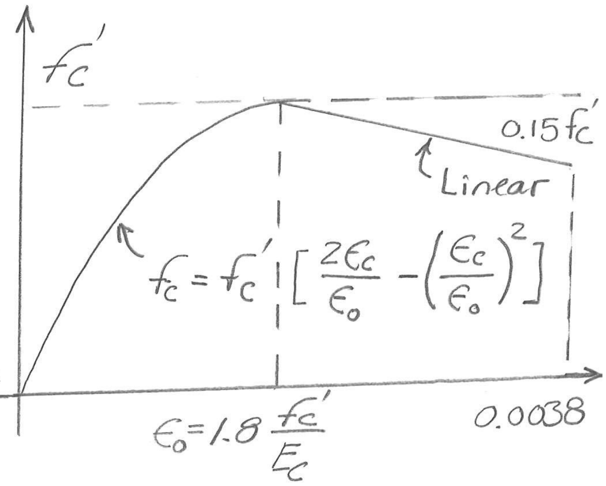
## Problem 2.1:

### (a) Using $\beta_1$ from ACI 318 equation:

Integrating the area under the Curve:




$$\alpha \cancel{f_c'} \epsilon_{cf} = \int_0^{\epsilon_o} \cancel{f_c'}\left[\frac{2\epsilon_c}{\epsilon_o} - \left(\frac{\epsilon_c}{\epsilon_o}\right)^2\right] d\epsilon_c$$

$$+ \int_{\epsilon_o}^{\epsilon_{cf}} \left[\cancel{f_c'} - \frac{0.15\cancel{f_c'}}{\epsilon_{cf} - \epsilon_o}(\epsilon_c - \epsilon_o)\right] d\epsilon_c$$

$$\alpha \epsilon_{cf} = \left[\frac{\epsilon_c^2}{\epsilon_o} - \frac{\epsilon_c^3}{3\epsilon_o^2}\right]_0^{\epsilon_o} + \left[\epsilon_c - \frac{0.15}{\epsilon_{cf} - \epsilon_o}\left(\frac{\epsilon_c^2}{2} - \epsilon_c \epsilon_o\right)\right]_{\epsilon_o}^{\epsilon_{cf}}$$

$$= \frac{2}{3}\epsilon_o + \left[\epsilon_{cf} - 0.15\epsilon_{cf}\frac{\epsilon_{cf}/2 - \epsilon_o}{\epsilon_{cf} - \epsilon_o} - \epsilon_o - \frac{0.15\epsilon_o^2}{2(\epsilon_{cf} - \epsilon_o)}\right]$$

$$\alpha = 1 - \frac{\epsilon_o}{3\epsilon_{cf}} - 0.15\frac{1}{(\epsilon_{cf} - \epsilon_o)}\left(\frac{\epsilon_{cf}}{2} - \epsilon_o + \frac{\epsilon_o^2}{2\epsilon_{cf}}\right)$$

$$= 1 - \frac{\epsilon_o}{3\epsilon_{cf}} - 0.15\frac{\epsilon_{cf} - \epsilon_o}{2\epsilon_{cf}} = 0.925 - 0.2583\frac{\epsilon_o}{\epsilon_{cf}}$$

substitute specific values:

$$\epsilon_o = 1.8\frac{f_c'}{E_c} = 1.8\frac{f_c}{57000\sqrt{f_c'}} = \frac{\sqrt{f_c'}}{31666.67}$$

$$\epsilon_{cf} = 0.0038$$

into $\alpha$ expression:

$$\alpha = 0.925 - 0.2583\frac{\sqrt{f_c'}}{120.33}$$

$$= 0.925 - 2.1465\times10^{-3}\sqrt{f_c'}$$

$$\gamma = \frac{\alpha}{\beta_1}$$

Results:

| $f_c'$ (psi) | $\beta_1$ | $\alpha$ | $\gamma$ |
|---|---|---|---|
| 3000 | 0.85 | 0.807 | 0.95 |
| 4000 | 0.85 | 0.789 | 0.928 |
| 5000 | 0.80 | 0.773 | 0.966 |
| 6000 | 0.75 | 0.759 | 1.012 |
| 7000 | 0.7 | 0.745 | 1.064 |
| 8000 | 0.65 | 0.733 | 1.128 |

Conclusion: Actual $\alpha$ for this selected curve does not vary as $\alpha$ values selected by Karr et al. (1978), see Table 2.2

(b) using $\beta_1$ from Park and Paulay model:

$$\beta_1 = 2\beta = \frac{\frac{2}{3} - \frac{\epsilon_{cf}}{6\epsilon_0}}{1 - \frac{\epsilon_{cf}}{3\epsilon_0}}$$

(derive $\beta$ as the centroid location of Hognestad's parabola measured from extreme compression fiber)

$$\beta_1 = \frac{\frac{2}{3} - \frac{0.0038}{6\sqrt{f_c'}/31666.67}}{1 - \frac{0.0038}{3\sqrt{f_c'}/31666.67}} = \frac{\frac{2}{3} - \frac{20.06}{\sqrt{f_c'}}}{1 - \frac{40.11}{\sqrt{f_c'}}}$$

Results

| $f_c'$ (psi) | $\beta_1$ | $\alpha$ | $\gamma$ |
|---|---|---|---|
| 3000 | 1.122 | 0.807 | 0.719 |
| 4000 | 0.955 | 0.789 | 0.826 |
| 5000 | 0.885 | 0.773 | 0.873 |
| 6000 | 0.846 | 0.759 | 0.897 |
| 7000 | 0.820 | 0.745 | 0.909 |
| 8000 | 0.802 | 0.733 | 0.914 |

Conclusion: Results for $\gamma$ are closer to the value selected by ACI318. However, $\gamma$ increases with $f_c'$ while $\gamma$ is assumed constant by ACI 318.

