Electricity and Electronics for Renewable Energy Technology, An Introduction

Chapter 2

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Chapter 2

Basic Mathematics and Systems of Measurement Units

Ratio and Percentage

Ratio shows the relative relationship between two measured values

- Ratio can be found by dividing the two measured values, one divided by the other;
- The measurements must have the same units
- Ratio is just a number; it does not have any unit.

Ratio and Percentage

- Ratio is normally a positive number; it can be smaller than 1 or larger

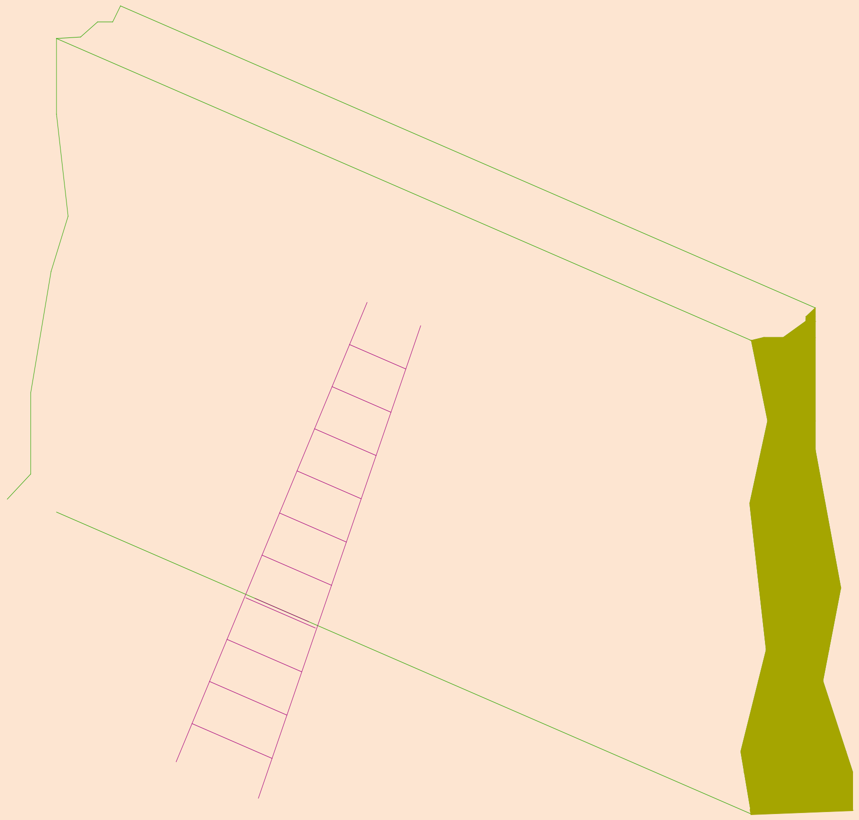
- The ratio of two numbers can be shown by percentage. For this the ratio is multiplied by 100

Examples: the ratio of A to B is 4/5 or 0.8

- The ratio of B to A then is 5/4 or 1.2
- Alternatively: A is 80% of B or B is 120% of A

Is the ladder taller or the wall?

To reach to top of the wall, should the ladder length be 100% of wall height?



Area and Volume Ratios

In dealing with area and volume the ratio of lengths does not directly apply.

For area the length ratio must be squared

For volume the length ratio must be cubed

Example: 1 yard is 90% of 1 m

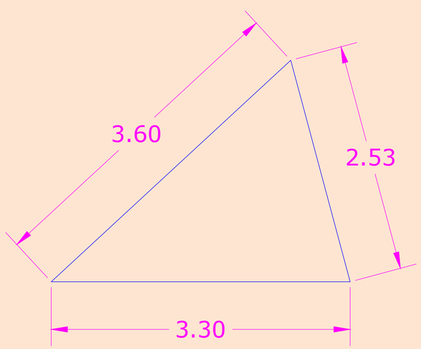
1 square yard is not 90% of 1 m^2 ; it is 81%

