Understanding Customer Demand: Forecasting

Factors that Influence Demand

- Product characteristics
- Past demand
- Economic condition
- Competition
- Planned marketing efforts
- Planned price discount

Forecasting Methods

- ***Qualitative***: rely on human judgment
  - – Market survey (customer response)
  - – Delphi technique (expert opinion)
- ***Causal***: demand is highly correlated with certain factors
- ***Time Series***: past demand is a good indicator of future demand
- ***Simulation***: mimic consumer behavior to conduct what-if analysis

Time-series Forecasting

- Constant process
  - Average
  - Moving average
  - Exponential smoothing
- Trend process
  - Regression
  - Double exponential smoothing
- Seasonal process

Characteristics of Forecast

- Forecasts are always wrong and thus should include an error analysis
- Long-term forecasts are usually less accurate than short-term forecasts
- Aggregate forecasts are usually more accurate than disaggregate forecasts

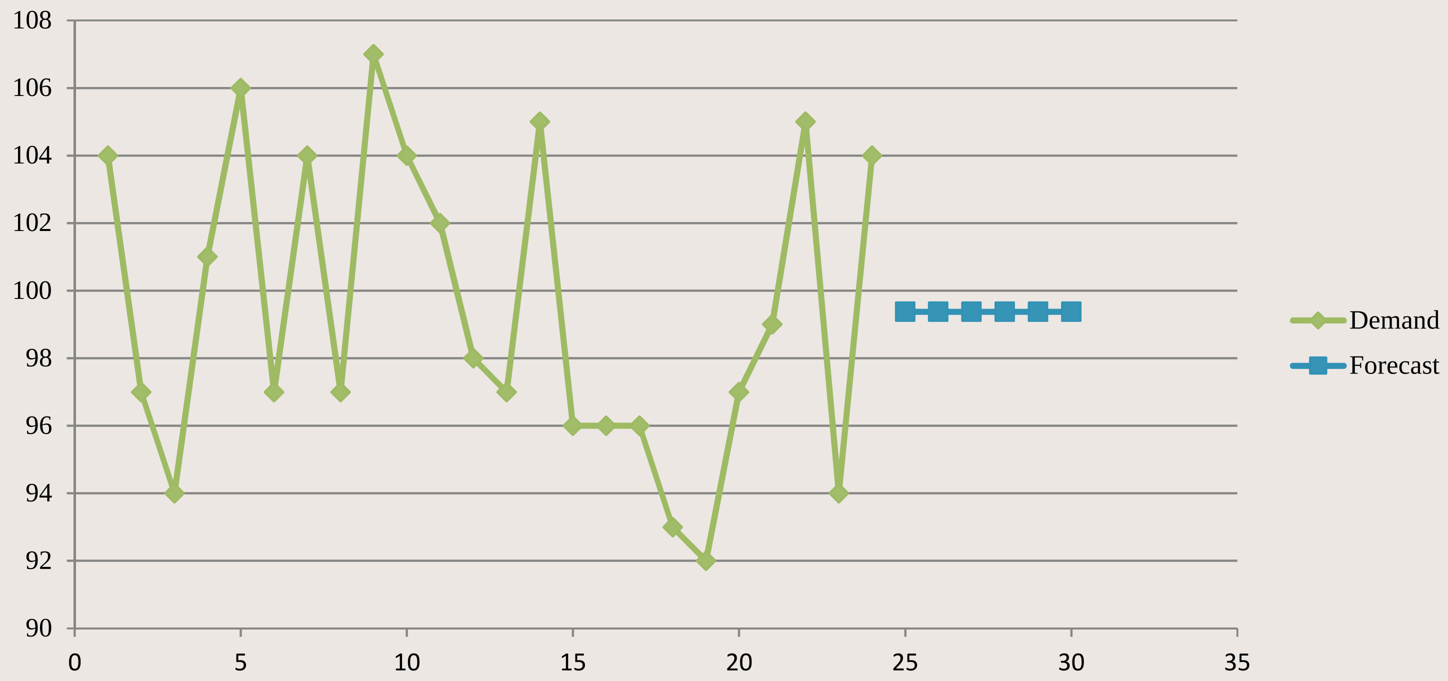
Constant Model: Average

- Constant model $d_t = a + \varepsilon_t$
- Forecast $\hat{a} = \frac{\sum_{t=1}^{N} d_t}{N}$
- Derived based on minimizing the sum of squared errors $e_t = d_t - \hat{a}$

$$\frac{d\left(\sum_{t=1}^{N} e_t^2\right)}{d\hat{a}} = \frac{d\left[\sum_{t=1}^{N}(d_t - \hat{a})^2\right]}{d\hat{a}} = -2\sum_{t=1}^{N}\left(d_t - \hat{a}\right) = 0$$

$$\sum_{t=1}^{N} d_t = N\hat{a}$$

Example: Average



Moving Average

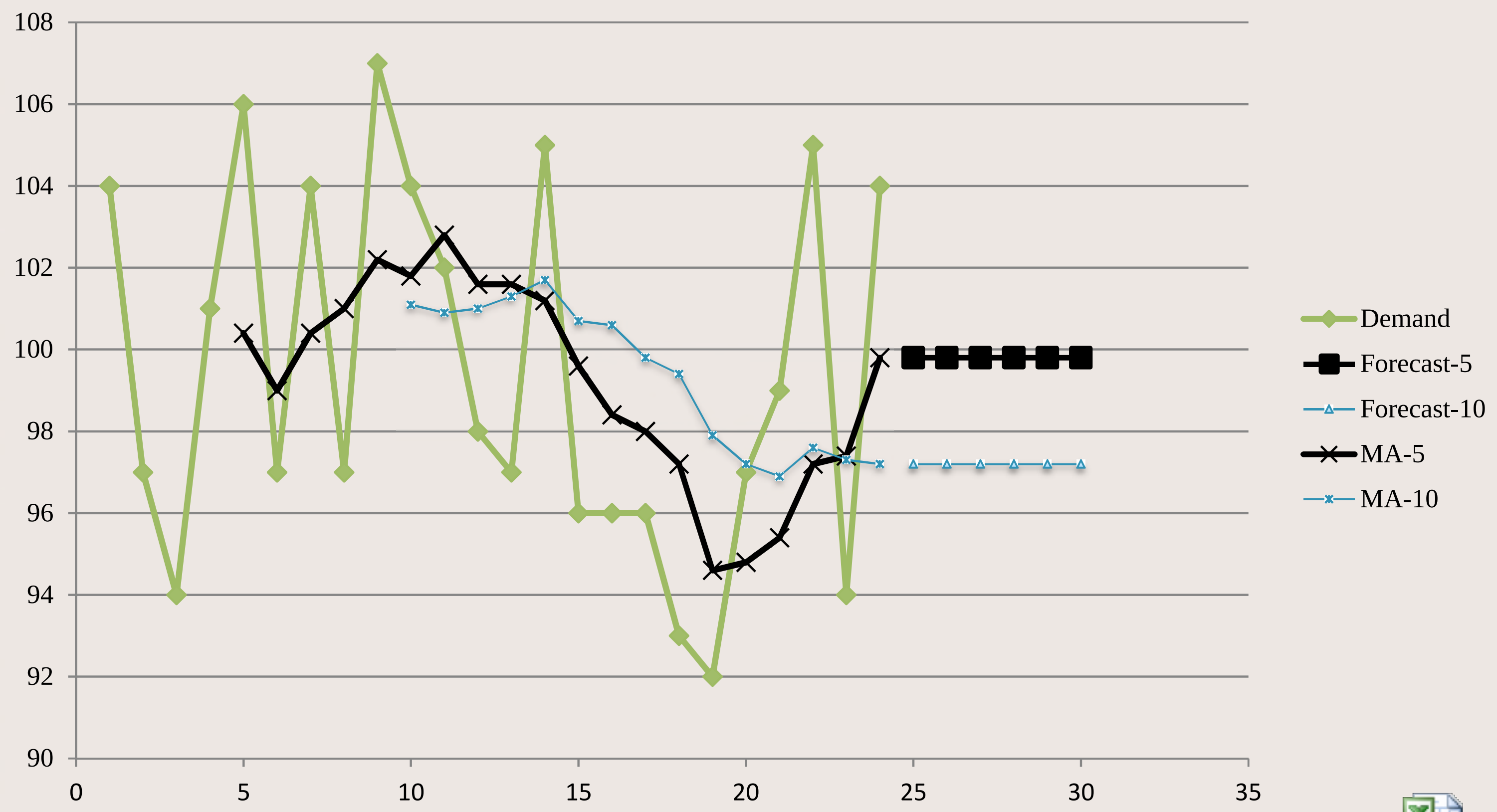
- Average only the most recent data points

$$M_t = \frac{\sum_{t-n+1}^{t} d_t}{n}$$

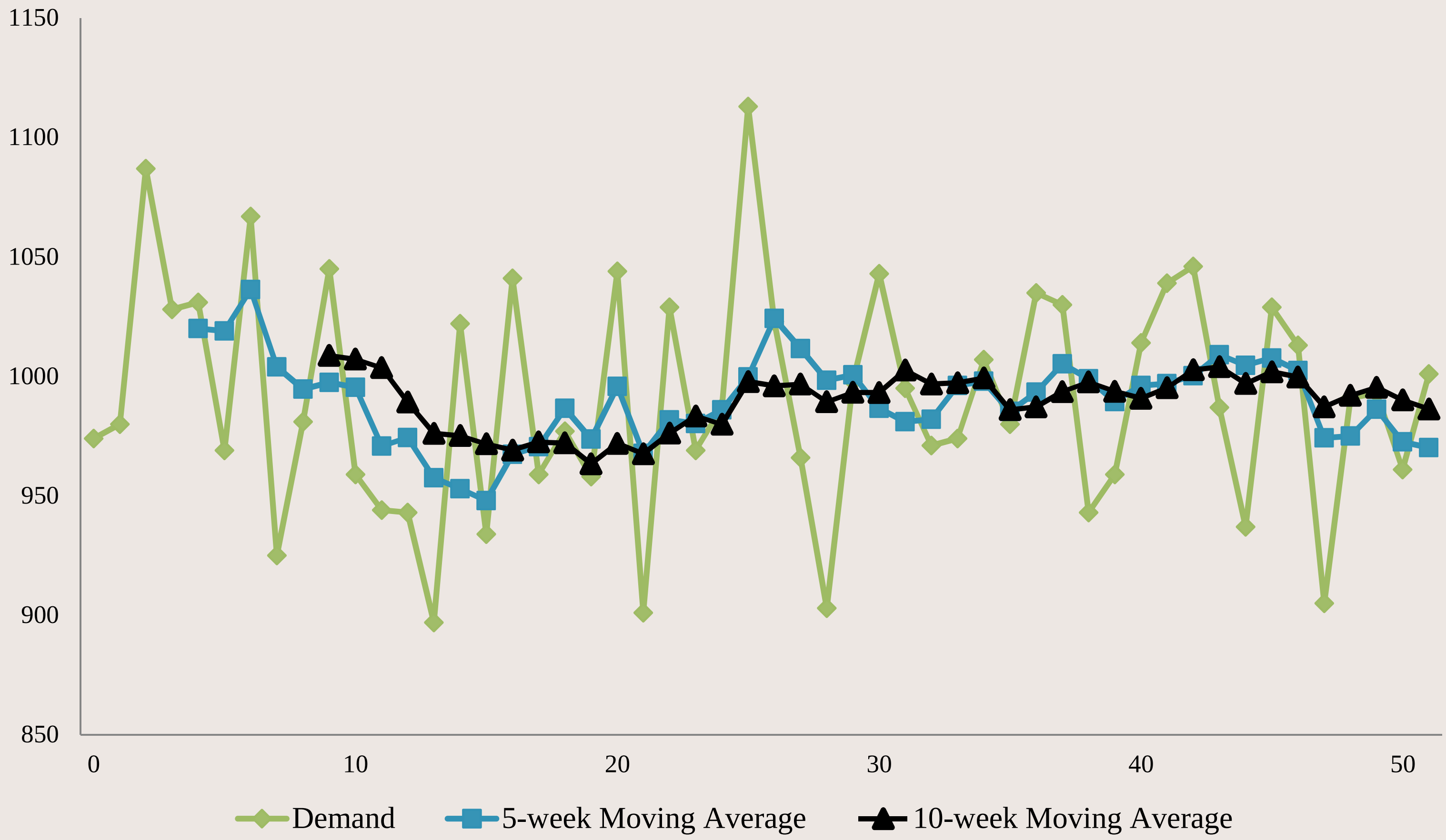
$$M_{t+1} = M_t + \frac{d_{t+1} - d_{t-n+1}}{n}$$

