Chapter 2 Solution

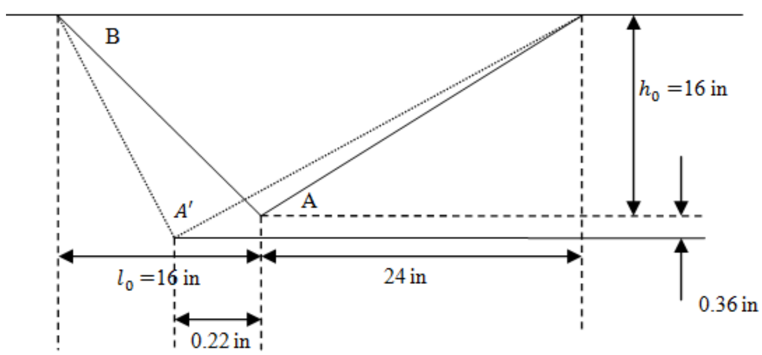
2.3

Given: Truss as shown, $\Delta_{ax} = 0.360$ $\Delta_{ay} = 0.220$

Find: ε_{ab}

Solution:

Defintion: Strain $\varepsilon = \frac{\Delta L}{L_0}$ (2.1)



$$\Delta_{AB} = \sqrt{{\Delta_{ax}}^2 + {\Delta_{ay}}^2} = \sqrt{0.36^2 + 0.22^2} = 0.4219in$$

$$L_{AB} = \sqrt{{L_0}^2 + {h_0}^2} = \sqrt{16^2 + 16^2} = 4.7568in$$

$$\varepsilon_{AB} = \frac{\Delta_{AB}}{L_{AB}} = \frac{0.4219}{4.7568} = 0.08869 \qquad \text{(ans)}$$

2.5

Given: Truss system shown $m = 2kg$ $\sigma_{safe} = 10Mpa$

Find:

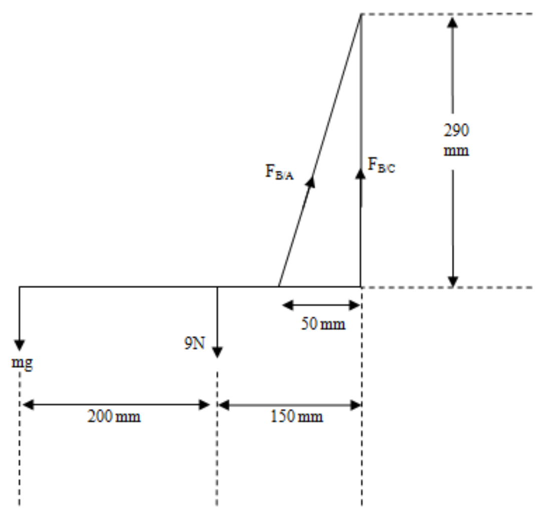
(a)Average normal stress in AB

(b)Largest suspended mass that can be supported

Solution:

Definition: Stress $\sigma = \frac{P}{A}$ (2.7)

FBD:



(a)

$$\theta = tan^{-1}(\frac{290}{50}) = 80.22°$$

$$\sum M_c = 2 \times 9.81 \times (0.2 + 0.15) + 9 \times 0.15 - 0.05 \times F_{B/A} sin\theta = 0$$

$$F_{B/A} = \frac{8.22}{0.05 sin\theta} = 166.76\text{N}$$

Therefore the normal stress

$$\sigma_{AB} = \frac{F_{B/A}}{A_{AB}} = \frac{166.76}{28 \times 10^{-6}} = 5.95\text{Mpa}$$

(b)

$$\sum M_c = 9.81m \times (0.2 + 0.15) + 9 \times 0.15 - 0.05 \times F_{B/A} sin\theta = 0$$
$$F_{B/A} = \frac{3.4335m + 1.35}{0.05sin(80.22^\circ)} = 69.6827(m + 0.39185)$$

$$\sigma_{safe} = \frac{F_{B/A}}{A_{AB}} = \frac{69.6827(m + 0.39185)}{28 \times 10^{-6}} = 10 \times 10^6 \qquad (1)$$

Solve eqn (1)

$$m = 3.625kg \qquad \text{(ans)}$$

2.6

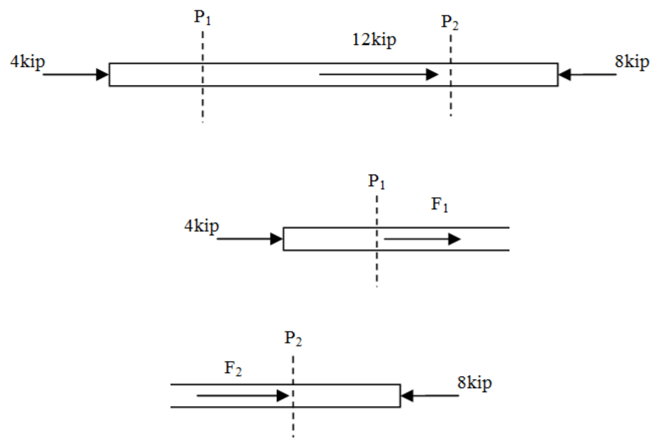
Given: Bar and loaded as shown

Find: Normal stress at P_1 and P_2

Solution:

Definition: Stress $\sigma = \frac{P}{A}$ (2.7)

