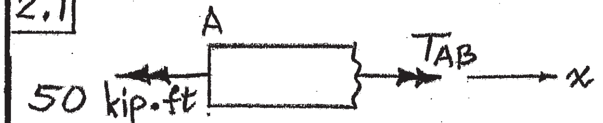
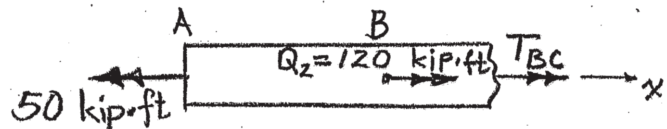
2.1



$\Sigma T_x = 0: \; T_{AB} - 50 = 0$

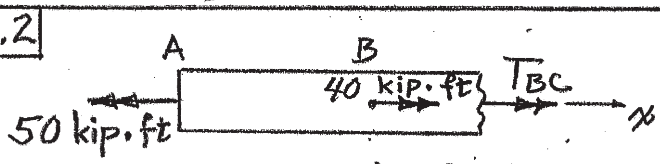
$T_{AB} = 50$ kip·ft ANS.



$\Sigma T_x = 0: \; T_{BC} + 120 - 50 = 0$

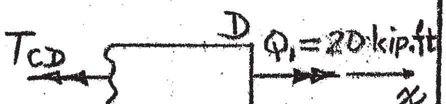
$T_{BC} = -70$ kip·ft ANS.

2.2



$\Sigma T_x = 0: \; T_{BC} + 40 - 50 = 0$

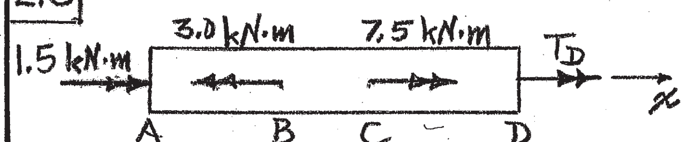
$T_{BC} = 10$ kip·ft ANS.



$\Sigma T_x = 0: \; 20 - T_{CD} = 0$

$T_{CD} = 20$ kip·ft ANS.

2.3

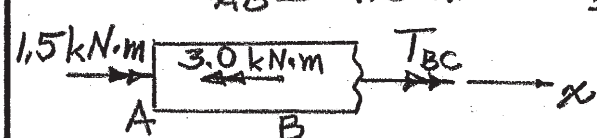


$\Sigma T_x = 0: \; T_D + 7.5 - 3.0 + 1.5 = 0$

$T_D = -6.0$ kN·m ANS.

$\Sigma T_x = 0: \; T_{AB} + 1.5 = 0$

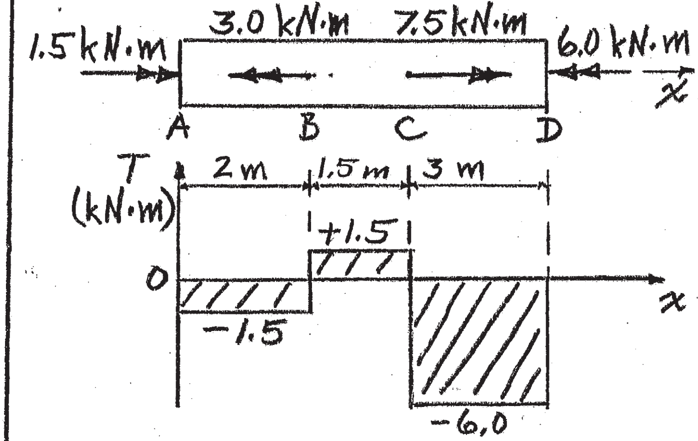
$T_{AB} = -1.5$ kN·m ANS.



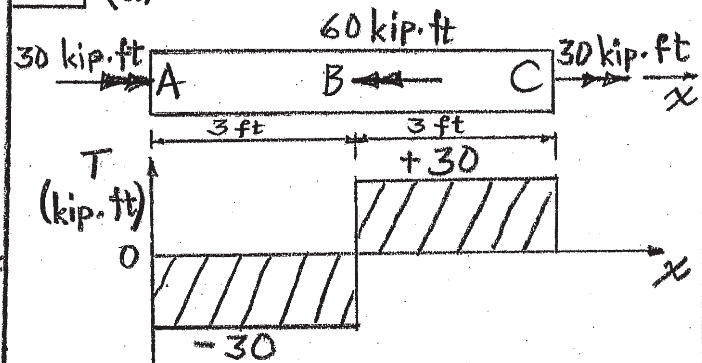
$\Sigma T_x = 0: \; T_{BC} - 3.0 + 1.5 = 0$

$T_{BC} = 1.5$ kN·m ANS.

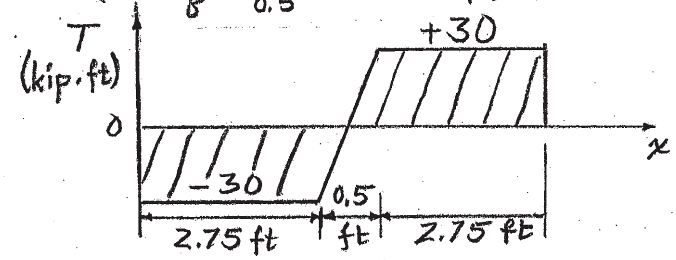
2.4



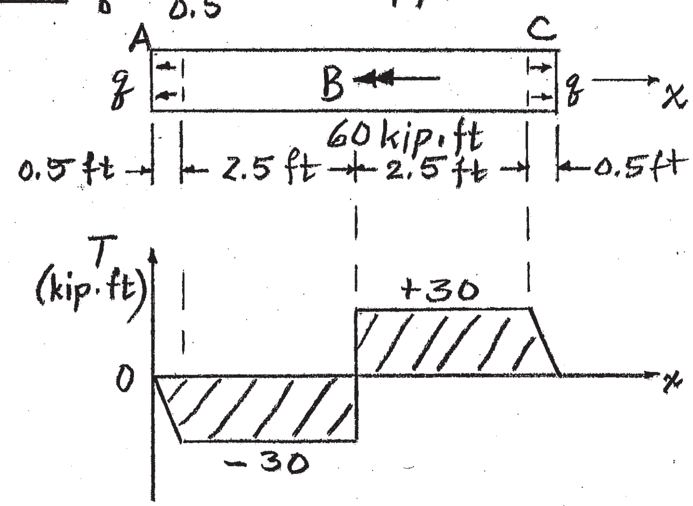
2.5 (a)



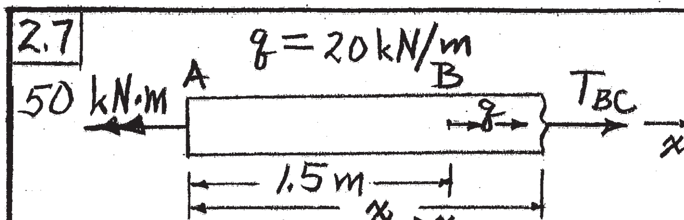
(b) $q = \frac{60}{0.5} = 120$ kip/ft



2.6 $q = \frac{30}{0.5} = 60$ kip/ft



2.7

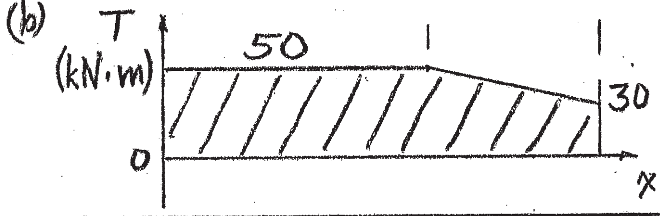


(a) $\sum T_x = 0: T_{BC} + \int_{1.5}^{x} 20\,dx - 50 = 0$

$T_{BC} = 80 - 20x$ ANS.

$T_C = 80 - 20(2.5) = 30$ kN·m ANS.

(b)



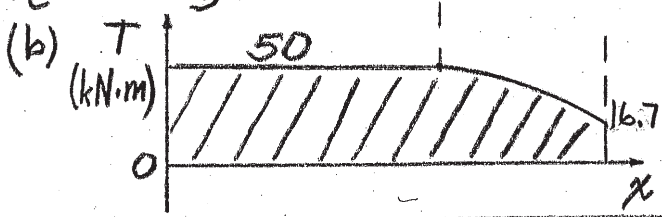
2.8 REFER TO THE F.B.D. IN PROB. 2.7. & MAKE $q = 100(x-1.5)^2$ kN/m.

(a) $\sum T_x = 0: T_{BC} + \int_{1.5}^{x} 100(x-1.5)^2 dx - 50 = 0$

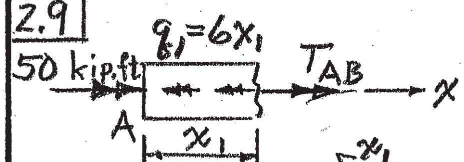
$T_{BC} = 50 - \frac{100}{3}(x-1.5)^3$ ANS.

$T_C = 50 - \frac{100}{3}(2.5-1.5)^3 = 16.7$ kN·m ANS.

(b)

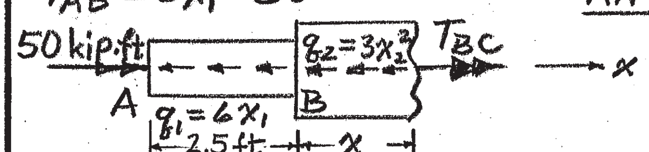


2.9



$\sum T_x = 0: T_{AB} - \int_0^{x_1} 6x_1\,dx_1 + 50 = 0$

$T_{AB} = 3x_1^2 - 50$ ANS.

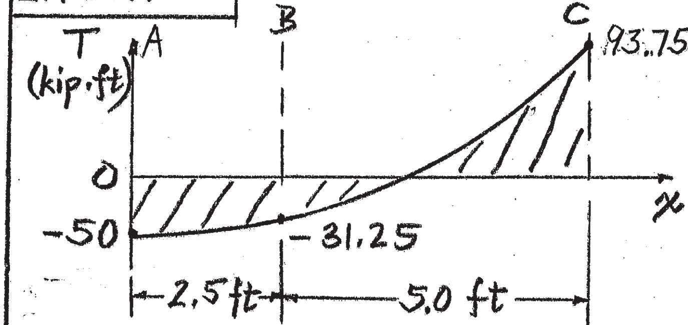


$T_B = 3(2.5^2) - 50 = -31.25$ kip.ft

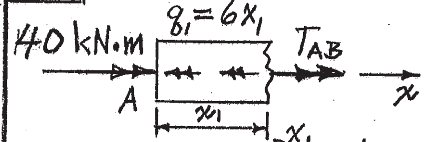
$\sum T_x = 0: T_{BC} - \int_0^{x_2} 3x_2^2\,dx_2 + 31.25 = 0$

$T_{BC} = x_2^3 - 31.25$ ANS.

2.9 CONT'D

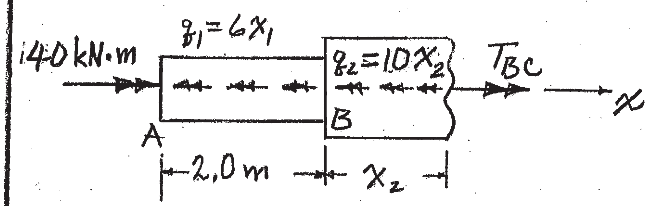


2.10



$\sum T_x = 0: T_{AB} - \int_0^{x_1} 6x_1^2\,dx_1 + 40 = 0$

$T_{AB} = 2x_1^3 - 40$ ANS.



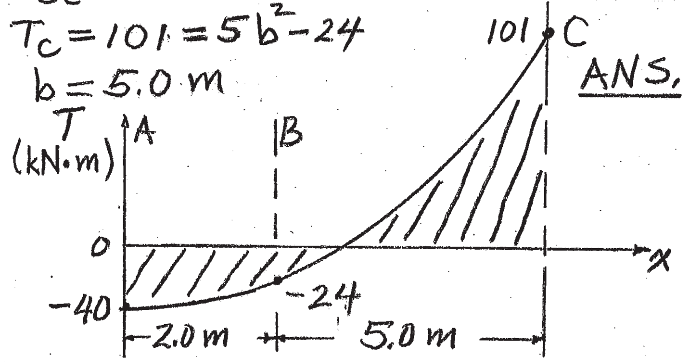
$T_B = 2(2.0^3) - 40 = -24.0$ kN·m

$\sum T_x = 0: T_{BC} - \int_0^{x_2} 10x_2\,dx_2 + 24.0 = 0$

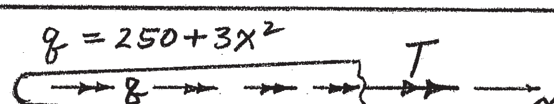
$T_{BC} = 5x_2^2 - 24$ ANS.

$T_C = 101 = 5b^2 - 24$

$b = 5.0$ m ANS.

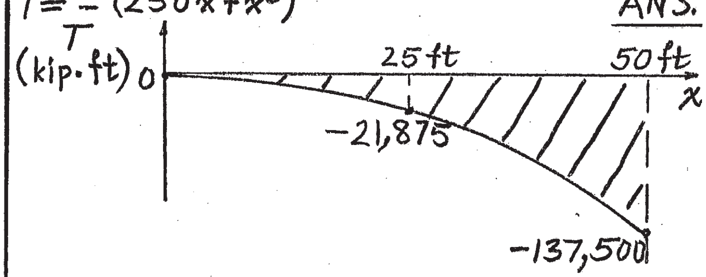


2.11



$\sum T_x = 0: T + \int_0^{x} (250 + 3x^2)dx = 0$

$T = -(250x + x^3)$ ANS.



2.12 REFER TO F.B.D. IN PROB. 2.11

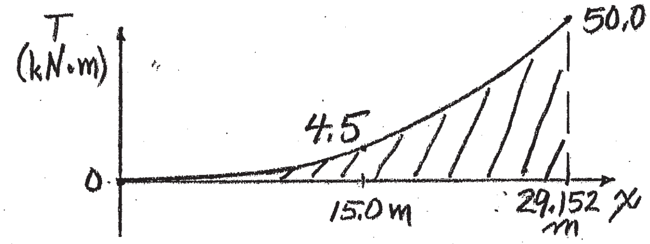
$q = 1.2 - 0.2x$

$\sum T_x = 0: T + \int_0^x (1.2 - 0.2x)dx = 0$

$T = 0.1x^2 - 1.2x$ ANS.

$T_B = 50.0 = 0.1L^2 - 1.2L$

$L^2 - 12L - 500 = 0;\ L = 29.152\text{ m}$ ANS.



2.13

(a) $G = \frac{TL}{J\theta} = \frac{24.85\times10^3(0.25)}{(\frac{\pi}{32})(0.1^4 - 0.092^4)(1.4\pi/180)}$

$G = 91.3\text{ GPa}$ ANS.

$\tau_U = \frac{T_U R}{J} = \frac{46.25\times10^3(0.05)}{(\pi/32)(0.1^4 - 0.0924^4)}$

$\tau_U = 830.6\text{ MPa}$ ANS.

(b) $\tau_p = \frac{TR}{J} = \frac{24.85\times10^3(0.05)}{(\pi/32)(0.1^4 - 0.0924^4)}$

$\tau_p = 446.3\text{ MPa}$ ANS.

$\gamma_p = \frac{\tau_p}{G} = \frac{446.3\times10^6}{91.3\times10^9}$

$\gamma_p = 4.89\times10^{-3}$ ANS.

2.14

(a) $G = \frac{TL}{J\theta} = \frac{62.8\times10^3(6.0)}{(\frac{\pi}{32})(2.0^4)(1.14\pi/180)}$

$G = 12.056\times10^6\text{ psi}$ ANS.

(b) $u = \frac{1}{2}\tau_p\gamma_p = \frac{1}{2}\left(\frac{TR}{J}\right)\left(\frac{R\theta}{L}\right)$

$= \left(\frac{1}{2}\right)\left[\frac{62.8\times10^3(1.0^2)(1.14\pi/180)}{(\pi/32)(2.0^4)(6.0)}\right]$

$u = 66.3\text{ lb.in./in.}^3$ ANS.

2.15

(a) $G = \frac{TL}{J\theta} = \frac{220(10.0)}{(\frac{\pi}{32})(4.0^4 - 3.6^4)(1.4\pi/180)}$

$G = 10.4\times10^3\text{ ksi}$ ANS.

$\tau_U = \frac{T_U R}{J} = \frac{456(2.0)}{(\frac{\pi}{32})(4.0^4 - 3.6^4)}$

$\tau_U = 105.5\text{ ksi}$ ANS.

(b) $\tau_p = \frac{TR}{J} = \frac{220(2.0)}{(\frac{\pi}{32})(4.0^4 - 3.6^4)}$

$\tau_p = 50.9\text{ ksi}$

$\gamma_p = \frac{\tau_p}{G} = \frac{50.9}{10.4\times10^3} = 4.894\times10^{-3}$

$u = \frac{1}{2}\tau_p\gamma_p = (\frac{1}{2})(50.9)(4.894\times10^{-3})$

$u = 0.125\text{ kip.in./in.}$ ANS.

2.16

(a) $G = \frac{TL}{J\theta} = \frac{7.35\times10^3(0.150)}{(\frac{\pi}{32})(0.04^4)(1.2\pi/180)}$

$G = 209.4\text{ GPa}$ ANS.

(b) $u = \frac{1}{2}\tau_p\gamma_p = \frac{1}{2}\left(\frac{TR}{J}\right)\left(\frac{R\theta}{L}\right)$

$= \frac{1}{2}\left[\frac{7.35\times10^3(0.04^2)(1.2\pi/180)}{(\frac{\pi}{32})(0.04^4)(0.150)}\right]$

$u = 3266.7\text{ kN·m/m}^3$ ANS.

(c) $\tau_U = \frac{T_U R}{J} = \frac{18.25\times10^3(0.04)}{(\frac{\pi}{32})(0.04^4)}$

$\tau_U = 2904.6\text{ MPa}$ ANS.

2.17 $T_{AB} = 5\text{ kip.in.};\ T_{BC} = 10\text{ kip.in.}$

(a) $\tau_{MAX} = \frac{TR}{J} = \frac{10(1.0)}{(\frac{\pi}{32})(2.0^4 - 1.25^4)}$

$\tau_{MAX} = 7.513\text{ ksi}$ ANS.

(b) $\tau_{MAX} = \frac{TR}{J} = \frac{10(1.0)}{(\frac{\pi}{32})(2.0^4)}$

$\tau_{MAX} = 6.366\text{ ksi}$ ANS.

$\boxed{2.18}$ $T_{AB} = 5$ kip.in.; $T_{BC} = 10$ kip.in.

(a) $\theta_A = \sum \frac{TL}{JG} = \theta_{AB} + \theta_{BC}$

$$= \frac{5\times10^3(10)}{(\frac{\pi}{32})(1.5^4-0.75^4)(6\times10^6)} - \frac{10\times10^3(15)}{(\frac{\pi}{32})(1.5^4-0.75^4)(6\times10^6)}$$

$$= 17.885\times10^{-3} - 53.654\times10^{-3}$$

$\theta_A = -35.769\times10^{-3}$ rad. CCW AS VIEWED FROM A TO C. ANS.

(b) $\theta_A = \sum \frac{TL}{JG} = \theta_{AB} + \theta_{BC}$

$$= \frac{5\times10^3(10)}{(\frac{\pi}{32})(1.5^4)(6\times10^6)} - \frac{10\times10^3(15)}{(\frac{\pi}{32})(1.5^4)(6\times10^6)}$$

$$= 16.767\times10^{-3} - 50.301\times10^{-3}$$

$\theta_A = -33.534\times10^{-3}$ rad. CCW AS VIEWED FROM A TO C. ANS.

$\boxed{2.19}$ $T_{AB} = 5$ kN.m; $T_{BC} = 10$ kN.m

(a) $\tau_{AB} = \frac{TR}{J} = \frac{5\times10^3(0.03)}{(\pi/32)(0.06^4)}$

$= 117.9$ MPa

$\tau_{BC} = \frac{TR}{J} = \frac{10\times10^3(0.04)}{(\pi/32)(0.08^4)}$

$= 99.5$ MPa

$\tau_{MAX} = \tau_{AB} = 117.9$ MPa ANS.

(b) $\theta_{A/C} = \sum \frac{TL}{JG} = \theta_{A/B} + \theta_{B/C}$

$$= \frac{5\times10^3(0.30)}{(\frac{\pi}{32})(0.06^4)(40\times10^9)} - \frac{10\times10^3(0.50)}{(\frac{\pi}{32})(0.08^4)(25\times10^9)}$$

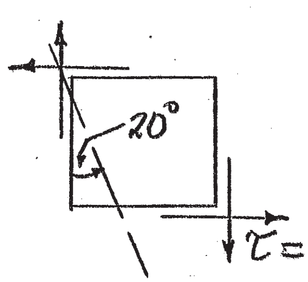
$$= 2.947\times10^{-2} - 4.974\times10^{-2}$$

$\theta_{A/C} = -2.027\times10^{-2}$ rad. CCW AS VIEWED FROM A TO C. ANS.

$\boxed{2.20}$ $T_{AB} = 5$ kN.m; $T_{BC} = 10$ kN.m

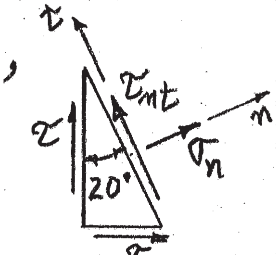
FROM PROB. 2.19, $\tau_{AB} = 117.9$ MPa

(a)



$\boxed{2.20 \text{ CONT'D}}$ (b)

BY EQS. (2.14) & (2.15),



$\sigma_n = \tau \sin 2\theta$

$= -117.9 \sin 40°$

$= -75.8$ MPa ANS.

$\tau_{nt} = \tau \cos 2\theta$

$= -117.9 \cos 40° = -90.3$ MPa ANS.

$\boxed{2.21}$ $\tau = \frac{T\rho}{J}$

$25 = \frac{50(D_1/2)}{(\pi/32)(D_1^4)}$; $D_1^3 = 10.186$

$D_1 = 2.168$ in.

AS DEDUCED FROM EQ. (2.14)

τ_{MAX} AT $\theta = 0° = (\sigma_n)_{MAX}$ AT $\theta = 45° = 40$ ksi.

$\therefore 40 = \frac{50(D_2/2)}{(\pi/32)(D_2^4)}$; $D_2^3 = 6.366$

$D_2 = 1.853$ in.

$D_{MIN} = D_1 = 2.168$ in. ANS.

$\boxed{2.22}$ EQs. (2.14) & (2.15) lead to the conclusion THAT τ_{MAX} AT $\theta = 0° = (\sigma_n)_{MAX}$ AT $\theta = 45°$ $= 100$ MPa.

$T_{MAX} = \frac{\tau J}{R} = \frac{100\times10^6(\pi/32)(0.05^4 - 0.03^4)}{0.025}$

$T_{MAX} = 2.136$ kN.m ANS.

$\boxed{2.23}$

$J = \frac{\pi}{32}(d_o^4 - d_i^4) = \frac{\pi}{32}(2.0^4 - d_i^4) = \frac{TL}{G\theta}$

$\frac{\pi}{32}(2.0^4 - d_i^4) = \frac{40(20)}{4\times10^3(10\pi/180)}$

$d_i^4 = 4.32780$; $d_i = 1.442$ in. ANS.

$\boxed{2.24}$ AB: $\tau = \frac{T\rho}{J}$

$25 = \frac{Q_1(1.0)}{(\pi/32)(2.0^4)}$; $Q_1 = 39.270$ kip.in.

$\therefore T_{BC} = 59.270$ kip.in.

BC: $\tau = \frac{TR}{J}$

$25 = \frac{59.270(D/2)}{(\pi/32)D^4}$; $D^3 = 12.07439$

$D = 2.294$ in. ANS.