1 cubic yard is not 90% of 1 m^2 ; it is 72.9%

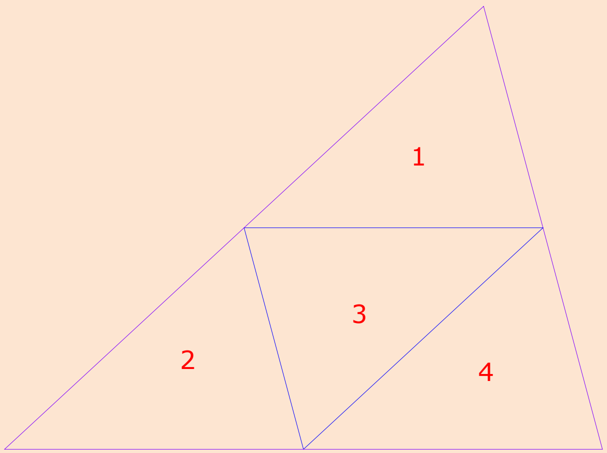
Area and Volume Ratios

Each side of the triangle on right is twice the corresponding side of the triangle on left

But its area is four times larger



(a)

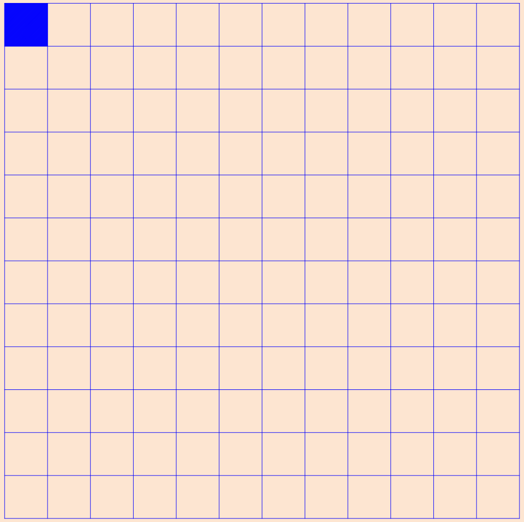


(b)

