<u>*Ans:*</u>
Since h has units of length and g has units of (length)(time)$^{-2}$, let us divide both sides of the above equation by $\sqrt{h/g}$:

$$\frac{t}{\sqrt{h/g}} = \frac{f\left(h, m, g\right)}{\sqrt{h/g}}$$

The left side of this equation is now dimensionless. Therefore, the right side must also be dimensionless, which implies that the time of flight cannot depend on the mass of the object. Thus dimensional analysis implies the following functional relationship:

$$t = \alpha\sqrt{\frac{h}{g}}$$

where α is a dimensionless constant. Only one experiment would be required to estimate α, but several trials at various heights might be advisable to obtain a reliable estimate of this constant. Note that $\alpha = \sqrt{2}$ according to Newton's laws of motion.

CHAPTER 2

2.1 According to Eq. (2.25), the energy required to increase the crack area a unit amount is equal to *twice* the fracture work per unit surface area, w_f. Why is the factor of 2 in this equation necessary?

<u>*Ans:*</u>
The factor of 2 stems from the difference between *crack area* and *surface area*. The former is defined as the projected area of the crack. The surface area is twice the crack area because the formation of a crack results in the creation of two surfaces. Consequently, the material resistance to crack extension = 2 w_f.

2.2 Derive Eq. (2.30) for both load control and displacement control by substituting Eq. (2.29) into Eqs. (2.27) and (2.28), respectively.

<u>*Ans:*</u>
(a) Load control.

$$\mathcal{G} = \frac{P}{2B}\left(\frac{d\Delta}{da}\right)_P = \frac{P}{2B}\left(\frac{d\left[CP\right]}{da}\right)_P = \frac{P}{2B}\frac{dC}{da}$$

(b) Displacement control.

$$\mathcal{G} = -\frac{\Delta}{2B}\left(\frac{dP}{da}\right)_{\Delta}$$

$$\left(\frac{dP}{da}\right)_{\Delta} = \Delta\frac{d\left(1/C\right)}{da} = -\frac{\Delta}{C^2}\frac{dC}{da}$$

$$\mathcal{G} = \frac{\left(\Delta/C\right)^2}{2B}\frac{dC}{da} = \frac{P^2}{2B}\frac{dC}{da}$$

2.3	Figure 2.10 illustrates that the driving force is linear for a through-thickness crack in an infinite plate when the stress is fixed. Suppose that a remote displacement (rather than load) were fixed in this configuration. Would the driving force curves be altered? Explain. (Hint: see Section 2.5.3).

Ans:

In a cracked plate where $2a$ << the plate width, crack extension at a fixed remote displacement would not effect the load, since the crack comprises a negligible portion of the cross section. Thus a fixed remote displacement implies a fixed load, and load control and displacement control are equivalent in this case. The driving force curves would not be altered if remote displacement, rather than stress, were specified.

Consider the spring in series analog in Fig. 2.12. The load and remote displacement are related as follows:

$$\Delta_T = (C + C_m)\,P \quad \Delta_T = \left(C + C_m\right)P$$

where C is the "local" compliance and C_m is the system compliance. For the present problem, assume that C_m represents the compliance of the uncracked plate and C is the additional compliance that results from the presence of the crack. When the crack is small compared to the plate dimensions, $C_m >> C$. If the crack were to grow at a fixed Δ_T, only C would change; thus load would also remain fixed.

2.4	A plate *2W* wide contains a centrally located crack *2a* long and is subject to a tensile load, *P*. Beginning with Eq. (2.24), derive an expression for the elastic compliance, $C\ (= \Delta/P)$ in terms of the plate dimensions and elastic modulus, *E*. The stress in Eq. (2.24) is the nominal value; i.e., $\sigma = P/2BW$ in this problem. (Note: Eq. (2.24) only applies when $a << W$; the expression you derive is only approximate for a finite width plate.)

