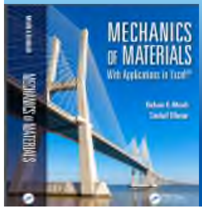
MECHANICS OF MATERIALS: WITH APPLICATIONS IN EXCEL
CHAPTER 2: TORSIONAL LOADS

LECTURE 8

2.3 STRESSES AND DEFORMATIONS IN CIRCULAR SHAFTS



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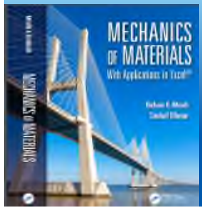
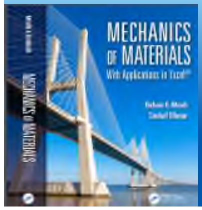
Lecture Outline

- Introduction
- Stresses and Deformation in Circular Shafts
- Material Properties in Shear
- Stress Element
- Examples



Introduction

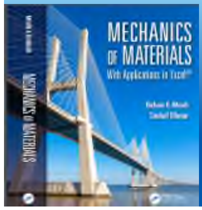
- The torsion of circular shafts represents the simplest of all torsion problems.
- Pure torsion causes shear strain and shear stress.



Stresses and Deformations in Circular Shafts

- Shear stress-strain relations are used to study the behavior of torsional shafts. Stress-Strain relations are governed by Hooke’s Law:

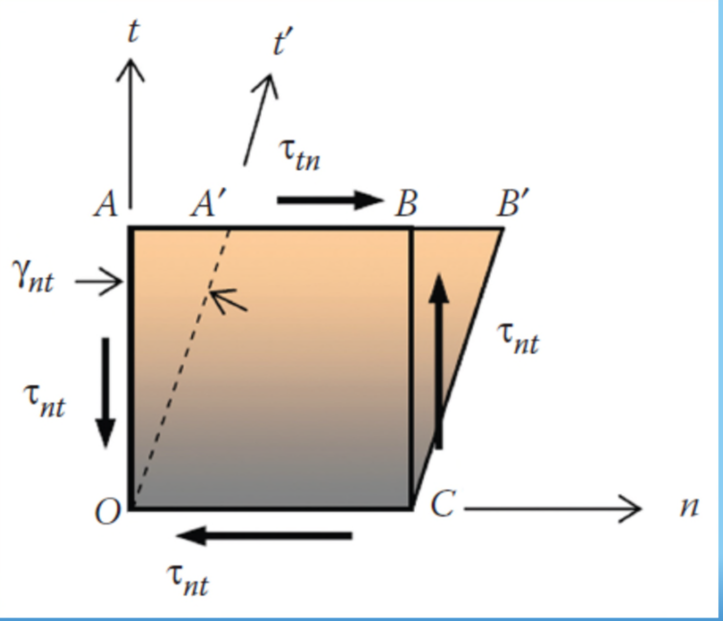
$$\tau = G\gamma$$



Shearing Strain

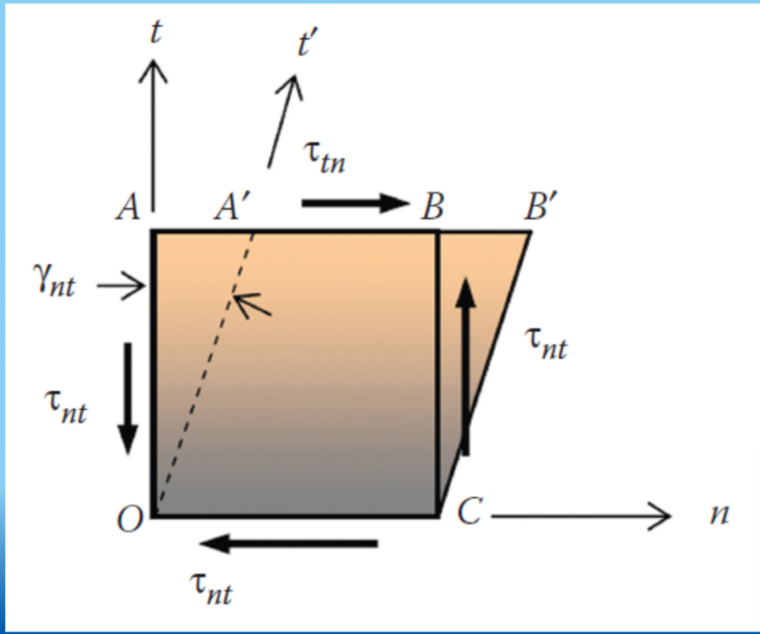
- Shear strain is a distortion – it represents a change in the initial shape of the body.
- Distortion can best be represented by the change in the angle between two initially perpendicular lines

