Solutions to Problems for Chapter 2

2.1 Find the momentum and velocity of an electron with mass of 9.1x10^{-31} kg and de Broglie wavelength of 20 nm.

$$\lambda_D = \frac{h}{p} = \frac{h}{m_o v} \Rightarrow v = \frac{h}{m_o \lambda_D} = \frac{6.63\times10^{-34}\,J.s}{9.1\times10^{-31}\,kg\times20\times10^{-9}\,m} = 0.36\times10^{5}\,m/s$$

$$p = m_o v = 9.1\times10^{-31}\,kg\times0.36\times10^{5}\,m/s = 3.31\times10^{-26}\,kg.m/s$$

2.2 Find the frequency f (Hz) radian frequency ω (rad/s) and period T of the infra-red radiation with wave length of 1 μm. Find the energy of the infra-red photon. Express its energy both in Joules and eV.

$$f = \frac{c}{\lambda} = \frac{3\times10^{8}\,m/s}{1\times10^{-6}\,m} = 3\times10^{14}\,Hz$$

$$\omega = 2\pi f = 2\pi\times3\times10^{14} = 1.885\times10^{15}\,rad/s$$

$$T = \frac{1}{f} = \frac{1}{3\times10^{14}\,Hz} = 3.33\times10^{-15}\,s$$

$$E(eV) = \frac{1240}{\lambda(nm)} = \frac{1240}{1000} = 1.24eV \quad E(J) = 1.24eV\times1.6\times10^{-19}\,J/eV = 1.98\times10^{-19}\,J$$

2.3 The potential (in V) due to a point charge q at a distance r from the charge is given by $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$. (a) Evaluate the potential of nuclear charge +q in the hydrogen atom at a distance r=r$_1$=0.053 nm (radius of the first Bohr orbit). (b) Obtain the potential energy (PE) U=-qV of the electron with charge –q in the potential of the nucleus. Calculate the kinetic energy K$E = \frac{1}{2}\frac{1}{4\pi\varepsilon_o}\frac{q^2}{r_1}$ of the electron revolving in the first orbit. (c) Calculate the total energy E=KE+PE. Express your answers for PE (U), KE, and total energy E in joules (J) and electron volts (eV). (d) What is the linear momentum and the angular momentum of the electron in the ground state? (e) What is the time required to complete one orbit.

Given $V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r}$ and r=r$_1$=0.053 nm

$$V = \frac{1}{4\pi\varepsilon_o}\frac{q}{r_1} = \frac{1.602\times10^{-19}}{4\pi\times8.854\times10^{-12}\times0.053\times10^{-9}} = 27.2V$$

Potential energy (PE),

$$U = -qV = -1.6\times10^{-19}\,C\times27.2V = -4.35\times10^{-18}\,J$$

$$= -4.35\times10^{-18}\,J/1.6\times10^{-19}\,J/eV = -27.2\,eV$$

Or

$$U = -qV = -27.2\,eV = -27.2\,eV \times 1.6\times10^{-19}\,J/eV = -4.35\times10^{-18}\,J$$

Kinetic energy,

$$KE = \frac{1}{2}\frac{1}{4\pi\varepsilon_o}\frac{q^2}{r_1} = \frac{1}{2}\frac{1.6\times10^{-19}}{4\pi\times8.854\times10^{-12}\times0.053\times10^{-9}}$$
$$= 2.18\times10^{-18}\,J = 2.18\times10^{-18}\,J/1.6\times10^{-19}\,J/eV$$
$$= +13.6\ eV$$

Total energy,

$$E = KE + PE = +13.6\ eV - 27.2\,eV = -13.6\ eV$$
$$= -13.6\ eV\times1.602\times10^{-19}\,J/eV = -2.18\times10^{-18}\,J$$

Linear momentum

$$\lambda_D = \frac{h}{p} \Rightarrow p = \frac{h}{\lambda_D} = \frac{h}{2\pi r_1} = \frac{\hbar}{r_1} = \frac{1.055\times10^{-34}\,J.s}{0.053\times10^{-9}\,m} = 2.0\times10^{-24}\,kg.m/s$$

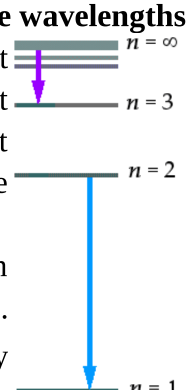
Angular momentum,

$$L = n\hbar = 1\hbar = 1.055\times10^{-34}\,J.s = 1.055\times10^{-34}\,kgm^2/s$$

