	Chapter 02
2.1	Obtain the expression for the electron density when the Fermi function is box-like.
Ans.	The carrier density is given by $$n = \int g(E) f(E) dE = N_C \int_{E_c}^{E_F} (E - E_c)^{1/2} dE = (2N_C/3)(E_F - E_c)^{3/2}$$ where we have used $f(E) = 1$ for $E_c < E < E_F$.
2.2	Obtain the expression for the density-of-states for electrons in Si. Show that Eq. (2.10) is valid if m_{de} is replaced by $6^{2/3}(m_l m_t^2)^{1/3}$.
Ans.	The density of state function is given by $N(E)dE = \frac{g_v}{2\pi^2}\left(\frac{2m_{de}}{\hbar^2}\right)^{3/2} E^{1/2} dE$, for Si g_v=6 .Therefore, $N(E)dE = \frac{3}{\pi^2}\left(\frac{2m_{de}}{\hbar^2}\right)^{3/2} E^{1/2} dE$
2.3	Calculate the total number of states in silicon between E_c and $E_c + k_BT$ at 300 K.
Ans.	the required number of states may be obtained by evaluating the following integral : $$\int_{E_c}^{E_c + k_BT} (E - E_c)^{1/2} dE = \frac{2}{3}(k_BT)^{3/2}$$ The number of states is therefore $$N = \frac{g_v}{2\pi^2}\left(\frac{2m_{de}}{\hbar^2}\right)^{3/2} \frac{2}{3}(k_BT)^{3/2}$$ Putting g_v = 6, $m_de = (m_l m_t^2)^{1/3}$, m_l = 0.92 m_0 and m_t = 0.19 m_0, one obtains N = 1.9 x 10^{25} m^{-3}.
2.4	Assume that a semiconductor becomes degenerate when the Fermi level touches the band edge. Assume also that Eq. (2.16) is valid. Calculate the donor density needed to make the electron system in GaAs degenerate at T = 300 K. Take $m_e = 0.067\ m_0$.

Ans	When Eq. 2.16 is valid, the donor density needed to make the electron system in GaAs degenerate at T=300K will be equal to the density of state function and its value will be 4.13×10^{22} m^{-3}
2.5	Calculate the maximum temperature of operation of a Si device doped with 10^{16} donor atoms/cm^3. Use the expression for band gap given by Eq. (2.27).
Ans.	Using eq. 2.27, we get the band gap of Si at 300K as 1.12 eV.: For device operation the semiconductor must be either n or p-type. This defines a maximum temperature of operation of any device at which temperature the extrinsic carrier density equals the intrinsic carrier density. Therefore, $N_D = n_i = \sqrt{N_c N_v}\,\exp\left(-E_g / 2k_B T_{\max}\right)$, This gives T_{max}=873K
2.6	Using the expression for temperature dependent band gap calculate the gap at 300 K and 1200 K. Take $E_g(0)$ = 1.17 eV, α= 4.73 x 10^{-4} eV/K, β = 636 K.
Ans.	The temperature dependent band gap is given by $E_g(T) = E_{g0} - \alpha T^2/(T+\beta)$, the band gap at T=300K will be 1.1245 eV and at T=1200K, the band gap will be 0.799 eV.
2.7	Obtain the general expression for electron density given as $n = N_c F_{1/2}(\eta)$, where $F_{1/2}(\eta) = (2/\sqrt{\pi})\int_0^\infty \frac{x^{1/2}dx}{[1+\exp(x-\eta)]}$, $\eta = (E_F - E_c)/k_B T$.
Ans.	The general expression for electron density is given by Eq. (2.11). Using Eq. (2.13) for the Fermi function, introducing the variables $x = (E - E_c)/k_B T$ and $\eta = (E_F - E_c)/k_B T$, and noting the expression for N_C, the required expression is easily obtained.
2.8	Show that when $\eta << -1$, $F_{1/2}(\eta) = \exp(\eta)$ and the electron density is expressed by Eq. (2.15) valid for nondegenerate semiconductors.
Ans.	Under the condition stated, $1 + \exp(x-\eta) \approx \exp(\eta - x)$. and $\int_0^\infty e^{-x} x^{1/2} dx = \sqrt{\pi}/2$. One obtains Eq. (2.16) valid for non-degenerate statistics.