## Problem 2.2:

$$\sigma_c = f_c' \left[ 2\frac{\epsilon_c}{\epsilon_c'} - \left(\frac{\epsilon_c}{\epsilon_c'}\right)^2 \right]$$

$$\alpha = \frac{\epsilon_{cf}}{\epsilon_c'} - \frac{\epsilon_{cf}^2}{3\epsilon_c'^2} \qquad \text{Eqn (2-20) in text}$$

$$\epsilon_{cf} = 0.003$$

$$\epsilon_c' = 1.71\frac{f_c'}{E_c} = \frac{1.71}{57000}\sqrt{f_c'} = 3 \times 10^{-5}\sqrt{f_c'}$$

$$\alpha = \frac{0.003}{3\times 10^{-5}\sqrt{f_c'}} - \frac{0.003^2}{3\times(3\times10^{-5}\sqrt{f_c'})^2} = \frac{100}{\sqrt{f_c'}} - \frac{3\,333.33}{f_c'}$$

$$\beta_1 = \frac{\frac{2}{3} - \frac{\epsilon_{cf}}{6\epsilon_c'}}{1 - \frac{\epsilon_{cf}}{3\epsilon_c'}} = \frac{\frac{2}{3} - \frac{0.003}{6\times3\times10^{-5}\sqrt{f_c'}}}{1 - \frac{0.003}{3\times3\times10^{-5}\sqrt{f_c'}}} = \frac{\frac{2}{3} - \frac{16.67}{\sqrt{f_c'}}}{1 - \frac{33.33}{\sqrt{f_c'}}}$$

| $f_c'$ (psi) | $\alpha$ | $\beta_1$ | $\gamma$ |
|---|---|---|---|
| 3000 | 0.715 | 0.926 | 0.772 |
| 4000 | 0.748 | 0.852 | 0.878 |
| 5000 | 0.748 | 0.815 | 0.918 |
| 6000 | 0.735 | 0.792 | 0.928 |
| 7000 | 0.719 | 0.777 | 0.925 |
| 8000 | 0.701 | 0.766 | 0.915 |

Conclusion: $\gamma$ is close to the value selected by ACI 318 (Karr et al. 1978) only at $f_c' = 4$ ksi.

## Problem 2.3:

$$d' = 1.5'' + \frac{4}{8} + \frac{1}{2} \times \frac{7}{8} = 2.44''$$

$$d_t = 18'' - 1.5'' - \frac{4}{8} - \frac{1}{2} \times \frac{7}{8} = 15.56''$$

$$d_2 = 15.56'' - \frac{7}{8} - 1'' = 13.69''$$

$$d_3 = 13.69 - \frac{7}{8} - 1'' = 11.81''$$

$$d = \frac{3 \times 0.6 \times 15.65 + 2 \times 0.6 \times 13.69 + 2 \times 0.6 \times 11.81}{7 \times 0.6} = 13.95''$$

Force equilibrium assuming yielding of Compression steel:

$$0.85 f_c' b \beta_1 C + A_s' f_y = A_s f_y$$

$$C = \frac{(A_s - A_s')}{0.85 b \beta_1} \frac{f_y}{f_c'} = 6.228''$$

$$\epsilon_s' = \frac{0.003}{C}(C - d') = 1.825 \times 10^{-3} < 0.00207$$

$\Rightarrow$ Compression steel does not yield

$$0.85 f_c' b \beta_1 C + A_s' f_s' = A_s f_y$$

$$0.85 f_c' b \beta_1 C + A_s' E_s \frac{0.003}{C}(C - d') = A_s f_y$$

$$28.9C^2 + 104.4C - 254.736 = 252C$$

$$28.9C^2 - 147.6C - 254.736 = 0$$

$$C = \frac{147.6 + \sqrt{147.6^2 + 4 \times 28.9 \times 254.736}}{2 \times 28.9} = 6.47''$$

$$\epsilon_s' = \frac{0.003}{6.47''}(6.47 - 2.44) = 1.869 \times 10^{-3}$$

$f_s' = E_s \, \epsilon_s' = 54.19^{ksi}$

check yielding of all tension steel

$$\epsilon_{s_3} = \frac{0.003}{c}(d_3 - c) = \frac{0.003}{6.47}(11.81 - 6.47)$$

$$= 2.476 \times 10^{-3} > \epsilon_{sy} = 0.00207$$

$$\frac{c}{d_t} = \frac{6.47}{15.56} = 0.416 \quad \text{transition zone}$$

$$\phi = 0.65 + 0.25\left[\frac{1}{0.416} - \frac{5}{3}\right] = 0.834$$

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right) + \phi A_s' f_s' \left(\frac{a}{2} - d'\right)$$

$$= 0.834 \times 7 \times 0.6 \times 60 \times \left(13.95 - \frac{0.85 \times 6.47}{2}\right)$$

$$+ 0.834 \times 2 \times 0.6 \times 54.19 \times \left(\frac{0.85 \times 6.47}{2} - 2.44\right)$$

$$= 2370.73^{k\text{-}in} = 197.56^{k\text{-}ft} < 220^{k\text{-}ft}$$

which was expected since $d$ is reduced and $d'$ is increased

Redesign the section for more tension reinforcement or just add a 3rd bar in the second layer of bars and repeat the analysis above.

