q}_0}{2}\,(1 - \cos\alpha)}$$

2.13 Consider the control volume shown in Figure 2.39. At any radial position, the equilibrium condition can be expressed by

$$\tau_{rz}\,(2\pi r)\, l = (p_1 - p_2)\; \pi\, r^2$$

$$\tau_{rz} = \boxed{\left(\frac{p_1 - p_2}{l}\right)\frac{r}{2}}$$

Given that,

$$\tau_{rz} = -\,\mu\left(\frac{dV_z}{dr}\right) = \left(\frac{p_1 - p_2}{l}\right)\frac{r}{2}$$

Therefore,

$$V_z = -\left(\frac{p_1 - p_2}{l}\right)\frac{1}{2\mu}\frac{r^2}{2} + \text{constant}$$

At $r = R$, $V_z = 0$, by no slip condition, thus we have,

$$V_z = \boxed{\left(\frac{p_1 - p_2}{l}\right)\frac{1}{4\mu}\left(R^2 - r^2\right)}$$

2.14 (a) The x-component of the Navier-Stokes equation for this case is zero, since $V_x = 0$.

The y-component of the Navier-Stokes equation becomes

$$0 = \rho\, g + \mu\, \frac{d^2 V_y}{dx^2}$$

Integrating, we get

$$\mu\, \frac{dV_y}{dx} + \rho\, g\, x = \text{constant} = c$$

But, at $x = h$,

$$\mu\, \frac{dV_y}{dx} = \tau$$

Thus, the constant becomes

$$c = \rho\, g\, h - \tau$$

Hence,

$$\mu\, \frac{dV_y}{dx} = \rho\, g\, (h - x) - \tau$$

Integrating this, we get

$$\mu V_y = \rho g \left(hx - \frac{x^2}{2}\right) - \tau x + \text{constant} \tag{1}$$

But at $x = 0$, $V_y = 0$, therefore, constant $= 0$. Thus,

$$V_y = \boxed{\frac{\rho g \left(hx - \frac{x^2}{2}\right) - \tau x}{\mu}}$$

(b) Integrating the Eq. (1) once more, we get

$$\frac{\mu^2 {V_y}^2}{2\tau} = \rho g \left(\frac{hx^2}{2} - \frac{x^3}{6}\right) - \frac{\tau x^2}{2} + c_3$$

The constant $c_3 = 0$ with the condition that at $x = 0$, $V_y = 0$.

Also, the volume flow rate $hV_y = 0$, since $V_y = 0$. Therefore,

$$\begin{aligned}
\rho g \left(\frac{h^3}{2} - \frac{h^3}{6}\right) - \frac{\tau h^2}{2} &= 0 \\
\frac{\tau h^2}{2} &= \frac{\rho g h^3}{3} \\
\tau &= \boxed{\frac{2}{3}\rho\, g\, h}
\end{aligned}$$

2.15 The volume flow rate through the pipe is

$$\begin{aligned}
\mathbb{V} &= \int_A V \cdot dA = \int_0^R u\, 2\pi r dr \\
&= \int_0^R \frac{1}{4\,\mu}\left(\frac{\partial p}{\partial x}\right)\left(r^2 - R^2\right) 2\pi r dr
\end{aligned}$$

since for pipe flow the local velocity u is given by

$$u = -\frac{R^2}{4\mu}\left(\frac{\partial p}{\partial x}\right)\left[1 - \left(\frac{r}{R}\right)^2\right]$$

where R is the pipe radius, x is the axial coordinate and r is the radial coordinate. Thus,

$$\mathbb{V} = -\frac{\pi R^4}{8\mu}\left(\frac{\partial p}{\partial x}\right)$$

In fully developed flow, the pressure gradient, $\partial p/dx$, is constant.