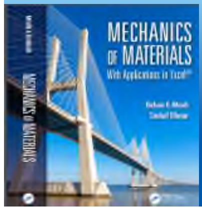
$$\gamma_{nt} \approx \tan \gamma_{nt} = \frac{AA'}{OA}$$



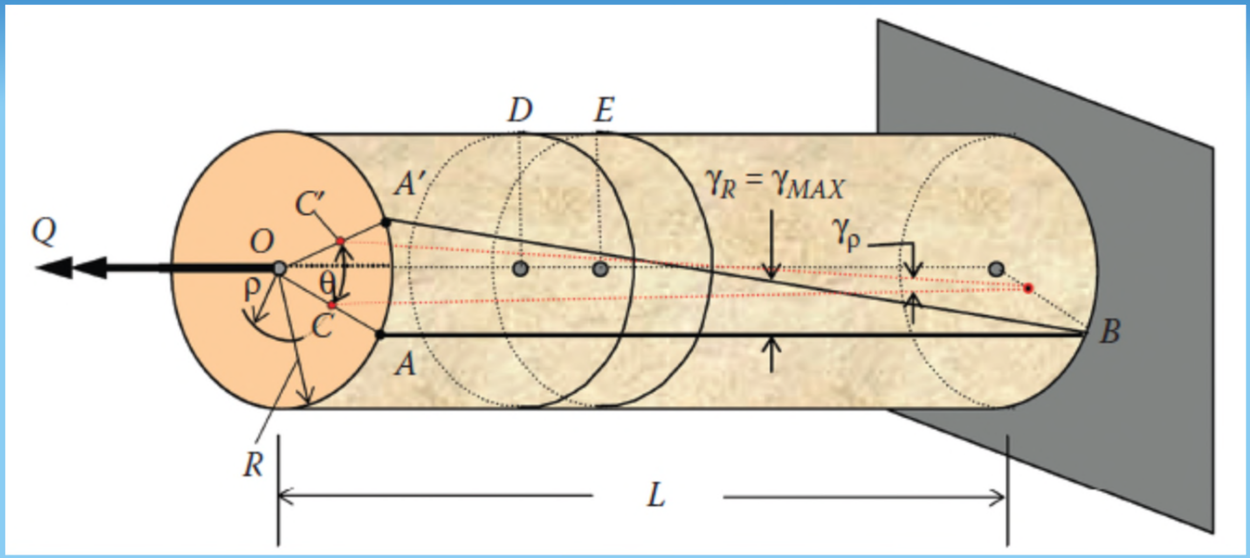
Shearing Strain – Sign Convention

- A positive shearing strain represents a decrease in the 90° angle and a negative shearing strain represents an increase in this angle.
- The shearing strain shown below is positive since it represents a decrease in the 90° angle between OA and OC.



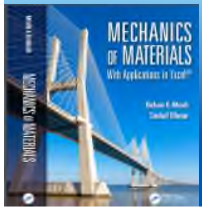


Shearing Stress and Shearing Deformation

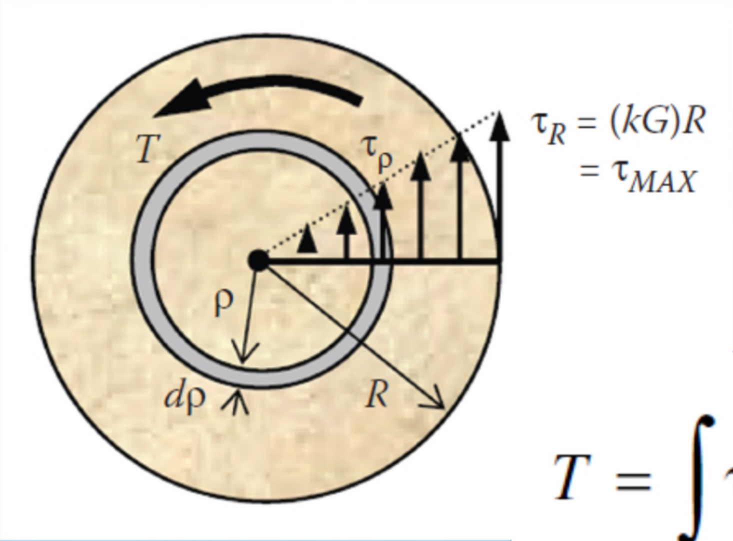


Assumptions:

1. Cross sections such as those at D and E are plane prior to twisting and remain plane after twisting of the circular shaft.
2. Straight line elements on the surface of the shaft such as line AB are assumed to remain straight after twisting occurs. Point A moves to A′ and line A′B is assumed to be straight for small angles θ even though its true shape is helical.
3. The material is assumed to behave elastically in order to be able to use Hooke's law.