(e)

$$T = \frac{1}{f} = \frac{2\pi r_1}{v_1} = \frac{2\pi r_1}{p_1/m_o}$$
$$= \frac{2\pi\times0.053\times10^{-9}\,m}{2.0\times10^{-24}\,kg.m/s/9.1\times10^{-31}\,kg} = 1.53\times10^{-16}\,s$$

2.4 Find (a) the longest wavelength in the Lyman series and (b) the shortest wavelength in the Paschen series. What is the energy of the photon in each case (a) and (b). Name the region of the electromagnetic spectrum in which these wavelengths are.



The image shows two electron transitions in hydrogen. The transition at lower right is a member of the Lyman series, and the transition at upper left is in the Paschen series. We want to calculate the longest wavelength in the Lyman series and the shortest wavelength in the Paschen series.

The longest wavelength in a series has the smallest change in energy. In the Lyman series this corresponds to the transition n_u = 2 to $n_\ell = 1$. The shortest wavelength in a series has the largest energy transition. In the Paschen series this transition corresponds to $n_u = \infty$ to $n_\ell = 3$ Use the following equation to calculate the corresponding wavelengths.

$$\lambda = \frac{1240\ nm}{13.6\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2}\right)} = \frac{91.18nm}{\left(\frac{1}{n_\ell^2} - \frac{1}{n_u^2}\right)}$$

Calculate the longest wavelength in the Lyman series:

$$\lambda == \frac{91.18nm}{\left(\frac{1}{2^2} - \frac{1}{3^2}\right)} = 121.5\ nm \qquad E(eV) = \frac{1240}{121.5} = 10.2\ eV$$

Calculate the longest wavelength in the Lyman series:

$$\lambda = \frac{91.18nm}{\left(\frac{1}{3^2} - \frac{1}{\infty^2}\right)} = 820.4\ nm \qquad E(eV) = \frac{1240}{820.4} = 1.51\ eV$$

The longest wavelength in the Lyman series is ultraviolet while the shortest in the Paschen is infrared. Visible light is therefore not possible from either series.

2.5 Calculate the wavelength and energy of a photon in the following transitions of an electron in a hydrogen atom. Which part of electromagnetic spectrum these photons of emitted light reside?

(a) $n=2 \to n=1$	$\Delta E = hf = 10.2eV \quad \lambda = 121.6nm$	UV
(b) $n=5 \to n=4$	$\Delta E = hf = 0.306eV \quad \lambda = 4054.5nm$	far infrared
(c) $n=10 \to n=9$	$\Delta E = hf = 0.0319eV \quad \lambda = 38.9\mu m$	microwave
(d) $n=8 \to n=2$	$\Delta E = hf = 3.188eV \quad \lambda = 389.2nm$	visible-violet
(e) $n=12 \to n=1$	$\Delta E = hf = 13.51eV \quad \lambda = 91.83nm$	UV
(f) $n=\infty \to n=1$	$\Delta E = hf = 13.6eV \quad \lambda = 91.23nm$	UV

2.6. (a) Find the de Broglie wavelength of the ground state (n=1) of the hydrogen atom. (b) What is the quantum number n of the hydrogen-atom orbit represented by the Figure F2.5? (c)What is the radius of the hydrogen-atom orbit represented by the Figure F2.5? (d) What is the velocity of the electron in the hydrogen-atom orbit represented by the Figure F2.5?

The Bohr model can be used to calculate the de Broglie wavelength of an electron in the ground state of a hydrogen atom. The orbit contains integer number of the de Broglie waves:

(a)

$$n\lambda_D = 2\pi r_n \Rightarrow \lambda_D = \frac{2\pi r_n}{n} = \frac{2\pi r_1}{1} = 2\pi \times 0.053\times 10^{-9} nm = 0.332\ nm$$

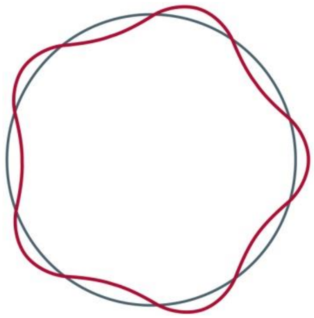


Fig. F2.5. Problem 2.6

(b) There are five wavelengths contained in the orbit shown in the figure, so we conclude the electron is in the n = 5 state.