2.25 $T_{AB} = 25$ kip·in; $T_{BC} = 35$ kip·in.

$$\theta_{B/C} = \left(\frac{TL}{JG}\right)_{BC} = \frac{35(10)}{(\pi/32)(3.5^4)(4\times10^3)}$$

$$= 5.93931\times10^{-3} \text{ rad.}$$

$$\theta_{A/B} = \left(\frac{TL}{JG}\right) = \frac{25(6)}{(\pi/32)D_{AB}^4(10\times10^3)}$$

$$= 5.93931\times10^{-3}$$

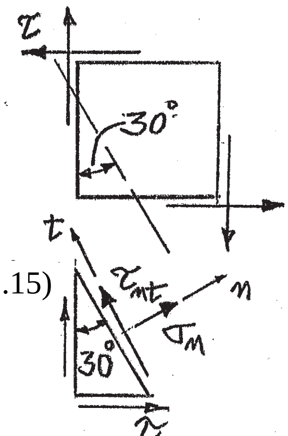
$D_{AB} = 2.252$ in. ANS.

2.26

$$\tau = \frac{TR}{J}$$

$$= \frac{20\times10^3(0.2)}{(\pi/32)(0.4^4 - 0.38^4)}$$

$\tau = 8.580$ MPa



BY EQS. EQs. (2.14) & (2.15)

$$\sigma_n = \tau \sin 2\theta$$

$$= -8.580 \sin 60^\circ$$

$$= -7.4 \text{ MPa}$$ ANS.

$$\tau_{mt} = \tau \cos 2\theta$$

$$= -8.580 \cos 60^\circ = -4.3 \text{ MPa}$$ ANS.

2.27 BY EQ. (2.15)

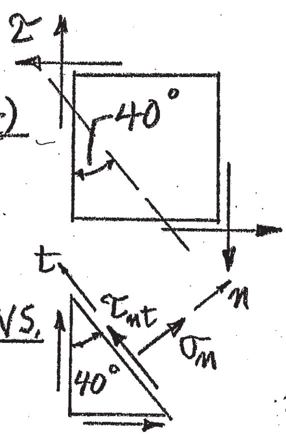
$$\tau = \frac{10}{\cos 2\theta} = 57.588$$

$$57.588 = \frac{50(12)(\frac{1}{2})(D_o + \frac{1}{2})}{(\pi/32)(D_o + \frac{1}{2})^4}$$

$$(D_o + \tfrac{1}{2})^3 = 53.06270$$

$D_o = 3.258$ in.

≈ 3.3 in. ANS.



BY EQ. (2.14)

$$\tau = \frac{\sigma_n}{\sin 2\theta}$$

$$= \frac{15}{\sin 80^\circ} = 15.231$$

$$15.231 = \frac{50(12)(\frac{1}{2})(D_o + \frac{1}{2})}{(\pi/32)(D_o + \frac{1}{2})^4}$$

$$(D_o + \tfrac{1}{2})^3 = 200.62865$$

$D_o = 5.354$ in.

$(D_o)_{MAX} = 5.354 \approx 5.4$ in.

2.28 $\tau = \dfrac{TR}{J}$; $T_{AB} = Q_1$

AB: $$100\times10^6 = \frac{Q_1(0.015/2)}{(\pi/32)(0.015^4)}$$

$Q_1 = 66.267 \approx 66.3$ N·m ANS.

BC: $T_{BC} = Q_2 - 66.267$

$$100\times10^6 = \frac{(Q_2 - 66.267)(0.03/2)}{(\pi/32)(0.03^4)}$$

$Q_2 = 596.411 \approx 596.4$ N·m ANS.

CD: $T_{CD} = 530.144 - Q_3$

$$100\times10^6 = \frac{(530.144 - Q_3)(0.045/2)}{(\pi/32)(0.045^4)}$$

$Q_3 = -1259.091 \approx -1259.1$ N·m ANS.

2.29 $T_{AB} = -5$ kip·in; $T_{BC} = 5$ kip·in.

$T_{CD} = -15$ kip·in.

$$\theta_A = \sum \frac{TL}{JG};\ -\frac{15\pi}{180} = \frac{-5(18)}{(\pi/32)D_1^4(10\times10^3)}$$

$$+ \frac{5(12)}{(\pi/32)(\frac{3}{2}D_1)^4(5\times10^3)} + \frac{-15(8)}{(\pi/32)(3D_1)^4(3\times10^3)}$$

$D_1 = 0.726$ in.; $D_2 = 1.088$ in.; AND

$D_3 = 2.177$ in. ANS.

2.30 BY INSPECTION, CONCLUDE THAT

$\tau_{BC} < \tau_{AB}$ AND $\tau_{CD} < \tau_{DE}$

∴ THE CHOICE IS BETWEEN SEGMENTS AB AND DE.

AB: $T_{AB} = Q_1 = 50$ kN·m

$$\tau_{AB} = \left(\frac{TR}{J}\right)_{AB} = \frac{50\times10^3(0.04)}{(\pi/32)(0.08^4 - 0.04^4)}$$

$$= 530.5 \text{ MPa}$$

DE: $T_{DE} = Q_1 + Q_2 - Q_3 = -50$ kN·m

$$|\tau_{DE}| = \left(\frac{TR}{J}\right)_{DE} = \frac{50\times10^3(0.06)}{(\pi/32)(0.12^4 - 0.04^4)}$$

$$= 149.2 \text{ MPa}$$

$\tau_{MAX} = \tau_{AB} = 530.5$ MPa ANS.

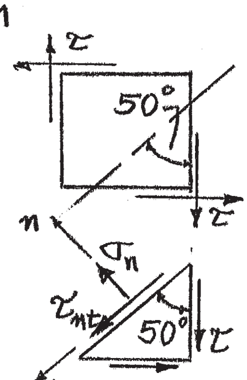
2.31 $T_{AB} = 50$ kN·m ; $T_{BC} = 75$ kN·m ;
$T_{CD} = T_{DE} = -50$ kN·m

$$\theta_A = \sum \frac{TL}{JG} = \theta_{A/B} + \theta_{B/C} + \theta_{C/E} + \theta_{E/F}$$
$$= \frac{50\times10^3(0.2)}{(\frac{\pi}{32})(0.08^4-0.04^4)(70\times10^9)} + \frac{75\times10^3(0.1)}{(\frac{\pi}{32})(0.08^4)(70\times10^9)}$$
$$- \frac{50\times10^3(0.15)}{(\frac{\pi}{32})(0.12^4)(70\times10^9)} - \frac{50\times10^3(0.25)}{(\frac{\pi}{32})(0.12^4-0.04^4)(70\times10^9)}$$
$= 0.05039$ rad CCW AS VIEWED FROM F TO A ANS.

2.32 $T_{AB} = 20$ kip·in.; $T_{BC} = 10$ kip·in.;
$T_{CD} = 5$ kip·in.; $T_{DE} = 0$.

$$\tau_{AB} = \left(\frac{TR}{J}\right)_{AB} = \frac{20(1.25)}{(\pi/32)(2.5^4)} = 6.5 \text{ ksi}$$
$$\tau_{BC} = \left(\frac{TR}{J}\right)_{BC} = \frac{10(1.0)}{(\pi/32)(2.0^4)} = 6.4 \text{ ksi}$$
$$\tau_{CD} = \left(\frac{TR}{J}\right)_{CD} = \frac{5(0.875)}{(\pi/32)(1.75^4)} = 4.8 \text{ ksi}$$
$$\tau_{DE} = 0$$
$\tau_{MAX} = \tau_{AB} = 6.5$ ksi ANS.

2.33 $T_{BC} = 1.5$ kN·m



$$\tau = \frac{1.5\times10^3(0.0325)}{(\pi/32)(0.065^4)}$$
$= 27.8$ MPa

BY EQS. (2.14) & (2.15),

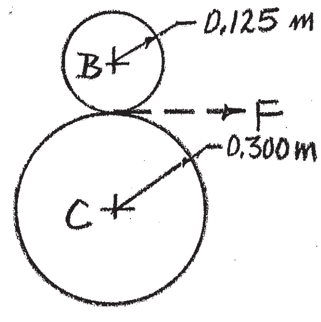
$\sigma_n = \tau \sin 2\theta$
$= -27.8 \sin(-100°)$
$= 27.4$ MPa ANS.

$\tau_{nt} = \tau \cos 2\theta$
$= -27.8 \cos(-100°)$
$= 4.8$ MPa ANS.

2.34 USE TORQUE STATED IN PROB. 2.32.

$$\theta_{E/A} = \theta_{E/D} + \theta_{D/C} + \theta_{C/B} + \theta_{B/A}$$
$$= \frac{20(3)(12)}{(\frac{\pi}{32})(2.5^4)(10\times10^3)} + \frac{10(3)(12)}{(\frac{\pi}{32})(2.0^4)(10\times10^3)} + \frac{5(3)(12)}{(\frac{\pi}{32})(1.75^4)(10\times10^3)} + 0$$
$= 0.06124$ rad CCW AS VIEWED FROM E TO A. ANS.

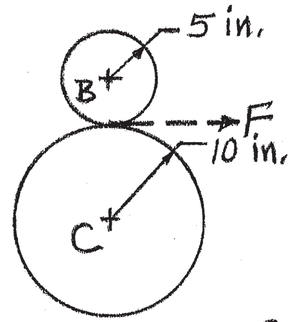
2.35



$Q = 10 = F(0.125)$
$F = 80$ kN
$T_{CD} = 80(0.3)$
$= 24$ kN·m

$$\tau_{MAX} = \tau_{CD} = \left(\frac{T\rho}{J}\right)_{CD} = \frac{24\times10^3(0.0375)}{(\pi/32)(0.075^4 - 0.045^4)}$$
$\tau_{MAX} = 332.9$ MPa ANS.

2.36



$Q = F(5)$
$F = \dfrac{Q}{5}$
$T_{CD} = F(10) = 2Q$

$$\theta_{C/D} = \left(\frac{TL}{JG}\right)_{CD} = \frac{2Q(12)}{(\pi/32)(1.75^4)(4\times10^3)} = 6.51627\times10^{-3}Q$$
$$\theta_B = 2\theta_{C/D} = 13.03254\times10^{-3}Q$$
$$\theta_{A/B} = \left(\frac{TL}{JG}\right)_{AB} = \frac{Q(12)}{(\pi/32)(1.75^4)(4\times10^3)} = 3.25814\times10^{-3}Q$$
$$\theta_{A/D} = \theta_B + \theta_{A/B} = 16.29068\times10^{-3}Q$$
$\therefore 16.29068\times10^{-3}Q = 5\pi/180$

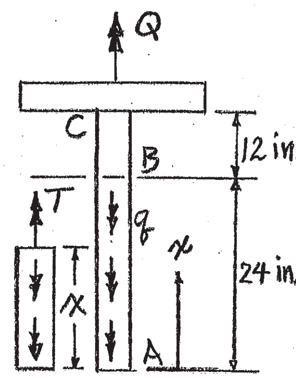
$Q = 5.4$ kip·in. ANS.

2.37 $\sum T_x = 0:$

$$T - \int_0^x \left(\tfrac{3}{4}\right) x\,dx = 0$$

$$T = \left(\tfrac{3}{8}\right) x^2$$

$$T_{MAX} = T_{x=24} = 216 \text{ lb.in.}$$



$$\tau_{MAX} = \frac{T_{MAX} R}{J}$$

$$= \frac{216(1)}{\left(\frac{\pi}{32}\right)(2^4 - 1.5^4)} = 201.2 \text{ psi}$$

ANY PLACE BETWEEN B & C ON OUTSIDE SURFACE ANS.

2.38 REFER TO THE F.B.D. IN PROB. 2.37.

(a) $\sum T_x = 0: 48 - q(0.7) = 0$

$$q = 68.571 \text{ N.m/m} \quad \text{ANS.}$$

(b) $\sum T_x = 0: T - \int_0^x (68.571)\,dx = 0$

$$T = 68.571 x$$

$$\theta_{A/C} = \int \frac{T\,dx}{JG} = \int_0^{0.7} \frac{68.571 x\,dx}{\left(\frac{\pi}{32}\right)(0.055^4 - 0.045^4)(25\times10^9)}$$

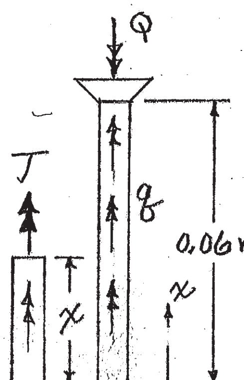
$$\frac{68.571}{1.2394\times10^{-4}} (x^2)\Big|_0^{0.7} = 27.108\times10^{-5} \text{ rad.} \quad \text{ANS.}$$

2.39

$\sum T_x = 0$

$$\int_0^x 3x^2\,dx - T = 0$$

$$T = x^3$$



$$T_{MAX} = T_{x=0.06} = Q$$

$$= 2.16\times10^{-4} \text{ kN.m}$$

$$\tau_{MAX} = \frac{T_{MAX}\,\rho}{J} = \frac{2.16\times10^{-4}\times10^3(0.003)}{(\pi/32)(0.006^4)}$$

$$= 5.1 \text{ MPa} \quad \text{ANS.}$$

2.40 REFER TO THE F.B.D. IN PROB. 2.39.

$\sum T_x = 0: T + qx = 0;\ T = -qx$

$$|T_{MAX}| = |T_{x=0.06}| = 0.06q = Q \quad \text{——— (1)}$$

$$\tau_{MAX} = \frac{T_{MAX} R}{J} = \frac{T_{MAX}(0.04)}{\left(\frac{\pi}{32}\right)(0.08^4)} = 10\times10^6$$

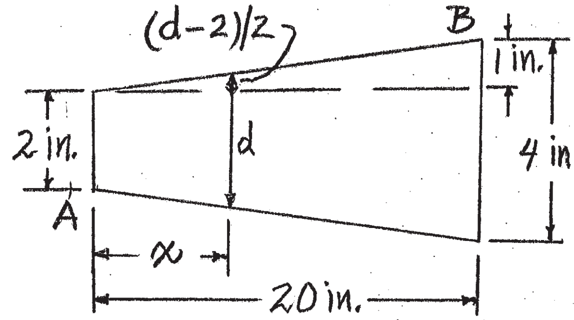
$$T_{MAX} = 1.005 \text{ kN.m} = Q \text{ —— (2) ANS.}$$

2.40 CONT'D

(2) INTO (1) $\Rightarrow$

$$q = \frac{1.005}{0.06} = 16.75 \text{ kN.m/m} \quad \text{ANS.}$$

2.41



USING SIMILAR TRIANGLES, WE HAVE

$$\frac{(d-2)/2}{x} = \frac{1}{20};\ d = \frac{20+x}{10}$$

(a)

$$\tau_{MAX} = \tau_A = \frac{TR}{J} = \frac{30(1)}{(\pi/32)(2^4)}$$

$$= 19.1 \text{ ksi} \quad \text{ANS.}$$

(b)

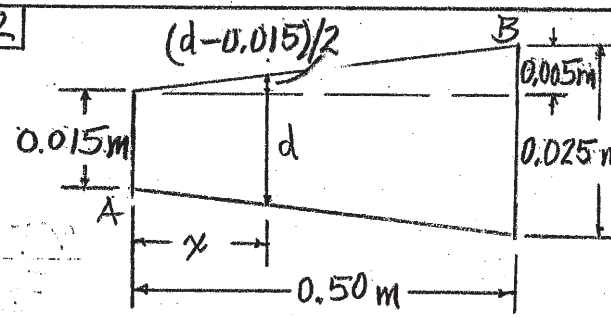
$$\theta_{A/B} = \frac{T}{G}\int_0^{20} \frac{dx}{J}$$

$$= \frac{30}{4\times10^6}\int_0^{20} \frac{dx}{\left(\frac{\pi}{32}\right)\left(\frac{20+x}{10}\right)^4}$$

$$= \frac{30(32)(10^4)}{\pi(4\times10^6)}\int_0^{20} (20+x)^{-4}\,dx$$

$$= 2.785\times10^{-5} \text{ rad} \quad \text{ANS.}$$

2.42



USING SIMILAR TRIANGLES, WE HAVE

$$\frac{(d-0.015)/2}{x} = \frac{0.005}{0.50};\ d = \frac{x+0.75}{50}$$

$$\tau = \frac{TR}{J};\ T = Q$$

$$70\times10^6 = \frac{Q(0.0075)}{(\pi/32)(0.015^4)}$$

$$Q = 46.387 \text{ N.m}$$

2.42 CONT'D

$\theta = \int \frac{T\,dx}{JG}$; $T = Q$

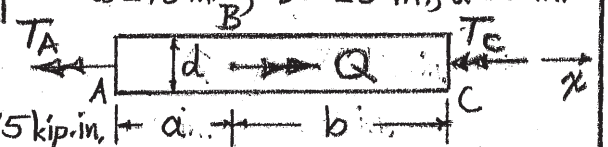
$\frac{45\pi}{180} = \frac{Q}{50\times10^9}\int_0^{0.5}\frac{dx}{(\frac{\pi}{32})(\frac{x+0.75}{50})^4}$

$= \frac{32(50^4)Q}{\pi(50\times10^9)}\int_0^{0.5}(x+0.75)^{-4}dx$

$Q = 88.5\ N\cdot m$

$\therefore Q_{MAX} = 46.4\ N\cdot m$ ANS.

2.43 $a = 10$ in.; $b = 20$ in.; $d = 3$ in.



$\sum T_x = 0: 75 - T_A - T_C = 0$ ——— (1)

$\theta_{B/A} = \theta_{B/C}: \frac{T_A(10)}{(\frac{\pi}{32})(3^4)(4\times10^3)} = \frac{T_C(20)}{(\frac{\pi}{32})(3^4)(4\times10^3)}$

$T_A = 2T_C$ ——— (2)

SOLVE (1) & (2) TO GET

$T_A = 50$ kip.in.; $T_C = 25$ kip.in.

(a) $\tau_{MAX} = (\tau)_{AB} = \frac{50(1.5)}{(\pi/32)(3^4)} = 9.4$ ksi ANS.

(b) $\theta_B = \theta_{B/C} = \frac{50(10)}{(\pi/32)(3^4)(4\times10^3)} = 0.01572$ rad ANS.

2.44 REFER TO THE F.B.D. IN PROB. 2.43.

$a = 0.5$ m; $b = 1.0$ m; $d = 0.07$ m

$\sum T_x = 0: Q - T_A - T_C = 0$ ——— (1)

$\theta_{B/A} = \theta_{B/C}: \frac{T_A(0.5)}{(\pi/32)(0.07^4)(25\times10^9)} = \frac{T_C(1.0)}{(\frac{\pi}{32})(0.07^4)(40\times10^9)}$

$T_A = (\frac{5}{4})T_C$ ——— (2)

$\therefore (\tau_{MAX})_A$ CONTROLS. THUS,

$150\times10^6 = \frac{T_A(0.035)}{(\pi/32)(0.07^4)}$

$T_A = 10.102\ kN\cdot m$

(2) $\Rightarrow T_C = (\frac{4}{5})(10.102) = 8.082\ kN\cdot m$

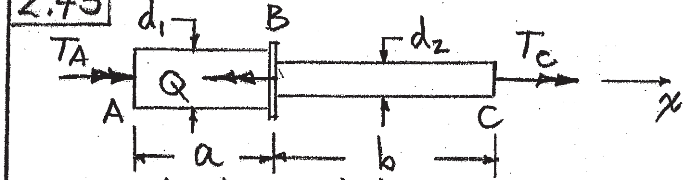
(1) $\Rightarrow Q = T_A + T_C$

$= 10.102 + 8.082$

$= 18.184$

$\approx 18.2\ kN\cdot m$ ANS.

2.45



$Q = 0.2(100) = 20\ kN\cdot m$

$a = 0.3$ m; $b = 0.5$ m; $d_1 = 0.050$ m

$d_2 = 0.030$ m.

$\sum T_x = 0: T_A + T_C - 20 = 0$ ——— (1)

$\theta_{B/A} = \theta_{B/C}: \frac{T_A(0.3)}{(\pi/32)(0.05^4)(40\times10^9)} = \frac{T_C(0.5)}{(\pi/32)(0.03^4)(40\times10^9)}$

$T_A = 12.860\,T_C$ ——— (2)

SOLVE (1) & (2) TO GET

$T_A = 18.557\ kN\cdot m$; $T_C = 1.443\ kN\cdot m$

$\tau_{MAX} = (\tau)_{AB} = \frac{18.557(0.025)}{(\pi/32)(0.05^4)}$

$= 756.1$ MPa ANS.

$\theta_{MAX} = \theta_{B/A} = \frac{18.557\times10^3(0.3)}{(\pi/32)(0.05^4)(40\times10^9)}$

$= 0.22682$ rad ANS.

2.46 REFER TO THE F.B.D. IN PROB. 2.45.

$Q = 8F$; $a = 12$ in.; $b = 20$ in.

$d_1 = 2.0$ in.; $d_2 = 1.0$ in.

$\sum T_x = 0: T_A + T_C - 8F = 0$ ——— (1)

$\theta_B = \theta_{B/A} = \frac{5\pi}{180} = \frac{T_A(12)}{(\pi/32)(2^4)(4\times10^6)}$

$T_A = 45.693$ kip.in. ——— (2)

$\theta_B = \theta_{B/C} = \frac{5\pi}{180} = \frac{T_C(20)}{(\pi/32)(1^4)(10\times10^6)}$

$T_C = 4.284$ kip.in. ——— (3)

(2) & (3) INTO (1) $\Rightarrow$

$F = 6.247 \approx 6.2$ kips ANS.

2.47 REFER TO THE F.B.D. IN PROB. 2.45.

$Q = 10F$; $a = 15$ in.; $b = 25$ in.

$d_1 = 3.0$ in.; $d_2 = 2.0$ in.

