

# Chapter 2

## NUMBER SYSTEMS AND CODES

2.1

(1) Base 3

(a)  $(245)_{10} = (?)_3$

$$\begin{array}{r}
 3 \overline{) 245} \\
 3 \overline{) 81} \\
 3 \overline{) 27} \\
 3 \overline{) 9} \\
 3 \overline{) 3} \\
 3 \overline{) 1} \\
 0
 \end{array}
 \begin{array}{l}
 2 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 1
 \end{array}
 \uparrow
 = (100002)_3$$

(b)  $(461)_{10} = (?)_3$

$$\begin{array}{r}
 3 \overline{) 461} \\
 3 \overline{) 153} \\
 3 \overline{) 51} \\
 3 \overline{) 17} \\
 3 \overline{) 5} \\
 3 \overline{) 1} \\
 0
 \end{array}
 \begin{array}{l}
 2 \\
 0 \\
 0 \\
 2 \\
 2 \\
 2 \\
 1
 \end{array}
 \uparrow
 = (122002)_3$$

(c)  $(76.5)_{10} = (?)_3$

$$\begin{array}{r}
 3 \overline{) 76} \\
 3 \overline{) 25} \\
 3 \overline{) 8} \\
 3 \overline{) 2} \\
 0
 \end{array}
 \begin{array}{l}
 1 \\
 1 \\
 2 \\
 2 \\
 2
 \end{array}
 \uparrow$$

$$\begin{array}{r}
 .5 \\
 \times 3 \\
 \hline
 1.5
 \end{array}
 \begin{array}{l}
 * \text{repeats} \\
 \\
 \\
 \\
 \end{array}
 = (2211.\overline{1})_3$$

(d)  $(46.45)_{10} = (?)_3$

$$\begin{array}{r}
 3 \overline{) 46} \\
 3 \overline{) 15} \\
 3 \overline{) 5} \\
 3 \overline{) 1} \\
 0
 \end{array}
 \begin{array}{l}
 1 \\
 0 \\
 2 \\
 1 \\
 1
 \end{array}
 \uparrow$$

$$\begin{array}{r}
 .45 \\
 \times 3 \\
 \hline
 1.35 \\
 \times 3 \\
 \hline
 1.05 \\
 \times 3 \\
 \hline
 0.15 \\
 \times 3 \\
 \hline
 0.45
 \end{array}
 \begin{array}{l}
 \\
 \\
 \\
 \\
 \\
 \\
 * \text{repeats}
 \end{array}
 = (1201.\overline{1100})_3$$

$$(e) (231.78)_{10} = (?)_3$$

$$\begin{array}{r} 3 \overline{) 231} \\ 3 \overline{) 77} \\ 3 \overline{) 25} \\ 3 \overline{) 8} \\ 3 \overline{) 2} \\ 0 \end{array} \begin{array}{l} 0 \uparrow \\ 2 \\ 1 \\ 2 \\ 2 \end{array}$$

$$\begin{array}{r} .78 \\ \times 3 \\ \hline 2.34 \\ \times 3 \\ \hline 1.02 \\ \times 3 \\ \hline 0.06 \\ \times 3 \\ \hline 0.18 \end{array}$$

$$=(22120.2100\dots)_3$$

$$(f) (1023.25)_{10} = (?)_3$$

$$\begin{array}{r} 3 \overline{) 1023} \\ 3 \overline{) 341} \\ 3 \overline{) 113} \\ 3 \overline{) 37} \\ 3 \overline{) 12} \\ 3 \overline{) 4} \\ 3 \overline{) 1} \\ 0 \end{array} \begin{array}{l} 0 \uparrow \\ 2 \\ 2 \\ 1 \\ 1 \\ 0 \\ 1 \\ 1 \end{array}$$

$$\begin{array}{r} .25 \\ \times 3 \\ \hline 0.75 \\ \times 3 \\ \hline 2.25 \end{array}$$

\*repeats

$$=(1101220.\overline{02})_3$$

(2) Base 5

$$(a) (245)_{10} = (?)_5$$

$$\begin{array}{r} 5 \overline{) 245} \\ 5 \overline{) 49} \\ 5 \overline{) 9} \\ 5 \overline{) 1} \\ 0 \end{array} \begin{array}{l} 0 \uparrow \\ 4 \\ 4 \\ 1 \end{array}$$

$$=(1440)_5$$

$$(b) (461)_{10} = (?)_5$$

$$\begin{array}{r} 5 \overline{) 461} \\ 5 \overline{) 92} \\ 5 \overline{) 18} \\ 5 \overline{) 3} \\ 0 \end{array} \begin{array}{l} 1 \uparrow \\ 2 \\ 3 \\ 3 \end{array}$$

$$=(3321)_5$$

$$(c) (76.5)_{10} = (?)_5$$

$$\begin{array}{r} 5 \overline{) 76} \\ 5 \overline{) 15} \\ 5 \overline{) 3} \\ 0 \end{array} \begin{array}{l} 1 \uparrow \\ 0 \\ 3 \end{array}$$

$$\begin{array}{r} .5 \\ \times 5 \\ \hline 2.5 \end{array}$$

\*repeats

$$=(301.\overline{2})_5$$

$$(d) (46.45)_{10} = (?)_5$$

$$\begin{array}{r} 5 \overline{)46} \quad 1 \uparrow \\ 5 \quad \underline{9} \quad 4 \\ 5 \quad \underline{1} \quad 1 \\ \quad \quad 0 \end{array} \quad \downarrow \quad \begin{array}{r} .45 \\ \times 5 \\ \hline 2.25 \\ \times 5 \\ \hline 1.25 \end{array} \quad \text{*repeats}$$

