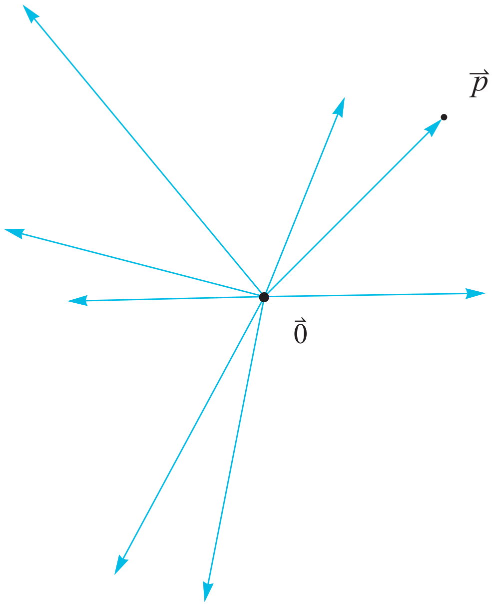

Figure 3.1 **Geometric vectors.** An arrow from the origin $\vec{0}$ to any point in the plane forms a *geometric vector.*

003x001.eps

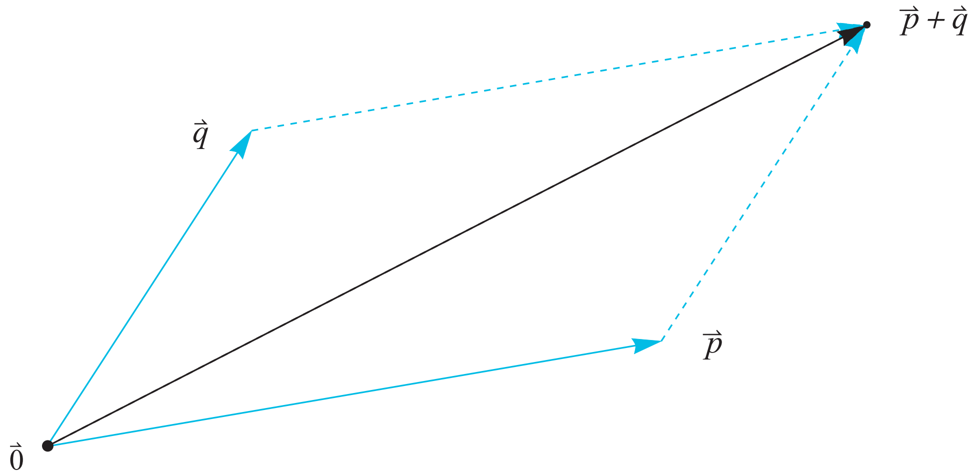

Figure 3.2 **Addition.** We form $\vec{p}+\vec{q}$ by sliding the tail of $\vec{q}$ to the tip of $\vec{p}$ (dashed arrow on the right) or vice-versa (dashed arrow above). Either way yields the black vector $\vec{p}+\vec{q}$.

003x002.eps

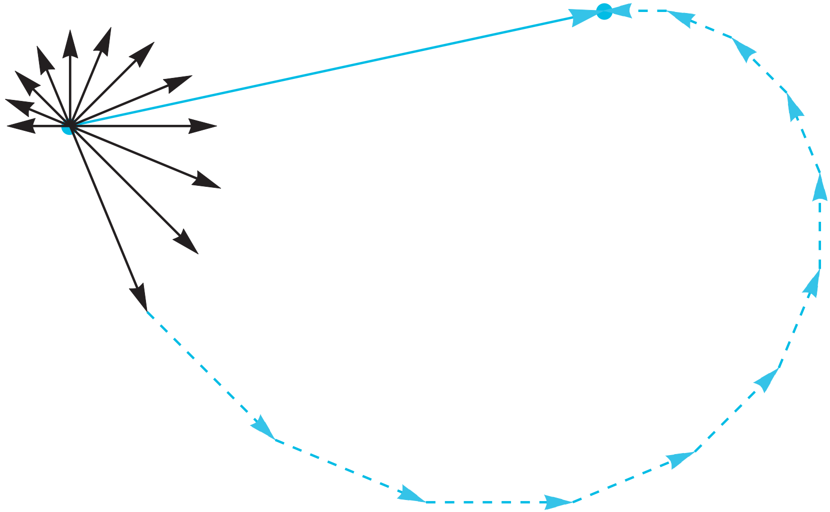

Figure 3.3 **Vector summation.** To sum the black vectors, we chain them tail-to-tip (dashed blue arrows). The solid blue arrow from the origin to the tip of the chain is the sum.

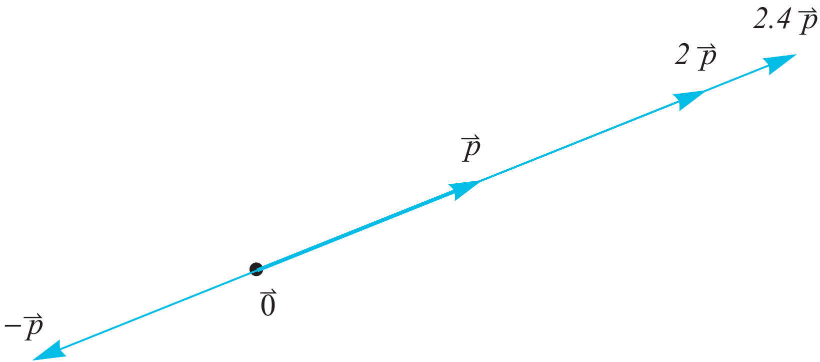

Figure 3.4 A geometric vector $\vec{p}$ and some of its scalar multiples.

