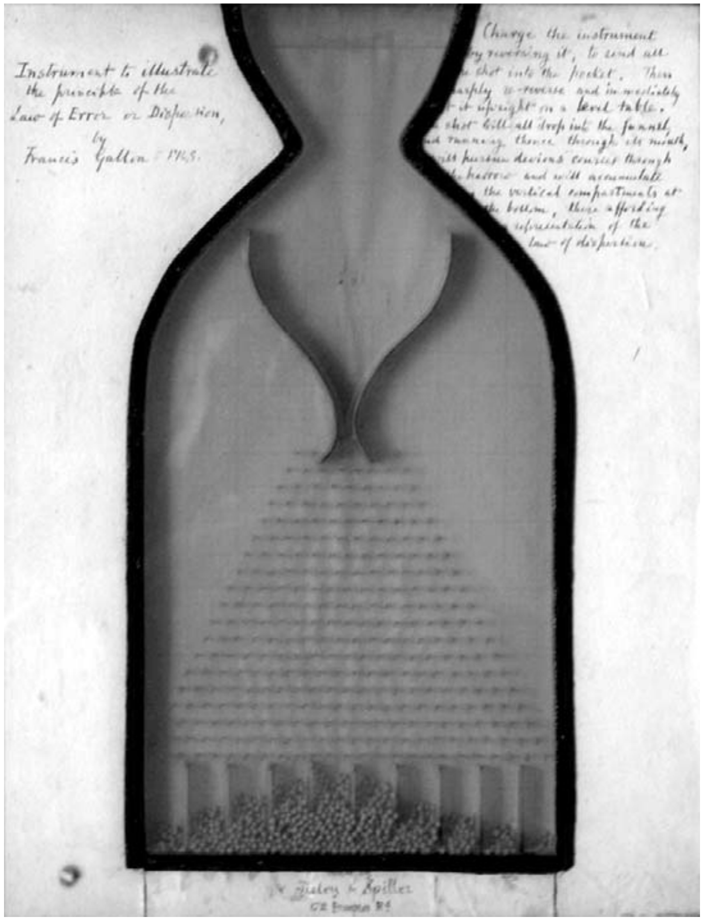
Chapter 7: PROBABILITY

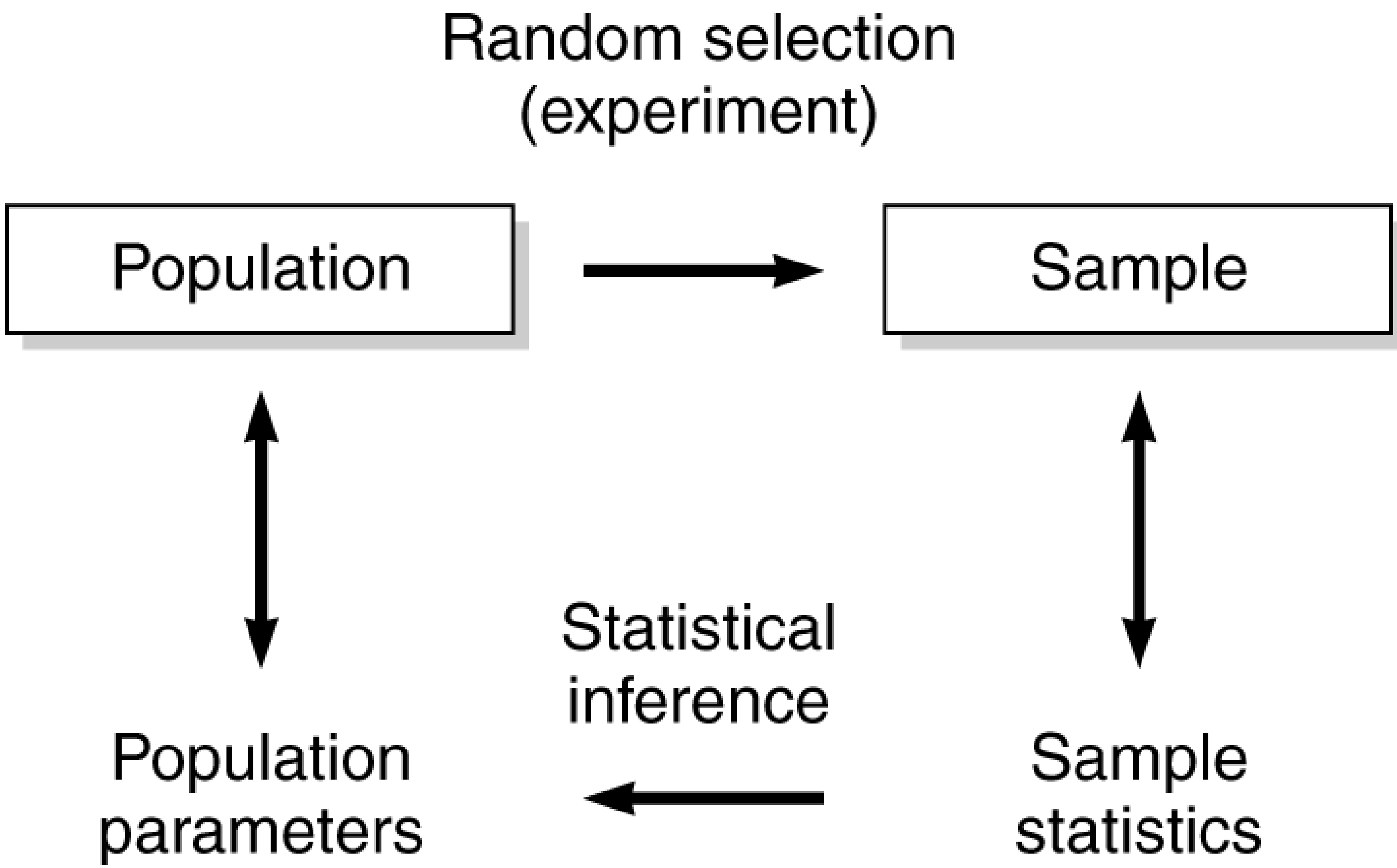


Sir Francis Galton's Quincunx

"When you deal in large numbers, probabilities are the same as certainties. I wouldn't bet my life on the toss of a single