Example of Ratios in Conversion

$1 \text{ m} = 1'000 \text{ mm}$

$1 \text{ m}^2 = 1'000'000 \text{ mm}^2$



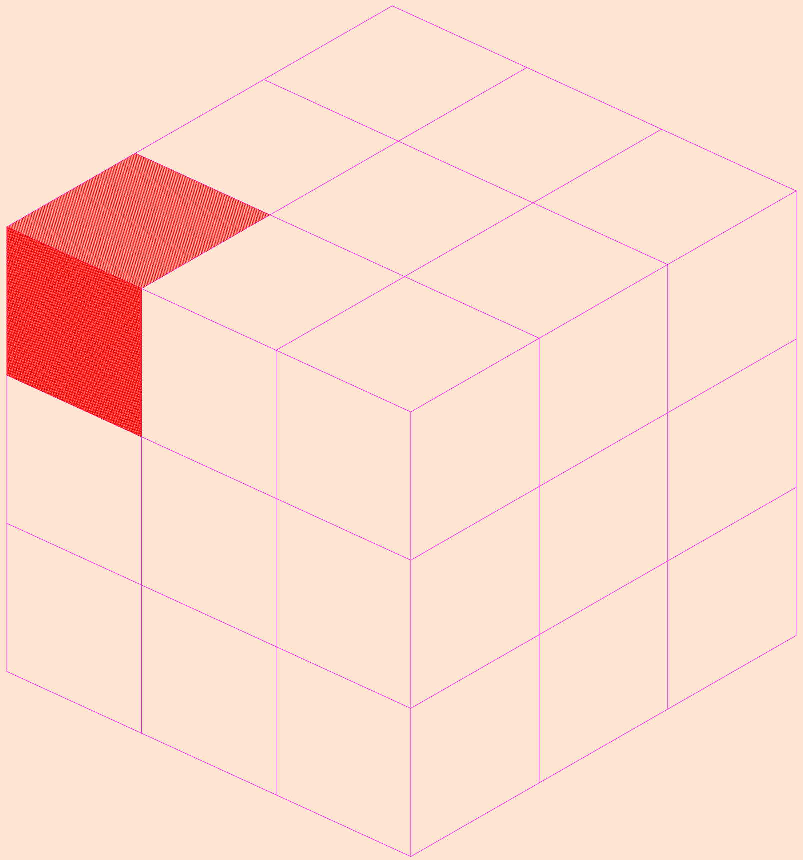
$1 \text{ ft} = 12 \text{ in}$

$1 \text{ ft}^2 = 144 \text{ in}^2$

Area and Volume Ratios

1 m = 100 cm

$1\ m^3 = 1'000'000\ cm^3$



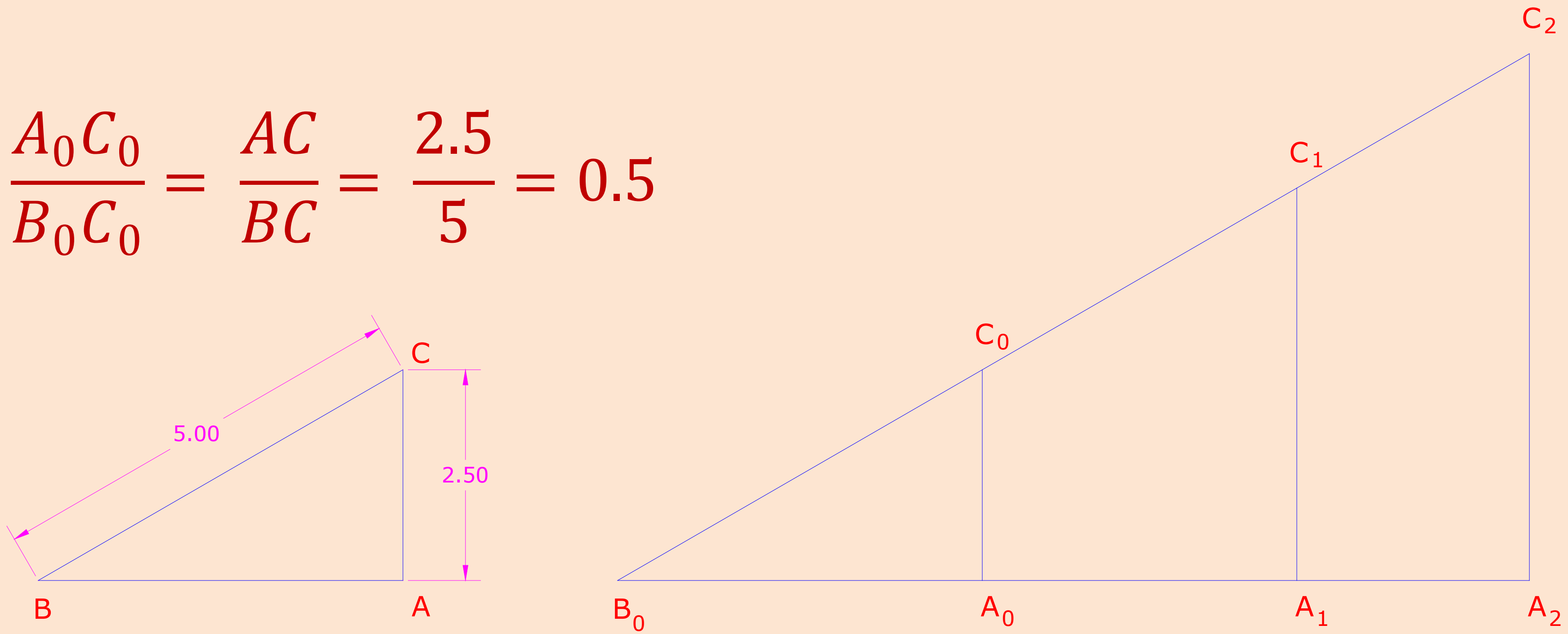
1 Yd = 3 ft

$1\ Yd^3 = 27\ ft^3$

Similar Triangle Relationships

$$\frac{A_0C_0}{B_0C_0} = \frac{A_1C_1}{B_0C_1} = \frac{A_2C_2}{B_0C_2}$$

$$\frac{A_0C_0}{B_0C_0} = \frac{AC}{BC} = \frac{2.5}{5} = 0.5$$

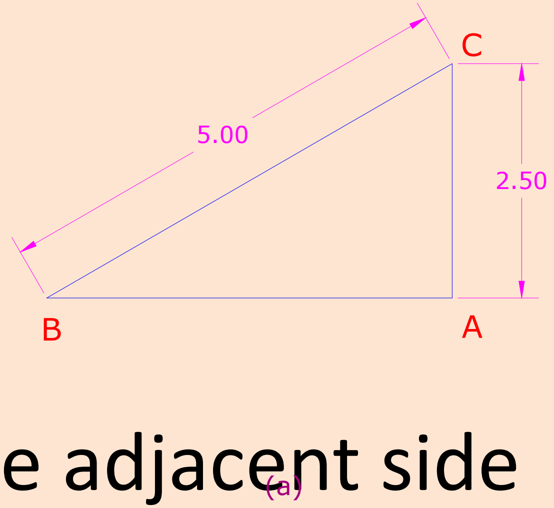


(a)

(b)

Angles, Triangles and Trigonometric Relationships

