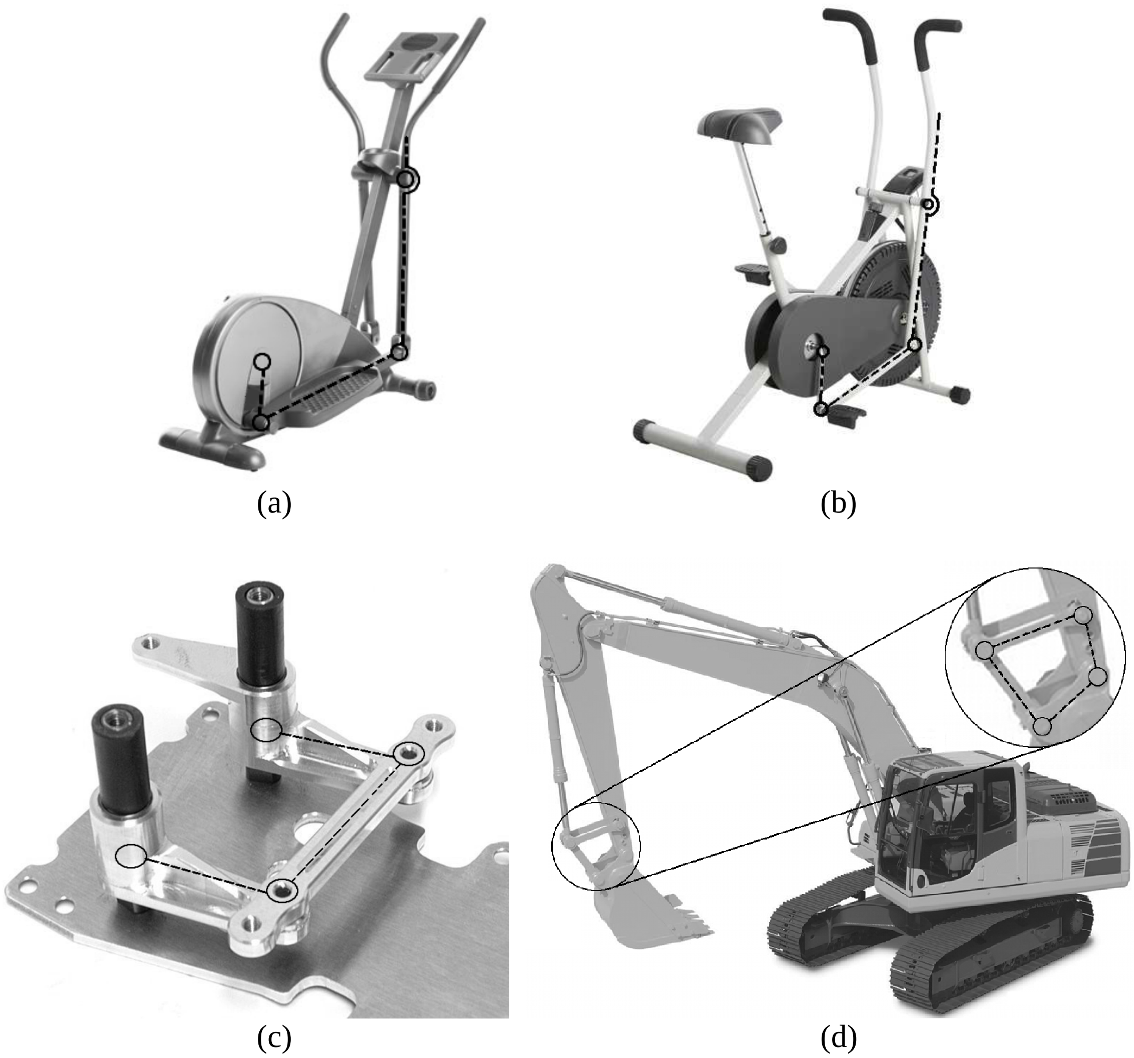
CHAPTER 2 MOBILITY OF MECHANISMS

Problem 2.1 Statement:

Planar four-bar linkages have many everyday applications (some are illustrated Figure 2.1). Identify and describe four additional everyday applications for the planar four-bar linkage.

