

# Higher-Order System Response

- Most measurement systems involve more than one instrument.
- Many instruments in a measurement system are linear.
- For linear systems comprised of more than one instrument, the overall system response can be determined as follows:
  1. The static sensitivity is the \_\_\_\_\_ of all static sensitivities.
  2. The magnitude ratio is the \_\_\_\_\_ of all magnitude ratios.
  3. The phase lag is the \_\_\_\_\_ of all phase lags.

# Example 5.5 in Text

*Statement:* A pressure transducer is connected through flexible tubing to a static pressure port on the surface of a cylinder that is mounted inside a wind tunnel. The structure of the flow local to the port is such that the static pressure,  $p(t)$ , varies as

$$p(t) = 15\sin 2t,$$

in which  $t$  is time. Both the tubing and the pressure transducer behave as second-order systems.

The natural frequencies of the transducer,  $\omega_{n,trans}$ , and the tubing,  $\omega_{n,tube}$ , are 2000 rad/s and

4 rad/s, respectively. Their damping ratios are  $\zeta_{trans} = 0.7$  and  $\zeta_{tube} = 0.2$ , respectively. Find

the magnitude attenuation and phase lag of the pressure signal, as determined from the output

of the pressure transducer, and then write the expression for this signal.

*Solution:* Because this measurement system is linear, the system's magnitude ratio,  $M_s(\omega)$ , is the product of the components' magnitude ratios and the phase lag,  $\phi_s(\omega)$ , is the sum of the components' phase lags, where  $\omega$  the circular frequency of the pressure. Thus,

$$M_s(\omega) = M_{tube}(\omega) \cdot M_{trans}(\omega)$$

and

$$\phi_s(\omega) = \phi_{tube}(\omega) + \phi_{trans}(\omega).$$

Also,  $\omega/\omega_{tube} = 2/4 = 0.5$  and  $\omega/\omega_{trans} = 2/2000 = 0.001$ . Application of Equations 5.62 and 5.64, noting  $\zeta_{trans} = 0.7$  and  $\zeta_{tube} = 0.2$ , yields  $\phi_{tube} = -21.8^\circ$ ,  $\phi_{trans} = -0.1^\circ$ ,  $M_{tube} = 1.86$  and  $M_{trans} = 1.00$ . Thus,  $\phi_s(2) = -21.8^\circ + -0.1^\circ = -21.9^\circ$  and  $M_s(2) = (1.86)(1.00) = 1.86$ . The pressure signal, as determined from the output of the transducer is  $p_s(t) = (15)(1.86)\sin[2t - (21.9)(\pi/180)] = 27.9\sin(2t - 0.38)$ . Thus, the magnitude of the pressure signal at the output of the measurement system will appear 186 % greater than the actual pressure signal and be delayed in time by 0.19 s [(0.38 s)/(2 rad/s)].