<u>*Ans:*</u>
The through-thickness crack has two tips; an increment of crack growth causes the crack area to increase by $2B\,da$. The compliance relationship for energy release rate must be modified accordingly:

$$\mathcal{G} = \frac{P^2}{2B}\frac{dC}{d(2a)} = \frac{P^2}{4B}\frac{dC}{da}$$

Equating the above expression with Eq. (2.24) gives

$$\mathcal{G} = \frac{\sigma^2 \pi a}{E} = \frac{P^2 \pi a}{4B^2 W^2 E} = \frac{P^2}{4B}\frac{dC}{da}$$

Solving for compliance leads to

$$C = \int dC = \frac{\pi}{BW^2 E}\int a\,da = \frac{\pi}{BE}\left(\frac{a}{W}\right)^2 + \text{ constant}$$

The constant corresponds to the compliance of the uncracked plate. Assuming a gage length L, the *total* compliance is given by

$$C_{tot} = \frac{\pi}{BE}\left(\frac{a}{W}\right)^2 + \frac{L\left(1-\nu^2\right)}{2BWE} = C + C_m$$

where C_m represents the compliance of the uncracked plate and C is the *additional* compliance due to the crack. When $a << W$ or $a << L$, the first term in the above expression is negligible. Recall the previous problem, where it was argued that displacement control is equivalent to load control in an infinite plate because $C << C_m$.

2.5 A material exhibits the following crack growth resistance behavior:

$$R = 6.95(a - a_o)^{0.5}$$

where a_o is the initial crack size. R has units of kJ/m^2 and crack size is in millimeters. Alternatively,

$$R = 200(a - a_o)^{0.5}$$

where R has units of in-lb/in^2 and crack size is in inches. The elastic modulus of this material = 207,000 MPa (30,000 ksi). Consider a wide plate with a through crack ($a << W$) that is made from this material.

(a) If this plate fractures at 138 MPa (20.0 ksi), compute the following:
   (i) The half crack size at failure (a_c).
   (ii) The amount of stable crack growth (at each crack tip) that precedes failure (a_c - a_o).

(b) If this plate has an initial crack length ($2a_o$) of 50.8 mm (2.0 in) and the plate is loaded to failure, compute the following:
   (i) The stress at failure.
   (ii) The half crack size at failure.
   (iii) The stable crack growth at each crack tip

<u>Ans:</u>

At instability, $\mathcal{G} = R$ and $d\mathcal{G}$/da = dR/da. Therefore,

$$\frac{\pi\sigma^2 a_c}{E} = 6.95\left(a_c - a_o\right)^{0.5} \qquad (1)$$

and

$$\frac{\pi\sigma^2}{E} = 3.48\left(a_c - a_o\right)^{-0.5} \qquad (2)$$

Thus we have two equations to relate σ, a_c and a_o, and we must specify one of these quantities.

(a) σ = 138 MPa

From Eq. (1) above,

$$\frac{\pi\left(138{,}000\text{ kPa}\right)^2}{2.07\times10^8\text{ kPa}} = 3.48\left(a_c - a_o\right)^{-0.5}$$

$$a_c - a_o = 145\text{ mm}$$

Substituting into (2) gives

$$\frac{\pi\left(138{,}000\text{ kPa}\right)^2 a_c}{2.07\times10^8\text{ kPa}} = 6.95\left(145\text{ mm}\right)^{0.5}$$

Thus

(i) a_c = 290 mm
(ii) a_c - a_o = 145 mm
(iii) a_o = 145 mm

(b) a_o = 25.4 mm

Dividing Eq. (1) by Eq. (2) leads to

$$a_c = 2(a_c - a_o)$$

Therefore, if a_o = 25.4 mm, a_c = 50.8 mm and (a_c - a_o) = 25.4 mm. We can solve for critical stress by substituting these results into Eq. (1):

$$\frac{\pi\sigma^2(0.0508\text{ m})}{2.07\times10^8\text{ kPa}} = 6.95(25.4\text{ mm})^{0.5}$$

Thus

(i) σ = 213,000 kPa = 213 MPa
(ii) a_c = 50.8 mm
(iii) a_c - a_o = 25.4 mm

2.6	Suppose that a double cantilever beam specimen (Fig. 2.9) is fabricated from the same material considered in Problem 2.5. Calculate the load at failure and the amount of stable crack growth. The specimen dimensions are as follows: B = 25.4 mm (1 in) h = 12.7 mm (0.5 in) a_o = 152 mm (6 in)