Problem 2.1 Solution:

As illustrated in Solution 2.1 Figure 1, the (a) elliptical trainer, (b) fan-style exercise bicycle, (c) model car steering linkage and (d) excavator bucket all include the planar four-bar linkage.



Solution 2.1 Figure 1

Problem 2.2 Statement:

a) Why is it important to know if a mechanism has a single degree of freedom?

b) Why is a crank-rocker mechanism more useful than a double-rocker mechanism?

c) Should the transmission angle for the planar four-bar linkage be close to 0°? Explain?

Problem 2.2 Solution:

a) Because the DOF of a mechanism is the number of mechanism parameters required to fully define its motion, a mechanism having a single DOF requires a single mechanism motion parameter to fully define mechanism motion. As a result, a single DOF condition provides maximum mechanism motion control.

b) Because the driving joint in a crank-rocker mechanism is grounded, it can be directly affixed to a grounded drive system (to compel mechanism motion). The driving joint in the double-rocker mechanism however is not grounded and cannot be directly affixed to a grounded drive system. When considering a grounded drive system, the crank-rocker mechanism is a more practical design option than the double-rocker mechanism.

c) Revolute joints by design are better suited to handle transverse loads (loads perpendicular to a link) than loads acting along the link length. For the follower link, the proportions these two follower load components are determined by the transmission angle. It can be determined from the equations in Figure 2.8b that the optimum transmission angle is 90° because it produces a zero force along the follower length.

Problem 2.3 Statement:

For the two linkages illustrated in Figure P.2.1, which (if any) of the links can undergo a complete rotation relative to the other links? How do you know?

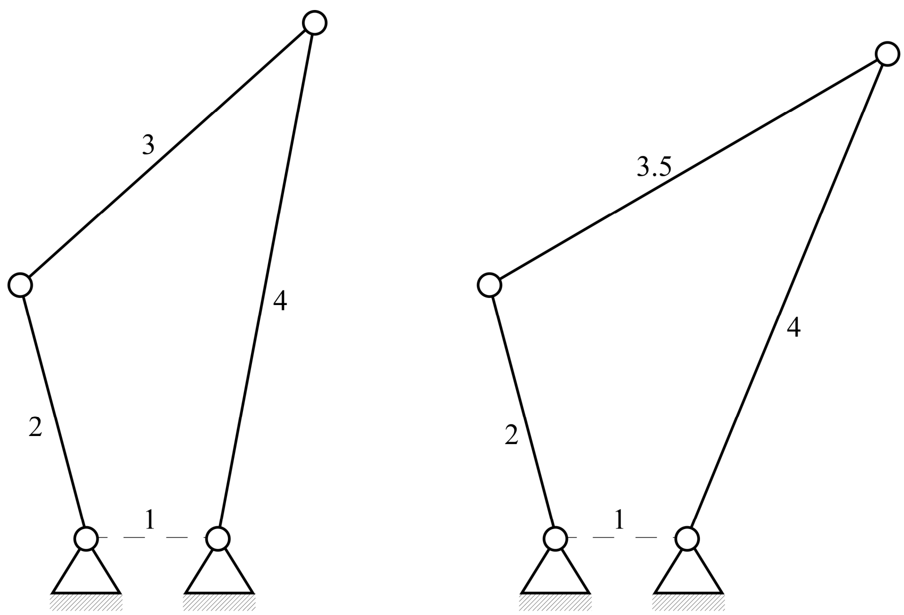


Figure P.2.1 Planar four-bar linkages with dimensionless link lengths

Problem 2.3 Solution:

For the first mechanism, the Grashof condition $S + L < P + Q$ holds true and the shortest link is ground. This makes the first mechanism a *Double-Crank* (or *Drag-Link*) mechanism and capable of full crank rotation. Because the Grashof condition $S + L < P + Q$ does not hold true for the second mechanism, it is a *Non-Grashof* mechanism and by definition incapable of full link rotation.

Problem 2.4 Statement:

Determine the number of links, joints and the mobility of each of the three planar mechanisms in Figure P.2.2.