$$=(141.2\overline{1})_5$$

$$(e) (231.78)_{10} = (?)_5$$

$$\begin{array}{r} 5 \overline{)231} \quad 1 \uparrow \\ 5 \quad \underline{46} \quad 1 \\ 5 \quad \underline{9} \quad 1 \\ 5 \quad \underline{1} \quad 4 \\ \quad \quad 0 \quad 1 \end{array} \quad \downarrow \quad \begin{array}{r} .78 \\ \times 5 \\ \hline 3.90 \\ \times 5 \\ \hline 4.50 \\ \times 5 \\ \hline 2.50 \end{array} \quad \text{*repeats}$$

$$=(1411.34\overline{2})_5$$

$$(f) (1023.25)_{10} = (?)_5$$

$$\begin{array}{r} 5 \overline{)1023} \quad 3 \uparrow \\ 5 \quad \underline{204} \quad 4 \\ 5 \quad \underline{40} \quad 0 \\ 5 \quad \underline{8} \quad 3 \\ 5 \quad \underline{1} \quad 1 \\ \quad \quad 0 \end{array} \quad \downarrow \quad \begin{array}{r} .25 \\ \times 5 \\ \hline 1.25 \end{array} \quad \text{*repeats}$$

$$=(13043.\overline{1})_5$$

(3) Base 8

$$(a) (245)_{10} = (?)_8$$

$$\begin{array}{r} 8 \overline{)245} \quad 5 \uparrow \\ 8 \quad \underline{30} \quad 6 \\ 8 \quad \underline{3} \quad 3 \\ \quad \quad 0 \end{array} \quad = (365)_8$$

$$(b) (461)_{10} = (?)_8$$

$$\begin{array}{r} 8 \overline{)461} \quad 5 \uparrow \\ 8 \quad \underline{57} \quad 1 \\ 8 \quad \underline{7} \quad 7 \\ \quad \quad 0 \end{array} \quad = (715)_8$$

$$(c) (76.5)_{10} = (?)_8$$

$$\begin{array}{r} 8 \overline{) 76} \quad 4 \uparrow \\ 8 \quad \underline{72} \quad 1 \\ 8 \quad \quad \underline{4} \quad 1 \\ \quad \quad \quad 0 \quad 1 \end{array} \quad \downarrow \quad \begin{array}{r} .5 \\ \times 8 \\ \hline 4.0 \end{array}$$

$$=(114.4)_8$$

$$(d) (46.45)_{10} = (?)_8$$

$$\begin{array}{r} 8 \overline{) 46} \quad 6 \uparrow \\ 8 \quad \underline{40} \quad 5 \\ \quad \quad \underline{6} \end{array} \quad \downarrow \quad \begin{array}{r} .45 \\ \times 8 \\ \hline 3.60 \\ \times 8 \\ \hline 4.8 \\ \times 8 \\ \hline 6.4 \\ \times 8 \\ \hline 3.2 \\ \times 8 \\ \hline 1.6 \end{array} \quad \begin{array}{l} \\ \\ \\ \\ \\ \end{array} \quad \begin{array}{l} \\ \\ \\ \\ \text{*repeats} \end{array}$$

$$=(56.\overline{34631})_8$$

$$(e) (231.78)_{10} = (?)_8$$

$$\begin{array}{r} 8 \overline{) 231} \quad 7 \uparrow \\ 8 \quad \underline{24} \quad 4 \\ 8 \quad \quad \underline{3} \quad 3 \\ \quad \quad \quad 0 \end{array} \quad \downarrow \quad \begin{array}{r} .78 \\ \times 8 \\ \hline 6.24 \\ \times 8 \\ \hline 1.92 \\ \times 8 \\ \hline 7.36 \\ \times 8 \\ \hline 2.88 \end{array}$$

$$=(347.6172\dots)_8$$

$$(f) (1023.25)_{10} = (?)_8$$

$$\begin{array}{r} 8 \overline{) 1023} \quad 7 \uparrow \\ 8 \quad \underline{127} \quad 7 \\ 8 \quad \quad \underline{15} \quad 7 \\ 8 \quad \quad \quad \underline{1} \quad 7 \\ \quad \quad \quad \quad 0 \quad 1 \end{array} \quad \downarrow \quad \begin{array}{r} .25 \\ \times 8 \\ \hline 2.00 \end{array}$$

$$=(1777.2)_8$$

(4) Base 16

(a)  $(245)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \overline{)245} \\ 16 \overline{)15} \quad 5 = 5 \\ \quad 0 \quad 15 = F \end{array} \uparrow = (F5)_{16}$$

(b)  $(461)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \overline{)461} \\ 16 \overline{)28} \quad 13 = D \\ 16 \overline{)1} \quad 12 = C \\ \quad 0 \quad 1 = 1 \end{array} \uparrow = (1CD)_{16}$$

(c)  $(76.5)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \overline{)76} \\ 16 \overline{)4} \quad 12 = C \\ \quad 0 \quad 4 = 4 \end{array} \uparrow \quad \downarrow \begin{array}{r} .5 \\ \times 16 \\ \hline 8.0 \end{array} = (4C.8)_{16}$$

(d)  $(46.45)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \overline{)46} \\ 16 \overline{)2} \quad 14 = E \\ \quad 0 \quad 2 = 2 \end{array} \uparrow \quad \downarrow \begin{array}{r} .45 \\ \times 16 \\ \hline 7.2 \\ \times 16 \\ \hline 3.2 \end{array} \text{ *repeats}$$

$= (2E.7\overline{3})_{16}$

(e)  $(231.78)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \overline{)231} \\ 16 \overline{)14} \quad 7 = 7 \\ \quad 0 \quad 14 = E \end{array} \uparrow \quad \downarrow \begin{array}{r} .78 \\ \times 16 \\ \hline (C=) 12.48 \\ \times 16 \\ \hline (7=) 7.68 \\ \times 16 \\ \hline (A=) 10.88 \end{array}$$

