Chapter 2

Resistance and Strain

2.1 Introduction and Objectives

The primary purpose of this exercise is to determine the relationship between the relative change in resistance of a fine wire and the relative change in its length. This concept is the fundamental principle by which nearly all strain gages operate. A strain gage (see Figure 2.1) basically consists of a metallic pattern bonded to an insulating backing. This can be attached to a surface to provide a method to measure strains induced by loading. The concepts of stress, strain, and the way in which structures are loaded will be explained further in your solid mechanics course. The important thing to remember in this exercise is that when a wire is stretched (strained), its resistance changes.

These objectives will be accomplished by stretching a wire and measuring its resistance at various lengths. In the process of performing this experiment you will learn to use a digital multimeter to measure resistance in a wire. You will examine your experimental results by plotting the relative change in resistance versus the relative change in length. From this information you will determine the "gage factor" for this wire. You will also determine the uncertainties in your measurements and relate them to your results.

As stated above, the goal of this lab is to relate a change in resistance to a change in length. This is done through a quantity known as the *local* gage factor, G_l, which is defined as the ratio of the relative resistance change to the relative length change,

$$G_l = \frac{dR/R}{dL/L}. \tag{2.1}$$

The denominator in the above equation is quickly recognized from solid mechanics to be the longitudinal (axial) strain, or ϵ_L. You will notice that the above expression relates *differential* changes in resistance and length. That is, it describes a local gage factor, only valid over a very small (local) range of strain in the neighborhood of interest. We shall see later that local gage factor is very much a function of how much the wire is strained.

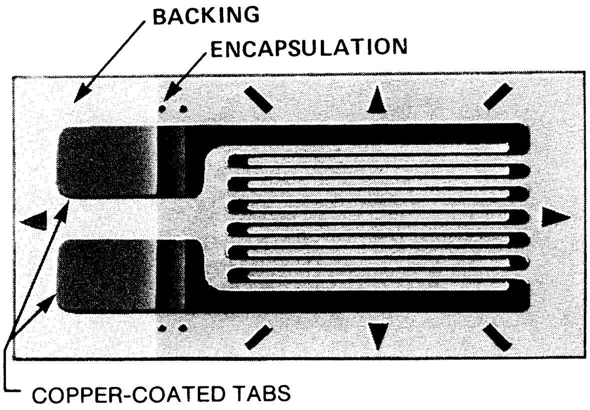

Figure 2.1: A strain gage (from Measurements Group Bulletin 309D).

Of much greater use is a quantity known as the *engineering* gage factor, defined to be

$$G_e = \frac{\Delta R/R}{\Delta L/L}. \tag{2.2}$$

It can be seen that this expression is based on small, but finite changes in resistance and length. The *local* gage factor can be thought of as the instantaneous slope of a plot of $\Delta R/R$ vs. $\Delta L/L$, whereas the *engineering* gage factor would be the slope based on the total resistance change through out the region of interest. It is nearly impossible to measure local changes in length and resistance. Thus, the engineering gage factor is typically the quantity of interest in strain applications, and is what we will measure in this exercise. For a more detailed explanation, see the Supplemental Information section at the end of this handout.

2.2 Instrumentation

In this exercise you will use the following instruments:

- Hewlett Packard 3468A Multimeter (resolution: 1 $\mu\Omega$ in the ohm range)
- Starrett dial indicator (resolution: 0.0005 in.)
- A metal meter stick (resolution: 0.5 mm)
- A wire stretcher for wires approximately 1 m in length

2.3 Measurements

First and foremost - a safety note. It is imperative that you wear safety (or your own) glasses when stretching the wire. It can break and snap back into your face.

In this lab, you are to mount a length of wire between the two clamps of the tension device and load it using the screw mechanism. First, cut a piece of wire approximately 4 feet long from the spool. Figure 2.2 shows a schematic of the clamping mechanism.