FBD:



$$\sum F_{x1} = F_1 + 4 = 0$$
$$F_1 = -4kip$$

$$\sum F_2 = F_2 - 8 = 0$$
$$F_2 = 8kip$$
$$A = \pi r^2 = \pi \times 2^2 = 12.5664in^2$$

$$\sigma_1 = \frac{F_1}{A} = \frac{4}{12.5664} = 0.318ksi \qquad \text{(ans)}$$
$$\sigma_2 = \frac{F_2}{A} = \frac{8}{12.5664} = 0.637ksi \qquad \text{(ans)}$$

2.7

Given: $P_{aw} = 1800psi$ $P_{ac} = 1250psi$ $W_c = 150Ibf/ft^3 = 1.0416Ibf/in^3$

Find:

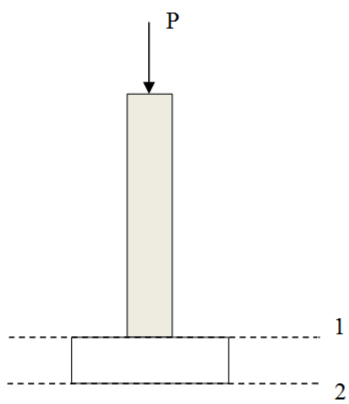
(a) specific weight of wood

(b) load P when the weights of wood and concrete are included

(c) load P when the weights of wood and concrete are not inculded

Solution:

Definition: Stress $\sigma = \frac{P}{A}$ (2.7)



(a)

$$S_w = {l_w}^2 = 4^2 = 16in^2$$
$$V = hS_w = 2 \times 12 \times 16 = 384in^3$$
$$w_0 = \frac{W}{V} = \frac{21}{384} = 0.05469Ibf/in^3$$

(b)

$$W_w = w_0 V_w = 0.05469 \times 4^2 \times 6 \times 12 = 63Ib$$
$$W_c = w_c V_c = 1.0416 \times 6^2 \times 1 \times 12 = 432Ib$$
$$S = L^2 = 4^2 = 16in^2$$
$$S_c = 6^2 = 36in^2$$

The max P wood can support

$$F_{y1} = SP_{aw} = 16 \times 1800 = 28800Ib$$

The P allowable is

$$P_1 = F_{y1} - W_w = 28800 - 63 = 28737Ib$$

The max P concrete can support at surface 1

$$F_{y2} = SP_{ac} = 16 \times 1250 = 20000Ib$$

The P allowable at surface 1

$$P_2 = F_{y2} - W_w - W_c = 20000 - 63 = 19937Ib$$

The max P concrete can support at surface 2

$$F_{y3} = S_c P_{ac} = 36 \times 1250 = 45000Ib$$

The P allowable at surface 2

$$P_3 = F_{y3} - W_w - W_c = 45000 - 63 - 432 = 44505Ib$$

Compare P_1, P_2 and P_3

$$P2 < P1 < P3$$

Therefore the P allowable is 19937Ib

(c)
When the weight of wood and concrete are not included

$$P_1 = F_{y1} = 28800Ib$$
$$P_2 = F_{y2} = 20000Ib$$
$$P_3 = F_{y3} = 45000Ib$$

$$P2 < P1 < P3$$

The supported load allowable is 20000Ib

2.8

Given: bolt cutter with dimensions and force F exerted on the handle as shown

Find:

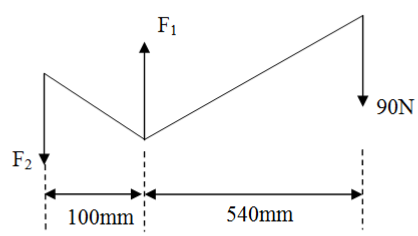
(a) average normal stress in each link

(b) average shear stress in the pins

Solution:

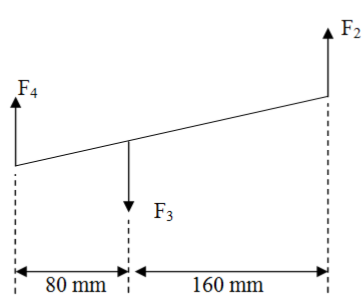
Definition: $\tau = \frac{P}{A_v}$ (2.7) $\sigma = \frac{P}{A}$ (2.1)

(a)



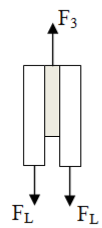
1.Handle

$$S_L = 750mm^2 = 7.5 \times 10^{-4}m^2$$
$$\sum M_1 = 100F_2 - 90 \times 540 = 0$$
$$F_2 = \frac{90 \times 540}{100} = 486N$$



2.Jaw side view

$$\sum M_4 = (80+160)F_2 - 80F_3 = 0$$
$$F_3 = \frac{80}{160+80}F_2 = 162N$$



3.Jaw front view

$$\sum F_y = F_3 - 2F_L = 0$$
$$F_L = \frac{F_3}{2} = \frac{162}{2} = 81N$$
$$\sigma = \frac{F_L}{A_L} = \frac{81}{7.5\times 10^{-4}} = 1.08\times 10^3 Pa$$

(b)

$$A = \pi R^2 = \pi(\frac{20}{2}\times 10^{-3}) = 3.142\times 10^{-4} m^2$$
$$\tau = \frac{P}{A} = \frac{F_L}{A} = \frac{81}{3.14\times 10^{-4}} = 2.58\times 10^3 Pa$$