$= (E7.C7A\dots)_{16}$

(f)  $(1023.25)_{10} = (?)_{16}$

$$\begin{array}{r} 16 \overline{)1023} \\ 16 \overline{)63} \quad 15 = F \\ 16 \overline{)3} \quad 15 = F \\ \quad 0 \quad 3 = 3 \end{array} \uparrow \quad \downarrow \begin{array}{r} .25 \\ \times 16 \\ \hline 4.00 \end{array} = (3FF.4)_{16}$$

- 2.2      Base 7  
             0, 1, 2, 3, 4, 5, 6, 10, 11, 12, 13, 14, 15, 16, 20  
             Base 9  
             0, 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15

2.3      Current Odometer reading is  $(24516)_8$   

$$= 2 \times 8^4 + 4 \times 8^3 + 5 \times 8^2 + 1 \times 8^1 + 6 \times 8^0$$

$$= 2 \times 4096 + 4 \times 512 + 5 \times 64 + 8 + 6$$

$$= (10574)_{10}$$

Since  $(23)_{10} = (27)_8$

$$\begin{array}{r} 8 \overline{)23} \\ 8 \overline{)2} \quad 7 \\ \underline{0} \quad 2 \end{array}$$

the new reading will be:  $(24516)_8 + (27)_8 = (24545)_8$

2.4       $(24516)_8 = (?)_{16}$

Added bit  
 $\downarrow$

Expanding each digit in to 3 bits:  $\underbrace{0 \ 010 \ 100 \ 101 \ 001 \ 110}_{\substack{= \quad 2 \quad 9 \quad 4 \quad E}}$

In 5 digits the reading is: 0294E.

Since  $(23)_{10} = (17)_{16}$ , the new reading is  $(0294E)_{16} + (17)_{16}$   

$$= (02965)_{16}$$

## 2.5

(1) one's complement

- (a) 10010 ----> 01101  
 (b) 110010 ----> 001101  
 (c) 0010101 ----> 1101010  
 (d) 10110.0101 ----> 01001.1010  
 (e) 1101.1100 ----> 0010.0011  
 (f) 111010.0011 ----> 000101.1100  
 (g) 1001.0001 ----> 0110.1110  
 (h) 110100.0100 ----> 001011.1011  
 (i) 1010110.111 ----> 0101001.000

(2) two's complement

- (a) 10010 ----> 01101 <--- complement each bit  
                   + 1 <--- add 1 to the LSB  
                   01110  
 (b) 110010 ----> 001101 + 1 = 001110  
 (c) 0010101 ----> 1101010 + 1 = 1101011  
 (d) 10110.0101 ----> 01001.1010 + 1 = 01001.1011  
 (e) 1101.1100 ----> 0010.0011 + 1 = 0010.0100  
 (f) 111010.0011 ----> 000101.1100 + 1 = 000101.1101  
 (g) 1001.0001 ----> 0110.1110 + 1 = 0110.1111  
 (h) 110100.0100 ----> 001011.1011 + 1 = 001011.1100  
 (i) 1010110.111 ----> 0101001.000 + 1 = 0101001.001

## 2.6

nine's complement

- (a) 465  
 $(465)_9 = 10^3 - (465)_{10} - 1 = 534$   
 (b) 09867  
 $09867_9 = 10^5 - (09867)_{10} - 1 = 90132$   
 (c) 42678  
 $(42678)_9 = 10^5 - (42678)_{10} - 1 = 57321$   
 (d) 8976  
 $(8976)_9 = 10^4 - (8976)_{10} - 1 = 1023$   
 (e) 423.76  
 $(423.76)_9 = 10^3 - (423.76)_{10} - 0.01 = 576.23$   
 (f) 561.876  
 $(561.876)_9 = 10^3 - (561.876)_{10} - 0.001 = 438.123$   
 (g) 463.90  
 $(463.90)_9 = 10^3 - (463.90)_{10} - 0.01 = 536.09$   
 (h) 1786.967  
 $(1786.967)_9 = 10^4 - (1786.967)_{10} - 0.001 = 8213.032$   
 (i) 12356.078  
 $(12356.078)_9 = 10^5 - (12356.078)_{10} - 0.001 = 87643.921$

Ten's complement

- (a)  $(465)_{10} = 10^3 - (465)_{10} = 535$   
 (b)  $(09867)_{10} = 10^5 - (09867)_{10} = 90133$   
 (c)  $(42678)_{10} = 10^5 - (42678)_{10} = 57322$   
 (d)  $(8976)_{10} = 10^4 - (8976)_{10} = 1024$   
 (e)  $(423.76)_{10} = 10^3 - (423.76)_{10} = 576.24$   
 (f)  $(561.876)_{10} = 10^3 - (561.876)_{10} = 438.124$   
 (g)  $(463.90)_{10} = 10^3 - (463.90)_{10} = 536.10$   
 (h)  $(1786.967)_{10} = 10^4 - (1786.967)_{10} = 8213.033$   
 (i)  $(12356.078)_{10} = 10^5 - (12356.078)_{10} = 87643.922$

2.7

(a)  $x+y$  
$$\begin{array}{r} 1101010 \\ + 10111 \\ \hline 10000001 \end{array}$$
  $x-y$  
$$\begin{array}{r} 1101010 \\ - 10111 \\ \hline 1010011 \end{array}$$

$xy$  
$$\begin{array}{r} 1101010 \\ \times 10111 \\ \hline 1101010 \\ 1101010 \\ 1101010 \\ 0000000 \\ \hline 1101010 \\ \hline 100110000110 \end{array}$$