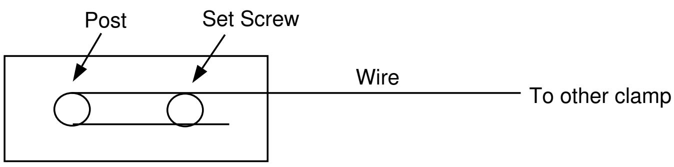

Figure 2.2: Clamping of wire in mechanism.

With the top of the clamp removed, loop the wire once about the end post before directing it to the other clamp. Then replace the top of the clamp and tighten the set screw to prevent slipping. Do likewise with the other clamp. Wire slippage can occur when tension is applied, resulting in an indication of displacement without an expected increase in resistance. When the wire is mounted correctly, it should be straight, not sagging, and the tension end with the brass thumb wheel should have about 1/2 inch of travel available.

Connect the multimeter to the wire for 4-wire resistance measurement as follows:

1. Using one pair of banana-alligator test leads, connect the HI and LO pair of terminals of the multimeter under INPUT to the wire under tension near the clamps, one at each end.

2. Using a second pair of test leads, connect the HI and LO terminals under Ω-Sense to the wire under tension just inside of the two leads of Step (1).

3. Place the multimeter into the 4-wire resistance mode by pressing the "4 WIRE" button (4 Ω should be appear on the LCD display). Also depress with the AUTO/MAN button to put the meter in the manual mode (M RNG should be appear on the display). This will yield a resistance reading resolution of 10 $m\Omega$. The KOHM range should be displayed. If not, depress the up arrow button to obtain its indication. Finally, depress the blue button, then the INT TRIG button to set the multimeter in the auto zero mode. If the auto zero mode is NOT set, then AZ OFF will appear on the display (no indication means it's set correctly). The indicated reading should be approximately 150 Ω to 200 Ω. If a negative resistance is indicated, you can switch the two inner wires to make it positive. Allow a couple of minutes for the meter to warm up before you start taking actual data.

For a brief explanation of resistance measurement methods, see the Supplemental Information section.

Slowly tension the wire by turning the brass thumb wheel until the resistance starts to increase (watch the least and second least significant digits for some consistent increase). This shall be your zero point. Record the reading on the dial gage and the resistance. (The dial indicator scale goes from 0 to 50, corresponding to 0.000 in. to 0.050 in. of travel). Measure the initial length of wire between the measuring points, the leads of Step (2), using the metal meter stick.

At increments of approximately 0.01 inches (increments of 10 on the dial indicator), record the elongation (in.) and the resistance (Ω) until the wire has been stretched about 0.20 in. The resistance changes approximately 0.1 Ω for every 0.01 in. of stretch. Return to a couple of data points and repeat those measurements to see if they have changed at all. Now, try repeating the experiment all the way out to failure of the wire. This should take on the order of 0.50 in. of travel. Try taking around 20 data points, with larger intervals at the beginning, becoming smaller as you get closer to the wire snapping.

When you are finished, turn off the multimeter, bring the dial indicator back to the zero starting point, disconnect the test leads from the wire and the multimeter, and remove your wire.

2.4 What to Report

Outside the lab after all of your data is collected, plot the relative resistance change vs. the relative length change, for both the first case and the case when you stretched the wire to failure. Estimate the uncertainties of $\Delta R/R$ and of $\Delta L/L$, following the procedures that you learned in freshman physics or those detailed in the class notes. Calculate the engineering gage factors for both cases. Are the values the same? Explain this in the context of your measurement uncertainties. Try approximating some local gage factors by calculating slopes over a few data points in your data sets, especially at lower strains. What can you say about the relation between these local gage factors and the extent to which the wire was strained at that point? Plot the local gage factor versus the strain to illustrate this. Compare the local with the engineering gage factors from the two cases, always being aware of the uncertainties involved.

Perform a least-squares linear regression analysis of the relative resistance change versus strain. Determine the correlation coefficient and the percent confidence associated with that correlation coefficient. How does the slope of the best fit line compare with some of the gage factors you calculated earlier?

