CHAPTER 2 MATERIALS

SOLUTION (**2.1**)

$$A_0 = \tfrac{\pi}{4}(12.5)^2 = 122.7 \text{ mm}^2, \quad A_f = \tfrac{\pi}{4}(12.5-0.006)^2 = 122.6 \text{ mm}^2$$

We have $\varepsilon_a = \frac{0.3}{200} = 1500\ \mu, \quad \varepsilon_t = \frac{0.006}{12.5} = 480\ \mu$

Thus

$$S_p = \frac{P}{A_0} = \frac{18(10^3)}{122.7} = 146.8 \text{ MPa}$$

$$E = \frac{S_p}{\varepsilon_a} = \frac{146.7(10^6)}{1500(10^{-6})} = 97.8 \text{ GPa}, \qquad \nu = \frac{\varepsilon_t}{\varepsilon_a} = 0.32 \quad ◄$$

Also

$$\%\ elongation = \tfrac{0.3}{200}(100) = 0.15$$

$$\%\ reduction\ \ in\ area = \tfrac{122.7-122.6}{122.7}(100) = 0.082 \quad ◄$$

SOLUTION (**2.2**)

Normal stress is

$$\sigma = \frac{P}{A} = \frac{2200}{\frac{\pi}{4}(3.125)^2} = 286.8 \text{ MPa}$$

This is below the yield strength of 350 MPa (Table B.1).
We have

$$\varepsilon = \frac{\delta}{L} = \frac{7.5}{5600} = 0.001339 = 1339\ \mu$$

Hence

$$E = \frac{\sigma}{\varepsilon} = \frac{286.8(10^6)}{1339(10^{-6})} = 214.2 \text{ GPa} \quad ◄$$

SOLUTION (**2.3**)

The cross-sectional area: $A = w_o t_o = 12.7(6.1) = 77.47 \text{ mm}^2$

(**a**) Axial strain and axial stress are

$$\varepsilon_a = \frac{0.0841}{63.5} = 0.001324 = 1324\ \mu$$

$$\sigma_a = \frac{P}{A} = \frac{21,500}{77.47(10^{-6})} = 277.5 \text{ MPa}$$

Because $\sigma_a < S_y$ (See Table B.1), Hooke's Law is valid.

(**b**) Modulus of elasticity,

$$E = \frac{\sigma_a}{\varepsilon_a} = \frac{277.5(10^6)}{1324(10^{-6})} = 209.6 \text{ GPa}$$

(**c**) Decrease in the width and thickness

$$\Delta w = \nu w_o = 0.3(12.7) = 3.81 \text{ mm} \quad ◄$$

$$\Delta t = \nu t_o = 0.3(6.1) = 1.83 \text{ mm}$$

SOLUTION (**2.4**)

Assume Hooke's Law applies. We have

$$\varepsilon_t = -\tfrac{1.5}{5} = -300\ \mu$$

$$\varepsilon_a = -\tfrac{\varepsilon_t}{\nu} = -\tfrac{-300}{0.34} = 822\ \mu$$

Thus,

$$\sigma = E\varepsilon_a = (105\times 10^9)(822\times 10^{-9}) = 92.61\ \mathrm{MPa}$$

Since $\sigma < S_y$, our assumption is valid.

So

$$P = \sigma A = (92.61)(\pi/4)(5)^2 = 1.818\ \mathrm{kN} \quad ◄$$

SOLUTION (**2.5**)

We obtain

$$L_{AC} = L_{BD} = \sqrt{15^2 + 15^2} = 21.21\ \mathrm{mm}$$

$$\varepsilon_x = \tfrac{\Delta L_{AC}}{L_{AC}} = \tfrac{21.17 - 21.21}{21.21} = -1886\ \mu$$

$$\varepsilon_y = \tfrac{\Delta L_{BD}}{L_{BD}} = \tfrac{21.22 - 21.21}{21.21} = 471\ \mu$$

(**a**) $E = \frac{\sigma_x}{\varepsilon_x} = \frac{100\,(10^6)}{-1886\,(10^{-6})} = 53\ \mathrm{GPa}$ ◄