$\sum T_x = 0: T_A + T_C - 10F = 0$ ——— (1)

$\theta_{B/A} = \theta_{B/C}: \frac{T_A(15)}{(\pi/32)(3^4)(4\times10^3)} = \frac{T_C(25)}{(\pi/32)(2^4)(10\times10^3)}$

$T_A = 3.375\,T_C$ ——— (2)

$\tau_A = 10 = \frac{T_A(1.5)}{(\pi/32)(3^4)}$; $T_A = 53.014$ kip.in. ——— (3)

$\tau_C = 15 = \frac{T_C(1.0)}{(\pi/32)(2^4)}$; $T_C = 23.562$ kip.in. ——— (4)

2.47 CONT'D

ASSUME $T_A = 53.014$ kip.in. CONTROLS. THEN, BY EQ.(2),

$$T_C = \frac{53.014}{3.375} = 15.708 \text{ kip.in.} \quad \text{———(5)}$$

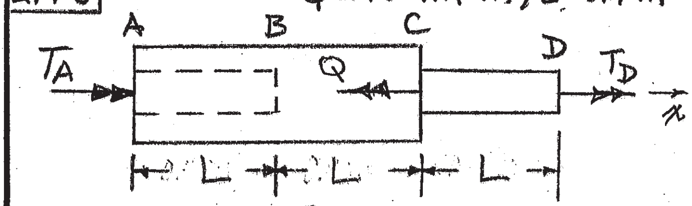
$$\tau_A = \frac{53.014(1.5)}{(\pi/32)(3^4)} = 10.0 \text{ ksi} = (\tau_{ALL})_A \quad \text{OK}$$

$$\tau_C = \frac{15.708(1.0)}{(\pi/32)(2^4)} = 10.0 \text{ ksi} < (\tau_{ALL})_S \quad \text{OK}$$

(3) & (5) INTO (1) $\Rightarrow$

$$F = 6.872 \approx 6.9 \text{ kips} \qquad \underline{\text{ANS.}}$$

2.48 $Q = 10$ kN.m; $L = 0.4$ m



$$\Sigma T_x = 0: \; T_A + T_D - 10 = 0 \quad \text{———(1)}$$

$$\theta_{C/A} = \theta_{C/D}: \; \frac{T_A(0.4)}{(\frac{\pi}{32})(0.075^4 - 0.04^4)(25\times10^9)} + \frac{T_A(0.4)}{(\frac{\pi}{32})(0.04^4)(25\times10^9)}$$

$$= \frac{T_D(0.4)}{(\frac{\pi}{32})(0.04^4)(25\times10^9)}$$

$$T_A = 5.919\, T_D \quad \text{———(2)}$$

SOLVE (1) & (2) TO GET

$$T_A = 8.555 \text{ kN.m}; \; T_D = 1.445 \text{ kN.m}$$

(a) $\tau_{MAX} = \tau_{CD} = \dfrac{1.445\times10^3(0.02)}{(\frac{\pi}{32})(0.04^4)}$

$$= 115.0 \text{ MPa} \qquad \underline{\text{ANS.}}$$

(b) $\theta_{MAX} = \theta_{C/D} = \dfrac{1.445\times10^3(0.4)}{(\frac{\pi}{32})(0.04^4)(25\times10^9)}$

$$= 0.0920 \text{ rad} \qquad \underline{\text{ANS.}}$$

2.49 REFER TO F.B.D. IN PROB. 2.48.
$L = 12$ in.

$$\Sigma T_x = 0: \; T_A + T_D - Q = 0 \quad \text{———(1)}$$

$$\theta_{C/A} = \theta_{C/D}: \; \frac{T_A(12)}{(\frac{\pi}{32})(4^4 - 2^4)(4\times10^3)} + \frac{T_A(12)}{(\frac{\pi}{32})(4^4)(4\times10^3)}$$

$$= \frac{T_D(12)}{(\frac{\pi}{32})(2^4)(6\times10^3)}$$

$$T_A = 5.161\, T_D \quad \text{———(2)}$$

$$\tau_{AB} = 30 = \frac{T_A(2)}{(\pi/32)(4^4 - 2^4)}$$

$$T_A = 353.429 \text{ kip.in.}$$

2.49 CONT'D

$$\tau_{CD} = 60 = \frac{T_D(1)}{(\pi/32)(2^4)}; \; T_D = 94.248 \text{ kip.in.}$$

ASSUME $T_A = 353.429$ kip.in. CONTROLS. THEN, BY EQ.(2),

$$T_D = \frac{353.429}{5.161} = 68.481 \text{ kip.in.}$$

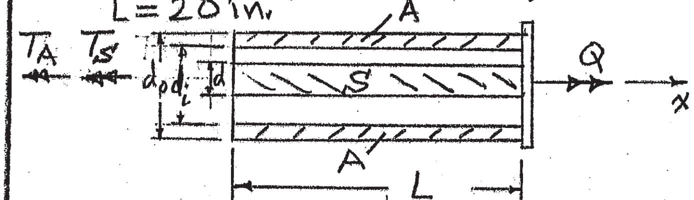
$$\tau_A = \frac{353.429(2)}{(\pi/32)(4^4 - 2^4)} = 30 \text{ ksi} = (\tau_{ALL})_A \quad \text{OK}$$

$$\tau_D = \frac{68.481(1)}{(\pi/32)(2^4)} = 43.6 \text{ ksi} < (\tau_{ALL})_T \quad \text{OK}$$

$\therefore$ (1) $\Rightarrow$ $Q = 353.429 + 68.481 = 421.91$

$$Q \approx 421.9 \text{ kip.in.} \qquad \underline{\text{ANS.}}$$

2.50 $Q = 50$ kip.in.; $d_o = 5$ in.; $d_i = 3$ in.; $d = 2$ in.
$L = 20$ in.



$$\Sigma T_x = 0: \; 50 - T_S - T_A = 0 \quad \text{———(1)}$$

$$\theta_S = \theta_A: \; \frac{T_S(20)}{(\pi/32)(2^4)(10\times10^3)} = \frac{T_A(20)}{(\pi/32)(5^4 - 3^4)(4\times10^3)}$$

$$T_A = 13.546\, T_S \quad \text{———(2)}$$

SOLVE (1) & (2) TO GET

$$T_A = 46.563 \text{ kip.in.}; \; T_S = 3.437 \text{ kip.in.}$$

(a) $\tau_A = \dfrac{46.563(2.5)}{(\frac{\pi}{32})(5^4 - 3^4)} \approx 2.2$ ksi $\qquad \underline{\text{ANS.}}$

$\tau_S = \dfrac{3.437(1)}{(\frac{\pi}{32})(2^4)} \approx 2.2$ ksi $\qquad \underline{\text{ANS.}}$

(b) $\theta_{PL} = \theta_S = \dfrac{3.437(20)}{(\frac{\pi}{32})(2^4)(10\times10^3)} = 0.00438$ rad $\qquad \underline{\text{ANS.}}$

2.51 REFER TO THE F.B.D. IN PROB. 2.50.

$d_o = 120$ mm; $d_i = 80$ mm; $d = 45$ mm; $L = 0.5$ m

$$\Sigma T_x = 0: \; Q - T_S - T_A = 0 \quad \text{———(1)}$$

$$\theta_S = \theta_A: \; \frac{T_S(0.5)}{(\frac{\pi}{32})(0.045^4)(75\times10^9)} = \frac{T_A(0.5)}{(\frac{\pi}{32})(0.12^4 - 0.08^4)(25\times10^9)}$$

$$T_A = 13.526\, T_S \quad \text{———(2)}$$

$$\tau_A = 60\times10^6 = \frac{T_A(0.06)}{(\frac{\pi}{32})(0.12^4 - 0.08^4)}; \; T_A = 16.336 \text{ kN.m}$$

$$\tau_S = 100\times10^6 = \frac{T_S(0.0225)}{(\frac{\pi}{32})(0.045^4)}; \; T_S = 1.789 \text{ kN.m}$$

2.51 CONT'D

ASSUME $T_A = 16.336$ kN·m CONTROLS. THEN, BY EQ. (2)

$$T_S = \frac{16.336}{13.526} = 1.208 \text{ kN·m}$$

$$\tau_A = \frac{16.336 \times 10^3 (0.06)}{(\frac{\pi}{32})(0.12^4 - 0.08^4)} = 60 \text{ MPa} = (\tau_{ALL})_A \quad \text{OK}$$

$$\tau_S = \frac{1.208 \times 10^3 (0.0225)}{(\frac{\pi}{32})(0.045^4)} = 67.5 \text{ MPa} < (\tau_{ALL})_S \quad \text{OK.}$$

BY EQ. (1) $Q = T_S + T_A = 1.208 + 16.336$

$Q \approx 17.5$ kN·m ANS.

2.52 AS IN PROB. 2.51

$$Q - T_S - T_A = 0 \quad \text{———— (1)}$$

$$\theta = \frac{TL}{JG}$$

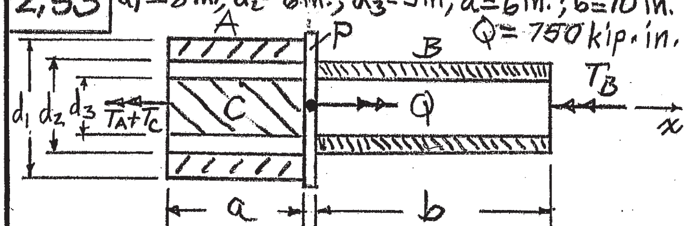
$$\frac{5\pi}{180} = \frac{T_S (0.5)}{(\frac{\pi}{32})(0.045^4)(75 \times 10^9)}; \; T_S = 5.269 \text{ kN·m}$$

$$\frac{5\pi}{180} = \frac{T_A (0.5)}{(\frac{\pi}{32})(0.12^4 - 0.08^4)(25 \times 10^9)}; \; T_A = 71.283 \text{ kN·m}$$

$(1) \Rightarrow Q = 5.269 + 71.283 = 76.552$

≈ 76.6 kN·m ANS.

2.53 $d_1 = 8$ in.; $d_2 = 6$ in.; $d_3 = 3$ in; $a = 6$ in.; $b = 10$ in. $Q = 750$ kip·in.



$$\Sigma T_x = 0: 750 - T_A - T_B - T_C = 0 \quad \text{———— (1)}$$

AT P, $\theta_A = \theta_B = \theta_C$

$$\frac{T_A (6)}{(\frac{\pi}{32})(8^4 - 5^4)(4 \times 10^6)} = \frac{T_B (10)}{(\frac{\pi}{32})(5^4 - 3^4)(6 \times 10^6)} = \frac{T_C (6)}{(\frac{\pi}{32})(3^4)(10 \times 10^6)}$$

$$T_B = 0.14105 T_A \quad \text{———— (2)}$$

$$T_C = 0.05834 T_A \quad \text{———— (3)}$$

$(1) \Rightarrow T_A + 0.14105 T_A + 0.05834 T_A = 750$

$T_A = 625.318$; $T_B = 88.201$; $T_C = 36.481$

(a) $$\tau_{MAX} = \tau_A = \left(\frac{TR}{J}\right)_A = \frac{625.318 (4)}{(\pi/32)(8^4 - 5^4)}$$

$= 7.340 \approx 7.3$ ksi ANS.

(b) $$\theta_P = \theta_C = \frac{36.481 \times 10^3 (6)}{(\pi/32)(3^4)(10 \times 10^6)}$$

$= 0.00273$ rad ANS.

2.54 REFER TO THE F.B.D. IN PROB. 2.53.

$a = 0.2$ m; $b = 0.3$ m

$d_1 = 250$ mm; $d_2 = 150$ mm; $d_3 = 70$ mm

$$\Sigma T_x = 0 \Rightarrow Q = T_A + T_B + T_C \quad \text{———— (1)}$$

AT P, $\theta_A = \theta_B = \theta_C$

$$\frac{T_A (0.2)}{(\frac{\pi}{32})(0.25^4 - 0.15^4)(25 \times 10^9)} = \frac{T_B (0.3)}{(\frac{\pi}{32})(0.15^4 - 0.07^4)(40 \times 10^9)}$$

$$= \frac{T_C (0.2)}{(\frac{\pi}{32})(0.07^4)(75 \times 10^9)}$$

$$T_B = 0.1513 T_A \quad \text{———— (2)}$$

$$T_C = 0.0212 T_A \quad \text{———— (3)}$$

ASSUME $\tau_A = 70$ MPa CONTROLS. THEN,

$$70 \times 10^6 = \frac{T_A (0.125)}{(\pi/32)(0.25^4 - 0.15^4)}$$

$T_A = 186.925$ kN·m

$(2) \Rightarrow T_B = 28.282$ kN·m

$(3) \Rightarrow T_C = 3.963$ kN·m

$$\tau_B = \frac{28.282 \times 10^3 (0.075)}{(\pi/32)(0.15^4 - 0.07^4)} = 44.8 \text{ MPa} < (\tau_{ALL})_B \quad \text{OK}$$

$$\tau_C = \frac{3.963 \times 10^3 (0.035)}{(\pi/32)(0.07^4)} = 58.8 \text{ MPa} < (\tau_{ALL})_C \quad \text{OK}$$

$\therefore (1) \Rightarrow Q = 186.925 + 28.282 + 3.963$

$= 219.170 \approx 219.2$ kN·m ANS.

2.55 REFER TO THE F.B.D. IN PROB. 2.53.

$a = 0.2$ m; $b = 0.3$ m

$d_1 = 250$ mm; $d_2 = 150$ mm; $d_3 = 70$ mm

$$\Sigma T_x = 0 \Rightarrow Q = T_A + T_B + T_C \quad \text{———— (1)}$$

$$\theta = \frac{TL}{JG}$$

$$\frac{0.5\pi}{180} = \frac{T_A (0.2)}{(\frac{\pi}{32})(0.25^4 - 0.15^4)(25 \times 10^9)}$$

$T_A = 364.113$ kN·m

$$\frac{0.5\pi}{180} = \frac{T_B (0.3)}{(\frac{\pi}{32})(0.15^4 - 0.07^4)(40 \times 10^9)}$$

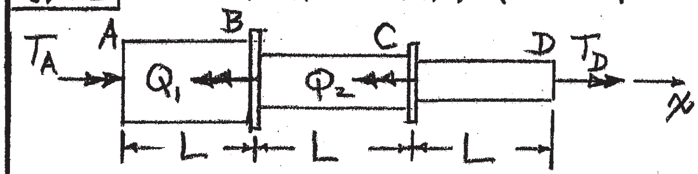
$T_B = 55.087$ kN·m

$$\frac{0.5\pi}{180} = \frac{T_C (0.2)}{(\pi/32)(0.07^4)(75 \times 10^9)}; \; T_C = 7.713 \text{ kN·m}$$

$(1) \Rightarrow Q = 364.113 + 55.087 + 7.713$

$= 426.913 \approx 426.9$ kN·m ANS.

2.56 $L = 1.5\text{ ft};\ Q_1 = 10\text{ kip}\cdot\text{ft};\ Q_2 = 15\text{ kip}\cdot\text{ft}$



$\Sigma T_x = 0:\ T_A + T_D - 25 = 0$ ——— (1)

ALSO, $T_{BC} = 10 - T_A = T_D - 15$ ——— (2)

$\theta_{B/A} = \theta_{B/C} + \theta_{C/D}$ $\quad (2) \Rightarrow T_D = 25 - T_A$ ——— (3)

$$\frac{T_A(1.5)}{(\frac{\pi}{32})(2.5^4)(4\times10^3)} = \frac{(10-T_A)(1.5)}{(\frac{\pi}{32})(1.5^4)(6\times10^3)} + \frac{(25-T_A)(1.5)}{(\frac{\pi}{32})(1.0^4)(10\times10^3)}$$

$T_A = 20.307;\ T_D = 4.693;\ T_{BC} = -10.307$

$$\tau_{MAX} = \tau_{CD} = \frac{4.693\times10^3(0.5)}{(\pi/32)(1.0^4)} = 23.901$$

$\tau_{MAX} = \tau_{CD} \approx 23.9\text{ ksi}$ ANS.

$$\theta_C = \theta_{C/D} = \frac{4.693\times10^3(1.5\times12)}{(\frac{\pi}{32})(1.0^4)(10\times10^6)} = 86.045\times10^{-3}$$

$\theta_C = \theta_{C/D} \approx 0.086\text{ rad}$ ANS.

2.57 REFER TO THE F.B.D. IN PROB. 2.56

$L = 1.0\text{ m};\ Q_1 = 50\text{ kN}\cdot\text{m};\ Q_2 = 75\text{ kN}\cdot\text{m}$

$\Sigma T_x = 0:\ T_A + T_D - 125 = 0$ ——— (1)

ALSO, $T_{BC} = 50 - T_A = T_D - 75$ ——— (2)

$(2) \Rightarrow T_D = 125 - T_A$ ——— (3)

$\theta_{B/A} = \theta_{B/C} + \theta_{C/D}$

$$\frac{T_A(1.0)}{(\frac{\pi}{32})d^4(30\times10^9)} = \frac{(50-T_A)(1.0)}{(\frac{\pi}{32})d^4(30\times10^9)} + \frac{(125-T_A)(1.0)}{(\frac{\pi}{32})d^4(30\times10^9)}$$

$T_A = 58.333;\ T_D = 66.667;\ T_{BC} = -8.333$

MINIMUM DIA. CONTROLLED BY SHEARING STRESS IN SEGMENT CD. THUS,

$$120\times10^6 = \frac{66.667\times10^3(d/2)}{(\pi/32)d^4};\ d = 0.14144\text{ m}$$

$d_{MIN} \approx 141.4\text{ mm}$ ANS.

2.58 $\Sigma T_{BA} = 0:$

$T_{BA} + 3F - 20 = 0$ —— (1)

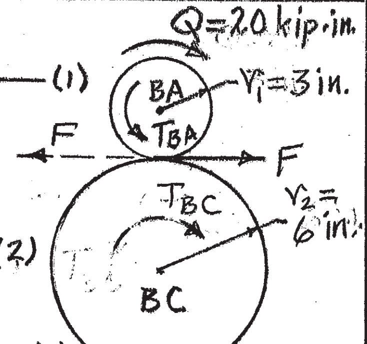
$\Sigma T_{BC} = 0:$

$6F - T_{BC} = 0$

$F = (\frac{1}{6})T_{BC}$ —— (2)

(2) INTO (1) $\Rightarrow$

$T_{BA} + (\frac{1}{2})T_{BC} = 20$ —— (3)



2.58 CONT'D

$3\theta_{B/A} = 6\theta_{B/C} \Rightarrow \theta_{B/A} = 2\theta_{B/C}$ ——— (4)

$$(4) \Rightarrow \frac{T_{BA}(15)}{(\frac{\pi}{32})(1.0^4)(10\times10^3)} = 2\left[\frac{T_{BC}(15)}{(\frac{\pi}{32})(2.0^4)(4\times10^3)}\right]$$

$T_{BC} = 3.2\,T_{BA}$ ——— (5)

SOLVE (3) & (5) TO GET

$T_{BC} = 24.614;\ T_{BA} = 7.692$

$$\tau_{AB} = \frac{7.692(0.5)}{(\pi/32)(1.0^4)} \approx 39.2\text{ ksi} \quad \text{ANS.}$$

$$\tau_{BC} = \frac{24.614(1.0)}{(\pi/32)(2.0^4)} \approx 15.7\text{ ksi} \quad \text{ANS.}$$

2.59 REFER TO THE F.B.D. IN PROB. 2.58

$\Sigma T_{BA} = 0:\ T_{BA} + 0.1F - Q = 0$ ——— (1)

$\Sigma T_{BC} = 0:\ 0.15F - T_{BC} = 0$

$F = (\frac{1}{0.15})T_{BC}$ ——— (2)

(2) INTO (1) $\Rightarrow$

$T_{BA} + (\frac{1}{1.5})T_{BC} = Q$ ——— (3)

$0.1\,\theta_{B/A} = 0.15\,\theta_{B/C};\ \theta_{B/A} = 1.5\,\theta_{B/C}$ ——— (4)

$$\theta_{B/A} = \frac{T_{BA}(0.75)}{(\pi/32)(0.075^4)(50\times10^9)} = 4.829\times10^{-6}\,T_{BA} \text{ ——— (5)}$$

$$\theta_{B/C} = \frac{T_{BC}(0.75)}{(\pi/32)(0.075^4)(50\times10^9)} = 4.829\times10^{-6}\,T_{BC} \text{ ——— (6)}$$

(5) & (6) INTO (4) $\Rightarrow T_{BA} = 1.5\,T_{BC}$ ——— (7)

(7) INTO (3) $\Rightarrow T_{BC} = 0.462Q$

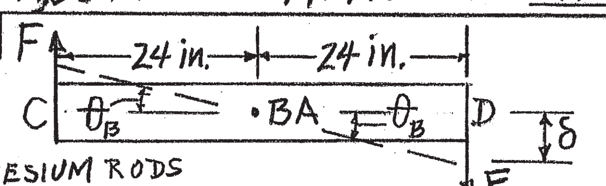
(7) $\Rightarrow T_{BA} = 0.692Q$

$\therefore$ SHEARING STRESS IN AB CONTROLS. THUS

$$120\times10^6 = \frac{0.692Q(0.0375)}{(\pi/32)(0.075^4)}$$

$Q = 14{,}364\text{ N}\cdot\text{m} = 14.4\text{ kN}\cdot\text{m}$ ANS.

2.60



MAGNESIUM RODS

$\delta = \alpha\Delta T L - FL/AE$

$= 14.5\times10^{-6}(150)(18) - F(18)/(1)(6.5\times10^6)$

$\delta = (39{,}150 - 2.769F)\times10^{-6}$

$\theta_B = \delta/24 = (1{,}631.25 - 0.11538F)\times10^{-6}$ ——— (1)

2.60 CONT'D

ALSO, $\theta_B = \left(\frac{TL}{JG}\right)_{BA} = \frac{T_{BA}(36)}{(\frac{\pi}{32})(4.0^4-3.5^4)(6\times10^6)}$

$\theta_B = 0.5769 T_{BA} \times 10^{-6}$ —— (2)

$\Sigma T_{BA}=0: T_{BA}-48F=0; F=\frac{T_{BA}}{48}$ —— (3)

(3) INTO (1) $\Rightarrow \theta_B = (1631.25-0.002400T_{BA})\times10^{-6}$ —— (4)

$(2)=(4) \Rightarrow$

$0.5769T_{BA}\times10^{-6} = (1631.25-0.00240T_{BA})\times10^{-6}$

$T_{BA} = 2815.89$ lb.in. $= 2.816$ kip.in.

$\tau_{BA} = \frac{2.816(2)}{(\frac{\pi}{32})(4.0^4-3.5^4)} = 0.542$ ksi   ANS.

$(2) \Rightarrow \theta_B = 0.5769(2815.89)\times10^{-6}$

$\theta_B = 0.00162$ rad   ANS.

2.61 REFER TO THE F.B.D. IN PROB. 2.60.

$\Sigma T_{BA}=0: T_{BA}-48F=0; F=\frac{T_{BA}}{48}$ —— (1)

ASSUME $\tau = 7$ ksi CONTROLS

$\tau = 7 = \frac{T_{BA}(1.5)}{(\frac{\pi}{32})(3.0^4-2.5^4)}; T_{BA}=19.214$ kip.in

$(1) \Rightarrow F = \frac{19.214}{48} = 0.400$ kip

$\sigma_{ROD} = \frac{F}{A} = \frac{0.400}{(\frac{\pi}{4})(0.5^2)} = 2.04 < (\sigma_{ALL})$ OK

∴ ASSUMPTION IS CORRECT.

$\delta = \alpha\Delta TL - FL/AE$

$= 13.0\times10^{-6}\Delta T(15) - \frac{0.400\times10^3(15)}{(\frac{\pi}{4})(0.5^2)(10\times10^6)}$

$\delta = (195\Delta T - 3055.775)\times10^{-6}$

$\theta_B = \frac{\delta}{24} = (8.125\Delta T - 127.324)\times10^{-6}$

ALSO, $\theta_B = \left(\frac{TL}{JG}\right)_{BA} = \frac{19.214\times10^3(15)}{(\frac{\pi}{32})(3.0^4-2.5^4)(4\times10^6)}$

$\theta_B = 17.5\times10^{-3}$ rad

$\therefore 17.5\times10^{-3} = (8.125\Delta T - 127.324)\times10^{-6}$

$\Delta T_{MAX} = 2.170\,°F$ ANS.

2.62 $P = T\omega;\ 120 = T\left(\frac{900\times2\pi}{60}\right)$

$T = 1.273$ kN·m

$\tau_{MAX} = \frac{Tr}{J} = \frac{1.273\times10^3(0.015)}{(\pi/32)(0.03^4)}$

$= 240.1$ MPa   ANS.

$\theta = \frac{TL}{JG} = \frac{1.273\times10^3(1.0)}{(\pi/32)(0.03^4)(70\times10^9)}$

$= 0.229$ rad   ANS.

2.63 SHEARING STRESS LIMITATION

$\tau_{ALL} = \frac{Tr}{J};\ T = \frac{\tau_{ALL}J}{r} = \frac{8(\frac{\pi}{32})(2.0^4-1.5^4)}{1.0}$

$T = 8.590$ kip. in.

ANGLE OF TWIST LIMITATION

$\theta_{ALL} = \frac{TL}{JG};\ T = \frac{\theta_{ALL}JG}{L}$

$T = \frac{(4\pi/180)(\pi/32)(2.0^4-1.5^4)(4\times10^6)}{3\times12}$

$= 8.329$ kip. in.

∴ ALLOWABLE ANGLE OF TWIST CONTROLS.

$P = T\omega = 8.329\times10^3\left(\frac{600\times2\pi}{60}\right)$

$= 523.327\times10^3$ lb. in./s

$= 79.3$ hp   ANS.

2.64 $P = T\omega;\ T = \frac{P}{\omega} = \frac{30\times10^3\times60}{1200(2\pi)}$

$T = 238.732$ N.m

SHEARING STRESS LIMITATION

$\tau_{ALL} = \frac{TR}{J};\ \frac{J}{R} = \frac{T}{\tau_{ALL}}$

$\frac{(\pi/32)d^4}{d/2} = \frac{238.732}{50\times10^6};\ d = 0.02897$ m

ANGLE OF TWIST LIMITATION

$\theta_{ALL} = \frac{TL}{JG};\ J = \frac{TL}{\theta_{ALL}G}$

$\left(\frac{\pi}{32}\right)d^4 = \frac{238.732(0.50)}{(0.08)(75\times10^9)}$

$d = 0.0212$ m

$\therefore d_{MIN} = 0.02897$ m ≈ 29.0 mm   ANS.

2.65 $P = T\omega$; $T = \dfrac{P}{\omega} = \dfrac{20\times10^3\times60}{1000\times2\pi}$

$T = 190.986\ N\cdot m$

SHEARING STRESS LIMITATION

$\tau_{ALL} = \dfrac{TR}{J}$; $J = \dfrac{TR}{\tau_{ALL}}$

$\left(\dfrac{\pi}{32}\right)(0.05^4 - d_i^4) = \dfrac{190.986\,(0.025)}{65\times10^6}$

$d_i = 0.0484\ m = 48.4\ mm$

ANGLE OF TWIST LIMITION

$\theta_{ALL} = \dfrac{TL}{JG}$; $J = \dfrac{TL}{\theta_{ALL}G}$

$\left(\dfrac{\pi}{32}\right)(0.05^4 - d_i^4) = \dfrac{190.986(0.75)}{0.07(75\times10^9)}$

$d_i = 0.0494\ m = 49.4\ mm$

$\therefore (d_i)_{MAX} = 48.4\ mm$ ANS.