- Smooth out noise
- Can respond to change in process

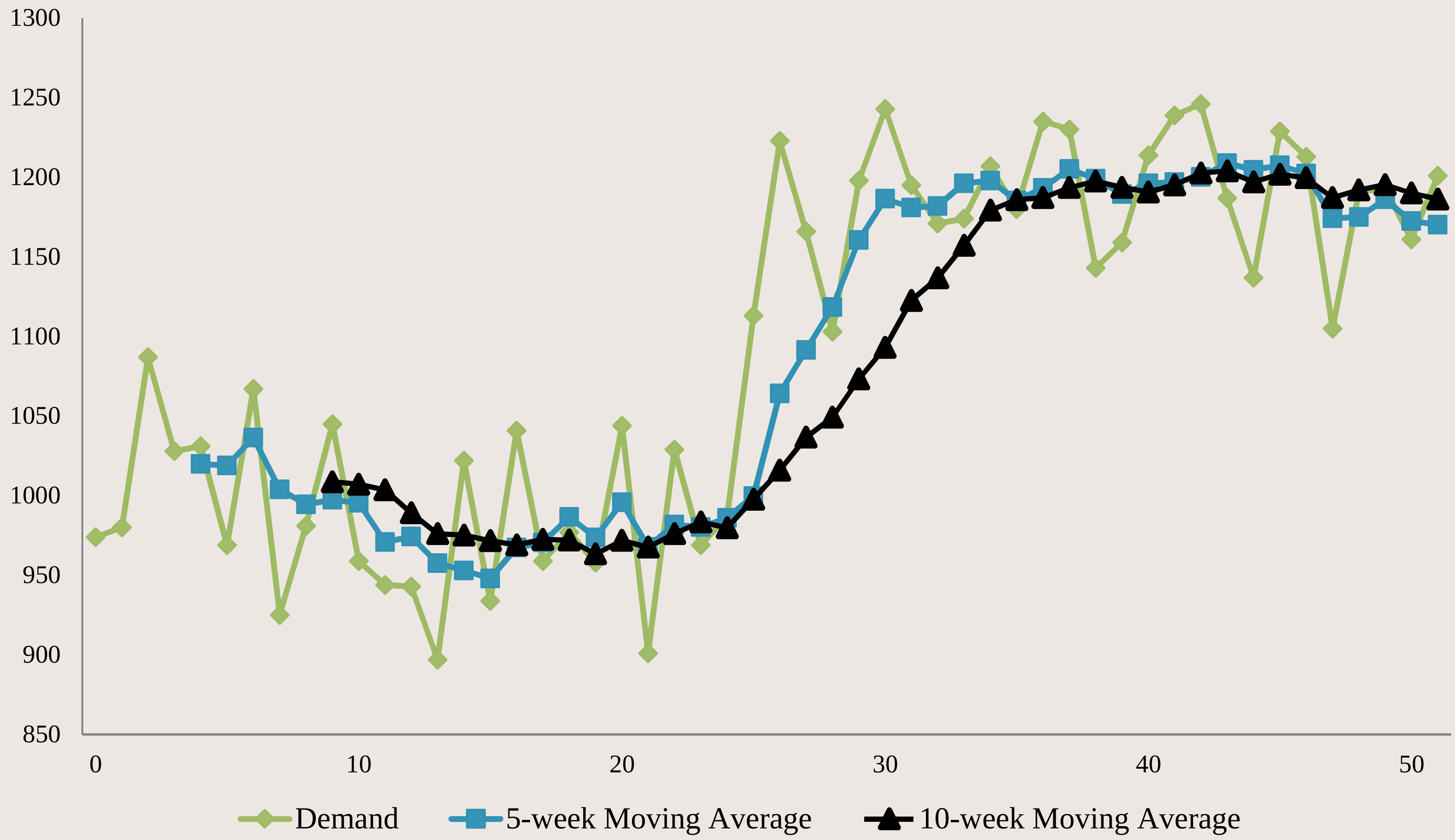
Example: Moving Average



Noise Smoothing



Response to Process Change



Exponential Smoothing

- Adjust forecast based on the most recent data point

$$S_t = \alpha \times d_t + (1-\alpha) \times S_{t-1}$$

$$= \alpha d_t + (1-\alpha)\left[\alpha d_{t-1} + (1-\alpha)S_{t-2}\right]$$

$$= \alpha d_t + \alpha(1-\alpha)d_{t-1} + (1-\alpha)^2\left[\alpha d_{t-2} + (1-\alpha)S_{t-3}\right]$$

$$= \alpha d_t + \alpha(1-\alpha)d_{t-1} + \alpha(1-\alpha)^2 d_{t-2} + \ldots + \alpha(1-\alpha)^{t-1} d_1 + (1-\alpha)^t S_0$$

- It is a weighted average of all historical data points, with the weight decreasing exponentially with the age of the data point

Exponential Smoothing Forecast for Constant Process

- Different initial estimates can be used – average of several past data points

$$E[S_t] = E[\alpha \sum_{i=0}^{t-1} (1-\alpha)^i d_{t-i} + (1-\alpha)^t S_0]$$

$$= E[\alpha \sum_{i=0}^{t-1} (1-\alpha)^i d_{t-i}] + E[(1-\alpha)^t S_0]$$

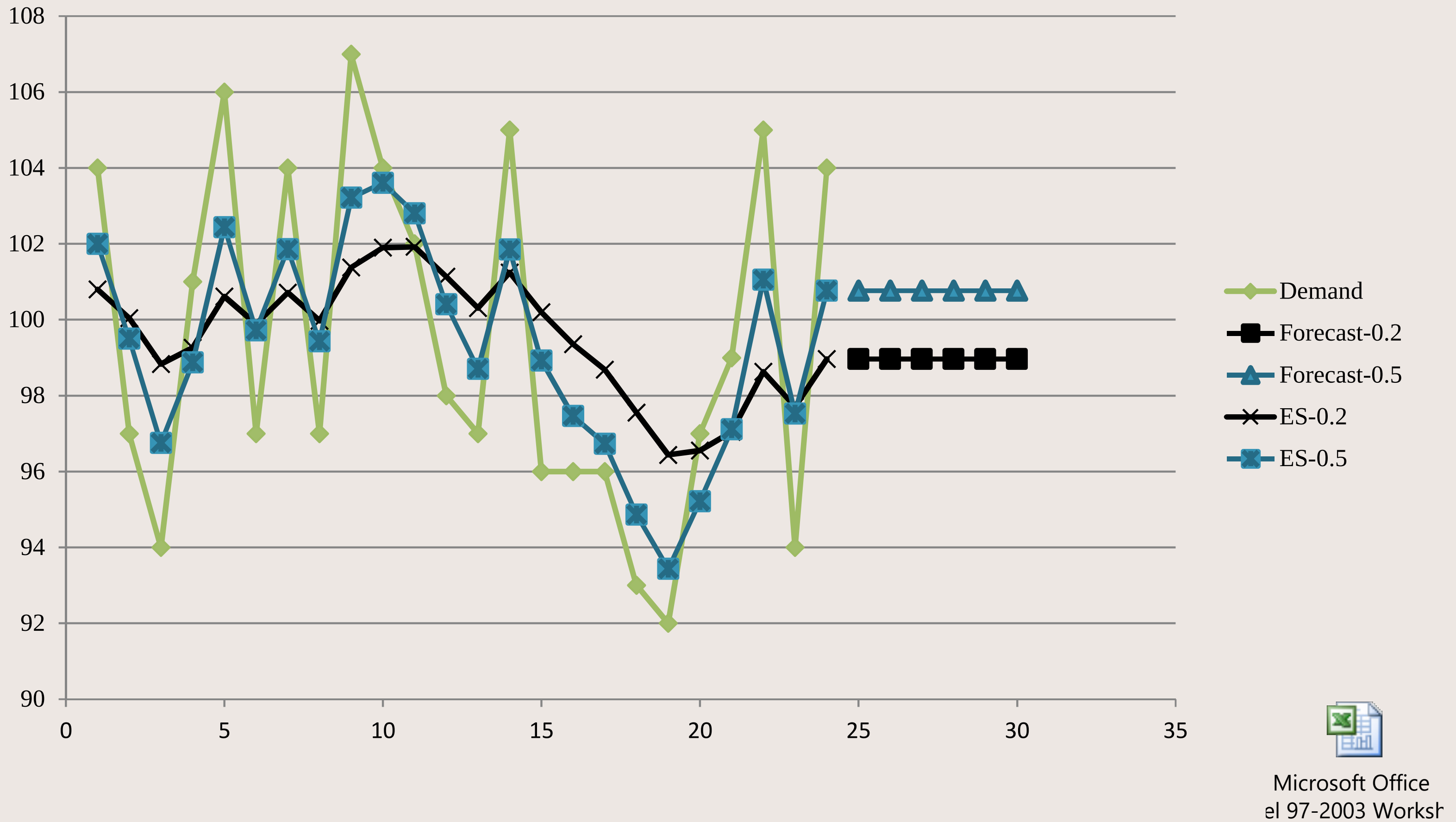
$$\lim_{t\to\infty} (1-\alpha)^t = 0$$

$$E[S_t] = E\left[\alpha \sum_{i=0}^{t-1} (1-\alpha)^i d_{t-i}\right] = \alpha \sum_{i=0}^{t-1} (1-\alpha)^i E[d_{t-i}]$$