(c) The de Broglie wavelength is proportional to the orbit number, and the number of wavelengths contained in the circumference is equal to the orbit number. These two facts require the radius to be proportional to n^2:

$$r_n = n^2 r_1$$

$$r_5 = 5^2\left(5.29\times10^{-11}\ \text{m}\right) = 1.32\ nm$$

(d)

$$v_n = \frac{n\hbar}{m_o r_n} \Rightarrow v_5 = \frac{5\hbar}{m_o r_5} = 0.44\times10^6\ \frac{m}{s}$$

Or

$$v_n = v_1\frac{Z}{n} \Rightarrow v_5 = 2.2\times10^6 m/s\,\frac{1}{5} = 0.44\times10^6 m/s$$

2.7. Hydrogen atom number 1 is known to be in the 4*f* state. (a) What is the energy of this atom? (b) What is the magnitude of this atom's orbital angular momentum? (c) Hydrogen atom number 2 is in the 5*d* state. Is this atom's energy greater than, less than, or the same as that of atom 1? Explain. (d) Is the magnitude of the orbital angular momentum of atom 1 greater than, less than, or the same as that of atom 2? Explain.

Hydrogen atom 1 is in the 4f state and hydrogen atom 2 is in the 5d state. The energy and orbital angular momentum of each are described by the quantum mechanical model of the atom. Calculate the energy in the 4f state with n = 4. Calculate the orbital angular momentum with $\ell = 3$ corresponding to the subshell f. To compare the energies of atoms 1 and 2, compare their principal quantum numbers. To compare the orbital angular momenta, compare their subshells.

$$E_4 = -\frac{(13.6\ \text{eV})}{4^2} = -0.850\ eV$$

$$L = \sqrt{\ell(\ell+1)}\ \frac{h}{2\pi} = \frac{\sqrt{3(4)}\left(6.63\times10^{-34}\ \text{J}\cdot\text{s}\right)}{2\pi} = 3.66\times10^{-34}\ \text{J}\cdot\text{s}$$

(c) The energy in atom 2 is greater than the energy in atom 1, because an n = 5 state is farther from the nucleus and has a smaller negative energy.

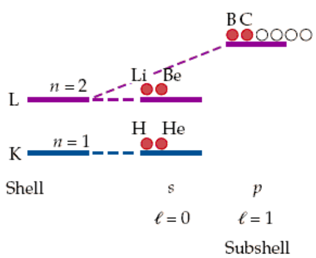
(d) The orbital angular momentum in atom 1 is greater than the orbital angular momentum in atom 2, because $\ell = 3$ for an f sublevel and $\ell = 2$ for a d sublevel.

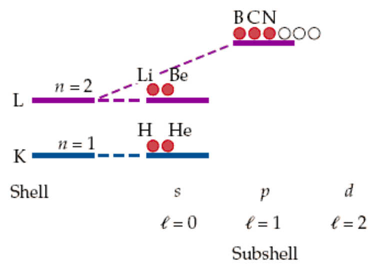
2.8. Give the electronic configuration for the ground state of carbon, nitrogen, silicon, phosphorus, boron, gallium and arsenic.

The figure shows how the levels are filled for a carbon atom. A carbon atom has six protons (Z = 6) and six electrons. Start filling at the n = 1 shell, for which there is only an s subshell. The 1s-subshell is filled with two electrons. Then start filling the n = 2 shell. First fill the 2s subshell with two electrons, and then place the remaining two electrons in the 2p subshell.

C (Z=6): $1s^2 2s^2 2p^2$

N (Z=7): $1s^2 2s^2 2p^3$





Nitrogen has seven protons (Z = 7) and seven electrons. Fill the 1s shell with the first two electrons. Fill the 2s subshell with the next two electrons. Place the last three electrons in the 2p subshell.

Silicon Si (Z=14) ➔ $1s^2 2s^2 2p^6 3s^2 3p^2$

Phosphorus P (Z= 15) ➔ $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^3$
Boron B (Z=5) ➔ $1s^2\ 2s^2\ 2p^1$
Gallium Ga (Z=31) ➔ $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10}\ 4s^2\ 4p^1$
Arsenic As (Z=33) ➔ $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 3d^{10} 4s^2\ 4p^3$

2.9. Suppose that the 5*d* subshell is filled in a certain atom. Write out the 10 sets of four quantum numbers (n, ℓ, m_ℓ, m_s) **for the electrons in this subshell.**