2.10

Given: allowable shear stress $\tau_{max} = 1MN/m^2$, allowable normal stress $\sigma_{max} = 4MN/m^2$

Find: The max compressive force the block can support

Solution:

Definition: $\sigma_{0-90^\circ} = \frac{P}{A} sin^2\theta$ (2.18a) $\tau_{0-90^\circ} = \frac{P}{A} sin\theta cos\theta$ (2.18b)

$$\theta = tan^{-1}(\frac{24}{7}) = 73.74^\circ$$
$$A = R^2 = 50^2 = 2500mm^2$$

Limited by max normal stress

$$F_1 = \frac{\sigma A}{sin^2\theta} = \frac{4 \times 2500}{sin^2(73.74^\circ)} = 10850N$$

Limited by max shear stress

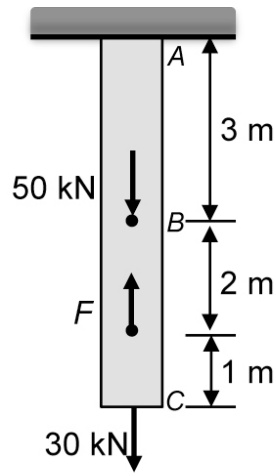
$$F_2 = \frac{\tau A}{sin\theta \cos\theta} = \frac{2500}{sin(73.74^\circ)cos(73.74^\circ)} = 9300N$$

Since $F_2 < F_1$, block will fail when F_1 is loaded

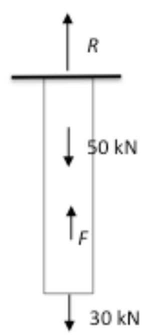
The max compressive force is $F_1 = 9300N$

2.11

Given: steel rod AC with cross-sectional area A = 50 mm^2, known forces applied at B and C.
Find: Allowable force F applied at point Q; deformations δ_{BC} and δ_{AC}
Assume: Hooke's law applies, yield strength 150 MN/m^2.



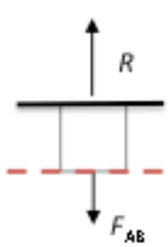
Solution: we begin, of course, with our FBD:



By applying equilibrium in the vertical direction, we can solve for the reaction force

$R = 50 + 30 - F = 80 - F$ kN.

We next use the method of sections to solve for the internal forces in segment AB of the bar.



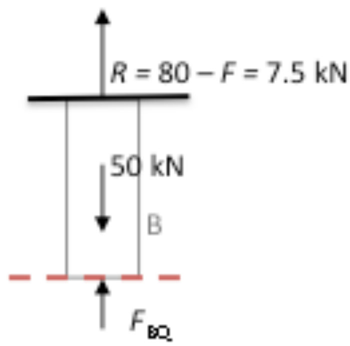
The internal force in segment AB is thus $F_{AB} = R = (80 - F)$ kN.

To keep the stress in this segment below the allowable limit, we must have:

$$\sigma_{AB} = \frac{F_{AB}}{A} = \frac{80 - F \text{ kN}}{50 \times 10^{-6} \text{m}^2} \le 150 \frac{\text{MN}}{\text{m}^2} = 150{,}000 \frac{\text{kN}}{\text{m}^2}$$

which we can solve for $\underline{F \ge 72.5 \text{ kN}}$ (Ans)

Examining segment BC will require two cuts, because there is a change in the internal force within this segment. First, to find the change in the length of segment BQ, we need to find the internal force in this segment, and use eqn (2.20).



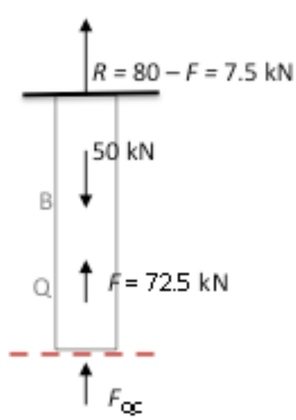
Equilibrium tells us that F_{BQ} = 50 -7.4 = 42.5 kN (Compressive)

The change in length of segment BQ is then:

$$\delta_{BQ} = \frac{F_{BQ} L_{BQ}}{AE} = \frac{(-42.5 \times 10^3 \text{N})(2\text{m})}{(50 \times 10^{-6} \text{m}^2)(200 \times 10^9 \text{Pa})} = -8.5 \times 10^{-3} \text{m}$$

That is, segment BQ experiences an 8.5 mm contraction.

Making another cut between point Q and point C, we can find the internal force, and then the deformation, of segment QC.



Equilibrium yields F_{QC} = 30 kN (Tensile)

The change in length of segment QC is then:

$$\delta_{QC} = \frac{F_{QC}L_{QC}}{AE} = \frac{(30\times10^{3}\,\mathrm{N})(1\mathrm{m})}{(50\times10^{-6}\,\mathrm{m}^2)(200\times10^{9}\,\mathrm{Pa})} = 3\times10^{-3}\,\mathrm{m}$$

We can combine the deformations of segments BQ and QC to find the deformation of segment BC:

$$\delta_{BC} = \delta_{BQ} + \delta_{QC} = \frac{F_{BQ}L_{BQ}}{AE} + \frac{F_{QC}L_{QC}}{AE} = -8.5 + 3\ \mathrm{mm} = \text{-}5.5\ \mathrm{mm}\ \text{(Ans)}$$

That is, <u>segment BC experiences a 5.5 mm contraction</u>.