coin, but I would, with great confidence, bet on heads appearing between 49 % and 51 % of the throws of a coin if the number of tosses was 1 billion."

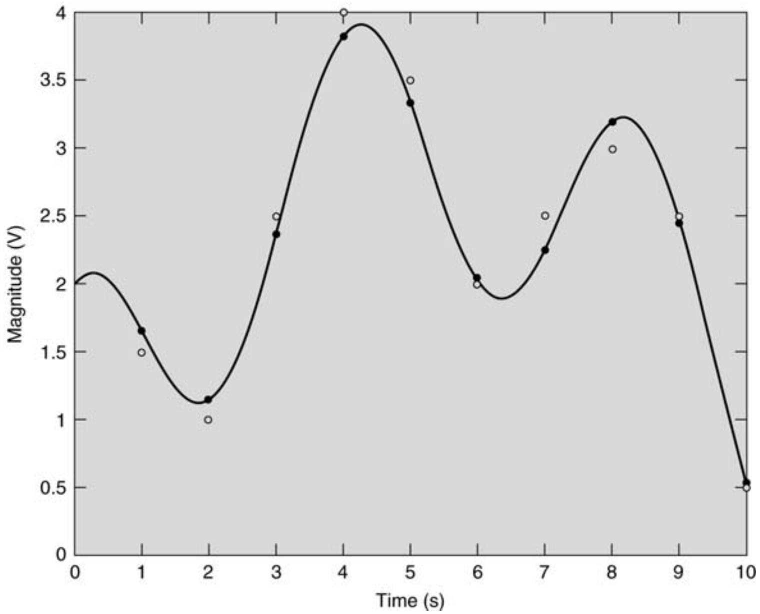
Brian Silver, 1998, *The Ascent of Science*, Oxford University Press.

