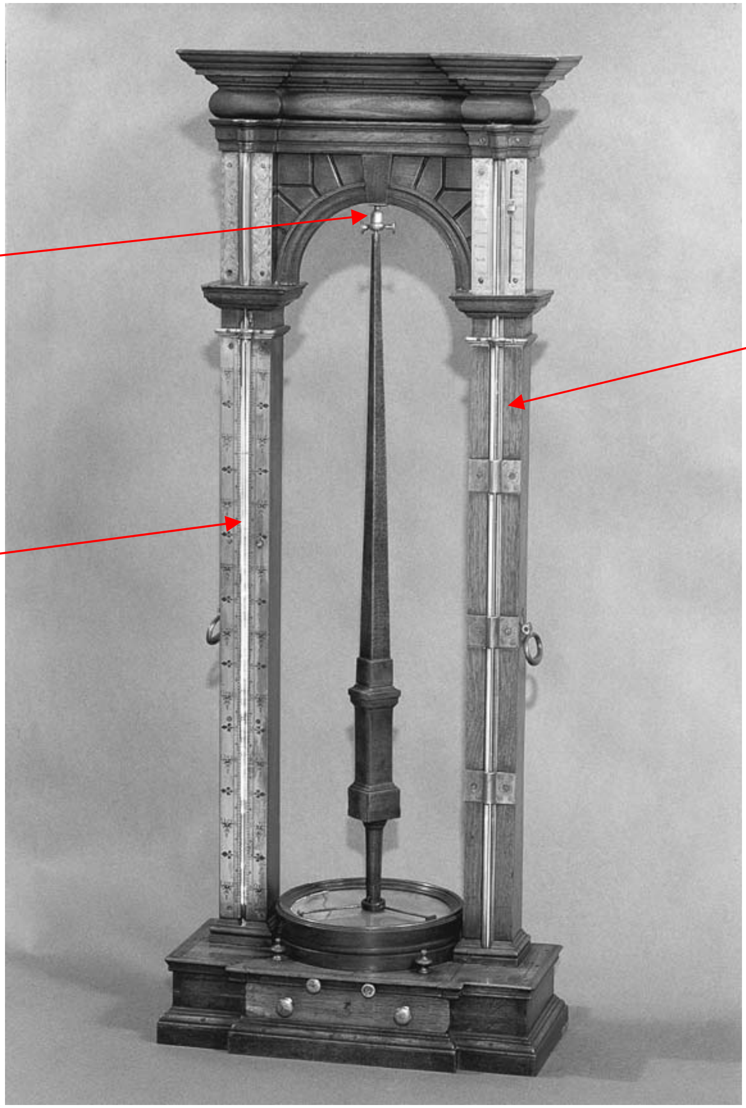
1700's T, p, RH Measurement System



First-Order System Dynamic Response

- The general expression for a first-order system is

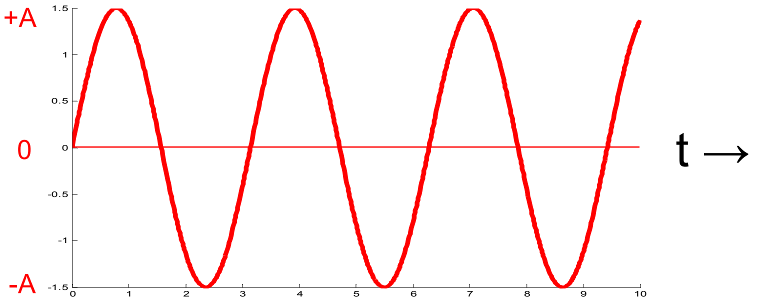
- This is a linear first-order ODE, which can be rearranged as

- The time constant of the system, τ, equals a_1/a_0. K ($=1/a_0$) is a constant that converts F(t) into units of y.

- The general expression is a physical law for the system, such as conservation of energy (Kirchhoff's voltage law) for a simple RC circuit.

