CHAPTER 2

Problem 2.1 Statement:

Formulate an equation for the vector loop illustrated in Figure P.2.1. Consider that vector $\mathbf{V}_j$ always lies along the real axis.

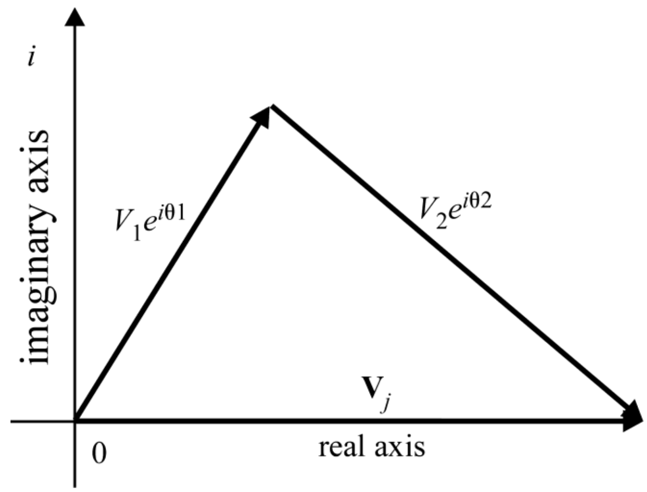

Figure P.2.1 Vector loop (3 vectors where $\mathbf{V}_j$ changes length) in 2-D complex space

Problem 2.1 Solution:

Taking the clockwise sum of the vector loop in Figure P.2.1 produces the equation

$$V_1 e^{i\theta_1} + V_2 e^{i\theta_2} - \mathbf{V}_j = 0 .$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} V_1 \cos\theta_1 + V_2 \cos\theta_2 - \mathbf{V}_j &= 0 \\ V_1 \sin\theta_1 + V_2 \sin\theta_2 &= 0 \end{aligned} .$$

Problem 2.2 Statement:

Formulate an equation for the vector loop illustrated in Figure P.2.2. Consider that vector $\mathbf{V}_j$ always lies along the real axis and vector $\mathbf{V}_3$ is always perpendicular to the real axis.

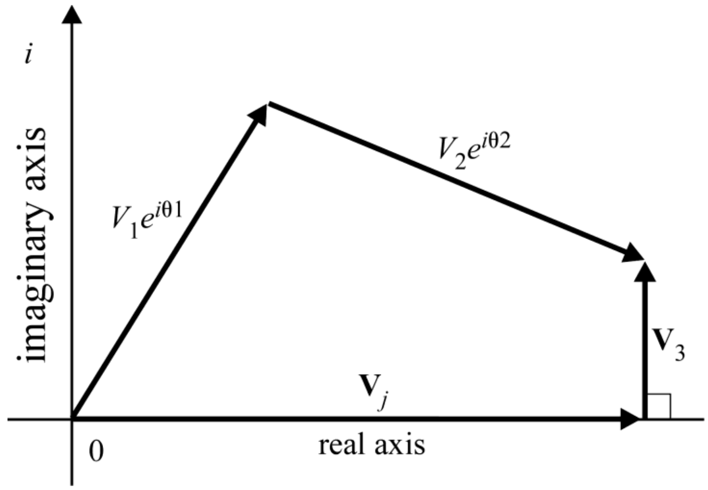

Figure P.2.2 Vector loop (4 vectors where $\mathbf{V}_j$ changes length) in 2-D complex space

Problem 2.2 Solution:

Taking the clockwise sum of the vector loop in Figure P.2.2 produces the equation

$$V_1 e^{i\theta_1} + V_2 e^{i\theta_2} - \mathbf{V}_3 - \mathbf{V}_j = 0 .$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} V_1 \cos\theta_1 + V_2 \cos\theta_2 - \mathbf{V}_j &= 0 \\ V_1 \sin\theta_1 + V_2 \sin\theta_2 - \mathbf{V}_3 &= 0 \end{aligned} .$$

Problem 2.3 Statement:

Calculate the first derivative of the vector loop equation solution from Problem 2.2. Consider only angles θ_1, θ_2 and vector $\mathbf{V}_j$ from Problem 2 to be time-dependent.