2.66 $P = T\omega$; $T = \dfrac{P}{\omega} = \dfrac{20(6600)60}{600\times2\pi}$

$T = 2100.845\ lb.in.$

$\tau_{ALL} = \dfrac{Tr}{J}$; $J = \dfrac{Tr}{\tau_{ALL}}$

$\left(\dfrac{\pi}{32}\right)d^4 = \dfrac{2100.845\,(d/2)}{15\times10^3}$

$d_{MIN} = 0.893\ in.$ ANS.

2.67 $P = T\omega$; $\omega = P/T$ ——(1)

SHEARING STRESS LIMITATION

$\tau_{ALL} = \dfrac{TR}{J}$; $T = \dfrac{\tau_{ALL}J}{R}$

$T = \dfrac{25\times10^6(\pi/32)(0.055^4)}{(0.055/2)}$

$T = 816.691\ N.m$

ANGLE OF TWIS LIMITATION

$\theta_{ALL} = \dfrac{TL}{JG}$; $T = \dfrac{\theta_{ALL}JG}{L}$

$T = \dfrac{(3\pi/180)(\pi/32)(0.055^4)(26\times10^9)}{(1)}$

$T = 1{,}222\ N.m$

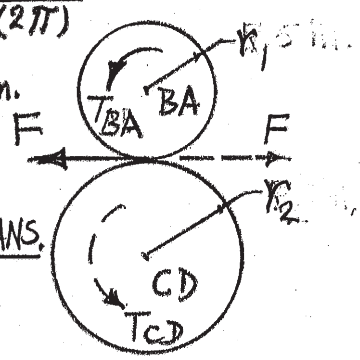
$\therefore$ SHEARING STRESS CONTROLS. THUS

(1) $\Rightarrow$ $\omega = \dfrac{150\times10^3}{816.691} = 183.668\ rad/s$

$= 1753.9\ rpm$ ANS.

2.68 $P = T\omega$; $r_1 = 2.5\ in$; $r_2 = 4.0\ in.$

$T_{BA} = \dfrac{P}{\omega} = \dfrac{20(6600)(60)}{300(2\pi)}$



$T_{BA} = 4201.690\ lb.in.$

$(\tau_{MAX})_{BA} = \dfrac{4201.690(0.75)}{\left(\frac{\pi}{32}\right)(1.5^4)}$

$(\tau_{MAX})_{BA} = 6.3\ ksi$ ANS.

$\theta_{BA} = \dfrac{4201.690(4\times12)}{\left(\frac{\pi}{32}\right)(1.5^4)(4\times10^6)}$

$\theta_{BA} = 0.101\ rad$ ANS.

$F = \dfrac{T_{BA}}{r_1} = \dfrac{4201.690}{2.5} = 1680.676\ lb$

$\therefore \dfrac{T_{CD}}{r_2} = 1680.676$; $T_{CD} = 1680.676(4.0)$

$T_{CD} = 6722.704\ lb.in.$

$(\tau_{MAX})_{CD} = \dfrac{6722.704(0.75)}{\left(\frac{\pi}{32}\right)(1.5^4)} = 10.1\ ksi$ ANS.

$\theta_{CD} = \dfrac{6722.704(3\times12)}{\left(\frac{\pi}{32}\right)(1.5^4)(4\times10^6)} = 0.12174\ rad$ ANS.

2.69 REFER TO THE F.B.D. IN PROB. 2.68.

$P = T\omega$; $T_{BA} = \dfrac{P}{\omega} = \dfrac{10\times10^3\times60}{400(2\pi)}$

$T_{BA} = 2.387\times10^2\ N.m.$

$(\tau_{ALL})_{BA} = \left(\dfrac{Tr}{J}\right)_{BA}$; $\dfrac{J}{r} = \dfrac{T}{\tau_{ALL}} = \dfrac{2.387\times10^2}{50\times10^6}$

$\dfrac{J}{r} = 4.774\times10^{-6}$

$\left(\dfrac{\pi}{32}\right)\left(\dfrac{d^4}{d/2}\right) = 4.774\times10^{-6}$

$d_{BA} = 0.02897\ m \approx 29.0\ mm$ ANS.

$F = \dfrac{T_{BA}}{r_1} = \dfrac{2.387\times10^2}{0.04} = 5967.5\ N$

$\therefore \dfrac{T_{CD}}{r_2} = 5967.5$; $T_{CD} = 5967.5(0.075)$

$T_{CD} = 447.6\ N.m$

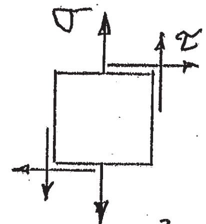
$(\tau_{ALL})_{CD} = \left(\dfrac{TR}{J}\right)_{CD}$; $\dfrac{J}{R} = \dfrac{T}{\tau_{ALL}} = \dfrac{447.6}{50\times10^6}$

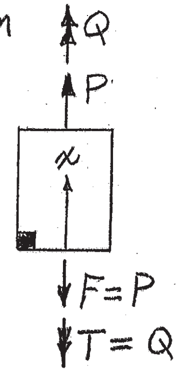
$\dfrac{J}{R} = 8.952\times10^{-6}$

$\left(\dfrac{\pi}{32}\right)\left(\dfrac{d^4}{d/2}\right) = 8.952\times10^{-6}$

$d_{CD} = 0.03572\ m \approx 35.7\ mm$ ANS.

2.70 $P = 650\,kN;\ Q = 20\,kN\cdot m$





$$\sigma = \frac{F}{A} = \frac{650\times10^3}{(\frac{\pi}{4})(0.12^2)}$$

$\sigma = 57.473 \approx 57.5\,MPa$ ANS.

$$\tau = \frac{TR}{J} = \frac{20\times10^3(0.06)}{(\frac{\pi}{32})(0.12^4)} = 58.946 \approx 58.9\,MPa$$ ANS.

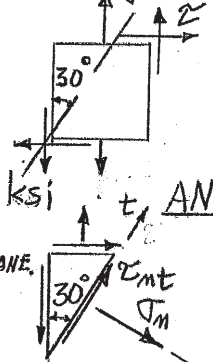
2.71 REFER TO THE F.B.D. IN PROB. 2.70.

$P = 150$ kips; $Q = 150$ kip·in.

(a) $\sigma = \frac{F}{A} = \frac{150}{(\frac{\pi}{4})(5^2)}$

$\sigma = 7.639 \approx 7.6$ ksi ANS.

$\tau = \frac{TR}{J} = \frac{150(2.5)}{(\frac{\pi}{32})(5^4)} = 6.112 \approx 6.1$ ksi ANS.



(b) LET A_n BE AREA OF INCINED PLANE.

$\Sigma F_n = 0$

$\sigma_n A + \tau(A_n\cos30)\sin30$
$+ \tau(A_n\sin30)\cos30 - \sigma(A_n\sin30)\sin30 = 0$

$\sigma_n = \sigma\sin^2 30 - 2\tau\sin30\cos30$

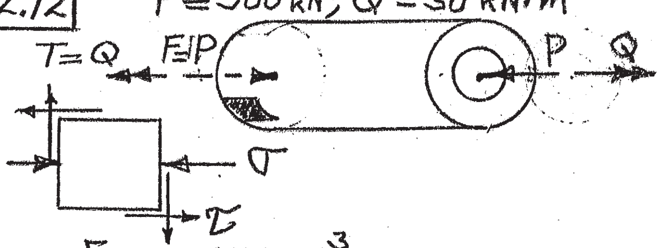
$\sigma_n = -3.383 \approx -3.4$ ksi ANS.

$\Sigma F_t = 0;\ \tau_{nt}A_n - \tau(A_n\cos30)\cos30$
$+ \tau(A_n\sin30)\sin30 + \sigma(A_n\sin30)\cos30 = 0$

$\tau_{nt} = \tau(\cos^2 30 - \sin^2 30) - \sigma\sin30\cos30$

$\tau_{nt} = -0.252 \approx -0.3$ ksi ANS.

2.72 $P = 500\,kN;\ Q = 30\,kN\cdot m$



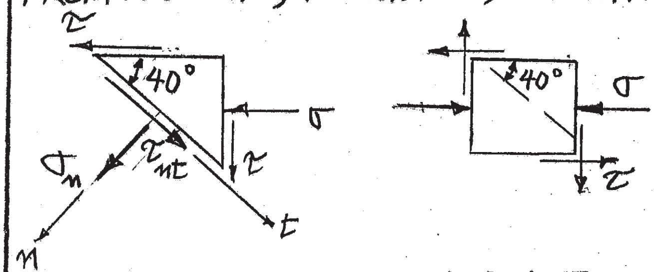
$$\sigma = \frac{F}{A} = \frac{500\times10^3}{(\pi/4)(0.15^2 - 0.006^2)} = 28.340$$

$\sigma \approx 28.3\,MPa$ ANS.

$$\tau = \frac{TR}{J} = \frac{30\times10^3(0.075)}{(\pi/32)(0.15^4 - 0.006^4)} = 45.271$$

$\tau \approx 45.3\,MPa$ ANS.

2.73 REFER TO THE F.B.D. IN PROB. 2.72. FROM PROB. 2.72; $\sigma = 28.340\,MP$; $\tau = 45.271\,MPa$.



LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0;\ \sigma_n A_n + \tau(A_n\cos40)\sin40$
$+ \tau(A_n\sin40)\cos40 + \sigma(A_n\sin40)\sin40 = 0$

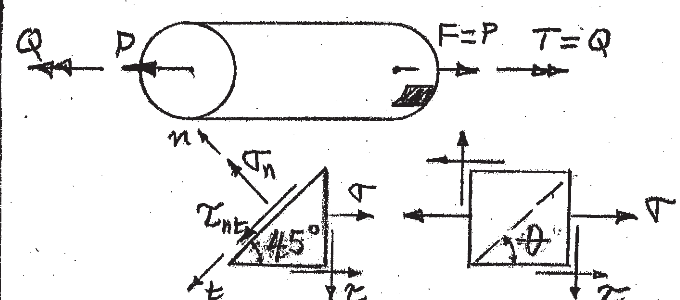
$\sigma_n = -2\tau\sin40\cos40 - \sigma\sin^2 40$
$= -56.293 \approx -56.3$ MPa ANS.

$\Sigma F_t = 0;\ \tau_{nt}A_n + \tau(A_n\cos40)\cos40$
$+ \tau(A_n\sin40)\sin40 - \sigma(A_n\sin40)\cos40 = 0$

$\tau_{nt} = \sigma\sin40\cos40 + \tau(\cos^2 40 - \sin^2 40)$

$= 21.816 \approx 21.8$ MPa ANS.

2.74 $P = 125$ kips; $Q = 150$ kip·in.; $\theta = 45°$



$\sigma = \frac{F}{A} = \frac{125}{(\pi/4)(3.5^2)} = 12.992$ ksi

$\tau = \frac{TR}{J} = \frac{150(1.75)}{(\pi/32)(3.5^4)} = 17.818$ ksi

LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0;\ \sigma_n A_n - \sigma(A_n\sin45)\sin45$
$- \tau(A_n\sin45)\cos45 - \tau(A_n\cos45)\sin45 = 0$

$\sigma_n = \sigma\sin^2 45 + 2\tau\sin45\cos45$
$= 24.314 \approx 24.3$ ksi ANS.

$\Sigma F_t = 0:\ \tau_{nt}A_n - \sigma(A_n\sin45)\cos45$
$+ \tau(A_n\sin45)\sin45 - \tau(A_n\cos45)\cos45 = 0$

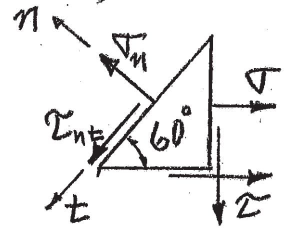
$\tau_{nt} = \sigma\sin45\cos45 + \tau(\cos^2 45 - \sin^2 45)$
$= 6.496 \approx 6.5$ ksi ANS.

2.75 REFER TO THE F.B.D. IN PROB. 2.74.

$P = 600\ kN;\ \varphi = 20\ kN\cdot m;\ \theta = 60°.$

$\sigma = \frac{F}{A} = \frac{600\times10^3}{\frac{\pi}{4}(0.1^2)}$

$= 76.394\ MPa$



$\tau = \frac{TR}{J} = \frac{20\times10^3(0.05)}{\frac{\pi}{32}(0.1^4)} = 101.859\ MPa$

LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0:\ \sigma_n A_n - \sigma(A_n \sin60)\sin60$

$-\tau(A_n\sin60)\cos60 - \tau(A_n\cos60)\sin60 = 0$

$\sigma_n = \sigma\sin^2 60 + 2\tau\sin60\cos60$

$= 145.508 \approx 145.5\ MPa$ ANS.

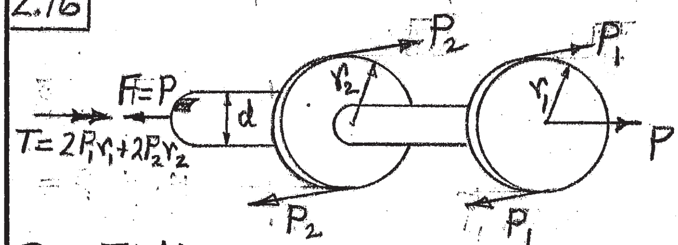
$\Sigma F_t = 0:\ \tau_{nt}A_n - \sigma(A_n\sin60)\cos60$

$+\tau(A_n\sin60)\sin60 - \tau(A_n\cos60)\cos60 = 0$

$\tau_{nt} = \sigma\sin60\cos60 + \tau(\cos^2 60 - \sin^2 60)$

$= -17.850 \approx -17.9\ MPa$ ANS.

2.76

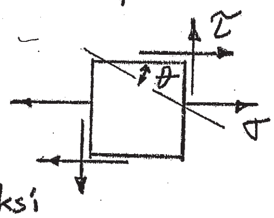


$P = 150\ kips;\ P_1 = 2\ kips;$

$P_2 = 3\ kips;\ r_1 = 0.75\ ft;$

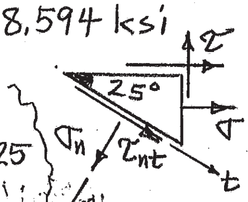
$r_2 = 1.0\ ft.;\ d = 4\ in.;$

$\theta = 25°;\ T = 9\ kip.ft$



$\sigma = \frac{F}{A} = \frac{150}{\frac{\pi}{4}(4^2)} = 11.937\ ksi$

$\tau = \frac{TR}{J} = \frac{9(12)(2)}{\frac{\pi}{32}(4^4)} = 8.594\ ksi$



LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0:\ \sigma_n A_n - \tau(A_n\cos25)\sin25$

$-\tau(A_n\sin25)\cos25$

$-\sigma(A_n\sin25)\sin25 = 0$

$\sigma_n = 2\tau\sin25\cos25 + \sigma\sin^2 25 =$

$= 8.715\ ksi \approx 8.7\ ksi$ ANS.

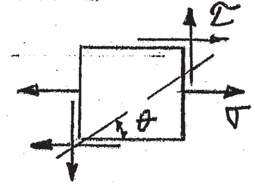
$\Sigma F_t = 0:\ \tau_{nt}A_n + \tau(A_n\cos25)\cos25$

$-\tau(A_n\sin25)\sin25 + \sigma(A_n\sin25\cos25) = 0$

$\tau_{nt} = \tau(\sin^2 25 - \cos^2 25) - \sigma\sin^2 25$

$= -7.656 \approx -7.7\ ksi$ ANS.

2.77 REFER TO F.B.D. IN PROB. 2.76.



$P = 550\ kN;\ P_1 = 8\ kN;$

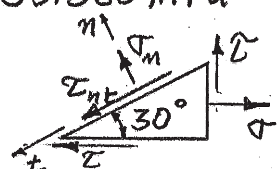
$P_2 = 12\ kN;\ r_1 = 0.375\ m;$

$r_2 = 0.25\ m;\ d = 120\ mm;$

$\theta = 30°;\ T = 12\ kN\cdot m.$

$\sigma = \frac{F}{A} = \frac{550\times10^3}{\frac{\pi}{4}(0.12^2)} = 48.631\ MPa$

$\tau = \frac{TR}{J} = \frac{12(10^3)(0.06)}{\frac{\pi}{32}(0.12^4)} = 35.368\ MPa$



LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0:\ \sigma_n A_n - \sigma(A_n\sin30)\sin30$

$+\tau(A_n\sin30)\cos30 + \tau(A_n\cos30)\sin30 = 0$

$\sigma_n = \sigma\sin^2 30 - 2\tau\sin30\cos30$

$= -18.472 \approx -18.5\ MPa$ ANS.

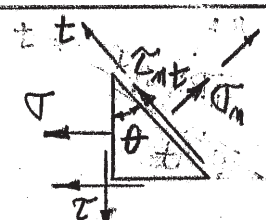
$\Sigma F_t = 0:\ \tau_{nt}A_n - \sigma(A_n\sin30)\cos30$

$-\tau(A_n\sin30)\sin30 + \tau(A_n\cos30)\cos30 = 0$

$\tau_{nt} = \sigma\sin30\cos30 + \tau(\sin^2 30 - \cos^2 30)$

$= 3.374 \approx 3.4\ MPa$ ANS.

2.78



$\sigma_n = 15\ ksi$

$\tau_{nt} = 10\ ksi$

$\theta = 35°$

LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0:\ 15A_n - \sigma(A_n\cos35)\cos35$

$-\tau(A_n\cos35)\sin35 - \tau(A_n\sin35)\cos35 = 0$

$0.6710\sigma + 0.9397\tau = 15$ ——— (1)

$\Sigma F_t = 0:\ 10A_n + \sigma(A_n\cos35)\sin35$

$-\tau(A_n\cos35)\cos35 + \tau(A_n\sin35)\sin35 = 0$

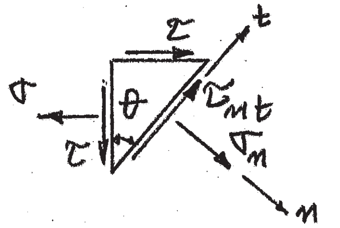
$-0.4698\sigma + 0.3420\tau = 10$ ——— (2)

SOLVE (1) & (2) TO GET

$\sigma = -6.357 \approx -6.4\ ksi$ ANS.

$\tau = 20.504 \approx 20.5\ ksi$ ANS.

2.79



$\sigma_n = 150$ MPa
$\tau_{nt} = 100$ MPa
$\theta = 40°$

LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0: 150A_n - \sigma(A_n \cos 40)\cos 40 + \tau(A_n \cos 40)\sin 40 + \tau(A_n \sin 40)\cos 40 = 0$

$0.5868\,\sigma - 0.9848\,\tau = 150$ ——(1)

$\Sigma F_t = 0: 100A_n - \sigma(A_n\cos 40)\sin 40 - \tau(A_n \cos 40)\cos 40 + \tau(A_n \sin 40)\sin 40 = 0$

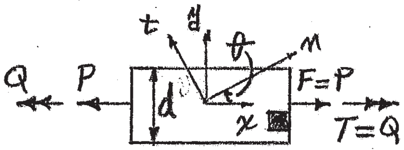
$0.4924\,\sigma + 0.1736\,\tau = 100$ ——(2)

SOLVE (1) & (2) TO GET

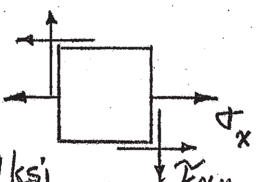
$\sigma = 212.207 \approx 212.2$ MPa ANS.

$\tau = -25.869 \approx -25.9$ MPa ANS.

2.80



$P = 60$ kips
$Q = 30$ kip.in.
$d = 2$ in.
$\theta = 30°$



$$\sigma_x = \frac{F}{A} = \frac{60}{\frac{\pi}{4}(2^2)} = 19.099 \text{ ksi}$$

$$\tau_{xy} = \frac{TR}{J} = \frac{30(1)}{\frac{\pi}{32}(2^4)} = 19.099 \text{ ksi}$$

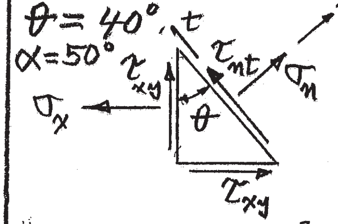
$$\epsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y) = \frac{1}{30\times10^3}(19.099 - 0)$$
$$= 0.6366\times10^{-3} = 636.6\,\mu \quad \text{ANS.}$$

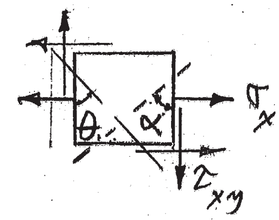
$$\epsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x) = \frac{1}{30\times10^3}(0 - 0.33\times19.099)$$
$$= -0.2101\times10^{-3} = 210.1\,\mu \quad \text{ANS.}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{\tau_{xy}}{E/2(1+\mu)} = \frac{19.099}{30\times10^3/2(1+0.33)}$$
$$= 1.6934\times10^{-3} = 1693.4\,\mu \quad \text{ANS.}$$

2.81 REFER TO THE F.B.D. IN PROB. 2.80.

$P = 150$ kN; $Q = 5$ kN.m; $d = 60$ mm;
$\theta = 40°$
$\alpha = 50°$





$$\sigma_x = \frac{F}{A} = \frac{150\times10^3}{\frac{\pi}{4}(0.06^2)} = 53.052 \text{ MPa}$$

$$\tau_{xy} = \frac{TR}{J} = \frac{5\times10^3(0.03)}{(\pi/32)(0.06^4)} = 117.893 \text{ MPa}$$

LET A_n BE AREA OF INCLINED PLANE.

$\Sigma F_n = 0: \sigma_n A_n - \sigma_x(A_n \cos^2 40) + \tau_{xy}(A_n \cos 40)\sin 40 + \tau_{xy}(A_n \sin 40)\cos 40 = 0$

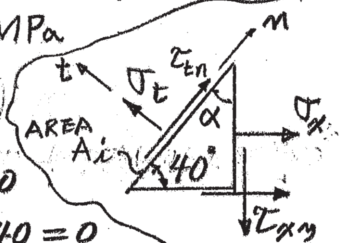
$\sigma_n = \sigma_x \cos^2 40 - 2\tau_{xy}\sin 40 \cos 40$

$\sigma_n = -84.970$ MPa

$\Sigma F_t = 0: \tau_{nt}A_n + \tau_{xy}(A_n \cos^2 40) - \tau_{xy}(A_n \sin^2 40) + \sigma_x(A_n \cos 40)\sin 40 = 0$

$\tau_{nt} = -\sigma_x \sin 40 \cos 40 + \tau_{xy}(\sin^2 40 - \cos^2 40)$

$\tau_{nt} = -46.595$ MPa



$\Sigma F_t = 0:$
$\sigma_t A_i - \sigma_x(A_i \sin^2 40) - \tau_{xy}(A_i \sin 40)\cos 40 - \tau_{xy}(A_i \cos 40)\sin 40 = 0$

$\sigma_t = \sigma_x \sin^2 40 + 2\tau_{xy}\sin 40 \cos 40$
$= 138.022$ MPa

(a) $$\epsilon_n = \frac{1}{E}(\sigma_n - \mu\sigma_t)$$
$$= \frac{1}{75\times10^9}[-84.970 - (0.28)(138.022)]\times10^6$$
$$= -1.648\times10^{-3} = -1648\,\mu \quad \text{ANS.}$$

$$\epsilon_t = \frac{1}{E}(\sigma_t - \mu\sigma_n)$$
$$= \frac{1}{75\times10^9}[138.022 - (0.28)(-84.970)]\times10^6$$
$$= 2.158\times10^{-3} = 2158\,\mu \quad \text{ANS.}$$

(b) $$\gamma_{nt} = \frac{\tau_{nt}}{G} = \frac{\tau_{nt}}{E/2(1+\mu)} = \frac{-46.595\times10^6}{75\times10^9/2(1+0.28)}$$
$$= -1.590\times10^{-3} = +1590\,\mu \quad \text{ANS.}$$

2.82 REFER TO THE F.B.D. IN PROB. 2.80.

$\epsilon_x = 400\mu$; $\epsilon_y = -120\mu$; $\gamma_{xy} = 800\mu$

$\sigma_x = \frac{E}{1-\mu^2}(\epsilon_x + \mu\epsilon_y)$

$= \left(\frac{10\times10^6}{1-0.3^2}\right)[400+0.3(-120)]\times10^{-6}$

$= 4{,}000$ psi

$\sigma_y = \frac{E}{1-\mu^2}(\epsilon_y + \mu\epsilon_x)$

$= \left(\frac{10\times10^6}{1-0.3^2}\right)[-120+0.3(400)]\times10^{-6}$

$= 0$

$P = F = \sigma_x A = 4{,}000\left(\frac{\pi}{4}\right)(2.5^2)$

$= 19{,}634.95$ lb ≈ 19.6 kips ANS.