Problem 2.4:

$$d = d_t = 12'' - 1'' - \frac{3}{8} - \frac{1}{2} \times \frac{7}{8} = 10.19''$$

$$d' = 1'' + \frac{3}{8} + \frac{1}{2} \times \frac{3}{8} = 1.56''$$

Assume $\phi = 0.9 \Rightarrow \epsilon_t = 0.005$

$$\frac{0.003}{c} = \frac{0.005}{d_t - c} \Rightarrow \frac{d_t}{c} = 2.67$$

$$\frac{c}{d_t} = 0.375 \Rightarrow c = 0.375 d_t = 0.375 \times 10.19 = 3.82''$$

$$0.85 f'_c b \beta_1 c = A_s f_y$$

$$A_{s_t} = \frac{0.85 \times 5 \times 6 \times 0.8 \times 3.82}{69} = 1.129 \text{ in}^2$$

$$M_{ut} = \phi A_s f_y \left(d - \frac{a}{2}\right) = 0.9 \times 1.129 \times 69 \times \left(10.19 - \frac{0.8 \times 3.82}{2}\right)$$

$$= 607.3^{k\text{-}in} = 50.61^{k\text{-}ft} < 60^{k\text{-}ft}$$

Use doubly reinforced section

$$R_{nt} = \frac{M_{ut}}{\phi b d_t^2} = \frac{607.3}{0.9 \times 6 \times 10.19^2} = 1.0831 \text{ ksi}$$

$$= 1083.1 \text{ psi}$$

Table 2.4 gives $R_{nt} = 1084$ psi

$$R_n = \frac{M_u}{\phi b d_t^2} = \frac{60 \times 12}{0.9 \times 6 \times 10.19^2} = 1.2841 \text{ ksi}$$

$$= 1284.1 \text{ psi} > 1083.1 \text{ psi}$$

$$M_{nt} = R_{nt} b d_t^2 = 1083.1 \times 6 \times 10.19^2 = 674{,}789.28 \text{ lb-in}$$

$$= 674.79^{k\text{-}in}$$

$$= 56.23^{k\text{-}ft}$$

$$M_n' = M_n - M_{nt} = \frac{M_u}{\phi} - M_{nt}$$

$$= \frac{60}{0.9} - 56.23 = 10.43^{k\text{-}ft}$$

Strain in Compression Steel

$$\frac{\epsilon_s'}{c-d'} = \frac{0.003}{c} \Rightarrow \epsilon_s' = \frac{3.82 - 1.56}{3.82} * 0.003 = 1.775 * 10^{-3}$$

$$< \frac{69}{29000} = 2.379 * 10^{-3}$$

compression steel does not yield

$$f_s' = E_s \epsilon_s' = 29000 * 0.001775 = 51.48^{ksi}$$

$$M_n' = A_s' f_s' (d - d')$$

$$10.43 * 12 = A_s' * 51.48 * (10.19 - 1.56)$$

$$A_s' = 0.2817^{in^2}$$

$$A_s = A_{st} + A_s' \frac{f_s'}{f_y} = 1.129 + 0.2817 \frac{51.48}{69}$$

$$= 1.339^{in^2}$$

OR use Table 2.4 to get $\rho_{st}$.

$\rho_{st} = 0.02125$ for $f_y = 60$ ksi

$\rho_{st} = 0.017$ for $f_y = 75^{ksi}$

$$\rho_{st} = 0.02125 - \frac{69-60}{75-60}(0.02125 - 0.017)$$

$$= 0.0187$$

$$A_s = \rho_{st} bd + A_s' \frac{f_s'}{f_y} = 1.143 + 0.2817 * \frac{51.48}{69} =$$

$$= 1.353^{in^2} \text{ (approximate)}$$

$A_s$ : #7 bars $\quad A_b = 0.6 \, in^2 \quad$ # of bars $= \frac{1.339}{0.6}$

$= 2.23$

Use 3 #7 bars for tension

$b = 1'' \times 2 + \frac{3}{8} \times 2 + 3 \times \frac{7}{8} + 2 \times 1 = 7.375'' > 6''$

Try #8 bars $\quad A_b = 0.79 \, in^2 \quad$ # of bars $= \frac{1.339}{0.79}$

$= 1.695$

Use 2 #8 bars for tension

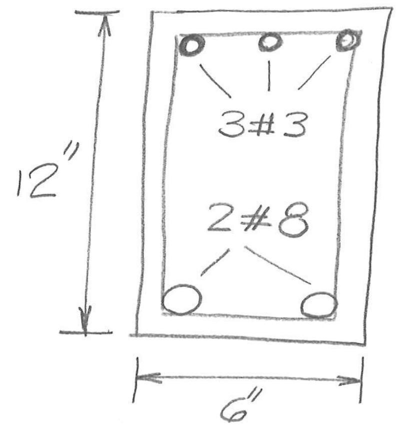
$b = 1'' \times 2 + \frac{3}{8} \times 2 + 2 \times 1'' + 1'' = 5.75'' < 6''$ O.K.