Sine of angle B = The ratio of the opposite side to B to the hypotenuse = $\frac{AC}{BC}$



- This is written: sin B = $\frac{AC}{BC}$

Cosine of angle B = The ratio of the adjacent side to B to the hypotenuse = $\frac{AB}{BC}$

- This is written: cos B = $\frac{AB}{BC}$

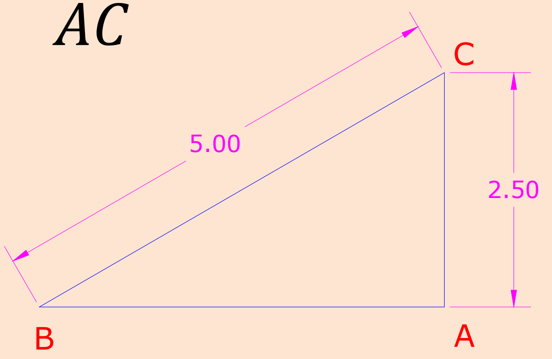
Angles, Triangles and Trigonometric Relationships

Tangent of angle B = The ratio of the opposite side to B to the adjacent side to B = $\frac{AC}{AB}$

- This is written: tan B = $\frac{AC}{AB}$

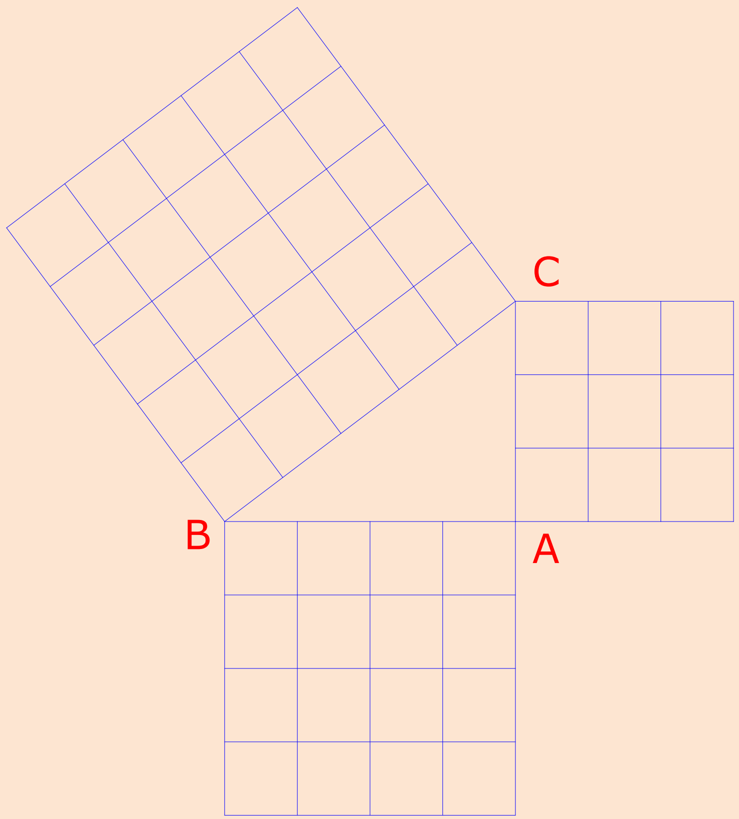
Cotangent of angle B = The ratio of the adjacent side to B to the opposite side to B = $\frac{AB}{AC}$