(**b**) $\nu = \frac{\varepsilon_y}{\varepsilon_x} = \left|\frac{471}{1886}\right| = 0.25$ ◄

(**c**) $G = \frac{53}{2(1+0.25)} = 21.2\ \mathrm{GPa}$ ◄

SOLUTION (**2.6**)

Use generalized Hooke’s law:

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \tfrac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) \qquad (1)$$

For a constant triaxial state of stress:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = \varepsilon, \quad \sigma_x = \sigma_y = \sigma_z = \sigma$$

Then, Eq. (1) becomes $\varepsilon = \frac{1-2\nu}{E}\sigma$. Since σ and ε must have identical signs:

$$1 - 2\nu \geq 0 \quad \text{or} \quad \nu = \tfrac{1}{2} \quad ◄$$

SOLUTION (**2.7**)

We have $\sigma_x = \frac{450(10^3)}{50(75)} = 120\ \mathrm{MPa}$

(CONT.)

2.7 (CONT.)

(a) $\varepsilon_x = \frac{0.5}{250} = 2000\ \mu, \quad \varepsilon_y = -\frac{0.025}{50} = -500\ \mu$

$\nu = \left|\frac{500}{2000}\right| = 0.25$ ◄

(b) $E = \frac{\sigma_x}{\varepsilon_x} = \frac{120(10^6)}{2000(10^{-6})} = 60\ \text{GPa}$ ◄

(c) $\varepsilon_z = -\frac{\nu\sigma_x}{E} = -0.25\frac{120(10^6)}{60(10^9)} = -500\ \mu$

$\Delta a = -500(10^{-6})75 = -37.5(10^{-3})\ \text{mm}; \qquad a' = 75 - 0.0375 = 74.9625\ \text{mm}$ ◄

(d) $G = \frac{60(10^9)}{2(1+0.25)} = 24\ \text{GPa}$ ◄

SOLUTION (**2.8**)

We have

$$\varepsilon_y = \varepsilon_z = 0 \qquad \sigma_x = \frac{25(10^3)}{20\times 10(10^{-6})} = 125\ \text{MPa}$$

Thus

$$\varepsilon_y = 0 = \tfrac{1}{E}[\sigma_y - \nu(\sigma_x + \sigma_z)] \qquad (1)$$

$$\varepsilon_z = 0 = \tfrac{1}{E}[\sigma_z - \nu(\sigma_x + \sigma_y)] \qquad (2)$$

$$\varepsilon_x = \tfrac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)] \qquad (3)$$

Equations (1) and (2) become

$$\sigma_y - \nu\sigma_z = \nu\sigma_x \qquad (1')$$

$$\sigma_z - \nu\sigma_y = \nu\sigma_x \qquad (2')$$

Adding : $\nu(\sigma_y + \sigma_z) = 2\nu^2\sigma_x \big/ (1-\nu)$. Then, Eq. (3):

$$\varepsilon_x = \frac{1-\nu-2\nu^2}{1-\nu}\frac{\sigma_x}{E}$$ ◄

Substituting the data:

$$\varepsilon_x = \frac{1-0.3-0.18}{0.7}\frac{125(10^6)}{70(10^9)} = 1327\ \mu$$ ◄

SOLUTION (**2.9**)

<u>Hooke's Law.</u> We have $\sigma_y = 0$ and

$$\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{10^6}{72\times10^9}[(80) - 0 - 0.3(140)] = 0.000528 = 528\ \mu$$

$$\varepsilon_y = -\frac{\nu\sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu\sigma_z}{E}$$

$$= \frac{10^6}{72\times10^9}[-0.3(80) + 0 - 0.3(140)] = -917\ \mu$$

$$\varepsilon_z = -\frac{\nu\sigma_x}{E} - \frac{\nu\sigma_y}{E} + \frac{\sigma_z}{E}$$

$$= \frac{10^6}{72\times10^9}[-0.3(80) - 0 + 140] = 1611\ \mu$$

(CONT.)

2.9 (CONT.)

