Chapter 2

DATA PRESENTATION

Problem 2.1
Explain what is meant by 'order tracking' and why it is an important concept for the monitoring of rotating machines.

On shutdown a large boiler feed pump runs down from 6000rpm at an initial rate of 120rpm/sec. The response up to four times shaft speed is required.

i) How would you sample to obtain maximum resolution, describing block size and sampling rate.

ii) What is the resulting frequency resolution?

Solution

a) Order tracking is the process of resolving vibration into contributions at shaft speed and multiples thereof. For modest slew rates the Short Time Fourier Transform is a suitable technique. An alternative for rapid transients is the Vold-Kalman approach which is explained in Chapter 8.

b) For an exponential speed decay, use Equation 2.16 to calculate the optimum number of revs.

$$N = \sqrt{\frac{\Omega}{2\pi\alpha}} \tag{2.16}$$

i) Using this value we see that, if the sampling time is T, then

$$\Delta f = \frac{1}{T} = \frac{\Omega}{2\pi N} = \sqrt{\frac{\alpha\Omega}{2\pi}} \tag{2.17}$$

From the question we infer that

$$\alpha = 120/6000 = 1/50$$

$$\therefore N = \sqrt{\frac{100 * 2\pi}{2\pi * 1/50}} = \sqrt{5000} \approx 71$$

Hence for maximum resolution, sample over 71 revs = (initially 0.71 seconds)

This means that to capture up to 4$^{\text{th}}$ harmonic at maximum speed, sampling rate must be at least 800hz, block size 800*0.71= 570 samples

ii) The maximum resolution is given by $\Delta f = \dfrac{1}{0.71} = 1.4\text{Hz}$

If sampling is fixed to shaft speed, resolution decreases as speed decays, but normally this is of little consequence.

Problem 2.2

Assuming an exponential rundown rate in question 1, how does the resolution vary during the rundown? If the rundown rate is linear, how is the resolution changed?

Solution

If the acquisition is fixed by rotor position then the resolution follows equation 2.17 and the resolution improved as the rotor decelerates. Hence

$$N = \sqrt{\frac{\Omega}{2\pi\alpha}}$$

and so

$$\Delta f = \frac{1}{T} = \frac{\Omega}{2\pi N} = \sqrt{\frac{\alpha\Omega}{2\pi}}$$

If it is at fixed time intervals then the resolution stays fixed at it initial values assuming that the block size is constant. However, using a fixed time interval can lead to problems of leakage.

For a linear slowing rate, the calculation is straightforward. The analysis follows the same logic as the exponential case, but gives different results. So letting the speed deceleration be

$$\frac{d\Omega}{dt} = -2\pi\beta$$

Hence the deceleration is *b* rev/sec^2 (or *b*/60 rev/min/sec.). Then, as in the main text, optimum resolution is obtained by considering the decay rate against the need for as long a sample as possible. This leads to

$$\Delta f = \frac{1}{T} = \beta T$$

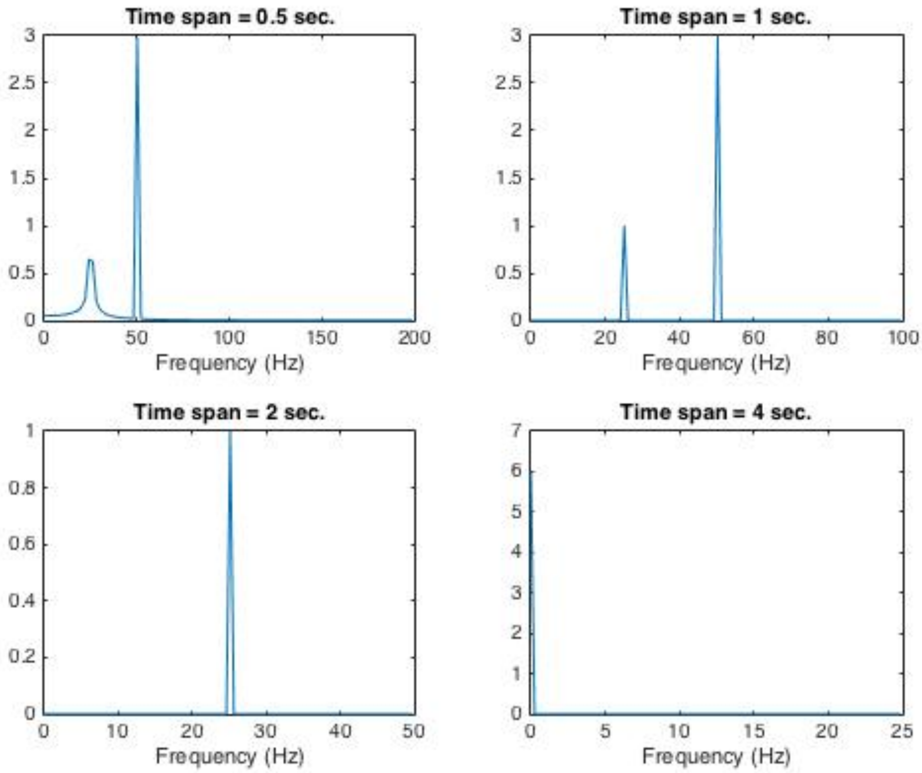
So for optimum sampling $T = 1/\sqrt{\beta}$ which is fixed throughout the speed range. Care is needed to avoid leakage problems.