The 5d notation refers to the subshell for which n = 5 and $\ell = 2$. The fact that $\ell = 2$ means that the magnetic quantum numbers m_ℓ range from −2 to +2. For each m_ℓ there are two states corresponding to $m_s = \pm\frac{1}{2}$. Use these facts to write out the ten possible states in tabular format.

	n	ℓ	m_ℓ	m_s
$5d^1$	5	2	–2	$-\frac{1}{2}$
$5d^2$	5	2	–2	$\frac{1}{2}$
$5d^3$	5	2	–1	$-\frac{1}{2}$
$5d^4$	5	2	–1	$\frac{1}{2}$
$5d^5$	5	2	0	$-\frac{1}{2}$
$5d^6$	5	2	0	$\frac{1}{2}$
$5d^7$	5	2	1	$-\frac{1}{2}$
$5d^8$	5	2	1	$\frac{1}{2}$
$5d^9$	5	2	2	$-\frac{1}{2}$
$5d^{10}$	5	2	2	$\frac{1}{2}$

2.10 (a) In an $Al_xGa_{1-x}As$ /GaAs/ $Al_xGa_{1-x}As$ quantum well with electrons, find the value of x for $\Delta E_c = E_c^{AlGaAs} - E_c^{GaAs}$ is 0.238 eV. The bandgap as a function of x is given by

$$E_g = 1.426 + 1.247x$$

(b) Assume 2/3rd of the bandgap difference of $Al_xGa_{1-x}As$ goes to conduction band discontinuity. Since $0.238 \gg k_B T$, it is justified to assume quantum well with infinite boundaries. What should be the thickness L of the semiconductor layer to ensure that the difference between the ground (i.e. the lowest) energy level and the first excited level is equal to the thermal energy (k_BT) at room temperature (T = 300 K)?

(a) $\Delta E_c = \frac{2}{3}\Delta E_g = \frac{2}{3}\times 1.47\,eV\,x \Rightarrow x = \frac{3}{2}\times 0.238/1.47 = 0.242$

(b)

$$E_n = n^2\frac{\pi^2\hbar^2}{2mL^2} \Longrightarrow \Delta E_{2-1} = \left(2^2 - 1^2\right) = 3E_1 = k_B T \Rightarrow E_1 = \frac{0.0259}{3} = 0.00863\,eV$$

$$E_1 = \frac{\pi^2\hbar^2}{2m^* L^2} \Rightarrow L = \sqrt{\frac{\pi^2\hbar^2}{2m^* E_1}} = \sqrt{\frac{\pi^2\left(1.055\times10^{-34}\right)^2}{2\times0.067\times9.1\times10^{-31}\times0.00863\times1.6\times10^{-19}}}$$

$$= 25.5\times10^{-9}\,m = 25.5\,nm$$

2.11 Exciton in the semiconductors lessen the bandgap by the exciton energy:

$$E_g^{'} = E_g(eV) - \frac{m_r^*}{m_o\varepsilon_r^2}13.6\,eV$$

(a) For GaAs, determine the required photon energy to create an exciton. The reduced effective mass for exciton is $m_r^* = \left[\frac{1}{m_n^*} + \frac{1}{m_p^*}\right]^{-1} = \frac{m_n^* m_p^*}{m_n^* + m_p^*} = 0.0502 m_o$.

(b) The application of a dc electric field tends to separate the electron and the hole. Using Coulomb's law, show that the magnitude of the electric field between the electron and hole is

$$|\mathscr{E}| = \frac{1}{4\pi\varepsilon_0\varepsilon_r}\frac{q}{r_{ex}^2}$$

(c) For GaAs, determine $|\mathscr{E}|$**, the magnitude of an electric field that would break apart the exciton.**

(a)

$$m_r^* = \left[\frac{1}{m_n^*} + \frac{1}{m_p^*}\right]^{-1} = \frac{m_n^* m_p^*}{m_n^* + m_p^*} = 0.0502 m_o$$

$$E_g^{'} = 1.43 eV - \frac{0.0502}{13.2^2} 13.6 eV = 1.426\ eV$$

$$r_{ex} = \frac{\varepsilon_r m_o}{m_r^*} r_1 = \frac{13.3}{0.0502} 0.053 nm = 14.0\ nm$$