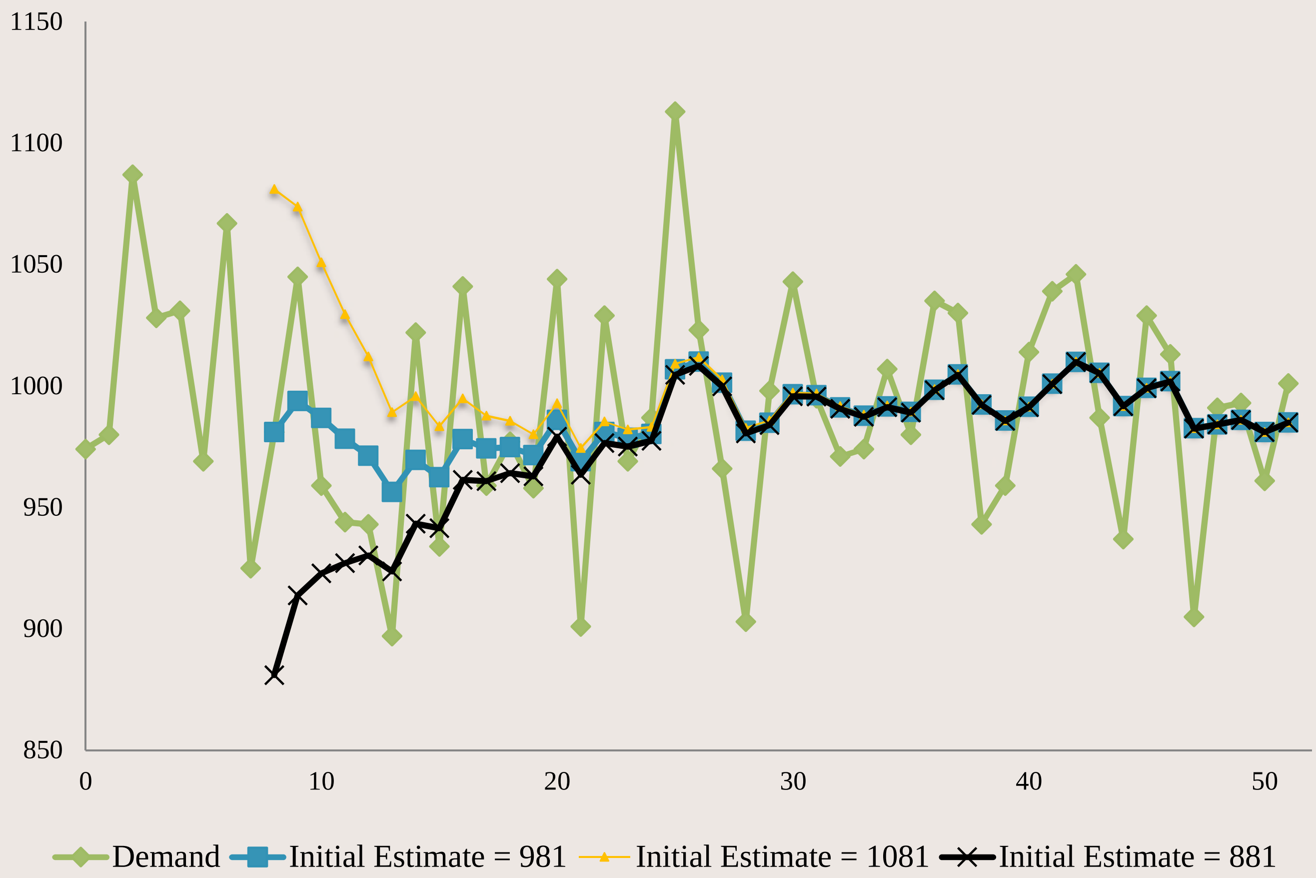
$$E[d_{t-i}] = E[a + \varepsilon_t] = a \qquad \lim_{t\to\infty} \alpha \sum_{i=0}^{t-1} (1-\alpha)^i = \frac{\alpha}{1-(1-\alpha)} = 1$$

$$E[S_t] = a$$

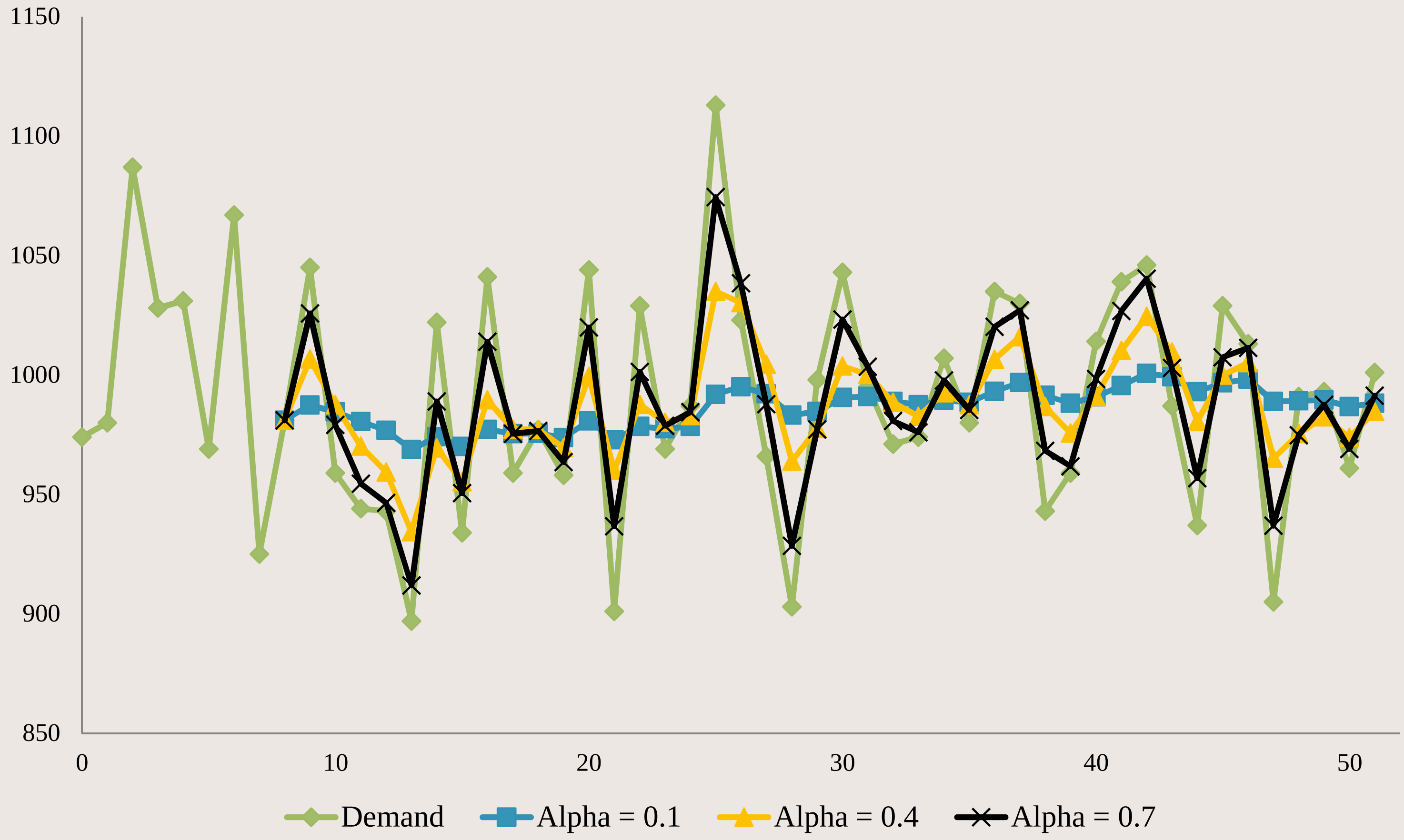
Example: Exponential Smoothing



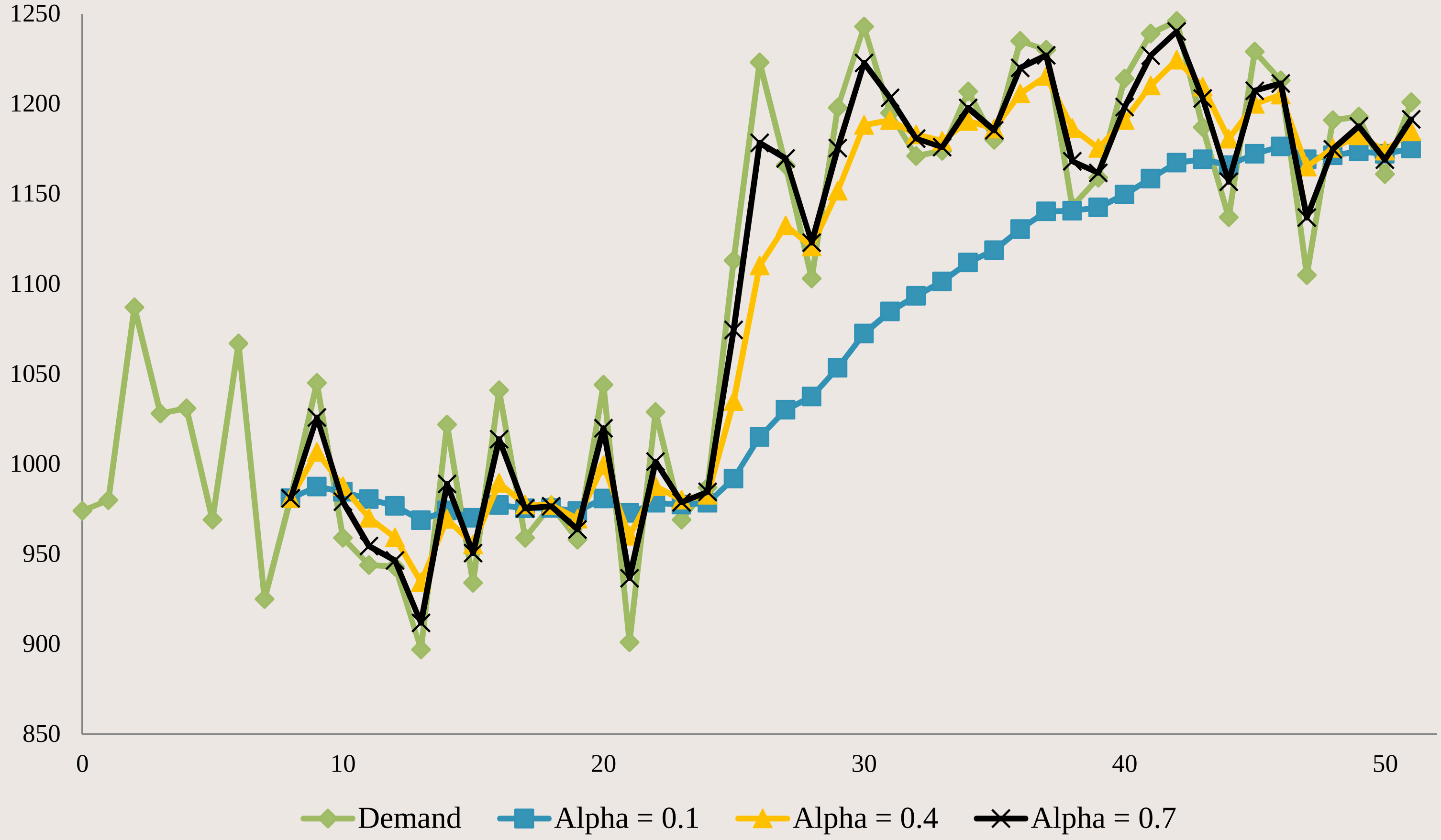
Insensitive to Initial Estimate



Effect of Weighting Factor



Response to Process Change



Trend Model: Regression

- Trend model $d_t = a + bt + \varepsilon_t$
- Model parameters: $\hat{a}$ (level) and $\hat{b}$ (slope)
- Sum of squared error

$$\sum_{t=1}^{N} e_t^2 = \sum_{t=1}^{N} \left(d_t - \hat{a} - \hat{b}t\right)^2$$

- Minimize sum of squared error

Trend Model: Regression (Cont.)