$A_s'$ : #3 bars $\quad A_b = 0.11 \, in^2 \quad$ # of bars $= \frac{0.2817}{0.11}$

$= 2.56$

Use 3 #3 bars for compression

$b = 1'' \times 2 + \frac{3}{8} \times 2 + 3 \times \frac{3}{8} + 2 \times 1'' = 5.875'' < 6''$ O.K.




Check the design

$d = d_t = 12'' - 1'' - \frac{3}{8}'' - \frac{1}{2}'' = 10.13''$

$d' = 1.56''$

$$A_s = 2 \times 0.79 = 1.58 \text{ in}^2 > A_{s\,min}$$

$$A_s' = 3 \times 0.11 = 0.33 \text{ in}^2$$

$$A_{s\,min} = \frac{3\sqrt{f_c'}}{f_y} b_w d = \frac{3\sqrt{5000}}{69000} \times 6 \times 10.13 = \underline{\underline{0.187 \text{ in}^2}}$$

$$\frac{200}{f_y} b_w d = 0.176 \text{ in}^2$$

Assume compression steel does not yield:

$$0.85 f_c' b \beta_1 c + A_s' f_s' = A_s f_y$$

$$0.85 \times 5 \times 6 \times 0.8c + 0.33 \times 29000 \frac{\times 0.003}{c} (c - 1.56)$$

$$= 1.58 \times 69$$

$$20.4c^2 + 28.71c - 44.79 = 109.02c$$

$$20.4c^2 - 80.31c - 44.79 = 0$$

$$c = \frac{80.31 + \sqrt{80.31^2 + 4(20.4)(44.79)}}{2 \times 20.4}$$

$$= 4.43''$$

$$\epsilon_s' = \frac{0.003}{4.43}(4.43 - 1.56) = 1.944 \times 10^{-3} < 2.379 \times 10^{-3}$$

$$\frac{c}{d_t} = \frac{4.43}{10.13} = 0.437$$

$$\phi = 0.65 + 0.25\left[\frac{1}{0.437} - \frac{5}{3}\right] = 0.805$$

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right) + \phi A_s' f_s' \left(\frac{a}{2} - d'\right)$$

$$M_u = 0.805 \times 1.58 \times 69 \times \left(10.13 - \frac{0.8 \times 4.43}{2}\right)$$

$$+ 0.805 \times 0.33 \times 29000 \times 1.944 \times 10^{-3} \times \left(\frac{0.8 \times 4.43}{2} - 1.56\right)$$

$$= 736.68^{k\text{-}in} = 61.39^{k\text{-}ft} > 60^{k\text{-}ft}$$

O.K.

## Problem 2.5:

### Solution steps:

1- Determine the flexural failure load (P).

2- Design for shear to resist at least the load (P).

Assume weight of concrete = 150 lb/ft³

$$DL = 150\frac{lb}{ft^3}\cdot\left(\frac{18''}{12}\cdot\frac{10''}{12}\right) = 187.5\ lb/ft$$

$$W_u = 1.2DL + 1.6LL$$

since DL is uniform and LL is concentrated

Use the load effect or moment

$$M_u = 1.2M_D + 1.6M_L$$

$$= 1.2\frac{W_D L^2}{8} + 1.6\frac{P_L L}{4}$$

$$= 1.2\frac{\times 0.1875\times 16^2}{8} + 1.6\frac{P\times 16}{4}$$

$$= 7.2^{k\text{-}ft} + 6.4P^{k\text{-}ft}$$

From Problem 2-3:

$$M_u = 197.56^{k\text{-}ft}$$

$$= 7.2 + 6.4P \Rightarrow P = 29.74^k$$

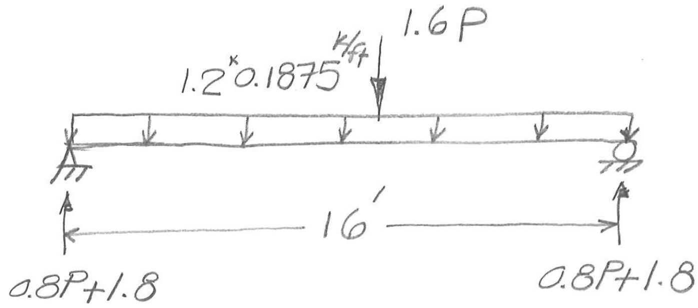
Assume $P = 30^k$ for shear design

Assume #3 stirrups

$d = 13.95''$ (from Problem 2-3)