<u>*Ans:*</u>

At instability, $\mathcal{G}$ = R and d$\mathcal{G}$/da = dR/da. Hence,

$$\mathcal{G}_c = \frac{12P_c^2 a_c^2}{B^2h^3E} = 6.95(a_c - a_o)^{0.5} \qquad (1)$$

$$\frac{2\mathcal{G}_c}{a} = 3.48(a_c - a_o)^{-0.5} \qquad (2)$$

Dividing (1) by (2) gives

$$\frac{a_c}{2} = 2(a_c - a_o)$$

Thus

$$a_c = \frac{4}{3}a_o = 203\text{ mm}$$

and

$$\mathcal{G}_c = \frac{12P_c^2\left(0.203\ \text{m}\right)^2}{\left(0.025\ \text{m}\right)^2\left(0.0127\ \text{m}\right)^3 2.07\times10^8\ \text{kPa}} = 6.95\left(203-152\right)^{0.5}$$

$P_c = 5.16$ kN

2.7	Consider a nominally linear elastic material with a rising R curve (e.g., Problems 2.5 and 2.6). Suppose that one test is performed on wide plate with a through crack (Fig. 2.3) and a second test on the same material is performed on a DCB specimen (Fig. 2.9). If both tests are conducted in load control, would the $\mathcal{G}_c$ values at instability be the same? If not, which geometry would result in a higher $\mathcal{G}_c$? Explain.

<u>*Ans*</u>

The driving force curve for the through crack is linear, while $\mathcal{G}$ varies with a^2 for the DCB specimen. Therefore, the two geometries would have different points of tangency on the R curve, as Fig. S1 illustrates. The $\mathcal{G}_c$value for the through crack would be higher, and this geometry would experience more stable crack growth prior to failure.

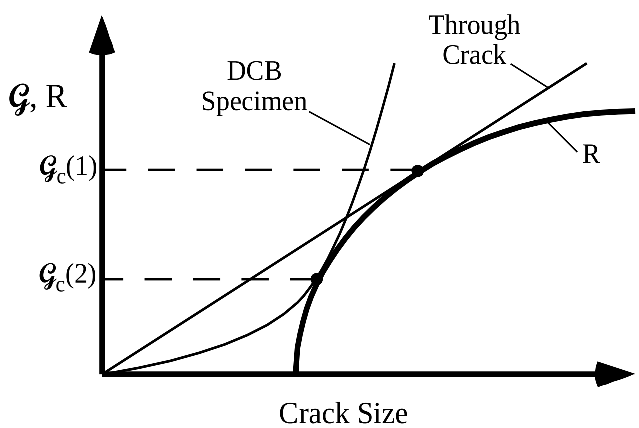

FIGURE S1 Effect of specimen geometry on instability (Problem 2.7)

2.8	Example 2.3 showed that the energy release rate, $\mathcal{G}$, of the double cantilever beam (DCB) specimen increases with crack growth when the specimen is held at a constant load. Describe (qualitatively) how you could alter the design of the DCB specimen such that a growing crack in load control would experience a constant $\mathcal{G}$.

<u>*Ans:*</u>
In a conventional DCB specimen, compliance varies with a^3, and energy release is proportional to a^2 when load is fixed. In order for $\mathcal{G}$ to remain constant with crack growth, compliance must vary linearly with crack length. One way to accomplish this is to taper the specimen width, as Fig. S2 illustrates. Alternatively, the thickness can be tapered. The latter method is not as effective as the former because compliance is less sensitive to the thickness dimension; recall that the moment of inertia of the cross section is proportional to Bh^3. Specimens such as illustrated in Fig. S2, where $\mathcal{G}$ is relatively constant over a range of crack lengths, have been used successfully in laboratory experiments.

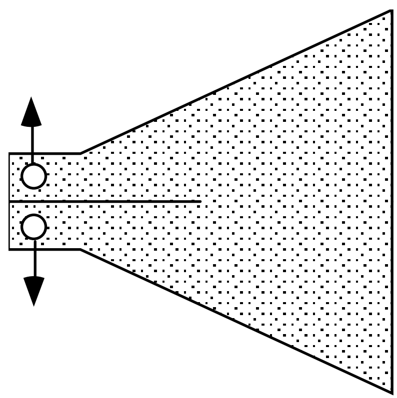

FIGURE S2 Tapered DCB specimen (Problem 2.8).