Therefore,

$$\frac{\partial p}{\partial x} = \frac{p_2 - p_1}{L} = -\frac{\Delta p}{L}$$

Therefore,

$$\begin{aligned}
\mathbb{V} &= -\frac{\pi R^4}{8\mu}\left[-\frac{\Delta p}{L}\right] \\
&= \frac{\pi \Delta p R^4}{8\mu L} = \frac{\pi \Delta p D^4}{128\mu L}
\end{aligned}$$

for laminar flow in a horizontal pipe. This gives,

$$\begin{aligned}
\Delta p &= \frac{128\mu L \mathbb{V}}{\pi D^4} \\
&= \frac{128\mu L \overline{V}\left(\frac{\pi D^2}{4}\right)}{\pi D^4} = 32\frac{L}{D}\frac{\mu \overline{V}}{D}
\end{aligned}$$

The head loss h_l is

$$h_l = \frac{p_1 - p_2}{\rho g} + (z_1 - z_2)$$

For horizontal pipe $z_1 = z_2$, thus,

$$\begin{aligned}
h_l &= \frac{p_1 - p_2}{\rho g} = \frac{\Delta p}{\rho g} \\
&= 32\frac{L}{D}\frac{\mu}{\rho g}\frac{\overline{V}}{D} = \frac{L}{D}\frac{\overline{V}^2}{2g}\left(64\frac{\mu}{\rho \overline{V} D}\right) \\
&= \boxed{\frac{64}{\mathrm{Re}}\frac{L}{D}\frac{\overline{V}^2}{2g}}
\end{aligned}$$

where $\overline{V} = V_{\mathrm{av}}$.

2.16 The pressure drop through the pipe is given by

$$p_1 - p_2 = \frac{\rho_{\mathrm{ave}}}{2}V^2\frac{L}{D}f$$

But by state equation, $p_1/\rho_1 = p_2/\rho_2$. Therefore, $p_2 = (\rho_2/\rho_1)\,p_1$. Thus,

$$p_1 - \frac{\rho_2}{\rho_1}p_1 = \frac{\rho_{\mathrm{ave}}}{2}V^2\frac{L}{D}f$$

or

$$\frac{p_1}{\rho_1}(\rho_1 - \rho_2) = \frac{\rho_{\text{ave}}}{2} V^2 \frac{L}{D} f \tag{1}$$

The mass flow rate is given by

$$\dot{m} = \rho_{\text{ave}} A V = \frac{\rho_1 + \rho_2}{2} \frac{\pi D^2}{4} V$$

Therefore,

$$V = \frac{8\dot{m}}{\pi D^2} \frac{1}{\rho_1 + \rho_2}$$

Substituting this V into Eq. (1), we get

$$\begin{aligned}
\frac{p_1}{\rho_1}(\rho_1 - \rho_2) &= \frac{\rho_{\text{ave}} f}{2} \frac{L}{D} \frac{64\,\dot{m}^2}{\pi^2 D^4} \frac{1}{(\rho_1 + \rho_2)^2} \\
\frac{p_1}{\rho_1}(\rho_1^2 - \rho_2^2) &= \boxed{\frac{16\, f\, L\, \dot{m}^2}{\pi^2\, D^5}}
\end{aligned}$$

2.17 The flow outside the boundary layer can be assumed to be isentropic. By energy equation, we have

$$h_A + \frac{V_A^2}{2} = h_B + \frac{V_B^2}{2}$$

where h_A and h_B are the static enthalpy at A and B, respectively, V_A and V_B are the corresponding velocities. Treating air to be a perfect gas, we have $h = c_p T$. Thus, the energy equation becomes

$$\begin{aligned}
c_p T_A + \frac{V_A^2}{2} &= c_p T_B + \frac{V_B^2}{2} \\
T_B &= T_A + \frac{V_A^2}{2c_p} - \frac{V_B^2}{2c_p} \\
&= 288 + \frac{1}{2 \times 1004.5}\left[\left(\frac{250}{3.6}\right)^2 - \left(\frac{470}{3.6}\right)^2\right] \\
&= \boxed{281.9\,\mathrm{K}}
\end{aligned}$$

since $c_p = 1004.5$ m^2/(s^2 K), for air. The Mach number at B is

$$\begin{aligned} M_B &= \frac{V_B}{a} \\ &= \frac{V_B}{\sqrt{\gamma R T_B}} \\ &= \frac{470/3.6}{\sqrt{1.4 \times 287 \times 281.9}} \\ &= \boxed{0.388} \end{aligned}$$