$x/y$  
$$\begin{array}{r} 1101010 \\ - 10111 \\ \hline 00111 \\ - 10111 \\ \hline 01110 \\ - 10111 \\ \hline 1110 \end{array}$$
  $11010 > 10111 \quad q_1 = 1 ; \text{subtract}$   
 $00111 < 10111 \quad q_2 = 0 ; \text{do not subtract}$   
 $01110 < 10111 \quad q_3 = 0 ; \text{do not subtract}$   
 $1110 \leftarrow \text{remainder}$   
 $100 \leftarrow \text{quotient}$

(b)  $x+y$  
$$\begin{array}{r} 101101 \\ + 1111 \\ \hline 111100 \end{array}$$
  $x-y$  
$$\begin{array}{r} 101101 \\ - 1111 \\ \hline 11110 \end{array}$$

$xy$  
$$\begin{array}{r} 101101 \\ \times 1111 \\ \hline 101101 \\ 101101 \\ 101101 \\ 101101 \\ \hline 101101 \\ \hline 1010100011 \end{array}$$

$x/y$  
$$\begin{array}{r} 101101 \\ - 1111 \\ \hline 10110 \\ - 1111 \\ \hline 1111 \\ - 1111 \\ \hline 0 \end{array}$$
  $1011 < 1111 \quad q_1 = 0 ; \text{do not subtract}$   
 $10110 > 1111 \quad q_2 = 1 ; \text{subtract}$   
 $1111 \geq 1111 \quad q_3 = 1 ; \text{subtract}$   
 $0 \leftarrow \text{remainder}$   
 $011 \leftarrow \text{quotient}$



(c)  $x+y$   $\begin{array}{r} 1001 \\ +1111 \\ \hline 11000 \end{array}$   $x-y$   $\begin{array}{r} 1001 \\ -1111 \\ \hline 0110 \end{array} \rightarrow -0110(x < y)$

$xy$   $\begin{array}{r} 1001 \\ \times 1111 \\ \hline 1001 \\ 1001 \\ 1001 \\ 1001 \\ \hline 1001 \\ \hline 10000111 \end{array}$

$x/y$   $\begin{array}{r} 1001 \\ -1111 \\ \hline 1001 \end{array} \leftarrow \text{remainder}$   $0 \leftarrow \text{quotient}$   $1001 < 1111 \quad q_1=0; \text{do not subtract}$

(d)  $x+y$   $\begin{array}{r} 110.11 \\ + 10.11 \\ \hline 1001.10 \end{array}$   $x-y$   $\begin{array}{r} 110.11 \\ - 10.11 \\ \hline 100.00 \end{array}$

$x*y$   $\begin{array}{r} 110.11 \\ \times 10.11 \\ \hline 11011 \\ 11011 \\ 00000 \\ \hline 11011 \\ \hline 100101001 \end{array}$

$x/y$   $\begin{array}{r} 110.11 \\ - 101.1 \\ \hline 0101 \\ - 1011 \\ \hline 1010 \\ - 1011 \\ \hline 10100 \\ - 1011 \\ \hline 1001 \end{array}$   $1101 > 1011 \quad q_1=1; \text{subtract}$   
 $0101 < 1011 \quad q_2=0; \text{do not subtract}$   
 $1010 < 1011 \quad q_3=0; \quad "$   
 $10100 > 1011 \quad q_4=1; \text{subtract}$   
 $0.1001 \leftarrow \text{remainder}$   
 $10.01 \leftarrow \text{quotient}$

(e)

$$\begin{array}{r}
 x+y \quad 1110.101 \\
 + 1011.100 \\
 \hline
 11010.001
 \end{array}$$

$$\begin{array}{r}
 x-y \quad 1110.101 \\
 - 1011.100 \\
 \hline
 0011.001
 \end{array}$$

x\*y

$$\begin{array}{r}
 \phantom{x} \quad 1110.101 \\
 x \quad 1011.100 \\
 \hline
 0000 \ 000 \\
 00000 \ 00 \\
 111010 \ 1 \\
 1110101 \\
 1110101 \\
 0000000 \\
 1110101 \\
 \hline
 10100110001 \ 100
 \end{array}$$

x/y

1110 101	1110101 > 1011100	q <sub>1</sub> =1; subtract
- 1011 100		
011 0010	0110010 < 1011100	q <sub>2</sub> =0; do not
101 1100		subtract
11 00100	1100100 > 1011100	q <sub>3</sub> =1; subtract
- 10 11100		
0 010000	0010000 < 1011100	q <sub>4</sub> =0; do not
		subtract
0.010 ←	remainder	
1.010 ←	quotient	

(f)

$$\begin{array}{r}
 x+y \quad 1011.00 \\
 + 1100.00 \\
 \hline
 10111.00
 \end{array}$$

$$\begin{array}{r}
 x-y \quad 1100.00 \\
 - 1011.00 \\
 \hline
 0001.00 \\
 = -0001.00 (X < Y)
 \end{array}$$

x\*y

$$\begin{array}{r}
 \phantom{x} \quad 1011.00 \\
 x \quad 1100.00 \\
 \hline
 10110000 \ 00 \\
 101100 \\
 \hline
 1000010000 \ 00
 \end{array}$$

x/y 1011 00	101100 < 110000	q <sub>1</sub> =0; do not
- 1100 00		subtract
1011 000	1011000 > 110000	q <sub>2</sub> =1; subtract
- 110 000		
101 0000	1010000 > 110000	q <sub>3</sub> =1; "
- 11 0000		
10 0000		
10.00 ←	remainder	
0.11 ←	quotient	