Representing Population Behavior

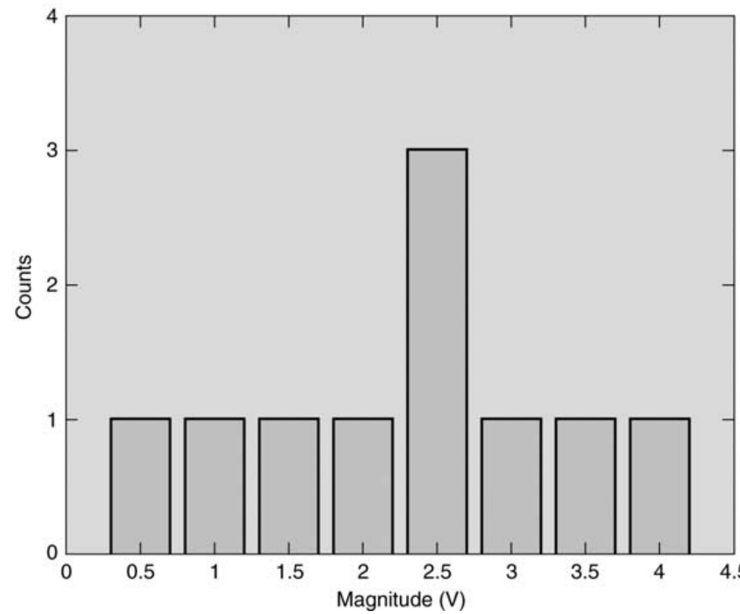


Histogram

Time record



Histogram



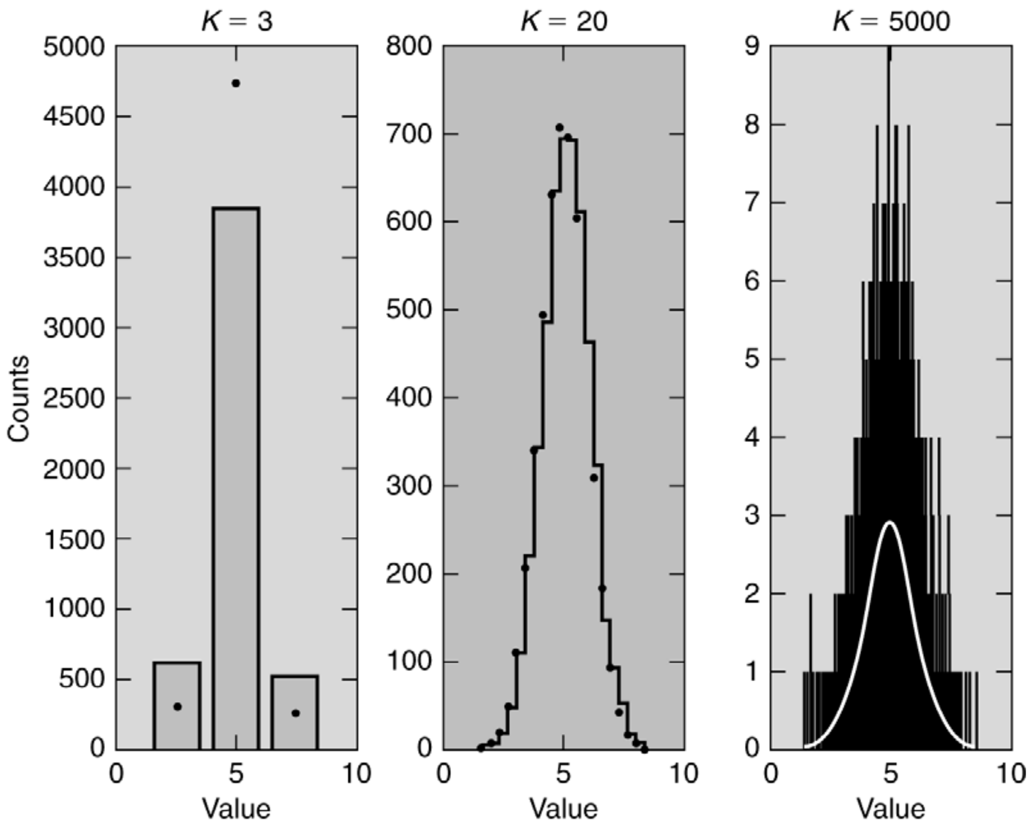
10 digital values: 1.5, 1.0, 2.5, 4.0, 3.5, 2.0, 2.5, 3.0, 2.5 and 0.5 V