(**a**) Change in length $\Delta L_{AB} = \varepsilon_x a$,

$$\Delta L_{AB} = (528\times 10^{-6})(320) = 0.169 \text{ mm}$$

(**b**) Change in thickness

$$\Delta t = \varepsilon_y t = (-917\times 10^{-6})(15) = -0.014 \text{ mm}$$

(**c**) Change in volume,

$$e = \varepsilon_x + \varepsilon_y + \varepsilon_z = 528 - 917 + 1611 = 1.222$$

$$\Delta V = eV_o = 1.222(320\times 320\times 15) = 1.877 \text{ mm}^3$$

SOLUTION (**2.10**)

By assumptions, rubber in triaxial stress:

$$\sigma_x = \sigma_z = -p, \quad \sigma_y = -\tfrac{F}{\pi d^2/4} = -\tfrac{4F}{\pi d^2}$$

Stains are $\varepsilon_x = \varepsilon_z = 0$. Hooke's law gives

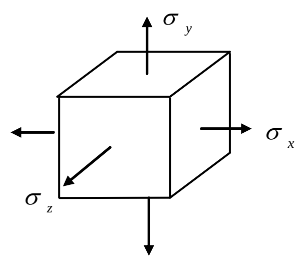
$$\varepsilon_x = 0 = \tfrac{1}{E}[\sigma_x - \nu(\sigma_y + \sigma_z)]$$

or

$$0 = p - \nu\tfrac{4F}{\pi d^2(1-\nu)}$$

Solving,

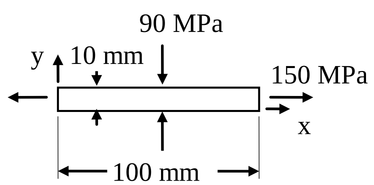
$$p = \tfrac{4\nu F}{\pi d^2(1-\nu)}$$ Q.E.D.



Substitute the data:

$$p = \tfrac{4(0.5)(10\times 10^3)}{\pi(62.5)^2(1-0.5)} = 3.26 \text{ MPa} \quad \text{(C)}$$

SOLUTION (**2.11**)



Hooke's law gives

$$\varepsilon_x = \tfrac{1}{E}(\sigma_x - \nu\sigma_y) = \tfrac{10^6}{100(10^9)}(150 + \tfrac{90}{3}) = 1800\ \mu$$

$$\varepsilon_y = \tfrac{1}{E}(\sigma_y - \nu\sigma_x) = \tfrac{10^6}{100(10^9)}(-90 - \tfrac{150}{3}) = -1400\ \mu$$

$$\varepsilon_z = -\tfrac{\nu}{E}(\sigma_x + \sigma_y) = -\tfrac{(1/3)10^6}{100(10^9)}(150 - 90) = -200\ \mu$$

Thus

$$\Delta L = 1800\ \mu(100) = 180\ \mu\text{m}$$

$$\Delta a = -1400\ \mu(50) = -70\ \mu\text{m}$$

$$\Delta b = -200\ \mu(10) = -2\ \mu\text{m}$$

and

$$L' = 100.018 \text{ mm}, \quad a' = 49.993 \text{ mm}, \quad b' = 9.9998 \text{ mm}$$

SOLUTION (**2.12**)

We have

$$\sigma_x = \sigma_y = \sigma_z = -p$$

Gen. Hooke's law:

$$\varepsilon_x = \varepsilon_y = \varepsilon_z = -\tfrac{p}{E}(1-2\nu) = -\tfrac{120(10^6)}{100(10^9)}\tfrac{1}{3} = -400 \ \mu$$

Thus

$$\Delta L = -400\ \mu(100) = -40\ \mu\text{m}$$

$$\Delta a = -400\ \mu(50) = -20\ \mu\text{m}$$

$$\Delta b = -400\ \mu(10) = -4\ \mu\text{m}$$

and

$$L' = 99.96\ \text{mm}, \quad a' = 49.98\ \text{mm}, \quad b' = 9.996\ \text{mm}$$ ◄

SOLUTION (**2.13**)

We have

$\sigma_x = \sigma_y = \sigma_z = -p$. The volume is

$$V_o = \tfrac{4}{3}\pi r^3 = \tfrac{4\pi}{3}(125)^3 = 8.181(10^6)\ \text{mm}^3$$