2.8

(a)

$$\begin{array}{r} x+y \quad 533 \\ +234 \\ \hline 767 \end{array}$$

$$\begin{array}{r} x-y \quad 533 \\ -234 \\ \hline 277 \end{array}$$

$\begin{array}{r} x*y \quad 533 \\ \times 234 \\ \hline 2554 \\ 2021 \\ \hline 1266 \\ 151564 \leftarrow \text{product} \end{array}$	$\leftarrow$ $\leftarrow$ $\leftarrow$ $\leftarrow$	$\begin{array}{l} \text{scratch pad} \\ 3 \times 4 = (14)_8 \\ 3 \times 4 = (14)_8 \\ 5 \times 4 = (24)_8 \\ \hline 2554 \\ 3 \times 3 = (11)_8 \\ 3 \times 3 = (11)_8 \\ 5 \times 3 = (17)_8 \\ \hline 2021 \\ 3 \times 2 = (6)_8 \\ 3 \times 2 = (6)_8 \\ 5 \times 2 = (12)_8 \\ \hline 1266 \end{array}$
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$$\begin{array}{r} x/y \quad 234 \overline{) 533} \\ \underline{470} \\ 43 \end{array}$$

(b)

$$\begin{array}{r} x+y \quad 46537 \\ + \quad 234 \\ \hline 46773 \end{array}$$

$$\begin{array}{r} x-y \quad 46537 \\ - \quad 234 \\ \hline 46303 \end{array}$$

$\begin{array}{r} x*y \quad 46537 \\ \times 234 \\ \hline 232574 \\ 164035 \\ \hline 115276 \\ 13622744 \leftarrow \text{product} \end{array}$	$\leftarrow$ $\leftarrow$ $\leftarrow$ $\leftarrow$	$\begin{array}{l} \text{scratch pad} \\ 7 \times 4 = (34)_8 \\ 3 \times 4 = (14)_8 \\ 5 \times 4 = (24)_8 \\ 6 \times 4 = (30)_8 \\ 4 \times 4 = (20)_8 \\ \hline 232574 \\ 7 \times 3 = (25)_8 \\ 3 \times 3 = (11)_8 \\ 5 \times 3 = (17)_8 \\ 6 \times 3 = (22)_8 \\ 4 \times 3 = (14)_8 \\ \hline 164035 \\ 7 \times 2 = (16)_8 \\ 3 \times 2 = (6)_8 \\ 5 \times 2 = (12)_8 \\ 6 \times 2 = (14)_8 \\ 4 \times 2 = (10)_8 \\ \hline 115276 \end{array}$
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$$\begin{array}{r} x/y \quad 234 \overline{) 46537} \\ \underline{234} \\ 2313 \\ \underline{2204} \\ 1077 \\ \underline{724} \\ 153 \end{array}$$

(c)  $x+y$   $\begin{array}{r} 26 \\ +533 \\ \hline 561 \end{array}$   $x-y$   $\begin{array}{r} 533 \\ - 26 \\ \hline 505 \end{array} \rightarrow -505(x < y)$

$x*y$   $\begin{array}{r} 26 \\ \times 533 \\ \hline 102 \\ 102 \\ 156 \\ \hline 16722 \end{array}$   $\leftarrow$  product

scratch pad

$6 \times 3 = (22)_8$	$102$
$2 \times 3 = (6)_8$	$102$
$6 \times 5 = (36)_8$	$156$
$2 \times 5 = (12)_8$	$156$

$x/y$   $\begin{array}{r} 0 \\ 533 \overline{) 26} \\ \underline{0} \\ 26 \end{array}$

(d)  $x+y$   $\begin{array}{r} 123.2 \\ + 234.0 \\ \hline 357.2 \end{array}$   $x-y$   $\begin{array}{r} 234.0 \\ - 123.2 \\ \hline 110.6 \\ = -110.6(X < Y) \end{array}$

$x*y$   $\begin{array}{r} 123.2 \\ \times 234.0 \\ \hline 000\ 0 \\ 5150 \\ 3716 \\ 2464 \\ \hline 312730\ 0 \end{array}$

scratch pad

$2 \times 4 =$	$10$
$3 \times 4 =$	$14$
$2 \times 4 =$	$10$
$1 \times 4 =$	$4$
	$\hline 5150$
$2 \times 3 =$	$6$
$3 \times 3 =$	$11$
$2 \times 3 =$	$6$
$1 \times 3 =$	$3$
	$\hline 3716$

$x/y$   $\begin{array}{r} 0.4 \\ 234 \overline{) 123.2} \\ \underline{234} \\ 123.2 \\ \underline{116.0} \\ 5.2 \end{array}$

(e)

$$\begin{array}{r} x+y \\ 234.6 \\ + 156.7 \\ \hline 413.5 \end{array}$$

$$\begin{array}{r} x-y \\ 234.6 \\ - 156.7 \\ \hline 055.7 \end{array}$$

$x*y$	$\begin{array}{r} 234.6 \\ \times 156.7 \\ \hline 2111\ 2 \\ 16544 \\ 14176 \\ 2346 \\ \hline 417435.2 \end{array}$	$\begin{array}{l} \leftarrow \\ \leftarrow \\ \leftarrow \end{array}$	$\begin{array}{r} \text{scratch pad} \\ 6x7 = \\ 4x7 = \\ 3x7 = \\ 2x7 = \\ \hline 6x6 = \\ 4x6 = \\ 3x6 = \\ 2x6 = \\ \hline 6x5 = \\ 4x5 = \\ 3x5 = \\ 2x5 = \end{array}$	$\begin{array}{r} \text{Octal} \\ 52 \\ 34 \\ 25 \\ 16 \\ \hline 21112 \\ 44 \\ 30 \\ 22 \\ 14 \\ \hline 16544 \\ 36 \\ 24 \\ 17 \\ 12 \\ \hline 14176 \end{array}$
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$$\begin{array}{r} x/y \\ 156.7 \overline{) 234.6} \\ \underline{156.7} \\ 55.70 \\ \underline{51.45} \\ 4.23 \end{array}$$

