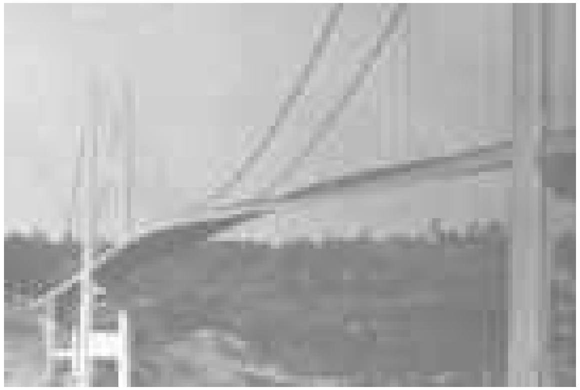
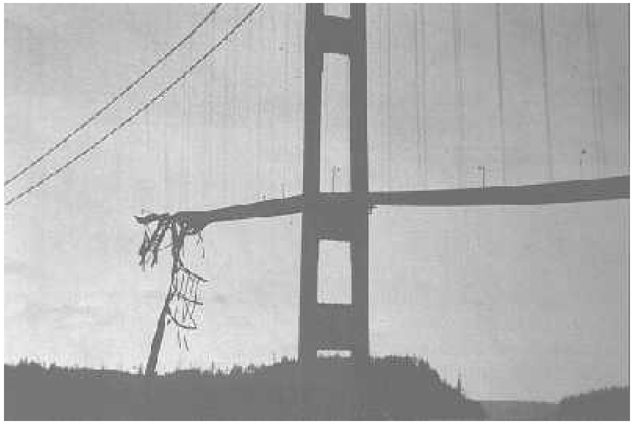
1940 Tacoma Narrows Bridge Collapse

(see www.enm.bris.ac.uk/research/nonlinear/tacoma/tacoma.html)





Second-Order System Dynamic Response

- The general expression for a 2nd-order system is

- This is a linear 2nd-order ODE, which can be rearranged as

$$\left(\frac{1}{\omega_n^2}\right)\ddot{y} + \left(\frac{2\xi}{\omega_n}\right)\dot{y} + y = KF(t)$$

- ω_n ($=\sqrt{a_0/a_2}$) is the *natural frequency*, which equals $\sqrt{1/LC}$ for a RLC circuit and $\sqrt{k/m}$ for a spring-mass-damper system.
- ζ ($=a_1/2\sqrt{a_0 a_2}$) is the *damping ratio*, which equals $R/\sqrt{4L/C}$ for a RLC circuit and $\gamma/\sqrt{4km}$ for a spring-mass-damper system.

Step-Input Forcing

- For step-input forcing, there are 3 specific solutions of the ODE because there are 3 different roots of the characteristic equation (see Appendix I).

1. $\zeta < 1$ is the *underdamped* case. Its solution is Eqn. 5.57.
2. $\zeta = 1$ is the *critically damped* case. Its solution is Eqn. 5.59.
3. $\zeta > 1$ is the *overdamped* case. Its solution is Eqn. 5.60.

- The two initial conditions used are $\dot{y}(0) = 0$ and y(0)=0.