(b) The electron and hole in an exciton rotate at the center of mass which is in the middle of the exciton. The electric field is the force on a unit positive charge that is given

$$|\mathscr{E}| = \frac{1}{4\pi\varepsilon_o\varepsilon_r}\frac{q}{r_{ex}^2}$$

A unit charge kept at the center of mass will be attracted by electron and repelled by hole, making it twice as large as that of a single electron. The electric field in the middle of the exciton is thus $|\mathscr{E}| = \frac{2}{4\pi\varepsilon_o\varepsilon_r}\frac{q}{r_{ex}^2}$

(c) $|\mathscr{E}| = \frac{2}{4\pi\varepsilon_o\varepsilon_r}\frac{q}{r_{ex}^2} = \frac{2\times 1.6\times 10^{-19}}{4\pi \times 8.85\times 10^{-12} \times 13.3 \times (14.0\times 10^{-9})^2} = 1.10\times 10^6 V/m$

2.12. The conduction band minima in GaP occur right at the first Brillouin zone boundary along <100> directions in k-space. Taking the constant energy surface to be ellipsoids with $m_\ell^* = 1.12 m_o$ **and** $m_t^* = 0.22 m_o$**, determine the density-of-states effective mass of electrons in GaP. What will be the conductivity effective mass?**

The DOS effective mass for electrons in GaP will be, $m_{dsn}^{*} = g_v^{2/3}\left(m_t^{*2} m_\ell^{*}\right)^{1/3}$

where, $g_v = 6$ is the valley degeneracy and $m_\ell^{*} = 1.12\,m_o$ and $m_t^{*} = 0.22\,m_o$

The calculated value is, $m_{dsn}^{*} = g_v^{2/3}\left(m_t^{*2} m_\ell^{*}\right)^{1/3} = 6^{2/3}(0.22^2 \times 1.12)^{1/3} m_0 = 1.28 m_0$

The conductivity effective mass of electrons in GaP is,

$$\frac{1}{m_{ce}^{*}} = \frac{1}{6}\left(\frac{2}{m_\ell^{*}} + \frac{4}{m_t^{*}}\right) = \frac{1}{6}\left(\frac{2}{1.12} + \frac{4}{0.22}\right)\frac{1}{m_o} = \frac{3.33}{m_o} \Rightarrow m_{ce}^{*} = 0.30 m_o$$

2.13. In Si, what fraction of the holes are heavy holes? How does this fraction change in a 2D quantum well? Discuss and obtain effective density of states effective mass.

The DOS effective mass of holes is given by,

$m_{dsp}^{*} = \left[m_{hh}^{*3/2} + m_{\ell h}^{*3/2}\right]^{2/3} = 0.54 m_o$, where $m_{hh} = 0.48 m_0$ and $m_{lh} = 0.16 m_0$ for Si.

The fraction of heavy holes present in the effective mass is,

$$f_{hh} = \frac{m_{hh}^{*3/2}}{m_{hh}^{*3/2} + m_{lh}^{*3/2}} = 0.83$$

In 2D quantum well, DOS is directly proportional to the effective mass. So the DOS effective mass will be,

$m_{dsp}^{*} = \left[m_{hh} + m_{lh}\right] = 0.64 m_o$

And the fraction of heavy holes will be,

$$f_{hh} = \frac{m_{hh}}{m_{hh} + m_{lh}} = 0.75$$

2.14 Estimate tunneling coefficient for an electron in gallium arsenide ($m^{*} = 0.067\,m_o$), tunneling through a rectangular barrier with a barrier height U_O =1 eV and a barrier width of 2.0 nm. The electron energy is 0.25 eV.

Given $m_e^{*} = 0.067 m_0$; $U_0 = 1eV$; d=2nm and E=0.25eV

The tunneling coefficient is given by,

$$T = \left|\frac{F}{A}\right|^2 = \frac{4}{4\cosh^2(\alpha d) + \left[\frac{\alpha}{k} - \frac{k}{\alpha}\right]^2 \sinh^2(\alpha d)} \qquad E < U_o$$

where

$$k = \sqrt{\frac{2m_e^{*}E}{\hbar^2}} = \sqrt{\frac{2 \times 0.067 \times 9.1 \times 10^{-31} \times 1.602 \times 10^{-19}}{\left(1.055 \times 10^{-34}\right)^2}} = 6.63 \times 10^{8}$$

$$\alpha = \sqrt{\frac{2m_s^*\left(U_0 - E\right)}{\hbar^2}} = \sqrt{\frac{2\times 0.067\times 9.1\times 10^{-31}\times\left(1-0.25\right)\times 1.602\times 10^{-39}}{\left(1.055\times 10^{-34}\right)^2}} = 1.148\times 10^9$$