2.9

$$\begin{array}{r} (a) \quad x+y \\ 11 \leftarrow \text{carry} \\ 1CF \\ + B6 \\ \hline 285 \end{array}$$

$$\begin{array}{r} x-y \\ 1CF \\ - B6 \\ \hline 119 \end{array}$$

$x*y$	$\begin{array}{r} 1CF \\ \times B6 \\ \hline ADA \\ 13E5 \\ \hline 1492A \end{array}$	$\begin{array}{l} \leftarrow \\ \leftarrow \end{array}$	$\begin{array}{r} \text{scratch pad} \\ Fx6 = (5A)_{16} \\ Cx6 = (48)_{16} \\ 1x6 = (6)_{16} \\ \hline ADA \\ FxB = (A5)_{16} \\ CxB = (84)_{16} \\ 1xB = (B)_{16} \\ \hline 13E5 \end{array}$
-------	---	---	--

$$\begin{array}{r} x/y \\ B6 \overline{) 1CF} \\ \underline{1B5} \\ 1A \leftarrow \text{remainder} \end{array}$$

$$\begin{array}{r} (b) \quad x+y \\ 1B59A \\ + C23 \\ \hline 1C1BD \end{array}$$

$$\begin{array}{r} x-y \\ 1B59A \\ - C23 \\ \hline 1A977 \end{array}$$

x*y	1859A	scratch pad	hexadecimal
	x C23	Ax3=	1E
	520CE <--P1	9x3=	1B
	36834 <--P2	5x3=	F
	148338 <--P3	Bx3=	21
	148FOCOE	1x3=	3
			520CE(P1)
		Ax2=	14
		9x2=	12
		5x2=	A
		Bx2=	16
		1x2=	2
			36834(P2)
		AxC=	78
		9xC=	6C
		5xC=	3C
		BxC=	84
		1xC=	C
			148338(P3)

x/y

C23	24
	1859A
	1846
	313A
	308C
	AE

(c) x+y

B6	1CF
+1CF	-B6
285	119 --> -119(x<y)

x*y	B6	scratch pad	hexadecimal
	x1CF	6xF=	5A
	AAA	BxF=	A5
	888		AAA(P1)
	B6	6xC=	48
	1492A	BxC=	84
			888(P2)
		6x1=	6
		Bx1=	B
			B6(P3)

x/y

1CF	0	<--- quotient
	B6	
	0	
	B6	<--- remainder

(d)

x+y

```
  2ECD
+ 4321
-----
 71EB
```

x-y

```
  4321
- 2ECD
-----
 1454
--1454 (X<Y)
```

x\*y

```
  2ECD
x 4321
-----
  2ECD
 7D9A (P1)
 8C67 (P2)
BB34 (P3)
-----
C47AF6D
```

scratch pad

Hexadecimal

Dx2 =	1A
Cx2 =	18
Ex2 =	1C
2x3 =	6
	<hr/>
Dx3 =	7D9A (P1)
Cx3 =	27
Ex3 =	24
2x3 =	2A
	6
	<hr/>
Dx4 =	8C67 (P2)
Cx4 =	34
Ex4 =	30
2x4 =	38
	8
	<hr/>
	BB34 (P3)

x/y

```
      0
4321 2ECD
     4321
     ----
     2ECD
```

(e)  
x+y

```

234F.16
+ 456E.00
-----
68BD.16

```

x-y

```

456E.00
- 234F.16
-----
221E.EA
=-221E.EA(X<Y)

```

x\*y

```

234F.16
x 456E.00
-----
0000 00
00000 0
1EE5334 (P1)
D3DA84 (P2)
B08B6E (P3)
8D3C58 (P4)
-----
8C37CE974 00

```

scratch pad hexadecimal

6xE =	54
1xE =	E
FxE =	D2
4xE =	38
3xE =	2A
2xE =	1C
	<hr/>
	1EE5334 (P1)
6x6 =	24
1x6 =	6
Fx6 =	5A
4x6 =	18
3x6 =	12
2x6 =	C
	<hr/>
	D3DA84 (P2)
6x5 =	1E
1x5 =	5
Fx5 =	4B
4x5 =	14
3x5 =	F
2x5 =	A
	<hr/>
	B08B6E (P3)
6x4 =	18
1x4 =	4
Fx4 =	3C
4x4 =	10
3x4 =	C
2x4 =	8
	<hr/>
	8D3C58 (P4)

x/y

```

0.82
456E 234F.16
      456E
      ----
      234F.1
      22B7.0
      ----
      98.16
      8A.DC
      ----
      D.3A

```



2.10

a)

2	234	
2	117	0
2	58	1
2	29	0
2	14	1
2	7	0
2	3	1
2	1	1
2	0	1

↑

=(11101010)<sub>2</sub>

(b) convert to base 10  
 $(3345)_6 = 3 \times 6^3 + 3 \times 6^2 + 4 \times 6^1 + 5 \times 6^0 = 648 + 108 + 24 + 5$   
 $= (785)_{10}$

convert to base 2

2	785	
2	392	1
2	196	0
2	98	0
2	49	0
2	24	1
2	12	0
2	6	0
2	3	0
2	1	1
2	0	1

↑

=(1100010001)<sub>2</sub>

(c) convert to base 10  
 $(875)_9 = 8 \times 9^2 + 7 \times 9^1 + 5 \times 9^0 = 648 + 63 + 5$   
 $= (716)_{10}$

convert to base 11

11	716	
11	65	1
11	5	10
	0	5

↑

=(5 10 1)<sub>11</sub>

(d)  $(0.3212)_4 = 3 \times 4^{-1} + 2 \times 4^{-2} + 1 \times 4^{-3} + 2 \times 4^{-4}$   
 $= \frac{3}{4} + \frac{2}{16} + \frac{1}{64} + \frac{2}{256}$   
 $= \frac{230}{256} = (\frac{115}{128})_{10}$