Problem 2.3

A signal is known to be $y = \sin(50\pi t) + 3\cos(100\pi t)$. *Using 200 sampling points, obtain the FFTs using sample lengths of 0.5, 1, 2 and 4 seconds. Comment on the results.*

Solution

This problem is most conveniently tackled in MATLAB. Note that the standard routine 'fft' does not scale either frequency or amplitudes. The scaling involves the number of samples. Scaling is done within the script 'fftscale' which can be found in the RotorDynamics toolbox.



There are two components at 25 and 50Hz respectively.

Case (a), time span 0.5sec, shows both components but with low resolution and the 25Hz component has suffered leakage

Case (b), time span 1 sec, no leakage, both components accurately reflected.

Case (c), time span 2 sec, sampling interval now 0.01s and so cut-off frequency is 50Hz and so the higher frequency component does not appear.

Case (d), time span 4s, both components are aliased to zero owing to an inadequate sample rate.

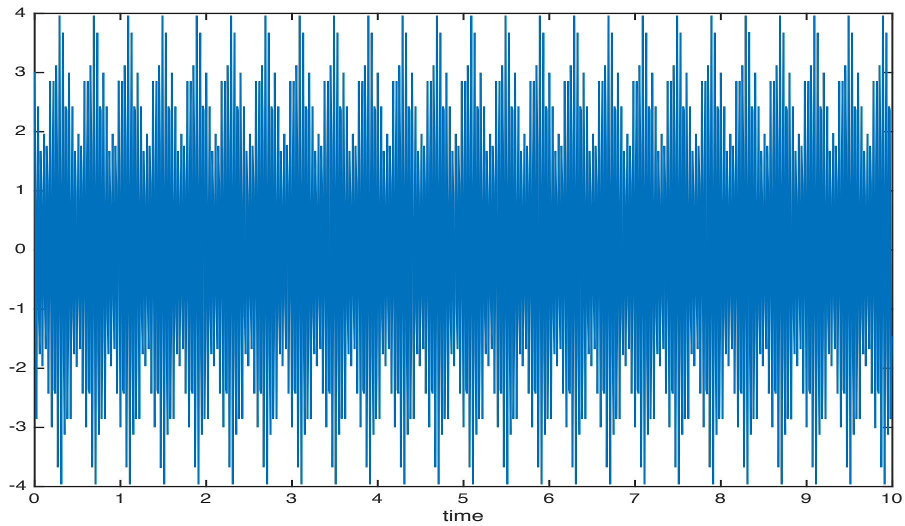
The MATLAB script for this problem is ‘Problem2p3.m’.