2.9	When $\eta >> 1$, $F_{1/2}(\eta) \approx [4\eta^{3/2}/3\sqrt{\pi}]$. Using this express Fermi energy in terms of electron density.
Ans.	Under the stated condition, $n = N_C \dfrac{4\eta^{3/2}}{3\sqrt{\pi}}$. To verify that this expression automatically follows from the general expression Eq. (2.11), use the approximation $f(E) \approx 1$. The integration leads to $(2/3)(E_F - E_c)^{3/2}$. After performing the algebra, the above expression is easily obtained.
2.10	Obtain the expressions for the electron and hole densities when both the donors and acceptors are present.
Ans.	Consider N_A acceptors and N_D donors per cm^3. Considering complete ionization, we can write $n - p = N_D^+ - N_A^- \approx N_D - N_A$. Again we know, $np = n_i^2$, therefore, substituting $p = \frac{n_i^2}{n}$, we have the quadratic equation $n^2 - (N_D - N_A)n - n_i^2 = 0$, from the solution, we get $n = \frac{(N_D-N_A)+\sqrt{(N_D-N_A)^2+4n_i^2}}{2}$. Similarly, substituting $n = \frac{n_i^2}{p}$, we get $p = \frac{(N_A-N_D)+\sqrt{(N_D-N_A)^2+4n_i^2}}{2}$
2.11	Calculate the wavelength of emission for transition from 2s to 1s level of P impurity in Si. The donor binding energy is 25 meV and the Bohr model is valid.
Ans.	According to Bohr's Model, if donor binding energy is 25 meV, the ionization energy of P in Si is 0.045 eV, the emission wavelength will be (1.24/0.045) µm, i.e., 27.55µm
2.12	Calculate the Bohr radius for donor ions in Ge using $m_e = 0.12\ m_0$ and ε_r=16. Calculate the approximate number of Ge atoms within the volume defined by the Bohr radius (a_{Ge} = 5.66 A). Does this number justify the use of effective mass in Eq. (2.28)?
Ans.	The Bohr radius for donor ions in Ge is given by $r = n^2\epsilon_r \frac{m_0}{m_r} a_0$, where $a_0 = 0.53$Å.

	Considering n=1, we get r=70.66 Å
2.13	Calculate the resistivity of intrinsic Ge at 300 K.
Ans.	The resistivity is given by $= \frac{1}{qn_i\,(\mu_n+\mu_p)}$, The intrinsic carrier concentration of Ge is 2.3 $\times 10^{13}$ cm^{-3}, electron mobility is 3900 cm^2/V-sec and hole mobility is 1900 cm^2/V-sec. So the resistivity is given by 46.85 Ω cm
2.14	The mobilities by two scattering processes vary with temperature *T* as *AT* and *B/T*. Prove that the combined mobility will show a maximum when $T^2 = B/A$.
Ans.	Let the mobility $\mu_1 \propto AT$ and $\mu_2 \propto {}^{B}/_{T}$. According to Mathiessen's rule, the over mobility $\mu = \frac{\mu_1\mu_2}{\mu_1+\mu_2}$, for maximum mobility , $d\mu/dT = 0$, then one can easily prove that$T^2 = B/A$
2.15	The electrons in a semiconductor are scattered by impurities (μ_{imp}= $AT^{3/2}$) and by phonons (μ_{ph} = $BT^{-2.5}$). Using Mathiessen rule show that the inverse mobility attains a minimum at a certain temperature. Obtain the expression for the temperature giving minimum 1/μ.
Ans.	$\mu_{imp} = AT^{3/2}$ and $\mu_{ph} = B\,T^{-2.5}$, According to Mathiessen's rule, the inverse mobility becomes$\frac{1}{\mu} = \frac{T^{5/2}}{B} + \frac{1}{A}T^{-3.2}$. Putting $\frac{d}{dT}\left(\frac{1}{\mu}\right) = 0$, one can obtain the expression for temperature as $T^4 = \frac{3B}{5A}$.
2.16	The current density for electrons in Si along the *x* direction is $J_x = ne\mu_{xx}F_{x,}$ where *F* is the field along *x* = (100) direction, and $\mu_{xx} = e\langle\tau\rangle/m_{xx}$. Consider all the six valleys and show that $J_x = ne^2\langle t\rangle F_x/m_c$, where m_c is the conductivity effective mass.
Ans.	$\mu_{xx} = e\langle\tau\rangle / m_{xx}$. Consider a valley with major axis along [100]