003x004.eps

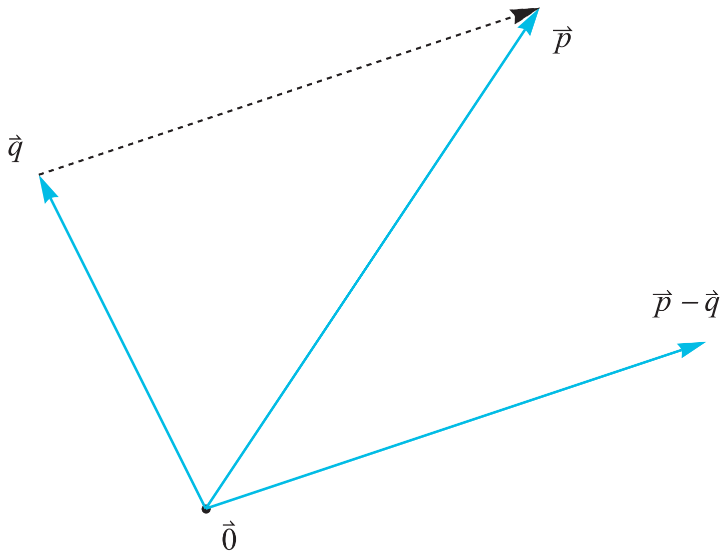

Figure 3.5 **Vector subtraction.** We find the difference $\vec{p} - \vec{q}$ by sketching an arrow from the tip of $\vec{q}$ to the tip of $\vec{p}$ (here dashed black), then translating its tail to the origin.

003x005.eps

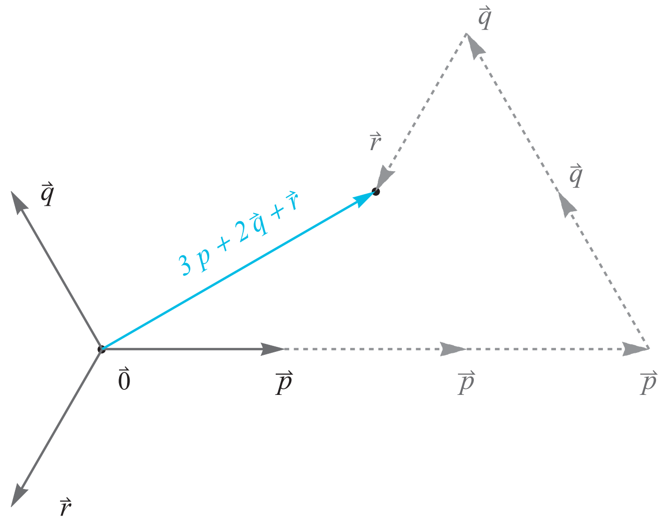

Figure 3.6 **Linear combination.** Multiplying $\vec{p}$, $\vec{q}$, and $\vec{r}$ by 3,2, and 1, respectively before summing, we get the linear combination $3\vec{p}+2\vec{q}+\vec{r}$ (blue arrow).

003x006.eps

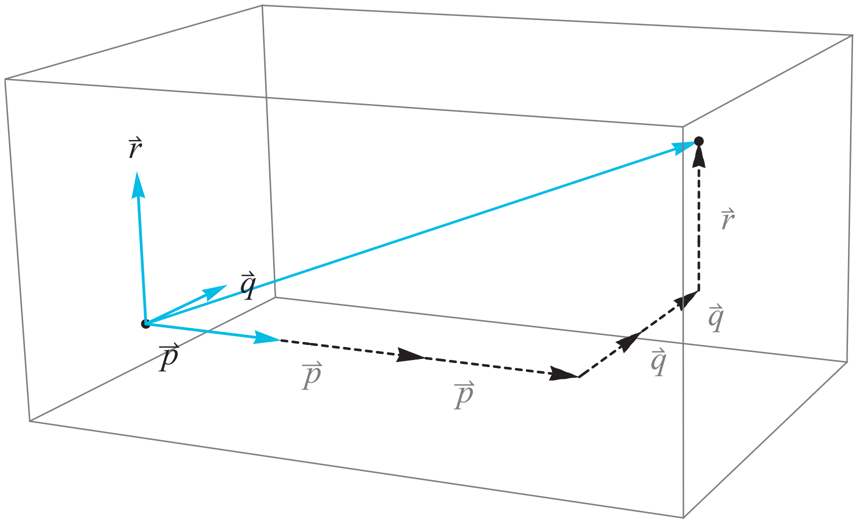

Figure 3.7 Linear combination in space works just as it does in the plane. Here the black dot marks the tip of $3\vec{p}+2\vec{q}+\vec{r}$ (not shown).

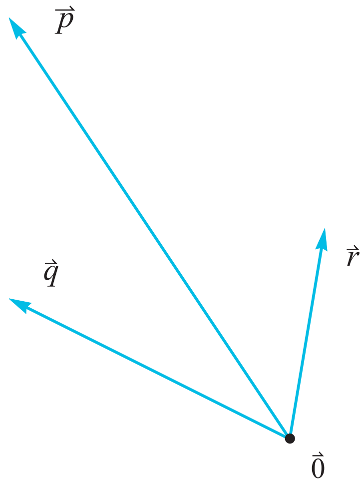

Figure 3.8 Template for Exercises 135 and 136.