With large α, the approximate expression given by $T \approx \frac{16E(U_o - E)}{U_o^2}e^{-2\alpha a}$ can be used, giving the same result:

$$T \approx \frac{16\times 0.25\times(1.0-0.25)}{1^2}e^{-2\times 1.15\times 10^9\times 2.0\times 10^{-9}} = 3\times 0.01 = 0.03$$

If WKB approximation is used, the expression is $T \approx e^{-2\alpha a} = 0.01$. Using MWKB approximation, you get $T \approx T_0 e^{-2\alpha a} = 0.03$

2.15 Estimate tunneling coefficient for free electrons with energy of 0.5 eV (m_o = 9.11x10^{-31} kg) incoming onto a parabolic barrier shown in the Fig. F2.11 with parabolic barrier $U_o = f_s x^2$ with f_s = 4x10^{16} eV/m^2 rising from x=0 to x=5.0 nm.

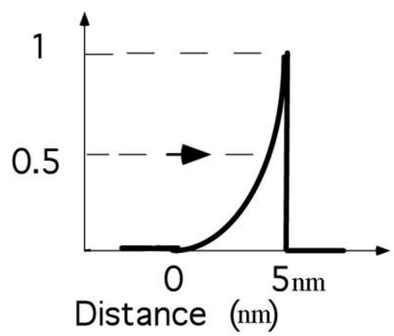


Fig. F2.11. Problem 2.15

The parabolic potential barrier is described by the following equation: $U_0 = f_s x^2$

where $f_s = 4\times 10^{16}\, eV/m^2 = 4\times 10^{16}\times 1.6\times 10^{-19}\, J/m^2 = 6.4\times 10^{-3}\, J/m^2$

WKB approximation gives the tunneling coefficient given by,

$$T \cong \exp\left[-2\int_{x_1}^{x_2}\left|\alpha(x)dx\right|\right]$$

where,

$$\alpha(x) = \sqrt{\frac{2m^*}{\hbar^2}(U_0 - E)}$$

x_1 is calculated using the approximation,

$$U_o\left(x_1\right)=E \Rightarrow 4\times10^{16}\times1.6\times10^{-19}x_1^2=0.5\times1.6\times10^{-19}\Rightarrow x_1=3.5\,nm,$$
$$x_2=5\,nm$$

$$\alpha(x)=\sqrt{\frac{2m^*}{\hbar^2}(U_0-E)}=\sqrt{\frac{2m^* f_s}{\hbar^2}(x^2-\frac{E}{f_s})}\approx\sqrt{\frac{2m^* f_s}{\hbar^2}}x,\ E\ll f_s$$

$$T\cong\exp\left[-2\int_{x_1}^{x_2}\left|\alpha(x)dx\right|\right]=\exp\left[-2\int_{3.5\times10^{-9}}^{5\times10^{-9}}\sqrt{\frac{2m^* f_s}{\hbar^2}}xdx\right]=\exp\left[-2\times6.37\times10^{-18}\times\sqrt{\frac{2m^* f_s}{\hbar^2}}\right]$$

$$T=\exp\left[-2\times6.37\times10^{-18}\times\sqrt{\frac{2\times9.1\times10^{-31}\times6.4\times10^{-3}}{\left(1.55\times10^{-34}\right)^2}}\right]=2.88\times10^{-4}$$

2.16. Obtain an expression for the tunneling coefficient of a rectangular barrier of width a using the WKB approximation and compare with the exact calculations

Tunneling coefficient for a rectangular barrier using the WKB approximation is,

$$T\approx e^{-2a\sqrt{\frac{2m^*}{\hbar^2}(U_o-E)}}$$

2.17. Calculate the tunneling probability for free electron incoming onto the triangular barrier and compare it with the tunneling probability of a rectangular barrier using MWKB approximation. Assume the height is each case is $U_o=1.0\ eV$. The triangular barrier is described by $U(x)=U_o\frac{x}{L}$ for $0\le x\le L=2.0\ nm$. The rectangular barrier has the same area as triangular barrier with $U(x)=U_o$ for $0\le x\le1.0\ nm$. The energy of the incoming electron $E=0.5\ eV$.

