CHAPTER TWO

MOTION IN ONE DIMENSION

2.1 With the origin, O (0.0, 0.0) placed at Indiana, the overall displacement (Figure Pb 2.1 below) consists of three displacements. These are

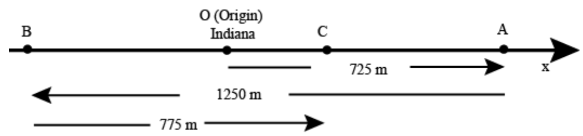

Figure Pb 2.1

$$\Delta x_{O\text{-}A} = x_A - x_O = 725 \text{ m},\ \Delta x_{A\text{-}B} = x_B - x_A = -1250 \text{ m},\ \Delta x_{B\text{-}C} = x_C - x_B = 775 \text{ m}$$

(a) The net displacement from Indiana is $\Delta x_{O\text{-}C} = \Delta x_{O\text{-}A} + \Delta x_{A\text{-}B} + \Delta x_{B\text{-}C}$

$$= (725 \text{ m}) + (-1250 \text{ m}) + (775 \text{ m})$$
$$= 250 \text{ m}$$

(b) The total distance d consists of three distances:

$$d_1 = (725 \text{ m}),\quad d_2 = 1250 \text{ m},\quad d_3 = 775 \text{ m}.$$

And $$d = d_1 + d_2 + d_3 = 725 \text{ m} + 1250 \text{ m} + 775 \text{ m} = 2750 \text{ m}$$

Analysis

The answer to **(a)** means that the bicycler destination is 250 m east of Indiana. However, calculation in part **(b)** shows the distance she traveled is 2750 m.

2.2

(a) From Equation (2.2) the average velocity is $\bar{v} = \dfrac{\Delta x}{\Delta t}$. Thus

$$\bar{v} = \frac{250\,\text{m}}{22\,\text{min} \times \dfrac{60\,\text{s}}{\text{min}}} = 0.19\,\text{m/s}.$$

(b) From Equation (2.5) the bicycler's speed is $v = \dfrac{d}{t}$. Thus $v = \dfrac{2750\,\text{m}}{22\,\text{min}\dfrac{60\,\text{s}}{\text{min}}} = 2.1\,\text{m/s}.$

Analysis

1. The average velocity is positive, meaning that the bicycler is moving away from the origin, Indiana.
2. The magnitude of the bicycler's average velocity is less than her speed. This is expected, since she spent some time during her backward motion from B to A.

2.3 The positions at the different points of the object's motion are:

$$x_A = 40.0 \text{ m} \quad x_B = 10.0 \text{ m}, \quad x_C = 10.0 \text{ m}, \quad x_D = 40.0 \text{ m}$$

(a) The displacement of the object in the interval between $t = 2.00$ s and $t = 8.00$ s is

$$\Delta x_{A\text{-}C} = x_C - x_A = 10.0 \text{ m} - 40.0 \text{ m} = -30.0 \text{ m},$$

(b) The displacement of the object in the interval between $t = 2.00$ s and $t = 16.0$ s is

$$\Delta x_{A\text{-}D} = x_D - x_A = 40.0 \text{ m} - 40.0 \text{ m} = 00.0 \text{ m}$$

(c) From From Equation (2.2), the average velocity is $\bar{v} = \dfrac{\Delta x}{\Delta t}$. Thus

$$\bar{v}_{A\text{-}C} = \frac{\Delta x_{A\text{-}C}}{\Delta t_{A\text{-}C}} = \frac{-30.0 \text{ m}}{6.00 \text{ s}} = -5.00 \text{ m/s}; \quad \bar{v}_{A\text{-}D} = \frac{\Delta x_{A\text{-}D}}{\Delta t_{A\text{-}D}} = \frac{00.0 \text{ m}}{14.0 \text{ s}} = 00.0 \text{ m/s}$$

Analysis

The average vbelocity $\bar{v}_{A\text{-}C}$ is negative; the object is moving toward the origin. This is also clear from the negative slope of the line AC (not connected). However, $\bar{v}_{A\text{-}D}$ is zero; the object in region AD ended at where it started, with no net displacement from its starting position. This is also clear from the zero slope of the line AD (not connected).