003x008.eps

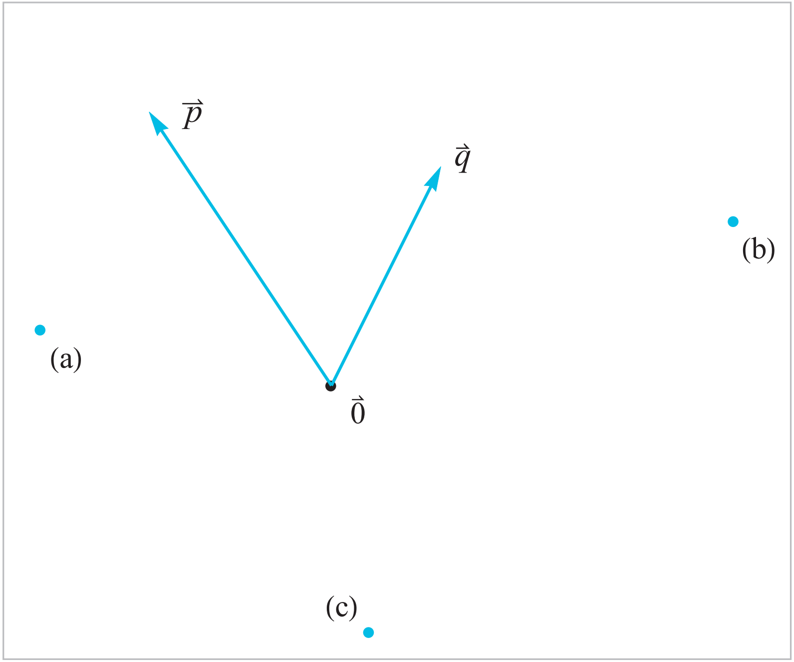

Figure 3.9 Template for Exercises 137 and 138.

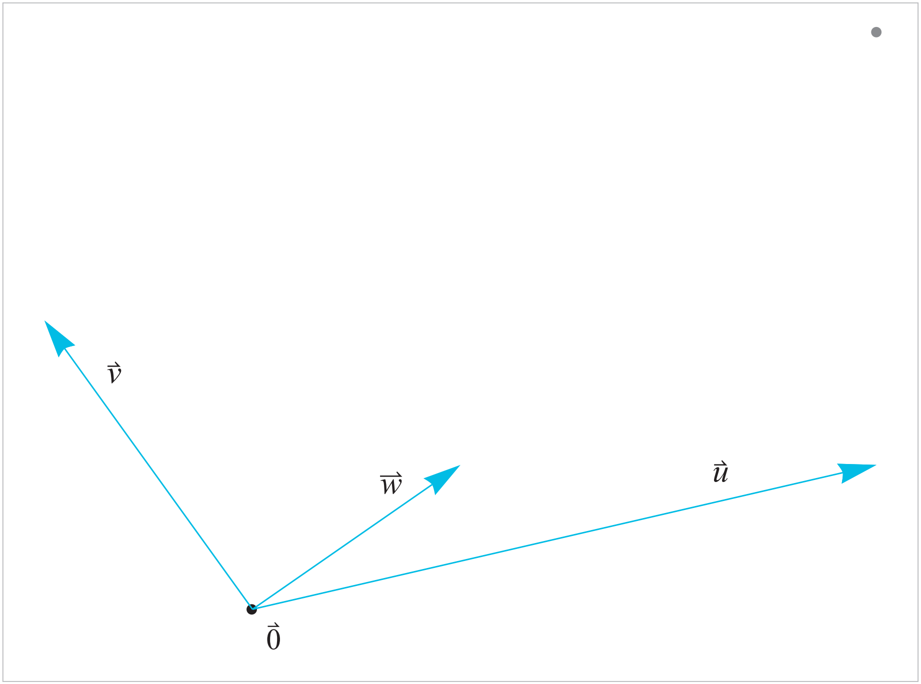

Figure 3.10 Template for Exercise 139.

003x010.eps

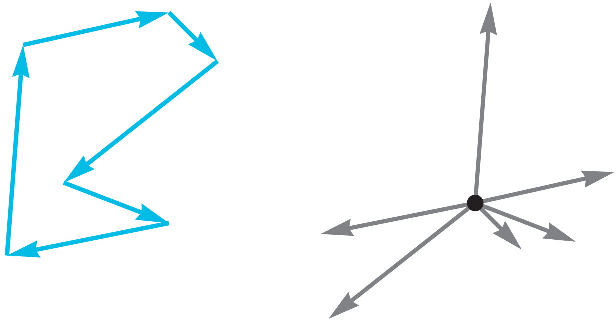

Figure 3.11 At left, a “tour” around a random polygon. At right, each edge is translated back to the origin to give a geometric vector. Why do the vectors at right sum to $\vec{0}$?

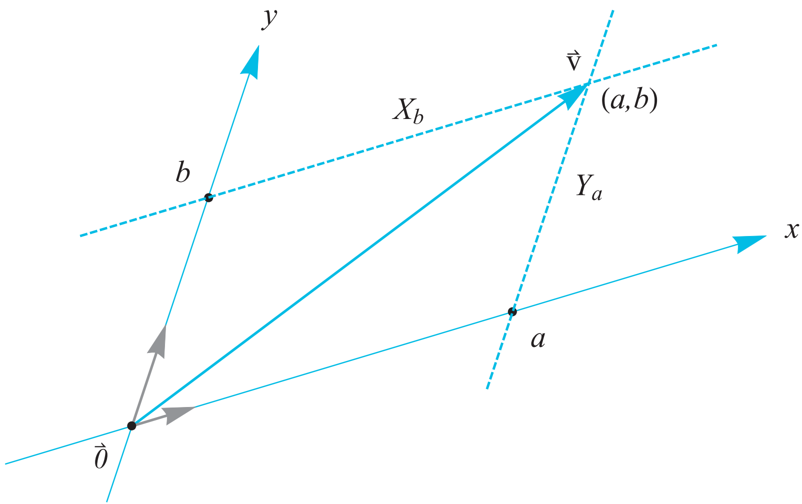

Figure 3.12 Coordinates. If we choose x and y axes, and introduce units (gray arrows), we can assign a numeric vector (a, b) to each geometric vector $\vec{v}$, and vice-versa.