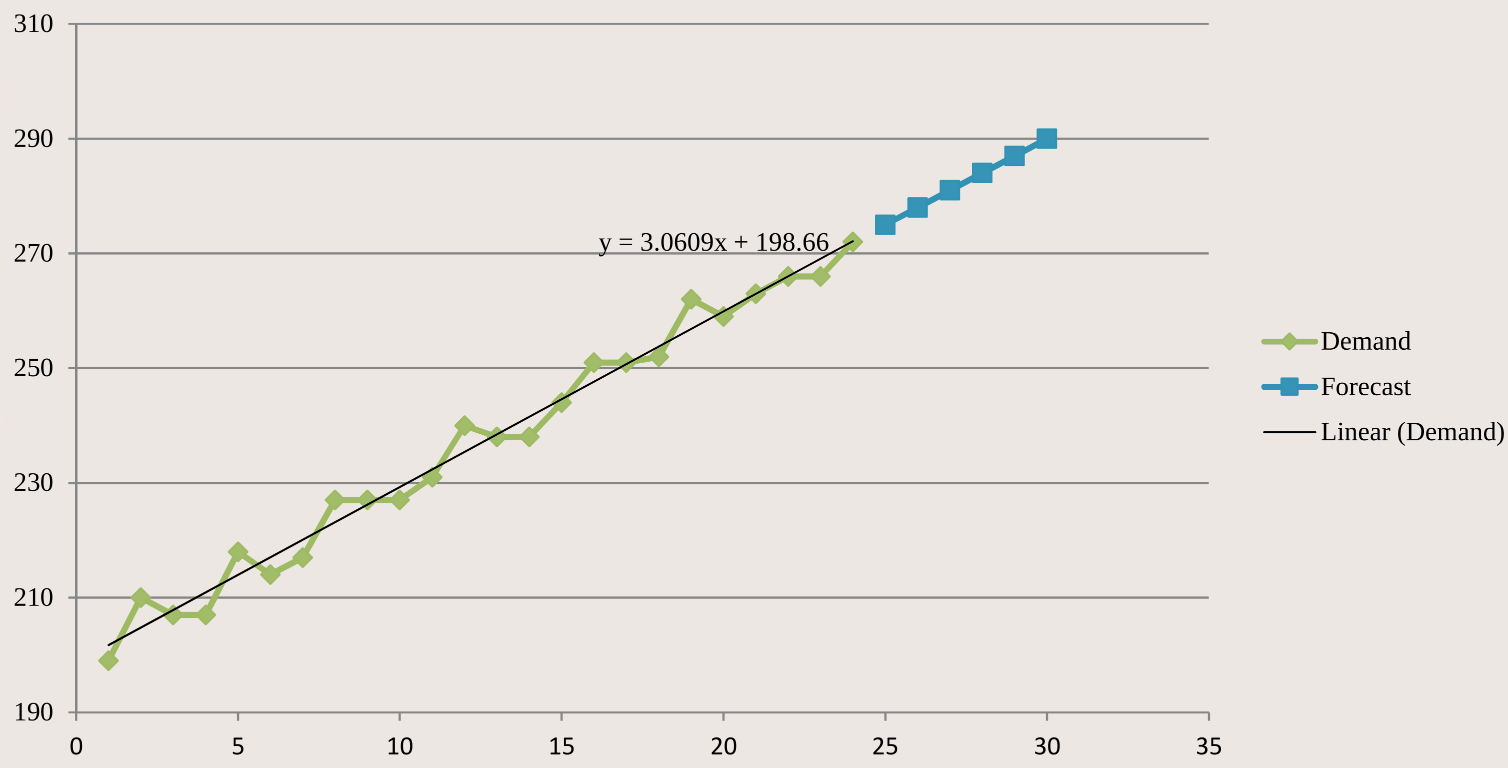
$$\frac{d\left[\sum_{t=1}^{N}\left(d_t - \hat{a} - \hat{b}t\right)^2\right]}{d\hat{a}} = -2\sum_{t=1}^{N}\left(d_t - \hat{a} - \hat{b}t\right) = 0$$

$$\frac{d\left[\sum_{t=1}^{N}\left(d_t - \hat{a} - \hat{b}t\right)^2\right]}{d\hat{b}} = 2\sum_{t=1}^{N}\left(d_t t - \hat{a}t - \hat{b}t^2\right) = 0$$

- Solving the two simultaneous equations

$$\hat{a} = \frac{\sum_{t=1}^{N} d_t - \hat{b}\sum_{t=1}^{N} t}{N} \qquad \hat{b} = \frac{N\sum_{t=1}^{N} t d_t - \sum_{t=1}^{N} d_t \sum_{t=1}^{N} t}{N\sum_{t=1}^{N} t^2 - \left(\sum_{t=1}^{N} t\right)^2}$$

Example: Regression



Double Exponential Smoothing

- Update level and slope

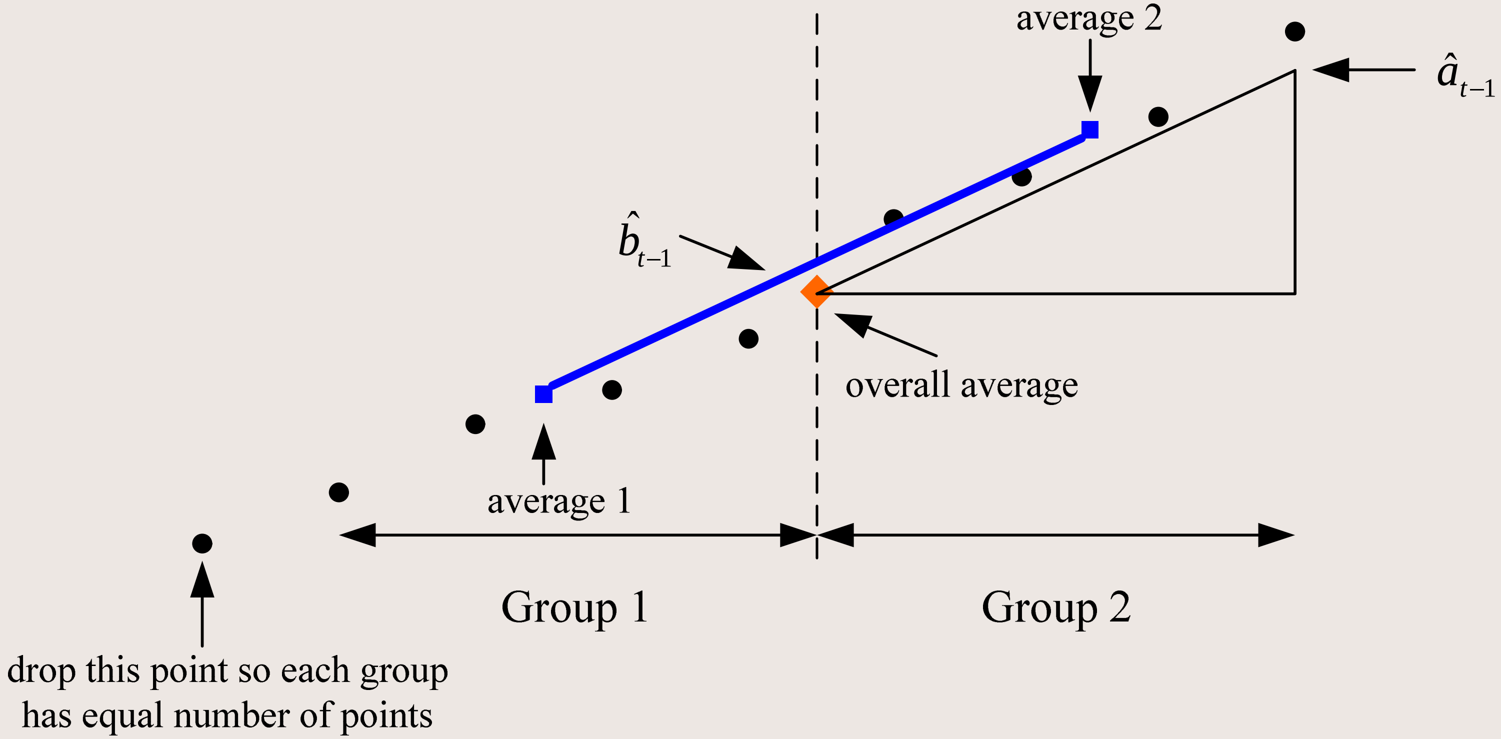
$$\hat{a}_t = \alpha \times d_t + (1-\alpha) \times \left(\hat{a}_{t-1} + \hat{b}_{t-1}\right)$$

$$\hat{b}_t = \beta \times \left(\hat{a}_t - \hat{a}_{t-1}\right) + (1-\beta)\hat{b}_{t-1}$$

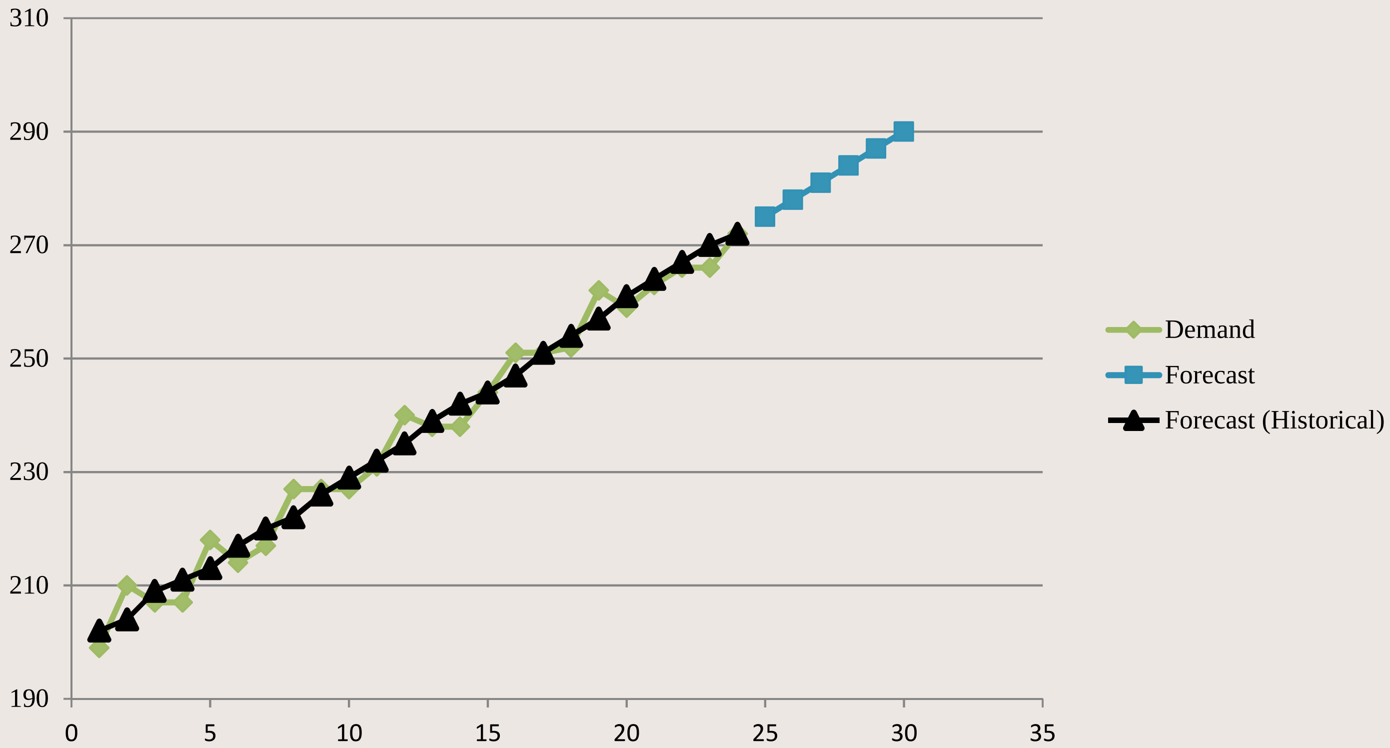
$$\hat{d}_{t+k} = \hat{a}_t + k\hat{b}_t$$

- The choice of α and β is a trade off between smoothing out noise and quick response