(e) convert to base 10  
 $(87.35)_9 = 8 \times 9^1 + 7 \times 9^0 + 3 \times 9^{-1} + 5 \times 9^{-2}$   
 $= 72 + 7 + \frac{3}{9} + \frac{5}{81}$   
 $= (79\frac{32}{81})_{10}$

convert to base 11

$$\begin{array}{r} 11 \overline{) 79} \\ 11 \overline{) 7} \quad 2 \uparrow \\ \quad 0 \quad 7 \end{array}$$

$$\begin{array}{r} .395 \\ \times 11 \\ \hline 4.345 \\ \times 11 \\ \hline 3.795 \\ \times 11 \\ \hline 8.745 \\ \times 11 \\ \hline 7.995 \\ \times 11 \\ \hline 10.945 \\ \times 11 \\ \hline 10.395 \\ \times 11 \\ \hline 4.345 \end{array} \quad \text{*repeats}$$

$$= (72.4387 \ 10 \ 10)_{11}$$

2.12

$$(a) \quad \frac{10}{2} \ \frac{11}{3} \ \frac{01}{1} \ \frac{00}{0} \ . \frac{00}{0} \ \frac{10}{2} \ \frac{10}{2} \\ = (2310.022)_4$$

$$(b) \quad \begin{array}{c} A \quad B \quad 1 \quad 4 \quad 3 \\ \frac{010}{2} \ \frac{10}{5} \ \frac{1}{3} \ \frac{011}{0} \ \frac{000}{5} \ \frac{1}{5} \ \frac{01}{0} \ \frac{00}{0} \ \frac{011}{3} \end{array} \\ = (2530503)_8$$

$$(c) \quad \begin{array}{c} 2 \quad 3 \quad 4 \quad 7 \quad 4 \quad 5 \\ \frac{010}{4} \ \frac{0}{E} \ \frac{11}{7} \ \frac{10}{7} \ \frac{0}{7} \ \frac{111}{7} \end{array} \cdot \begin{array}{c} 4 \quad 5 \\ \frac{100}{9} \ \frac{1}{4} \ \frac{01}{4} \ \frac{00}{4} \end{array} \\ = (4E7.94)_{16}$$

$$(d) \quad \begin{array}{c} 0001 \quad 1011 \quad 1110 \\ \frac{1}{1} \quad \frac{B}{8} \quad \frac{E}{8} \end{array} \cdot \begin{array}{c} 0100 \quad 0001 \quad 1000 \\ \frac{4}{4} \quad \frac{1}{1} \quad \frac{8}{8} \end{array} \\ = (1BE.418)_{16}$$

2.11

$$(2574)_9 = (?)_3$$

Since  $9 = 3^2$ , each digit of the base-9 number is first expanded in to 2 digits; the right most digit will have a weight of 1 and the left most digit will have a weight of 3. Hence,

$$(2 \ 5 \ 7 \ 4)_9 = (02 \ 12 \ 21 \ 11)_3$$

2.13

$$\begin{aligned}
 (130)_x &= (28)_{10} \\
 1 \cdot x^2 + 3 \cdot x + 0 \cdot x^0 &= 2 \cdot 10 + 8 \cdot 10^0 \\
 \rightarrow x^2 + 3x - 28 &= 0 \\
 \rightarrow (x-4)(x-7) &= 0 \\
 x &= 4 \text{ or } x = -7 \\
 \text{We pick up } x &= 4 \text{ since } x \text{ is positive.}
 \end{aligned}$$

2.14

$$\begin{array}{r}
 \text{(a)} \quad 11101 \\
 + 1111 \\
 \hline
 101100 \\
 + 1011 \\
 \hline
 110111
 \end{array}$$

$$\begin{array}{r}
 \text{(b)} \quad 111000 \\
 - 10101 \\
 \hline
 100011
 \end{array}$$

$  \begin{array}{r}  \text{(c)} \quad 11001101 \\  - 101 \\  \hline  010 \\  - 101 \\  \hline  101 \\  - 101 \\  \hline  001 \\  - 101 \\  \hline  010 \\  - 101 \\  \hline  101 \\  - 101 \\  \hline  0 \\  101001  \end{array}  $	$110 > 101 \quad q_1 = 1; \text{subtract}$ $010 < 101 \quad q_2 = 0; \text{do not subtract}$ $101 \geq 101 \quad q_3 = 1$ $001 < 101 \quad q_4 = 0$ $010 < 101 \quad q_5 = 0$ $101 < 101 \quad q_6 = 1$  $0 \quad \leftarrow \text{remainder}$ $101001 \quad \leftarrow \text{quotient}$
---	--

$$\begin{array}{r}
 \text{(d)} \quad 11010 \\
 \times 11001 \\
 \hline
 11010 \\
 00000 \\
 00000 \\
 11010 \\
 + 11010 \\
 \hline
 1010001010
 \end{array}$$

2.15 (a) two's complement

	<u>2's complement</u>	
1011010	1011010	
<u>- 10101</u>	<u>1101011</u>	
	11000101	
	= 1000101	
10101	0010101	
<u>-1011010</u>	<u>+0100110</u>	
	0111011	=(1000101)

(b) one's complement

	<u>1's complement</u>	
1011010	1011010	
<u>- 10101</u>	<u>+1101010</u>	
	11000100	
	+ 1	
	1000101	=(1000101)
10101	0010101	
<u>-1011010</u>	<u>+0100101</u>	
	0111010	=(1000101)

2.16 (a) nine's complement

	<u>9's complement</u>
1875	1875
<u>-924</u>	<u>+9075</u>
	<u>10950</u>
	<u>+ 1</u>
	0951 = 951
924	0924
<u>-1875</u>	<u>+8124</u>
	9048 = -(951)

(b) ten's complement

	<u>10's complement</u>
1875	1875
<u>-924</u>	<u>+9076</u>
	<u>10951</u> = 951
924	0924
<u>-1875</u>	<u>+8125</u>
	9049 = -(951)