- This is written: cotan B = $\frac{AB}{AC}$



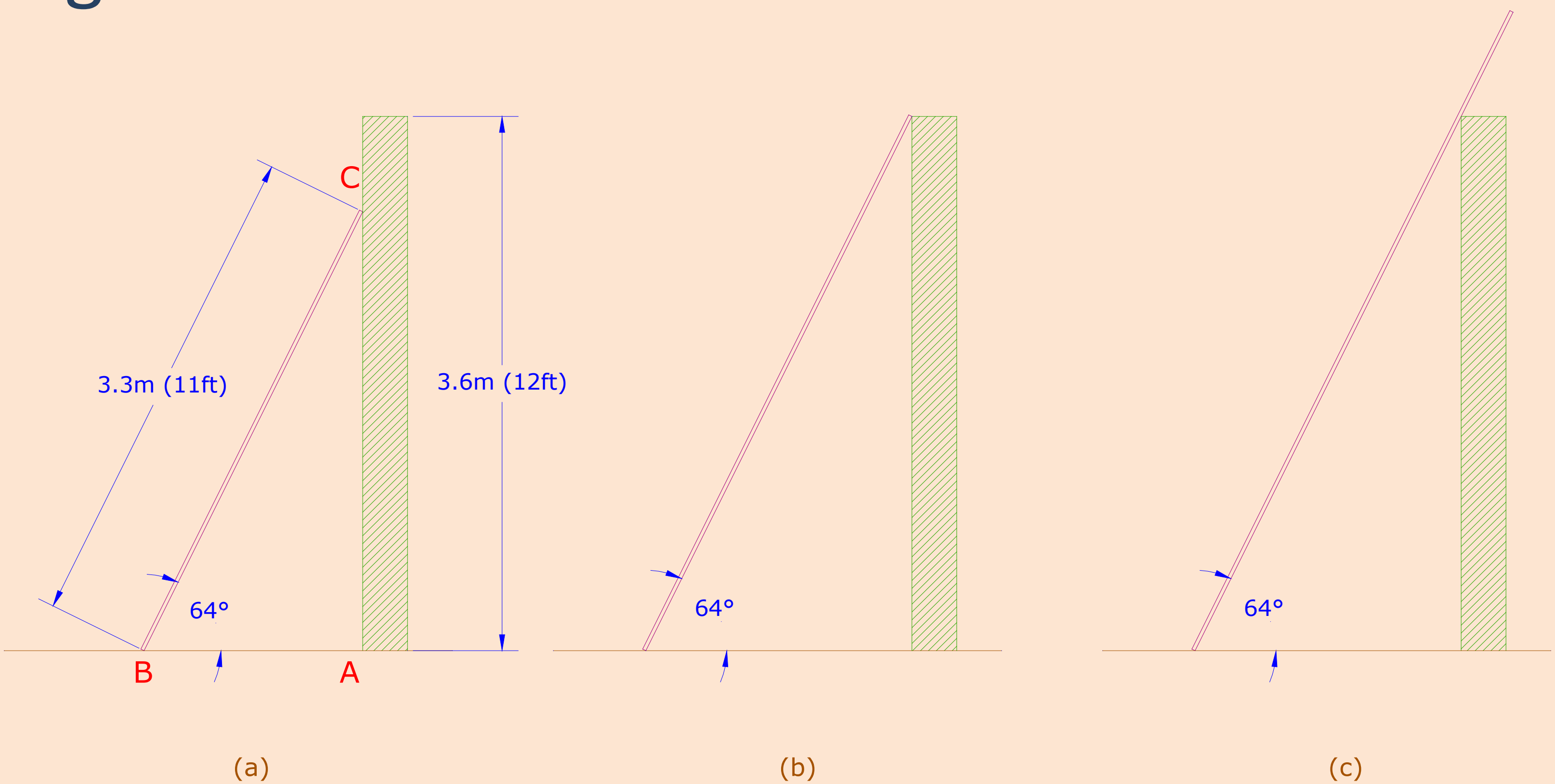
Observation of the Pythagorean Theorem

$a^2 = b^2 + c^2$ $\qquad$ $b = 3, c = 4 \longrightarrow a = 5$