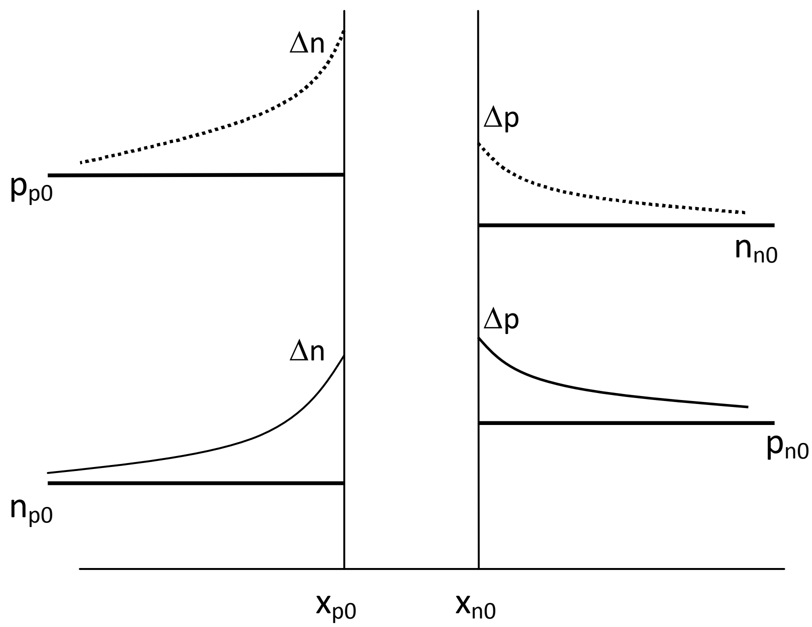
	$[\mu] = e\langle\tau\rangle \begin{bmatrix} 1/m_{xx} & 0 & 0 \\ 0 & 1/m_{yy} & 0 \\ 0 & 0 & 1/m_{zz} \end{bmatrix}; m_{xx} = m_l, m_{yy} = m_t, m_{zz} = m_t$ For valleys with major axis along [010] $[\mu] = e\langle\tau\rangle \begin{bmatrix} 1/m_t & 0 & 0 \\ 0 & 1/m_l & 0 \\ 0 & 0 & 1/m_t \end{bmatrix}.$ Since all the six valleys have minima with same energy, the concentration in all the valleys are equal, i.e., $n^{[100]} = n^{[010]} = n^{[001]} = n^{[\bar{1}00]}, \text{etc} = n_0/6$, where n_0 is the total electron concentration. Summing the contributions by each valley to the conductivity, one may obtain its expression as $[\sigma] = n_0 e^2 \langle\tau\rangle \begin{bmatrix} \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right) & 0 & 0 \\ 0 & \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right) & 0 \\ 0 & 0 & \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right) \end{bmatrix}.$ The conductivity tensor is thus isotropic and the current density along x direction is $J_x = ne^2\langle\tau\rangle F_x / m_c$, where $1/m_c = \frac{1}{3}\left(\frac{1}{m_l} + \frac{2}{m_t}\right)$.
2.17	Prove that the imputity scattering limited mobility follows the $T^{3/2}$ law.
Ans.	At low temperature, scattering of electrons by thermal vibration is weaker than the scattering by ionized impurity. As an electron passes by an ionized donor, it is attracted and thus deflected from its straight path. The potential energy of an electron at a distance r from the donor ion is due to Coulomb attraction and its magnitude is given by