(**a**) $\varepsilon_x = -\tfrac{1}{E}[\sigma - \nu(\sigma+\sigma)] = -\tfrac{\sigma}{E}(1-2\nu)$

$$= \tfrac{168(10^6)}{70(10^9)}(1-0.5) = -1200\ \mu$$

Change in diameter,

$$\Delta d = \varepsilon_x d = -1200(10^{-6})250 = -0.3\ \text{mm}$$

Decrease in circumference:

$$\pi(\Delta d) = -0.3\pi = -0.9425\ \text{mm}$$ ◄

(**b**) $\Delta V = eV_o = (1-2\nu)\varepsilon_x V_o$

$$= (0.5)(-1200\times 10^{-6})(8.181\times 10^6) = -4909\ \text{mm}^3$$ ◄

SOLUTION (**2.14**)

From Fig.2.3b and Eq.2.20:

$$U_t = \tfrac{S_y + S_u}{2}\varepsilon_f \approx \tfrac{250+440}{2}(0.27) \approx 93\ \text{MPa}$$ ◄

We have $L_f = 50 + 50(0.27) = 63.5\ \text{mm}$

Using Eq.(2.1): $\%\ elongation = \tfrac{63.5-50}{50}(100) = 27\ \%$ ◄

SOLUTION (**2.15**)

Table B.1: $S_y = 260$ MPa , $E = 70$ GPa

We have

$$V = AL = \tfrac{\pi}{4}(0.005)^2(3) = 58.9 \times 10^{-6} \text{ m}^3$$

$$U_r = \frac{S_y^2}{2E} = \frac{(260 \times 10^6)^2}{2(70 \times 10^9)} = 482.9 \text{ kJ/m}^3$$

$$U_{app} = U_r V = 482.9 \times 10^3 (58.9 \times 10^{-6}) = 28.44 \text{ J}$$

For $U_{app} = 9$ J :

$$n = \frac{28.44}{9} = 3.16$$ ◄

SOLUTION (**2.16**)

(**a**) <u>ASTM-A242.</u> $E = 200$ GPa and $\sigma_y = 345$ MPa

$$U_o = \frac{S_y^2}{2E} = \frac{(345\times10^6)^2}{2(200\times10^9)} = 298 \ \frac{\text{kJ}}{\text{m}^3}$$ ◄

(**b**) <u>Stainless (302).</u> $E = 190$ GPa and $S_y = 520$ MPa

$$U_o = \frac{S_y^2}{2E} = \frac{(520\times10^6)^2}{2(200\times10^9)} = 712 \ \frac{\text{kJ}}{\text{m}^3}$$ ◄

SOLUTION (**2.17**)

(**a**) <u>Aluminum 2014-T6.</u> $E = 72$ GPa and $\sigma_y = 410$ MPa

$$U_o = \frac{S_y^2}{2E} = \frac{(410\times10^6)^2}{2(72\times10^9)}$$

$$= 1167 \ \frac{kJ}{m^3}$$ ◄

(**b**) <u>Annealed yellow brass.</u> $E = 105$ GPa and $S_y = 105$ MPa

$$U_o = \frac{S_y^2}{2E} = \frac{(105\times10^6)^2}{2(105\times10^9)} = 52.5 \ \frac{\text{kJ}}{\text{m}^3}$$ ◄

SOLUTION (**2.18**)

Referring to Fig. P2.18: $E = \frac{105(10^6)}{0.0025} = 42$ GPa, $S_y = 192.5$ MPa

(**a**) $U_o = \frac{S_y^2}{2E} = \frac{(192.5\times10^6)^2}{2(42\times10^9)} = 441.5 \text{ kJ/m}^3$ ◄

(**b**) Total area under $\sigma - \varepsilon$ diagram:

$$U_t \approx 262.5(10^6)(0.176) = 46.2 \text{ MJ/m}^3$$ ◄

SOLUTION (**2.19**)