Example of the Ladder and Wall

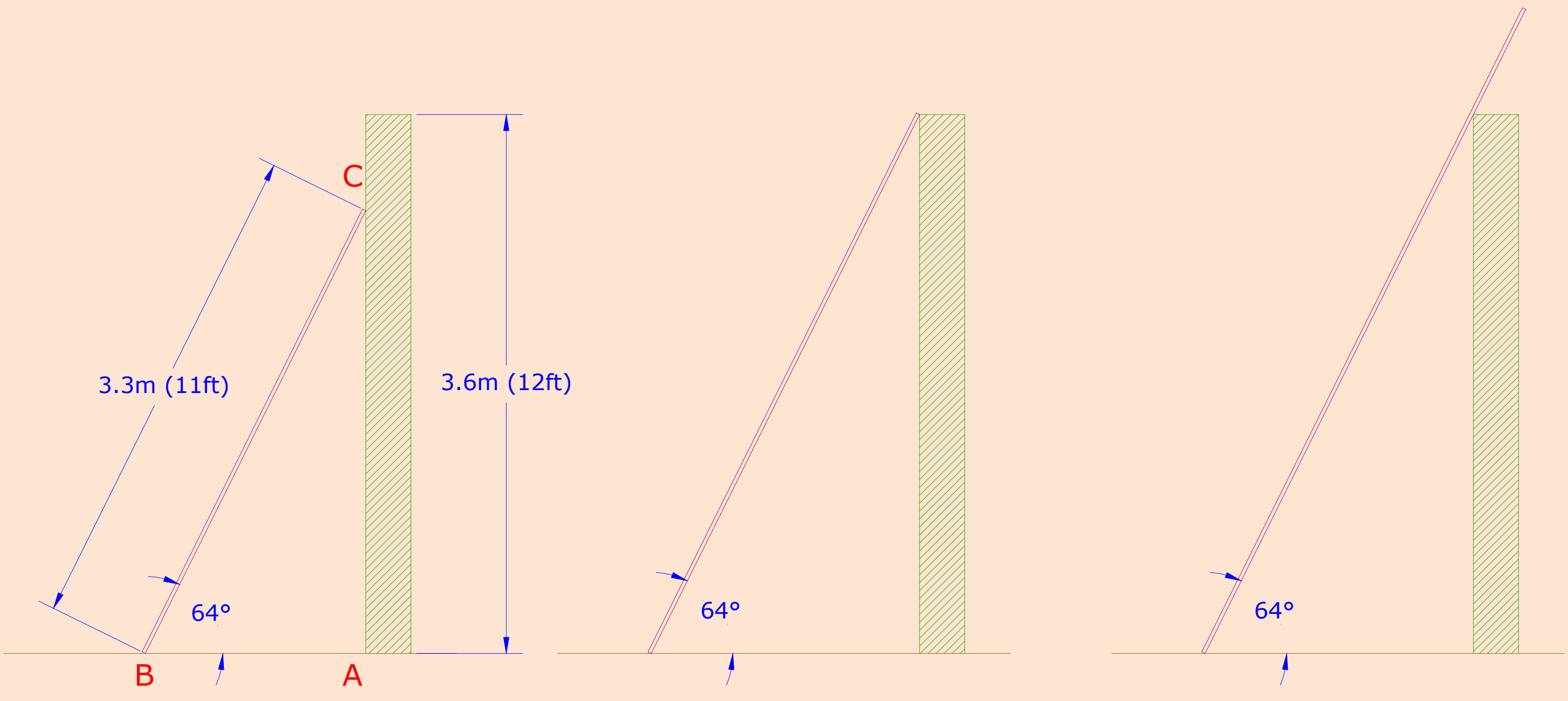
In figure (b) the wall is 3.6m. The ladder makes 64° angle with horizontal. What is the ladder length?