2.4

(a) The x-t Plot in each of the AB, BC, and CD in Figure Pb 2.4 is a straight line. The value of the instantaneous velocity at any instant of a region is equal to the slope of the line at that very instant. While each of these lines has a different slope, the slope at all points on any line has the same value. Thus

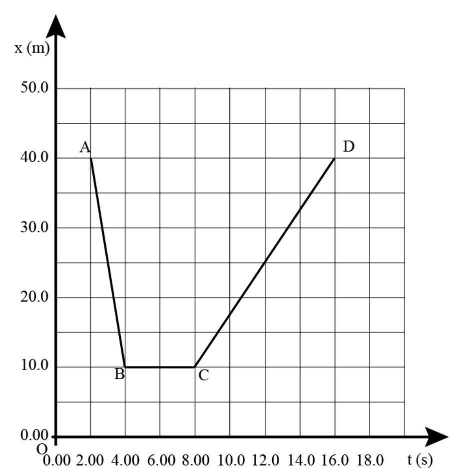

Figure Pb 2.4

$$v_{t=3.00s} = \text{slope of AB at } t = 3.00 \text{ s. That is, } v_{t=3.00s} = \frac{10.0 \text{ m} - 40.0 \text{ m}}{4.00\text{s} - 2.00\text{s}} = -15.0 \text{ m/s}$$

$$v_{t=6.00s} = \text{slope of BC at } t = 6.00\text{ s. That is, } v_{t=6.00s} = \frac{10.0\text{ m} - 10.0\text{ m}}{8.00\text{s} - 4.00\text{s}} = 00.0\text{ m/s}$$

$$v_{t=11.00s} = \text{slope of CD at } t = 11.0\text{ s. That is, } v_{t=11.00s} = \frac{40.0\text{ m} - 10.0\text{ m}}{16.0\text{s} - 8.00\text{s}} = 3.75\text{ m/s}$$

(b) The distance the object has covered in the whole motion is

$$d = d_{A\text{-}B} + d_{B\text{-}C} + d_{C\text{-}D} = 30.0\text{ m} + 00.0\text{ m} + 30.0\text{ m} = 60.0\text{ m}.$$

From Equation (2.5) the average speed is $v = \frac{d}{t} = \frac{60.0\text{ m}}{14.0\text{ s}} = 4.28\text{ m/s}$.

Analysis

The average vbelocity $\bar{v}_{A\text{-}B}$ is negative, meaning that the object is moving toward the origin. This is also clear from the plot being a straight line of negative slope, while $\bar{v}_{B\text{-}C}$ is zero, meaning that the object in segment BC ended at C where it had started at B; the CD plot is a straight line of positive slope; the object is moving away from the origin.

2.5 We first read from the plot the object's positions at points O, A, B, C, and D that define the segments OA, AB, BC, and AD. These are:

$$x_O = 00.0\text{ m},\ x_A = 10.0\text{ m},\ x_B = 35.0\text{ m},\ x_C, = 35.0\text{ m, and } x_D = 00.0\text{ m}$$

(a) The displacements asked for in part **(a)** are:

$$\Delta x_{O\text{-}A} = x_A - x_O = 10.0\text{ m} - 00.0\text{ m} = 10.0\text{ m},$$
$$\Delta x_{A\text{-}B} = x_B - x_A = 35.0\text{ m} - 10.0\text{ m} = 25.0\text{ m}$$
$$\Delta x_{B\text{-}D} = x_D - x_B = 00.0\text{ m} - 35.0\text{ m} = -35.0\text{ m}$$