Problem 2.3 Solution:

Differentiating the vector loop equation solution from Problem 2.2 produces the equation

$$i\dot{\theta}_1 V_1 e^{i\theta_1} + i\dot{\theta}_2 V_2 e^{i\theta_2} - \dot{\mathbf{V}}_j = 0 .$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} -\dot{\theta}_1 V_1 \sin\theta_1 - \dot{\theta}_2 V_2 \sin\theta_2 - \dot{\mathbf{V}}_j &= 0 \\ \dot{\theta}_1 V_1 \cos\theta_1 + \dot{\theta}_2 V_2 \cos\theta_2 &= 0 \end{aligned} .$$

Problem 2.4 Statement:

Calculate the second derivative of the vector loop equation solution from problem 2.2. Consider only angles θ_1, θ_2 and vector $\mathbf{V}_j$ from Problem 2 to be time-dependent.

Problem 2.4 Solution:

Differentiating the vector loop equation solution from Problem 2.3 produces the equation

$$-\dot{\theta}_1^2 V_1 e^{i\theta_1} + i\ddot{\theta}_1 V_1 e^{i\theta_1} - \dot{\theta}_2^2 V_2 e^{i\theta_2} + i\ddot{\theta}_2 V_2 e^{i\theta_2} - \ddot{\mathbf{V}}_j = 0 .$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} -\dot{\theta}_1^2 V_1 \cos\theta_1 - \ddot{\theta}_1 V_1 \sin\theta_1 - \dot{\theta}_2^2 V_2 \cos\theta_2 - \ddot{\theta}_2 V_2 \sin\theta_2 = 0 \\ -\dot{\theta}_1^2 V_1 \sin\theta_1 + \ddot{\theta}_1 V_1 \cos\theta_1 - \dot{\theta}_2^2 V_2 \sin\theta_2 + \ddot{\theta}_2 V_2 \cos\theta_2 - \ddot{\mathbf{V}}_j = 0 \end{aligned} .$$

Problem 2.5 Statement:

Formulate an equation for the vector loop illustrated in Figure P.2.3.

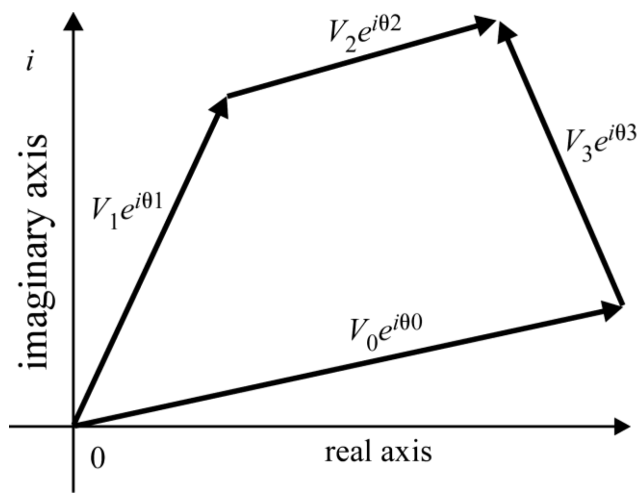

Figure P.2.3 Vector loop (4 vectors) in 2-D complex space

Problem 2.5 Solution:

Taking the clockwise sum of the vector loop in Figure P.2.3 produces the equation

$$V_1 e^{i\theta_1} + V_2 e^{i\theta_2} - V_3 e^{i\theta_3} - V_0 e^{i\theta_0} = 0 .$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} V_1 \cos\theta_1 + V_2 \cos\theta_2 - V_3 \cos\theta_3 - V_0 \cos\theta_0 &= 0 \\ V_1 \sin\theta_1 + V_2 \sin\theta_2 - V_3 \sin\theta_3 - V_0 \sin\theta_0 &= 0 \end{aligned}.$$

Problem 2.6 Statement:

Calculate the first derivative of the vector loop equation solution from Problem 2.5. Consider only angles θ_1, θ_2 and θ_3 from Problem 5 to be time-dependent.