$$V_{u\,support} = 0.8(30) + 1.8$$

$$= 25.8^{K}$$




$$V_u(d) = 25.8 - 0.225\left(\frac{13.95''}{12}\right)$$

$$= 25.54^{K}$$

$$\phi V_c = \phi 2\sqrt{f_c'}\, b_w d = 0.75 * 2\sqrt{4000}\,(10'')(13.95'')$$

$$= 13{,}234.13^{Lb} = 13.234^{K}$$

$$V_u = \phi(V_c + V_s)$$

$$\phi V_s = V_u - \phi V_c = 25.54 - 13.234 = 12.306^{K}$$

$$= \frac{\phi A_v f_{yt} d}{S} = \frac{0.75 * 0.22^{in^2} * 60 * 13.95''}{S}$$

$$S = 11.22''$$

check Limits:

$$\phi V_s = 12.306^{K} < \phi 8\sqrt{f_c'}\, b_w d = 52{,}937^{Lb} = 52.937^{K}$$

$$\phi V_s = 12.306^{K} < \phi 4\sqrt{f_c'}\, b_w d = 26.468^{K}$$

$$\Rightarrow S_{max} = \min\left(\frac{d}{2}, 24''\right) = \frac{13.95}{2} = 6.975'' \text{ controls}$$

Use #3 stirrups at 7''

check minimum shear in beam

$$V_{u\,℄} = 0.8P = 24^{K}$$

$$\phi V_s = V_u - \phi V_c = 24^{K} - 13.234 = 10.766^{K} < \phi 4\sqrt{f_c'}\, b_w d$$

$$\Rightarrow S_{max} = 6.975'' \approx 7''$$

use #3 stirrups @ 7'' for the entire beam.

## Problem 2.6:

Given:

$L = 24'$ $\qquad w_{DL} = 1^{k/ft}$

$b = 12''$ $\qquad w_{LL} = 1.2^{k/ft}$

$h = 24''$ $\qquad w_u = 3.12^{k/ft}$

$d = 21.5''$ $\qquad \bar{w}_u = 4.08^{k/ft}$

$f'_c = 5\ ksi$ $\qquad M_u = 224^{k\text{-}ft}$

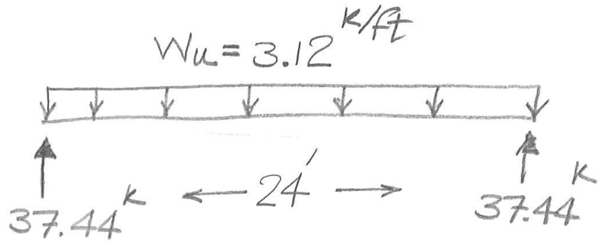
$f_y = 60^{ksi}$ $\qquad \bar{M}_u = 294.4^{k\text{-}ft}$

solution steps:

1 – Design the beam for shear under original loads.

2 – check adequacy of shear design under upgraded loads.

$$V_{u\,support} = \frac{w_u L}{2} = \frac{3.12 * 24'}{2} = 37.44^k$$




$d = 21.5''$

$$V_u(d) = 37.44 - 3.12 * \frac{21.5''}{12} = 31.85^k$$

$$V_c = 2\lambda\sqrt{f'_c}\, b_w d = 2 * 1.0 * \sqrt{5000} * 12'' * 21.5''$$

$$= 36{,}486.71^{lb} = 36.486^k$$

$$\phi V_c = 0.75 * 36.486^k = 27.365^k$$

Region I: $V_u \geqslant \phi V_c$

$$\phi V_s = V_u^{(d)} - \phi V_c = 31.85 - 27.365 = 4.485^k$$

$$= \frac{\phi A_v f_{yt} d}{S} \Rightarrow S = \frac{0.75 * 0.22 * 60 * 21.5}{4.485} = 47.46''$$

$$\phi V_s = 4.485^k < \phi 4\sqrt{f_c'}\, b_w d = 54.73^k$$

$$\Rightarrow S_{max} = \min\left(\frac{d}{2}, 24''\right)$$

$$= \frac{21.5}{2} = 10.75''$$

Use #3 stirrups @ 10" c/c for Region I

<u>Region II:</u> $V_u \geq \frac{\phi V_c}{2}$

$\leq \phi V_c$

find distance $x_c$ (where $V_{u_{x_c}} = \phi V_c$)

$$\phi V_c = 27.365^k = V_u - w_u x_c$$

$$= 37.44 - 3.12 x_c$$

$$x_c = \frac{37.44 - 27.365}{3.12} = 3.23'$$

find distance $x_m$ (where $V_{u_{x_m}} = \phi \frac{V_c}{2}$)

$$\frac{\phi V_c}{2} = 13.683^k = 37.44 - 3.12 x_m$$

$$x_m = \frac{37.44 - 13.683}{3.12} = 7.61'$$

Determine minimum shear reinforcement

$$S_{req'd} = \min. \left\{ \begin{array}{l} \dfrac{A_v f_{yt}}{\phi \sqrt{f_c'}\, b_w} = \dfrac{0.22 * 60000}{0.75\sqrt{5000} * 12''} = 20.74'' \\ \dfrac{A_v f_{yt}}{50 b_w} = \dfrac{0.22 * 60000}{50 * 12''} = 22'' \end{array} \right\} = 20.74''$$