Step-Input Forcing

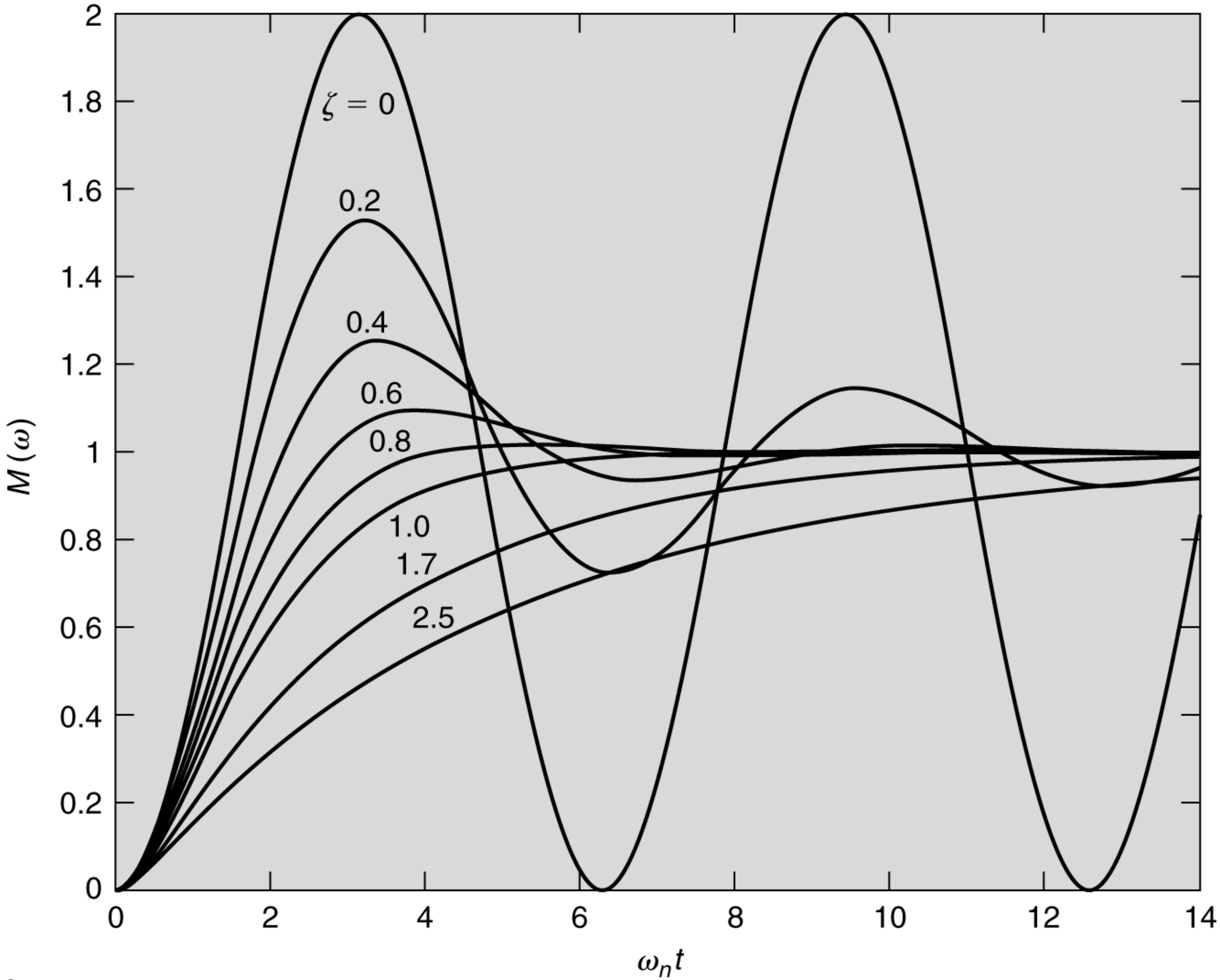


Figure 5.6

Step-Input Forcing Terminology

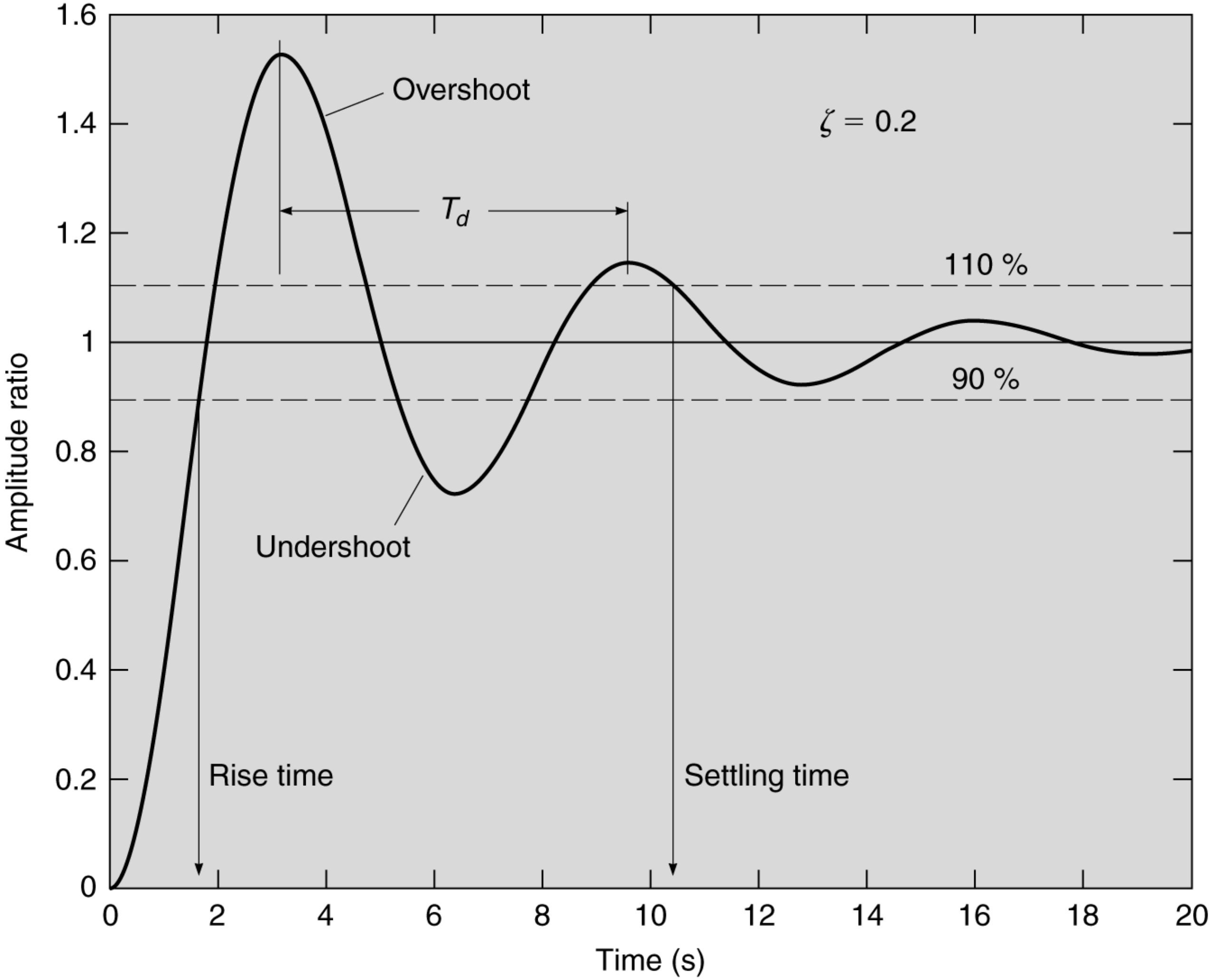


Figure 5.7

In-Class Example

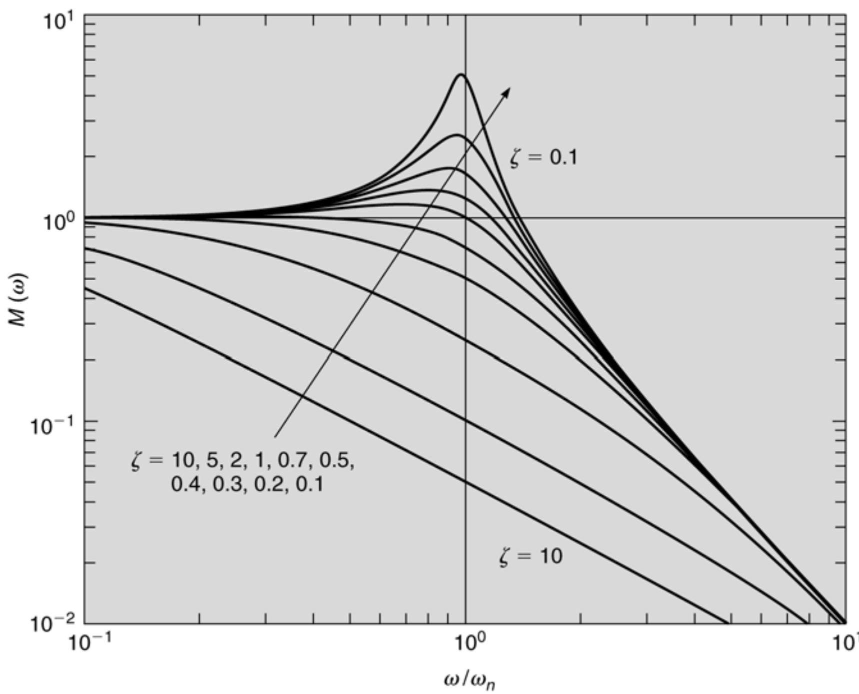