The triangular potential barrier (TPB) is described by $U(x)=q\mathcal{E}x$ where $\mathcal{E}=\frac{V_o}{L}=\frac{U_o}{qL}$. The

turning points for the TPB is $E=U(x_1)=U_o\frac{x_1}{L}\Rightarrow x_1=\frac{E}{U_o}L$ and $x_2=L$.

$$\alpha(x)=\sqrt{\frac{2m^*}{\hbar^2}(q\mathcal{E}x-E)}=\frac{\sqrt{2m^*q\mathcal{E}}}{\hbar}\sqrt{(x-x_1)}$$

$$T=T_o\exp(-2\int_{x_1}^{x_2}|\alpha(x)|\,dx)=T=T_o\exp(-2\int_{x_1}^{L}\frac{\sqrt{2m^*q\mathcal{E}}}{\hbar}\sqrt{(x-x_1)}\ dx$$

$$T=T_o\exp(-2\frac{\sqrt{2m^*q\mathcal{E}}}{\hbar}\int_{x_1}^{L}\sqrt{(x-x_1)}\ dx\ T=T_o\exp(-2\frac{\sqrt{2m^*q\mathcal{E}}}{\hbar}\frac{2}{3}(L-x_1)^{3/2}$$

For given parameters $T_o = 4$, $x_1 = 1.0\,nm$, $L = 2.0\,nm$, $\mathcal{E} = 5\times10^8 \frac{V}{m}$, T is found to be $T = 0.032$.

For the rectangular barrier T=0.00284.

2.18. The potential energy of electrons in a metal with a surface electric field is shown in the Fig. P2.12. The electron concentration is 10^{23} cm-3, the electronic effective mass is $9.11\text{x}10^{-31}$ kg. The velocity of electrons impinging on the metal surface is $\frac{1}{4}\sqrt{\frac{2E}{m_o}}$ with E= 4eV. (a) Find the strength of the electric field at the surface. (b) Calculate the electric current density J=nqv T (in A/m^2) of electrons escaping the metal.

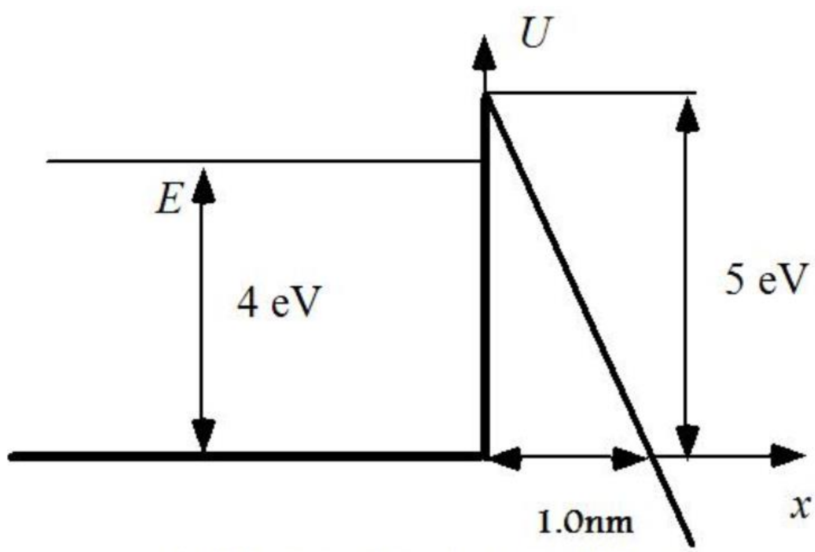


Fig. P2.12. Triangular barrier of 1.0 nm thickness

The strength of the electric field at the surface is $\mathcal{E} = 5/10^{-8} V/m = 5\times10^8 V/m$.

The strength of the electric field at the surface is $\mathcal{E}$ = $5/10^{-9}$ = $5\text{x}10^9$ V/m. The tunneling probability for a triangular barrier is given by

$$T = \exp\left(-\frac{4\sqrt{2m_n^* q\mathcal{E}(d - d_o)^3}}{3\hbar} \right)$$

Substituting the parameter values, we find T= $2.34\text{x}10^{-6}$, v^n = $2.97\text{x}10^5$ m/s,

J = $0.56\text{x}10^4$ A/m^2 = 0.56 A/cm^2

2.19. Al-SiO_2-Al, a metal-insulator-metal hetrojunction is a practical example of a rectangular barrier. Its barrier height is $q\phi_b = q\phi - q\chi = 4.1eV - 0.9eV = 3.2\ eV$, where $q\phi = 4.1eV$ is the work function of Aluminium and $q\chi = 0.9eV$ is electron affinity for SiO_2. Determine the tunnelling probability if the barrier width is 1 nm and the electron energy is 3.5 eV. State if any approximation used. Repeat for barrier width of 2 nm, 3nm, and 10 nm.