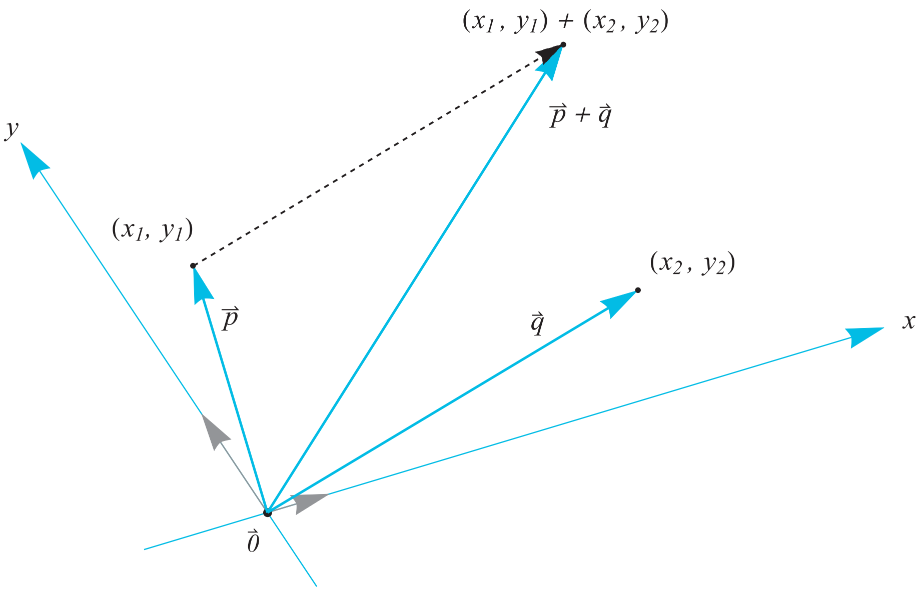

Figure 3.13 **Geometric/numeric duality.** We can add numeric vectors and then interpret geometrically, or add geometric vectors and interpret numerically—either way, we get the same result.

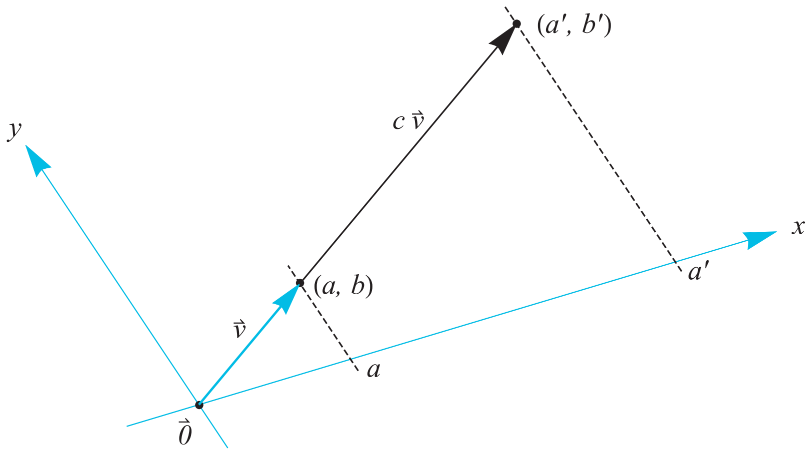

Figure 3.14 Scalar multiplication respects numeric/ geometric duality because the triangles cut off by the dashed lines are similar.

003x014.eps

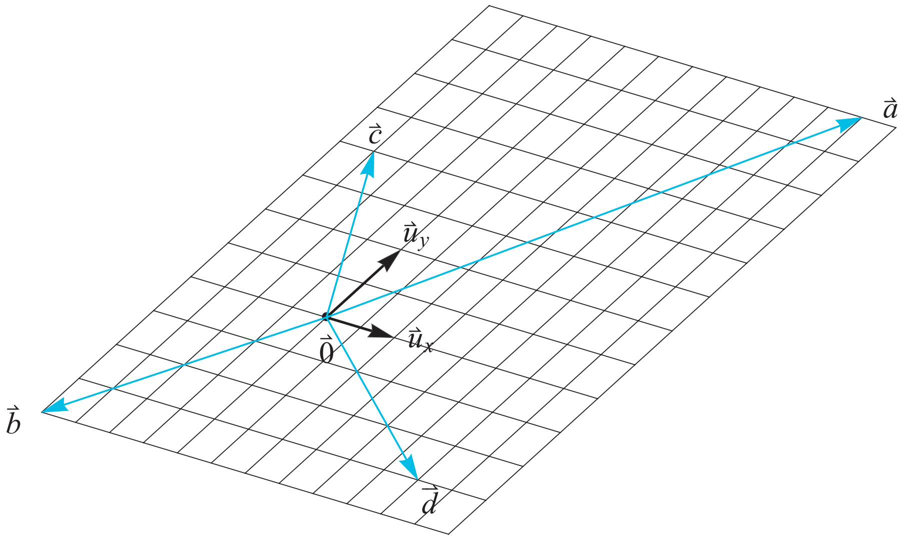

Figure 3.15 Graphic for Exercise 144.