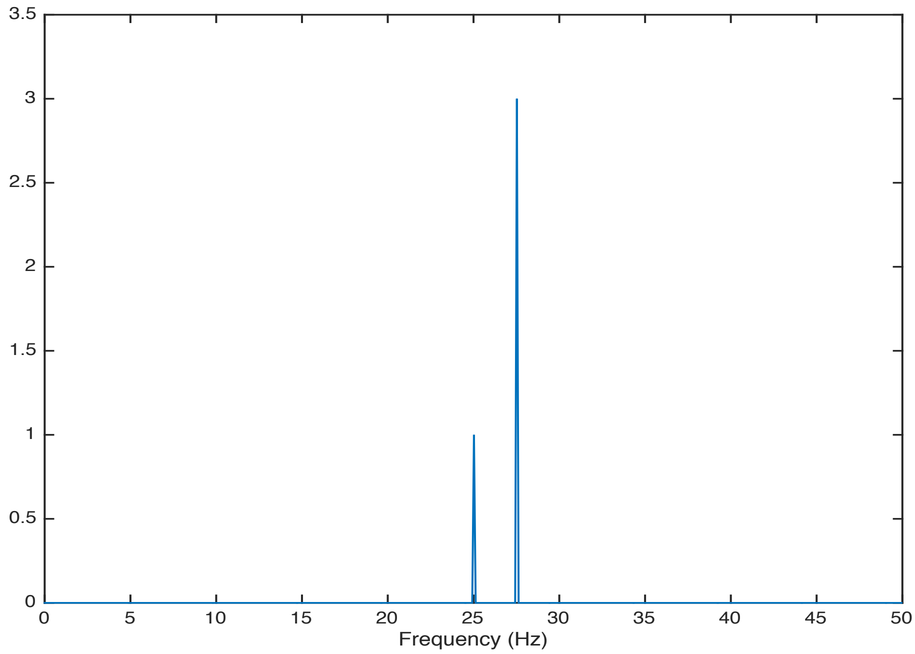
Problem 2.4

A signal has the form $y = \sin(50\pi t) + 4\cos(55\pi t)$. *Determine appropriate sampling rates and the number of samples to obtain adequate separation of the two components. Obtain the FFT and discuss your results.*

Solution

The resolution must beat least 2.5 Hz, preferably 1.25 Hz which means a sampling period of at least 0.8 seconds. The two frequencies are 25Hz and 27.5 Hz and to avoid leakage there must be an exact number of cycles of both signals, this suggests a 10 sec sample, sampling at 55Hz. But this gives a cut-off frequency of 27.5 and an aliased term at 30Hz, which is confusing. So acquire at a higher frequency, say 100Hz. The two figures show the time signal and the resulting FFT.





The script for this analysis is ‘Problem2p4.m’.

Problem 2.5.

A signal which is band limited in frequency can be expressed as

$$y(t) = \frac{1}{2\Delta\omega} \int_{\omega_0 - \Delta\omega}^{\omega_0 + \Delta\omega} \cos\omega t d\omega$$

Where $2\Delta\omega$ *is the total bandwidth. Calculate the time response and plot this function for values of* $\Delta\omega = \pi$ and 50π*, for a constant* $\omega_0 = 100\pi$.

Solution

$$y(t) = \frac{1}{2\Delta\omega} \int_{\omega_0 - \Delta\omega}^{\omega_0 + \Delta\omega} \cos\omega t d\omega$$

Expanding this expression gives

$$y(t)=\frac{1}{2\Delta\omega}\left[\frac{\sin\omega t}{t}\right]_{\omega_0-\Delta\omega}^{\omega_0+\Delta\omega}$$

$$y(t)=\frac{1}{2\Delta\omega t}\left[\sin(\omega_0+\Delta\omega)t-\sin(\omega_0-\Delta\omega)t\right]$$

Hence

$$y(t)=\frac{1}{2\Delta\omega t}\left[\sin\omega_0 t\cos\Delta\omega t+\cos\omega_0 t\sin\Delta\omega t\right]-$$
$$\frac{1}{2\Delta\omega t}\left[\sin\omega_0 t\cos\Delta\omega t-\cos\omega_0 t\sin\Delta\omega t\right]$$

This reduces to

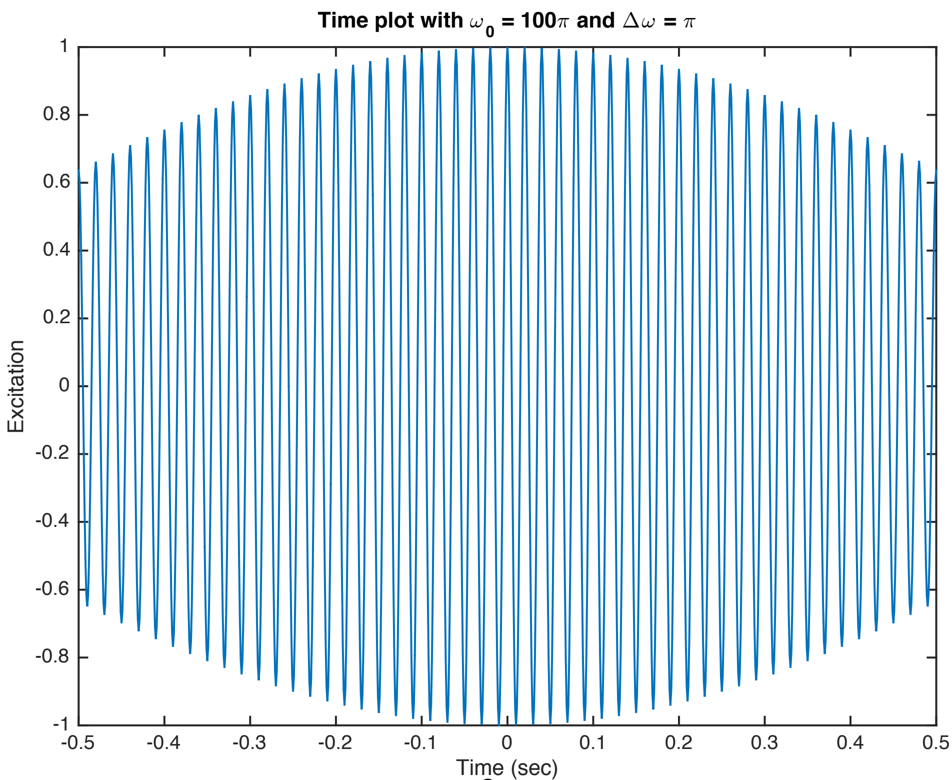
$$y(t)=\frac{1}{\Delta\omega t}\left[\cos\omega_0 t\sin\Delta\omega t\right]$$