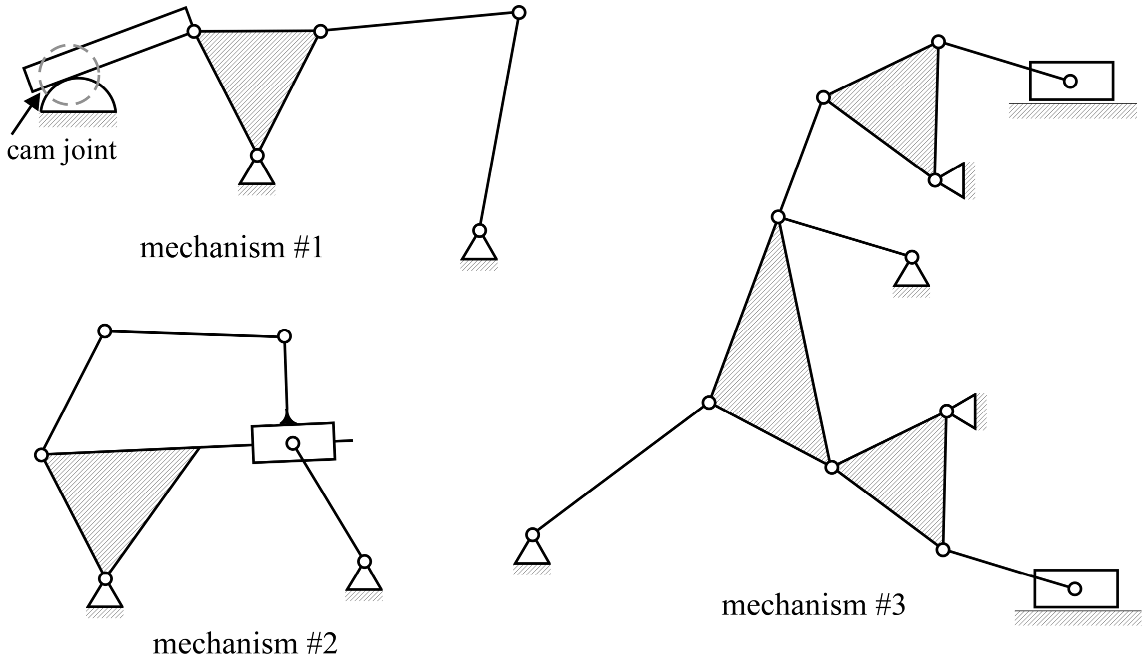


Figure P.2.2 Planar mechanisms

Problem 2.4 Solution:

Mechanism #1 includes 5 links ($L = 5$), 5 1-DOF joints ($J_1 = 5$) and 1 2-DOF joint ($J_2 = 1$). Using Equation (2.1) mechanism #1 has a mobility of 1. Mechanism #2 includes 6 links ($L = 6$) and 7 1-DOF joints ($J_1 = 7$). Using Equation (2.1) mechanism #2 has a mobility of 1.

Mechanism #3 includes 11 links ($L = 11$) and 14 1-DOF joints ($J_1 = 14$). Using Equation (2.1) mechanism #3 has a mobility of 2.

Problem 2.5 Statement:

For the planar four-bar linkage illustrated in Figure P.2.3, $L_2/L_1 = 1.5$ and $L_3/L_1 = 1.2$. Find the range of L_0/L_1 required for a *drag link mechanism*.

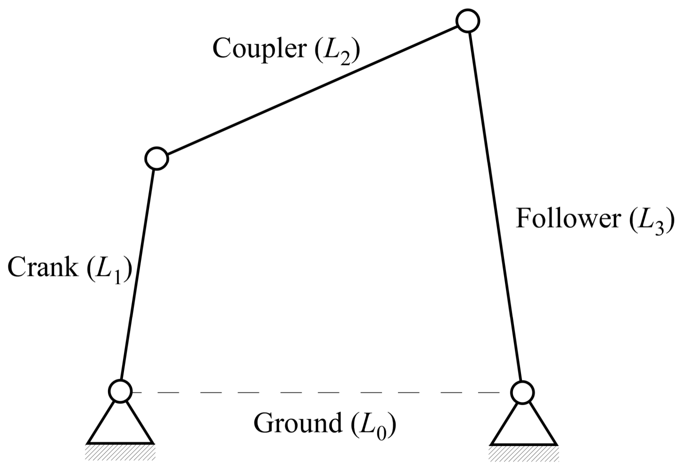


Figure P.2.3 Planar four-bar linkage

Problem 2.5 Solution:

From the link length ratios given in the problem statement we know that $L_2 = 1.5L_1$, $L_3 = 1.2L_1$ and $L_0 = xL_1$ (where the value of the unknown variable x is to be determined). Because the shortest link in a Drag-Link mechanism is the ground link, the Grashof link length relationship $1.5L_1 + xL_1 < 1.2L_1 + L_1$ is formulated. Solving for x produces $x < 0.7$. Therefore, for a Drag-Link mechanism, $L_0/L_1 < 0.7$.

Problem 2.6 Statement:

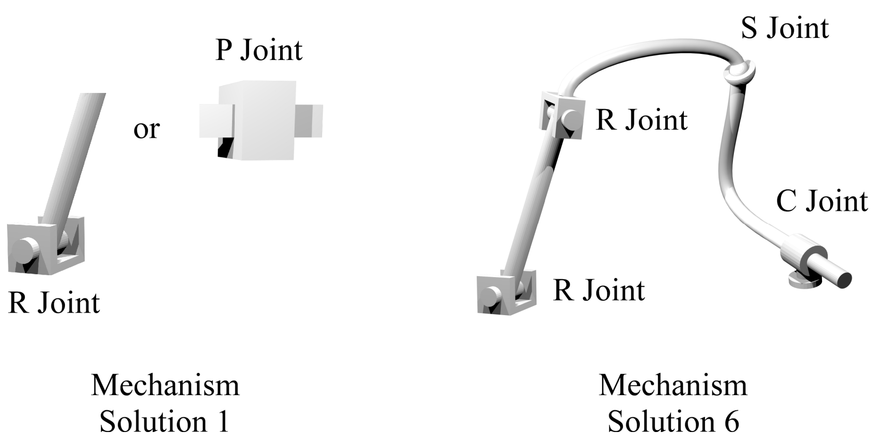
Compile a table of 1 DOF spatial mechanisms having 2, 3 and 4 links (let $J_4 = J_5 = 0$ in Equation (2.2)). Illustrate some of these mechanism solutions.

Problem 2.6 Solution:

Solution 2.6 Table 1 includes the 1 DOF mechanism solutions produced having 2, 3 and 4 links from Equation (2.4). Solutions 1 and 6 are illustrated in Solution 2.6 Figure 1.

Solution 2.6 Table 1

Mechanism Solution	L	J_1	J_2	J_3
1	2	1	0	0
2	3	0	2	1
3	3	1	0	2
4	4	0	2	3
5	4	1	3	0
6	4	2	1	1



Solution 2.6 Figure 1

Problem 2.7 Statement:

7. Figure P.2.4 illustrates 9 spatial mechanisms that include revolute (R), prismatic (P), cylindrical (C) and spherical (S) joints. Calculate the mobility of these mechanisms.