(b) From From Equation (2.2) the average velocity is $\bar{v} = \frac{\Delta x}{\Delta t}$. Thus

$$\bar{v}_{O\text{-}A} = \frac{\Delta x_{O\text{-}A}}{\Delta t_{O\text{-}A}} = \frac{10.0\text{ m}}{4.00\text{ s}} = 2.50\text{ m/s};\quad \bar{v}_{A\text{-}B} = \frac{\Delta x_{A\text{-}B}}{\Delta t_{A\text{-}B}} = \frac{25.0\text{ m}}{2.00\text{ s}} = 12.5\text{ m/s}$$

$$\bar{v}_{B\text{-}D} = \frac{\Delta x_{B\text{-}D}}{\Delta t_{B\text{-}D}} = \frac{-35.0\text{ m}}{8.00\text{ s}} = -4.38\text{ m/s}$$

2.6 From the diagram, Figure Pb 2.6 (depiction of Figure 2.13 in the text):

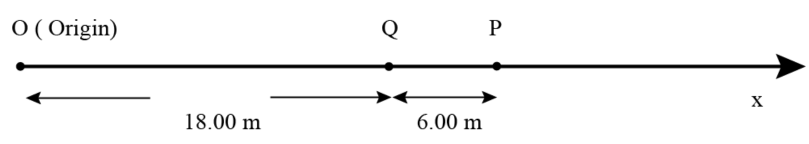

Figure Pb 2.6

$\Delta x_{O\text{-}P} = x_P - x_O = 24.0\text{ m} - 0.00\text{ m} = 24.0\text{ m},\quad \Delta t_{O\text{-}P} = 6.0\text{ s},$

$\Delta x_{P\text{-}Q} = x_Q - x_P = 18.0\text{ m} - 24.0\text{ m} = -6.0\text{ m}$, $\Delta t_{P\text{-}Q} = 2.0\text{ s}$

$\Delta x_{O\text{-}Q} = x_Q - x_O = 18.0\text{ m} - 0.0\text{ m} = 18.0\text{ m}$, $\Delta t_{O\text{-}Q} = \Delta t_{O\text{-}P} + \Delta t_{P\text{-}Q} = 6.0\text{ s} + 2.0\text{s} = 8.0\text{ s}$

(a) From From Equation (2.2) the average velocity is $\bar{v} = \dfrac{\Delta x}{\Delta t}$. Thus

$$\bar{v}_{O\text{-}P} = \frac{\Delta x_{O\text{-}P}}{\Delta t_{O\text{-}P}} = \frac{24.0\text{ m}}{6.0\text{ s}} = 4.0\text{ m/s}; \quad \bar{v}_{P\text{-}Q} = \frac{\Delta x_{P\text{-}Q}}{\Delta t_{P\text{-}Q}} = \frac{-6.0\text{ m}}{2.0\text{ s}} = -3.0\text{ m/s};$$

(b) $$\bar{v}_{O\text{-}Q} = \frac{\Delta x_{O\text{-}Q}}{\Delta t_{O\text{-}Q}} = \frac{18.0\text{ m}}{8.0\text{ s}} = 2.3\text{ m/s};$$

(c) From Equation (2.5) the average speed is $v = \dfrac{d}{t} = \dfrac{30.0\text{ m}}{8.0} = 3.8\text{ m/s}$.

Analysis

Notice that the average velocity in region OP was positive, while in region PQ it was negative. In the overall displacement between her initial and final destinations the average velocity was positive, all in consistence with the player's direction of motion.