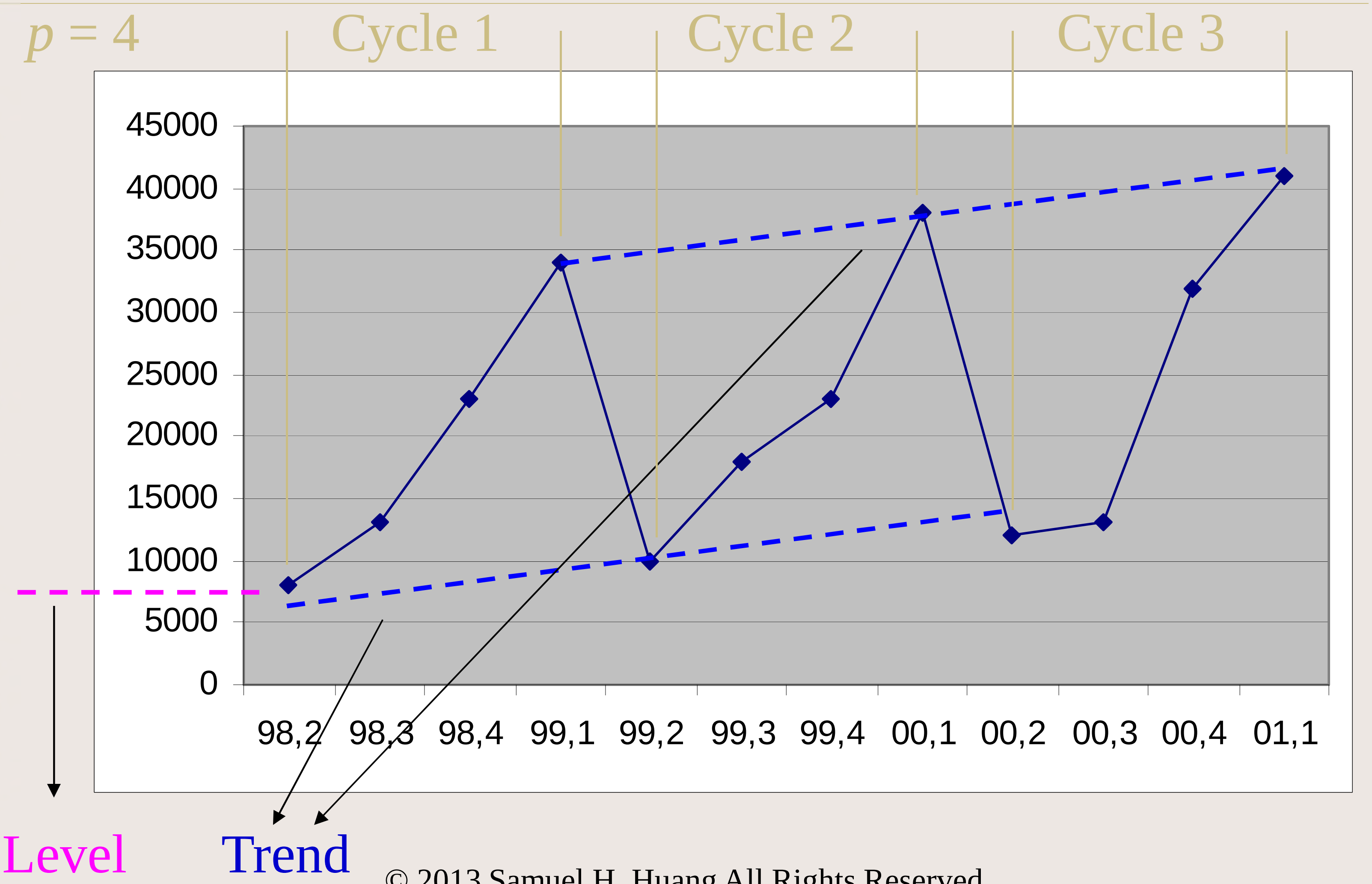
Initial Estimate



Example: Double Exponential Smoothing



Seasonal Process: Illustration



Multiplicative Model

- Seasonal model $d_t = (a + bt)c_t + \varepsilon_t$
- C_t: seasonal factor for time period t
- Assumptions:
  - The seasonal factor remains unchanged from cycle to cycle
  - The average of each cycle follows a trend process or a constant process

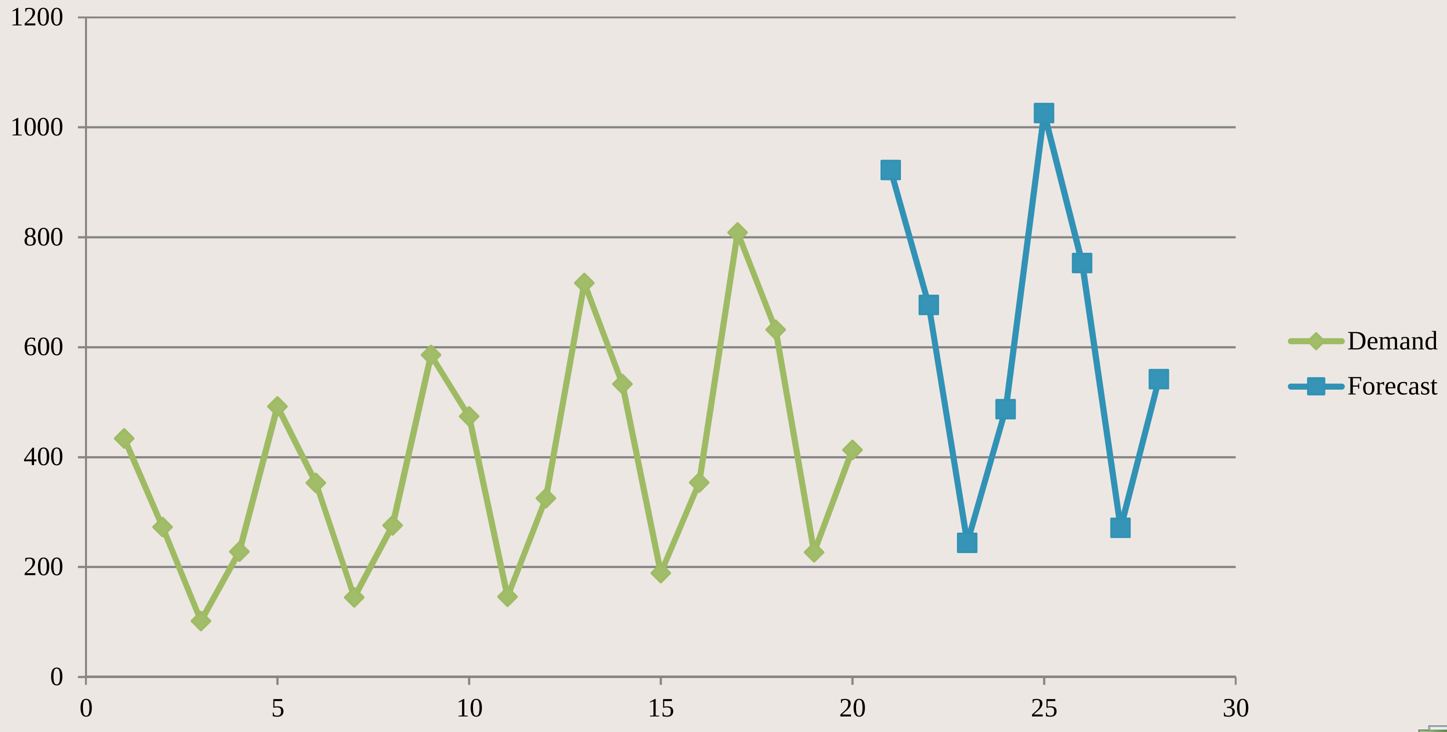
Seasonal Forecasting

- Identify seasonal factor and deseasonalize the data
  - Determine the average for each cycle
  - For each cycle, divide the data value by the average
  - Average over each season to compute the seasonal indices
  - For each season, divide the original data by the corresponding seasonal index

Seasonal Forecasting (Cont.)

- Fit the deseasonalized data
  - Calculate the average for each cycle using the deseasonalized data
  - Fit the cycle average data using an appropriate model
- Make seasonal forecast
  - Use the model to forecast the cycle average
  - Multiply the cycle average by the seasonal indices to make seasonal forecast

Example: Seasonal Forecasting



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Forecast Error Analysis

- Forecast error

$$e_t = d_t - \hat{d}_t$$

- Looking at the error for an isolated period does not provide useful information
- The performance of a forecasting model should be evaluated by studying the errors over the history of the entire forecast period

Bias

$$bias_n = \sum_{t=1}^{n} e_t$$

- Should fluctuate around 0
- If deviates significantly from 0, then the model is either underestimating (positive bias) or overestimating (negative bias) the demand

Mean Absolute Deviation (MAD)

$$MAD = \frac{1}{n}\sum_{t=1}^{n}\left|e_t\right|$$

- Measures dispersion of error; smaller is better
- For a normal distribution the MAD and the error standard deviation (σ_ε) are related by

$$MAD = \sqrt{\frac{2}{\pi}}\sigma_\varepsilon \approx 0.8\sigma_\varepsilon \qquad \sigma_\varepsilon = \frac{MAD}{0.8}$$

Mean Squared Error (MSE)

$$MSE = \frac{1}{n}\sum_{t=1}^{n} e_t^2$$

- Increased penalty for large errors
- Incorporates both model bias and variance
- A regression based forecasting model minimizes its MSE

Mean Absolute Percent Error (MAPE)

$$MAPE = \frac{1}{n}\left(\sum_{t=1}^{n} \frac{|e_t|}{d_t} \times 100 \right)$$

- Error relative to the magnitude of demand
- Periods of low demand may distort the performance of the forecasting model (e.g., division by 0 if there is no demand in a certain time period)

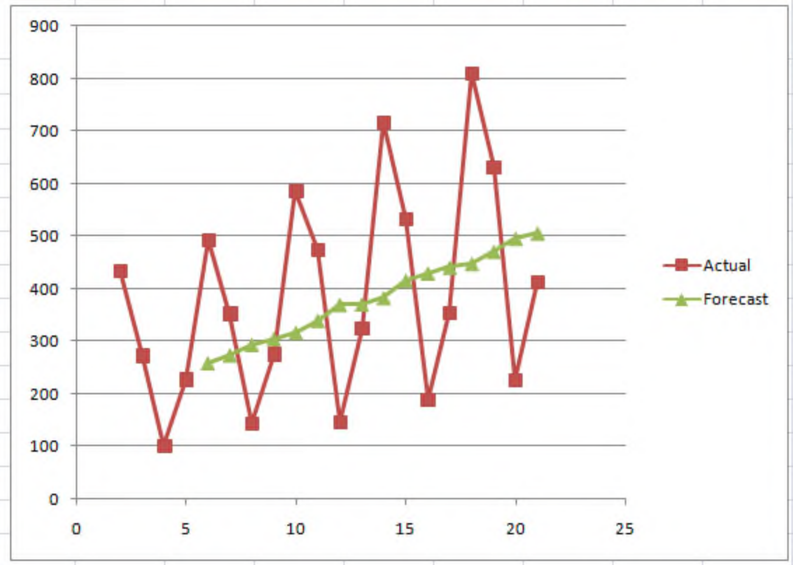
Tracking Signal (TS)

$$TS_t = \frac{bias_t}{MAD_t}$$

- Time series measure to monitor the randomness of the forecast error
- It is compared to predefined control limits (±4 or ±6) to determine if the actual demand reflects the assumptions of the forecast model