resorted in order: 0.5, 1.0, 1.5, 2.0, 2.5, 2.5, 3.0, 3.5, 4.0 V

N = 9 occurrences; j = 8 cells; n_j = occurrences in j-th cell

Proper Choice of Δx

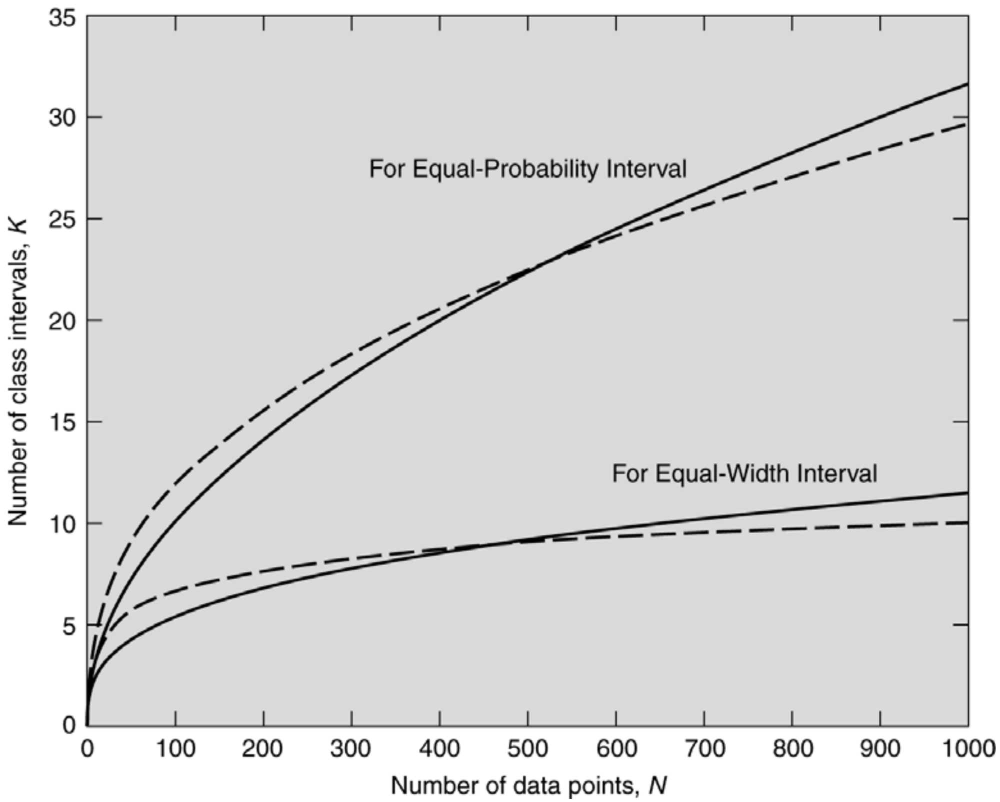
The choice of Δx is critical to the interpretation of the histogram.



← made using 3histos.m

Δx Choices

Typically, we construct equal-width-interval histograms.