- A second-order system with $\zeta = 0.5$ and a natural frequency equal to $1/\pi$ Hz is subjected to a step input of magnitude A. Determine the system's time constant.

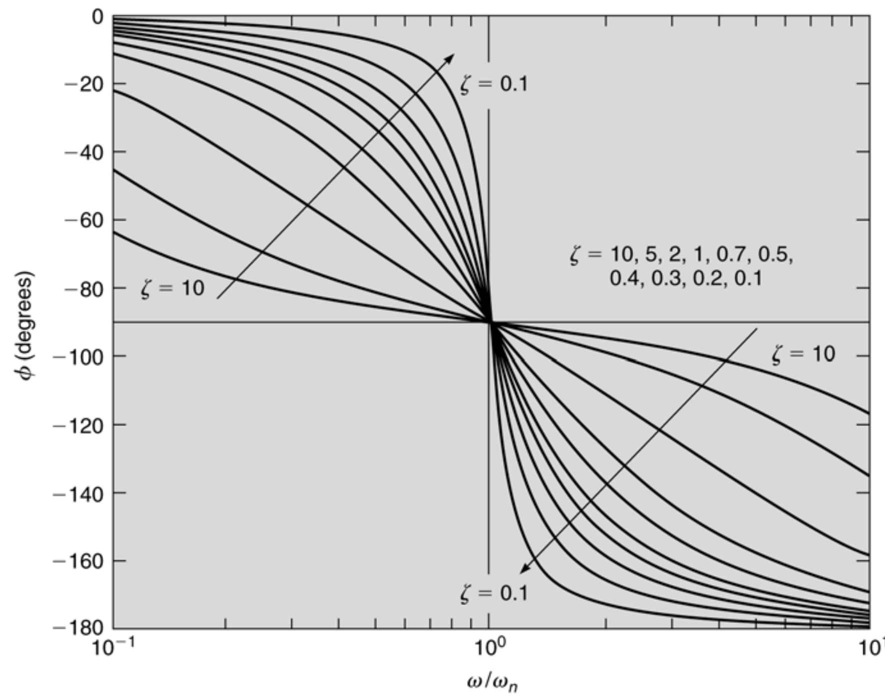
For an underdamped system

$$y(t) = KA\left\{1 - e^{-\zeta\omega_n t}\left[\frac{1}{\sqrt{1-\zeta^2}}\sin(\omega_n\sqrt{1-\zeta^2} + \varphi)\right]\right\}$$

Sinusoidal-Input Forcing

- For sinusoidal-input forcing, the solution typically is recast into expressions for M(ω) and φ(ω) (see Eqns. 5.62-5.64 and Appendix I).