$\tau_{xy} = \gamma_{xy} G = \gamma_{xy}\left(\frac{E}{2(1+\mu)}\right)$

$= 800\times10^{-6}\left(\frac{10\times10^6}{2(1+0.3)}\right) = 3{,}077$ psi

$Q = T = \frac{\tau_{xy} J}{R} = \frac{3{,}077(\pi/32)(2.5^4)}{1.25}$

$= 9440.1$ lb.in. ≈ 9.4 kip.in. ANS.

2.83 REFER TO THE F.B.D. IN PROB. 2.80.

$\epsilon_n = -336.3\mu$; $\epsilon_t = 858.6\mu$; $\gamma_{nt} = -1{,}939.8\mu$;

$\theta = 30°$

$\sigma_n = \frac{E}{1-\mu^2}(\epsilon_n + \mu\epsilon_t) = \frac{200\times10^9}{1-0.3^2}[-336.3+0.3(858.6)]\times10^{-6}$

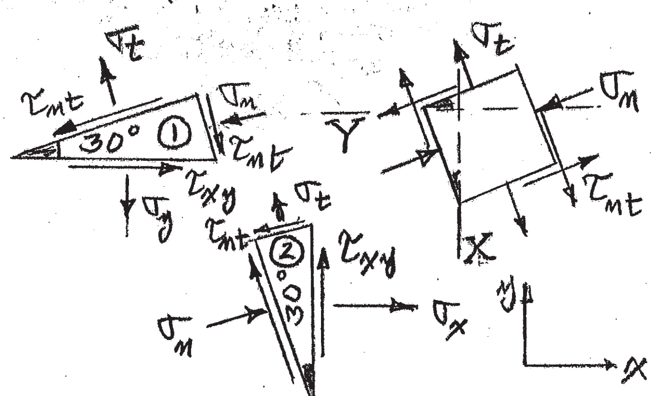
$= -17.301$ MPa

$\sigma_t = \frac{E}{1-\mu^2}(\epsilon_t + \mu\epsilon_n) = \frac{200\times10^9}{1-0.3^2}[858.6+0.3(-336.3)]\times10^{-6}$

$= 166.530$ MPa

$\tau_{nt} = G\gamma_{nt} = \left(\frac{E}{2(1+\mu)}\right)\gamma_{nt}$

$= \left(\frac{200\times10^9}{2(1+0.3)}\right)(-1{,}939.8)\times10^{-6} = -149.215$ MPa



2.83 CONT'D

F.B.D. ①: A IS AREA OF Y PLANE.

$\Sigma F_y = 0$

$\sigma_y A + \tau_{nt}(A\cos 30)\sin 30 - \sigma_t(A\cos^2 30)$

$+ \sigma_n(A\sin^2 30) + \tau_{nt}(A\cos 30)\sin 30 = 0$

$\sigma_y = \sigma_t\cos^2 30 - \sigma_n\sin^2 30 - 2\tau_{nt}\sin 30\cos 30$

$= 0.001 \approx 0$ OK

F.B.D. ②: A IS AREA OF X PLANE.

$\Sigma F_x = 0: \sigma_x A + \sigma_n(A\cos^2 30) - \tau_{nt}(A\cos 30)\sin 30$

$- \tau_{nt}(A\sin 30)\cos 30 - \sigma_t(A\sin^2 30) = 0$

$\sigma_x = 2\tau_{nt}\sin 30\cos 30 + \sigma_t\sin^2 30 - \sigma_n\cos^2 30$

$= -74.616$ MPa

$\Sigma F_y = 0: \tau_{xy} A + \sigma_t(A\sin 30)\cos 30 - \tau_{nt}(A\sin^2 30)$

$+ \sigma_n(A\cos 30)\sin 30 + \tau_{nt}(A\cos^2 30) = 0$

$\tau_{xy} = \tau_{nt}(\sin^2 30 - \cos^2 30) - \sigma_t\sin 30\cos 30$

$- \sigma_n\sin 30\cos 30$

$= -9.989$ MPa

$P = F = \sigma_x A = -74.616\left(\frac{\pi}{4}\right)(0.08^2)(10^6)$

$= -375.061 \approx -375.1$ kN ANS.

$Q = T = \frac{\tau_{xy} J}{R} = \frac{-9.989(10^6)\left(\frac{\pi}{32}\right)(0.08^4)}{0.04}$

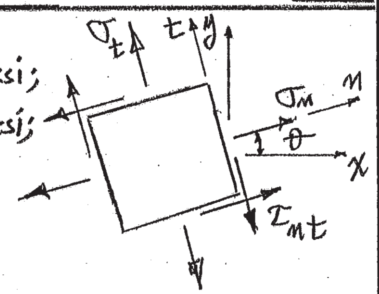
$= -1.004 \approx -1.0$ kN.m ANS.

2.84

$\sigma_n = 60$ ksi; $\sigma_t = -30$ ksi;

$\tau_{nt} = 20$ ksi; $E = 10\times10^3$ ksi;

$\mu = 0.3$; $\theta = 30°$.



(a)

$\epsilon_n = \frac{1}{E}(\sigma_n - \mu\sigma_t)$

$= \frac{1}{10\times10^3}[60-0.3(-30)] = 0.0069$

$= 6{,}900\mu$ ANS.

$\epsilon_t = \frac{1}{E}(\sigma_t - \mu\sigma_n) = \frac{1}{10\times10^3}[-30-0.3(60)]$

$= -0.0048 = -4{,}800\mu$ ANS.

(b)

$\gamma_{nt} = \frac{\tau_{nt}}{G} = \frac{\tau_{nt}}{E/2(1+\mu)} = \frac{20}{10\times10^3/2(1+0.3)}$

$= 0.0052 = 5{,}200\mu$ ANS.

2.85 REFER TO THE STRESS ELEMENT IN PROB. 2.84.

$\epsilon_n = 500\mu;\ \epsilon_t = 400\mu;\ \gamma_{nt} = 0;\ \theta = 30^\circ$

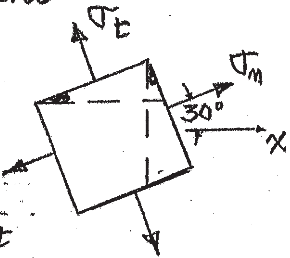
$$\sigma_n = \frac{E}{1-\mu^2}(\epsilon_n + \mu\epsilon_t) = \frac{75\times10^9}{1-0.33^2}[500+0.33(400)]\times10^{-6}$$

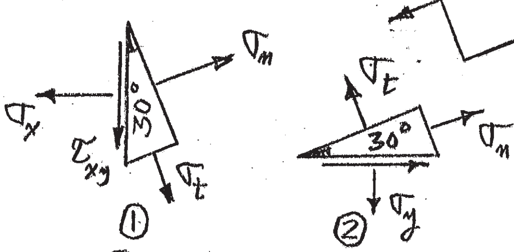
$= 53.193$ MPa

$$\sigma_t = \frac{E}{1-\mu^2}(\epsilon_t + \mu\epsilon_n) = \frac{75\times10^9}{1-0.33^2}[400+0.33(500)]\times10^{-6}$$

$= 47.554$ MPa

$\tau_{nt} = \gamma_{nt} G = 0$





F.B.D. ①:

$\Sigma F_x = 0:\ \sigma_x A_x - \sigma_n(A_x\cos^2 30) - \sigma_t(A_x \sin^2 30) = 0$

$\sigma_x = \sigma_n \cos^2 30 + \sigma_t \sin^2 30$

$= 51.783 \approx 51.8$ MPa ANS.

$\Sigma F_y = 0:\ \tau_{xy} A_x - \sigma_n(A_x \sin 30)\cos 30 + \sigma_t(A_x \sin 30)\cos 30 = 0$

$\tau_{xy} = (\sigma_n - \sigma_t)\sin 30 \cos 30$

$= 2.442 \approx 2.4$ MPa ANS

F.B.D. ②:

$\Sigma F_y = 0:\ \sigma_y A_y - \sigma_t(A_y \cos^2 30) - \sigma_n(A_y \sin^2 30) = 0$

$\sigma_y = \sigma_t \cos^2 30 + \sigma_n \sin^2 30$

$= 48.964 \approx 49.0$ MPa ANS.

2.86 REFER TO THE STRESS ELEMENT IN PROB. 2.84.

$\epsilon_x = 450\mu;\ \epsilon_y = 600\mu;\ \gamma_{xy} = -500\mu;\ \theta = 20^\circ$

$$\sigma_x = \frac{E}{1-\mu^2}(\epsilon_x + \mu\epsilon_y) = \frac{30\times10^3}{1-0.3^2}[450+0.3(600)]\times10^{-6}$$

$= 20.769$ ksi

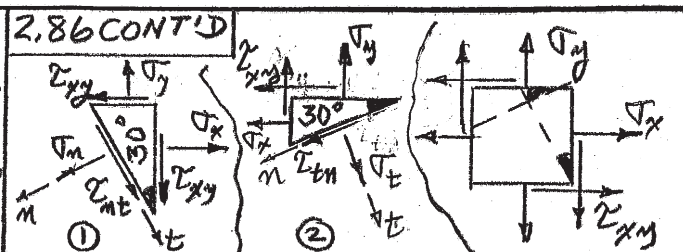
$$\sigma_y = \frac{E}{1-\mu^2}(\epsilon_y + \mu\epsilon_x) = \frac{30\times10^3}{1-0.3^2}[600+0.3(450)]\times10^{-6}$$

$= 24.231$ ksi

$$\tau_{xy} = \gamma_{xy} G = \gamma_{xy}\left(\frac{E}{2(1+\mu)}\right) = -500(10^{-6})\left(\frac{30\times10^3}{2(1+0.3)}\right)$$

$= -5.769$ ksi

2.86 CONT'D



LET A_n BE AREA OF INCLINED PLANE.

F.B.D. ①:

$\Sigma F_n = 0:\ \sigma_n A_n - \sigma_x(A_n \cos^2 20) + \tau_{xy}(A_n \cos 20)\sin 20 + \tau_{xy}(A_n \sin 20)\cos 20 - \sigma_y(A_n \sin^2 20) = 0$

$\sigma_n = \sigma_x \cos^2 20 + \sigma_y \sin^2 20 - 2\tau_{xy}\sin 20 \cos 20$

$= 24.882 \approx 24.9$ ksi ANS.

$\Sigma F_t = 0:\ \tau_{nt} A_n + \tau_{xy}(A_n \cos^2 20) + \sigma_x(A_n \cos 20)\sin 20 - \sigma_y(A_n \sin 20)\cos 20 - \tau_{xy}(A_n \sin^2 20) = 0$

$\tau_{nt} = (\sigma_y - \sigma_x)\sin 20 \cos 20 + \tau_{xy}(\sin^2 20 - \cos^2 20)$

$= 5.532 \approx 5.5$ ksi ANS.

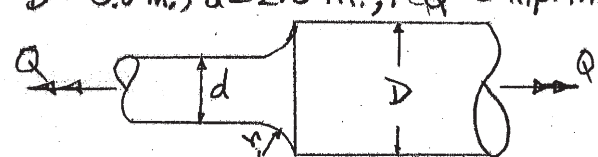
F.B.D. ②:

$\Sigma F_t = 0:\ \sigma_t A_n - \sigma_y(A_n \cos^2 20) - \tau_{xy}(A_n \cos 20)\sin 20 - \tau_{xy}(A_n \sin 20)\cos 20 - \sigma_x(A_n \sin^2 20) = 0$

$\sigma_t = \sigma_x \sin^2 20 + \sigma_y \cos^2 20 + 2\tau_{xy}\sin 20 \cos 20$

$= 20.118 \approx 20.1$ ksi ANS.

2.87 $D = 3.0$ in.; $d = 2.0$ in.; $T = Q = 5$ kip·in.



(a) $r = 0.15$ in.;

$\frac{D}{d} = 1.5;\ \frac{r}{d} = 0.075 \Rightarrow k \approx 1.5$

$$\tau_{max} = k\left(\frac{TR}{J}\right) = 1.5\left(\frac{5(1)}{\frac{\pi}{32}(2^4)}\right)$$

$= 4.775 \approx 4.8$ ksi ANS.

(b) $r = 0.05$ in.;

$\frac{D}{d} = 1.5;\ \frac{r}{d} = 0.025 \Rightarrow k \approx 2.0$

$$\tau_{max} = k\left(\frac{Tr}{J}\right) = 2.0\left(\frac{5(1)}{\frac{\pi}{32}(2^4)}\right)$$

$= 6.366 \approx 6.4$ ksi ANS.

2.88 REFER TO THE SKETCH IN PROB. 2.87.

$D = 100$ mm; $d = 50$ mm; $\tau_{ALL} = 45$ MPa; $\omega = 1200$ rpm;

(a) $r = 3.0$ mm

$$\frac{D}{d} = 2;\ \frac{r}{d} = 0.06 \Rightarrow k \approx 1.65$$

$$\tau_{MAX} = k\left(\frac{TR}{J}\right);\ T = \frac{\tau_{MAX} J}{kR}$$

$$T = \frac{45\times10^6(\pi/32)(0.05^4)}{1.65(0.025)} = 0.669\ \text{kN}\cdot\text{m}$$

$$P = T\omega = 0.669\left(\frac{1200(2\pi)}{60}\right) = 84.1\ \text{kW} \quad \underline{\text{ANS.}}$$

(b) $r = 1.0$ mm

$$\frac{D}{d} = 2;\ \frac{r}{d} = 0.02 \Rightarrow k \approx 3.20$$

$$T = \frac{45\times10^6(\pi/32)(0.05^4)}{3.20(0.025)} = 0.345\ \text{kN}\cdot\text{m}$$

$$P = T\omega = 0.345\left(\frac{1200(2\pi)}{60}\right) = 43.4\ \text{kW} \quad \underline{\text{ANS.}}$$

2.89 REFER TO THE SKETCH IN PROB. 2.87.

$D = 2.0$ in.; $d = 1.0$ in.

$$\tau_{MAX} = k\left(\frac{TR}{J}\right);\ T = \frac{\tau_{MAX} J}{kR}$$

$$T = \frac{10\times10^3(\pi/32)(1.0^4)}{k(0.5)} = \frac{1963.5}{k} \quad \text{—(1)}$$

$$P = T\omega;\ T = \frac{P}{\omega} = \frac{15(6600)}{900(2\pi/60)}$$

$$T = 1050.4\ \text{lb}\cdot\text{in.} \quad \text{—— (2)}$$

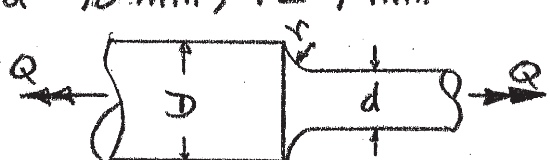
EQUATE (1) & (2) TO GET $k = 1.87$.

ALSO, $\frac{D}{d} = 2.0 \Rightarrow \frac{r}{d} \approx 0.03$

$\therefore r \approx 0.03d \approx 0.03(1.0)$

≈ 0.03 in. $\quad \underline{\text{ANS.}}$

2.90 $d = 70$ mm; $r = 4$ mm



$$P = T\omega;\ T = \frac{P}{\omega} = \frac{50\times10^3}{180(2\pi/60)}$$

$$T = 2652.6\ \text{N}\cdot\text{m}$$

$$\tau_{MAX} = k\left(\frac{TR}{J}\right);\ k = \frac{\tau_{MAX} J}{TR}$$

$$k = \frac{60\times10^6(\pi/32)(0.07^4)}{2652.6(0.035)} = 1.52$$

2.90 CONT'D

$$\frac{r}{d} = 0.057 \Rightarrow \frac{D}{d} \approx 1.25$$

$$D = 1.25d = 87.5\ \text{mm} \quad \underline{\text{ANS.}}$$

2.91 REFER TO THE SKETCH IN PROB. 2.90.

$D = 3.0$ in.; $d = 2.0$ in.

$$P = T\omega;\ T = \frac{P}{\omega} = \frac{50(6600)}{600(2\pi/60)}$$

$$T = 5252.1\ \text{lb}\cdot\text{in.}$$

$$\tau_{MAX} = k\left(\frac{TR}{J}\right);\ k = \frac{\tau_{MAX} J}{TR}$$

$$k = \frac{6\times10^3(\pi/32)(2.0^4)}{5252.1(1.0)} = 1.79$$

$$\frac{D}{d} = 1.5 \Rightarrow \frac{r}{d} \approx 0.04$$

$r \approx 0.04d \approx 0.08$ in. $\quad \underline{\text{ANS.}}$

2.92 REFER TO THE SKETCH IN PROB. 2.90.

$D = 100$ mm; $d = 70$ mm; $r = 5$ mm

$$\frac{D}{d} = 1.43;\ \frac{r}{d} = 0.071 \Rightarrow k \approx 1.55$$

$$\tau_{MAX} = k\left(\frac{TR}{J}\right);\ T = \frac{\tau_{MAX} J}{kR}$$

$$T = \frac{70\times10^6(\pi/32)(0.07^4)}{1.55(0.035)} = 3042\ \text{N}\cdot\text{m}$$

$$P = T\omega;\ \omega = \frac{P}{T} = \frac{100\times10^3}{3042}$$

$\omega = 32.873$ rad/s ≈ 314 rpm $\quad \underline{\text{ANS.}}$

2.93 $\Delta = \theta b = \left(\frac{TL}{JG}\right)b = \frac{Fb^2L}{JG}$

$$\Delta = \frac{F(15^2)(20)}{(\pi/32)(4.0^4 - 3.0^4)(4\times10^6)} = 65.481\times10^{-6}F$$

$$0.85W(h + \Delta) = \frac{1}{2}F\Delta$$

$$0.85(100)(1.2 + 65.481\times10^{-6}F) = \frac{1}{2}F(65.481\times10^{-6}F)$$

$$F^2 - 169.997F - 3.115\times10^6 = 0$$

$$F = 1851.981\ \text{lb}$$

(a) $\tau_{MAX} = \frac{TR}{J} = \frac{FbR}{J} = \frac{1851.981(15)(2)}{(\pi/32)(4.0^4 - 3.0^4)}$

$= 3233.8$ psi ≈ 3.2 ksi $\quad \underline{\text{ANS.}}$

(b) $\theta = \frac{TL}{JG} = \frac{1851.981(20)}{(\pi/32)(4.0^4 - 3.0^4)(4\times10^6)}$

$= 5.340\times10^{-4} \approx 0.00053$ rad $\quad \underline{\text{ANS.}}$

2.94

$$\tau_{MAX} = \frac{TR}{J}; \quad T_1 = \frac{\tau_{MAX} J}{R}$$

$$T_1 = \frac{100\times10^6(\pi/32)(0.05^4)}{0.025} = 2454.4 \text{ N}\cdot\text{m}$$

$$\theta_{MAX} = \frac{TL}{JG}; \quad T_2 = \frac{\theta_{MAX} JG}{L}$$

$$T_2 = \frac{(5\pi/180)(\pi/32)(0.05^4)(75\times10^9)}{0.75} = 5354 \text{ N}\cdot\text{m}$$

$\therefore T_1$ CONTROLS.

NOW, $T = Fb$; $F = \frac{T}{b}$

$$F = \frac{2454.4}{1.25} = 1963.5 \text{ N}$$

$$\Delta = \theta b = \left(\frac{TL}{JG}\right)b = \left(\frac{(Fb)L}{JG}\right)b = \frac{Fb^2L}{JG}$$

$$W(h+\Delta) = \frac{1}{2}F\Delta; \quad W\left(h+\frac{Fb^2L}{JG}\right) = \frac{1}{2}\left(\frac{F^2b^2L}{JG}\right)$$

$$W\left[0.075 + \frac{1963.5(1.25^2)(0.75)}{(\pi/32)(0.05^4)(75\times10^9)}\right] = \frac{1}{2}\left[\frac{1963.5^2(1.25^2)(0.75)}{(\pi/32)(0.05^4)(75\times10^9)}\right]$$

$W = 392.7$ N ANS.

2.95

$$\tau_{MAX} = \frac{TR}{J}; \quad T_1 = \frac{\tau_{MAX} J}{R}$$

$$T_1 = \frac{12(\pi/32)(6.0^4-5.5^4)}{3} = 149.6 \text{ kip}\cdot\text{in.}$$

$$\theta_{MAX} = \frac{TL}{JG}; \quad T_2 = \frac{\theta_{MAX} JG}{L}$$

$$T_2 = \frac{0.2(\pi/32)(6.0^4-5.5^4)(4\times10^3)}{3(12)} = 831.1 \text{ kip}\cdot\text{in.}$$

$\therefore T_1$ CONTROLS,

NOW, $T = Fb$; $F = \frac{T}{b}$

$$F = \frac{149.6}{5(12)} = 2.493 \text{ kips} \approx 2493 \text{ lb}$$

AS IN PROB. 2.94, $\Delta = \frac{Fb^2L}{JG}$.

$$W\left(h + \frac{Fb^2L}{JG}\right) = \frac{1}{2}\left(\frac{F^2b^2L}{JG}\right)$$

$$150\left[h + \frac{2493(60^2)(36)}{(\pi/32)(6.0^4-5.5^4)(4\times10^6)}\right] = \frac{1}{2}\left[\frac{2493^2(60^2)(36)}{(\pi/32)(6.0^4-5.5^4)(4\times10^6)}\right]$$

$h = 15.788 \approx 15.8$ in. ANS.

2.96 $W(h+\Delta) = \frac{1}{2}F\Delta$

AS IN PROB. 2.94, $\Delta = \frac{Fa^2(L/2)}{JG}$

$$\Delta = \frac{F(2.5^2)(1.0)}{(\frac{\pi}{32})(0.15^4-0.13^4)(75\times10^9)} = 3.847\times10^{-6} F$$

$$\therefore 800(0.125 + 3.847\times10^{-6}F) = \frac{1}{2}F(3.847\times10^{-6}F)$$

$$F^2 - 1600F - 51.989\times10^6 = 0$$

$F = 8055$ N

$$\tau_{MAX} = \frac{TR}{J} = \frac{(Fa)R}{J} = \frac{(8055\times2.5)(0.075)}{(\pi/32)(0.15^4-0.13^4)}$$

$= 69.724$ MPa ≈ 69.7 MPa ANS.

$$\theta_{MAX} = \frac{TL}{JG} = \frac{(Fa)L}{JG} = \frac{(8055\times2.5)(1.0)}{(\frac{\pi}{32})(0.15^4-0.13^4)(75\times10^9)}$$

$= 0.0124$ rad ANS.

2.97 $W(h+\Delta) = \frac{1}{2}F\Delta$

AS IN PROB. 2.94, $\Delta = \frac{Fa^2(L/2)}{JG}$

$$\Delta = \frac{F(10\times12)^2(8\times12)}{(\pi/32)d^4(4\times10^6)} = 3.520(F/d^4)$$

$$\therefore 185[6 + 3.520(F/d^4)] = \frac{1}{2}F[3.520(F/d^4)]$$

$$1110 + 651.2\left(\frac{F}{d^4}\right) = 1.76\left(\frac{F^2}{d^4}\right) \text{ ——— (1)}$$

$$\tau_{ALL} = \frac{TR}{J} = \frac{(Fa)R}{J}$$

$$\therefore 12\times10^3 = \frac{F(10\times12)(d/2)}{(\pi/32)d^4}$$

$$F = 19.635\, d^3 \text{ ——— (2)}$$

(2) INTO (1) $\Longrightarrow d^3 - 1.636d - 18.844 = 0$

A TRIAL-&-ERROR SOLUTION LEADS TO

$d = 2.865 \approx 2.9$ in. ANS.