Dynamic Response to F(t)

- The exact solution of $\tau\dot{y} + y = KF(t)$ depends upon the specific type of forcing function, F(t).
- We will study the responses to two different forcing functions.
- Step: F(t) = 0 for t ≤ 0 and F(t) = A for t > 0
- Sinusoidal: F(t) = A sin(ωt)



Step-Input Forcing

- The general solution for step-input forcing is of the form

- Substitution of this equation into the governing equation gives

- The *initial condition,* $y(t = 0) = y_0 = c_0 + c_1$, gives

- Thus, the specific solution is

Step-Input Forcing

$$y(t) = KA + (y_0 - KA)e^{-t/\tau}$$

- Also, $y_\infty = y(t = \infty) = KA$.
- Using the above expressions and defining the *magnitude ratio*, M, results in

- The *dynamic error*, $\delta(t)$, relates to the magnitude ratio as

Step-Input Forcing

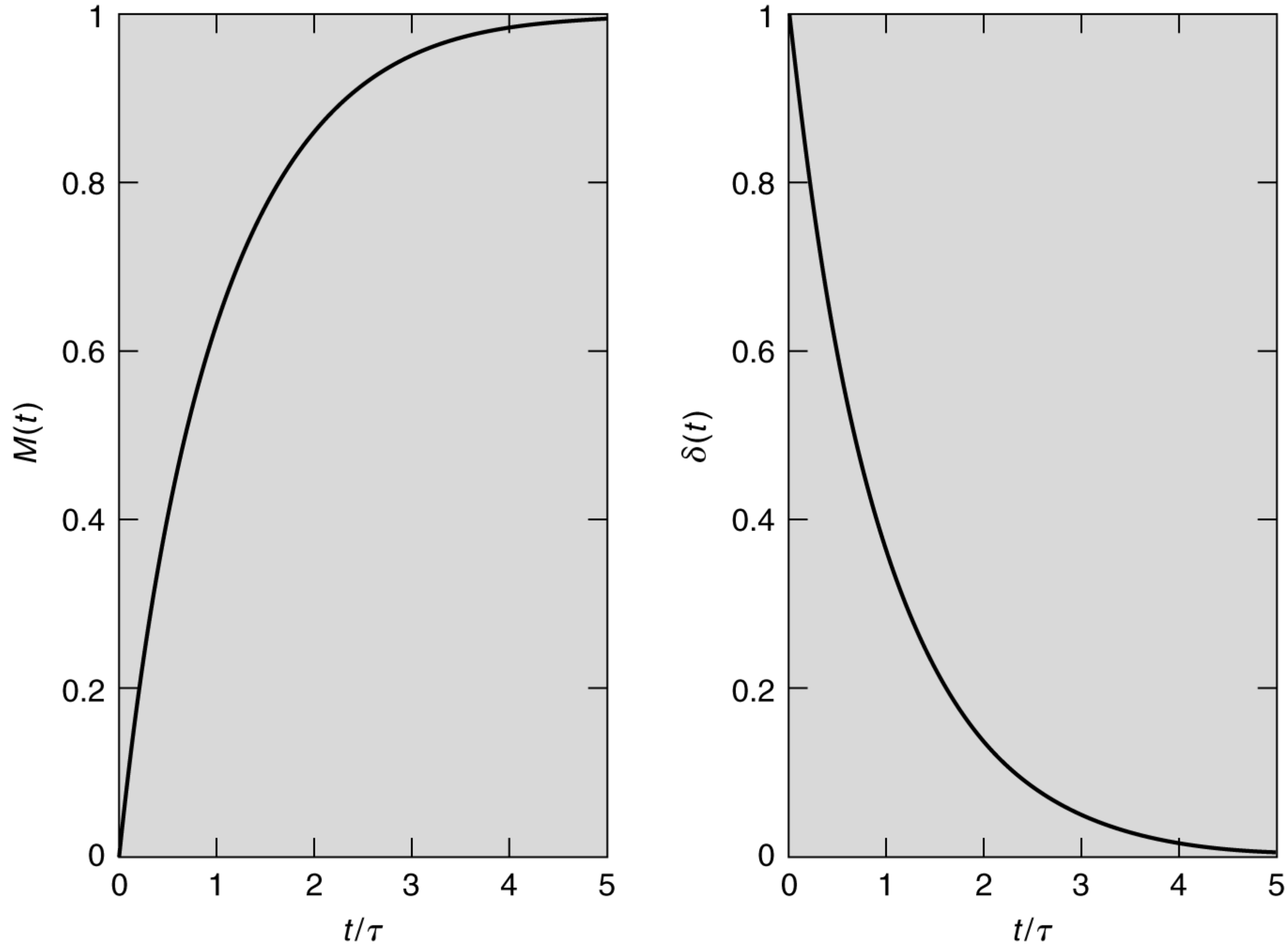


Figure 5.2

In-Class Example

- For a RC circuit (R = 2 Ω; C = 0.5 F) with step input forcing from 0 V to 1 V:

What is the V of the circuit at 1 s ?