2.18 The velocity at any point on the cylinder surface can be expressed, by Equation (2.51), as

$$V = -\left(2u\sin\theta + \frac{\Gamma}{2\pi a}\right)$$

where Γ is the circulation and θ varies from 0 to 2π. Let the top of the cylinder is at $\theta = \pi/2$. Therefore, the bottom is at $\theta = 3\pi/2$. Thus, the velocities at the top and bottom of the cylinder are

$$\begin{aligned} V_{\text{top}} &= -\left(2u + \frac{\Gamma}{2\pi a}\right) \\ V_{\text{bottom}} &= -\left(-2 + \frac{\Gamma}{2\pi a}\right) \end{aligned}$$

By Bernoulli equation, we have

$$p_{\text{top}} + \frac{1}{2}\rho V_{\text{top}}^2 = p_{\text{bottom}} + \frac{1}{2}\rho V_{\text{bottom}}^2$$

Therefore,

$$\begin{aligned} p_{\text{top}} - p_{\text{bottom}} &= \frac{1}{2}\rho\left(V_{\text{bottom}}^2 - V_{\text{top}}^2\right) \\ &= \frac{1}{2}\rho\left[-4u\,\frac{\Gamma}{2\pi a} - 4u\,\frac{\Gamma}{2\pi a}\right] \\ &= -\frac{2}{\pi a}\,\rho u \Gamma \end{aligned}$$

The circulation if given by

$$\Gamma = \zeta \times A$$

where ζ is the vorticity and also, $\zeta = 2\omega$.

Thus,

$$\Gamma = 2\omega\pi a^2$$

Therefore,

$$\begin{aligned}
\frac{p_{\text{top}} - p_{\text{bottom}}}{\frac{1}{2}\rho u^2} &= -\frac{2}{\pi a}\rho u \frac{(2\omega\pi a^2)}{\frac{1}{2}\rho u^2} \\
&= \boxed{-\frac{8a\omega}{u}}
\end{aligned}$$

2.19 The relation between the local and material rates of change is given by

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + V \cdot \bigtriangledown$$

Therefore,

$$\begin{aligned}
\frac{DT}{Dt} &= \frac{\partial T}{\partial t} + V_x \frac{\partial T}{\partial x} + V_y \frac{\partial T}{\partial y} + V_z \frac{\partial T}{\partial z} \\
&= 5 + (x)[1] + \left(3y + 3t^2 y\right)[2yz] + 12\left[y^2\right]
\end{aligned}$$

At (3,5,2) and t = 2,

$$\begin{aligned}
\frac{DT}{Dt} &= 5 + 3 \times 1 + [3 \times 5 + 3 \times 4 \times 5](2 \times 5 \times 2) + 12 \times 5 \times 5 \\
&= \boxed{1808}
\end{aligned}$$

2.20 Given, $d = 10$ m, $V = 5.5$ m/s, $p = 10^5$ Pa, $T = 18 + 273.15 = 291.15$ K, $C_D = 1.4$.

The air density is

$$\begin{aligned}\rho &= \frac{p}{RT}\\ &= \frac{10^5}{287 \times 291.15}\\ &= 1.197\,\mathrm{kg/m^3}\end{aligned}$$

The drag experienced by the parachute is

$$\begin{aligned}D &= \frac{1}{2}\rho V^2 S C_D\\ &= \frac{1}{2} \times 1.197 \times 5.5^2 \times \left(\frac{\pi\, D^2}{4}\right) \times 1.4\\ &= \frac{1}{2} \times 1.197 \times 5.5^2 \times \left(\frac{\pi\, 10^2}{4}\right) \times 1.4\\ &= 1990.7\,\mathrm{N}\end{aligned}$$

At steady descend, $D = W$, thus

$$W = \boxed{1.991\,\mathrm{kN}}$$