1.2 Solutions to Exercises in Chapter 2

Show that there is no solution to the problem of minimizing the function

$$g(t_1, t_2) = \frac{2}{t_1 t_2} + t_1 t_2 + t_1,$$

over $t_1 > 0$, $t_2 > 0$. *Can* $g(t_1, t_2)$ *ever be smaller than* $2\sqrt{2}$? The DGP is inconsistent in this case. We can still ask if there is a positive greatest lower bound to the values that g can take on. Show that, if $t_1 \geq 1$, then $g(t_1, t_2) \geq 3$, while, if $t_2 \leq 1$, then $g(t_1, t_2) \geq 4$. Therefore, our hunt for the greatest lower bound is concentrated in the region described by $0 < t_1 < 1$, and $t_2 > 1$. Since there is no minimum, we must consider values of t_2 going to infinity, but such that $t_1 t_2$ does not go to infinity and $t_1 t_2$ does not go to zero; therefore, t_1 must go to zero. Let $t_2 = \frac{f(t_1)}{t_1}$, for some function $f(t)$ such that $f(0) > 0$. Then, as t_1 goes to zero, $g(t_1, t_2)$ goes to $\frac{2}{f(0)} + f(0)$. Determine how small this limiting quantity can be.

1.3 Solutions to Exercises in Chapter 3

Exercise 3.1 *Let S and T be nonempty subsets of the real line, with $s \leq t$ for every s in S and t in T. Prove that* $\mathrm{lub}(S) \leq \mathrm{glb}(T)$. Since $s \leq t$, for every s and t, every t is an upper bound for the set S. Therefore, $\mathrm{lub}(S) \leq t$, for all t. This tells us that $\mathrm{lub}(S)$ is a lower bound for the set T and, therefore, $\mathrm{lub}(S)$ cannot be larger than the greatest lower bound for T.

Exercise 3.2

Let $f(x, y) : \mathbb{R}^2 \to \mathbb{R}$, and, for each fixed y, let $\inf_x f(x, y)$ denote the greatest lower bound of the set of numbers $\{f(x, y) | x \in \mathbb{R}\}$. Show that

$$\inf_x \left(\inf_y f(x, y) \right) = \inf_y \left(\inf_x f(x, y) \right).$$

Hint: note that

$$\inf_y f(x, y) \leq f(x, y),$$

for all x and y.