P2-1

$$A = \begin{bmatrix} 2 & 1 \\ .4 & 2 \end{bmatrix} \qquad B = \begin{bmatrix} 2.4 \\ 1 \end{bmatrix}$$

$$C = \begin{bmatrix} 2 & 0 \end{bmatrix} \qquad d = .4$$

$$G(z) = C(zI - A)^{-1}B + d$$

$$G(z) = \begin{bmatrix} 2 & 0 \end{bmatrix} \begin{bmatrix} z-2 & 1 \\ .4 & z-2 \end{bmatrix}^{-1} \begin{bmatrix} 2.4 \\ 1 \end{bmatrix} + .4$$

$$G(z) = \frac{.4z^2 + 3.2z - 10.16}{z^2 - 4z + 3.6}$$

P2.2

$$A = \begin{bmatrix} 2 & 0 & 4 \\ 4 & 2 & -1 \\ 0 & 6 & 2 \end{bmatrix}$$

$$h_{max} = \frac{1}{|\lambda_{max}(A)|}$$

$$\text{eigenvalues}(A) = \{6.14,\ -.072 \pm 4.34j\}$$ using TI-89

$$h = \frac{0.5}{6.14} = 0.0814$$

$$h_{max} = \frac{1}{6.14} = 0.1629$$

P2.3

$$\dot{x} = -2x + 3u \qquad a = -2$$
$$y = x \qquad b = 3$$

1) $G(s) = \dfrac{3}{s+2}$

2) $\phi = e^{-2h}$

$$\psi = \int_0^h e^{-2\sigma} d\sigma = \frac{(1 - e^{-2h})}{2}$$

$$\Gamma = 3\psi = \frac{3(1 - e^{-2h})}{2}$$

$$\tilde{G}(z) = \frac{\Gamma}{z - \phi} = \frac{3(1-e^{-2h})/2}{z - e^{-2h}}$$

P2.4

$$A = \begin{bmatrix} 0 & 1 \\ 0 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$
$$d = 0$$

$$\phi = e^{Ah} = \mathcal{L}^{-1}[(sI - A)^{-1}]_{t=h} = \mathcal{L}^{-1}\left[\begin{bmatrix} s & -1 \\ 0 & s+3 \end{bmatrix}^{-1}\right]_{t=h}$$

$$= \mathcal{L}^{-1}\begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+3)} \\ 0 & \frac{1}{s+3} \end{bmatrix}_{t=h} = \begin{bmatrix} 1 & \frac{1-e^{-3h}}{3} \\ 0 & e^{-3h} \end{bmatrix}$$

$$\psi = \int_0^h e^{A\sigma} d\sigma = \begin{bmatrix} h & \frac{3h - 1 + e^{-3h}}{9} \\ 0 & \frac{1 - e^{-3h}}{3} \end{bmatrix}$$

$$\Gamma = \psi B = \begin{bmatrix} \frac{3h - 1 + e^{-3h}}{9} \\ \frac{1 - e^{-3h}}{3} \end{bmatrix}$$

$$\tilde{G}(z) = C(zI - \phi)^{-1}\Gamma = \frac{z[3he^{3h} - e^{3h} + 1]}{3(z-h)(3ze^{3h} - e^{3h} + 1)}$$

P. 2 – 5

$$G(s) = \frac{1}{s(s+1)} = \frac{1}{s^2+s}$$

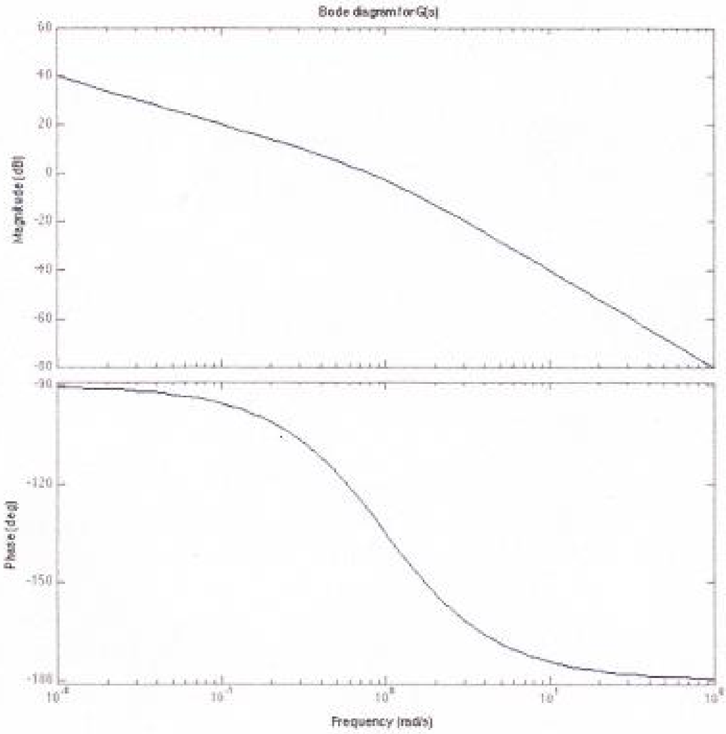
$$A = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