Finally, to find the change in length of the whole bar AC, we linearly combine the deformation of its subsections:

$$\delta_{AC} = \delta_{AB} + \delta_{BQ} + \delta_{QC} = \frac{F_{AB}L_{AB}}{AE} + \frac{F_{BQ}L_{BQ}}{AE} + \frac{F_{QC}L_{QC}}{AE} = -3.25\mathrm{mm}\ \text{(contraction)}\ \ \text{(Ans.)}$$

2.12

Given:W=10kip E=30000ksi $\sigma_y = 36$ksi $L_1 = 10ft$ $L_2 = 40$

Find:

(a)the area for each cable A_1 and A_2

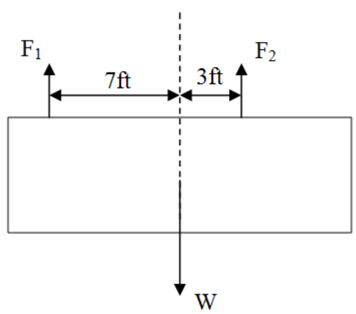
(b)normal stress σ and shear stress τ when grain angle θ is 15°

Solution:

Definition: Axial deformation $\delta = \frac{FL}{AE}$ (2.20),Stress $\sigma = \frac{P}{A}$ (2.7), $\sigma_{0-90^\circ} = \frac{P}{A}sin^2\theta$ (2.18a), $\tau_{0-90^\circ} = \frac{P}{A}sin\theta cos\theta$ (2.18b)

(a)

FBD:



$$\sum M_1 = -7W + (7+3)F_2 = 0$$
$$F_2 = \frac{7}{10}W = \frac{7}{10} \times 10 = 7kip$$
$$\sum M_2 = -(7+3)F_1 + 3W = 0$$
$$F_1 = \frac{3}{10} \times 10 = 3kip$$

Limited by the yield strength

$$A_{allow1} = \frac{F_1}{\sigma_y} = \frac{3}{36} = 0.083in^2$$
$$A_{allow2} = \frac{F_2}{\sigma_y} = \frac{7}{36} = 0.194in^2$$

The score hangs level
Therefoe

$$\delta_1 = \delta_2$$
$$\frac{F_1 L_1}{A_1 E} = \frac{F_2 L_2}{A_2 E} \tag{1}$$

When A_1 has the max $A_1 = A_{allow1}$
by eqn(1)

$$\frac{3 \times 10}{0.083 \times 30000} = \frac{7 \times 40}{A_2 \times 30000}$$
$$A_2 = 0.777 in^2 > A_{allow2}$$

Fails

When A_2 has the max $A_2 = A_{allow2}$
by eqn(1)

$$\frac{3 \times 10}{A_1 \times 30000} = \frac{7 \times 40}{0.194 \times 30000}$$
$$A_1 = 0.0208 < 0.083$$

Therefore $A_1 = 0.0208 in^2$ and $A_2 = 0.194 in^2$

(b)

$$\sigma = \frac{F_2}{A_2} sin^2\theta = \frac{3}{0.194} sin^2 15^\circ = 1.04 ksi \tag{ans}$$
$$\tau = \frac{F_2}{A_2} sin\theta cos\theta = \frac{3}{0.194} sin 15^\circ cos 15^\circ = 3.87 ksi \tag{ans}$$

2.13

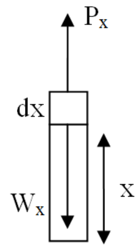
Given: pipe hanging vertically unit weight w_0, cross section area A, Moduli E

Find: the movement Δ of the tip of pipe for (a)L_1=3500ft (b)L_2=35000ft

Solution:

Definition:$\delta = \frac{FL}{AE}$ (2.20)

FBD:



$$
\begin{aligned}
W_x =& w_0 x \\
\sum F_y =& P_x - W_x = 0 \\
P_x =& W_x \\
\delta_i =& \frac{P_{xi} dx}{AE} \\
\sum \delta = \int_0^L \frac{w_0 x dx}{AE} &= \frac{w_0 x^2}{2AE}
\end{aligned}
$$

For $L_1 = 3500 ft$,

$$\delta = 1.225 \times 10^7 \frac{w_0}{2AE} \quad \text{(ans)}$$

For $L_2 = 35000 ft$

$$\delta = 1.225 \times 10^9 \frac{w_0}{2AE} \quad \text{(ans)}$$

2.14

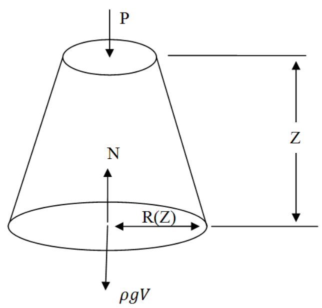
Given:

$$r(z) = r_0 e^{\frac{\rho g z}{2\sigma}}$$

Find: Stress $\sigma(z)$ is constant as z changing

Solution:

FBD:



$$\begin{aligned}
P &= \sigma \pi {r_0}^2 \\
\sum F_y &= -P - \rho g V + N = 0 \\
N &= P + \rho g V \\
V &= \int_0^z \pi r(h)^2 dh \\
\text{Let } k = \frac{\rho g}{2\sigma} & \\
V &= \frac{1}{2k} \pi {r_0}^2 (e^{2kz} - 1) \\
N &= P + \rho g \frac{1}{2k} \pi {r_0}^2 (e^{2kz} - 1) \\
\sigma(z) &= \frac{N}{\pi r(z)^2} = \frac{N}{\pi (r_0 e^{kz})^2} \\
&= \frac{\sigma \pi {r_0}^2 + \rho g \frac{\sigma}{\rho g} \pi {r_0}^2 (e^{\frac{\rho g}{\sigma}} - 1)}{\pi (r_0 e^{\frac{\rho g}{2\sigma}})^2} \\
&= \sigma && \text{(constant)}
\end{aligned}$$

2.15

Given:Bar and load as shown

Find:Reaction at both walls

Solution:

FBD:

$$\sum F_x = F_b + P - F_a = 0$$

$$F_a - F_b = p \quad (1)$$

$$\delta_a + \delta_b = 0$$

$$\frac{F_a a}{AE} + \frac{F_b b}{AE} = 0$$

$$F_a a + F_b b = 0 \quad (2)$$

From (1)

$$F_a = p + F_b$$

Substitute F_a with $p + F_b$ in (2)

$$(F_b + p)a + F_b b = 0$$

$$F_b(a + b) = -pa$$

$$F_b = \frac{-pa}{a + b} \quad \text{(ans)}$$

$$F_a = \frac{pb}{a + b} \quad \text{(ans)}$$

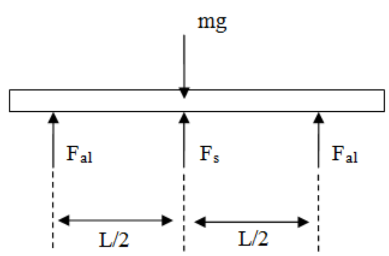
2.16

Given:A rigid slab $m = 15000kg$ support by three columns as shwon

Find:Compressive force of the columns F_{al}, F_s

Solution:

FBD



$$\sum F_y = 2F_{al} + F_s - mg = 0$$

$$F_s = mg - 2F_{al} \tag{1}$$

$$\delta_{al} = \delta_s$$

$$\frac{F_{al}L}{AE_{al}} = \frac{F_sL}{AE_s}$$

From Appendix C $E_{al} = 70Gpa$ and $E_s = 200Gpa$

$$\frac{F_{al}}{70} = \frac{F_s}{200} \tag{2}$$

Substitute F_s with $mg - 2F_{al}$ in (2)

$$\frac{F_{al}}{70} = \frac{mg - 2F_{al}}{200}$$
$$200F_{al} = 70mg - 140F_{al}$$

$$F_{al} = \frac{70mg}{340} = \frac{70 \times 15000 \times 9.8}{340}$$
$$= 30260N \qquad \text{(ans)}$$

$$F_s = mg - 2F_al = 15000 \times 9.8 - 2 \times 30260$$
$$= 86470N \qquad \text{(ans)}$$

2.17

Given:Bar loaded as shown $A_{BC} = A$ and $A_{AB} = mA$

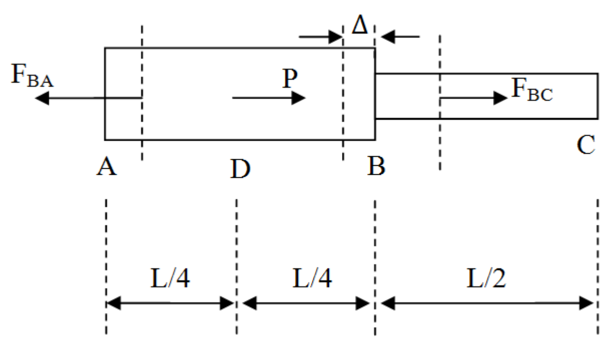
Find:

(a)Reactions at both walls

(b)The displacement of the point D, Δ_{AD} at which the load P acts

Solution:

FBD:



$$\sum F_y = F_{BC} + P - F_{BA} = 0$$
$$F_{BA} = P + F_{BC} \tag{1}$$

$$\delta_{AB} + \delta_{BC} = 0$$
$$\frac{F_{AB}\frac{L}{2}}{mAE} + \frac{F_{BC}\frac{L}{2}}{AE} = 0$$
$$\frac{F_{AB}}{m} + F_{BC} = 0 \tag{2}$$

Use (1) substitute F_{AB} with $F_{BC} + P$ in (2)

$$\frac{F_{BC} + P}{m} + F_{BC} = 0$$
$$F_{BC} = -\frac{1}{1+m}P$$
$$F_{AB} = F_{BC} + P = \frac{m}{1+m}P$$

$$R_A = F_{AB} = F_{BC} + P = \frac{m}{1+m}P \qquad \text{(ans)}$$
$$R_C = -F_{BC} = -\frac{1}{1+m}P = \frac{1}{1+m}P \qquad \text{(ans)}$$

2.18

Given:Bar loaded as shown $A_{BC} = A$ and $A_{AB} = mA$

Find:

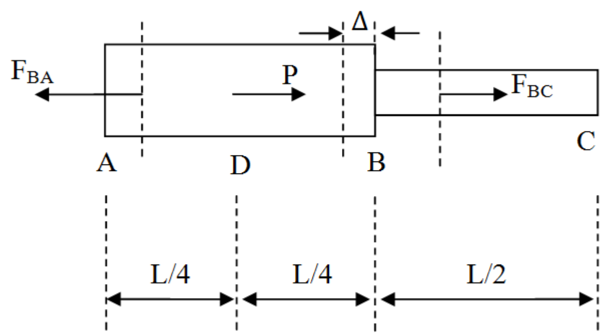
(a)Reactions at both walls

(b)The displacement of the point D, Δ_{AD} at which the load P acts

Solution:

Definition: $-k_1 - k_2 + P = 0$ (2.33a)

FBD:



Stifness $k = \frac{AE}{L}$

$$\sum F_y = -k_{AB}\Delta - k_{BC}\Delta + P = 0$$

$$\Delta = \frac{P}{k_{AB} + k_{BC}}$$

$$R_A = F_{BA} = \frac{k_{AB}}{k_{AB} + k_{BC}} = \frac{m}{1+m}P \qquad \text{(ans)}$$

$$R_C = F_{BC} = \frac{k_{BC}}{k_{AB} + k_{BC}} = \frac{1}{1+m}P \qquad \text{(ans)}$$

$$\Delta_{AD} = \frac{F_{AB}L_{AD}}{A_{AB}E} = \frac{\frac{m}{1+m}P \times \frac{1}{4}L}{(mA)E} = \frac{PL}{4AE(1+m)} \qquad \text{(ans)}$$

2.19

Given: Two bar with $d = 30mm = 0.03m$ one made of brass and the other of stainless steel $F_B = 40KN = 40000N$. There is a gap, $\Delta = 0.20mm = 2 \times 10^{-7}m$ between end E and the wall.

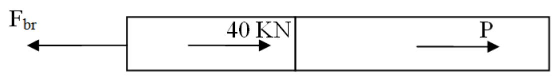
Find:

(a)The smallest force P needed to close the gap

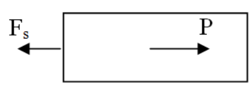
(b)The reaction at A and E if P is twice as calculated in (a)

Solution: (a)

FBD:



1. Segment AB



2. Segment BD

$$\sum F_{AE} = P + 40000 - F_{AB} = 0$$
$$F_{AB} = P + 40000$$
$$\sum F_{CE} = P - F_{BD} = 0$$
$$F_{BD} = P$$

From Appendix C $E_{br} = 105Gpa$ and $E_s = 190Gpa$

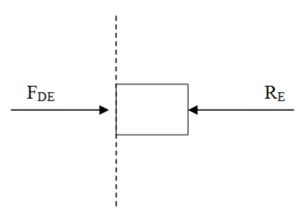
$$\begin{aligned}
A &= \frac{\pi d^2}{4} = \frac{\pi 0.03^2}{4} = 7.069 \times 10^{-4} m^2 \\
\Delta &= \delta_{AB} + \delta_{BC} + \delta_{CD} \\
&= \frac{F_{AB}L_{AB}}{AE_{br}} + \frac{F_{BD}L_{BC}}{AE_{br}} + \frac{F_{BD}L_{CD}}{AE_s} \\
&= \frac{(P + 40000) \times 0.1}{(7.069 \times 10^{-4})(105 \times 10^9)} + \frac{P \times 0.1}{(7.069 \times 10^{-4}) \times (105 \times 10^9)} \\
&+ \frac{P \times 0.1}{(7.069 \times 10^{-4}) \times (190 \times 10^9)} \\
&= 2 \times 10^{-4}
\end{aligned}$$

Solve equation for P

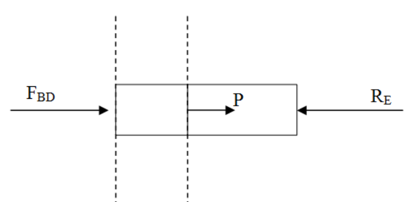
$$P = 42490N \qquad \text{(ans)}$$

(b)

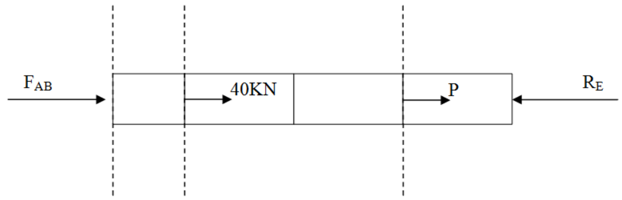
FBD:



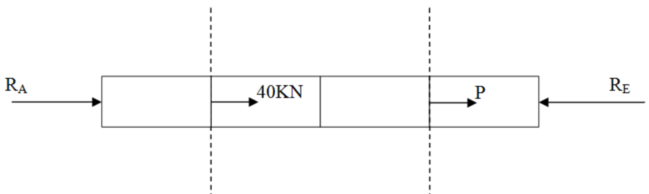
1. Segment DE



2. Segment BD



3. Segment AB



4. Segment AE

In segment DE

$$\sum F_x = F_{DE} - R_E = 0$$
$$F_{DE} = R_E$$

In segment AD

$$\sum F_x = P + F_{BD} - R_E = 0$$
$$F_{BD} = R_E - P$$

In segment AB

$$\sum F_x = P + 40000 + F_{AB} - R_E = 0$$
$$F_{AB} = R_E - P - 40000$$

For the bar AE

$$\sum F_x = R_A + 40000 + P - R_E = 0$$
$$R_A = R_E - P - 40000 \tag{1}$$

$$\begin{aligned}
\Delta &= \delta_{AB} + \delta_{BD} + \delta_{DE} \\
&= \frac{F_{AB}L_{AB}}{AE_s} + \frac{F_{BD}L_{CD}}{AE_s} + \frac{F_{BD}L_{BD}}{AE_{br}} + \frac{F_{AB}L_{AB}}{AE_{br}} \\
&= \frac{0.1(R_E - P - 40000)}{(7.069 \times 10^{-4}) \times (105 \times 10^9)} + \frac{0.1(R_E - P)}{(7.069 \times 10^{-4}) \times (105 \times 10^9)} \\
&+ \frac{0.1(R_E - P)}{(7.069 \times 10^{-4}) \times (190 \times 10^9)} + \frac{0.1R_E}{(7.069 \times 10^{-4}) \times (190 \times 10^9)} \\
&= 2 \times 10^{-4} \qquad (2) \\
P &= 2P_{(a)} = 2 \times 42490 = 84980N
\end{aligned}$$

Solve equation (2)

$$R_E = 130500N \qquad \text{(ans)}$$

By (1)

$$R_A = R_E - P - 40000 = 130500 - 84980 - 40000 = 5520N \qquad \text{(ans)}$$

2.20

Given:$E = 1.1GPa$ and $\alpha = 190 \times 10^{-6}/°C^{-1}$ $\sigma_y = 23Mpa$ $T_0 = 15°C$ $T = 37°C$

Find:

(a)Thermal strain ε

(b)If the pip had remained straight would the yield stress have been exceeded?

Solution:

Defintiion: Thermal strain $\varepsilon_T = \alpha \Delta T$ (2.36)

$$\varepsilon_T = \alpha(T - T_0) = 190 \times 10^{-6} \times (37 - 15) = 4.75 \times 10^-3 = 0.475\% \qquad \text{(ans)}$$
$$\sigma = E\varepsilon = 1.1 \times 10^9 \times 4.75 \times 10^-3 = 5.23 \times 10^6 pa = 5.23Mpa < \sigma_y$$

It would not be exceeded

While the pipe is safe from yielding, it still buckled.

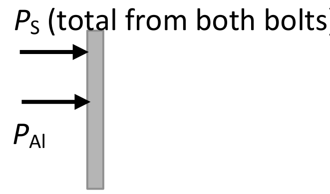
2.21

Given: system of aluminum cylinder and steel bolts rigidly joined by shared end plates

Find: stress in steel bolts following temperature increase of 50°C

Assume: OK to neglect thermal deformation of end plates

Solution:



Equilibrium requires that $P_{Al} = -P_s$ as shown at right.

We are combining the effects of both bolts into a single resultant force P_s.

Compatibility requires that the aluminum and steel deform by the same amount:

$$\Delta L_s = \Delta L_{Al}$$

$$\frac{P_S L_S}{A_S E_S} + \alpha_S \Delta T L_S = \frac{P_{Al} L_{Al}}{A_{Al} E_{Al}} + \alpha_{Al} \Delta T L_{Al}$$

Note that since we are combining the effects of both bolts, A_S here is the combined area of both bolts as well.

Substituting $P_{Al} = -P_S$ into this expression and doing some rearranging, we have:

$$P_S \left(\frac{L_S}{A_S E_S} + \frac{L_{Al}}{A_{Al} E_{Al}} \right) = \Delta T (\alpha_{Al} L_{Al} - \alpha_S L_S)$$

We can then solve for the force of the two bolts:

$$P_S = \frac{\Delta T (\alpha_{Al} L_{Al} - \alpha_S L_S)}{\left(\frac{L_S}{A_S E_S} + \frac{L_{Al}}{A_{Al} E_{Al}} \right)} = \frac{(50C)[(23\times10^{-6})(0.21\text{m}) - (12\times10^{-6})(0.25\text{m})]}{\left(\frac{0.25\text{m}}{(2\times10^{-4}\text{m}^2)(200\text{GPa})} + \frac{0.21\text{m}}{(30\times10^{-4}\text{m}^2)(70\text{GPa})} \right)} = 12.6 \text{ kN}$$

Then the stress in the bolts is this total force over the total bolt cross-sectional area, and each bolt will experience the same stress:

$$\sigma_S = \frac{P_S}{A_S} = \frac{12.6 \text{ kN}}{2\times10^{-4} \text{ m}^2} = 63 \text{ MPa} \quad \text{(Ans.)}$$

2.22

Given: Stepped bar as shown with d = 2.4 in, fillet radius 0.2 or hole radius 0.2

Find: comparison of load carrying capacity between fillet and hole cases

Solution relies on the graphed stress concentration factors in Fig 2.25:

$\sigma_{yield} = K\dfrac{P_{\max}}{A}$, so for the same yield strength and the same area (in this case, 2 in. times an unknown thickness), the case with the larger K will have the more reduced "load carrying capacity," $P_{\max}$.