2.98 BECAUSE OF SYMMETRY, THE APPLIED TORQUE IS DIVIDED EQUALLY BY THE TWO SEGMENTS OF THE SHAFT. THUS,

$W(h+\Delta) = \frac{1}{2}F\Delta$ WHERE

$$\Delta = \left[\frac{(\frac{1}{2}T)L/2}{JG}\right]a = \frac{\frac{1}{2}Fa^2(L/2)}{JG}$$

$$= \frac{F(10\times12)^2(10\times12)}{2(\frac{\pi}{32})(4.0^4-3.5^4)(10\times10^6)} = 8.307\times10^{-3}F$$

$$\therefore 180(5 + 8.307\times10^{-3}F) = \frac{1}{2}F(8.307\times10^{-3}F)$$

$$F^2 - 360F - 216{,}685 = 0$$

$F = 679.084$ lb

2.98 CONT'D

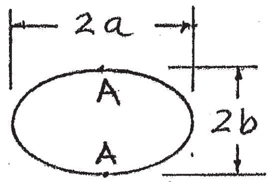
$$\tau = \frac{\frac{1}{2}TR}{J} = \frac{\frac{1}{2}(679.084)(10\times12)(2.0)}{(\pi/32)(4.0^4-3.5^4)}$$

$$\tau = 7.835 \approx 7.8\text{ ksi} \qquad \text{ANS.}$$

$$\theta = \frac{\frac{1}{2}T(L/2)}{JG} = \frac{\frac{1}{2}(679.084)(10\times12)(10\times12)}{(\pi/32)(4.0^4-3.5^4)(10\times10^6)}$$

$$\theta = 0.047\text{ rad} \qquad \text{ANS.}$$

2.99 $a = 2$ in.; $b = 1$ in.



(a)

$$\tau_{MAX} = \frac{2T}{\pi a b^2} \text{ AT A.}$$

$$= \frac{2(30)}{\pi(2)(1^2)} = 9.549 \approx 9.5\text{ ksi} \qquad \text{ANS}$$

(b)

$$K = \frac{\pi a^3 b^3}{a^2+b^2} = \frac{\pi(2^3)(1^3)}{2^2+1^2} = 5.0265$$

$$\theta = \frac{TL}{KG} = \frac{30(1)}{5.0265(4\times10^3)} = 0.00149\text{ rad/in.} \qquad \text{ANS}$$

2.100 REFER TO THE SKETCH IN PROB. 2.99.

$a = 60$ mm ; $b = 40$ mm ; $L = 1.5$ m

$$\tau_{MAX} = \frac{2T}{\pi a b^2};\quad T_1 = \frac{\pi a b^2 \tau_{MAX}}{2}$$

$$T_1 = \frac{\pi(0.06)(0.04^2)(60\times10^6)}{2} = 9.048\text{ kN}\cdot\text{m}$$

$$K = \frac{\pi a^3 b^3}{a^2+b^2} = \frac{\pi(0.06^3)(0.04^3)}{0.06^2+0.04^2} = 8.351\times10^{-6}$$

$$\theta_{MAX} = \frac{TL}{KG};\quad T_2 = \frac{KG\theta}{L}$$

$$T_2 = \frac{8.351\times10^{-6}(75\times10^9)(12\pi/180)}{1.5} = 7.288\text{ kN}\cdot\text{m}$$

$\therefore \theta_{MAX}$ CONTROLS AND

$$Q = T_2 = 7.288 \approx 7.3\text{ kN}\cdot\text{m} \qquad \text{ANS.}$$

2.101 REFER TO THE SKETCH IN PROB. 2.99.

$a = 3.0$ in. ; $b = ?$; $L = 3.5$ ft

$$\tau_{MAX} = \frac{2T}{\pi a b^2};\quad b = \sqrt{\frac{2T}{\pi a \tau_{MAX}}}$$

$$b = \sqrt{\frac{2(40)}{\pi(3.0)8}} = 1.030\text{ in.} \qquad \text{ANS.}$$

2.102 $T = 2V$ (CONST.)

$$= 2(3.0\times10^{-6})(80\times10^6) = 480\text{ N}\cdot\text{m}$$

$$\tau_{MAX} = \frac{2T}{\pi a b^2} = \frac{2(480)}{\pi(0.035)(0.025^2)}$$

$$\tau_{MAX} = 13.969 \approx 14.0\text{ MPa} \qquad \text{ANS.}$$

2.102 CONT'D

$$K = \frac{\pi a^3 b^3}{a^2+b^2}$$

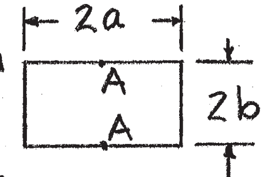
$$K = \frac{\pi(0.035^3)(0.025^3)}{0.035^2+0.025^2} = 1.137\times10^{-6}$$

$$\theta = \frac{TL}{KG} = \frac{480(1.5)}{1.137\times10^{-6}(25\times10^9)}$$

$$\theta = 0.02533\text{ rad} \qquad \text{ANS.}$$

2.103

$a = 50$ mm; $b = 30$ mm



$$\text{(a)}\ \tau_{MAX} = \frac{T(3a+1.8b)}{8a^2b^2}$$

$$\tau_{MAX} = \frac{500[3(0.05)+1.8(0.03)]}{8(0.05^2)(0.03^2)} = 5.667$$

$$\approx 5.7\text{ MPa AT A} \qquad \text{ANS.}$$

$$\text{(b)}\ K = ab^3\left[\frac{16}{3} - 3.36\left(\frac{b}{a}\right)\left(1-\frac{b^4}{12a^4}\right)\right]$$

$$K = (0.05)(0.03^3)\left[\frac{16}{3} - 3.36\left(\frac{0.03}{0.05}\right)\left(1-\frac{0.03^4}{12(0.05^4)}\right)\right]$$

$$= 4.507\times10^{-6}$$

$$\theta = \frac{TL}{KG} = \frac{500(1.20)}{4.507\times10^{-6}(30\times10^9)}$$

$$\theta = 0.00444\text{ rad} \qquad \text{ANS.}$$

2.104 REFER TO THE SKETCH IN PROB. 2.103.

$a = 2.5$ in. ; $b = 1.5$ in. ; $L = 3$ ft

$$\tau_{MAX} = \frac{T(3a+1.8b)}{8a^2b^2};\quad T_1 = \frac{8a^2b^2\tau_{MAX}}{(3a+1.8b)}$$

$$T_1 = \frac{8(2.5^2)(1.5^2)(15)}{3(2.5)+1.8(1.5)} = 165.441\text{ kip}\cdot\text{in.}$$

$$K = ab^3\left[\frac{16}{3} - 3.36\left(\frac{b}{a}\right)\left(1-\frac{b^4}{12a^4}\right)\right]$$

$$K = (2.5)(1.5^3)\left[\frac{16}{3} - 3.36\left(\frac{1.5}{2.5}\right)\left(1-\frac{1.5^4}{12(2.5^4)}\right)\right]$$

$$= 28.174$$

$$\phi_{MAX} = \frac{T}{KG};\quad T_2 = KG\phi_{MAX}$$

$$T_2 = 28.174(4\times10^3)\left(\frac{0.02}{12}\right)$$

$$= 187.827\text{ kip}\cdot\text{in.}$$

$\therefore \tau_{MAX}$ CONTROLS AND

$$Q = T_1 = 165.441 \approx 165.4\text{ kip}\cdot\text{in.} \qquad \text{ANS.}$$

2.105

$$\tau = \frac{T(3a+1.8b)}{8a^2b^2} \Rightarrow$$

$$\frac{a^2b^2}{3a+1.8b} = \frac{T}{8\tau}$$

$$\frac{0.045b^2}{3(0.045)+1.8b} = \frac{15\times10^3}{8(35\times10^6)} = 5.35714\times10^{-5}$$

SIMPLIFYING, WE OBTAIN

$$b^2 - 2.143\times10^{-3}b - 1.607\times10^{-4} = 0$$

SOLVING THIS QUADRATIC EQ. LEADS TO

$b = 0.01379$ m $= 13.8$ mm ANS.

2.106

$T = 2V(2G\phi/\ell p/P) = 2(2.5)(20.5) = 102.5$ kip.in.

$$\tau = \frac{T(3a+1.8b)}{8a^2b^2} = \frac{102.5[3(2.5)+1.8(1.75)]}{8(2.5^2)(1.75^2)}$$

$= 7.129$ ksi ANS.

$$\theta = \frac{TL}{KG};\ K = ab^3\left[\frac{16}{3} - 3.36\left(\frac{b}{a}\right)\left(1 - \frac{b^4}{12a^4}\right)\right]$$

$$K = (2.5)(1.75^3)\left[\frac{16}{3} - 3.36\left(\frac{1.75}{2.5}\right)\left(1 - \frac{1.75^4}{12(2.5^4)}\right)\right]$$

$= 40.576$ in.^{4}

$$\therefore \theta = \frac{7.129(5\times12)}{40.576(10\times10^3)} = 1.054\times10^{-3} \text{ rad}$$ ANS.

2.107

$$\tau = \frac{20T}{a^3} = \frac{20(7.5\times10^3)}{(0.08^3)}$$

$= 292.969$ MPa AT MIDPOINT OF THREE SIDES ANS.

$$\theta = \frac{TL}{KG};\ K = \frac{a^4\sqrt{3}}{80} = \frac{(0.08^4)\sqrt{3}}{80} = 8.86\times10^{-7}\text{ m}^4$$

$$\therefore \theta = \frac{7.5\times10^3(2.0)}{8.86\times10^{-7}(27\times10^9)} = 0.627 \text{ rad}$$ ANS.

2.108

$$\tau = \frac{20T}{a^3} \Rightarrow T_1 = \frac{\tau a^3}{20}$$

$$T_1 = \frac{12(3^3)}{20} = 16.2 \text{ kip.in.}$$

$$\theta = \frac{TL}{KG} = T_2 = \frac{KG\theta}{L}$$

$$K = \frac{a^4\sqrt{3}}{80} = \frac{3^4\sqrt{3}}{80} = 1.754 \text{ in.}^4$$

$$\therefore T_2 = \frac{1.754(4\times10^3)(0.15)}{3(12)} = 29.233 \text{ kip.in.}$$

$\therefore \tau$ CONTROLS AND

$Q = T_1 = 16.2$ kip.in. ANS.

2.109

$$\tau = \frac{20T}{a^3};\ a = \sqrt[3]{\frac{20T}{\tau}}$$

$$a_1 = \sqrt[3]{\frac{20(10\times10^3)}{20\times10^6}} = 0.2154 \text{ m}$$

$$\theta = \frac{TL}{KG};\ K = \frac{TL}{\theta G} = \frac{10\times10^3(2.0)}{(0.2)(27\times10^9)} = 3.7037\times10^{-6}\text{ m}^4$$

$$\therefore K = \frac{a^4\sqrt{3}}{80} = 3.7037\times10^{-6}$$

$a_2 = 0.1144$ m

$\therefore \tau$ CONTROLS AND

$a = a_1 = 0.2154$ m $= 215.4$ mm ANS.

2.110

CIRCLE: $R = 1.5$ in.

$$\tau = \frac{TR}{J};\ T_1 = \frac{\tau J}{R} = \frac{15(\pi/2)(1.5^4)}{1.5}$$

$T_1 = 79.522$ kip.in.

$A = \pi R^2 = \pi(1.5^2) = 7.06858$ in.^{2}

ELLIPSE: $b = 1.25$ in.

$A = \pi ab = 7.06858$

$$a = \frac{7.06858}{\pi(1.25)} = 1.80 \text{ in.}$$

$$\tau = \frac{2T}{\pi ab^2};\ T_2 = \frac{\tau(\pi ab^2)}{2} = \frac{15\pi(1.80)(1.25^2)}{2}$$

$T_2 = 66.268$ kip.in.

$\therefore$ THE CIRCULAR SHAFT IS CHOSEN BECAUSE IT CARRIES ABOUT 20% MORE TORQUE.

2.111

SQUARE: $2a = 65$ mm $= 0.065$ m

$$\tau = \frac{T(3a+1.8b)}{8a^2b^2} = \frac{T(4.8a)}{8a^4};\ T_1 = \frac{8a^3\tau}{4.8}$$

$$T_1 = \frac{8(0.0325^3)(75\times10^6)}{4.8} = 4291 \text{ N.m}$$

$A = 4a^2 = 4(0.0325^2) = 0.004225$ m^2

EQUILATERAL TRIANGLE: b ON A SIDE

$$A = \frac{\sqrt{3}b^2}{4} = 0.004225$$

$b = 0.09878$ m

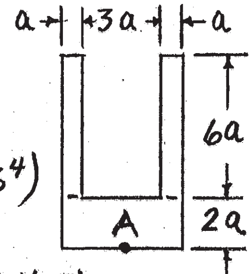
$$\tau = \frac{20T}{b^3};\ T_2 = \frac{\tau b^3}{20}$$

$$T_2 = \frac{75\times10^6(0.09878^3)}{20} = 3614 \text{ N.m}$$

$\therefore$ THE SQUARE SHAFT IS CHOSEN BECAUSE IT CARRIES ABOUT 19% MORE TORQUE.

2.112

$$K=\frac{1}{3}\sum Lt^3$$



$$=\frac{1}{3}\left[2(6a)a^3+5a(2a)^3\right]$$
$$=\frac{1}{3}(52a^4)=\frac{1}{3}(52)(1.5^4)$$
$$=87.75\ \text{in}^4$$

(a) $\tau_{MAX}=\frac{Tt}{K}=\frac{150(2)(1.5)}{87.75}$

$=5.128$ ksi AT A. ANS.

(b) $\theta=\frac{TL}{KG}=\frac{150(4\times12)}{87.75(4\times10^3)}$

$=20.5\times10^{-3}$ rad ANS.

2.113 AS IN PROB. 2.112, $K=\frac{1}{3}(52a^4)$

$$K=\frac{1}{3}(52)(0.05^4)=1.083\times10^{-4}\ m^4$$

$\tau_{MAX}=\frac{Tt}{K}$; $T_1=\frac{\tau K}{t}=\frac{40\times10^6(1.083\times10^{-4})}{2(0.05)}$

$T_1=433.2\times10^2$ N.m

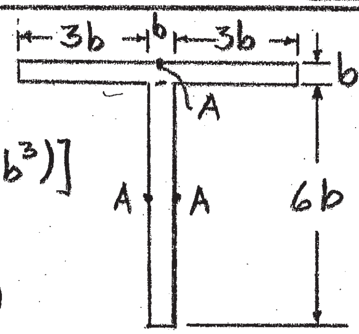
$\theta_{MAX}=\frac{TL}{KG}$; $T_2=\frac{KG\theta}{L}$

$T_2=\frac{1.083\times10^{-4}(25\times10^9)(6\pi/180)}{3.0}=945.1\times10^2$ N.m

$\therefore$ τ CONTROLS AND

$Q=T_1=433.2\times10^2$ N.m ≈ 43.3 kN.m ANS.

2.114 (a)



$$K=\frac{1}{3}\sum Lt^3$$
$$=\frac{1}{3}\left[7b(b^3)+6b(b^3)\right]$$
$$=\frac{1}{3}(13b^4)$$
$$=\frac{1}{3}(13)(1.25^4)$$
$$=10.579\ \text{in}^4$$ ANS.

$\tau_{MAX}=\frac{Tt}{K}=\frac{100(1.25)}{10.579}$

$=11.816$ ksi AT POINTS A ANS.

(b)

$\theta=\frac{TL}{KG}=\frac{100(3\times12)}{10.579(10\times10^3)}$

$=3.403\times10^{-2}$ rad ANS.

2.115 AS IN PROB. 2.114, $K=\frac{1}{3}(13b^4)$

$$K=\frac{1}{3}(13)(0.04^4)=11.093\times10^{-6}\ m^4$$

$\tau_{MAX}=\frac{Tt}{K}$; $T_1=\frac{K\tau}{t}=\frac{11.093\times10^{-6}(50\times10^6)}{0.04}$

$T_1=13{,}866$ N.m

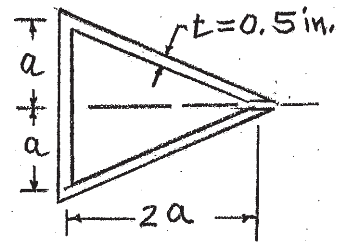
$\theta_{MAX}=\frac{TL}{KG}$; $T_2=\frac{KG\theta}{L}$

$T_2=\frac{11.093\times10^{-6}(27\times10^9)(5\pi/180)}{2}=13{,}069$ N.m

$\therefore$ θ CONTROLS AND

$Q=T_2=13{,}069$ N.m ≈ 13.1 kN.m ANS.

2.116 $K=\frac{1}{3}\sum Lt^3$;



$$K=\frac{1}{3}\left[2(2.236a)t^3+(2a)t^3\right]$$
$$=\frac{1}{3}(6.472at^3)$$
$$=\frac{1}{3}(6.472)(0.5^3)a$$
$$=0.26967a$$

$\tau=\frac{Tt}{K}$; $K=\frac{Tt}{\tau}=\frac{12(0.5)}{12}=0.5\ \text{in}^4$

$\therefore 0.26967a=0.5$

$a=1.854$ in. ANS.

2.117 AS IN PROB. 2.116, $K=\frac{1}{3}(6.472at^3)$

$$K=\frac{1}{3}(6.472)(0.1)(0.015^3)=7.28\times10^{-7}\ m^4$$

$\tau_{MAX}=\frac{Tt}{K}$; $T_1=\frac{\tau K}{t}=\frac{80\times10^6(7.28\times10^{-7})}{0.015}$

$T_1=3{,}882.7$ N.m

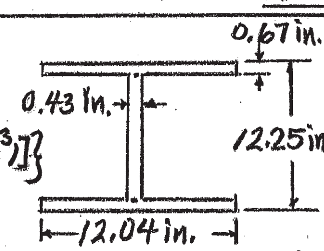
$\theta_{MAX}=\frac{TL}{KG}$; $T_2=\frac{KG\theta}{L}=\frac{7.28\times10^{-7}(77\times10^9)(0.10)}{1.5}$

$T_2=3{,}737.1$ N.m

$\therefore$ θ CONTROLS AND

$Q=T_2=3{,}737.1$ N.m ≈ 3.737 kN.m ANS.

2.118 $K=\frac{1}{3}\sum Lt^3$

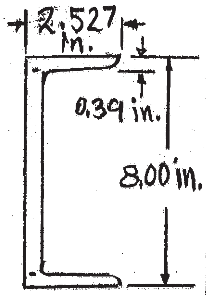


$$K=\frac{1}{3}\left[2(12.04)(0.67^3)+[12.25-2(0.67)](0.43^3)\right]$$
$$K=2.70326\ \text{in}^4$$

(a) $\tau_{MAX}=\frac{Tt}{K}=\frac{10\times12(0.67)}{2.70326}=29.742$ ksi ANS.

(b) $\theta=\frac{TL}{KG}=\frac{10\times12(5\times12)}{2.70326(10\times10^3)}=0.266$ rad ANS.

2.119 $K=\frac{1}{3}\sum Lt^3$



$K=\frac{1}{3}\{2(2.527)(0.39^3)$
$+[(8.00-2(0.39))(0.487^3)]\}$
$=0.37791\ in.^4$

$\tau_{MAX}=\frac{Tt}{K};\ T_1=\frac{\tau K}{t}$

$T_1=\frac{10(0.37791)}{0.487}=7.760$ kip·in.

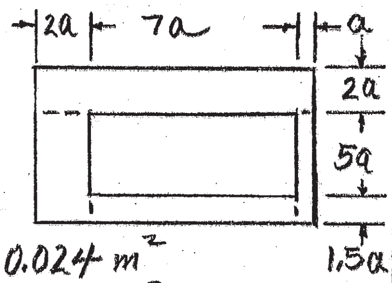
$\theta_{MAX}=\frac{TL}{KG};\ T_2=\frac{KG\theta}{L}$

$T_2=\frac{0.37791(10\times10^3)(10\pi/180)}{4\times12}=13.741$ kip·in.

∴ τ CONTROLS AND

$Q=T_1=7.760$ kip·in. ANS.

2.120 $a=20$ mm



$A_m=\frac{1}{2}[10a(8.5a)+5a(7a)]$

$A_m=60a^2$
$=60(0.02^2)=0.024\ m^2$

(a) $\tau=\frac{T}{2A_m t}=\frac{7.5\times10^3}{2(0.024)(0.02)}=7.813$ MPa ANS.

(b) $q=t\tau=(0.02)(7.813\times10^6)$
$=156.3$ kN/m ANS.

(c) $\theta=\frac{TL}{4GA_m^2}\sum(\frac{s}{t})$

$\theta=\frac{7.5\times10^3(1.0)}{4(27\times10^9)(0.024^2)}[\frac{10\cancel{a}}{2\cancel{a}}+\frac{6.5\cancel{a}}{2\cancel{a}}+\frac{7\cancel{a}}{1.5\cancel{a}}+\frac{6.5\cancel{a}}{\cancel{a}}]$

$=120.563\times10^{-6}(19.417)=2.34\times10^{-3}$ rad ANS.

2.121 AS IN PROB. 2.120, $A_m=60a^2$

$\tau=\frac{T}{2A_m t};\ A_m=\frac{T}{2\tau t}=\frac{5}{2(8)(a)}=\frac{5}{16a}$

∴ $\frac{5}{16a}=60a^2\Rightarrow a_1=0.173$ in.

$\theta=\frac{TL}{4GA_m^2}\sum(\frac{s}{t})=\frac{5(5\times12)}{4(10\times10^3)A_m^2}[\frac{16.5\cancel{a}}{2\cancel{a}}+\frac{7\cancel{a}}{1.5\cancel{a}}+\frac{6.5\cancel{a}}{\cancel{a}}]$

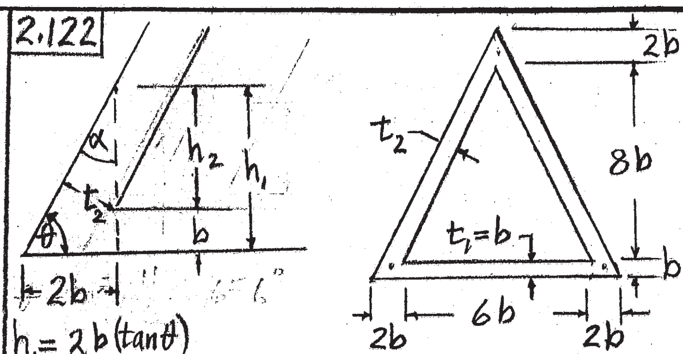
∴ $0.15=\frac{7.5\times10^{-3}}{A_m^2}(19.41667);\ A_m^2=0.97083$

$A_m=0.9853=60a^2;\ a_2=0.128$ in.

∴ τ CONTROLS AND

$a=a_1=0.173$ in. ANS.

2.122



$h_1=2b(\tan\theta)$
$=2b(\frac{11}{5})=4.4b$

∴ $h_2=3.4b$; $\alpha=\tan^{-1}(\frac{2}{4.4})=24.444°$

$t_2=3.4b\sin\alpha=1.407b.$

∴ τ_{MAX} IN RECTANGLE OF THICKNESS $t_1=b$.

$A_m=\frac{1}{2}[\frac{1}{2}(10b)(11b)+\frac{1}{2}(6b)(8b)]=39.5b^2$
$=39.5\ in.^2$

(a) $\tau_{MAX}=\frac{T}{2A_m t_1}=\frac{10(12)}{2(39.5)(1.0)}=1.519$ ksi ANS.

(b) $\theta=\frac{TL}{4GA_m^2}\sum(\frac{s}{t})$

$\theta=\frac{10(12)(6\times12)}{4(6.4\times10^3)(39.5)}\underbrace{[\frac{8b}{b}+\frac{\sqrt{(9.5b)^2+(4.0b)^2}}{1.407b}]}_{15.326}$

$=0.131$ rad. ANS.