	$\|PE\| = \dfrac{e^2}{4\pi\varepsilon_0\varepsilon_r r}$. If the kinetic energy of the electron approaching an donor ion is greater than its potential energy at a distance r from the donor, then the electron will essentially continue without feeling the potential energy and therefore without being deflected and we can say that it is not scattered.On the other hand, if the kinetic energy of the electron is less than the the potential energy at a distance r from the donor ion, then potential energy of the Coulomb interaction will be so strong that the electronwill be deflected. When the electron is just scattered, we can write $KE \approx \|PE(r_c)\|$. where, $r_c$is the critical radius.But average kinetic energy is 3/2 kT, so that at r=r_c, $\dfrac{3}{2}kT = \|PE(r_c)\| = \dfrac{e^2}{4\pi\varepsilon_0\varepsilon_r r}$ $r_c = \dfrac{e^2}{6\pi\varepsilon_0\varepsilon_r kT}$. As T increases, the scattering radius decreases. The scattering cross section $S = \pi r_c^2$ is given by $S = \dfrac{\pi e^4}{(6\pi\varepsilon_0\varepsilon_r kT)^2} \propto T^{-2}$. Thus the impurity scattering limited relaxation time is $\tau_{imp} = \dfrac{1}{S v_{th} N_1} \propto \dfrac{1}{T^{-2}T^{1/2}N_1} \propto \dfrac{T^{3/2}}{N_1}$
2.18	Using kinetic theory for the electron gas prove $D/\mu = k_B T/e$.

	The diffusion current density can be written as $J = qv_{th}l\,\frac{dn}{dx}$, Therefore, we can write the diffusion coefficient $D = v_{th}l$ The mean free path, $l = v_{th}\tau$, $\therefore D = (v_{th})^2\tau = (v_{th})^2\frac{e\tau}{m}\frac{m}{e} = (v_{th})^2\mu\frac{m}{e}$ Now, from kinetic theory gases, we know $mv_{th}^2 = kT$ Therefore, we write, $D = kT\frac{\mu}{e}$ $$\therefore \frac{D}{\mu} = \frac{kT}{e}$$
2.19	Prove that τ_n is the mean time spent by an excess electron before being lost by recombination.
Ans.	The variation of excess electron density with time is $\delta n(t) = \Delta n \exp(-t/\tau_n)$. (1) . This means that the density of electrons that does not suffer recombination after time *t* is given by the above equation. Similarly the density of electrons that do not suffer recombination over time $t + \delta t$ is given by $\delta n(t+\delta t) = \Delta n \exp[-(t+\delta t)/\tau_n) = \delta n(t)[1-(\delta t/\tau_n)]$, (2) where the approximation $e^{-x} \approx 1 - x$ has been employed. The difference between (1) and (2) denotes the density of electrons that suffer recombination between t and $t+\delta t$, and has a lifetime *t.* To calculate the average lifetime we write $\langle t \rangle = \Delta n \int_0^\infty t \exp(-t/\tau_n)(dt/\tau_n)/\Delta n$. The integral may easily be evaluated and we get $\langle t \rangle = \tau_n$.
2.20	The field-dependent mobility is expressed as $\mu(F) = \mu_0/[1+(F/F_c)]$. How you would obtain the expression for the saturation drift velocity?
Ans.	$\mu(F) = \mu_0/[1+(F/F_c)]$, i.e $v_d(F) = \mu_0 F/[1+(F/F_c)]$, or $v_d(F) = \mu_0/[1/F + (1/F_c)]$, if F>F$_c$, 1/F can be neglected, we get saturation drift velocity, as $\mu_0 F_c$
2.21	In the empirical expression for the drift velocity at high fields given in Example 2.11, find out the values of low field mobility and saturation drift velocity for electrons in Si.

Ans.	In example 2.11, the drift velocity is given as $v_d = v_1 \dfrac{F}{F_c}\left(\dfrac{1}{1+\left(\dfrac{F}{F_c}\right)^{\beta}}\right)^{1/\beta}$, For low field,F<<F_c, so the drift velocity comes out as v_1, which is equal to $1.07\times10^{7}\, cm/\sec$ In the similar way, the low fiels mobility will be v_1 / F
2.22	Under a high electric field, the electrons become "hot" with an effective temperature T_e>T_L, T_L being the lattice temperature. The energy balance equation $e\mu F^2 = (3/2)k_B(T_e - T_L)/\tau_e$ is used to determine the electron temperature. Prove that when the electric field is along the <100> direction, the two conduction band valleys having longitudinal mass along field direction will be colder than the remaining four valleys.
Ans.	Let the field be applied along z-direction, i.e, [001] direction. As seen, the corresponding effective mass along the field direction is m_l and the mobility is $\mu^{001} = e\tau / m_l$. However for the remaining four valleys the mass parallel to the field direction is m_t and the mobility is $_c$. Since m_l> m_t, electrons in valleys 1 and 2 will be less heated than the electrons in valleys 3-6.
2.23	Calculate the value of the injected electron density which will raise the quasi Fermi level 0.01 eV above the conduction band edge of GaAs.
Ans.	We know the carrier density, $n = N_C \exp[(E_F - E_C)/kT]$ At T=300 K, for GaAs, N_c=4.7x10^{17} /cm^3 E_F-E_C=0.01 eV, So n=6.9x10^{17} /cm^3
2.24	The Fermi level is constant in a p-n junction under equilibrium. Using this, show that the