$$S_{max} = \min\left(\frac{d}{2}, 24''\right) = 10.75''$$

use #3 stirrups @ 10" c/c for Region II

Region III: $V_u \leq \frac{\phi V_c}{2}$

No stirrups needed

For $X \leq 7.61' = 91.32'' \approx 92''$

use #3 stirrups @ 10" c/c

For $92'' \leq x \leq 144''$

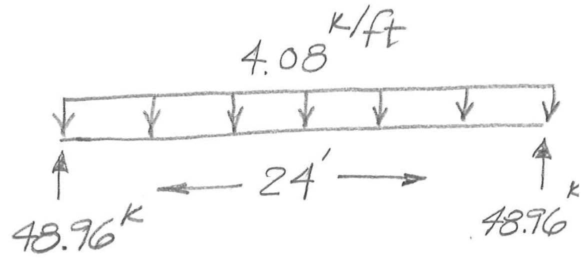
No stirrups required

symmetry applies

Upgraded Beam:

$V_{u\,support} = \frac{\bar{w}_u L}{2}$

$= \frac{4.08 \times 24'}{2} = 48.96^k$




$V_u(d) = 48.96 - 4.08 \times \frac{21.5}{12} = 41.65^k$

$\phi V_c = 27.365^k$ (as before)

Region I: $V_u \geq \phi V_c$

$\phi V_s = V_u(d) - \phi V_c = 41.65^k - 27.365^k = 14.285^k$

$= \frac{\phi A_v f_{yt} d}{S} \Rightarrow S = \frac{0.75 \times 0.22 \times 60 \times 21.5}{14.285} = 14.9''$

$\phi V_s = 14.285^k < \phi 4\sqrt{f_c'} b_w d = 54.73^k$

$\Rightarrow S_{max} = \min.\left(\frac{d}{2}, 24''\right)$

$= \frac{21.5}{2} = 10.75''$

Use #3 stirrups @ 10" c/c. No need for shear strengthening in Region I.

Region II: $V_u \geq \frac{\phi V_c}{2}$

$\leq \phi V_c$

find distance $x_c$ (where $V_{u x_c} = \phi V_c$)

$\phi V_c = 27.365^k = V_{u\,support} - w_u x_c$

$= 48.96 - 4.08 x_c$

$$x_c = \frac{48.96 - 27.365}{4.08} = 5.29' = 63.51''$$

find distance $x_m$ (where $V_{u x_m} = \phi \frac{V_c}{2}$)

$\frac{\phi V_c}{2} = 13.683^k = 48.96 - 4.08 x_m$

$$x_m = \frac{48.96 - 13.683}{4.08} = 8.65' = 103.76''$$

Determine minimumshear reinforcement

$$S_{req'd} = \min. \left\{ \begin{array}{l} \dfrac{A_v f_{yt}}{\phi\sqrt{f'_c}\, b_w} = 20.74'' \\ \\ \dfrac{A_v f_{yt}}{50\, b_w} = 22'' \end{array} \right\} = 20.74''$$

$S_{max} = \min\left(\frac{d}{2}, 24''\right) = 10.75''$

use #3 stirrups @10" c/c for Region II

Region III: $V_u \leq \frac{\phi V_c}{2}$

No stirrups required

For $0 \leq x \leq 7.61' = 91.32'' \approx 92''$

#3 stirrups @ $10''$ c/c are used

No need for shear strengthening

For $92'' \leq x \leq 103.76 \approx 104''$

No stirrups are provided

Their is a need for shear strengthening

For $104'' \leq x \leq 144''$

No stirrups required

No need for shear strengthening

Thus, there is only $12''$ distance requiring shear strengthening

use one layer of FRP with $12''$ width at that location

or perform shear strengthening design there.

## Problem 2.7:

Given: $L = 24'$     $W_{DL} = 1^{k/ft}$

$b = 12''$     $W_{LL} = 1.2^{k/ft}$

$h = 24''$     $A_s = 3\#9$ bars

$d = 21.5''$     $M_{DL} = 72^{k\text{-}ft}$

$f_c' = 5^{ksi}$     $M_{LL} = 86^{k\text{-}ft}$

$f_y = 60^{ksi}$

Find: At the time of strengthening

1- Extreme compression fiber stress

2- steel stress

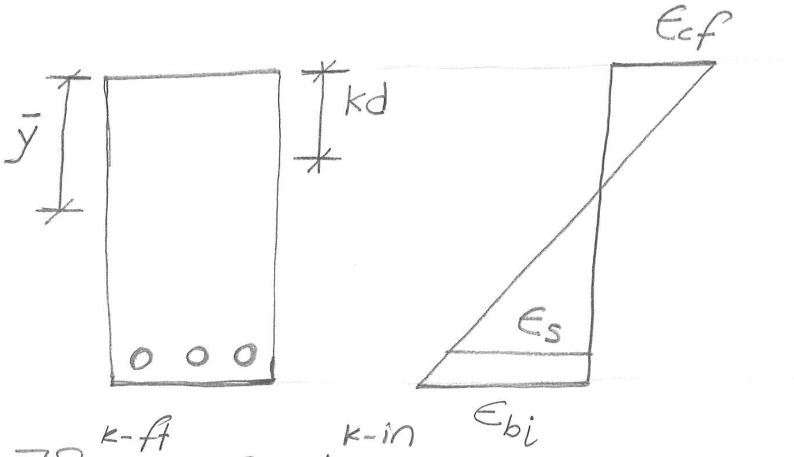
3- Extreme tensile strain in concrete at midspan ($\epsilon_{bi}$)

for the following cases:

a- Dead Load Only.