2.9	Beginning with Eq. (2.20), derive an expression for the potential energy of a plate subject to a tensile stress σ with a penny-shaped flaw of radius a. Assume that a << plate dimensions.

<u>*Ans:*</u>
At fracture,

$$-\frac{d\Pi}{d\mathcal{A}} = \frac{dW_S}{d\mathcal{A}}$$

For the penny-shaped crack,

$$W_S = 2\pi\gamma_s a^2$$

and

$$\frac{dW_S}{d\mathcal{A}} = 2\gamma_s$$

Combining the above results with Eq. (2.20) gives

$$2\gamma_s = \frac{4\left(1-\nu^2\right)a\sigma_f^{\,2}}{\pi E} = -\frac{d\Pi}{d\mathcal{A}}$$

The above equation must be integrated with respect to crack area to infer the potential energy. The crack area can be written in terms of the the crack radius, *a*:

$$dA = 2\pi a da$$

and

$$\frac{d}{dA} = \frac{1}{2\pi a}\frac{d}{da}$$

Therefore,

$$\frac{d\Pi}{da} = -\frac{8\sigma^2 a^2\left(1-\nu^2\right)}{E}$$

and

$$\Pi = \Pi_o - \frac{8\sigma^2 a^3\left(1-\nu^2\right)}{3E}$$

where Π_o is the potential energy of the uncracked solid.

2.10	Beginning with Eq. (2.20), derive expressions for the energy release rate and Mode I stress intensity factor of a penny-shaped flaw subject to a remote tensile stress. (Your K_I expression should be identical to Eq. (2.44).)

<u>*Ans:*</u>
At fracture in an ideally brittle material, $\mathcal{G} = \mathcal{G}_c = 2\gamma_s$. Rearranging Eq. (2.20) leads to

$$2\gamma_s = \frac{4\left(1-\nu^2\right)a\sigma_f^{\,2}}{\pi E} = \mathcal{G}_c$$

Thus

$$\mathcal{G} = \frac{4\left(1-\nu^2\right)a\sigma^2}{\pi E} \quad \mathcal{G} = \frac{4(1 - \nu^2)\, a\, \sigma^2}{\pi\, E}$$

Invoking the relationship between K_I and $\mathcal{G}$ (Eq. (2.56)) gives

$$K_I = \frac{2}{\pi}\sigma\sqrt{\pi a}$$

which agrees with Eq. (2.44). Note that the plane strain K_I–$\mathcal{G}$ relationship is appropriate in this case. The strain parallel to the crack front is zero because the crack is axisymmetric.

2.11 Calculate K_I for a rectangular bar containing an edge crack loaded in three point bending.

P = 35.0 kN (7870 lb); W = 50.8 mm (2.0 in); B = 25 mm (1.0 in); a/W = 0.2; S = 203 mm (8.0 in).

Ans:

The K_I solutions in Table 2.4 have the following form:

$$K_I = \frac{P}{B\sqrt{W}} f\left(a/W\right)$$

Inserting a/W = 0.2 and S/W = 4 into the appropriate polynomial in Table 2.4 gives $f(a/W)$ = 4.70. Thus

$$K_I = \frac{(35 \text{ kN})(4.70)}{0.025 \text{ m}\sqrt{0.050 \text{ m}}} = 29,400 \text{ kPa}\sqrt{\text{m}} = 29.4 \text{ MPa}\sqrt{\text{m}}$$

2.12 Consider a material where K_{IC} = 35 MPa $\sqrt{\text{m}}$ (31.8 ksi $\sqrt{\text{in}}$). Each of the five specimens in Table 2.4 and Fig. 2.23 have been fabricated from this material. In each case, B = 25.4 mm (1 in), W = 50.8 mm (2 in), and a/W = 0.5. Estimate the failure load for each specimen. Which specimen has the highest failure load? Which has the lowest?

Ans:

Failure load is inversely proportional to the geometry correction factor, f(a/W):

$$P_{crit} = \frac{K_{Ic}B\sqrt{W}}{f\left(a/W\right)}$$

Thus it is obvious from Fig. 2.23 that the CCT and DENT geometries have the highest failure load and the SE(B) geometry the lowest (for fixed B, W, and a). The calculated failure loads are tabulated below.