2.7 Denoting the first part of trip by Δx_1 covered in Δt_1 and the second by Δx_2 covered in Δt_2, then

$$\Delta x_1 = 111\text{ km}, \quad \Delta t_1 = 80.0\text{ min}; \quad \Delta x_2 = 111\text{ km}, \quad \Delta t_2 = 40.0\text{ min}.$$

The total displacement of the motorist is $\Delta x = \Delta x_1 + \Delta x_2 = 222\text{ km}$;

and the total time duration is $\Delta t = \Delta t_1 + \Delta t_2 = 80.0\text{ min} + 40.0\text{ min} = 120.0\text{ min}$

(a) From the Equation, $\bar{v} = \dfrac{\Delta x}{\Delta t}$, the average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{222\text{ km}}{120.0\text{ min}\,(1\text{ hr}/60\text{ min})} = 111\text{ km/hr}$$

(b) From the Equation, $v = \dfrac{d}{t}$, the average speed is

$$v = \frac{d}{t} = \frac{222\text{ km}}{120.0\text{ min}\,(1\text{ hr}/60\text{ min})} = 111\text{ km/hr}.$$

Analysis

Notice that the problem demonstrates a case where the average velocity and average speed are equal in magnitude and sign (!).

2.8

(a) Denoting the first part of the trip by Δx_1 (= 226 km), covered in a duration Δt_1, and the second by Δx_2 (= 416 km –226 km = 190 km) coverd in Δt_2, then

$$\Delta t_1 = \frac{\Delta x_1}{\bar{v}_1} = \frac{226\,\text{km}}{62.0\,\text{km/hr}} = 3.65\ \text{hrs}; \quad \Delta t_2 = \frac{\Delta x_2}{\bar{v}_2} = \frac{190\,\text{km}}{68.0\,\text{km/hr}} = 2.8\ \text{hrs},$$

and $\Delta t_{rest} = (15\ \text{min}/(60\ \text{min/hr})) = 0.25\ \text{hr}.$

The total time the driver took between the two cities is:

$$\Delta t = \Delta t_1 + \Delta t_{rest} + \Delta t_2 = 3.65\ \text{hrs} + 0.250\ \text{hr} + 2.8\ \text{hrs} = 6.7\ \text{hrs}.$$

(b) From Equation (2.2) the average velocity is $\bar{v} = \frac{\Delta x}{\Delta t}$. Thus

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{416\,\text{k m}}{6.7\,\text{hrs}} = 62\ \text{km/hr};$$

(c) From the Equation, $v = \frac{d}{t}$, the average speed is

$$v = \frac{d}{t} = \frac{416\,\text{km}}{6.7\,\text{hrs}} = 62\ \text{km/hr}.$$

Analysis

Notice that the average speed in the whole trip was of a value equal to the value of the average velocity the driver had in the whole trip.

2.9 From the statement, the player's final velocity is zero, and his displacement upward is 1.20 m. Taking the axis y positively upward with the origin at the floor, we can use the equation

$$v^2 = v_o^2 + 2\,a\,y; \quad a = -g = -9.80\ \text{m/s}^2$$

Thus $(0.00\ \text{m/s})^2 = v_o^2 + 2(-9.80\frac{\text{m}}{\text{s}^2})(1.20\,\text{m})$, giving $v_o^2 = 23.5\frac{\text{m}^2}{\text{s}^2}$.

Thus $v_o = \pm 4.85\frac{\text{m}}{\text{s}}$

Since the direction of v_o is upward, then the positive sign in the above answer is the relevant one. Accordingly, $v_o = 4.85\frac{\text{m}}{\text{s}}$.

2.10

(a) Taking the y axis popsitively upward with the origin at the balcony, the ball's initial velocity would be $v_o = -8.00$ m/s. The duration is $t = 3.20$ s, and $a = -9.80\ \text{m/s}^2$. We can use the equation

$$y = v_o t + \frac{1}{2} a\, t^2$$

Thus $y = (-8.00\frac{\text{m}}{\text{s}})(3.20\,\text{s}) + \frac{1}{2}(-9.80\frac{\text{m}}{\text{s}^2})\ (3.20)^2 = -75.8\ \text{m}$

Thus the ground is 75.8 m below the balcony, where the origin was placed.