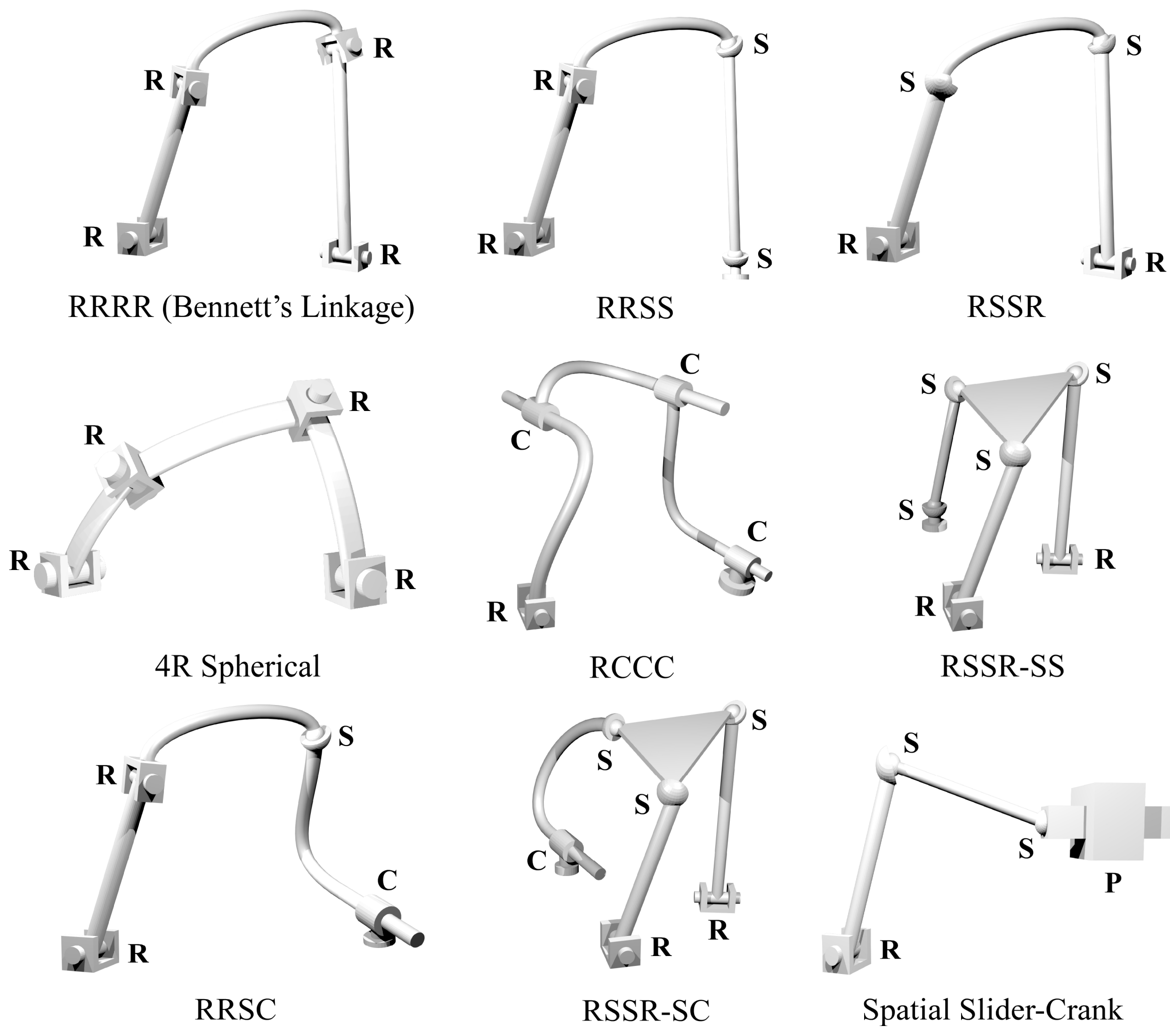


Figure P.2.4 Spatial mechanisms comprised of R, P, C and S joints

Problem 2.7 Solution:

Both the RRRR and 4R Spherical mechanisms have 4 links ($L = 4$) and 4 1-DOF joints ($J_1 = 4$).

Using Equation (2.2), both the RRRR and 4R Spherical mechanisms have a mobility of −2.

Both the RRSS and RSSR mechanisms have 4 links ($L = 4$), 2 1-DOF joints ($J_1 = 2$) and 2 3-

DOF joints ($J_3 = 2$) . Using Equation (2.2), both the RRSS and RSSR mechanisms have a mobility of 2.

The RCCC mechanism has 4 links ($L = 4$), 1 1-DOF joint ($J_1 = 1$) and 3 2-DOF joints ($J_2 = 3$) Using Equation (2.2), the RCCC mechanism has a mobility of 1.

The RRSC mechanism has 4 links ($L = 4$), 2 1-DOF joints ($J_1 = 2$), 1 2-DOF joint ($J_2 = 1$) and 1 3-DOF joint ($J_3 = 1$). Using Equation (2.2), the RRSC mechanism has a mobility of 1.

The RSSR-SS mechanism has 5 links ($L = 5$), 2 1-DOF joints ($J_1 = 2$) and 4 3-DOF joints ($J_3 = 4$). Using Equation (2.2), the RSSR-SS mechanism has a mobility of 2.