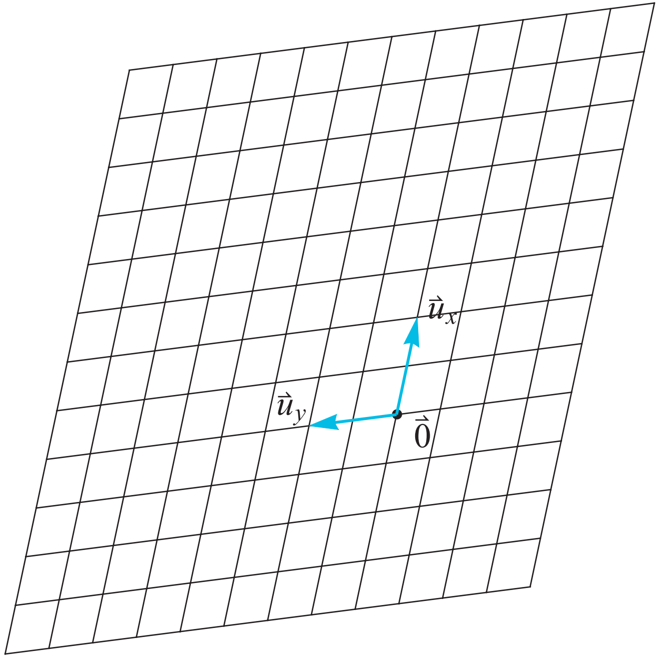

Figure 3.16 Template for Exercise 145.

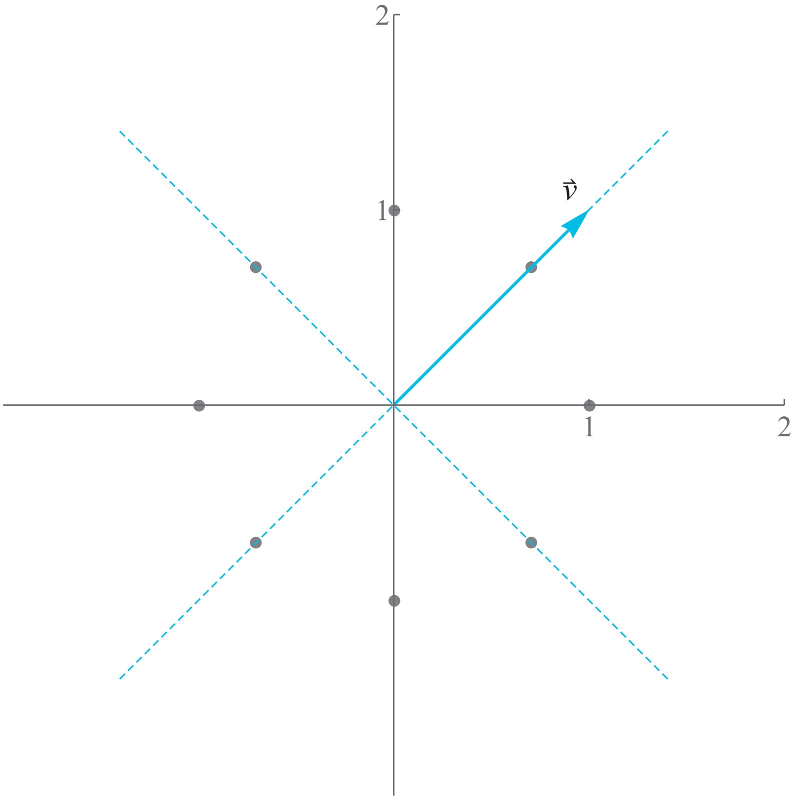

Figure 3.17 Using standard axes and units, the geometric vector $\vec{v}$ corresponds to the numeric vector (1,1). What numeric vector corresponds to $\vec{v}$ if we use the dashed blue axes? (Gray dots mark "1 unit of length").

003x017.eps

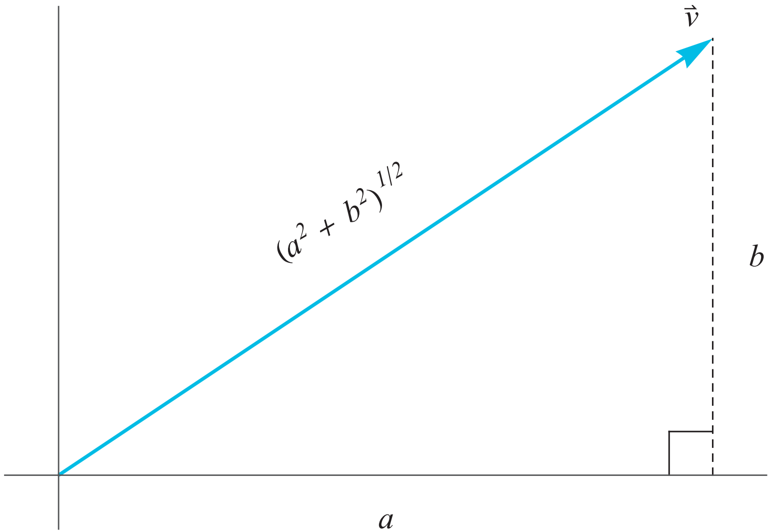

Figure 3.18 **Dot products and lengths.** In a euclidean coordinate system, a geometric vector $\vec{v}$ with numeric "address" $\mathbf{v} = (a, b)$ has length $|\vec{v}| = \left|\vec{v}\right| = \sqrt{a^2 + b^2} = \sqrt{\mathbf{v} \cdot \mathbf{v}}$.