Example of the Ladder and Wall

Wall height = Ladder length x sin 64°

Ladder length = Wall height/sin 64° = 4 m



(a) (b) (c)

Principal Measurement Entities

When a unit for measuring length or distance is defined, such as meter, other units can be defined

Example: m^2, m^2, km, mm

Also, if second is defined as a unit for measuring time other units for measuring speed and acceleration can be defined

Example: m/s, m/s^2

Principal Measurement Entities

In the above examples meter and second are principal units and other units are defined in terms of the principal units

Among all the units of measurement three of them are principal units and the rest are derived from them

Systems of Measurement

There are numerous systems of measurement

The most widely used system is metric system

In United States and some other countries the Imperial system is also widely used

In this course we consider only these two systems and their relationship

Principal Measurement Units

In metric system the principal units are:

Meter (for length and distance), Kilogram (for mass) and second (for time)

In Imperial system the principal units are:

foot (for length and distance), pound (for force) and second (for time)

Each entity can have smaller or larger units, such as ton, km, mile and inch. But the formal units are as above

Derivation of Other Measurement Units

Other units are directly derived according to their definition, but sometimes they have a separate name

For example:

Force = mass x acceleration (Based on Newton's law)

The unit for force in metric system then is $kg.m/s^2$, which is called Newton

Derivation of Other Measurement Units

Another example:

Pressure = Force/area

The unit for pressure is N/m^2, also called Pascal

But,

Work = Force x displacement

The unit for work is, thus, Newton-Meter or NM

Basic Units in Metric and US Customary Systems

	Metric		US customary	
	Main	Other/remark	Main	Other/remark
Length	Meter (m)	Micron, mm, cm, dm, km	Foot (ft)	Mil, inch, yard, mile
Force	Newton (N)	Kilo newton (kN) = 1000 N	Pound (lb)	(pound-force, lbf)
Time	Second (s)	min., hour, day, week, month, year, decade	Second (s)	min., hour, day, week, month, year, decade
Area	Sq. m	Sq. km	Sq. ft	Acre, sq. mile
Volume	Cubic meter	Liter (L) = 1/1000 m^3, CC	Cubic ft	Gallon, ounce (Oz)

Basic Units in Metric and US Customary Systems, Cont'd

Speed	m/s	Km/hr	ft/sec	in/sec, mph, knot
Power	Watt (W)- Nm/s	kW, MW, horsepower	lb.ft/sec	horsepower
Energy/work	Joule(J), N.m	Watt-hr, kW-hr, MW-hr	ft.lb	
Pressure	Pascal (Pa)	Bar = 10^5Pa	lb/in^2 (PSI)	Bar, kips (1000 PSI)
Mass	Kilogram (kg)	Ton = 1000Kg	slug	lbm, Ton (short) = 2000lbm
Density	Kg/m^3	Gram/liter	slug/ft^3	lbm/ft^3
Torque *	Nm		ft.lb †	ft.in, oz-in
Heat	Calorie	1 Cal = 4.1868 joule	BTU	(British Thermal Unit)