(b) To determine the final velocity of the ball just before it hits the ground, we use the equation $v = v_o + a\,t$, which upon substitution becomes

$$v = (-8.00\frac{m}{s}) + (-9.80\frac{m}{s^2})(3.20\,s) = -39.4\ m/s$$

Analysis

The relevance of the negative sign is that the ball's final velocity is directed down, which is what the situation is.

2.11 From Figure Pb 2.11.1 (depiction of Figure 2.14a in text), the displacements in the regions AB, BC, CD and DE are

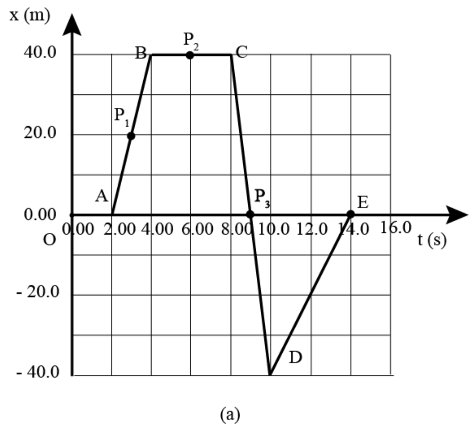

Figure Pb 2.11.1

$\Delta x_{A\text{-}B} = x_B - x_A = 40.0\ m - 00.0\ m = 40.0\ m,\qquad \Delta t_{A\text{-}B} = 2.00\ s;$
$\Delta x_{B\text{-}C} = x_C - x_B = 40.0\ m - 40.0\ m = 00.0\ m,\qquad \Delta t_{B\text{-}C} = 4.00\ s$
$\Delta x_{C\text{-}D} = x_D - x_C = -40.0\ m - 40.0\ m = -80.0\ m,\qquad \Delta t_{C\text{-}D} = 2.00\ s$
$\Delta x_{D\text{-}E} = x_E - x_D = 00.0\ m - (-40.0\ m) = 40.0\ m,\qquad \Delta t_{D\text{-}E} = 4.00\ s$

As the average velocity is $\bar{v} = \dfrac{\Delta x}{\Delta t}$, then

$$\bar{v}_{A\text{-}B} = \frac{\Delta x_{A\text{-}B}}{\Delta t_{A\text{-}B}} = \frac{40.0\,m}{2.00\,s} = 20.0\,m/s\,; \qquad \bar{v}_{B\text{-}C} = \frac{\Delta x_{B\text{-}C}}{\Delta t_{B\text{-}C}} = \frac{00.0\,m}{4.00\,s} = 00.0\,m/s\,;$$

$$\bar{v}_{C\text{-}D} = \frac{\Delta x_{C\text{-}D}}{\Delta t_{C\text{-}D}} = \frac{-80.0\,m}{2.0\,0s} = -40.0\,m/s\,. \qquad \bar{v}_{D\text{-}E} = \frac{\Delta x_{D\text{-}E}}{\Delta t_{D\text{-}E}} = \frac{40.0\,m}{4.0\,0s} = 10.0\,m/s\,.$$

The values of the average velocities in regions AB, BC, CD, and DE are sketched (heavy lines) in Figure Pb 2.11.2 below

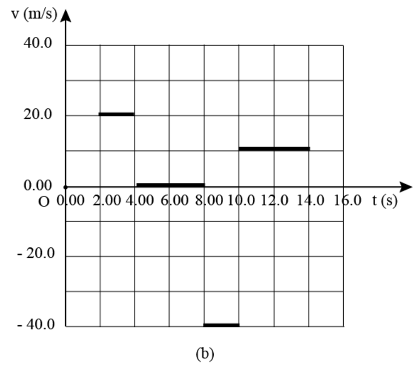

Figure Pb 2.11.2

Analysis

The velocity of the object in each region is constant. So without further calculations, it can be noticed that the acceleration in each of the specified regions should be zero.