003x018.eps

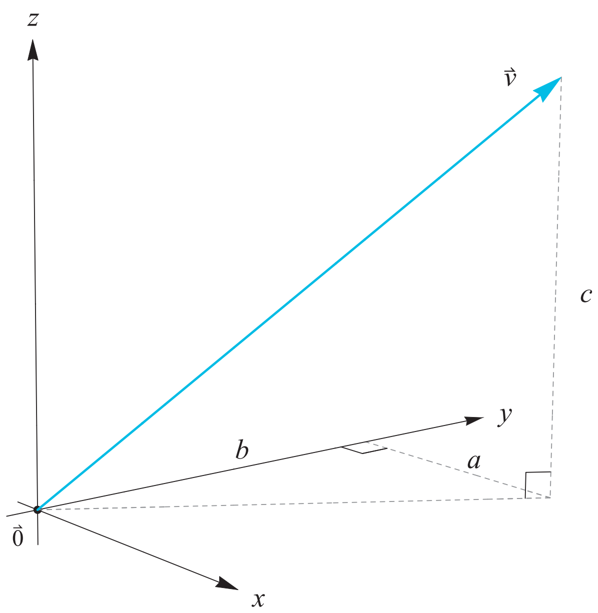

Figure 3.19 **Lengths in 3D.** In a 3D euclidean system that assigns numeric coordinates $\mathbf{v} = (a, b, c)$ to a geometric vector $\vec{v}$, the Pythagorean theorem, applied *twice*, gives $|\vec{v}| = \sqrt{a^2 + b^2 + c^2} = \sqrt{\mathbf{v} \cdot \mathbf{v}}$. Can you see why?

003x019.eps

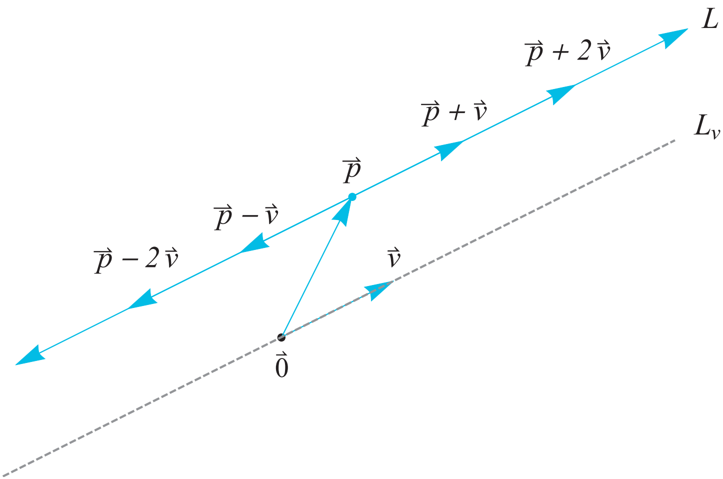

Figure 3.20 By adding all multiples of $\vec{v}$ to $\vec{p}$, we trace out a line (solid blue) through the tip of $\vec{p}$ and parallel to the (dashed) line generated by $\vec{v}$.

003x020.eps

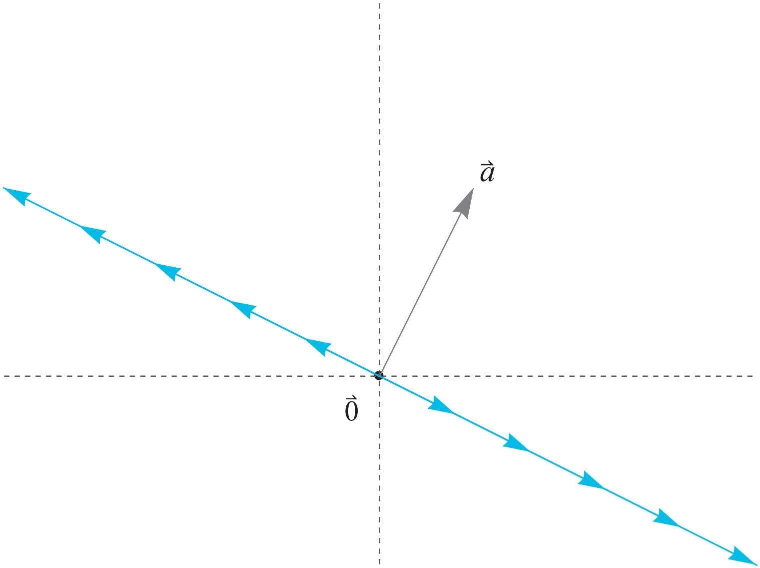

Figure 3.21 The vectors perpendicular to $\vec{a}$ form a *line* perpendicular to $\vec{a}$.

003x021.eps

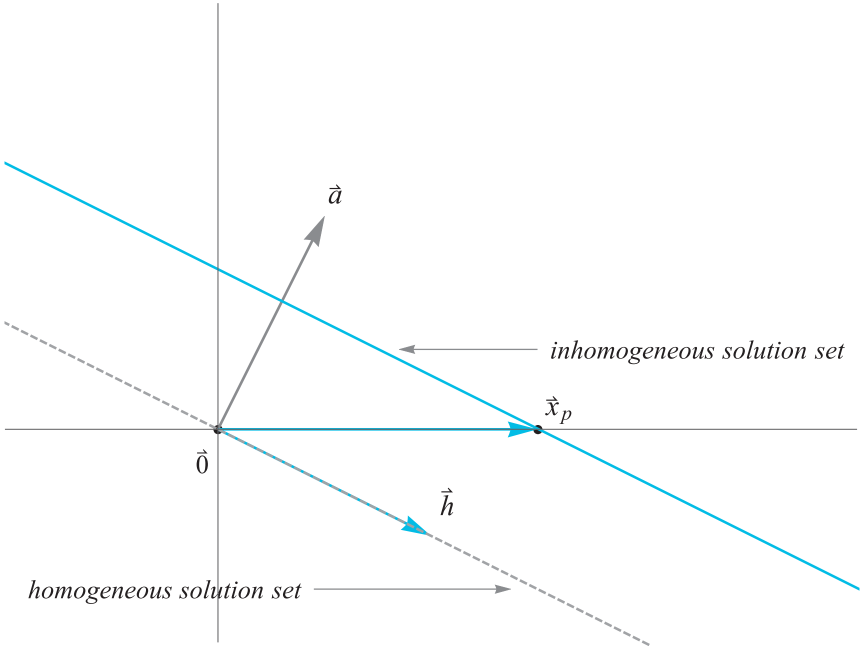

Figure 3.22 The solution set of an inhomogeneous equation $\mathbf{a} \cdot \mathbf{x} = b$ is a line that goes through the tip of $\vec{x}_p$, and runs parallel to the solution set of the homogeneous equation.