Conversion between Systems of Measurement

1 meter	3.28 ft	39.37 in	1.0936 Yd	6.215×10^{-4} mile
1 kilo-meter	3,280 ft	39,370 in	1,093.6 Yd	0.6215 mile
1 square meter	10.764 ft^2	1550. in^2	1.196 Yd^2	2.471×10^{-4} Acre
1cAubic meter	35.306 ft^3	61023.378 in^3	1.308 Yd^3	264.173 Gal
1 m/s	3.28 ft/s	39.37 in/s	3.6 km/hr	2.237 mph
1 km/hr	0.911 ft/s	10.936 in/s	0.278 m/s	0.6215 mph
1 newton	0.2248 lb			
1 watt	0.7376 lb.ft/sec	1.34×10^{-3} hp	3.412 BTU/hr	1×10^{-6} MW
1 joule = 1 Nm	0.7376 lb.ft	0.2388 cal	1/3,600,000 kW-hr	9.48×10^{-4} BTU
1 kg (mass)	2.2075 lbm	0.0686 slug		
1 pascal	1.45×10^{-4} PSI	0.02088 lb/ft^2	1 N/m^2	1×10^{-5} bar
1 kg/m^3	1.94×10^{-3} $slug/ft^3$	0.0624 lbm/ft^3		
1 calorie	0.003968 BTU	3.088 lb-ft	4.1868 joule	1.163×10^{-3} Whr

Conversion between Systems of Measurement

1 ft	0.3048 m	1/3 Yd	12 in	12,000 mil
1 mile	1609.34 m	1.609 km	5280 ft	1760 Yd
1 Square ft	0.0929 m^2	929 cm^2	144 in^2	0.1111 Yd^2
1 Cubic ft	0.0283 m^3	7.4805 gallons	28.317 L	28316.685 mm^3
1 ft/s	0.3048 m/s	1.0973 km/hr	0.6818 mph	1.6874 knot
1 mph	0.447 m/s	1.609 km/hr	1.4667 ft/s	2.475 knot
1 lb (force)	4.448 N			
1 lb-ft	1.3558 Nm	1.3558 joule	0.3238 cal	1.285×10^{-3} BTU
1 lb-ft/s	1.3558 joule/s	1.3558 W	0.001817 hp	4.626 BTU/hr
1 slug (mass)	14.594 Kg	32.17 lbm		
1 PSI	6894.757 Pascal	0.00694 lb/ft^2	0.06894 bar	1 $pound/in^2$
1 US Gallon	0.003785 m^3	3.785 L	0.13368 ft^3	3785 CC (mL)
1 BTU	252.0 cal	1055.06 joule	778.1 lb-ft	1055.06 Nm

Formulas, Relationships and Equations

A formula indicates the relationship between two or more variables in the form of an equation

The values for variables are subject to change, otherwise a variable whose value does not change can be replaced by a constant

Example: $X = 2\pi fL$ 3 variables X, f and L

Formulas, Relationships and Equations

$X = 2\pi fL$

This equation shows the relationship between the three variables X, f and L

It is written as a formula to find X in terms of the other two.

We may extract two other formulas to find f or L in terms of others $f = \dfrac{X}{2\pi L}$ and $L = \dfrac{X}{2\pi f}$

Formulas, Relationships and Equations

$X = 2\pi fL$ and $f = \dfrac{X}{2\pi L}$ and $L = \dfrac{X}{2\pi f}$

All above formulae denote one equation (Not more)

The following rules always apply:

1. For a relationship between 3 variables two must be known in order to determine the third one
2. Both sides of an equation have the same units. For instance, if on the left the unit is m/s the same must appear on the right after calculation