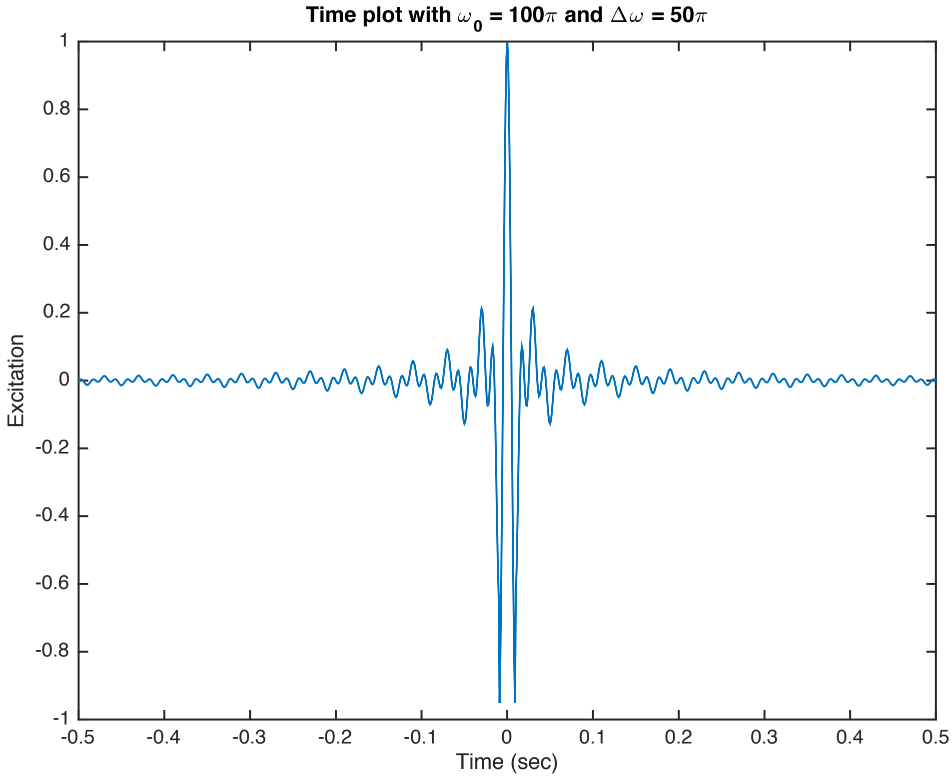
and this can be re-grouped as

$$\therefore\, y(t)=\frac{1}{\Delta\omega t}\left\{\cos(\omega_0 t)\sin(\Delta\omega t)\right\}=\cos(\omega_0 t)\left(\frac{\sin\Delta\omega t}{\Delta\omega t}\right)$$

The last term on the right hand side of this equation is often called the 'sinc' function.

The figures show plots of *y(t)* with $\omega_0=100\pi,\,\Delta\omega=\pi$, and $\omega_0=100\pi,\,\Delta\omega=50\pi$





The script for this solution is 'Problem2p5.m'.

Problem 2.6

A scatter diagram contains 1000 data points which are described by the parameters x_i and y_i. It has been established that

$$\sum_{i=1}^{1000}\left(x_i-\bar{x}\right)^2=73570 \qquad \sum_{i=1}^{1000}\left(x_i-\bar{x}\right)\left(y_i-\bar{y}\right)=-23600 \qquad \sum_{i=1}^{1000}\left(y_i-\bar{y}\right)^2=53860$$

where $\bar{x}=-3, \bar{y}=2$, are mean values. Derive the size and orientation of the ellipse which defines the standard deviation of this set of data. What is the probability of observing a point on the diagram within the interval $x=7\pm 1,\ y=2\pm\frac{1}{2}$?