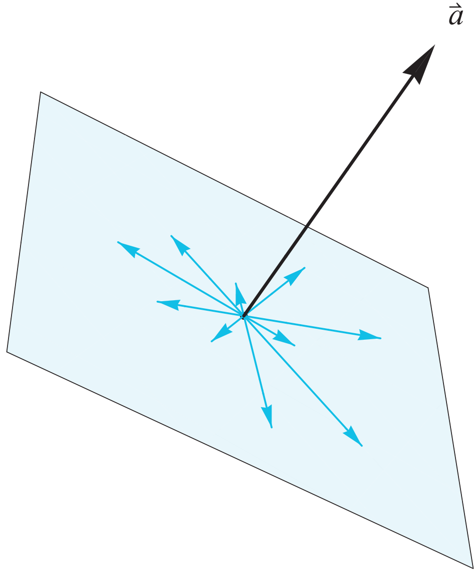

Figure 3.23 The vectors orthogonal to a non-zero vector $\vec{a}$ in space form a plane.

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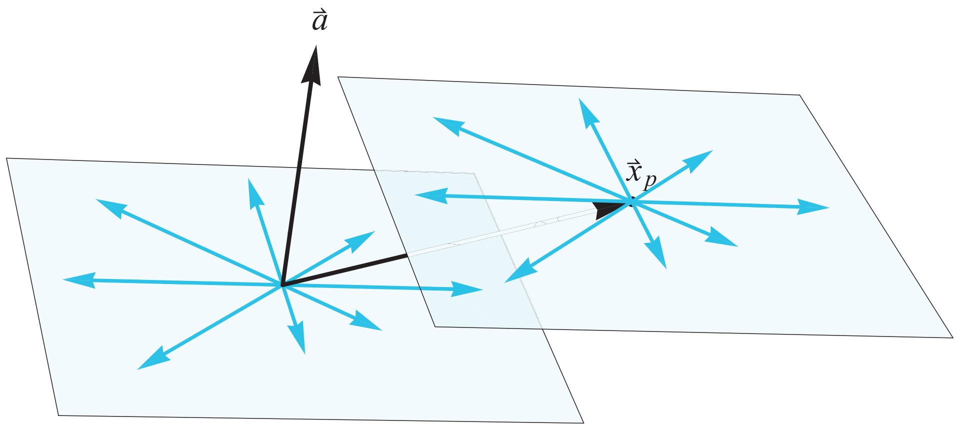

Figure 3.24 The solution set of $\mathbf{a} \cdot \mathbf{x} = b$ represents a plane orthogonal to the line generated by $\vec{a}$, but through $\vec{x}_p$ rather than the origin.

003x024.eps

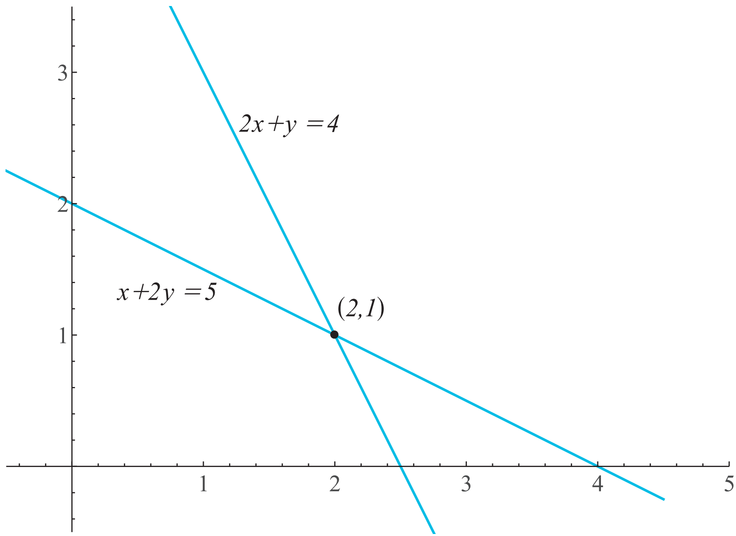

Figure 3.25 The solution of a linear system of two equations in two variables locates the intersection of two lines in the plane.

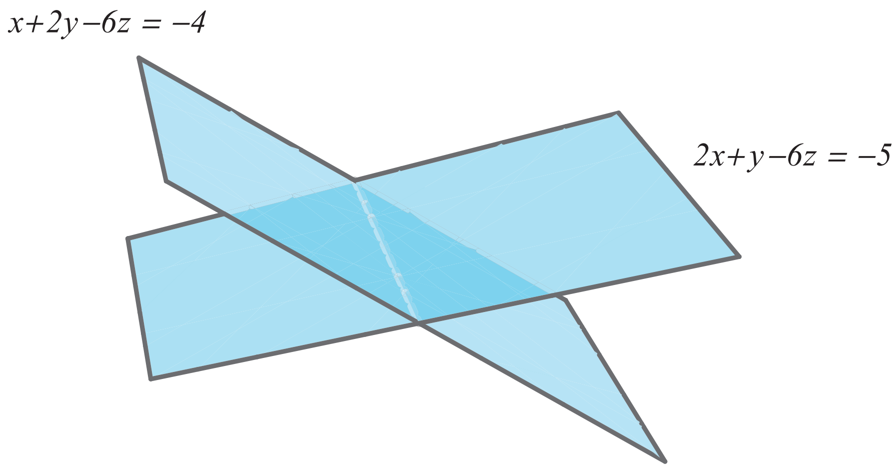

Figure 3.26 A system of two equations in 3 variables corresponds to a pair of planes in space. The solutions of the system are the vectors whose tips lie on the intersection of the planes—in this case, a line. The *row* interpretation of the system asks for this intersection.