	built-in potential in the junction is expressed by Eq. (2.77).
Ans.	The built in potential is actually the difference in the positions of the intrinsic levels in the n and p region of the junction. In the p region, the position of the Fermi level is given by $E_{Fp} = E_{ip} - kTln\frac{N_A}{n_i}$, in the n region , the position of the Fermi level is given by $E_{Fn} = E_{in} - kTln\frac{N_D}{n_i}$. Since the Fermi level is constant throughout, $E_{Fp} = E_{Fn}$, therefore, we have, $E_{ip} - E_{in} = kTln\frac{N_A N_D}{n_i^2} = qV_{bi}$, where V_{bi} is the built in potential. Assuming complete ionization, we can write $p_{p0} = N_A, n_{n0} = N_D, n_{p0} = \frac{n_i^2}{p_{p0}}$ and $p_{n0} = \frac{n_i^2}{n_{n0}}$, putting all these expressions we get the built in potential as given in Eq. (2.77)
2.25	Obtain the expressions for the potential, field and junction capacitance of a linearly graded p-n junction.
	Let us consider a linearly graded junction where the doping concentration is a linear function of x, i.e., $N_D - N_A = ax$, where a is the constant of proportionality. From the solution of Poisson's equation, we get the expression for the electric field as $\mathcal{E}(x) = \frac{qax^2}{2\varepsilon_s} + C$, Where C is the constant of integration to be evaluated from the boundary condition. The boundary conditions are, at $x = \pm W/2$, $\mathcal{E}(x) = 0$ and $x_n = x_p = W/2$, where W is the depletion layer width. Solving we get $\mathcal{E}(x) = \frac{qa}{2\varepsilon_s}\left(x^2 - \frac{W^2}{4}\right)$. Integrating this relation of electric field, the built in potential will be obtained as $V_{bi} = \int_{-x_p}^{x_n} \mathcal{E}(x)\,dx = \frac{qaW^3}{12\varepsilon_s}$
2.26	The depletion layer of a p-n junction acts as a parallel plate capacitor in which the –ve (+ve) charges are assumed to be placed at x_{p0} (x_{n0}) : the edges of the depletion layer on p(n) side. Net N_a = 10^{18} cm^{-3} and N_d = 10^{17} cm^{-3} and x_{p0} = 0.1 µm. Calculate the

	depletion layer capacitance per unit area.[$\varepsilon_0 = 8.84 \times 10^{-12}$ Farads/m].
Ans.	The depletion layer capacitance per unit area at zero bias is given by $C_d = \left(\frac{q\varepsilon_s}{2V_0}\frac{N_A N_D}{N_A+N_D}\right)^{1/2}$, where $V_0 = 0.026 \times ln\frac{N_A N_D}{n_i^2}$,for Si $V_0 = 0.026 \times ln\frac{N_A N_D}{n_i^2} =$ $0.877\ V$, $\varepsilon_s = 11.8$, $C_d = 9.3 \times 10^{-7}$cm^{-2}
2.27	Give a sketch of variation of electron and hole densities in a forward biased p-n junction, infinitely long in both sides.
Ans.	 The carrier distribution obeys the law :exp(-*x/L*), where *L* is the diffusion length.
2.28	From the diode equation prove that the dynamic resistance of a forward biased p-n junction diode is *dV/dI* = 26 ohm for forward current of 1 mA.
Ans.	From the diode equation we know, $I = I_0\left(e^{qV/kT} - 1\right)$. So $\frac{dV}{dI} = \frac{kT}{q}\frac{1}{I+I_0}$, as I_0 is negligible, putting $I = 1\ mA$, the dynamic resistance comes out as 26 Ω.

