Chapter 2

2.1 Discuss the dichotomy of rocket science in the modern era.

In fact, modern rocketry had two starts. The first major start could be traced to the Treaty of Versailles that officially ended World War I in 1919. The treaty was between the Allied and Central Powers and the German Empire. Among many things this Treaty would prevent Germany from being able to develop long-range artillery technology. From that point on the Germans became very interested in developing rocket technology to take the place of the long-range artillery. It was the impact of the Treaty of Versailles that sparked the V2 missile development and successful launches. World War II saw over 3000 V2 missile launches by the Germans. The success of the V2 led the Americans and the Soviets to long-range missile development efforts of their own that continued throughout the Cold War.

The second part of the dichotomy of modern rocketry development was sparked by the launch of Sputnik and the advent of the space race between the Americans and the Soviets. While the missile development efforts improved the rapid launch technologies, guidance and control, and throw weight versus range capabilities the space race led the development of rocketry toward placing payloads into orbit and even safely returning them. The space race added an element from a scientific curiosity standpoint in that science teams began seeing rockets as a means for sending payloads to orbit, deep space, and even extra-terrestrial bodies such as the Moon, Venus, and Mars. The combination of these closely coupled, yet parallel, efforts is what led to the modern era of rocketry.

2.2 In your own words give a definition for a rocket mission.

The mission is the reason for conducting the rocket flight. It usually includes the need to have some payload reach a particular mission location, velocity, or other requirement that only the rocket can manage.

2.3 What is a payload?

The payload is really the means for which a mission can be accomplished. In other words, the payload truly is the “means to an end” for the mission.

2.4 What is the so-called “smad”?

Space Mission Analysis and Design, Third Edition (there are later editions now available and the book is known as the SMAD pronounced “smad”) edited by James R. Wertz and Wiley J. Larson

2.5 Give the four basic assumptions required for understanding the basics of projectile motion.

1. acceleration due to gravity is assumed constant
2. neglect air resistance
3. assume the Earth is flat
4. assume the Earth’s rotation has no impact on the motion of the projectile.

2.6 Define MECO.

Main Engine Cut-off

2.7 Equation 2.9 gives the parabolic flight path of a rocket trajectory as height, *y*, as a function of range, *x*, or *y(x).* Use the quadratic equation to solve for *x* as a function of y to give a range equation as a function of height.

Starting with Equation 2.9

$$y(x) = y_{bo} + (\tan\theta)x - \left(\frac{g}{2v_{bo}^2\cos^2\theta}\right)x^2 .$$

Rewrite the above equation as

$$0 = y_{bo} - y + (\tan\theta)x - \left(\frac{g}{2v_{bo}^2\cos^2\theta}\right)x^2 ,$$

Let

$$a = \frac{g}{2v_{bo}^2\cos^2\theta}$$
$$b = \tan\theta$$
$$c = y_{bo} - y$$

Then using the quadratic formula results in the following

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-\tan\theta \pm \sqrt{(\tan\theta)^2 - 4\frac{g}{2v_{bo}^2\cos^2\theta}(y_{bo} - y)}}{\frac{g}{v_{bo}^2\cos^2\theta}} .$$

2.8 A rocket is launched with a burnout velocity of 75 m/s, burnout altitude of 300 m, and a burnout range of 100 m. Assuming a flight path angle of 75° calculate the final range of the rocket when it impacts the ground.

Use Equation 2.14 from the text:

$$x_{\max} = x_{bo} + (v_{bo}\cos\theta)\left[\frac{v_{bo}\sin\theta + \sqrt{v_{bo}^2\sin^2\theta + 2gy_{bo}}}{g}\right]$$

$$= 100\text{m} + (75\text{m/s})(\cos(75°))\left[\frac{(75\text{m/s})(\sin(75°)) + \sqrt{(75\text{m/s})(\sin(75°))^2 + 2(9.8\text{m/s}^2)300\text{m}}}{9.8\text{m/s}^2}\right]$$

$$= 452.45\text{m}$$

2.9 Calculate the maximum altitude reached by the rocket in Exercise 2.8.

Use Equation 2.11 from the text:

$$y_{\max} = y_{bo} + \frac{v_{bo}^2\sin^2\theta}{2g}$$

$$= 300m + \frac{(75m/s)^2(\sin(75°))^2}{2(9.8m/s^2)}$$

$$= 567.77m$$

2.10 Redo Exercise 2.8 to determine the range at MECO altitude. What is the range at MECO if the initial flight path angle is 15°?

Use Equation 2.14 from the text with x_{bo}=0:

$$x_{\max} = x_{bo} + (v_{bo}\cos\theta)\left[\frac{v_{bo}\sin\theta + \sqrt{v_{bo}^2\sin^2\theta + 2gy_{bo}}}{g}\right]$$

$$= 0\text{m} + (75\text{m/s})(\cos(75°))\left[\frac{(75\text{m/s})(\sin(75°)) + \sqrt{(75\text{m/s})(\sin(75°))^2 + 2(9.8\text{m/s}^2)300\text{m}}}{9.8\text{m/s}^2}\right]$$

$$= 352.45\text{m}$$

Since 15 and 75 degrees are complementary angles the maximum range at MECO are the same for both angles.