T=0.791 for d= 1 nm; T=0.515 for d= 2 nm; T=0.293 for d= 3 nm; and T=0.901 for d= 10 nm

2.20. In a GaAs HEMT (High Electron Mobility Transistor), the gate electric field with triangular potential well is $5\times10^7 V/m$. Calculate the first two energy levels.

$$\varepsilon_1 \approx \left[\frac{\hbar^2}{2m_3}\right]^{\frac{1}{3}} \left[\frac{3\pi q}{2}\mathscr{E}_t\left(0+\frac{3}{4}\right)\right]^{\frac{2}{3}}$$

$$\varepsilon_1 = \left[\frac{\left(1.055\times10^{-34}\right)^2}{2\times0.067\times9.11\times10^{-31}}\right]^{\frac{1}{3}} \left[\frac{9\pi\times1.6\times10^{-19}}{8}5\times10^7\right]^{\frac{2}{3}}$$
$$= 276.5\,meV$$

Through scaling of $\left(i+\frac{3}{4}\right)^{\frac{2}{3}} = \left(0+\frac{3}{4}\right)^{\frac{2}{3}}$ to $\left(1+\frac{3}{4}\right)^{\frac{2}{3}}$, 113.2 meV is multipled by $\left(\frac{7}{3}\right)^{\frac{2}{3}}$ to obtain $\varepsilon_2 = 276.5\,meV\times\left(\frac{7}{3}\right)^{\frac{2}{3}} = 486.4\ meV$

2.21. Silicon crystal has $5\times10^{22}\,atoms/cm^{-3}$. It is doped so 1 in 10,000 atoms are replaced by phosphorus. A donor impurity like phosphorus must replace a silicon atom (substitutional impurity) for silicon to become n-type. The interstitial phosphorus in the empty space between silicon atoms does not donate an electron. Assuming 30% of the implanted phosphorus is substitutional, determine the effective doping level and the electron concentration. Determine the average distance between the phosphorus atoms and between the mobile electrons.

The total number of active donors are $0.3\frac{5.0\times10^{22}}{10,000}\,atoms/cm^{-3} = 5.0\times10^{18}\,cm^{-3}$

Assuming at room temperature, every donor has donated an electron $n_3 = N_D = 5.0\times10^{18}\,cm^{-3}$.

Assuming that each atom or electron are length L apart, each one will occupy volume L3. Therefore number density should be 1/ L3=number desnity. For silicon atom 1/ L3=NSi giving $L = \left(\frac{1}{N_{Si}}\right)^{1/3} = \left(\frac{1}{5\times10^{28}m^3}\right)^{1/3} = 0.272\,nm$. For electrons interatomic spacing is $L = \left(\frac{1}{N_{Si}}\right)^{1/3} = \left(\frac{1}{1.5\times10^{24}m^3}\right)^{1/3} = 8.8\,nm$

Each electron is many atomic spacings apart.

2.22 An electron leaves a heated cathode with kinetic energy of 1 eV in free space. Determine the velocity, the wave number, the wavelength, and the frequency of the electron wave. Repeat if the electron acquires an energy of 10 keV while acclerted through a potential drop of 10 kV.

$$v = \sqrt{\frac{2\,KE}{m}} = \sqrt{\frac{2\times1eV\times1.6\times10^{-19}J/eV}{9.1\times10^{-31}kg}} = 5.93\times10^5\,m/s$$

$$p = mv = 9.1\times10^{-31}kg\times5.93\times10^5\,m/s = 5.40\times10^5\,kgm/s$$

$$\lambda_D = \frac{h}{p} = 1.23\,nm \qquad k = \frac{2\pi}{\lambda_D} = 5.12\times10^9\,m^{-1} \quad f = \frac{v}{\lambda_D} = 0.48\times10^{15}\,Hz$$

For KE=10,000 eV, 10^4, all factors are scaled by 10^2 giving

$$v = 5.93\times10^7\,m/s \quad p = 5.40\times10^7\,kgm/s$$

$$\lambda_D = \frac{h}{p} = 0.0123\,nm \quad k = \frac{2\pi}{\lambda_D} = 5.12\times10^{11}\,m^{-1} \quad f = \frac{v}{\lambda_D} = 0.48\times10^{19}\,Hz$$