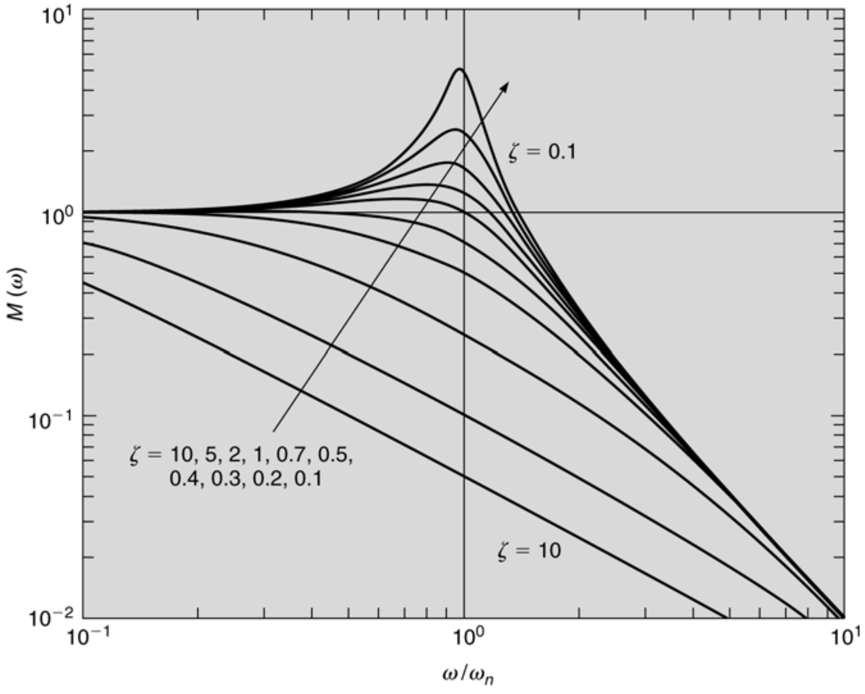
Figures 5.9 and 5.10

In-Class Example

- Is the RLC circuit (R = 2 Ω; C = 0.5 F; L = 0.5 H) underdamped, critically damped, or overdamped ?

- With sine input forcing of 3sin2t:

  What is its magnitude ratio ?



The Decibel

- The decibel is defined in terms of a power (P) ratio, or base measurand ratio (Q), or magnitude ratio by