Torque to Shearing Stress Relationship



$$\left.\begin{aligned} T &= \int \tau_\rho \rho \, dA = 2\pi \int_0^R \tau_\rho \rho^2 d\rho = 2\pi \int_0^R \left(\frac{\tau_\rho}{\rho}\right) \rho^3 \, d\rho \\ T &= 2\pi \left(\frac{\tau_\rho}{\rho}\right) \int_0^R \rho^3 d\rho = \left(\frac{\tau_\rho}{\rho}\right)\left(\frac{\pi R^4}{2}\right) = \left(\frac{\tau_\rho}{\rho}\right) J \end{aligned}\right\}$$

Maximum Shearing Stress and Angle of Twist

- Maximum Shearing Stress:

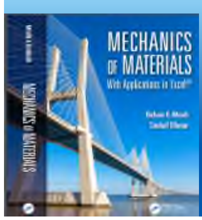
$$\tau_{MAX} = \tau_R = \frac{TR}{J}$$

- Angle of Twist:

$$\theta = \frac{TL}{JG}$$

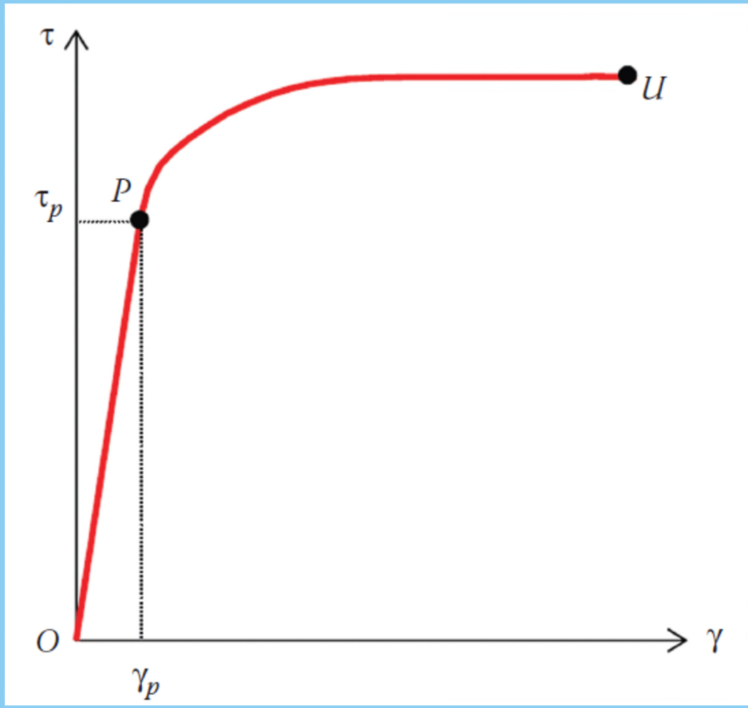
For a shaft with multiple segments: $\theta = \sum \frac{TL}{JG}$

J= Polar Moment of Inertia of the Cross-Section



Material Properties in Shear

Shear Stress-Strain Diagram:



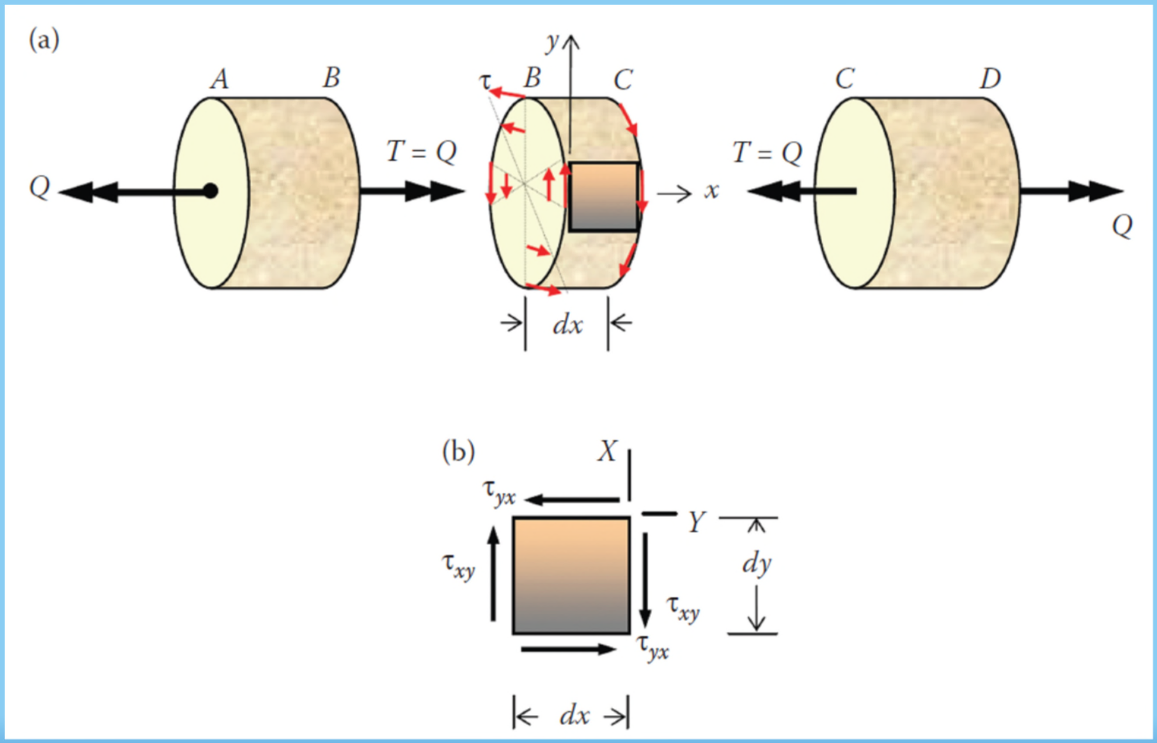
Hooke's Law applies in the linear-elastic region O-P:

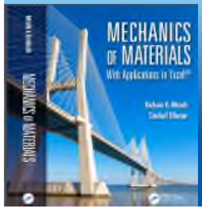
$$\tau = G\gamma$$

G: Shear Modulus of Elasticity

Stress Element

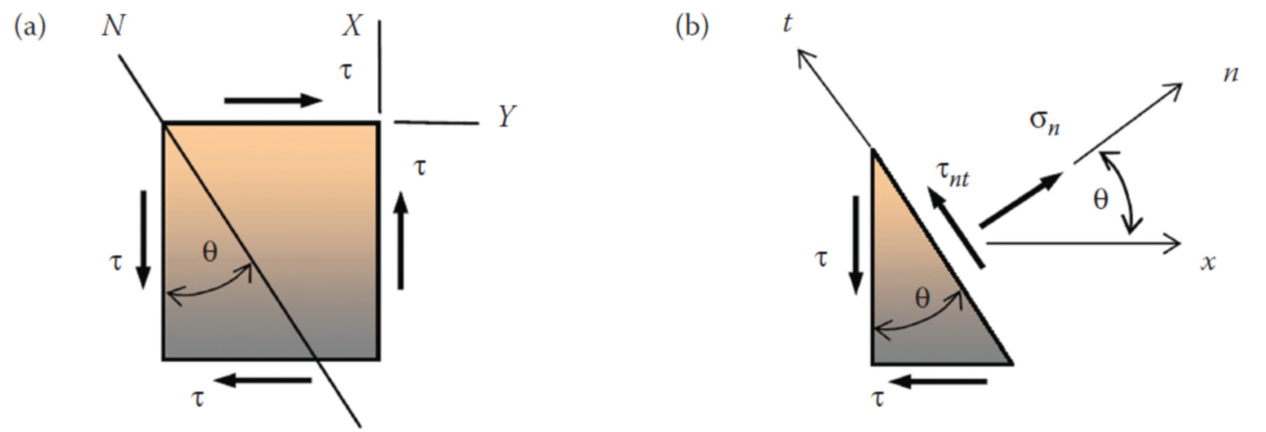
- State of pure shear in a shaft subjected to a torque Q





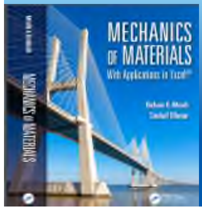
Stresses on Inclined Planes

- From a state of pure shear to the determination of normal and shearing stresses on an inclined plane



$$\sigma_n = 2\tau\sin\theta\cos\theta = \tau\sin 2\theta$$

$$\tau_{nt} = \tau(\cos^2\theta - \sin^2\theta) = \tau\cos 2\theta$$



Examples