b- Dead Load + 20% Live Load

solution:




$$M_{DL} = 72^{k\text{-}ft} = 864^{k\text{-}in}$$

$$M_{DL+0.2LL} = 72^{k\text{-}ft} + 0.2 * 86^{k\text{-}ft} = 89.2^{k\text{-}ft} = 1070.4^{k\text{-}in}$$

Finding Cracking Moment: $A_s = 3 \times 1.0 = 3^{in^2}$

$$n = \frac{E_s}{E_c} = \frac{29000}{57\sqrt{5000}} = 7.2$$

$$\bar{y} = \frac{bh^2/2 + (n-1)A_s d}{bh + (n-1)A_s} = \frac{12 \times \frac{24^2}{2} + 6.2 \times 3 \times 21.5}{12 \times 24 + 6.2 \times 3}$$

$$= 12.58''$$

$$I_{gt} = \frac{bh^3}{12} + bh\left(\bar{y} - \frac{h}{2}\right)^2 + (n-1)A_s(d - \bar{y})^2$$

$$= \frac{12 \times 24^3}{12} + 12 \times 24 (12.58 - 12)^2 + 6.2 \times 3 \times (21.5 - 12.58)^2$$

$$= 15{,}400.82^{in^4}$$

$$f_r = 7.5\lambda\sqrt{f_c'} = 7.5 \times 1.0\sqrt{5000} = 530.33^{psi}$$

$$= 0.53^{ksi}$$

$$f_r = \frac{M_{cr}(h - \bar{y})}{I_{gt}} \Rightarrow M_{cr} = \frac{f_r I_{gt}}{h - \bar{y}}$$

$$= \frac{0.53 \times 15400.82}{24 - 12.58}$$

$$M_{cr} = 715.2^{k\text{-}in}$$

a) Dead Load Only

$$M_{DL} = 864^{k\text{-}in} > M_{cr}$$

Compute cracked section properties.

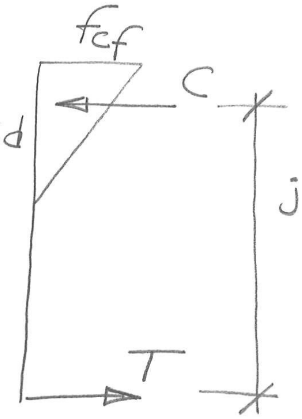
$$K = \sqrt{2n\rho + (n\rho)^2} - n\rho$$

$$\rho = \frac{3}{12 \times 21.5} = 0.01163$$

$$k = \sqrt{2 \times 7.2 \times 0.01163 + (7.2 \times 0.01163)^2} - 7.2 \times 0.01163$$

$$K = 0.334 \qquad Kd = 7.18''$$

$$M_{DL} = \frac{1}{2} f_{cf}\, Kd\, b\, j\, d$$




$$j = 1 - \frac{K}{3} = 0.889$$

$$jd = 19.11''$$

$$f_{cf} = \frac{2 M_{DL}}{b\, kd\, jd} = \frac{2 \times 864}{12 \times 7.18 \times 19.11}$$

$$= 1.05^{ksi} < 0.7 f_c'$$

$\therefore$ Linear-elastic analysis holds

$$M_{DL} = T jd = A_s f_s jd$$

$$f_s = \frac{M_{DL}}{A_s\, jd} = \frac{864}{3 \times 19.11} = 15.07^{ksi} \ll 60^{ksi}$$

$\therefore$ Linear-elastic analysis holds

Using strain compatibility

$$\frac{\epsilon_{bi}}{h - Kd} = \frac{\epsilon_{cf}}{Kd} \Rightarrow \epsilon_{bi} = \frac{\epsilon_{cf}}{Kd}(h - Kd)$$

$$= \frac{f_{cf}}{E_c\, Kd}(h - Kd)$$

$$\epsilon_{bi} = \frac{1.05}{57\sqrt{5000} \times 7.18}(24 - 7.18) = 0.00061$$

b) Dead Load + 20% Live Load:

since Linear-elastic analysis is expected to hold

$$I_{cr} = b\frac{kd^3}{3} + nA_s(d - kd)^2$$

$$= 12 \times \frac{7.18^3}{3} + 7.2 \times 3 \times (21.5 - 7.18)^2$$

$$= 5909.93 \text{ in}^4$$

$$f_{cf} = \frac{M_{DL+0.2LL} \times kd}{I_{cr}} = \frac{1070.4 \times 7.18}{5909.93} = 1.30^{ksi} < 0.7f_c'$$

$$f_s = n\frac{M_{DL+0.2LL} \times (d - kd)}{I_{cr}} = 7.2 \times \frac{1070.4 \times (21.5 - 7.18)}{5909.93}$$

$$= 18.67^{ksi} < f_y$$

$$\epsilon_{bi} = \frac{M_{DL+0.2LL} \times (h - kd)}{E_c I_{cr}} = \frac{1070.4(24 - 7.18)}{57\sqrt{5000} \times 5909.93}$$

$$= 0.00076$$

## Problem 2.8:

Given: $f_c' = 4\,ksi$ $\quad D = 20''$

$f_y = 60\,ksi$ $\quad$ clear cover $= 1.5''$

$f_{yt} = 50\,ksi$ $\quad A_s$: 9 #9 bars

#3 spirals

Find:

Increase in core capacity vs. loss of cover for

a) $s = 1.75''$

b) $s = 1.0''$

Solution:

$$\rho = \frac{9 \times 1^{in^2}}{\frac{\pi}{4} \times 20^2} = 0.0286 > 0.01$$

$$d_c = 20'' - 1.5'' \times 2 - \frac{3}{8}'' = 16.63''$$

$$\rho_{cc} = \frac{A_s}{\frac{\pi}{4} d_c^2} = \frac{9\,in^2}{\frac{\pi}{4} \times 16.63^2} = 0.0414$$

a) Cover loss:

$$A_{cover} = A - A_c = \frac{\pi}{4}(20)^2 - \frac{\pi}{4}(16.63)^2 = 96.95\,in^2$$

$$0.85 f_c' A_{cover} = 0.85 \times 4 \times 96.95 = 329.63^K$$

$$s' = s - d_{sp} = 1.75 - \frac{3}{8} = 1.375''$$

$$\rho_s = \frac{4 A_{sp}}{d_c\, s} = \frac{4\,(0.11)}{16.63 \times 1.75} = 0.0151$$

$$k_e = \frac{1 - \frac{s'}{2 d_c}}{1 - \rho_{cc}} = \frac{1 - \frac{1.375}{2 \times 16.63}}{1 - 0.0414} = 1.00$$

$$f_l' = \frac{1}{2} K_e \rho_s f_{y_t} = \frac{1}{2}(1.00)(0.0151)(50) = 0.3775^{ksi}$$

$$f_{cc}' = f_c'\left(-1.254 + 2.254\sqrt{1+\frac{7.94 f_l'}{f_c'}} - 2\frac{f_l'}{f_c'}\right)$$

$$= 4\left(-1.254 + 2.254\sqrt{1+\frac{3}{4}} - 2\frac{0.3775}{4}\right)$$

$$= 6.156$$

increase in core capacity:

$$0.85(f_{cc}' - f_c')A_c = 0.85(6.156-4)\times\frac{\pi}{4}\times 16.63^2 = 398.05^K$$

$$\frac{398.05}{329.63} = 1.2 = \frac{\text{increase in core capacity}}{\text{Loss of Cover}}$$

b) $s = 1.0''$ $\quad s' = 1.0 - \frac{3}{8} = 0.625''$

$$\rho_s = \frac{4(0.11)}{16.63\times 1} = 0.0265$$

$$K_e = \frac{1-\frac{0.625''}{2(16.63)}}{1-0.0414} = 1.024$$ due to the small spiral pitch

$$f_l' = \frac{1}{2}(1.024)(0.0265)(50) = 0.678^{ksi}$$

$$f_{cc}' = 4\left(-1.254 + 2.254\sqrt{1+\frac{7.94\times 0.678}{4}} - 2\frac{0.678}{4}\right)$$

$$= 7.437^{ksi}$$

$$0.85(f_{cc}' - f_c')A_c = 0.85(7.437-4)\times\frac{\pi}{4}\times 16.63^2 = 634.56$$

$$\frac{634.56}{329.63} = 1.925 = \frac{\text{increase in core capacity}}{\text{Loss of Cover}}$$

## Problem 2.9:

Given: $b = 300^{mm}$ $\quad f'_c = 25 MPa$

$h = 700^{mm}$ $\quad f_y = 414 MPa$

$d = 641^{mm}$ $\quad L_a = 4^{m}$

$A_s : 3\phi 22^{mm}$ $\quad M_{cr} = 85.2^{KN \cdot m}$

$P = 60^{KN}$

Find:
sectional stresses based on $I_{cr}$

solution:

$$M_{max} = \frac{P}{2} \times L_a = 120^{KN.m} > 85.2^{KN.m}$$

$\Rightarrow$ cracked section

$Kd = 175^{mm}$ (example 2.7)

$$I_{cr} = b\frac{kd^3}{3} + n A_s (d - kd)^2$$

$$= 300 \times \frac{175^3}{3} + 8.51 \times 1164 \times (641 - 175)^2$$

$$= 2,687,006,660^{mm^4}$$

$$f_{cc} = \frac{M_{max} \times Kd}{I_{cr}} = \frac{120 \times 10^3 \times 175}{2,687,006,660} = 7.815 \times 10^{-3} \frac{KN}{mm^2}$$

$$= 7.815\ MPa = 7.85^{MPa}$$

$$f_s = n\frac{M_{max}(d - Kd)}{I_{cr}} = 8.51 \frac{\times 120 \times 10^3 \times (641 - 175)}{2,687,006,660} = 0.1771^{GPa}$$

$$= 177.1^{MPa} \approx 176.93^{MPa}$$

Thus, Linear elastic analysis is valid