2.12

(a) The x-t Plot in each of the regions AB, BC, and CD in Figure pb 2.12.1 below (Figure 2.14a in text) is a straight line. Since the slope of a straight line at all of its points is the same, then the value of the average velocity in each of the regions AB, BC, and CD is equal to the instantaneous velocity at the middle point of that region. Thus

$$v_{P1} = \frac{\Delta x_{A\text{-}B}}{\Delta t_{A\text{-}B}} = \frac{40.0\,\text{m}}{2.00\,\text{s}} = 20.0\,\text{m/s}\,; \quad v_{P2} = \frac{\Delta x_{B\text{-}C}}{\Delta t_{B\text{-}C}} = \frac{00.0\,\text{m}}{4.00\,\text{s}} = 00.0\,\text{m/s}\,;$$

$$v_{P3} = \frac{\Delta x_{C\text{-}D}}{\Delta t_{C\text{-}D}} = \frac{-80.0\,\text{m}}{2.0\,0\text{s}} = -40.0\,\text{m/s}\,.$$

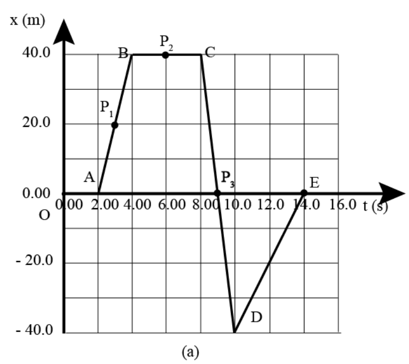

Figure Pb 2.12.1

(b) As the velocity in each of regions AB, BC, and CD is constant, then the acceleration in those is zero.

$$a_{P1} = \frac{\Delta\, v_{A\text{-}B}}{\Delta\, t_{A\text{-}B}} = \frac{00.0\,\text{m/s}}{2.00\,\text{s}} = 00.0\,\text{m/s}^2\,; \quad a_{P2} = \frac{\Delta\, v_{B\text{-}C}}{\Delta\, t_{B\text{-}C}} = \frac{00.0\,\text{m/s}}{4.00\,\text{s}} = 00.0\,\text{m/s}^2\,;$$

$$a_{P3} = \frac{\Delta v_{C\text{-}D}}{\Delta t_{C\text{-}D}} = \frac{00.0\text{ m/s}}{2.0\,0\text{s}} = 00.0\text{ m/s}^2 .$$

(c) As stated in (b) the v-t Plot in each of the regions AB, BC, and CD is a straight line, the value of the average acceleration in each of these regions is equal to the instantaneous acceleration at the middle point of the region, and that is equal to the slope of the line describing the region. In the figure below, a and b the v-t and a-t plots, respectively are depicted.

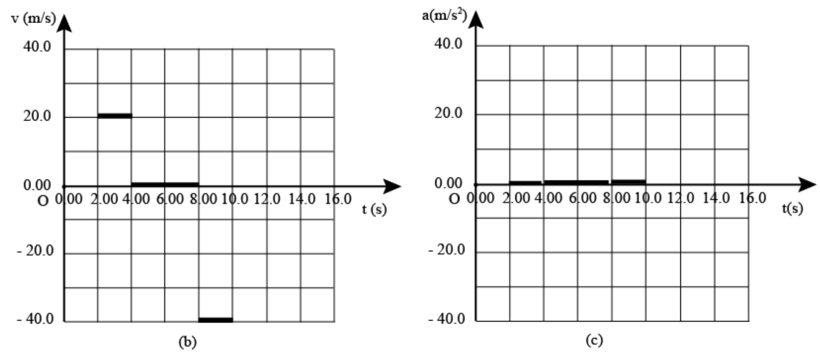

Figure Pb 2.12.2

Analysis

This is another illustration where a constant velocity of the object in a region means zero acceleration in that region.