(**a**) $V = 50 \times 50 \times 1{,}500 = 3.75\,(10^6)$ mm^3

Thus $nU = \frac{S_y^2}{2E}V$

or $S_y = [\frac{2EnU}{V}]^{\frac{1}{2}}$

$$= [\frac{2\times 200\times 10^9 \times 1.5\times 400}{3.75\,(10^{-3})}]^{\frac{1}{2}} = 253 \quad \text{MPa} \qquad \blacktriangleleft$$

(**b**) $U_r = \frac{S_y^2}{2E} = \frac{(253\times 10^6)^2}{2(200\times 10^9)} = 160$ kPa ◀

SOLUTION (**2.20**)

Table B.1: $S_y = 250$ MPa, $E = 200$ GPa

We have

$$U = nU_{app} = 5(17) = 85 \quad \text{N}\cdot\text{m}$$

$$U_r = \frac{S_y^2}{2E} = \frac{(250\times 10^6)^2}{2(200\times 10^9)} = 156.25 \quad \text{kN}\cdot\text{m}/\text{m}^3$$

Therefore

$$V = \frac{U}{U_r} = \frac{85}{156.25(10^3)} = 0.544(10^{-3}) \quad \text{m}^3$$

Also $V = AL: \quad 0.544\,(10^{-3}) = \frac{\pi}{4}d^2(2.4)$

or

$$d = 0.017 \quad \text{m} = 17 \text{ mm} \qquad \blacktriangleleft$$

SOLUTION (**2.21**)

Refer to Fig. P2.21. We have

$$E = \frac{190(10^6)}{0.001} = 190 \text{ GPa}, \quad S_y \approx 245 \text{ MPa}$$

(**a**) $U_o = \frac{S_y^2}{2E} = \frac{(245\times 10^3)^2}{2(190\times 10^9)} = 158 \ \frac{\text{kJ}}{\text{m}^3}$ ◀

(**b**) Total area under $\sigma - \varepsilon$ diagram:

$$U_t \approx 350\times 10^3(0.28) = 98 \ \frac{\text{MJ}}{\text{m}^3} \qquad \blacktriangleleft$$

SOLUTION (**2.22**)

$V = (0.05)(0.05)(1.2) = 0.003 \ m^3$ and $S_y \approx S_p$

(**a**) $n\cdot U = \frac{S_p^2}{2E}V, \quad S_p^2 = \frac{nU(2E)}{V}$

Substituting the data given,

(CONT.)

2.22 (CONT.)

$$S_p^{\,2} = \frac{1.8(150)(2\times 210\times 10^9)}{0.003} = 37.8\times 10^{15} \quad ◄$$

or

$$S_p = 194.4 \text{ MPa}$$

(b) $U_o = \frac{S_p^{\,2}}{2E} = \frac{37.8\times 10^{15}}{2(210\times 10^9)} = 90 \ \frac{\text{kJ}}{\text{m}^3}$ ◄

SOLUTION (**2.23**)

Applying Eq. (2.22), we find

$$S_u = 3.45 H_B \text{ MPa} = 3.45(149) = 514 \text{ MPa} \quad ◄$$

Equation (2.24):

$$S_y = 3.62(149) - 207 = 332.4 \text{ MPa} \quad ◄$$

SOLUTION (**2.24**)

Using Eq. (2.22),

$$S_u = 3.45 H_B \text{ MPa} = 3.45(179) = 618 \text{ MPa} \quad ◄$$

Formula (2.24):

$$S_y = 3.62(179) - 207 = 441 \text{ MPa} \quad ◄$$

SOLUTION (**2.25**)

Formula (2.22),

$$S_u = 3.45 H_B \text{ MPa} = 3.45(156) = 538 \text{ MPa} \quad ◄$$

Equation (2.24):

$$S_y = 3.62(156) - 207 = 378 \text{ MPa} \quad ◄$$

SOLUTION (**2.26**)

Equation (2.22) gives

$$S_u = 3.45 H_B \text{ MPa} = 3.45(293) = 1010.9 \text{ MPa} \quad ◄$$

Formula (2.24):

$$S_y = 3.62(293) - 207 = 853.7 \text{ MPa} \quad ◄$$

End of Chapter 2