2.29	Prove that in a p^+-n junction, the forward bias current is predominantly due to holes injected into the n-region.
Ans.	We know that the diode current is given by $I = I_0\left[\exp\left(\frac{qV}{k_B T}\right)-1\right]$, where $I_0 = Aq\left(\frac{D_p p_{n0}}{L_p}+\frac{D_n n_{p0}}{L_n}\right)$. For p^+-n junction, p side is doped much more heavily than the n side. So, the equilibrium electron concentration n_{p0} in the p side can be neglected compared to the equilibrium hole concentration p_{n0} in the n side.So $I_0 = Aq\left(\frac{D_p p_{n0}}{L_p}\right)$. Therefore, the forward bias current is predominantly due to holes injected into the n-region.
2.30	Prove that the stored charge in n-region due to injected holes is $Q_p = I\tau_p$. Obtain the expression for the small signal diffusion capacitance dQ_p / dV.
Ans.	The total excess hole charge stored in the n-region is given by $Q_p = eA\Delta p\int_0^{\infty}\exp(-x/L_p)dx = eAL_p\Delta p$. Since the average lifetime of a hole in n-side is τ_p, the entire charge Q_p recombine in time τ_p and this charge must be supplied in order to maintain steady state. Thus steady state current is $I_p = Q_p/\tau_p$. Further , using the relation $\Delta p = p_{n0}[\exp(eV/k_B T)-1] \approx p_{n0}\exp(eV/k_B T)$, we may write $Q_p = I\tau_p = eAL_p p_{n0}\exp(eV/k_B T)$. The capacitance due to small changes in the stored charge is $C_{diffusion} = dQ_p/dV = (e/k_B T)I\tau_p$.

Chapter 03

3.1	Find the wavelength range over which GaAlAs alloy can be used as a laser.
Ans.	The laser material must have direct gap. The alloy is direct gap for x<0.45 and the band gap is 1.985eV for x = 0.45. Since wavelength is $\lambda = 1.24/E_8$, the range of wavelength (energy) is from 0.625 (1.985 eV) to 0.871 µm (1.424eV).
3.2	For DH laser you need type I alignment. Find the maximum value of x for GaAlAs that ensures that $E_{g\Gamma}$inGaAlAs will be at least 0.2 eV below the E_{gX} or E_{gL}.
Ans.	$E_{gX} - E_{g\Gamma} = 1.900+0.125x+0.143x^2-1.424-1.247x=0.2$. Solving the quadratic equation $0.276- 1.122x+0.143x^2 = 0$, we obtain x = 0.132. Again $E_{gL} - E_{g\Gamma} = 1.708+0.642x-1.424-1.247x = 0.2$. This gives x = 0.1388 ~ 0.14.
3.3	Show that $In_{0.52}Al_{0.48}As$ is lattice matched to $In_{0.53}Ga_{0.47}$ As.
Ans.	a($In_{0.53}Ga_{0.47}$ As.)= 5.8688 A; a(InAs) = 6.0584, a(AlAs) = 5.6612. a($In_{1-x}Al_xAs$)= (1-x) 6.0584+ x 5.6612 = 5.8688. Solving x = 0.48.
3.4	Find the composition of In_{1-x} $Ga_xAs_yP_{1-y}$ lattice matched to InP for emission at 1.3 and 1.55 µms.
Ans.	The band gap for 1.3 µm is $E_g = 1.24/1.3 = 0.954$ eV. Putting this in the expression for $E_g(y) = 1.35 - 0.72y + 0.12y^2$ and solving one obtains y = 0.64 and x = 0.29. The composition is $In_{0.71}Ga_{0.29}As_{0.64}P_{0.36}$. For 1.55 µm the band gap is 0.8 eV. Proceeding similarly, the composition becomes $In_{0.58}Ga_{0.42}As_{0.9}P_{0.1}$.
3.5	Show that the conduction band offset in GaAs/AlGaAsheterojunction increases with x, attains a maximum and then decreases. Use the 65:35 ratio for calculating the band offsets.
Ans.	From the expressions for the band gap $\Delta E_{g\Gamma}$=1.247x. Therefore $\Delta E_{c\Gamma}$ will increase with x. However for x> 0.45, the X valley in AlGaAs becomes the lowest conduction band valley. The conduction band offset is now ΔE_{cX} and it decreases with x.
3.6	Show that the heterostructure $Al_xGa_{1-x}As/Al_yGa_{1-y}As$ can show type II band alignment for x <y .
Ans	deleted