Geometry	f(0.5)	P_{crit}, kN
SENT	3.543	5.57
SE(B)	10.65	1.85
CCT	1.051	18.76
DENT	1.049	18.80
Compact	9.659	2.04

2.13 A large block of material is loaded to a stress of 345 MPa (50 ksi). If the fracture toughness (K_{Ic}) is 44 MPa$\sqrt{\text{m}}$ (40 ksi$\sqrt{\text{in}}$), determine the critical radius of a buried penny-shaped crack.

Ans:

At fracture, $K_I = K_{Ic}$. Substituting the above data into Eq. (2.44) gives

$$44\ \text{MPa}\sqrt{\text{m}} = \frac{2}{\pi}\left(345\ \text{MPa}\right)\sqrt{\pi a_c}$$

$$a_c = 12.8\ \text{mm}$$

2.14 A semicircular surface crack in a pressure vessel is 10 mm (0.394 in) deep. The crack is on the inner wall of the pressure vessel and is oriented such that the hoop stress is perpendicular to the crack plane. Calculate K_I if the local hoop stress = 200 MPa (29.0 ksi) and the internal pressure = 20 MPa (2900 psi). Assume that the wall thickness >> 10 mm.

Ans:

Applying the principle of superposition (see Example 2.5) results in the following stress intensity solution for this case:

$$K_I = \lambda_s\left(\sigma + p\right)\sqrt{\frac{\pi a}{Q}}\, f\left(\phi\right)$$

$$= \left(1.14\right)\left(220\ \text{MPa}\right)\sqrt{\frac{\pi\left(0.01\ \text{m}\right)}{2.64}} = 28.4\ \text{MPa}\sqrt{\text{m}}$$

at $\phi = 0°$.

2.15 Calculate K_I for a semi-elliptical surface flaw at ϕ = 0°, 30°, 60°, 90°.

σ = 150 MPa (21.8 ksi); a = 8.00 mm (0.315 in); 2c = 40 mm (1.57 in).

Ans:
From Fig. 2.19,

$$K_I = \lambda_s \left(150 \text{ MPa}\right)\sqrt{\frac{\pi\left(0.008 \text{ m}\right)}{\text{Q}}}\, f\left(\phi\right)$$

ϕ, Degrees	λ_S	f(ϕ)	K_I, MPa$\sqrt{\text{m}}$
0.00	1.20	0.632	15.74
30.00	1.12	0.780	18.08
60.00	1.10	0.943	21.36
90.00	1.09	1.000	22.62

2.16 Consider a plate subject to biaxial tension with a through crack of length *2a*, oriented at an angle β from the σ_2 axis (Fig. 13.1). Derive expressions for K_I and K_{II} for this configuration. What happens to each K expression when $\sigma_1 = \sigma_2$?

Ans:
We can apply the principle of superposition separately to K_I and K_{II}:

$$K_I = \sigma_1 \cos^2\left(\beta\right)\sqrt{\pi a} + \sigma_2 \cos^2\left(90+\beta\right)\sqrt{\pi a}$$

$$= \left[\sigma_1 \cos^2\left(\beta\right) + \sigma_2 \sin^2\left(\beta\right)\right]\sqrt{\pi a}$$

$$K_{II} = \sigma_1 \cos\left(\beta\right)\sin\left(\beta\right)\sqrt{\pi a} + \sigma_2 \cos\left(90+\beta\right)\sin\left(90+\beta\right)\sqrt{\pi a}$$

$$= \left(\sigma_1 - \sigma_2\right)\cos\left(\beta\right)\sin\left(\beta\right)\sqrt{\pi a}$$

when $\sigma_1 - \sigma_2$, $K_I = \sigma_1\sqrt{\pi a}$ and K_{II} = 0.

2.17 A wide flat plate with a through-thickness crack experiences a nonuniform normal stress which can be represented by the following crack face traction:

$$p(x) = p_o e^{-x/\beta}$$

where p_o = 300 MPa and β = 25 mm. The origin (x = 0) is at the left crack tip, as illustrated in Fig. 2.27. Using the weight function derived in Example 2.6, calculate K_I at each crack tip for $2a$ = 25, 50, and 100 mm. You will need to integrate the weight function *numerically*.