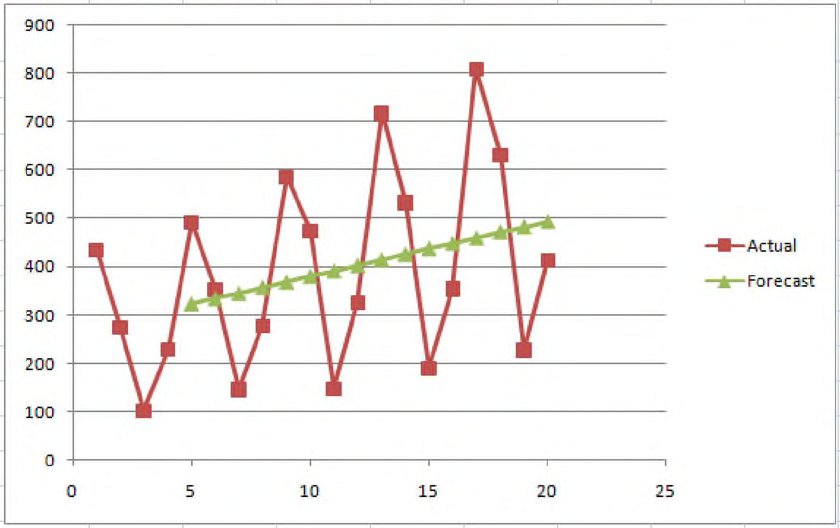
Example of Forecast Error Analysis: Moving Average

Quarter	d_t	$\hat{d}_t$	e_t	$\lvert e_t \rvert$	e_t^2	$\lvert e_t \rvert / d_t$
1	434					
2	273					
3	102					
4	228					
5	492	259	233	233	54289	0.47358
6	353	274	79	79	6241	0.2238
7	145	294	-149	149	22201	1.02759
8	276	305	-29	29	841	0.10507
9	586	317	269	269	72361	0.45904
10	474	340	134	134	17956	0.2827
11	146	370	-224	224	50176	1.53425
12	325	371	-46	46	2116	0.14154
13	717	383	334	334	111556	0.46583
14	533	416	117	117	13689	0.21951
15	189	430	-241	241	58081	1.27513
16	354	441	-87	87	7569	0.24576
17	809	448	361	361	130321	0.44623
18	632	471	161	161	25921	0.25475
19	227	496	-269	269	72361	1.18502
20	413	506	-93	93	8649	0.22518
			Bias	MAD	MSE	MAPE
			550	176.6	40895.5	53.5311



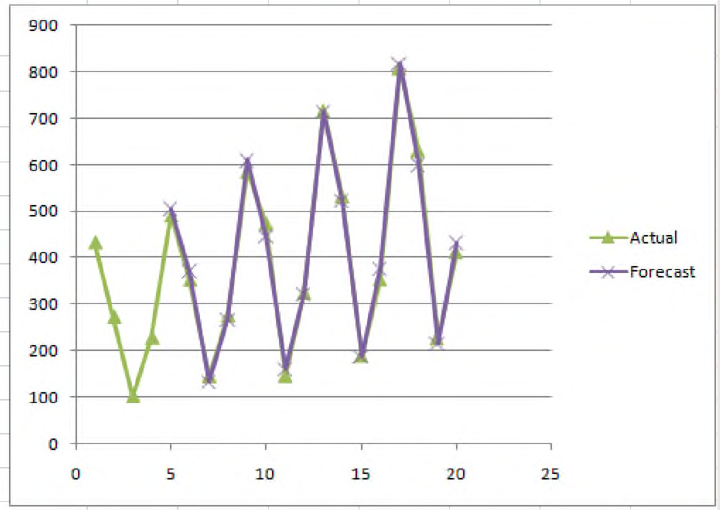
Example of Forecast Error Analysis: Regression

Quarter	d_t	$\hat{d}_t$	e_t	$\lvert e_t \rvert$	e_t^2	$\lvert e_t \rvert / d_t$
1	434					
2	273					
3	102					
4	228					
5	492	323	169	169	28561	0.3435
6	353	334	19	19	361	0.05382
7	145	345	-200	200	40000	1.37931
8	276	357	-81	81	6561	0.29348
9	586	368	218	218	47524	0.37201
10	474	380	94	94	8836	0.19831
11	146	391	-245	245	60025	1.67808
12	325	402	-77	77	5929	0.23692
13	717	414	303	303	91809	0.42259
14	533	425	108	108	11664	0.20263
15	189	437	-248	248	61504	1.31217
16	354	448	-94	94	8836	0.26554
17	809	459	350	350	1E+05	0.43263
18	632	471	161	161	25921	0.25475
19	227	482	-255	255	65025	1.12335
20	413	493	-80	80	6400	0.1937
			Bias	MAD	MSE	MAPE
			142	168.9	36966	54.7675



Example of Forecast Error Analysis

Quarter	Seasonal Index	d_t	$\hat{d}_t$	e_t	$\lvert e_t \rvert$	e_t^2	$\lvert e_t \rvert / d_t$
1	1.583	434					
2	1.162	273					
3	0.418	102					
4	0.837	228					
5	1.583	492	506	-14	14	196	0.02846
6	1.162	353	372	-19	19	361	0.05382
7	0.418	145	134	11	11	121	0.07586
8	0.837	276	268	8	8	64	0.02899
9	1.583	586	610	-24	24	576	0.04096
10	1.162	474	448	26	26	676	0.05485
11	0.418	146	161	-15	15	225	0.10274
12	0.837	325	323	2	2	4	0.00615
13	1.583	717	714	3	3	9	0.00418
14	1.162	533	524	9	9	81	0.01689
15	0.418	189	189	0	0	0	0
16	0.837	354	378	-24	24	576	0.0678
17	1.583	809	818	-9	9	81	0.01112
18	1.162	632	601	31	31	961	0.04905
19	0.418	227	216	11	11	121	0.04846
20	0.837	413	433	-20	20	400	0.04843
				Bias	MAD	MSE	MAPE
				-24	14.13	278.3	3.98597



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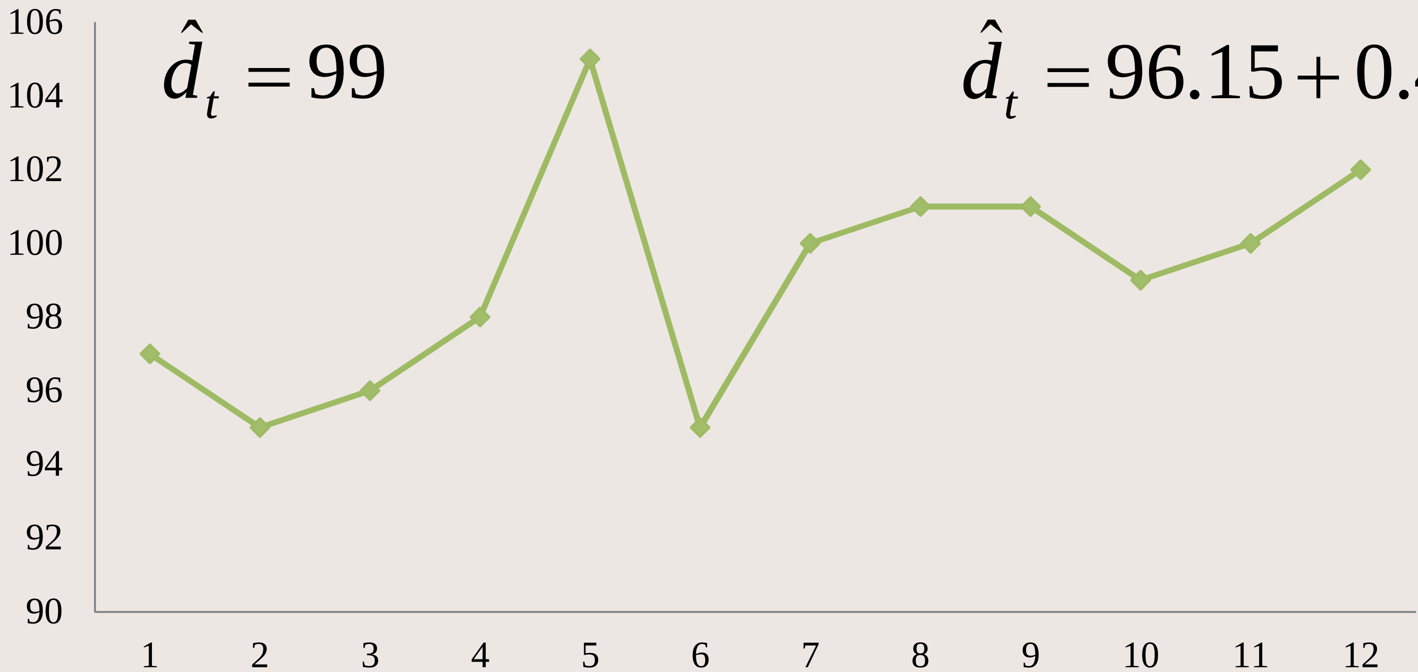
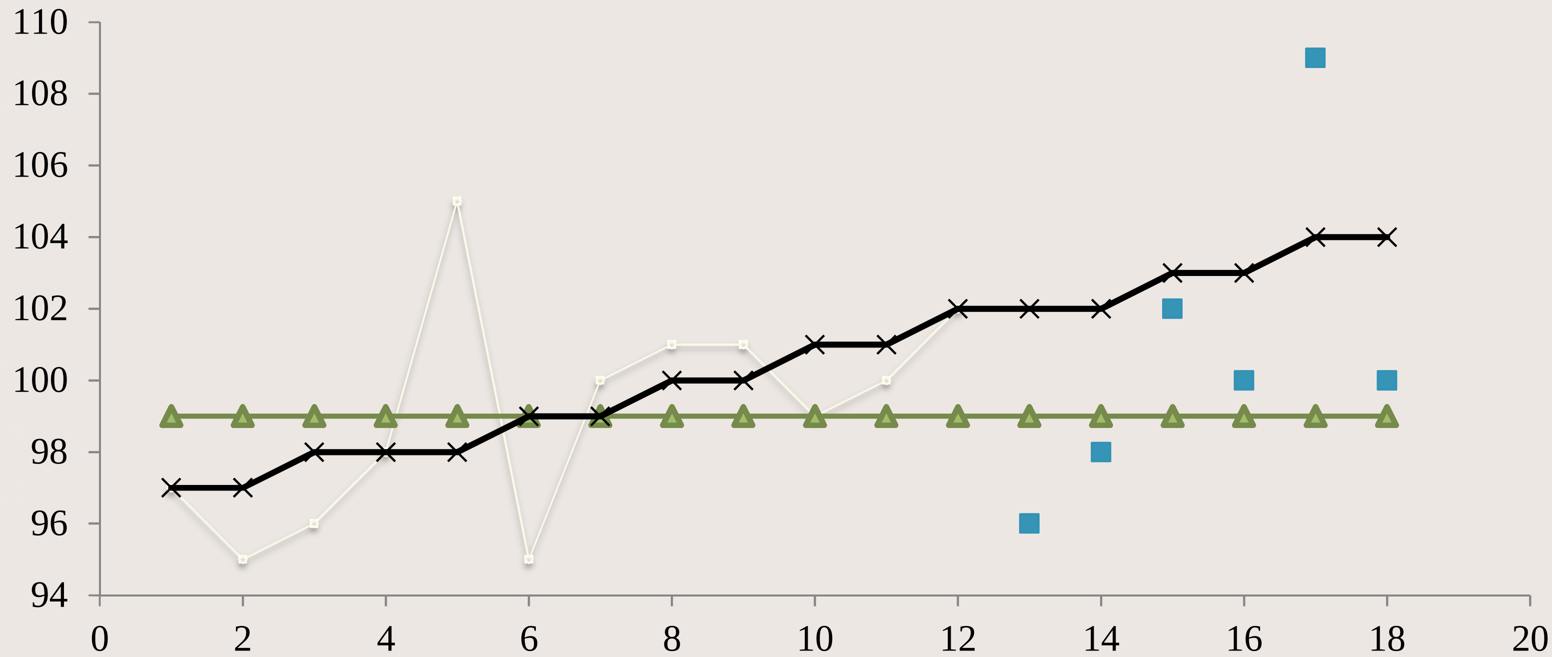
Constant or Trend?