The RSSR-SC mechanism has 5 links ($L = 5$), 2 1-DOF joints ($J_1 = 2$), 1 2-DOF joint ($J_2 = 1$) and 3 3-DOF joints ($J_3 = 3$). Using Equation (2.2), the RSSR-SC mechanism has a mobility of 1.

The Spatial Slider-Crank mechanism has 4 links ($L = 4$), 2 1-DOF joints ($J_1 = 2$) and 2 3-DOF joints ($J_3 = 2$). Using Equation (2.2), the Spatial Slider-Crank mechanism has a mobility of 2.

Problem 2.8 Statement:

Euler's buckling load (F) for a columnar member with pinned ends is

$$F = \frac{\pi^2 EI}{L^2}$$

where E, I and L are the modulus of elasticity, moment of inertia and length respectively of the columnar member. Formulate an equation from Euler's equation and Figure P.2.5 to calculate the transmission angle corresponding to follower link buckling. Calculate the transmission angle for follower buckling given a follower length of 12 in (0.3048m), a ¼ in (0.635cm) square follower cross section, a modulus of elasticity (for aluminum) of 10,000,000psi (68.05GPa) and a follower load $\left(\mathbf{F}_{\text{follower}}\right)$ of 250 lbf (1112.05N).

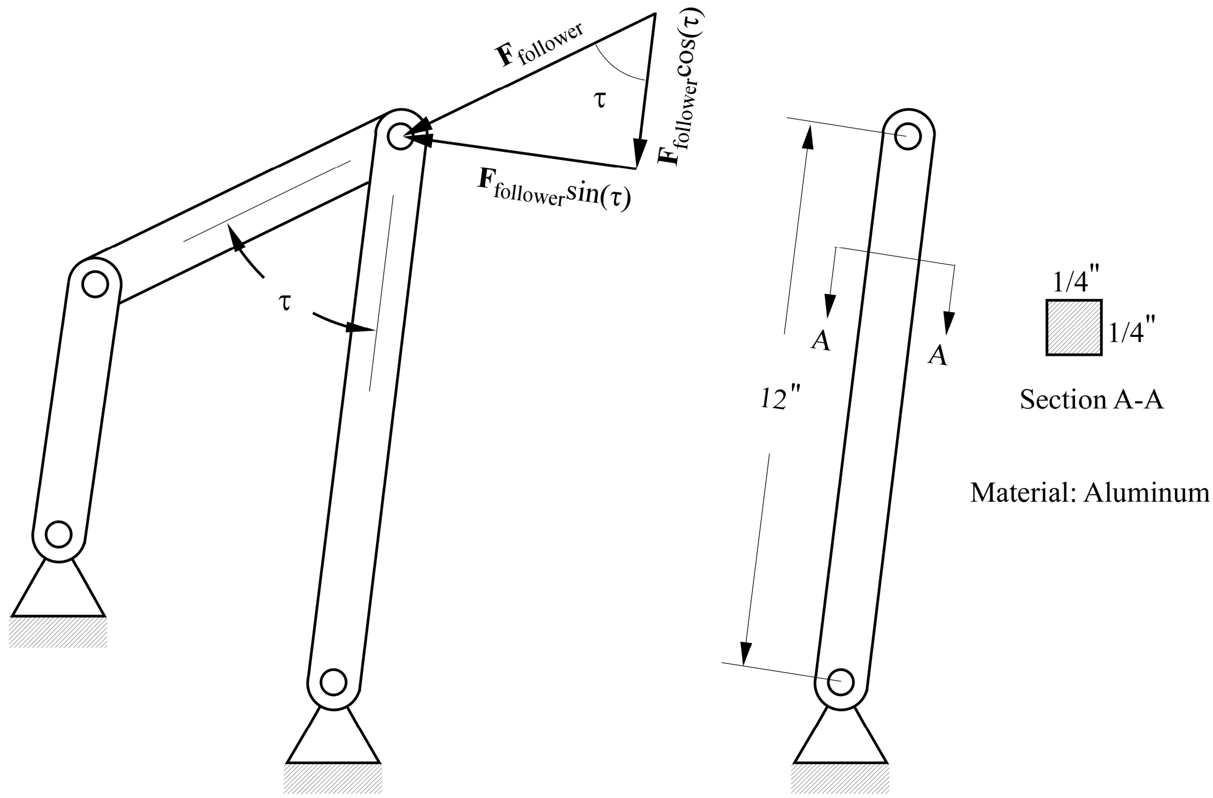


Figure P.2.5 Planar four-bar mechanism with transmission angle and crank and follower loads