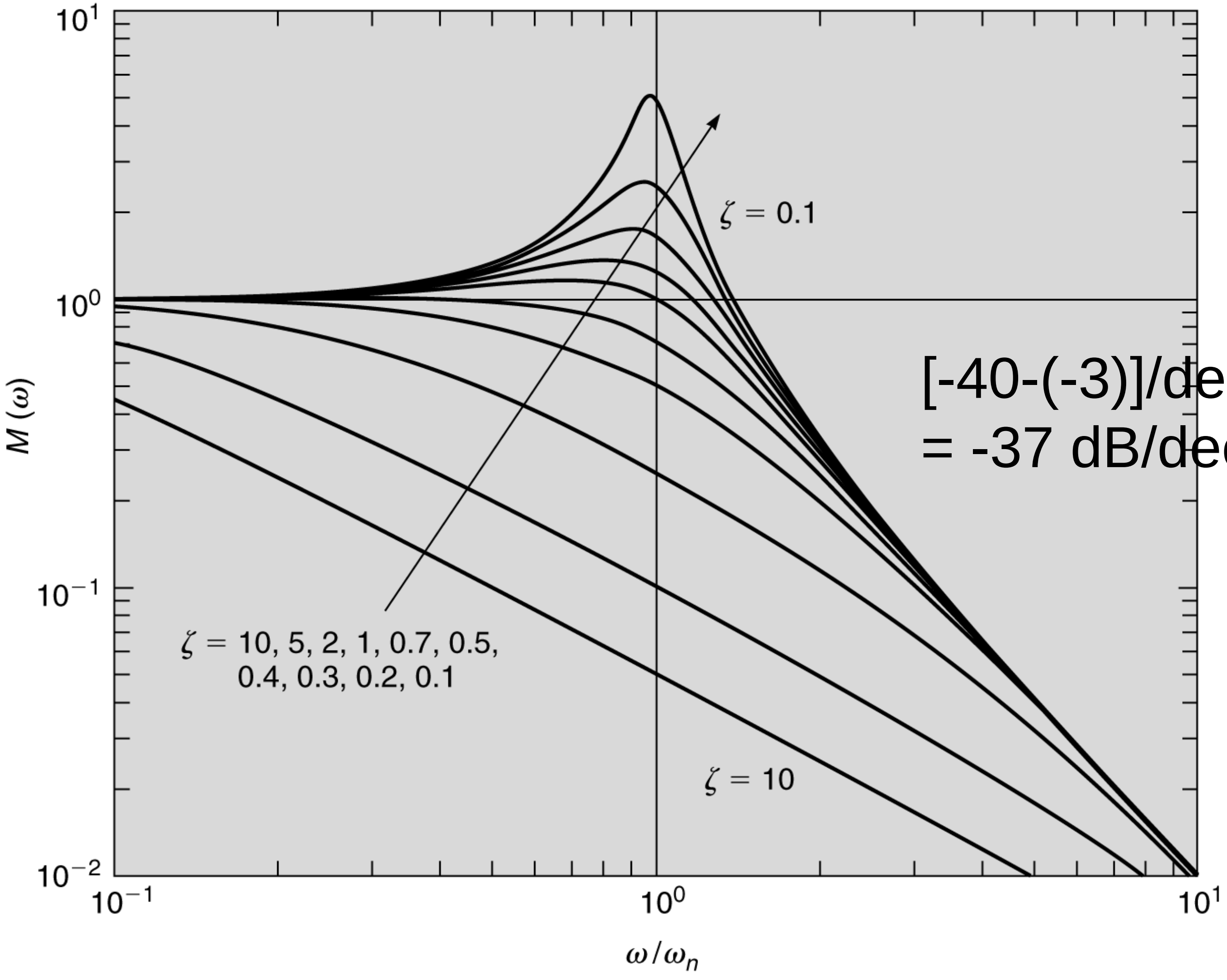
$$dB = 10\log_{10}\left(\frac{P_2}{P_1}\right) = 10\log_{10}\left(\frac{Q_2^2}{Q_1^2}\right) = 20\log_{10}\left(\frac{Q_2}{Q_1}\right) = 20\log_{10} M(\omega)$$

- The *one-half power point*, at which the output power is one-half of the input power, corresponds to

$$10\log_{10}(1/2)\ \text{dB} = -3\ \text{dB}$$

- This corresponds to where $M(\omega) = (1/2)^{1/2} = 0.707$
[or find $M(\omega)$ by solving: $-3\ \text{dB} = 20\log_{10} M(\omega)$]

Roll off (dB/decade)