All of your important experimental results and answers to the posed questions must be presented as a technical memo. Your answers to the posed questions should be contained in the explanation of your results and *not* listed item-for-item.

2.5 Supplemental Information

2.5.1 The Strain Gage

So how exactly does straining a wire change its resistance? From your elementary physics class, you might remember that the resistance of a conductor depends on its resistivity, ρ, its length, L, and its cross sectional area, A. For a wire of circular cross section, the resistance R can be written as

$$R = \rho \frac{L}{\pi r^2}, \tag{2.3}$$

where r is the radius of the wire. Differentiating the above expression and cleaning up some terms, the following result for the relative resistance change can be determined by

$$\frac{dR}{R} = (1 + 2\nu)\epsilon_L + \frac{d\rho}{\rho}. \tag{2.4}$$

In the above relation ϵ_L is the longitudinal strain and ν is Poisson's ratio (the ratio of transverse to longitudinal strains), both of which you'll learn more about in solid mechanics. Poisson's ratio is a material property that relates an axial strain to a radial strain (in the case of a wire). In other words, as you strain a wire by pulling it, its diameter will decrease. The extent to which it will decrease is found from Poisson's ratio. In the above expression, it can be seen that the relative resistance change is clearly dependent on the strain in the wire.

Equation 2.4 can be rewritten in terms of the engineering gage factor

$$G_e = 1 + 2\nu + [\frac{\Delta\rho}{\rho} \cdot \frac{1}{\epsilon_L}]. \tag{2.5}$$

For most metals $\nu \approx 0.3$ and the value of the last term in brackets representing the strain-induced changes in the resistivity (a piezoresistive effect) is around 0.4. Thus, the value of the engineering gage factor is typically around 2 or higher (sometimes up to 3 or 4).

Now, electrical currents and resistance are concepts related to free electrons moving about within a conductor. By making some arguments using atomic physics and materials science, the relative resistance change can be rewritten to eliminate Poisson's ratio from the above expression as

$$\frac{dR}{R} = 2\epsilon_L + \frac{dv_0}{v_0} - \frac{d\lambda}{\lambda} - \frac{dN_0}{N_0}. \tag{2.6}$$

In this equation, v_0 is the average number of electrons in the material in motion between ions, λ is the average distance traveled by an electron between collisions, and N_0 is the total number of conduction electrons. It would appear that things have just taken a turn for the worse. However, getting rid of Poisson's ratio has a nice result: it means that the differential resistance change (and thus the gage factor) is not a function of the material properties of the conductor. This is good, because material properties in a metal will change with strain as the material transitions from the elastic to the plastic regime on a stress-strain curve.

All that is left is to recognize that gage factor is dependent on the strain in the wire (ϵ_L) and any strain-induced changes at the atomic level of the number of free electrons present, their distance between collisions, and their velocity. Unfortunately, this means that gage factor will only be constant when the sum of these changes is either zero or is directly proportional to the strain producing the changes, which is seldom the case.

All is not lost, however. There are some materials in which the gage factor does not vary too much with strain, and therefore can be used for strain gage applications. Hopefully, this supplement provided you just a small understanding of why resistance changes with length change (the fundamental operating principle of strain gages), and also gave some clue as to why the gage factor is not constant over all levels of strain.

2.5.2 Resistance Measurement Methods

There are two ways to measure the resistance using a multimeter: the 2-wire method and the 4-wire method. The 2-wire method is straightforward. Simply connect two test leads to two points on the wire, between which is the desired resistance. The multimeter outputs a known current through the test leads and then measures the total voltage drop across the resistor *and* the test leads. This is no problem provided that the desired resistance is much larger than the resistances of the test leads. However, for this laboratory exercise, this is *not* the case .

The 4-wire method requires the use of two additional test leads. Two of the leads carry a known current to the resistance to be measured and then back to the meter, while the other two leads measure the resulting voltage drop across the resistance. Internally the meter determines (using Ohm's law) and then displays the measured resistance. This method obviously is more accurate and is the one that you will we use in this exercise.