$$\hat{d}_t = 99 \qquad \hat{d}_t = 96.15 + 0.45t$$



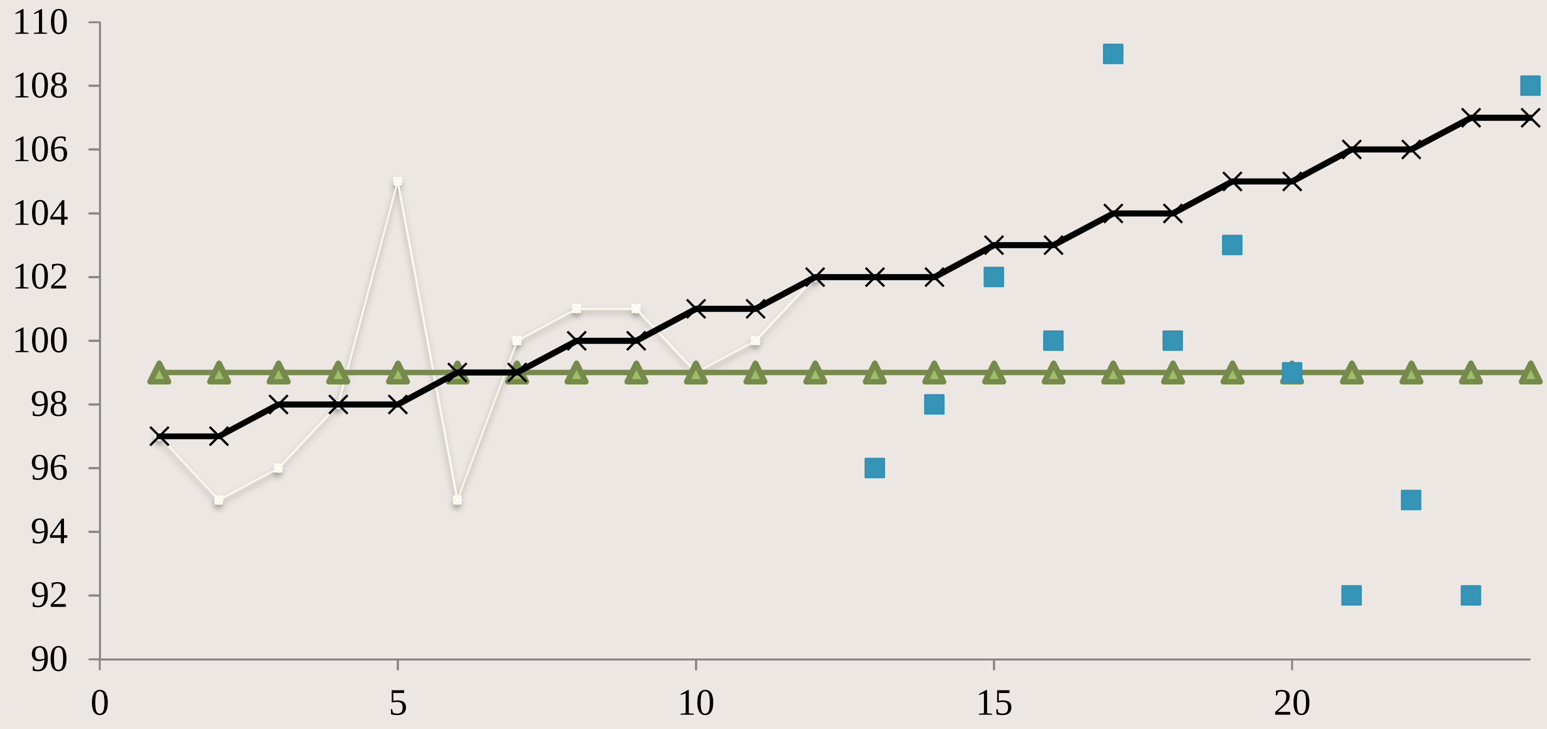
	Bias	MAD	MSE	MAPE
Trend Model	-1	1.75	6.75	1.76
Constant Model	-1	2.42	8.42	2.44

Validation Error Analysis

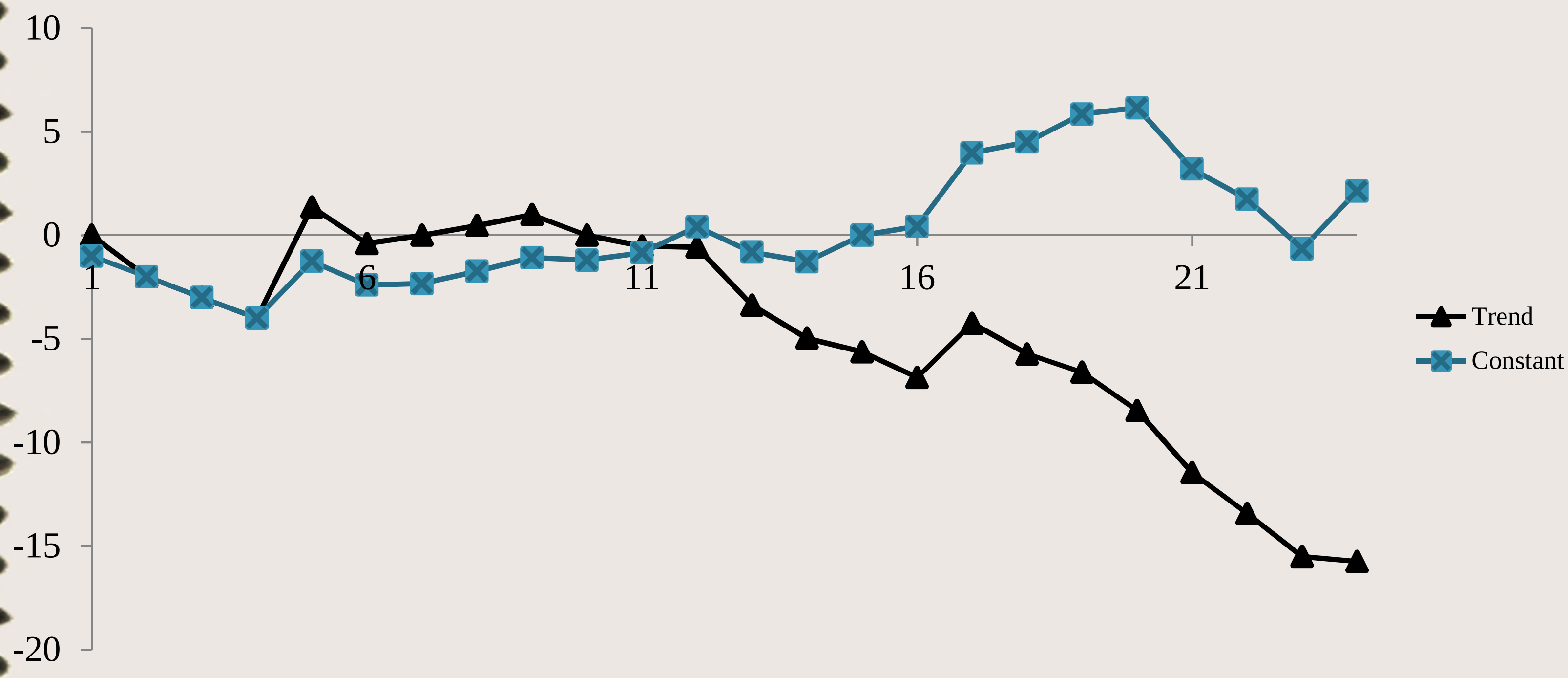


	Bias	MAD	MSE	MAPE
Trend Model	-13	3.83	17.17	3.82
Constant Model	11	3.17	20.17	3.04

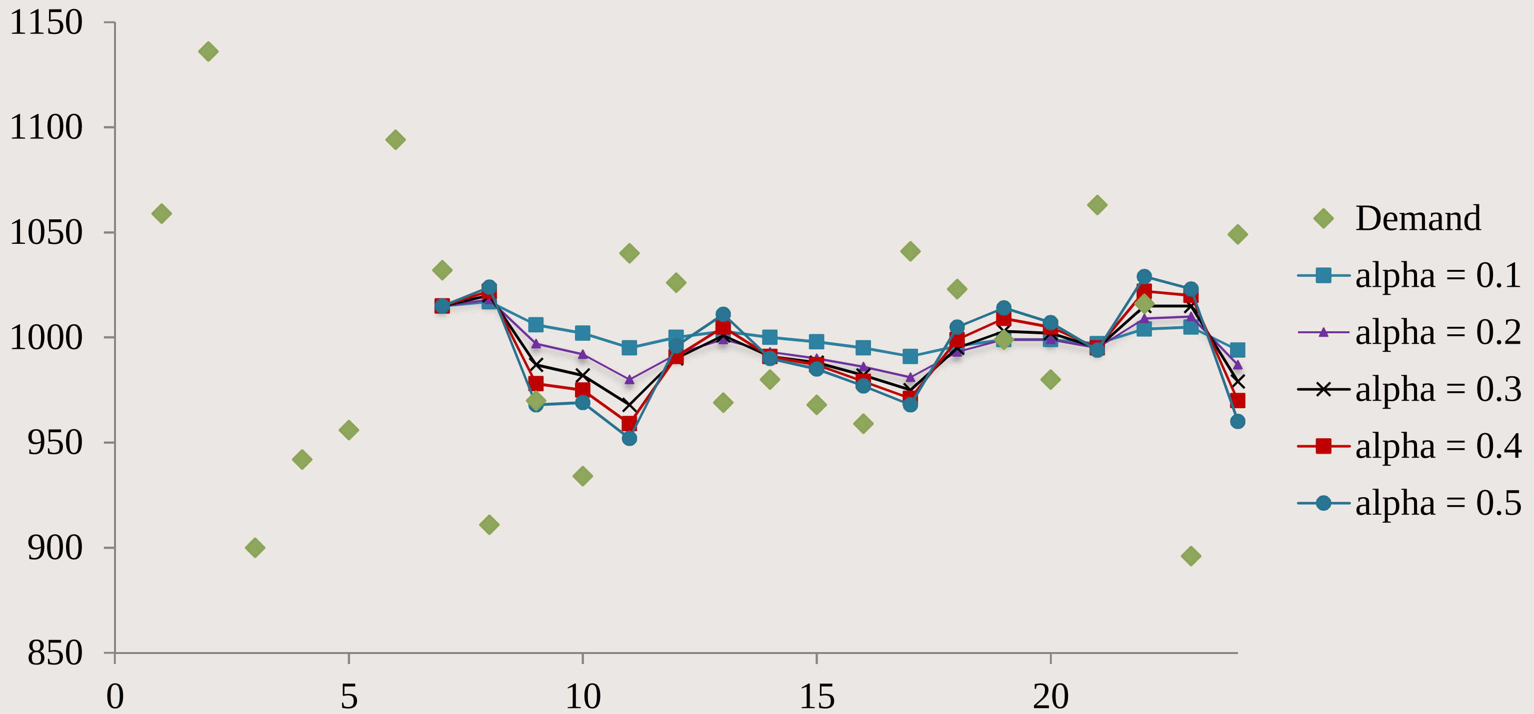
Monitoring Model Performance



Tracking Signal Analysis



Which Weighting Factor?