Problem 2.6 Solution:

Differentiating the vector loop equation solution from Problem 2.5 produces the equation

$$i\dot{\theta}_1 V_1 e^{i\theta_1} + i\dot{\theta}_2 V_2 e^{i\theta_2} - i\dot{\theta}_3 V_3 e^{i\theta_3} = 0.$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} -\dot{\theta}_1 V_1 \sin\theta_1 - \dot{\theta}_2 V_2 \sin\theta_2 + \dot{\theta}_3 V_3 \sin\theta_3 &= 0 \\ \dot{\theta}_1 V_1 \cos\theta_1 + \dot{\theta}_2 V_2 \cos\theta_2 - \dot{\theta}_3 V_3 \cos\theta_3 &= 0 \end{aligned}.$$

Problem 2.7 Statement:

Calculate the second derivative of the vector loop equation solution from Problem 2.5. Consider only angles θ_1, θ_2 and θ_3 from Problem 5 to be time-dependent.

Problem 2.7 Solution:

Differentiating the vector loop equation solution from Problem 2.6 produces the equation

$$-\dot{\theta}_1^2 V_1 e^{i\theta_1} + i\ddot{\theta}_1 V_1 e^{i\theta_1} - \dot{\theta}_2^2 V_2 e^{i\theta_2} + i\ddot{\theta}_2 V_2 e^{i\theta_2} + \dot{\theta}_3^2 V_3 e^{i\theta_3} - i\ddot{\theta}_3 V_3 e^{i\theta_3} = 0.$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} -\dot{\theta}_1^2 V_1 \cos\theta_1 - \ddot{\theta}_1 V_1 \sin\theta_1 - \dot{\theta}_2^2 V_2 \cos\theta_2 - \ddot{\theta}_2 V_2 \sin\theta_2 + \dot{\theta}_3^2 V_3 \cos\theta_3 + \ddot{\theta}_3 V_3 \sin\theta_3 &= 0 \\ -\dot{\theta}_1^2 V_1 \sin\theta_1 + \ddot{\theta}_1 V_1 \cos\theta_1 - \dot{\theta}_2^2 V_2 \sin\theta_2 + \ddot{\theta}_2 V_2 \cos\theta_2 + \dot{\theta}_3^2 V_3 \sin\theta_3 - \ddot{\theta}_3 V_3 \cos\theta_3 &= 0 \end{aligned}.$$

Problem 2.8 Statement:

Formulate an equation for the vector loop illustrated in Figure P.2.4.

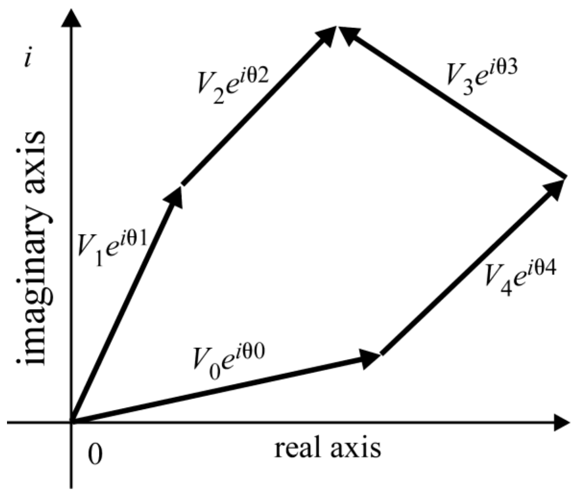

Figure P.2.4 Vector loop (5 vectors) in 2-D complex space

Problem 2.8 Solution:

Taking the clockwise sum of the vector loop in Figure P.2.4 produces the equation

$$V_1e^{\theta_1} + V_2e^{\theta_2} - V_3e^{\theta_3} - V_4e^{\theta_4} - V_0e^{\theta_0} = 0\,.$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned} V_1\cos\theta_1 + V_2\cos\theta_2 - V_3\cos\theta_3 - V_4\cos\theta_4 - V_0\cos\theta_0 = 0 \\ V_1\sin\theta_1 + V_2\sin\theta_2 - V_3\sin\theta_3 - V_4\cos\theta_4 - V_0\sin\theta_0 = 0 \end{aligned}\,.$$

Problem 2.9 Statement:

Calculate the first derivative of the vector loop equation solution from Problem 2.8. Consider only angles θ_1, θ_2, θ_3 and θ_4 from Problem 8 to be time-dependent.

Problem 2.9 Solution:

Differentiating the vector loop equation solution from Problem 2.8 produces the equation

$$i\dot{\theta}_1V_1e^{i\theta_1} + i\dot{\theta}_2V_2e^{i\theta_2} - i\dot{\theta}_3V_3e^{i\theta_3} - i\dot{\theta}_4V_4e^{i\theta_4} = 0\,.$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned}
-\dot{\theta}_1 V_1 \sin\theta_1 - \dot{\theta}_2 V_2 \sin\theta_2 + \dot{\theta}_3 V_3 \sin\theta_3 + \dot{\theta}_4 V_4 \sin\theta_4 &= 0 \\
\dot{\theta}_1 V_1 \cos\theta_1 + \dot{\theta}_2 V_2 \cos\theta_2 - \dot{\theta}_3 V_3 \cos\theta_3 - \dot{\theta}_4 V_4 \cos\theta_4 &= 0
\end{aligned}.$$

Problem 2.10 Statement:

Calculate the second derivative of the vector loop equation solution from Problem 2.8. Consider only angles θ_1, θ_2, θ_3 and θ_4 from Problem 8 to be time-dependent.

Problem 2.10 Solution:

Differentiating the vector loop equation solution from Problem 2.9 produces the equation

$$-\dot{\theta}_1^2 V_1 e^{i\theta_1} + i\ddot{\theta}_1 V_1 e^{i\theta_1} - \dot{\theta}_2^2 V_2 e^{i\theta_2} + i\ddot{\theta}_2 V_2 e^{i\theta_2} + \dot{\theta}_3^2 V_3 e^{i\theta_3} - i\ddot{\theta}_3 V_3 e^{i\theta_3} + \dot{\theta}_4^2 V_4 e^{i\theta_4} - i\ddot{\theta}_4 V_4 e^{i\theta_4} = 0.$$

When expanded and separated into real and imaginary terms, the vector loop equation becomes

$$\begin{aligned}
&-\dot{\theta}_1^2 V_1 \cos\theta_1 - \ddot{\theta}_1 V_1 \sin\theta_1 - \dot{\theta}_2^2 V_2 \cos\theta_2 - \ddot{\theta}_2 V_2 \sin\theta_2 + \dot{\theta}_3^2 V_3 \cos\theta_3 + \ddot{\theta}_3 V_3 \sin\theta_3 + \dot{\theta}_4^2 V_4 \cos\theta_4 \\
&+\ddot{\theta}_4 V_4 \sin\theta_4 = 0 \\
&-\dot{\theta}_1^2 V_1 \sin\theta_1 + \ddot{\theta}_1 V_1 \cos\theta_1 - \dot{\theta}_2^2 V_2 \sin\theta_2 + \ddot{\theta}_2 V_2 \cos\theta_2 + \dot{\theta}_3^2 V_3 \sin\theta_3 - \ddot{\theta}_3 V_3 \cos\theta_3 + \dot{\theta}_4^2 V_4 \sin\theta_4 \\
&-\ddot{\theta}_4 V_4 \cos\theta_4 = 0
\end{aligned}.$$