Ans:

Figure S3 is a plot of K_I versus crack length for the through crack with the exponential stress distribution given above. Values for three crack lengths are tabulated below.

$2a$, mm	K_I, MPa$\sqrt{\text{m}}$	
	$x = 2a$	$x = 0$
25	29.21	48.15
50	21.74	57.37
100	11.06	62.65

2.18 Repeat Problem 2.17 with the following crack face pressure profile:

$$p(x) = p_o \cos\left(\frac{\pi x}{50 \text{ mm}}\right)$$

At what crack length(s) does K_I = 0 at the right tip?

Ans.

Figure S4 is a plot of K_I versus crack length for the right tip ($x = 2a$). The curve passes through zero at $2a$ = 33 mm and 88 mm.

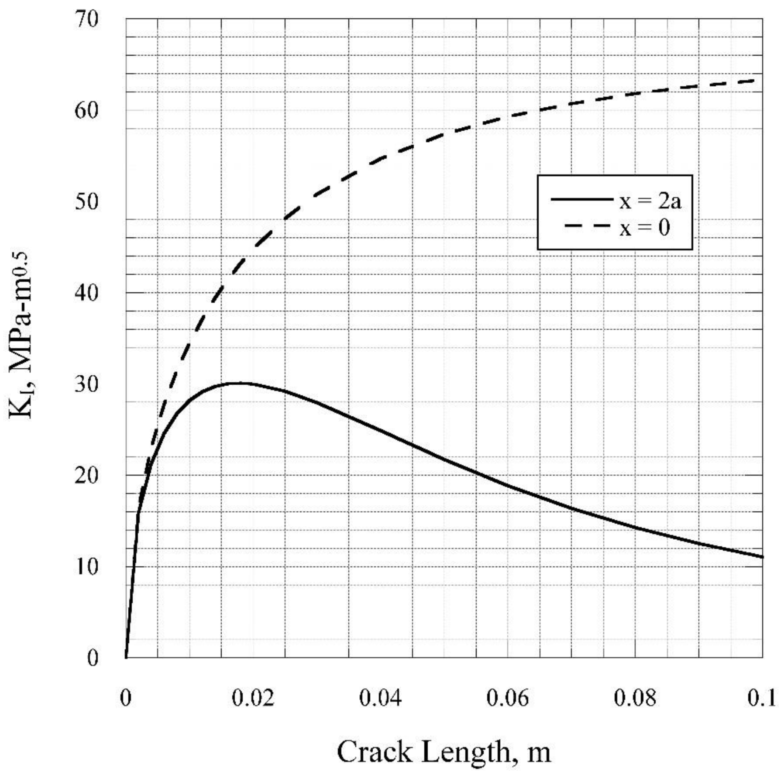

FIGURE S3. Solution to Problem 2.17.