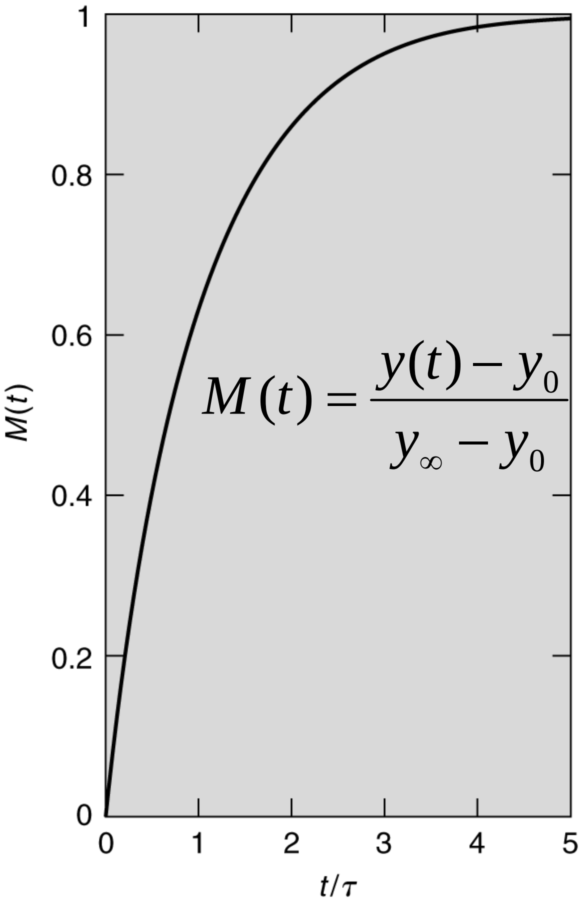


[-40-(-3)]/decade
= -37 dB/decade

1st-Order System Summary

Step Input [F(t)=KA]:

$$y(t) = KA + (y_0 - KA)e^{-t/\tau}$$

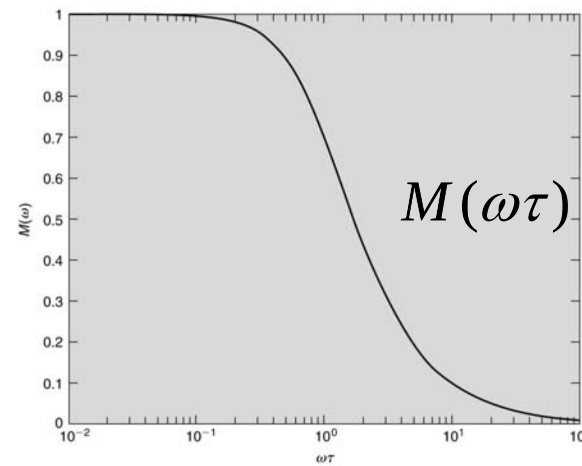


$$M(t) = \frac{y(t) - y_0}{y_\infty - y_0} = 1 - e^{-t/\tau}$$

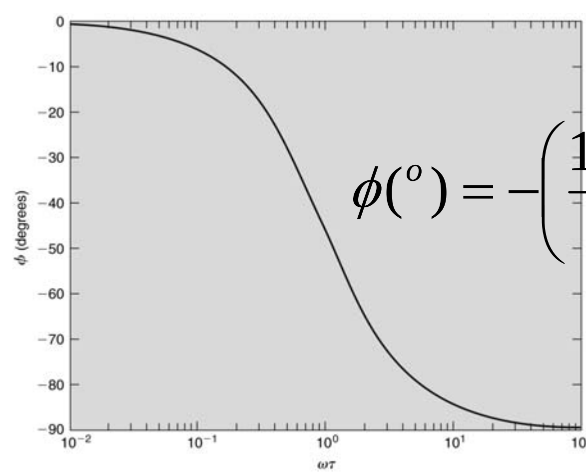
t/τ

Sine Input [F(t)=KAsin(ωt)]:

$$y(t) = \left(y_0 + \frac{\omega\tau KA}{\omega^2\tau^2 + 1} \right) e^{-t/\tau} + \frac{KA}{\sqrt{\omega^2\tau^2 + 1}} \sin(\omega t + \phi)$$