Using Figure 2.25:

For fillets, $\dfrac{r}{d} = \dfrac{0.2}{2} = 0.1 \rightarrow K_f \approx 1.8$

For hole, $\dfrac{r}{d} = \dfrac{0.2}{2.4 - 0.4} = 0.1 \rightarrow K_h \approx 2.5$

The hole case has a higher stress concentration factor *K*, so the load carrying capacity <u>is reduced</u> by this change in geometry.

2.23

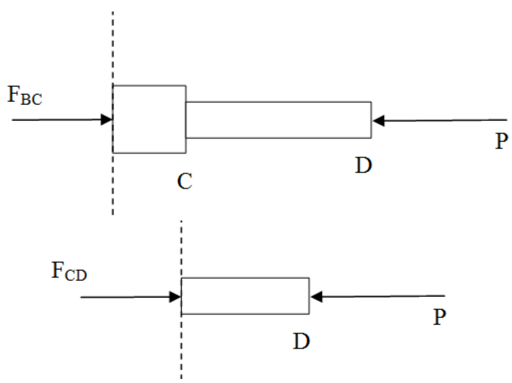
Given: Bar length L, cross section area in segment CD $A_{CD} = A$ and $A_{CB} = n^2 A$ modulus E

Find:The strain engergy of the bar when P is loaded

Solution:

Definition: complementary energy denstiy $U_0 = \frac{1}{2}E\varepsilon$ (2.41)

FBD:



$$
\begin{aligned}
\sum F_x &= F_{BC} - P = 0 \\
F_{BC} &= P \\
\sum F_x &= F_{CD} - P = 0 \\
F_{CD} &= P \\
U &= \frac{1}{2}E\varepsilon^2 V = \frac{1}{2}E(\frac{F}{AE})^2 LA = \frac{F^2 L}{2AE}
\end{aligned}
$$

$$
\begin{aligned}
U_{BD} &= U_{BC} + U_{CD} \\
&= \frac{F_{BC}L_{BC}}{n^2 AE} + \frac{F_{AB}L_{AB}}{AE} \\
&= \frac{(1+n^2)PL}{n^2 AE} && \text{(ans)}
\end{aligned}
$$

2.24

Given:$d = 1cm = 0.01m$ $L = 45cm = 0.45m$ $E = 1.4Gpa$ $\varepsilon = 2\% = 0.02$ $m = 40kg$

Find: The strain energy E, and how high this amount of energy could lift the kangaroo

Solution:

$$\begin{aligned} U &= \frac{1}{2}E\varepsilon^2 V \\ &= \frac{1}{2}E\varepsilon^2(\pi R^2 L) \\ &= \frac{1}{2} \times (1.4 \times 10^9) \times 0.02^2 \times (\pi \times (\frac{0.01}{2})^2 \times 0.45) \\ &= 9.896J \\ \sum U &= 2U = 2 \times 9.896 = 19.79J \qquad \text{(ans)} \end{aligned}$$

$$\begin{aligned} h &= \frac{\sum U}{mg} \\ &= \frac{19.79}{40 \times 9.8} \\ &= 5.049 \times 10^{-2}m \qquad \text{(ans)} \end{aligned}$$

The energy from **both** tendons will lift the kangaroo just 5 cm. The kangaroo can actually jump much higher, since there are other elastic tissues that store energy, and the kangaroo's *muscles* exert some effort as well!

Case Study 1 Solution

CS 1.1

Given: $p = 4.79kPa = 100psf$, $L = 9.1m = 30.0ft$ and $b = 2m = 6.56ft$

Find:Force loaded on the hanger rod P

Solution:

$$A = Lb = 9.1 \times 2 = 18.2m^2$$
$$P = \frac{pA}{2} = \frac{1}{2} \times 4.79 \times 18.2 = 43.59kN \qquad \text{(ans)}$$

CS 1.2

Given:Design load is $90kN$

Find:Dead load, and the intensity of it

Solution:

$$2P = (w + W)bL$$
$$2 \times 90 = (w + 4.79) \times 18.2$$
$$w = 5.10kPa \qquad \text{(ans)}$$
$$P_d = wA = 5.10 \times 18.2 = 92.82kN \qquad \text{(ans)}$$

CS 1.3

Given: Cover thickness $t = 80mm = 8 \times 10^{-2}m$

Find: Specific weight of the cover

Solution:

$$w = 5.10kPa$$
$$w = \frac{G}{A} = \frac{\rho g A h}{A} = \rho g h = \rho \times 9.8 \times 0.08 = 5.10 \times 10^3 Pa$$
$$\rho = 6.505 \times 10^3 kg/m^3 \qquad \text{(ans)}$$

CS 1.4
Given: $d = 32mm$
Find:stress in the rod

Solution:

$$A_r = \pi R^2 = \pi \times (\frac{32 \times 10^{-3}}{2})^2 = 8.042 \times 10^{-4} m^2$$
$$\sigma_r = \frac{P}{A_r} = 111900kPa = 111.9MPa < P_{steel} \qquad \text{(ans)}$$

It is a reasonal stress

CS 1.5
Given: $H = 4.57m$
Find:The displacement of H under stress

Solution: From appendix $E = 200GPa$

$$\varepsilon = \frac{\sigma_r}{E} = \frac{111.9}{200 \times 10^3} = 5.595 \times 10^{-4}$$
$$\Delta H = H\varepsilon = 2 \times 4.57 \times 5.595 \times 10^{-4} = 5.114 \times 10^4 m \qquad \text{(ans)}$$