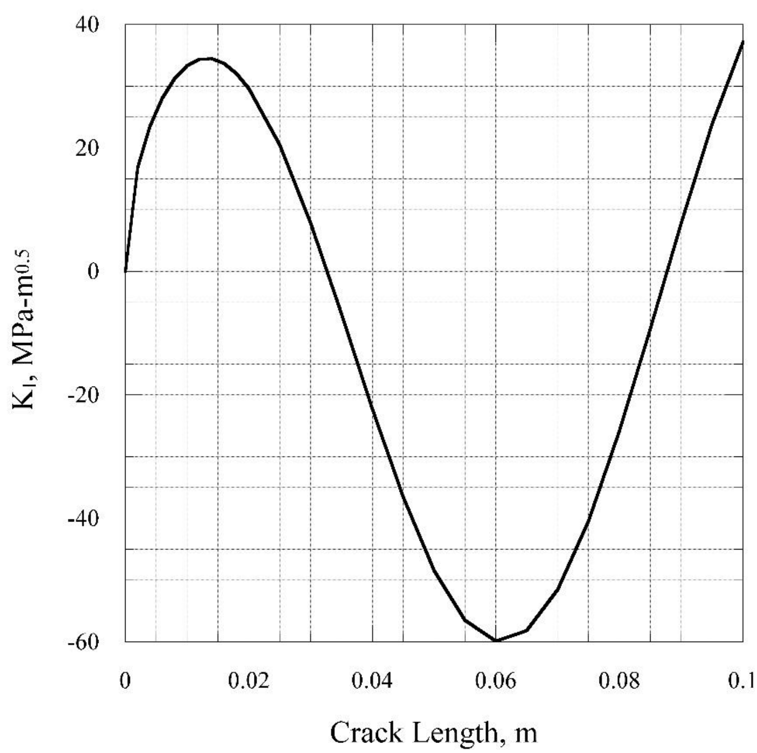

FIGURE S4. Solution to Problem 2.17

2.19 For an infinite plate with a through crack 50.8 mm (2.0 in) long, compute and tabulate K_{eff} $_{v}$. stress using the three methods indicated below. Assume σ_{YS} = 250 MPa (36.3 ksi).

Ans:
Applying Eqs. (2.41), (2.70), and (2.81) to the values above results in the following stress intensity factors for the LEFM, Irwin, and strip yield methods respectively:

	K_{eff}, MPa $\sqrt{\text{m}}$ or ksi $\sqrt{\text{in}}$		
Stress, MPa (ksi)	LEFM	Irwin Correction	Strip Yield Model
25 (3.63)	7.06	7.08	7.08
50 (7.25)	14.1	14.3	14.2
100 (14.5)	28.2	29.5	29.3
150 (21.8)	42.4	46.8	46.3
200 (29.0)	56.5	68.5	68.9
225 (32.6)	63.6	82.4	86.6
249 (36.1)	70.3	99.1	143
250 (36.3)	70.6	99.9	∞

2.20 A material has a yield strength of 345 MPa (50 ksi) and a fracture toughness of 110 MPa $\sqrt{\text{m}}$ (100 ksi $\sqrt{\text{in}}$). Determine the minimum specimen dimensions (B, a, W) required to perform a valid K_{Ic} test on this material, based on the traditional size requirements in Eq. (2.88). Comment on the feasibility of testing a specimen of this size.

Ans:
From Eq. (2.88),

$$a, B, (W-a) \geq 2.5\left(\frac{110 \text{ MPa}\sqrt{\text{m}}}{345 \text{ MPa}}\right)^2 = 0.254 \text{ m } (10.0 \text{ in})$$

Therefore,

$$W \geq 0.508 \text{ m (20.0 in)}$$

Testing such a large specimen would impractical because:

- Machining costs would be very high.
- A very large test machine would be required.
- Materials are usually not available in such large section thicknesses. Even if a section of sufficient size could be produced, its metallurgical properties would not be representative of a thinner plate of the same material.

2.21 You have been given a set of fracture mechanics test specimens, all of the same size and geometry. These specimens have been fatigue precracked to various crack lengths. The stress intensity of this specimen configuration can be expressed as follows:

$$K_I = \frac{P}{B\sqrt{W}} f(a/W)$$

where P is load, B is thickness, W is width, a is crack length, and $f(a/W)$ is a dimensionless geometry correction factor.

Describe a set of experiments you could perform to determine $f(a/W)$ for this specimen configuration. Hint: you may want to take advantage of the relationship between K_I and energy release rate for linear elastic materials.

Ans:
The stress intensity factor can be inferred from compliance measurements as follows:

$$\frac{K_I{}^2}{E'} = \mathcal{G} = \frac{P^2}{2B}\frac{dC}{da} = \frac{P^2}{B^2WE'} f^2\left({}^{a}\!/\!_{W}\right)$$

assuming the specimen contains an edge crack, such that $d\mathcal{A} = Bda$. Solving for $f(a/W)$ gives

$$f\left({}^{a}\!/\!_{W}\right) = \sqrt{\frac{BE'}{2}\frac{dC}{d\left(a/W\right)}}$$

Thus $f(a/W)$ for the geometry of interest can be inferred by measuring the elastic compliance as a function of crack length, evaluating $dC/d(a/W)$, and inserting the result into the above expression. Note that the absolute compliance depends on specimen size and material properties, but the quantity ($B\,E'\,C$) is dimensionless, and depends only on a/W.