2.11 What is the force due to gravitational attraction between the Earth and the Moon? Assume the Moon is 400,000 km from Earth and the mass of the Earth is 5.99 x 10^{24} kg, and the mass of the Moon is 7.36 x 10^{22} kg.

Use Equation 2.15 from text:

$$F = \frac{Gm_1m_2}{r^2}$$
$$= 6.67x10^{-11}\frac{Nm^2}{kg^2}\frac{(5.99x10^{24}kg)(7.36x10^{22}kg)}{(400{,}000{,}000m)^2}$$
$$= 1.838x10^{20}N$$

2.12 A satellite is in a circular orbit at 100 km above the Earth. What is the orbital velocity of the satellite? How long does it take for the satellite to make one complete orbit around the Earth?

Use Equation 2.73 from the text:

$$v_{circ}(r) = \sqrt{\frac{\mu}{r}}$$
$$= \sqrt{\frac{398{,}600\text{km}^3/\text{s}^2}{(100+6370)\text{km}}}$$
$$= 7.85\text{km/s}$$

2.13 What is the semi-latus rectum?

p = semi-latus rectum which is the distance from the focus of the ellipse to the ellipse itself

2.14 Give the equation for a conic section.

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

2.15 A spacecraft is traveling in an orbit with periapsis at 100 km and apoapsis at 1000 km. What is the eccentricity of the orbit? This orbit is what type of conic section?

Use Equation 2.44 from the text:

$$e = \sqrt{1 - \frac{b^2}{a^2}} = \frac{r_a - r_p}{r_a + r_p}$$
$$= \frac{1000 - 100}{1000 + 100}$$
$$= 0.818$$

This is an ellipse since $0 < e < 1$.

2.16 Calculate the semi-latus rectum of the spacecraft orbit in Exercise 2.15.

Use Equation 2.45 from the text and Figure 2.11 where it is shown that $2a = r_a + r_p$ then:

$$p = a(1 - e^2)$$
$$= \frac{(100 + 6370 + 1000 + 6370)}{2}\text{km}(1 - 0.669)$$
$$= 2290.5\text{km}$$

Refer back to Figure 2.11 and note that the semi-latus rectum is inside the planet so this orbit will not work!

2.17 What is the period of the orbit described in Exercise 2.15?

First we realize from Exercise 2.16 that the orbit will not work because it would have to pass through the Earth at the semi-latus rectum. But if we ignore that the period would be found as below. Use Equation 2.51 where *r* = *a*:

$$T^2 = \frac{4\pi^2 r^3}{GM_{Earth}}$$

$$T = \sqrt{\frac{4\pi^2 a^3}{GM_{Earth}}} = \sqrt{\frac{4\pi^2\left(\frac{(100{,}000 + 6{,}370{,}000)\text{m} + (1{,}000{,}000 + 6{,}370{,}000)}{2}\text{m}\right)^3}{6.67x10^{-11}\,\frac{\text{Nm}^2}{\text{kg}^2}\left(5.99x10^{24}\,\text{kg}\right)}}$$

$$T = 5719.3\text{s} = 1.6\text{hours}$$

2.18 What is the velocity of the spacecraft in Exercise 2.15?

We must use the *vis viva* equation of the form in Equation 2.71b:

$$v(r) = \sqrt{\frac{2\mu}{r} - \frac{\mu}{a}}$$

$$= \sqrt{\frac{2\left(398{,}600\ \text{km}^3/\text{s}^2\right)}{r} - \frac{398{,}600\ \text{km}^3/\text{s}^2}{(6920)\,\text{km}}}$$

We must realize here that this is as far as we can go because the velocity of the spacecraft in the elliptical orbit is different for different values of r along the orbit. The fastest point will be at r_p and the slowest at r_a.

2.19 Calculate the Δv needed to circularize an elliptical orbit with an apoapsis at 500 km above the Earth and a periapsis at 325 km above the Earth. (Hint: see Example 2.4)

The solution is the second half of Example 2.4 and is found as:

$$\begin{aligned}
\Delta v &= \sqrt{\frac{2\mu}{r_a + r_p} - \frac{\mu}{r_a}} \\
&= \sqrt{\frac{2\left(398{,}600\ \text{km}^3/\text{s}^2\right)}{(500+6370)\text{km} + (325+6370)\text{km}} - \frac{398{,}600\ \text{km}^3/\text{s}^2}{(500+6370)\ \text{km}}} \\
&= 2.076\ \text{km/s}
\end{aligned}$$

2.20 Calculate the Δv burns needed to conduct a Hohmann transfer from a 300 km circular orbit around Earth to a 35,000 km circular orbit around Earth.

Follow the same procedure in Example 2.4.