2.123 AS IN PROB. 2.122, $A_m=39.5b^2$

$A_m=39.5(0.02^2)=0.0158\ m^2$

$\tau_{MAX}=\frac{T}{2A_m t_1};\ T_1=2\tau_{MAX}A_m t_1$

$T_1=2(40\times10^6)(0.0158)(0.02)=25.280$ kN.m

$\theta_{MAX}=\frac{TL}{4GA_m^2}\sum(\frac{s}{t});\ T_2=\frac{4\theta GA_m^2}{L}[\frac{1}{\sum(\frac{s}{t})}]$

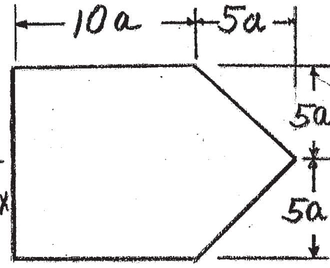
$T_2=\frac{4(6\pi/180)(50\times10^9)(0.0158^2)}{1.5}[\frac{1}{15.326}]=37.905$ kN·m

NOTE THAT $\sum(\frac{s}{t})=15.326$ WAS OBTAINED IN PROB. 2.122.

∴ τ_{MAX} CONTROLS AND

$Q=T_1=25.280$ kN·m ANS.

2.124



$A_m=(10a)(10a)+\frac{1}{2}(10a)(5a)$
$=125a^2$

$\tau_{MAX}=\frac{T}{2A_m t}$; $A_m=\frac{T}{2t\tau_{MAX}}$

$A_m=\frac{100}{2(a)(12)}=125a^2$

$\therefore a=\sqrt[3]{\frac{100}{2(12)(125)}}=0.322$ in.

$\theta_{MAX}=\frac{TL}{4GA_m^2}\sum\left(\frac{s}{t}\right)$; $A_m^2=\frac{TL}{4G\theta}\sum\left(\frac{s}{t}\right)$

$$A_m^2=\frac{100(4\times12)}{4(4\times10^3)(10\pi/180)}\underbrace{\left[\frac{30\cancel{a}}{\cancel{a}}+2\left(\frac{\sqrt{5^2+5^2}\,\cancel{a}}{\cancel{a}}\right)\right]}_{44.142}$$

$A_m^2=75.875=(125a^2)^2$

$a=0.264$ in.

$\therefore \tau$ CONTROLS AND $a=0.322$ in. ANS.

2.125 AS IN PROB. 2.124, $A_m=125a^2$

$A_m=125(0.015^2)=0.028125\ m^2$.

ALSO, FROM PROB. 2.124, $\sum\left(\frac{s}{t}\right)=44.142$.

$\tau_{MAX}=\frac{T}{2A_m t}$; $T_1=2A_m t\tau_{MAX}$

$T_1=2(0.028125)(0.015)(25\times10^6)$
$=21.094$ kN·m

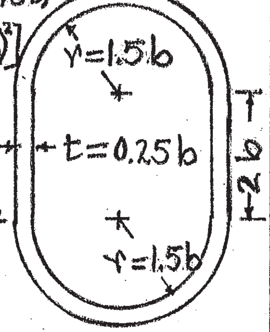
$\theta_{MAX}=\frac{TL}{4GA_m^2}\sum\left(\frac{s}{t}\right)$; $T_2=\frac{4GA_m^2\theta}{L\sum\left(\frac{s}{t}\right)}$

$T_2=\frac{4(27\times10^9)(0.028125^2)(0.05)}{2(44.142)}$
$=48.388$ kN·m

$\therefore \tau_{MAX}$ CONTROLS AND

$Q=T_1=21.094$ kN·m ANS.

2.126 $A_m=\frac{1}{2}[3.5b(2b)+2(\frac{\pi}{2})(1.75b)^2$
$+(3.0b)(2b)+2(\frac{\pi}{2})(1.5b)^2]$



$A_m=14.845b^2$

$\tau_{MAX}=\frac{T}{2A_m t}$; $A_m=\frac{T}{2\tau_{MAX}t}$

$A_m=\frac{15}{2(10)(0.25b)}=14.845b^2$

$b_1=0.587$ in.

2.126 CONT'D

$\theta=\frac{TL}{4GA_m^2}\sum\left(\frac{s}{t}\right)$; $A_m^2=\frac{TL}{4G\theta}\sum\left(\frac{s}{t}\right)$

$$A_m^2=\frac{15(3\times12)}{4(5.5\times10^3)(15\pi/180)}\underbrace{\left[2\left(\frac{2b}{0.25b}\right)+2\left(\frac{\pi(1.625b)}{0.25b}\right)\right]}_{56.841}$$

$A_m=2.309\text{ in.}^2=14.845b^2$

$b_2=0.394$ in.

$\therefore \tau_{MAX}$ CONTROLS AND

$b_{MIN}=b_1=0.587$ in. ANS.

2.127 AS IN PROB. 2.126, $A_m=14.845b^2$

$A_m=14.845(0.03^2)=1.336\times10^{-2}\ m^2$

$\tau_{MAX}=\frac{T}{2A_m t}$; $T_1=2\tau_{MAX}A_m t$

$T_1=2(30\times10^6)(1.336\times10^{-2})(7.5\times10^{-3})$
$=6.012$ kN·m

$\theta_{MAX}=\frac{TL}{4GA_m^2}\sum\left(\frac{s}{t}\right)$; $T_2=\frac{4\theta_{MAX}GA_m^2}{L\sum\left(\frac{s}{t}\right)}$

$\sum\left(\frac{s}{t}\right)=56.841$ FROM PROB. 2.126.

$\therefore T_2=\frac{4(4\pi/180)(26\times10^9)(1.336\times10^{-2})^2}{1.5(56.841)}$
$=16.199$ kN·m

$\therefore \tau_{MAX}$ CONTROLS AND

$Q=T_1=6.012$ kN·m ANS.

2.128 $\tau_y=12.0$ ksi; $\gamma_y=2\times10^{-3}$

(a) $G=\frac{\tau_y}{\gamma_y}=\frac{12.0}{2\times10^{-3}}=6\times10^3$ ksi ANS.

(b) $\tau_y=\frac{T_yR_o}{J}$; $T_y=\frac{\tau_yJ}{R_o}$

$T_y=\frac{12.0(\pi/32)(2.5^4-1.5^4)}{2.5}$
$=16.022$ ksi ANS.

(c) $\theta_y=\frac{T_yL}{JG}=\frac{16.022(2\times12)}{(\pi/32)(2.5^4-1.5^4)(6\times10^3)}$

$\theta_y=19.200\times10^{-3}$ rad ANS.

(d) $u_r=\frac{1}{2}\tau_y\gamma_y=\frac{1}{2}(12.0)(2\times10^{-3})$
$=12.0\times10^{-3}$ kip·in./in.³ ANS.

2.129 $T_y = 30.0\ \text{kN·m};\ \tau_y = 350\ \text{MPa}$

(a) $\tau_y = \frac{T_y R}{J} = \frac{T_y(\frac{D}{2})}{(\pi/32)D^4} = \frac{16\,T_y}{\pi D^3}$

$D^3 = \frac{16\,T_y}{\pi \tau_y} = \frac{16(30.0\times10^3)}{\pi(350\times10^6)} = 0.436539\times10^{-3}$

$D = 0.0759\ \text{m} = 75.9\ \text{mm}$ ANS.

(b) $\theta = \frac{TL}{JG};\ G = \frac{TL}{J\theta} = \frac{30.0\times10^3(1.5)}{(\pi/32)(0.0759^4)(8\pi/180)}$

$G = 98.919\times10^9 \approx 98.9\ \text{GPa}$ ANS.

2.130 $T_y = \frac{\tau_y J}{R} = \frac{17(\pi/32)(3.5^4)}{3.5} = 71.557\ \text{kip·in}$

(a) $\frac{T}{T_y} = \left(\frac{4}{3}\right)\left[1-\left(\frac{1}{4}\right)\left(\frac{\rho_y}{R}\right)^3\right];\ \frac{T}{T_y} = \frac{90}{71.557} = 1.258$

$\therefore 1.258 = \left(\frac{4}{3}\right)\left[1-\left(\frac{1}{4}\right)\left(\frac{\rho_y}{R}\right)^3\right]$

$\left(\frac{\rho_y}{R}\right)^3 = 4-3\left(\frac{T}{T_y}\right) = 4-3(1.258) = 0.226$

$\frac{\rho_y}{R} = 0.609;\ \rho_y = 0.609R = 0.609(1.75)$

$\rho_y = 1.066\ \text{in.}$

$\therefore d = 1.75-1.066 = 0.684\ \text{in.}$ ANS.

(b) AS IN PART (a),

$\left(\frac{\rho_y}{R}\right)^3 = 4-3\left(\frac{T}{T_y}\right) = 4-3\left(\frac{80}{71.557}\right) = 0.646$

$\frac{\rho_y}{R} = 0.864;\ \rho_y = 0.864R = 0.864(1.75)$

$\rho_y = 1.512\ \text{in.}$

$\therefore d = 1.75-1.512 = 0.238\ \text{in.}$ ANS.

2.131 $T = \frac{\pi \tau_y}{6}(4R^3-\rho_y^3)$

(a) $\rho_y = 2.0-0.5 = 1.5\ \text{in.}$

$T = \frac{\pi(20)}{6}[4(2.0)^3-(1.5)^3] = 299.760\ \text{kip·in}$ ANS.

(b) $\rho_y = 2.0-1.0 = 1.0\ \text{in.}$

$T = \frac{\pi(20)}{6}[4(2.0)^3-(1.0)^3] = 324.631\ \text{kip·in.}$ ANS.

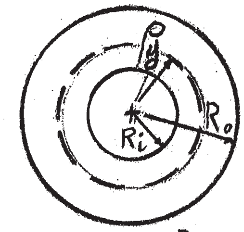
(c) $\rho_y = 2.0-1.5 = 0.5\ \text{in.}$

$T = \frac{\pi(20)}{6}[4(2.0)^3-(0.5)^3] = 333.794\ \text{kip·in.}$ ANS.

(d) $\rho_y = 2.0-2.0 = 0$

$T = \frac{\pi(20)}{6}[4(2.0)^3-0] = 335.103\ \text{kip·in.}$ ANS.

2.132 SEE DEVELOPMENT OF EQ. (2.38).



$T = \left(\frac{2\pi\tau_y}{\rho_y}\right)\int_{R_i}^{\rho_y}\rho^3 d\rho + (2\pi\tau_y)\int_{\rho_y}^{R_o}\rho^2 d\rho$

$T = \left(\frac{2\pi\tau_y}{\rho_y}\right)\left[\frac{\rho^4}{4}\right]_{R_i}^{\rho_y} + (2\pi\tau_y)\left[\frac{\rho^3}{3}\right]_{\rho_y}^{R_o}$

$= \left(\frac{2\pi\tau_y}{\rho_y}\right)\left(\frac{\rho_y^4-R_i^4}{4}\right) + 2\pi\tau_y\left(\frac{R_o^3-\rho_y^3}{3}\right)$

$T = \frac{\pi\tau_y}{2}\left[\frac{4R_o^3-\rho_y^3}{3} - \frac{R_i^4}{\rho_y}\right]$ ANS.

2.133 FROM STATEMENT OF PROB. 2.132,

$T = \frac{\pi\tau_y}{2}\left[\frac{4R_o^3-\rho_y^3}{3} - \frac{R_i^4}{\rho_y}\right]$

$32\times10^3 = \pi(175\times10^6)\left[\frac{4(0.05^3-\rho_y^3}{6} - \frac{0.03^4}{2\rho_y}\right]$

SIMPLIFYING, WE OBTAIN

$\rho_y^4 - 15.078\times10^{-5}\rho_y + 0.243\times10^{-5} = 0$

A TRIAL-AND-ERROR SOLUTION YIELDS

$\rho_y = 0.0461\ \text{m}$

$d = 0.05-0.0461 = 0.0039\ \text{m}$

$= 3.9\ \text{mm}$ ANS.

2.134 $\frac{T}{T_y} = \frac{4}{3}\left[1-\frac{1}{4(\theta/\theta_y)^3}\right]$

$T_y = \frac{\pi\tau_y R^3}{2} = \frac{\pi(15)(3.5^3)}{2} = 1010.218\ \text{kip·in.}$

$\theta_y = \frac{T_y L}{JG} = \frac{1010.218(2.0\times12)}{(\pi/32)(3.5^4)(10.5\times10^3)} = 0.157\ \text{rad}$

$T = \left(\frac{4}{3}\right)T_y\left[1-\frac{1}{4(\theta/\theta_y)^3}\right]$

(a) $\theta = 0.2\ \text{rad}$

$T = \left(\frac{4}{3}\right)(1010.218)\left[1-\frac{1}{4(0.2/0.157)^3}\right]$

$= 1084.064\ \text{kip·in.}$ ANS.

(b) $\theta = 0.4\ \text{rad}$

$T = \left(\frac{4}{3}\right)(1010.218)\left[1-\frac{1}{4(0.4/0.157)^3}\right]$

$= 1326.596\ \text{kip·in.}$ ANS.

2.135 $\dfrac{T}{T_y}=\left(\dfrac{4}{3}\right)\left[1-\left(\dfrac{1}{4}\right)\left(\dfrac{\rho_y}{R}\right)^3\right]$

$\rho_y = R\left[4-3\left(\dfrac{T}{T_y}\right)\right]^{1/3}$; $R=3.5$ in.

(a) FROM PROB. 2.134, $T=1084.064$ kip.in

$T_y = 1010.218$ kip.in, FOR $\theta=0.20$ rad.

$\therefore \rho_y = 3.5\left[4-3\left(\dfrac{1084.064}{1010.218}\right)\right]^{1/3}$

$=3.223$ in. ANS.

(b) FROM PROB. 2.134, $T=1326.596$ kip.in

$T_y = 1010.218$ kip.in, FOR $\theta=0.40$ rad.

$\therefore \rho_y = 3.5\left[4-3\left(\dfrac{1326.596}{1010.218}\right)\right]^{1/3}$

$=1.374$ in. ANS.

2.136 $T=\dfrac{\pi\tau_y}{6}(4R^3-\rho_y^3)$

$\rho_y=\left[4R^3-\dfrac{6T}{\pi\tau_y}\right]^{1/3}=\left[4(0.045^3)-\dfrac{6(25\times10^3)}{\pi(150\times10^6)}\right]^{1/3}$

$=0.0359$ m $=35.9$ mm ANS.

$\dfrac{\rho_y}{R}=\dfrac{\theta_y}{\theta}$; $\theta=\left(\dfrac{R}{\rho_y}\right)\theta_y$

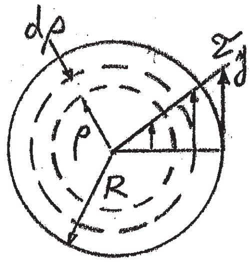
$\theta_y=\dfrac{T_yL}{JG}=\dfrac{(\tau_yJ/R)L}{JG}=\dfrac{\tau_yL}{RG}$

$=\dfrac{150\times10^6(0.75)}{0.045(75\times10^9)}=0.0333$ rad.

$\therefore \theta=\left(\dfrac{0.75}{0.0359}\right)(0.0333)$

$=0.696$ rad ANS.

2.137 (a)



$dF_y=(2\pi\rho\,d\rho)\tau$

$=(2\pi\rho\,d\rho)\left(\dfrac{\rho}{R}\tau_y\right)$

$=(2\pi\tau_y\rho^2d\rho)/R$

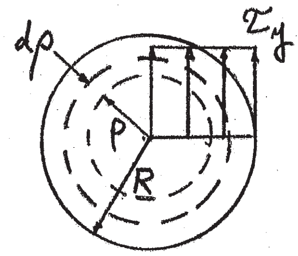
$dT_y=(2\pi\tau_y\rho^3d\rho)/R$

$T_y=\dfrac{2\pi\tau_y}{R}\displaystyle\int_0^R\rho^3d\rho=\dfrac{2\pi\tau_y}{R}\left[\dfrac{\rho^4}{4}\right|_0^R$

$T_y=\dfrac{\pi\tau_yR^3}{2}=\dfrac{\pi(25)(2^3)}{2}$

$=314.159$ kip.in. ANS.

2.137 CONT'D



$dF_p=(2\pi\rho\,d\rho)\tau_y$

$dT_p=(2\pi\tau_y\rho^2d\rho$

$T_p=2\pi\tau_y\displaystyle\int_0^R\rho^2d\rho$

$=2\pi\tau_y\left[\dfrac{\rho^3}{3}\right|_0^R=\left(\dfrac{2}{3}\right)\pi R^3\tau_y$

$T_p=\left(\dfrac{2}{3}\right)\pi(2^3)(25)=418.879$ kip.in. ANS.

(b)

$T=1.15T_y=1.15(314.159)=361.283$ kip.in

$T=\dfrac{\pi\tau_y}{6}(4R^3-\rho_y^3)$

$\rho_y=\left[4R^3-\dfrac{6T}{\pi\tau_y}\right]^{1/3}=\left[4(2^3)-\dfrac{6(361.283)}{\pi(25)}\right]^{1/3}$

$\rho_y=1.639$ in.

$d=2.0-1.639=0.361$ in. ANS.

$T=1.25T_y=1.25(314.159)=392.699$ kip.in.

$\rho_y=\left[4(2^3)-\dfrac{6(392.699)}{\pi(25)}\right]^{1/3}=1.260$ in.

$d=2.0-1.260=0.740$ in. ANS.

2.138 (a)

FROM PROB. 2.137, $T_y=\dfrac{\pi\tau_yR^3}{2}$

$T_y=\dfrac{\pi(160\times10^6)(0.04^3)}{2}=16.085$ kN.m

FOR $d=0.02$ m; $\rho_y=0.04-0.02=0.02$ m

$T=\dfrac{\pi\tau_y}{6}(4R^3-\rho_y^3)=\dfrac{\pi(160\times10^6)}{6}[4(0.04^3)-(0.02^3)]$

$T=20.776$ kN.m

$\dfrac{T}{T_y}=\dfrac{20.776}{16.085}=1.292$ ANS.

FOR $d=0.04$ m; $\rho_y=0.04-0.04=0$

$T=T_p=\left(\dfrac{2}{3}\right)\pi R^3\tau_y$ (SEE PROB. 2.137)

$=\left(\dfrac{2}{3}\right)(\pi)(0.04^3)(160\times10^6)=21.447$ kN.m

$\dfrac{T}{T_y}=\dfrac{21.447}{16.085}=1.333$ ANS.

(b)

$\dfrac{\rho_y}{R}=\dfrac{\theta_y}{\theta}$; $\theta=\left(\dfrac{R}{\rho_y}\right)\theta_y$; $\theta_y=\dfrac{T_yL}{JG}$

2.138 CONT'D

$\therefore \theta = \frac{R T_y L}{\rho_y J G}$; FOR $d=0.02 m$; $\rho_y = 0.02 m$

$\theta = \frac{(0.04)(16.085\times10^3)(0.80)}{(0.02)(\pi/32)(0.08^4)(70\times10^9)} = 0.0914$ rad

$\theta = 0.0914$ rad ANS.

FOR $d = 0.04 m$; $\rho_y = 0$

$\theta = \infty$ ANS.

2.139 FROM PROB. 2.132,

$$T = \frac{\pi \tau_y}{2}\left[\frac{4R_o^3 - \rho_y^3}{3} - \frac{R_i^4}{\rho_y}\right]$$

$$R_o = \left[\frac{\rho_y^3 + (6T/\pi\tau_y) + 3R_i^4/\rho_y}{4}\right]^{1/3}$$

$$= \left[\frac{(3.5)^3 + 6(250)/\pi(20) + 3(3.0)^4/3.5}{4}\right]^{1/3}$$

$= 3.241$ in. ANS.

2.140 FROM PROB. 2.132,

$$T = \frac{\pi \tau_0}{2}\left[\frac{4R_o^3 - \rho_y^3}{3} - \frac{R_i^4}{\rho_y}\right]$$

$$R_i = \left\{\rho_y\left[\left(\frac{4R_o^3 - \rho_y^3}{3}\right) - \left(\frac{2T}{\pi\tau_y}\right)\right]\right\}^{1/4}$$

$$= \left\{3.3\left[\frac{4(3.5)^3 - (3.3)^3}{3} - \frac{2(200)}{\pi(10)}\right]\right\}^{1/4}$$

$= 3.217$ in. ANS.

2.141 FROM PROB. 2.132,

$$T = \frac{\pi \tau_y}{2}\left[\frac{4R_o^3 - \rho_y^3}{3} - \frac{R_i^4}{\rho_y}\right]$$

(a) $d = 0.010 m$; $\rho_y = 0.05 - 0.01 = 0.04 m$

$$T = \frac{\pi(150\times10^6)}{2}\left[\frac{4(0.05)^3 - (0.04)^3}{3} - \frac{(0.02)^4}{0.04}\right]$$

$= 33.301$ kN.m ANS.

$\theta = \frac{R_o \theta_y}{\rho_y} = \frac{R_o(T_y L/JG)}{\rho_y}$; $T_y = \frac{\tau_y J}{R_o}$

$\therefore \theta = \frac{L\tau_y}{G\rho_y} = \frac{(0.6)(150\times10^6)}{75\times10^9(0.04)}$

$= 0.030$ rad ANS.

(b) $d = 0.020 m$; $\rho_y = 0.05 - 0.02 = 0.03$ m

$$T = \frac{\pi(150\times10^6)}{2}\left[\frac{4(0.05)^3 - (0.03)^3}{3} - \frac{(0.02)^4}{0.03}\right]$$

2.141 CONT'D

$T = 35.893$ kN·m ANS.

AS IN PART (a), $\theta = \frac{L\tau_y}{G\rho_y}$

$\theta = \frac{(0.6)(150\times10^6)}{75\times10^9(0.03)} = 0.040$ rad ANS.

2.142

(a) $T_y = \frac{\tau_y J}{R_o} = \frac{(160\times10^6)(\frac{\pi}{2})(0.07^4 - 0.04^4)}{0.07}$

$T_y = 77.014$ kN.m. ANS.

FROM PROB. 2.132,

$$T = \frac{\pi\tau_y}{2}\left[\frac{4R_o^3 - \rho_y^3}{3} - \frac{R_i^4}{\rho_y}\right] \quad \ldots (a)$$

$T = T_p$ WHEN $\rho_y = R_i$

$\therefore T_p = \frac{2\pi\tau_y}{3}(R_o^3 - R_i^3)$

$= \frac{2\pi(160\times10^6)}{3}\left[(0.07)^3 - (0.04)^3\right]$

$= 93.494$ kN.m ANS.

$\theta_y = \frac{T_y L}{JG} = \frac{77.014\times10^3(0.6)}{\frac{\pi}{2}(0.07^4 - 0.04^4)(70\times10^9)}$

$= 0.0196$ rad ANS.

$\theta_p = \infty$ (SEE PROB. 2.138) ANS.

(b) $\frac{T}{T_y} = 1.20$

$T = 1.20 T_y = 1.20(77.014) = 92.417$ kN.m

FROM EQ. (a) ABOVE, WE CONCLUDE THAT

$$\rho_y^4 + \left[\frac{6T}{\pi\tau_y} - 4R_o^3\right]\rho_y + 3R_i^4 = 0$$

SUBSTITUTING NUMERICAL VALUES WE OBTAIN

$$\rho_y^4 - 2.689\times10^{-4}\rho_y + 7.68\times10^{-6} = 0$$

A TRIAL-AND-ERROR SOLUTION YIELDS

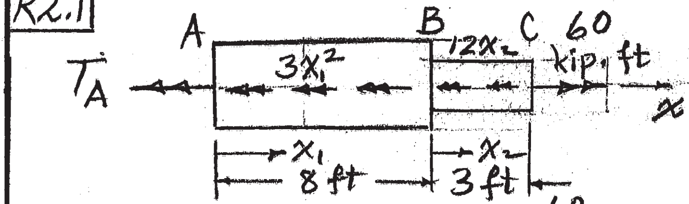
$\rho_y = 0.0475$ m; $\frac{\rho_y}{R_o} = \frac{\theta_y}{\theta}$

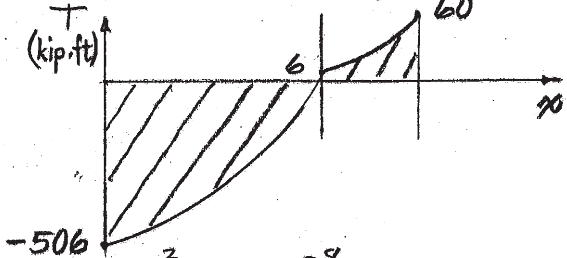
$\therefore \theta = \frac{R_o\theta_y}{\rho_y}$

$= \frac{(0.07)(0.0196)}{0.0475}$

$= 0.02888$ rad ANS.