2.22 Derive the Griffith-Inglis result for the potential energy of a through crack in an infinite plate subject to a remote tensile stress (Eq. (2.16)). Hint: solve for the work required to close the crack faces; Eq. (A2.43) gives the crack opening displacement for this configuration.

The crack opening displacement at a distance x from the center of the crack (assuming the coordinate system in Fig. 13.2) is given by

$$2u_y = \frac{4\sigma}{E}\sqrt{a^2 - x^2}$$

for *plane stress* loading. The incremental closure work done at a point is as follows:

$$d\Pi = \frac{1}{2} \bullet 2\sigma u_y\left(x\right)Bdx \; d\Pi = \frac{1}{2} \bullet 2\sigma \, u_y(x) \, B \, dx$$

Thus the decrease in potential energy due to the formation of the crack is given by

$$\Pi - \Pi_o = \frac{4\sigma^2 B}{E}\int_0^a \sqrt{a^2 - x^2}\,dx = \frac{\pi\sigma^2 a^2 B}{E}$$

which agrees with Eq. (2.16).

2.23 Using the Westergaard stress function approach, derive the stress intensity factor relationship for an infinite array of collinear cracks in a plate subject to biaxial tension (Fig. 2.21).

<u>*Ans:*</u>
Substituting z* = z - a into Eq. (A2.39) and re-arranging gives

$$Z\left(z^*\right) = \frac{\sigma \sin\left[\dfrac{\pi\left(a+z^*\right)}{2W}\right]}{\sqrt{\left[\sin\left(\dfrac{\pi\left(a+z^*\right)}{2W}\right)\right]^2 - \left[\sin\left(\dfrac{\pi a}{2W}\right)\right]^2}}$$

Let us now perform a series expansion about z* = 0 on the $\sin^2$ term on the left side of the denominator:

$$\left[\sin\left(\frac{\pi\left(a+z^*\right)}{2W}\right)\right]^2 = \left[\sin\left(\frac{\pi a}{2W}\right)\right]^2 + \sin\left(\frac{\pi a}{2W}\right)\cos\left(\frac{\pi a}{2W}\right)\frac{\pi z^*}{W} + \dots$$

Substituting this result into the stress function and taking a limit leads to

$$\lim_{z^*\to 0}\left[Z\left(z^*\right)\right]=\frac{\sigma\sqrt{\tan\left(\frac{\pi a}{2W}\right)}}{\sqrt{\frac{\pi z^*}{W}}}=\frac{K_I}{\sqrt{2\pi z^*}}$$

Solving for stress intensity gives

$$K_I=\sigma\sqrt{2W}\sqrt{\tan\left(\frac{\pi a}{2W}\right)}=\sigma\sqrt{\pi a}\sqrt{\frac{2W}{\pi a}\tan\left(\frac{\pi a}{2W}\right)}$$

which agrees with Eq. (2.45).

CHAPTER 3

3.1	Repeat the derivation of Eqs. (3.1) to (3.3) for the plane strain case.

<u>*Ans:*</u>
In plane strain, the displacement of the crack face a distance r_y behind the tip is given by

$$u_y=\frac{4\left(1-\nu^2\right)}{E}K_I\sqrt{\frac{r_y}{2\pi}}$$

Substituting Eq. (2.63) into the above expression leads to

$$\delta=2u_y=\frac{8\left(1-\nu^2\right)}{E}K_I\sqrt{\frac{K_I^2}{12\pi^2\sigma_{YS}^2}}=\frac{4}{\sqrt{3}\pi}\frac{K_I^2\left(1-\nu^2\right)}{\sigma_{YS}E}=\frac{4}{\sqrt{3}\pi}\frac{\mathcal{G}}{\sigma_{YS}}$$

3.2	A *CTOD* test is performed on a three point bend specimen. Figure 13.3 shows the deformed specimen after it has been unloaded. That is, the displacements shown are the *plastic components*. (a) Derive an expression for plastic *CTOD (δ_p)* in terms of Δ_p and specimen dimensions. (b) Suppose that V_p and Δ_p are measured on the same specimen, but that the plastic rotational factor, r_p, is unknown. Derive an expression for r_p in terms of Δ_p, V_p and specimen dimensions, assuming the angle of rotation is small.