2.17 (a)      113      0 1110001  
                  -87      +1 0101001  
                       10 0011010      = +(0011010)<sub>2</sub>

(b)      87      0 1010111  
             -113      +1 0001111  
                  1 1100110      = -(0011010)<sub>2</sub>

(c)      43      0 0101011  
             +26      +0 0011010  
                  0 1000101      = +(1000101)<sub>2</sub>

(d)      96      0,1100000  
             -22      +1 1101010  
                  10 1001010      = +(1001010)<sub>2</sub>

(e)      46      0 0101110  
             -77      +1 0110011  
                  1 1100001      = -(0011111)<sub>2</sub>

2.18 (a)	7256	<u>scratch pad</u>	<u>octal</u>
	x 23	6x3=	22
	26012 <--P1	5x3=	17
	16534 <--P2	2x3=	6
	213352	7x3=	25
			26012 <-- P1
		6x2=	14
		5x2=	12
		2x2=	4
		7x2=	16
			16534 <--P2

$$(b) (AF)_{16} = (10101111)_2 = (257)_8$$

	scratch pad	octal
56		52
$\times 257$	$6 \times 7 =$	
502 $\leftarrow P1$	$5 \times 7 =$	43
346 $\leftarrow P2$		502 $\leftarrow P1$
134 $\leftarrow P3$	$6 \times 5 =$	36
$(17562)_8$	$5 \times 5 =$	31
		346 $\leftarrow P2$
	$6 \times 2 =$	14
	$5 \times 2 =$	12
		134 $\leftarrow P3$

$$(17562)_8 = (001111101110010)_2$$

$$= (1331302)_4$$

2.19 (a)  $(11010.010)_2 = (0.11010010) \times 2^5$

sign bit = 0  
 exponent =  $5 + 127 = (1000\ 0100)$   
 fraction = 1010 0100...0

0	1	8	9	31
0	10000100	10100100	....	0

(b)

2	432		.26
2	216	0	$\times 2$
2	108	0	0.52
2	54	0	$\times 2$
2	27	0	1.04
2	13	1	$\times 2$
2	6	1	0.08
2	3	0	$\times 2$
2	1	1	0.16
2	0	1	$\times 2$
			0.32
			$\times 2$
			0.64
			$\times 2$
			1.28
			$\times 2$
			0.56

$$(432.26)_{10} = (110110000.010000101000111)_2$$

$$= (0.110110000010000101000111) \times 2^9$$

sign bit = 0  
 exponent =  $9 + 127 = 136 = (10001000)_2$   
 fraction =  $(10110000010000101000111)_2$

0	1	8	9	31
0	10001000	10110000010000101000111		

1	10000111	010011110010...0
---	----------	------------------

1	10000111	11011001100010100011110
---	----------	-------------------------

$$(2) \begin{array}{cccc} 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{array}$$

```
2.21      6 3 -1 1      <--weighted
          0 0 0 1 1
          1 0 0 0 1
          2 0 1 1 0
          3 0 1 1 1
          4 0 1 0 1
          5 1 0 1 0
          6 1 0 0 0
          7 1 0 0 1
          8 1 1 1 0
          9 1 1 0 0
```

```
2.22      3 2 -1 1  <--weighted
          0 0 0 1 1
          1 0 0 0 1
          2 0 1 0 0
          3 1 0 1 1
          4 1 1 1 0
          5 1 1 0 0
```

2.23 (a) (0100 1001 0101 0111)<sub>bcd</sub> = (4 9 5 7)<sub>bcd</sub>  
 (b) (0100 1001 0101 0111)<sub>2</sub> = (18772)<sub>10</sub>  
 (c) Excess-3 = (1 6 2 4)  
 (d) ASCII Code = ) 7

(b)  $(356)_{bcd} = (0011 \ 0101 \ 0110)_{bcd}$   
 $= (0000 \ 0011 \ 0101 \ 0110)$  in 16-bit register

2-23

2.25 (a)  $(746)_{10} = (0111\ 0100\ 0110)_{BCD}$   
 (b)  $(746)_{10} = (1011101010)_2$   
 (c)  $(746)_{10} = (01010111\ 01010100\ 01010110)_{ASCII}$

2.26 (a) Sign bit = 0  
 Exponent =  $10000010 = (130)_{10} = 127 + 3$   
 Binary =  $(1.1000\ 0010) \times 2^3$   
 $= (1100.0001)_2$

(b) Sign bit = 1  
 Exponent =  $01111000 = (120)_{10} = 127 + (-7)$   
 Binary =  $(1.01000110) \times 2^{-7}$   
 $= (0.00000010100011)_2$

2.27 Correction factor

(i) -0011 if sum  $\leq 9$

Ex.            0            0011  
               + 1            + 0100  
               1            0111  
                               - 0011    <--- minus 3  
                               0100

(ii) 0011 if sum  $> 9$

Ex.            3            0110  
               + 9            + 1100  
               12            1 0010  
                               0011 0011    <--- add 3  
                               0100 0101



2.28 a. Fixed point: Implementation dependent. But usually with one bit used as a sign, we get  $-2^{63}$  to  $2^{64}$ .

Floating point: sign(1)exponent(11)mantissa(52) is the bit distribution for representation.

So, the range is  $4.9 * 10^{-307}$  to  $1.8 * 10^{+308}$

b. Floating point has the larger range.

c. When the values are confirmedly integers, fixed point representation gives faster operations

2.29 a. sign(1)exponent(5)fraction(10) is the format.

0 0 0 10 1110110011

b. 3. 69921875

c. We can see that in both cases, a rational number in one system is also a rational system (a number with a terminating or repetitive fractional part) in another. But the representation varies in precision depending on the base for a given number of places.

d. Since 3.7 is a representable by just one number after the radix point in base 10 number system, it takes non terminating repetitive set of numbers (0011) after radix point in base 2 system.