$$M(\omega\tau) = \frac{1}{\sqrt{\omega^2\tau^2 + 1}}$$



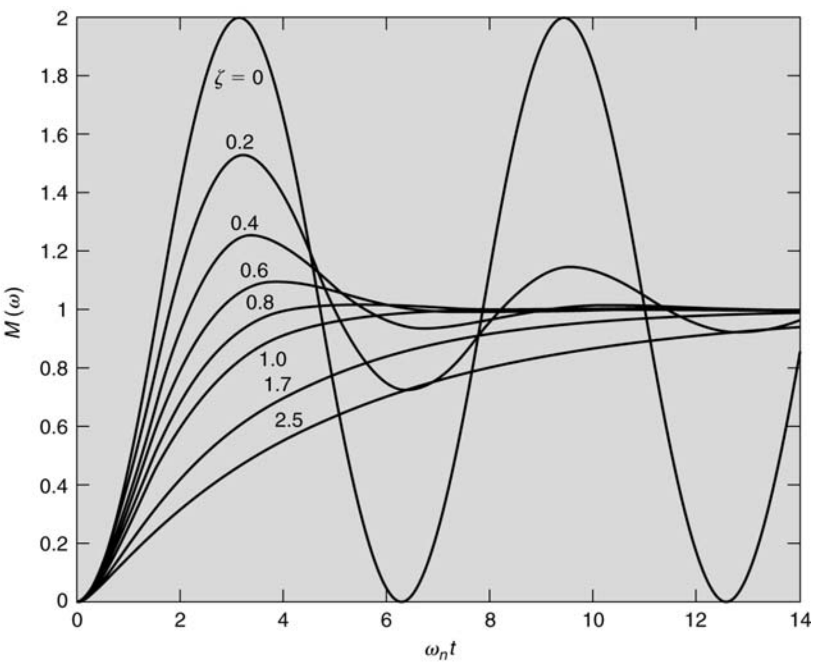
$$\phi(^o) = -\left(\frac{180}{\pi} \right) \tan^{-1}(\omega\tau)$$

ωτ

2nd-Order System Summary

Step Input [F(t)=KA]:

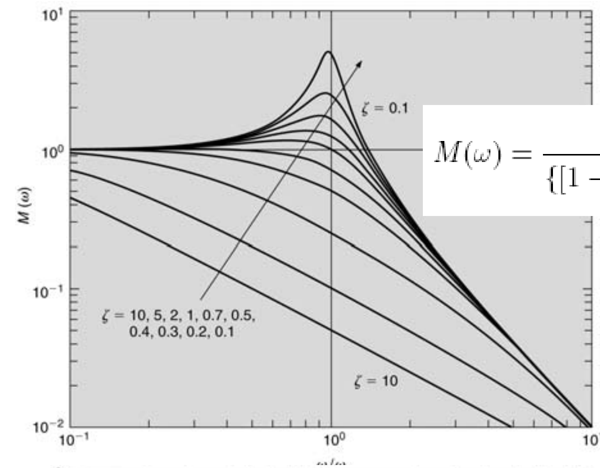
$$y(t) = f(KA,\ \omega_n t,\ \zeta)$$



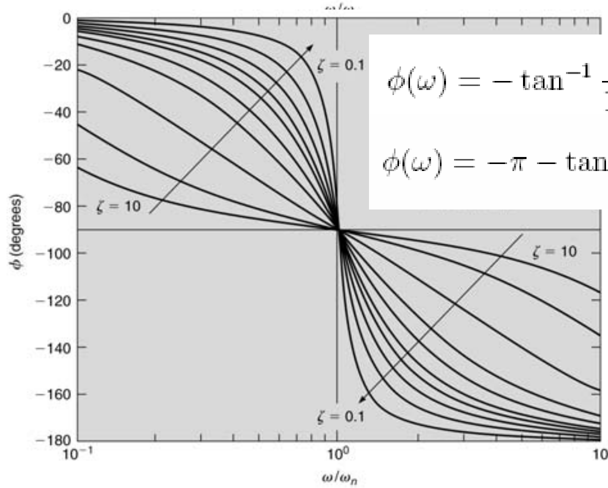
$\omega_n t$

Sine Input [F(t)=KAsin(ωt)]:

$$y(t) = f(KA,\ \omega/\omega_n,\ \zeta)$$



$$M(\omega) = \frac{1}{\{[1-(\omega/\omega_n)^2]^2 + [2\zeta\omega/\omega_n]^2\}^{1/2}}$$



$$\phi(\omega) = -\tan^{-1}\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2} \quad \text{for } \frac{\omega}{\omega_n} \le 1$$

$$\phi(\omega) = -\pi - \tan^{-1}\frac{2\zeta\omega/\omega_n}{1-(\omega/\omega_n)^2} \quad \text{for } \frac{\omega}{\omega_n} > 1.$$

ω/ω_n