2.13

(a) Taking the y axis popsitively upward with the origin at where the pencil started falling, then $y = 0$ at $t = 0$, and $v_o = 00.0$ m/s. For the floor, $y = -0.490$ m. Thus ,

$$y = v_o t + \frac{1}{2} a t^2 ,$$

after substituting for the known values becomes

$$-0.490\text{ m} = (00.0\text{ m/s})\,t + \frac{1}{2}(-9.80\text{ m/s}^2)\,t^2$$

This reduces to $t^2 = 0.100\text{ s}^2$, giving $t = \pm 0.316$ s

Thus $t = 0.316$ s the negative root is excluded since time has to be positive.

(b) For finding the velocity of the pencil just before it reaches the floor, we use the equation $v = v_o + a\,t$.

That is, $v = (00.0\text{ m/s}) + (-9.80\text{ m/s}^2)(0.316\text{ s}) = -3.10$ m/s

(c) The average velocity of the pencil can be found using Equation (2.2) for y in place of x as the variable,

$$\bar{v} = \frac{\Delta y}{\Delta t} = \frac{y_{floor} - y_o}{\Delta t} = \frac{-0.490\text{ m} - 00.0}{0.316} = -1.55\text{ m/s}$$

The relevance of the negative sign is to describe the direction of the pencil's velocity downward.

Analysis

Notice that the average velocity is half of the object's final instantaneous velocity. As the initial velocity is zero, this is consistent with the formula, $\bar{v} = \frac{1}{2}(v_o + v)$;

2.14

(a) Taking the y axis positively upward (Figure Pb 2.14a) with the origin at the ground level, then $y = 0$ m at $t = 0$ s, and $v_o = 14.0$ m/s. The ball's final velocity in its upward motion is zero. Substituting for the known values in the equation

$$v^2 = v_o^2 + 2ay, \qquad \text{makes it} \qquad (00.0\text{ m/s})^2 = (14.0\text{ m/s})^2 + 2(-9.80\text{ m/s}^2)\,y,$$

which gives for the maximum height $y = 10.0$ m.

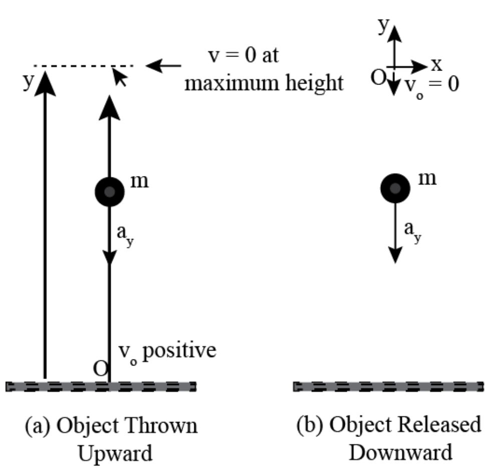

Figure Pb 2. 14

(b) To find the time it takes the ball to reach its maximum height use $v = v_o + a\,t$.

That is, $0 = (14.0\text{ m/s}) + (-9.80\text{ m/s}^2)(t)$, giving $t = 1.43$ s

To find the time it took the stone in its round trip, we work out the problem for the downward part of its motion. Taking the origin now at the stone's highest point (sketch **(b)**), then on its way down, $v_o = 00.0$ m/s, and y for the ground is $y = -\ 10.0$ m.

Thus upon substitution in Equation (2.15-b), $y = v_o t + \frac{1}{2} a t^2$, it becomes

$$-\ 10.0\ \text{m} = (00.0\ \text{m/s})\ t + \frac{1}{2}(-9.80\ \text{m/s}^2)\ t^2 .$$

This reduces to $t^2 = 2.05\ s^2$, giving $t = \pm\ 1.43$ s

That is, $t = 1.43$s, because the negative root for time is rejected
So the time it takes the stone in its round trip is $2.00 \times 1.43\ \text{s} = 2.86\ \text{s}$

Analysis

Ignoring air resistance, an object projected upward with an initial velocity v_o gets to its highest point in a time t equal to the time that elapses for the object to fall back to the ground.