Solution

Using the information given, the variance matrix is given by

$$\sum = \begin{bmatrix} 73.57 & -23.6 \\ -23.6 & 53.86 \end{bmatrix}$$

This is diagonalised with the eigenfunctions from the problem

$$\begin{bmatrix} 73.57-\lambda & -23.6 \\ -23.6 & 53.86-\lambda \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix}$$

To establish the eigenvalues

$$(73.57-\lambda)(53.86-\lambda)-23.6*23.6=0$$

Leading to eigenvalues $\lambda = 38.14,\ 89.92$

The mode shapes (eigenvectors) are given by $\begin{Bmatrix} 0.5544 \\ 0.8323 \end{Bmatrix}, \ \begin{Bmatrix} 0.8323 \\ -0.5544 \end{Bmatrix}$.

From this the standard deviations (semi-axes of the ellipse) are given by

$$\sigma_1 = \sqrt{38.14} = 6.18 \qquad\qquad \sigma_2 = \sqrt{89.29} = 9.45$$

The angle of axis orientation is given by $\theta = \dfrac{180}{\pi} * \cos^{-1}(0.5544) = -56^{\circ}$

The minus sign arises from either considering the sine term or simply reference to the original plot.

The probability of a reading within the specified region is given by the probability density at the specified point multiplied by the area, i.e. *dxdy*

The probability density is

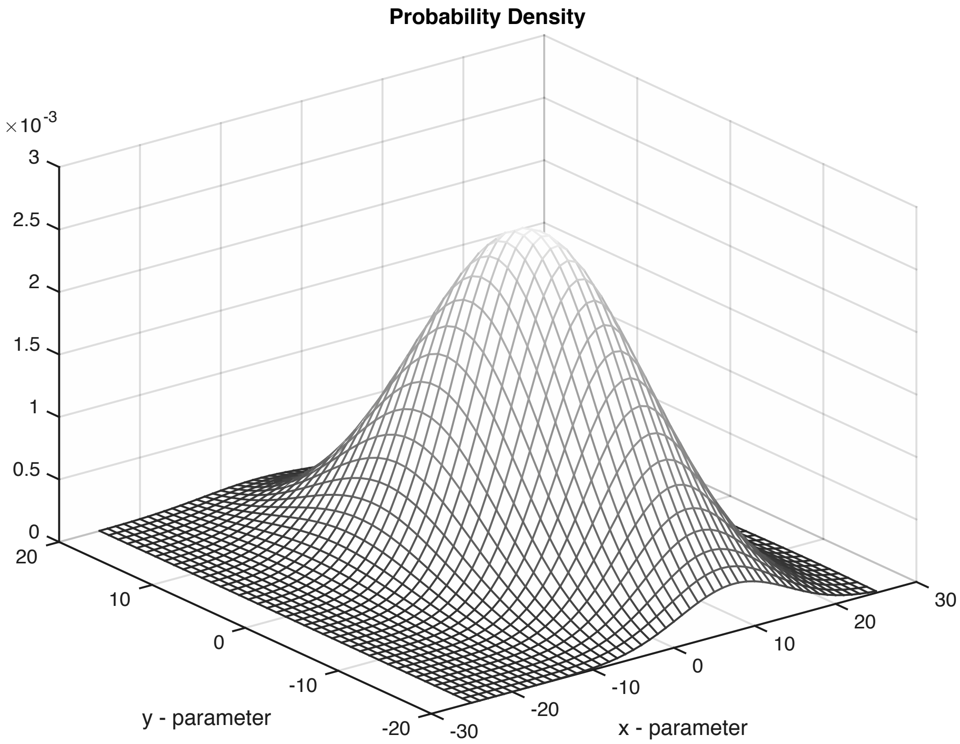
$$P(x_0, y_0) = \exp\left(-\begin{Bmatrix} 10 & 0 \end{Bmatrix} \Sigma^{-1} \begin{Bmatrix} 10 \\ 0 \end{Bmatrix}\right) = e^{-1.58} / c$$

where c is the normalisation constant given by $c = \frac{\sqrt{|\Sigma|}}{2\pi}$

The region of interest has area $dxdy$=2 . Hence Pr = 0.0025

i.e. Probability is about 0.25%

The figure shows a plot of the probability function.



The MATLAB script for this problem is ‘Problem2p6.m’.