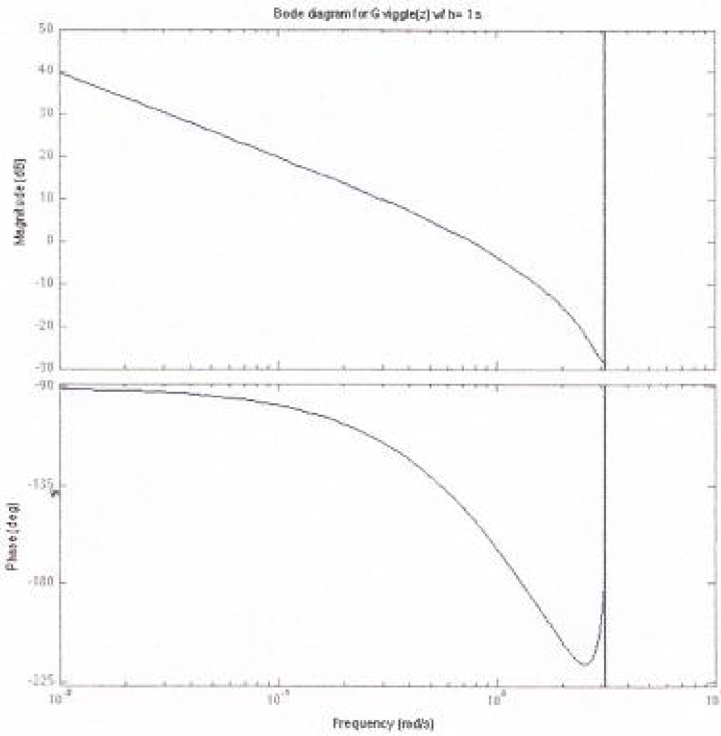
$$C = \begin{bmatrix} 0 & 1 \end{bmatrix} \quad d = 0$$

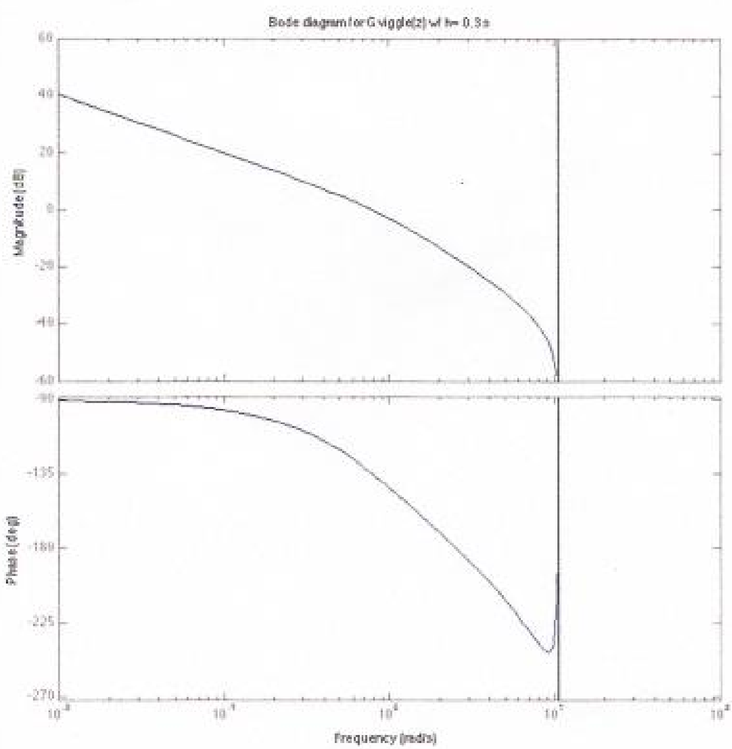
Code:

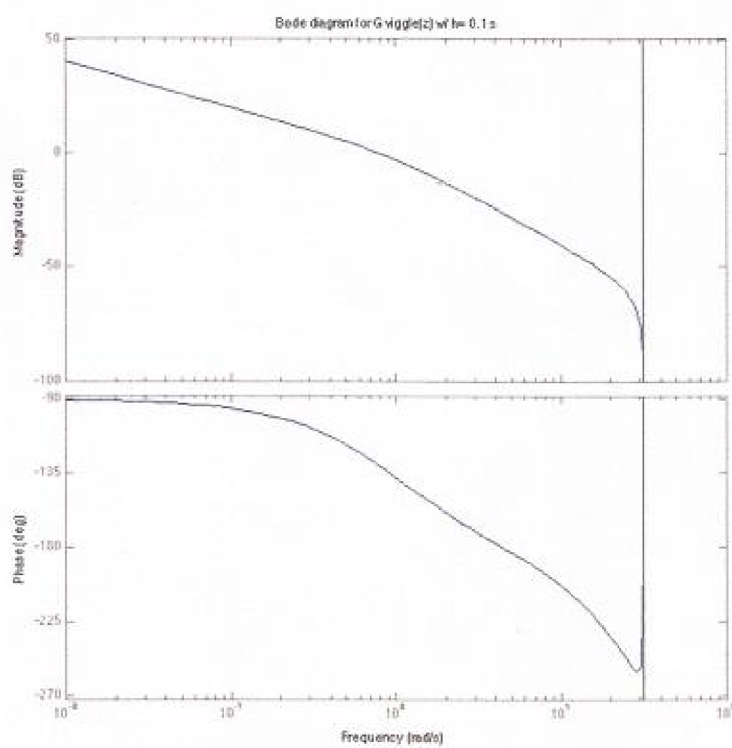
```
clear;
clf
h=input('Choose a value of h (1, 0.5, 0.3, or 0.1):');

A=[-1 0;1 0];
B=[1;0];
C=[0 1];
d=0;

sys=ss(A,B,C,d);
tf(sys)

sysd=c2d(sys,h);
tf(sysd)

bode(sys)
title('Bode diagram for G(s)');

bode(sysd)
title(['Bode diagram for G viggle(z) w/ h= ',num2str(h),' s']);
```

Continue P.2.5









P.2-6

$$G(s) = \frac{1}{s+a} e^{-Mhs} e^{-\epsilon s}$$

① $X(k+1) = \phi X(k) + \Gamma_1 u(k-1) + \Gamma_0 (k)$

$$\phi = e^{-ah} \qquad \Gamma_0 = \int_0^{h-\epsilon} e^{-a\sigma} d\sigma = \frac{1-e^{-a(h-\epsilon)}}{a}$$

$$\Gamma_1 = e^{-a(h-\epsilon)} \int_0^{\epsilon} e^{a\sigma} d\sigma = \frac{e^{-ah}(e^{a\epsilon}-1)}{a}$$

$$\tilde{G}(z) = \frac{1}{z^M} \frac{1}{z} \left[\frac{z\Gamma_0 + \Gamma_1}{z-\phi} \right]$$

$$\tilde{G}(z) = \frac{1}{z^{M+1}} \left[\frac{z(1-e^{-a(h-\epsilon)}) + e^{-ah}(e^{a\epsilon}-1)}{a(z-e^{-ah})} \right]$$

② $a=2 \quad M=4 \quad \epsilon=0.3 \quad h=0.5$

$$\tilde{G}(z) = \frac{1}{z^5} \left[\frac{z(1-e^{-0.4}) + e^{-1}(e^{0.6}-1)}{2(z-e^{-1})} \right]$$

$$\tilde{G}(z) = \frac{0.3297z + 0.3024}{2z^5(z-0.3679)}$$

P.2-7

ODE: $F = M\ddot{X} + D\dot{X} + KX$

where $\dot{x} = \frac{dx}{dt}$ $\ddot{x} = \frac{d^2x}{dt}$

$$\ddot{X} = \frac{F}{M} - \frac{D}{M}\dot{x} - \frac{K}{M}x$$

State model: $\dot{\underline{X}} = A\underline{X} + BF$

Measure. model: $y = C\underline{X} + dF$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{K}{M} & -\frac{D}{M} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}, \quad d = 0$$

Transfer function

$$T(s) = C(SI - A)^{-1}B$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} S & -1 \\ \frac{K}{M} & S + \frac{D}{M} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ \frac{1}{M} \end{bmatrix}$$

$$= \frac{1}{MS^2 + DS + K} = \frac{1/M}{S^2 + \frac{D}{M}S + \frac{K}{M}}$$

let $M = 3, \; D = 8, \; K = 2$

$$T(s) = \frac{1/3}{S^2 + \frac{8}{3}S + \frac{2}{3}} = \frac{0.333}{S^2 + 2.667S + 0.667}$$

MATLAB Code:

```
% Mass-Spring-Damper system analysis using MATLAB

clear;
clf

M= input('Enter a value of the mass of body M:');
D= input('Enter a value of the fractional constant D:');
K= input('Enter a value of the spring constant K:');

A=[0 1;-K/M -D/M];
B=[0;1/M];
C=[1 0];
d=0;

sys=ss(A,B,C,d);
tf(sys)
```