Burn #1:

$$\Delta v = \sqrt{\frac{\mu}{r_p} - \frac{2\mu}{r_a + r_p}}$$

$$= \sqrt{\frac{398{,}600\ \text{km}^3/\text{s}^2}{(300 + 6370)\ \text{km}} - \frac{2(398{,}600\ \text{km}^3/\text{s}^2)}{(300 + 6370)\text{km} + (35{,}000 + 6370)\text{km}}}$$

$$= 6.57\ \text{km/s}$$

Burn #2:

$$\Delta v = \sqrt{\frac{2\mu}{r_a + r_p} - \frac{\mu}{r_a}}$$

$$= \sqrt{\frac{2(398{,}600\ \text{km}^3/\text{s}^2)}{(35{,}000 + 6370)\text{km} + (300 + 6370)\text{km}} - \frac{398{,}600\ \text{km}^3/\text{s}^2}{(35{,}000 + 6370)\ \text{km}}}$$

$$= 2.64\ \text{km/s}$$

2.21 Calculate the transfer time for the Hohmann transfer given in Exercise 2.20.

Use Equation 2.83:

$$T = \sqrt{\frac{4\pi^2\ \text{a}^3}{\mu}} = \sqrt{\frac{4\pi^2\ (24020\ \text{km})^3}{398600\ \text{km}^3 / s^2}} = 37{,}029\ \text{s} = 10.3\text{hours}$$

2.22 A Space Shuttle is in a 325 km circular orbit in a 28° inclination. How much Δv is needed to move the shuttle to a 51° inclination?

Use Equation 2.84:

$$\Delta v_{plane\ change} = 2v\sin\left(\frac{\Delta i}{2}\right)$$

Since the Shuttle is in a circular orbit then

$$v_{circ}(r) = \sqrt{\frac{\mu}{r_p}} = \sqrt{\frac{398{,}600\,\text{km}^3/\text{s}^2}{(325+6370)\,\text{km}}} = 7.7\text{km/s}$$

Then

$$\Delta v_{plane\ change} = 2(7.7\text{km/s})\sin\left(\frac{51^\circ - 28^\circ}{2}\right) = 3.1\text{km/s}$$

2.23 What is C3?

C_3 is called the *characteristic energy* and sometimes the *launch energy*. It is equal to the square of the *hyperbolic excess velocity*. Quite often orbital mechanics are heard discussing the "see three" of a mission profile and it is this characteristic energy to which they are referring. *C_3* is a measure of the amount of speed a spacecraft needs to lose before it can achieve an orbit around a particular planet. It is also a measure of the amount of speed a spacecraft must gain in order to leave a circular orbit and achieve escape velocity.

2.24 A Mars probe leaves Earth's sphere of influence with a C3 of 16 km^2/s^2. How much Δv is required for the probe to enter a Mars orbit with periapsis at 100 km and apoapsis at 1000 km?

First we must realize that the coasting velocity of the probe is the square root of the C3 before it reaches the Martian sphere of influence. So, the velocity as the probe enters Mars space is 4 km/s. Use 3394 km for the radius of Mars and μ_{mars} = 42,828 km^3/s^2.

$$\Delta v = \sqrt{C_3 - \frac{2\mu}{r_a + r_p}}$$

$$= \sqrt{16\ \text{km}^2/\text{s}^2 - \frac{2\left(42{,}828\ \text{km}^3/\text{s}^2\right)}{(1000 + 3394)\text{km} + (100 + 3394)\text{km}}}$$

$$= 2.27\ \text{km/s}$$

2.25 In order to go from Equation 2.88 to 2.89 (as well as 2.90 and 2.91) some algebra was needed. Do this algebra showing all steps.

The first trick is to rewrite Equation 2.88 as a momentum equation and add the momentum of the ball being thrown from the boat.

$$m_{boat} v_{boat\ final} = m_{ball+boat} v_{ball+boat} + m_{ball} v_{ball}$$

$$= (m_{ball+boat}) \frac{m_{ball} v_{ball} + m_{boat} v_{boat}}{m_{ball+boat}} + m_{ball} v_{ball}$$

$$= m_{ball} v_{ball} + m_{boat} v_{boat} + m_{ball} v_{ball}$$

$$= 2 m_{ball} v_{ball} + m_{boat} v_{boat}$$

Divide through by m_{boat}

$$v_{boat\ final} = \frac{2 m_{ball} v_{ball} + m_{boat} v_{boat}}{m_{boat}}$$

$$v_{boat\ final} = \frac{2 m_{ball} v_{ball}}{m_{boat}} + v_{boat}$$

2.26 A ballistic missile has a powered flight range angle of 4° and a re-entry range angle of 5°. If the missile has a total ground range of 8000 km, what is its free-flight range angle? (Hint: assume the 8000 km range is the distance the missile travels around the circumference of the Earth. The radius of the Earth is 6370 km.)

We first must realize that the free-flight range angle is the arc distance covered by the missile divided by the circumference of the Earth. Therefore,

$$\Psi = \frac{8{,}000\text{km}}{2\pi r_{Earth}} = \frac{8{,}000\text{km}}{40{,}003.6\text{km}} = 0.2\text{rad}$$

$$= 0.2\text{rad}\left(\frac{180°}{\pi\text{rad}}\right) = 11.46°$$