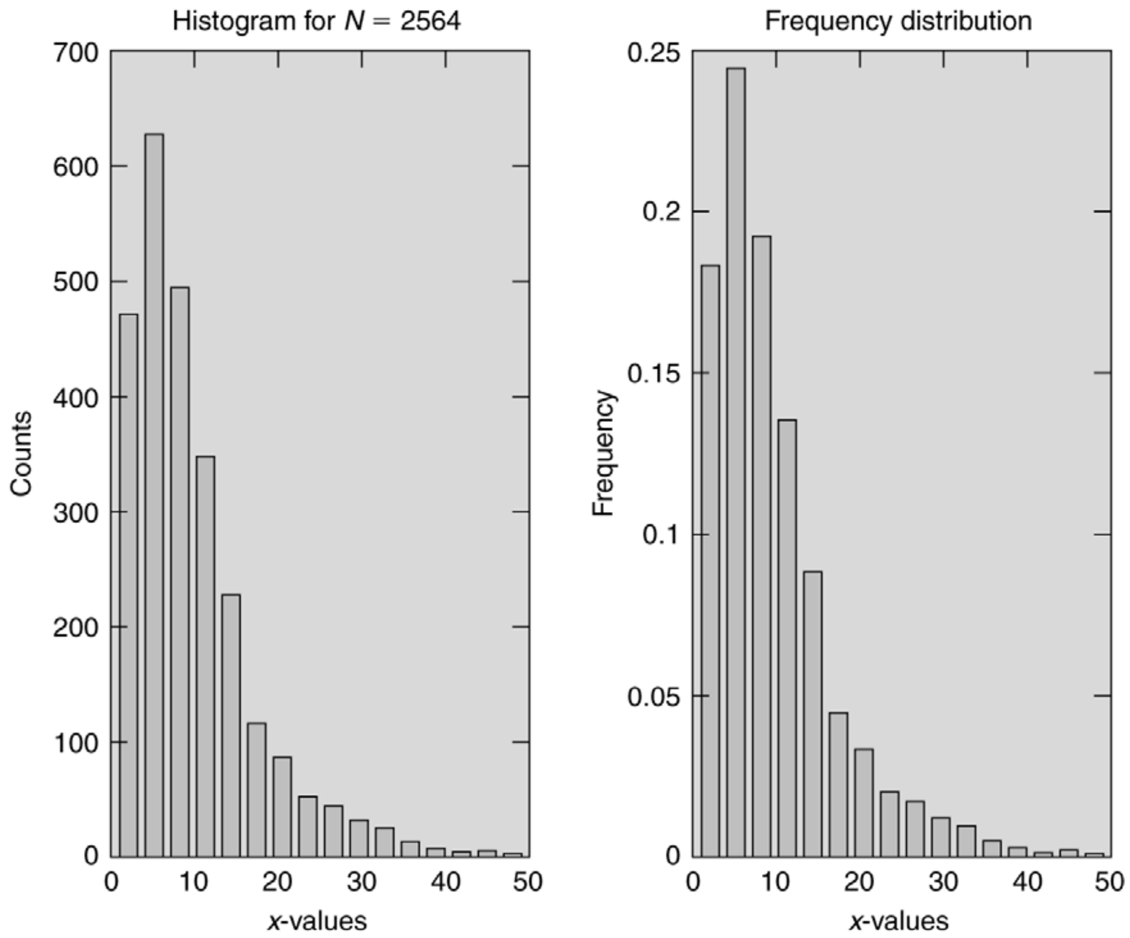
Histogram Construction Rules

To construct equal-width histograms:

1. Identify the minimum and maximum values of x and its range where $x_{range} = x_{max} - x_{min}$.
2. Determine K class intervals (usually use $K = 1.15N^{1/3}$).
3. Calculate $\Delta x = x_{range} / K$.
4. Determine n_j (j = 1 to K) in each Δx interval. Note $\sum n_j = N$.
5. Check that $n_j > 5$ and $\Delta x \geq u_x$.
6. Plot nj versus xm_j,where xm_j is the midpoint value of each interval.

Frequency Distribution

The **frequency distribution** is a plot of n_j /N versus magnitude.
It is *very similar to* the histogram.



← made using hf.m

Figure 7.7

Probability Density Function Concept

- Consider a signal that varies in time.