Results:

```
Please enter a value of the mass of body M:3
Please enter a value of the fractional constant D:8
Please enter a value of the spring constant K:2

tf(sys )

        0.3333
  ----------------------
  s^2 + 2.667 s + 0.6667
```

P.2-8

a) $G(s) = \dfrac{5(s+6)}{(s+3)(s+4)} = \dfrac{5s+30}{s^2+7s+12}$ $\quad |\lambda_{max}(A)| = 4$

$A = \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 30 \end{bmatrix} \quad d = 0$

$\dfrac{0.5}{|\lambda_{max}(A)|} \leq T_s \leq \dfrac{1}{|\lambda_{max}(A)|} \Rightarrow \dfrac{1}{8} \leq T_s \leq \dfrac{1}{4}$

Code:

```
clear;
clf
A=[-7 -12;1 0];
B=[1;0];
C=[5 30];
d=0;
Ts=0.5;
sys=ss(A,B,C,d);
tf(sys)
sysd=c2d(sys,Ts);
tf(sysd)
```

Results:

```
tf(sys)
   5 s + 30
 --------------
 s^2 + 7 s + 12
 Continuous-time transfer function.
tf(sysd)
   1.723 z - 0.04336
 -----------------------
 z^2 - 0.3585 z + 0.0302
 Sample time: 0.5 seconds
Discrete-time transfer function.
```

b) $G(s) = \dfrac{5(s+6)}{(s+3)(s+4)}$ $\quad \dfrac{1}{8} \leq T_s \leq \dfrac{1}{4}$

$A = \begin{bmatrix} -7 & -12 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad C = \begin{bmatrix} 5 & 30 \end{bmatrix} \quad d = 0$

Code:

```
clear;
clf
A=[-7 -12;1 0];
B=[1;0];
C=[5 30];
d=0;
Ts=1;
sys=ss(A,B,C,d);
tf(sys)
sysd=c2d(sys,Ts);
tf(sysd)
```

Results:

```
tf(sys)
   5 s + 30
 --------------
 s^2 + 7 s + 12
 Continuous-time transfer function.
tf(sysd)
    2.297 z + 0.03517
 --------------------------
 z^2 - 0.0681 z + 0.0009119
 Sample time: 1 seconds
Discrete-time transfer function.
```

Continue P.2-8

c) $G(s) = \dfrac{5(s+7)}{s(s+2)(s+4)} = \dfrac{5s+35}{s^3+6s^2+8s}$

$A = \begin{bmatrix} -6 & -8 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad C = [0 \; 5 \; 35] \quad d = 0$

$|\lambda_{max}(A)| = 4$

$\dfrac{0.5}{|\lambda_{max}(A)|} \leq T_s \leq \dfrac{1}{|\lambda_{max}(A)|} \Rightarrow \dfrac{1}{8} \leq T_s \leq \dfrac{1}{4}$

code:

```
clear;
clf
A=[-7 -12;1 0];
B=[1;0];
C=[5 30];
d=0;
Ts=0.5;
sys=ss(A,B,C,d);
tf(sys)
sysd=c2d(sys,Ts);
tf(sysd)
```

Results:

```
tf(sys)
    5 s + 35
  -----------------
  s^3 + 6 s^2 + 8 s
 Continuous-time transfer function.
tf(sysd)
  0.6174 z^2 + 0.5875 z - 0.009323
  ----------------------------------
  z^3 - 1.503 z^2 + 0.553 z - 0.04979
Sample time: 0.5 seconds
Discrete-time transfer function.
```

P.2-9

$$A = \begin{bmatrix} -2 & 3 & 0 \\ 0 & -1 & 0 \\ 5 & -2 & -4 \end{bmatrix} \quad B = \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix} \quad c = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \quad d = 0$$

$$|\lambda_{max}(A)| = 4 \qquad \frac{1}{8} \leq T_s \leq \frac{1}{4}$$

$$G(s) = C[sI - A]^{-1}B = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} s+2 & -3 & 0 \\ 0 & s+1 & 0 \\ -5 & 2 & s+4 \end{bmatrix}^{-1} \begin{bmatrix} 6 \\ -2 \\ 0 \end{bmatrix}$$

$$G(s) = \frac{-2}{s+1}$$

Code:

```
clear;
clf

A=[-2 3 0;0 -1 0;5 -2 -4];
B=[6;-2;0];
C=[0 1 0];
d=0;
Ts=.125;

sys=ss(A,B,C,d);
tf(sys)
sysd=c2d(sys,Ts,'zoh');
tf(sysd)
```

Results:

```
tf(sys)

  -2
 -----
 s + 1

Continuous-time transfer function.

tf(sysd)

   -0.235
 ----------
 z - 0.8825

Sample time: 0.125 seconds
Discrete-time transfer function.
```