Weighting Factor Selection

	A	B	C	D	E	F	G	H	I	J	K	L
1					α					MAPE		
2	Month	Demand	0.1	0.2	0.3	0.4	0.5	α = 0.1	α = 0.2	α = 0.3	α = 0.4	α = 0.5
3	1	1059										
4	2	1136										
5	3	900										
6	4	942										
7	5	956										
8	6	1094										
9	7	1032	1015	1015	1015	1015	1015	0.016473	0.016473	0.016473	0.016473	0.016473
10	8	911	1017	1018	1020	1022	1024	0.116356	0.117453	0.119649	0.121844	0.12404
11	9	970	1006	997	987	978	968	0.037113	0.027835	0.017526	0.008247	0.002062
12	10	934	1002	992	982	975	969	0.072805	0.062099	0.051392	0.043897	0.037473
13	11	1040	995	980	968	959	952	0.043269	0.057692	0.069231	0.077885	0.084615
14	12	1026	1000	992	990	991	996	0.025341	0.033138	0.035088	0.034113	0.02924
15	13	969	1003	999	1001	1005	1011	0.035088	0.03096	0.033024	0.037152	0.043344
16	14	980	1000	993	991	991	990	0.020408	0.013265	0.011224	0.011224	0.010204
17	15	968	998	990	988	987	985	0.030992	0.022727	0.020661	0.019628	0.017562
18	16	959	995	986	982	979	977	0.037539	0.028154	0.023983	0.020855	0.01877
19	17	1041	991	981	975	971	968	0.048031	0.057637	0.063401	0.067243	0.070125
20	18	1023	996	993	995	999	1005	0.026393	0.029326	0.02737	0.02346	0.017595
21	19	999	999	999	1003	1009	1014	0	0	0.004004	0.01001	0.015015
22	20	980	999	999	1002	1005	1007	0.019388	0.019388	0.022449	0.02551	0.027551
23	21	1063	997	995	995	995	994	0.062088	0.06397	0.06397	0.06397	0.064911
24	22	1016	1004	1009	1015	1022	1029	0.011811	0.00689	0.000984	0.005906	0.012795
25	23	896	1005	1010	1015	1020	1023	0.121652	0.127232	0.132813	0.138393	0.141741
26	24	1049	994	987	979	970	960	0.052431	0.059104	0.06673	0.07531	0.084843
27												
28								**3.881836**	**3.822104**	**3.92178**	**4.155509**	**4.37046**

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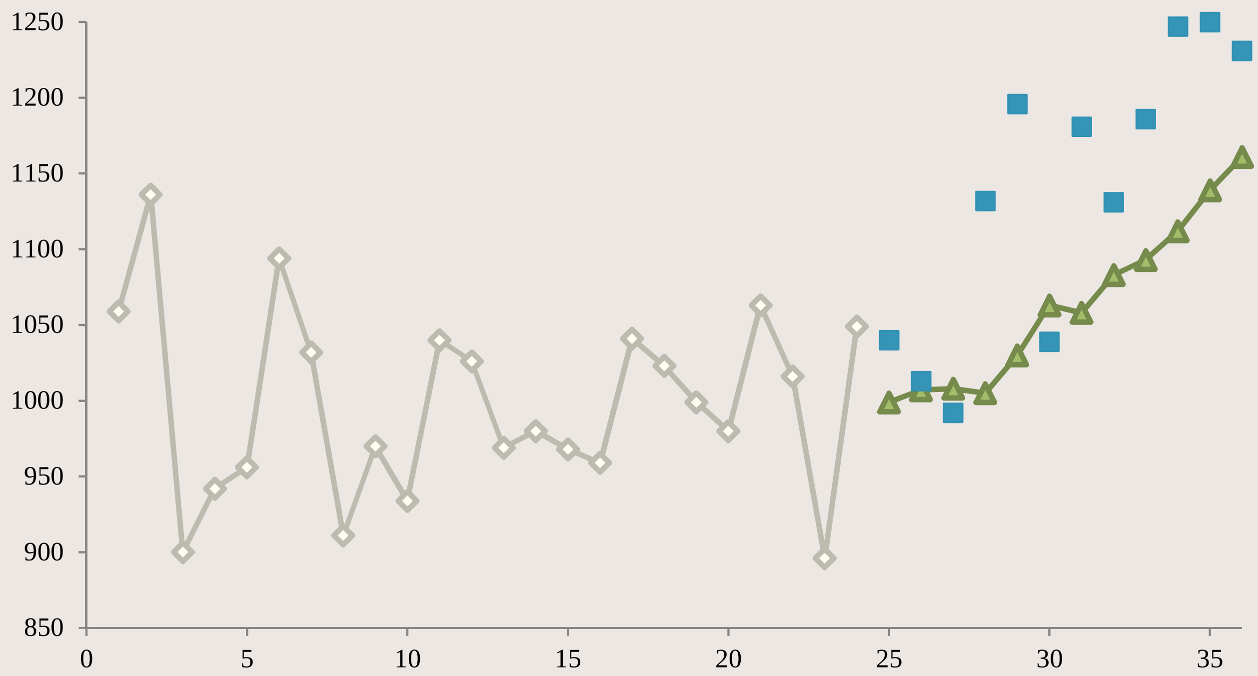
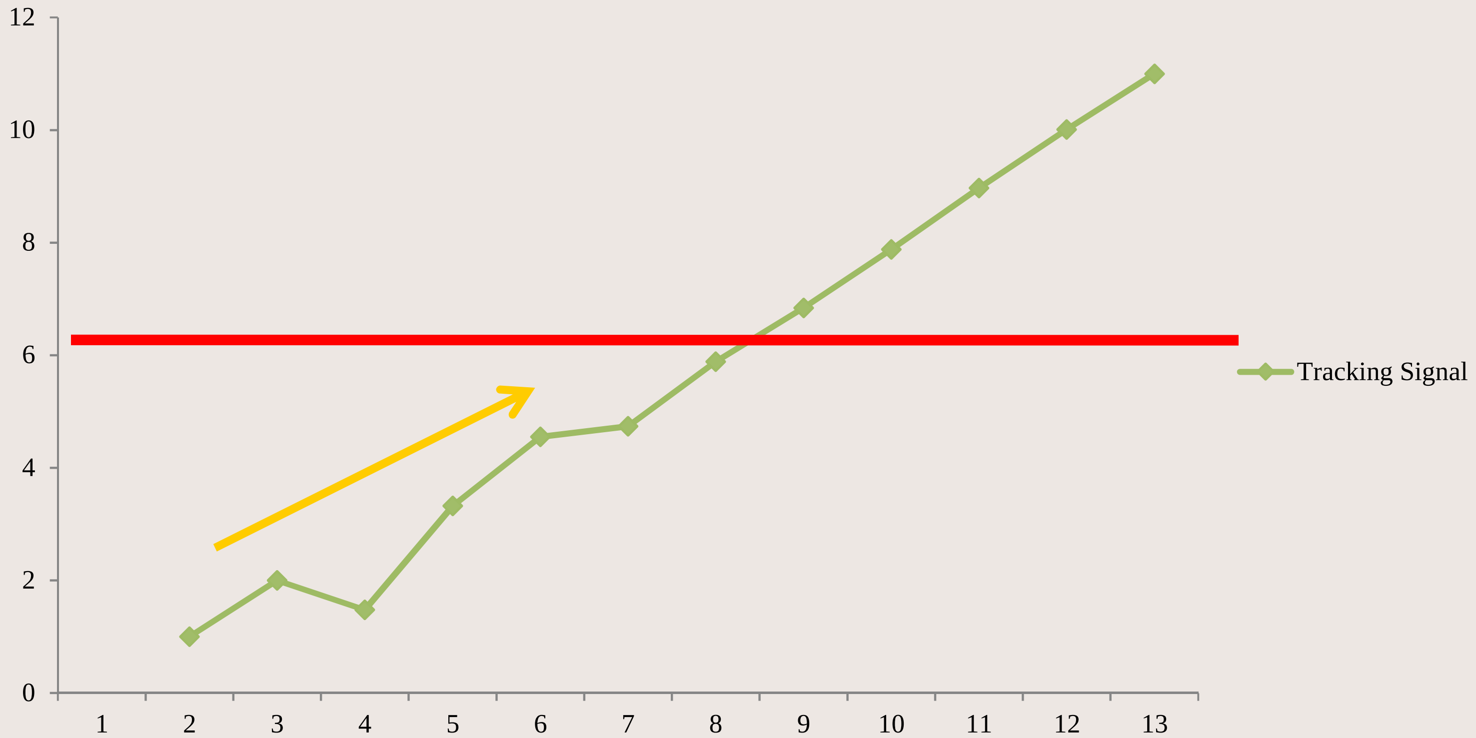
Process Change?

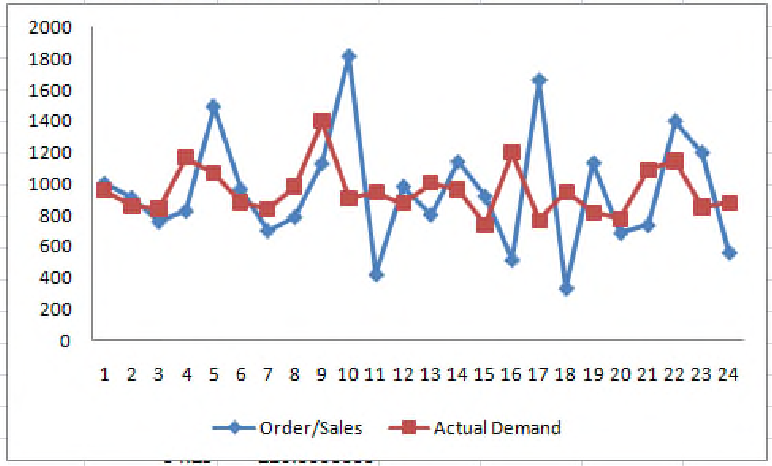


Detecting Process Change



Bullwhip Effect

	A	B	C	D	E	F	G	H
1								|e|
2	Month	Order/Sales	backlog	Inventory	Actual Demand		Order/Sales	Actual Demand
3	1	1000	0	46	954			
4	2	908	0	99	855			
5	3	756	0	15	840			
6	4	825	326	0	1166			
7	5	1492	0	101	1065			
8	6	964	0	183	882			
9	7	699	0	48	834			
10	8	786	147	0	981			
11	9	1128	416	0	1397			
12	10	1813	0	489	908			
13	11	419	36	0	944			
14	12	980	0	72	872			
15	13	800	133	0	1005			
16	14	1138	0	44	961			
17	15	917	0	225	736			
18	16	511	462	0	1198		440.25	245.6666667
19	17	1660	0	434	764		708.75	188.3333333
20	18	330	184	0	948		621.25	4.333333333
21	19	1132	0	132	816		180.75	136.3333333
22	20	684	0	41	775		267.25	177.3333333
23	21	734	312	0	1087		217.25	134.6666667
24	22	1399	55	0	1142		447.75	189.6666667
25	23	1197	0	292	850		245.75	102.3333333
26	24	558	26	0	876		393.25	76.33333333
27								
28	Average	951.25			952.3333333	MAD	286.6041667	119.3888889
29						sigma	358.2552083	149.2361111



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Bullwhip Effect: Lessons Learned

- Fluctuations in orders increase as they move up the supply chain from retailers to wholesalers to manufacturers to suppliers
- The further up the supply chain a company is, the larger is its forecast error
- A company should consider collaborating with its supply chain partners to develop forecasts based on sales to the end customer to reduce forecast error