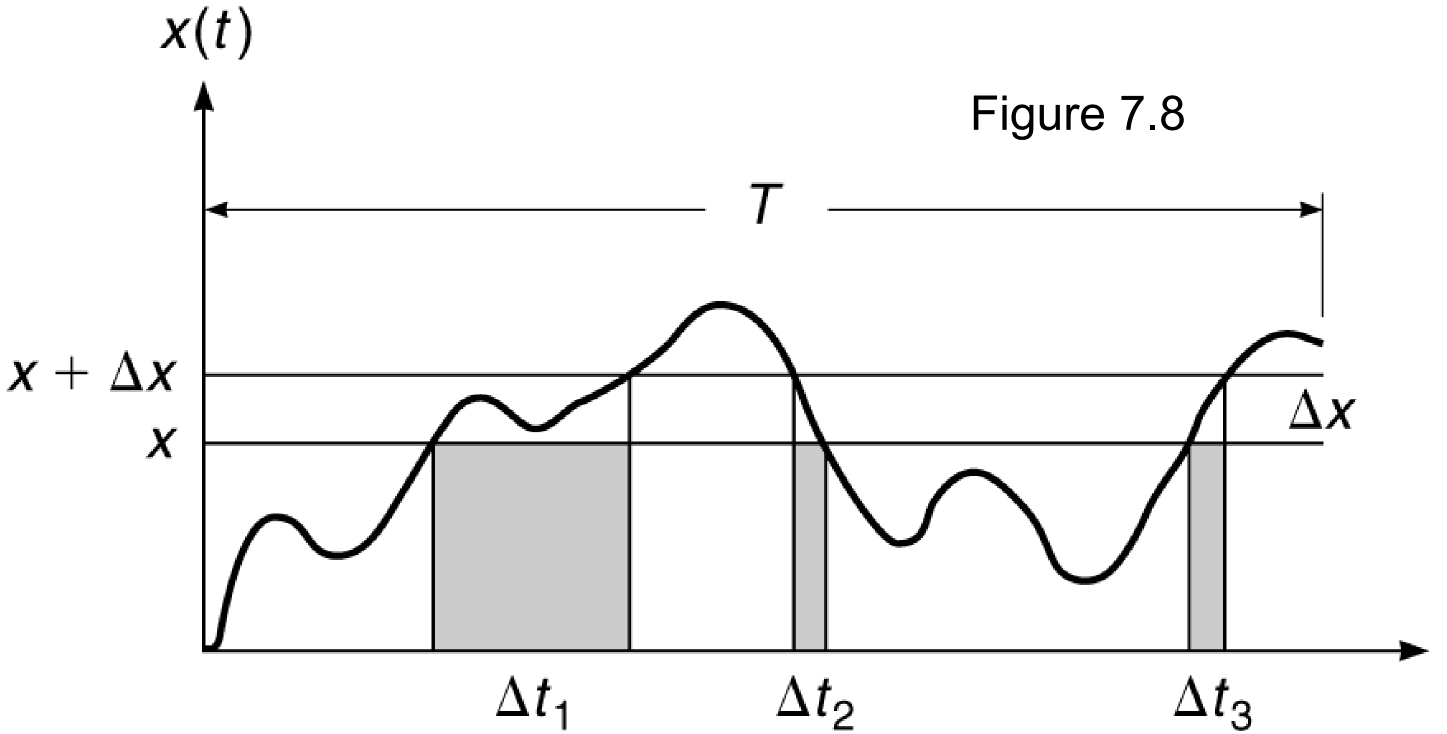


Figure 7.8

- What is the probability that the signal at a future time will reside between x and x + Δx?

Probability Density Function (pdf)

- Definition: $$p(x) = \lim_{\Delta x \to 0} \frac{Pr[x^* < x(t) \leq x^* + \Delta x]}{\Delta x}$$

- For x(t):

$$Pr[x < x(t) \leq x + \Delta x] = \lim_{T \to \infty} \left[\frac{T_x}{T} \right] = \lim_{T \to \infty} \frac{1}{T} \sum_{j=1}^{m} \Delta t_j \qquad T_x = \sum_{j=1}^{m} \Delta t_j$$

$$p(x) = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[\lim_{T \to \infty} \frac{1}{T} \sum_{j=1}^{m} \Delta t_j \right] = \lim_{\Delta x \to 0, T \to \infty} \left[\frac{T_x / T}{\Delta x} \right]$$

- For n occurrences:

$$p(x) = \lim_{\Delta x \to 0} \frac{1}{\Delta x} \left[\lim_{N \to \infty} \sum_{j=1}^{m} \frac{n_j}{N} \right] = \lim_{\Delta x \to 0, N \to \infty} \sum_{j=1}^{m} \left[\frac{n_j / N}{\Delta x} \right]$$