What is the V of the circuit at 5 s ?

What is the % dynamic error at 1 s ?

Sinusoidal-Input Forcing

- The general solution for sinusoidal-input forcing is of the form

$$y(t) = c_0 e^{-t/\tau} + c_1 + c_2 \sin(\omega t) + c_3 \cos(\omega t)$$

- Substitution into the governing equation, comparing like terms (see Exmpl. 5.4), and using the initial condition $y(0) = y_0$ with input forcing $F(t)=A\sin(\omega t)$ gives

$$y(t) = \left(y_0 + \frac{\omega \tau K A}{\omega^2 \tau^2 + 1} \right) e^{-t/\tau} + \frac{KA}{\sqrt{\omega^2 \tau^2 + 1}} \sin(\omega t + \phi)$$

- The *phase lag*, ϕ (in radians), shifts the output in time from the input, where $\phi = -\tan^{-1}(\omega\tau)$.

Sinusoidal-Input Forcing

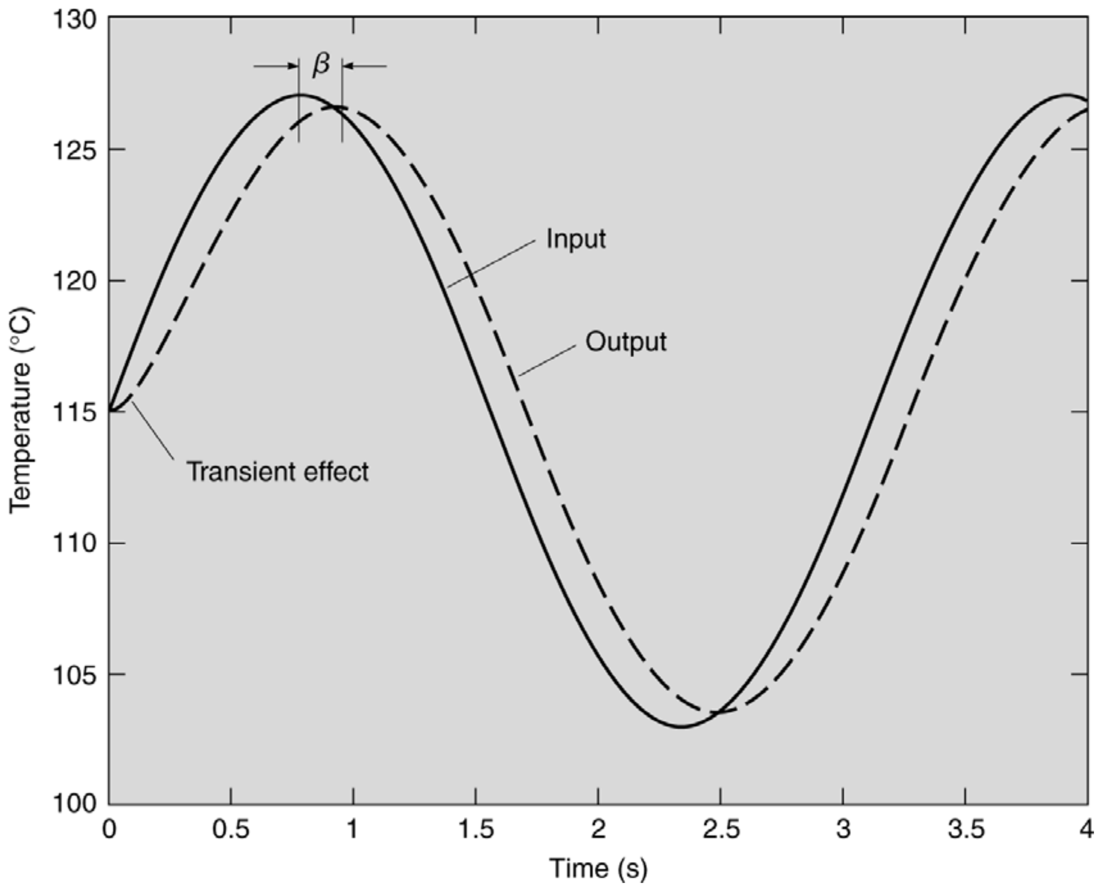
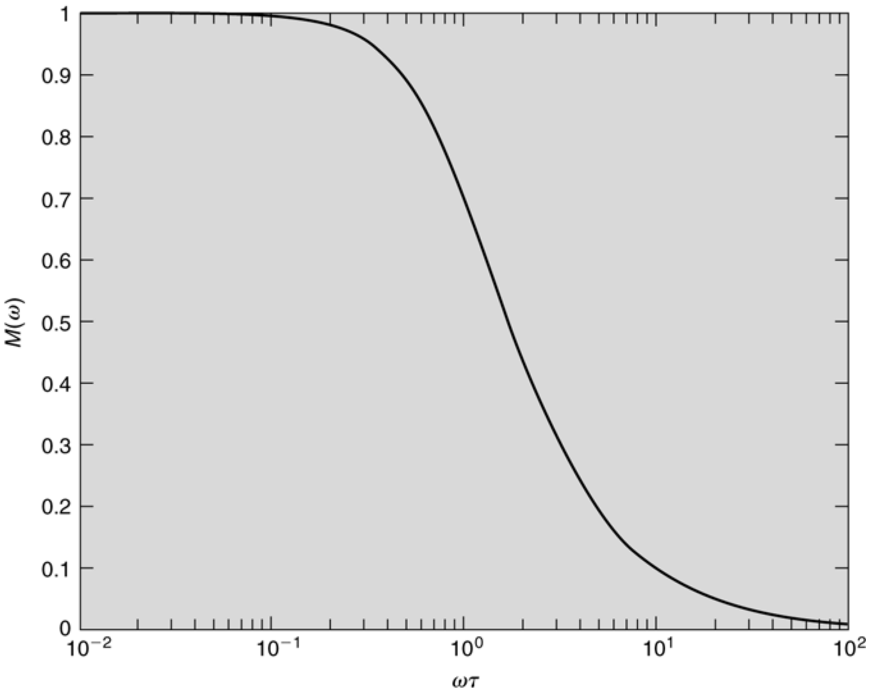
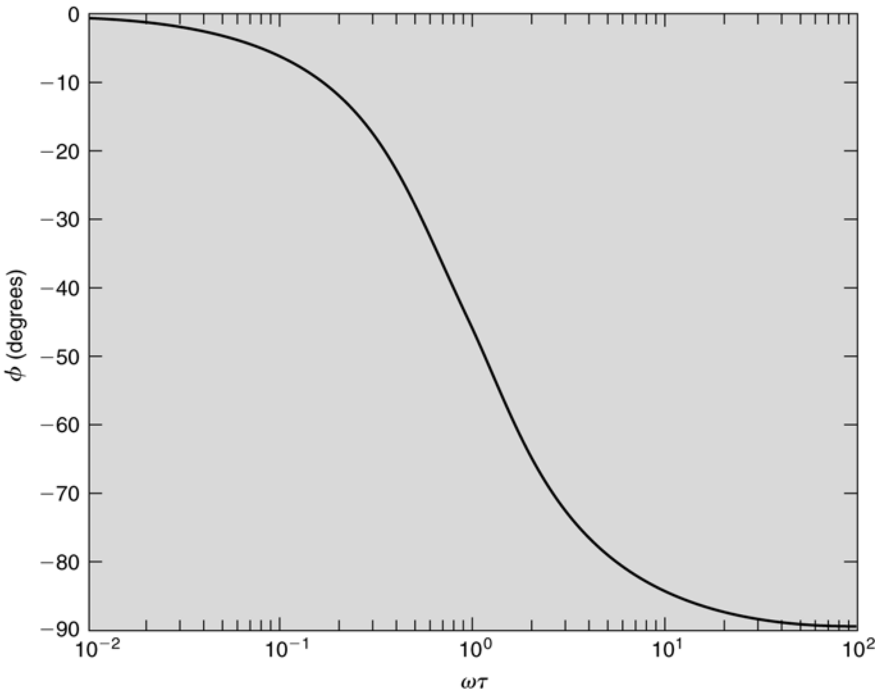


Figure 5.5

Sinusoidal-Input Forcing





$$M(\omega\tau) = \frac{1}{\sqrt{\omega^2\tau^2 + 1}}$$

$$\phi(^o) = -\left(\frac{180}{\pi}\right)\tan^{-1}(\omega\tau)$$

Figures 5.3 and 5.4

In-Class Example

- For a RC circuit (R = 2 Ω; C = 0.5 F) with sine input forcing of 3sin(2t) from 0 V to 1 V:

  What is its phase lag in degrees?

  What is its phase lag in s ?

  What is its magnitude ratio ?

- NOTE: The minus sign in the phase lag simply means that the output *lags* the input in time.

Sinusoidal-Input Forcing