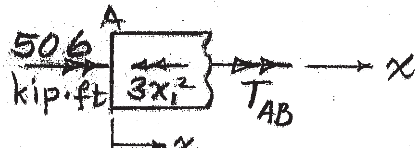
R2.1





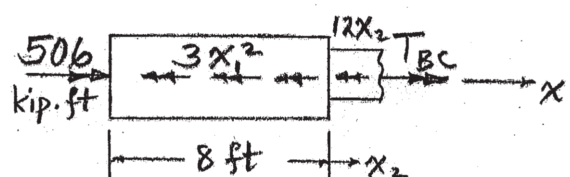
$$\Sigma T_x = 0: 60 - \int_0^3 12x_2\,dx_2 - \int_0^8 3x_1^2\,dx_1 - T_A = 0$$

$$T_A = 60 - 54 - 512 = -506 \text{ kip·ft}$$



$$\Sigma T_x = 0: 506 - \int_0^{x_1} 3x_1^2\,dx_1 + T_{AB} = 0$$

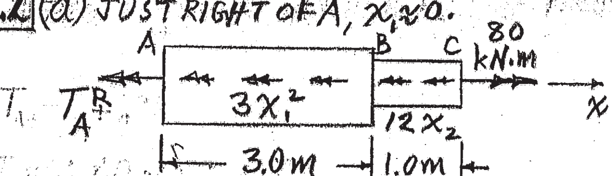
$$T_{AB} = x_1^3 - 506$$ ANS.



$$\Sigma T_x = 0: 506 - \int_0^8 3x_1^2\,dx_1 - \int_0^{x_2} 12x_2\,dx_2 + T_{BC} = 0$$

$$T_{BC} = 6x_2^2 + 6$$ ANS.

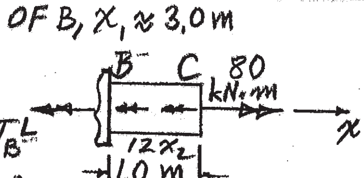
R2.2 (a) JUST RIGHT OF A, $x_1 \approx 0$.



$$\Sigma T_x = 0: 80 - \int_0^{1.0} 12x_2\,dx_2 - \int_0^{3.0} 3x_1^2\,dx_1 - T_A^R = 0$$

$$T_A^R = 47 \text{ kN·m}$$ ANS.

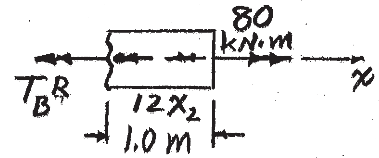
(b) JUST LEFT OF B, $x_1 \approx 3.0$ m



$$\Sigma T_x = 0: 80 - \int_0^{1.0} 12x_2\,dx_2 - T_B^L = 0$$

$$T_B^L = 74 \text{ kN·m}$$ ANS.

R2.2 CONT'D (c) JUST RIGHT OF B, $x_2 \approx 1.0$ m

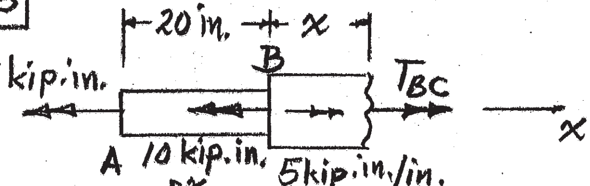


$\Sigma T_x = 0:$

$$80 - \int_0^{1.0} 12x_2\,dx_2 - T_B^R = 0$$

$$T_B^R = 74 \text{ kN·m}$$ ANS.

R2.3



$$\Sigma T_x = 0: T_{BC} + \int_0^x 5\,dx - 10 - 15 = 0$$

$$T_{BC} = 25 - 5x\Big|_0^x = 25 - 5x$$

(a) $\theta_A = \theta_{AB} + \theta_{BC}$

$$\theta_{AB} = \left(\frac{TL}{JG}\right)_{AB} = \frac{15(20)}{(\pi/32)(2^4)(12\times10^3)}$$

$$= 0.015915 \text{ rad}$$

$$\theta_{BC} = \int\left(\frac{T\,dx}{JG}\right)_{BC} = \frac{1}{JG}\int_0^{30}(25 - 5x)\,dx$$

$$= \frac{1}{(\pi/32)(3^4)(4\times10^3)}\left[25x - 2.5x^2\right|_0^{30}$$

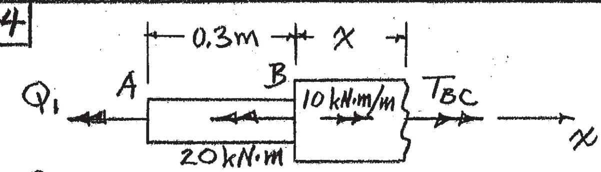
$$= -0.047157 \text{ rad}$$

$\theta_A = -0.0312$ rad WHEN VIEWED FROM A TO C. ANS.

(b) $(T_{BC})_{x=10''} = 25 - 5(10) = -25$ kip·in.

$$\left|\tau_{x=10''}\right| = \left|\frac{TR}{J}\right| = \frac{25(1.5)}{(\pi/32)(3^4)} = 4.716 \text{ ksi}$$ ANS.

R2.4



$T_{AB} = Q_1$

$$\tau_{AB} = \left(\frac{TR}{J}\right)_{AB};\ 120\times10^6 = \frac{Q_1(0.03)}{(\pi/32)(0.06^4)}$$

$$Q_1 = 5.089 \text{ kN·m}$$

$$\Sigma T_x = 0: T_{BC} + \int_0^x 10\,dx - 20 - Q_1 = 0$$

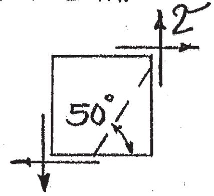
$T_{BC} = 20 + Q_1 - 10x$; $(T_{BC})_{MAX} = 20 + Q_1$ JUST RIGHT OF B.

$$\tau_{BC} = \left(\frac{TR}{J}\right)_{BC};\ 70\times10^6 = \frac{(20+Q_1)(0.06)}{(\pi/32)(0.120^4)}$$

$Q_1 = 3.750$ kN·m; $\therefore (Q_1)_{MAX} = 3.750$ kN·m ANS.

R2.5 $T = Q;\ R = 12 + 0.25 = 12.25$ in.

$$\tau = \frac{TR}{J} = \frac{Q(12.25)}{(\pi/32)(24.5^4)} = 3.463\times10^{-4}Q$$



$$\sigma_n = \tau \sin 2\theta$$

$$20 = (3.463\times10^{-4}Q)\sin(100°)$$

$$Q_1 = 58{,}644.3 \text{ kip}\cdot\text{in.}$$

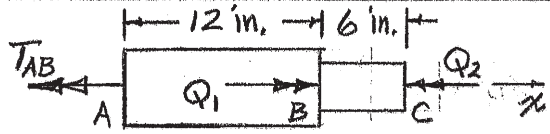
$$\tau_{nt} = \tau\cos 2\theta$$

$$14 = (3.463\times10^{-4}Q)\cos(100°)$$

$$|Q_2| = 232{,}812.0 \text{ kip}\cdot\text{in.}$$

$$\therefore Q_{MAX} = Q_1 = 58{,}644.3 \text{ kip}\cdot\text{in.} \quad \text{ANS.}$$

R2.6



$$\Sigma T_x = 0:\ Q_1 - Q_2 - T_{AB} = 0$$

$$T_{AB} = Q_1 - Q_2$$

$$\tau_{MAX} = \frac{TR}{J};\quad \theta = \frac{TL}{JG}$$

$$\underline{AB}: 15 = \frac{(Q_1-Q_2)(2)}{(\pi/32)(4^4)};\ Q_1 - Q_2 = 188.496 \text{ kip.in.} \text{ — ①}$$

$$\theta_C = \theta_{C/B} + \theta_{B/A}$$

$$0.03 = \frac{Q_2(6)}{(\pi/32)(2^4)(4\times10^3)} + \frac{(Q_1-Q_2)(12)}{(\pi/32)(4^4)(4\times10^3)}$$

$$0.03 = 0.000955Q_2 + 0.000119(Q_1 - Q_2)$$

$$\therefore 300 = 8.36Q_2 + 1.19Q_1 \text{ — ②}$$

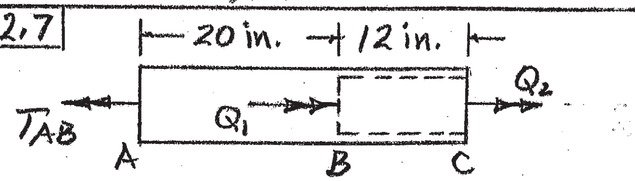
SOLVE ① AND ② TO GET

$$Q_1 = 196.4 \text{ kip.in.} \quad \text{ANS.}$$

$$Q_2 = 7.9 \text{ kip.in.} \quad \text{ANS.}$$

$$\underline{BC}: \tau = \frac{7.9(1)}{(\pi/32)(2^4)} = 5.0 \text{ ksi} < \tau_{ALL} \text{ OK}$$

R2.7



$$\tau_{BC} = \frac{Q_2(2)}{(\pi/32)(4.0^4-3.5^4)} = 0.19230Q_2$$

$$\tau_{AB} = \frac{(Q_1+Q_2)(2)}{(\pi/32)(4.0^4)} = 0.07958(Q_1+Q_2)$$

$$\therefore 0.19230Q_2 = 0.07958(Q_1+Q_2)$$

R2.7 CONT'D

$$\therefore 0.11272Q_2 = 0.07958Q_1$$

$$Q_1/Q_2 = 1.416 \quad \text{ANS.}$$

$$\theta_{C/B} = \frac{Q_2(12)}{(\pi/32)(4.0^4-3.5^4)G} = \frac{1.1538Q_2}{G}$$

$$\theta_{B/A} = \frac{(Q_1+Q_2)(20)}{(\pi/32)(4^4)G} = \frac{0.7958(Q_1+Q_2)}{G}$$

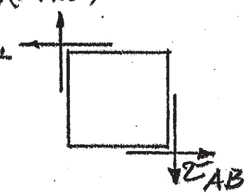
$$\frac{\theta_{B/A}}{\theta_{C/B}} = \frac{0.7958(Q_1+Q_2)}{1.1538Q_2} = \frac{0.7958\left(\frac{Q_1}{Q_2}+1\right)}{1.1538}$$

$$= \frac{0.7958(1.416+1)}{1.1538} = 1.666 \quad \text{ANS.}$$

R2.8 REFER TO THE SKETCH IN PROB. R2.7.

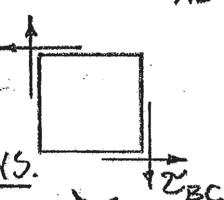
$$(a)\ \tau_{AB} = \left(\frac{TR}{J}\right)_{AB} = \frac{(25+15)(10^3)(0.06)}{(\pi/32)(0.120^4)}$$

$$\tau_{AB} = 117.893 \text{ MPa} \quad \text{ANS.}$$

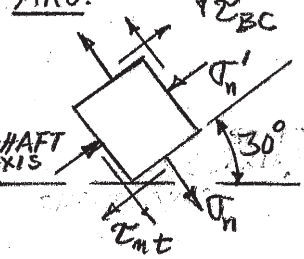


$$(b)\ \tau_{BC} = \left(\frac{TR}{J}\right)_{BC} = \frac{15(10^3)(0.06)}{(\pi/32)(0.120^4-0.100^4)}$$

$$= 85.389 \text{ MPa} \quad \text{ANS.}$$



$$\sigma_n = \tau\sin 2\theta = 85.389\sin 60° = 73.949 \text{ MPa} \quad \text{ANS.}$$



$$\sigma_n' = \tau\sin 2(\theta+90°) = 85.389\sin 240° = -73.949 \text{ MPa} \quad \text{ANS.}$$

$$\tau_{nt} = \tau\cos 2\theta = 85.389\cos 60 = 42.695 \text{ MPa}$$

R2.9 $\Sigma T_{CD} = 0$

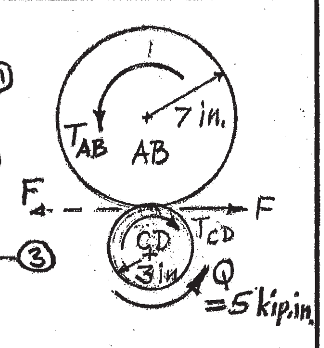
$$5 - T_{CD} - 3F = 0 \text{ — ①}$$

$$\Sigma T_{AB} = 0:\ T_{AB} - 7F = 0$$

$$F = (1/7)T_{AB} \text{ — ②}$$

② INTO ① YIELDS

$$5 - T_{CD} - (3/7)T_{AB} = 0 \text{ — ③}$$



$$3\theta_{C/D} = 7\theta_{B/A}$$

$$3\left[\frac{T_{CD}(24)}{(\pi/32)d^4G}\right] = 7\left[\frac{T_{AB}(20)}{(\pi/32)d^4G}\right]$$

$$T_{CD} = (140/72)T_{AB} \text{ — ④}$$

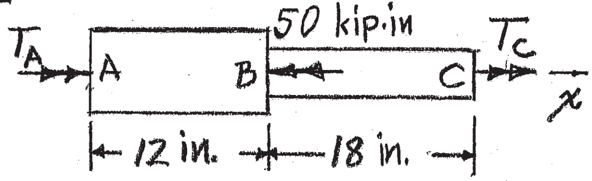
R2.9 CONT'D SOLVE ③ AND ④ TO GET

$T_{AB} = 2.107$ kip.in.; $T_{CD} = 4.097$ kip.in.

$\therefore \tau_{MAX} = 20$ ksi IN CD.

$20 = \frac{4.097(d/2)}{(\pi/32)d^4}$; $d = 1.014$ in. ANS.

R2.10



$\Sigma T_x = 0$: $T_A + T_C = 50$ ——— ①

$\theta_{B/A} = \theta_{B/C}$; $\left(\frac{TL}{JG}\right)_{AB} = \left(\frac{TL}{JG}\right)_{BC}$

$\frac{T_A(12)}{(\pi/32)d^4(4\times10^3)} = \frac{T_C(18)}{(\pi/32)(3.0^4)(6.4\times10^3)}$

$86.400\, T_A = d^4 T_C$ ——— ②

ASSUME BRASS CONTROLS.

$\therefore 18 = \frac{T_C(1.5)}{(\pi/32)(3.0^4)}$; $T_C = 42.412$ kip.in.

① $\Rightarrow T_A = 50 - 42.412 = 7.588$ kip.in.

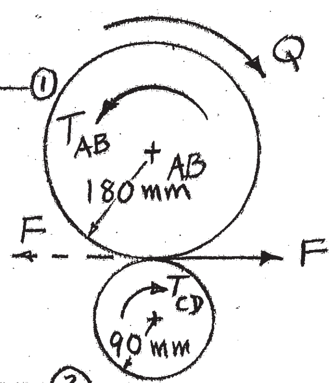
② $\Rightarrow 86.400(7.588) = d^4(42.412)$

$d = 1.983$ in.

$\tau_{AL} = \frac{7.588(1.983/2)}{(\pi/32)(1.983^4)} = 4.956$ ksi $< \tau_{ALL}$

$\therefore d_{MIN} = 1.983$ in. ANS.

R2.11 $\Sigma T_{AB} = 0$



$T_{AB} + 0.18F - Q = 0$ ——— ①

$\Sigma T_{CD} = 0$

$0.09F - T_{CD} = 0$

$F = 11.111\, T_{CD}$ ——— ②

② INTO ① $\Rightarrow$

$T_{AB} + 2T_{CD} = Q$ ——— ③

NOW $0.18\,\theta_{AB} = 0.09\,\theta_{CD}$; $\theta_{CD} = 2\theta_{AB}$

$\therefore \frac{T_{CD}(0.3)}{(\pi/32)(0.06^4)(25\times10^9)} = 2\left[\frac{T_{AB}(0.3)}{(\pi/32)(0.03^4)(70\times10^9)}\right]$

$T_{CD} = 11.429\, T_{AB}$ ——— ④

R2.11 CONT'D

ASSUME ALUMINUM CONTROLS. THUS,

$80\times10^6 = \frac{T_{CD}(0.03)}{(\pi/32)(0.06^4)}$; $T_{CD} = 3.393$ kN.m

④ $\Rightarrow T_{AB} = 0.297$ kN.m

$\tau_{AB} = \frac{(0.297)(10^3)(0.015)}{(\pi/32)(0.03^4)} = 56.0$ MPa $< \tau_{ALL}$

③ $\Rightarrow Q = 0.297 + 2(3.393) = 7.083$ kN.m ANS.

R2.12 $P = T\omega$; $T = \frac{P}{\omega}$

$T = \frac{500(6600)}{(400)(2\pi/60)} = 78{,}781.697$ lb.in.

$\tau = \frac{78{,}781.697(3.0)}{(\pi/32)(6.0^4 - d_i^4)} = 10\times10^3$

$d_i = 5.6995 \approx 5.7$ in.

$\theta = \frac{78{,}781.697(12)}{(\pi/32)(6.0^4 - d_i^4)(4\times10^6)} = 0.015$

$d_i = 5.8049 \approx 5.8$ in

$\therefore (d_i)_{MAX} = 5.7$ in. ANS.

R2.13 $U = \frac{1}{2}F\Delta$

$500(0.05 + \Delta) = \frac{1}{2}F\Delta$ ——— ①

$\Delta = \theta b = \left(\frac{TL}{JG}\right)b = \frac{Fb^2L}{JG} = \frac{F(0.4^2)(0.6)}{(\frac{\pi}{32})(0.11^4 - d_i^4)(25\times10^9)}$

$\Delta = \frac{3.91139\times10^{-11}F}{X}$ WHERE $X = (0.11^4 - d_i^4)$

SUBSTITUTING INTO ① AND SIMPLIFYING YIELDS

$F^2 - 1\times10^3 F - 12.78317\times10^{11} X = 0$ ——— ②

$\tau = \frac{TR}{J}$; $100\times10^6 = \frac{0.4F(0.055)}{(\pi/32)X}$

$X = 2.241\times10^{-9}F$ ——— ③

③ INTO ② LEADS TO

$F^{\cancel{2}} - 3.8647\times10^3 \cancel{F} = 0$; $F = 3{,}865$ N

③ $\Rightarrow X = 2.241\times10^{-9}(3865) = 866.1\times10^{-8}$

$\therefore 0.11^4 - d_i^4 = 866.1\times10^{-8}$; $d_i = 0.108$ m

$\theta = \frac{TL}{JG}$; $0.05 = \frac{0.4F(0.6)}{(\pi/32)X(25\times10^9)}$

$X = 1.956\times10^{-9}F$ ——— ④

④ INTO ② YIELDS

$F^{\cancel{2}} - 26\times10^3 \cancel{F} = 0$; $F = 3{,}500$ N

④ $\Rightarrow X = 6.825\times10^{-6}$

$\therefore 0.11^4 - d_i^4 = 6.825\times10^{-6}$; $d_i = 0.109$ m

$(d_i)_{MAX} = 0.108$ m $= 108$ mm ANS.

R2.14

$$\sigma_n = \frac{75\times10^9}{(1-0.33^2)}\left[500+0.33(-300)\right]\times10^{-6}$$
$$= 33.750 \text{ MPa}$$

$$\sigma_t = \frac{75\times10^9}{(1-0.33^2)}\left[-300+0.33(500)\right]\times10^{-6}$$
$$= -11.362 \text{ MPa}$$

FROM EXAMPLE 2.12, WE HAVE

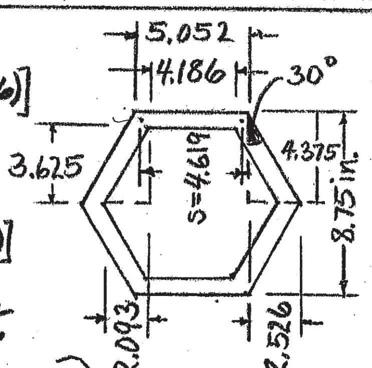
$$\sigma_m = \frac{1}{2}\sigma_x(1+\cos2\theta)+\tau_{xy}\sin2\theta$$
$$33.750 = \frac{1}{2}\sigma_x(1+\cos100)+\tau_{xy}\sin100$$
$$33.750 = 0.413\,\sigma_x + 0.985\,\tau_{xy} \quad\text{—— ①}$$
$$\sigma_t = \frac{1}{2}\sigma_x(1+\cos2(\theta+90))+\tau_{xy}\sin2(\theta+90)$$
$$-11.362 = \frac{1}{2}\sigma_x(1+\cos280)+\tau_{xy}\sin280$$
$$-11.362 = 0.587\,\sigma_x - 0.985\,\tau_{xy} \quad\text{—— ②}$$

SOLVE ① & ② TO GET

$$\sigma_x = 22.388 \text{ MPa};\quad \tau_{xy} = 24.883 \text{ MPa}$$
$$\sigma_x = \frac{P}{A};\quad P = \sigma_x A = 22.388\times10^6\left(\frac{\pi}{4}\right)(0.1^2)$$
$$P \approx 175.8 \text{ kN}$$ ANS.

$$\tau_{xy} = \frac{TR}{J};\quad T = \frac{\tau_{xy}J}{R} = \frac{24.883\times10^6(\pi/32)(0.1^4)}{0.05}$$
$$Q = T = 4.886 \text{ kN·m}$$ ANS.

R2.15



$$A_o = 2\left[\left(\tfrac{1}{2}\right)(8.75)(2.526)\right] + 5.052(8.75)$$
$$A_o = 66.3075 \text{ in}^2$$
$$A_i = 2\left[\left(\tfrac{1}{2}\right)(7.25)(2.093)\right] + 4.186(7.25)$$
$$A_i = 45.5228 \text{ in}^2$$
$$A_m = \frac{1}{2}(A_o+A_i) = 55.915 \text{ in}^2$$
$$\tau = \frac{T}{2tA_m};\quad T = 2tA_m\tau$$
$$T = 2(0.75)(55.915)(12) = 1006.5 \text{ kip·in.}$$
$$\theta = \frac{TL}{4GA_m^2}\sum\left(\frac{s}{t}\right);\quad T = \frac{4GA_m^2\theta}{L\sum(s/t)}$$
$$T = \frac{4(5.0\times10^3)(55.915^2)(2\pi/180)}{4(12)\left[6(4.619/0.75)\right]} = 1230.6 \text{ kip·in}$$
$$Q_{MAX} = 1006.5 \text{ kip·in.}$$ ANS.

R2.16

$$K = \frac{1}{3}\sum Lt^3 = \frac{1}{3}\left[6(4.619)(0.75^3)\right] = 3.897 \text{ in}^4$$
$$\tau = \frac{Tt}{K};\quad T = \frac{\tau K}{t} = \frac{12(3.897)}{0.75} = 62.352 \text{ kip·in}$$
$$\theta = \frac{TL}{KG};\quad T = \frac{KG\theta}{L} = \frac{3.897(5\times10^3)(2\pi/180)}{4(12)}$$
$$= 14.170 \text{ kip·in.}$$
$$\therefore Q_{MAX} = 14.170 \text{ kip·in.}$$ ANS.

R2.17

FROM PROBLEM 2.132 WE HAVE

$$T = \frac{\pi\tau_y}{2}\left[\frac{4R_o^3-\rho_y^3}{3} - \frac{R_i^4}{\rho_y}\right]$$

REWRITING THIS EQ. WE OBTAIN THE FOLLOWING POLYNOMIAL:

$$\rho_y^4 + \left(\frac{6T}{\pi\tau_y} - 4R_o^3\right)\rho_y + 3R_i^4 = 0$$

SUBSTITUTING GIVEN NUMERICAL VALUES, THE ABOVE EQ. REDUCES TO

$$\rho_y^4 - 5.313\times10^{-4}\rho_y + 7.68\times10^{-6} = 0$$

A TRIAL-AND-ERROR SOLUTION YIELDS

$$\rho_y = 0.07545 \text{ m}$$
$$d = R_o - \rho_y = 0.08 - 0.07545$$
$$= 0.00455 \text{ m} = 4.55 \text{ mm}$$ ANS.