2.15 Taking the y axis popsitively upward with the origin at the balcony the ball's initiall velocity is $v_o = 21.0$ m/s. The position of the ground y with respect to the origin at the balcony is -15.0 m.

This motion, shown in the Figure pb 2.15.1, can be divided into two parts, (a) is the motion of the stone upward, in which the stone ends at its highest position with a final velocity $v = 00.0$ m/s, and (b) is the stone's downward motion, falling toward the ground.

(a) For the first part, Figure pb 2.15.1a, $v_o = 21.0$ m/s. $v = 00.0$ m/s, and $a = -\ 9.80\ \text{m/s}^2$.

Using Equation (2.15-c), $v^2 = v_o^2 + 2\,a\,y$

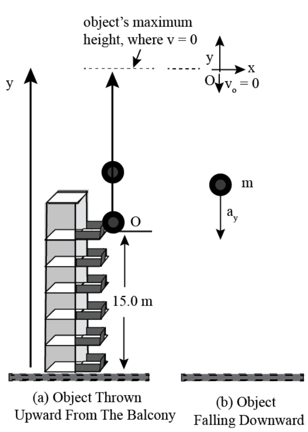

Figure Pb 2.15.1

we then have $(0)^2 = (21\ \text{m/s})^2 + 2\,(-9.80\ \text{m/s}^2)\ y$, giving $y = 22.5$ m

as the maximum height above the balcony the stone would reach.

(b) The time of flight above the balcony can be determined from Equation (2.15-a),

$v = v_o + a\,t\,.$ Thus $00.0\ \text{m/s} = (21.0\,\frac{\text{m}}{\text{s}}) + (-9.80\,\frac{\text{m}}{\text{s}^2})\,(t)\,,$

giving $t = 2.14\ \text{s}$

As the time the stone takes to fall down to the balcony's level is also 2.14 s, and its velocity downward at that level would be 21.0 m/s, then for the second part of the motion (Figure Pb 2.15.2).

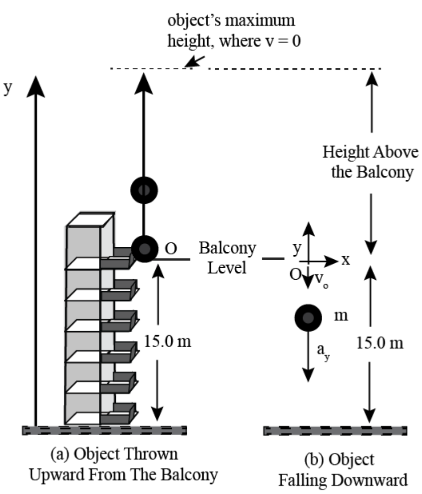

Figure Pb 2.15.2

$v_o = -21.0$ m/s. $y = -15.0$ m, and $a = -9.80\ \text{m/s}^2$.

From Equation (2.15-c), $v^2 = v_o^2 + 2\,a\,y\,,$

upon substitution for the known values becomes

$$v^2 = (-21.0\,\text{m/s})^2 + 2\,(-9.80\,\text{m/s}^2)\,(-15.0)\,\text{m} = 735\ \text{m}^2$$

This gives $v = \pm\, 27.1\ \text{m/s}$

Since the stone is moving down, the positive sign is dropped and the negative sign is considered. Therefore, the final velocity of the stone just before it hits the ground is

$$v = -\,27.1\ \text{m/s}$$

Finally, we can find the time it takes the stone to cover its last part of motion. The equation

$$v = v_o + a\,t\,,$$

after substituting for the already known values becomes

$(-27.1\,\text{m/s}) = (-21.0\,\text{m/s}) + (-9.80\,\text{m/s}^2)\,t$. This gives $t = 0.622\ \text{s}$

The total time of the stone in air is $t = 2 \times (2.14\text{s}) + 0.622\ \text{s} = 4.90\ \text{s}$