Problem 2.8 Solution:

It can be concluded from the buckling force equation provided and the follower column force vector in Figure P.2.5 that $\mathbf{F}_{\text{follower}} \cos(\tau) = F = \pi^2 EI/L^2$. Solving for the transmission angle in this equation produces $\tau = \cos^{-1}\left(\pi^2 EI/L^2\mathbf{F}_{\text{follower}}\right)$. Using the values provided for E, I, L and $\mathbf{F}_{\text{follower}}$, a transmission angle of $\tau = 26.8236^{\circ}$ is calculated.

Problem 2.9 Statement:

Maverick Mechanisms are mechanisms that defy Grubler's Equation (Grubler's Equation will produce misleading results for Maverick Mechanisms). *Passive Degrees of Freedom* are localized DOFs that have no effect on the overall mechanism kinematics. Determine which mechanism from Figure P.2.4 are maverick mechanisms and locate the passive DOFs from among the mechanisms in Figure P.2.4.

Problem 2.9 Solution:

Bennett's linkage (the RRRR mechanism) and the 4R Spherical mechanism are Maverick Mechanisms because contrary to Grubler's Equation, both mechanisms have a single DOF. The RRSS, RSSR, RSSR-SS and Spatial Slider-Crank mechanisms each have a passive degree of freedom in their S-S links. The passive DOF for these links allow for the free rotation about the link lengths.

Problem 2.10 Statement:

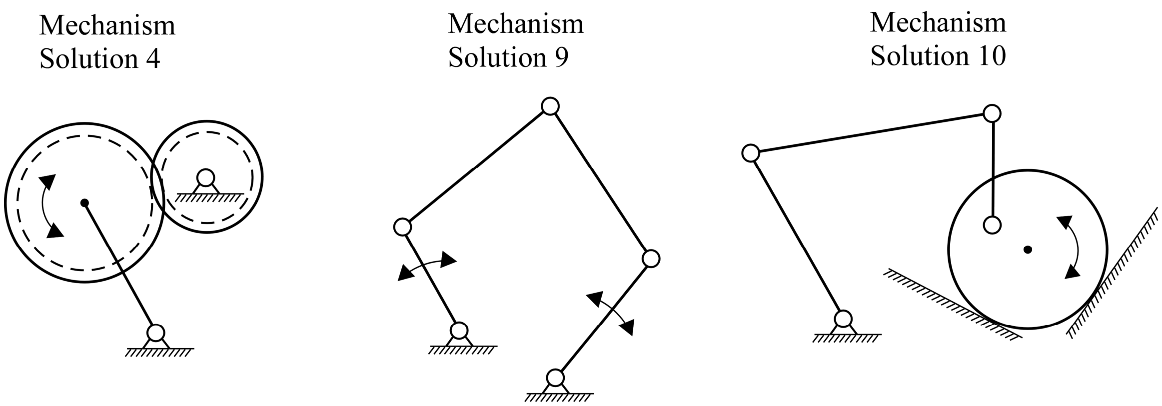
Compile a table of 2 DOF planar mechanisms having 4 and 5 links (considering J_2-cam and gear joints). Illustrate one four-bar mechanism solution and one five-bar mechanism solution

Problem 2.10 Solution:

Solution 2.10 Table 1 includes the 1 DOF mechanism solutions produced having 4 and 5 links from Equation (2.3). Solutions 4, 9 and 10 are illustrated in Solution 2.10 Figure 1.

Solution 2.10 Table 1

Mechanism Solution	L	J_1	J_2
1	4	0	7
2	4	1	5
3	4	2	3
4	4	3	1
5	5	0	10
6	5	1	8
7	5	2	6
8	5	3	4
9	5	4	2
10	5	5	0



Solution 2.10 Figure 1