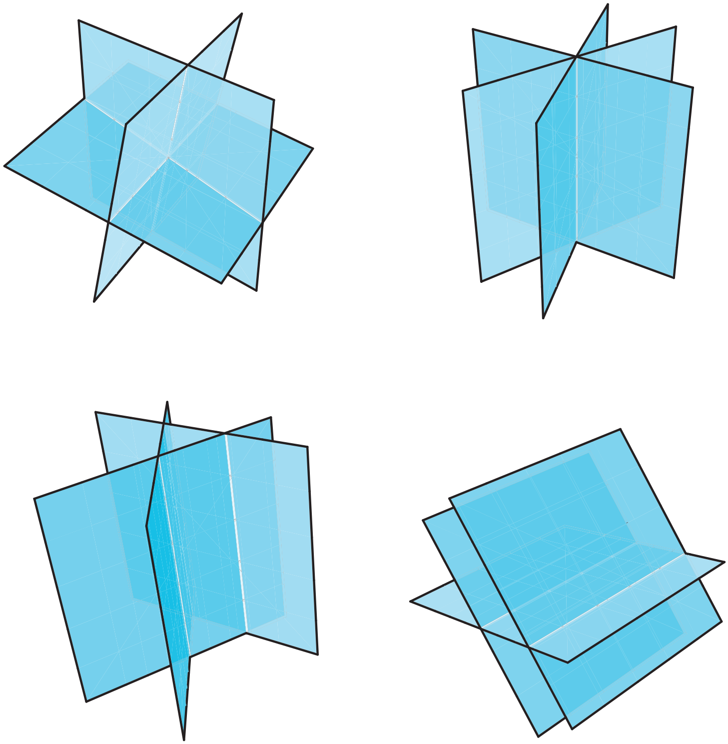

Figure 3.27 Typically, three planes in space intersect in a single point (upper left), corresponding to a 3-by-3 linear system with exactly one solution. But such a system may have an entire line of solutions (upper right), or none at all (bottom left and right).

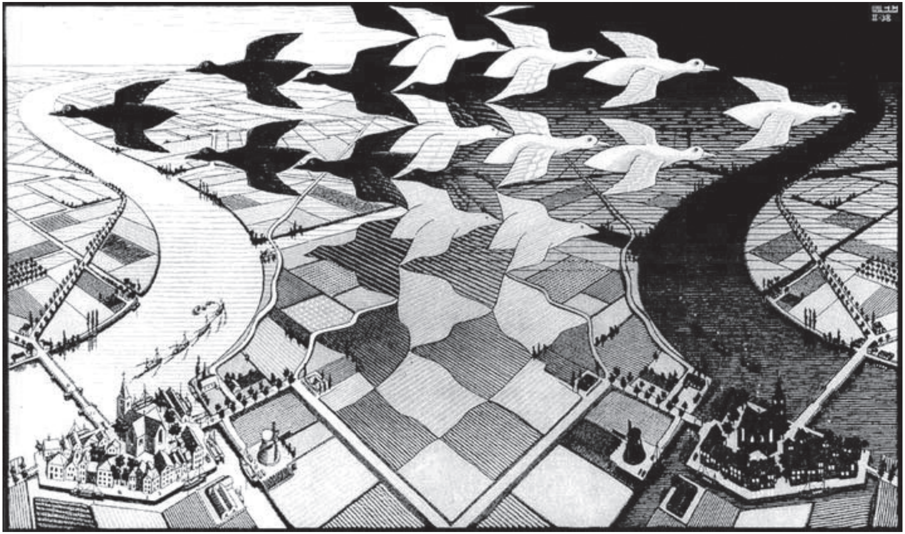

Figure 3.28 Which is it? White birds flying east, or black birds flying west? M.C. Escher's *Day and Night.*

003x028.eps

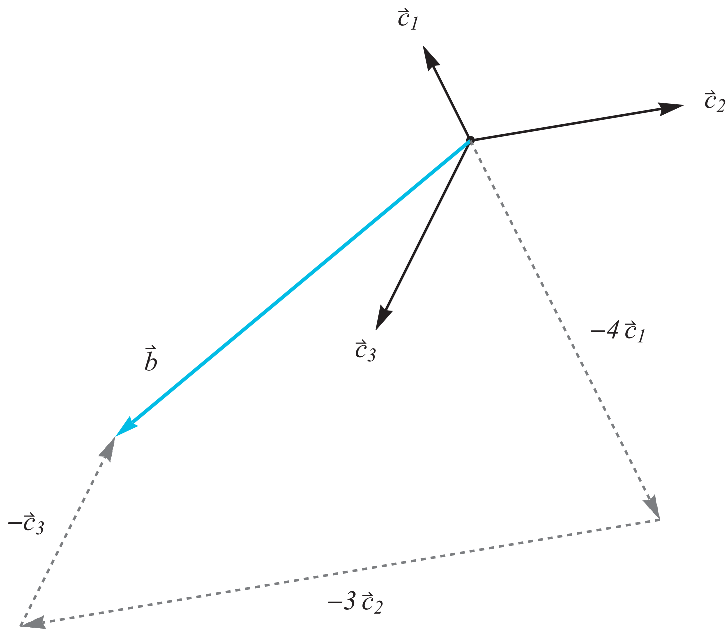

Figure 3.29 The **column problem**. This interpretation of the system (25) asks which scalar multiples of the columns $\mathbf{c}_i$ (black arrows) add up to $\mathbf{b}$ (blue arrow). The dashed gray arrows illustrate one of the many solutions, namely (−4, −3, −1).