Chapter 2: Component Replacement Decisions

Problem 1 The following table contains cumulative losses, total costs and average monthly costs of operation for $n = 1, 2, 3, 4$. Here

$$AC(n) = \frac{\sum_{i=1}^{n} L_i + R_n}{n}$$

where L_i stands for loss in productivity during year i with respect to the first year's productivity, R_i stands for replacement cost (constant)

Month	Productivity	Losses	Replacement	Total Cost	Average Cost
1	10000	0	1200	1200	1200
2	9700	300	1200	1500	750
3	9400	600+300	1200	2100	700
4	8900	1100+600+300	1200	3200	800

Clearly, the optimal replacement time is 3 months since the pump is new.

Problem 2 One can use the model from section 2.5 (see 2.5.2). In this problem $C_p = 100$, $C_f = 200$,

$$R(t_p) = 1 - F(t_p) = 1 - \int_0^{t_p} f(z)\,dz = 1 - \frac{t_p}{40000} = \frac{40000 - t_p}{40000}$$

According to the model,

$$C(t_p) = \frac{C_p R(t_p) + C_f(1 - R(t_p))}{t_p R(t_p) + M(t_p)(1 - R(t_p))} =$$

$$= \frac{100 \times \frac{40000 - t_p}{40000} + 200 \times \frac{t_p}{40000}}{t_p \times \frac{40000 - t_p}{40000} + \int_0^{t_p} z f(z)\,dz} = \frac{100(80000 + 2t_p)}{80000 t_p - t_p^2}$$

$$C(t_p) = \begin{cases} 0.0143 & , t_p = 10000 \\ 0.01 & , t_p = 20000 \\ 0.0093 & , t_p = 30000 \\ 0.01 & , t_p = 40000 \end{cases}$$

Calculations above indicate that the optimal age is 30000 km.

Problem 3 Firstly, one can find $f(t)$. Since the area below the probability density curve is equal to 1, the area of each rectangle on the Figure 2.40 is $\frac{1}{5}$. It follows then, that

$$f(t) = \begin{cases} \frac{1}{25000} & , t \in [0..15000] \\ \frac{2}{25000} & , t \in [15000..25000] \\ 0 & , \text{elsewhere} \end{cases}$$

Secondly,

$$M(t_p) \times (1 - R(t_p)) = \int_0^{t_p} z f(z)\,dz = \begin{cases} \frac{t_p^2}{50000} & , t_p \in [0..15000] \\ \frac{15000^2}{50000} + 2\int_{15000}^{t_p} \frac{z}{25000}\,dz & , t_p \in [15000..20000] \end{cases}$$

To find $R(t)$ for the given values of t_p one can use Figure 2.40 ($R(t)$ is the area under $f(z)$ for $z > t$).

$$M(t_p) \times (1 - R(t_p)) = \begin{cases} 500 & , t_p = 5000 \\ 2000 & , t_p = 10000 \\ 4500 & , t_p = 15000 \\ 11500 & , t_p = 20000 \end{cases}, R(t_p) = \begin{cases} 0.8 & , t_p = 5000 \\ 0.6 & , t_p = 10000 \\ 0.4 & , t_p = 15000 \\ 0 & , t_p = 20000 \end{cases}$$

Using the suggested model $C(t_p) = \frac{C_p R(t_p) + C_f (1 - R(t_p))}{t_p R(t_p) + M(t_p)(1 - R(t_p))}$ for the given values of C_f, C_p yields

$$C(t_p) = \begin{cases} 0.093 & , t_p = 5000 \\ 0.067 & , t_p = 10000 \\ 0.063 & , t_p = 15000 \\ 0.078 & , t_p = 20000 \end{cases}$$

Therefore 15000 km is the optimal preventive replacement age.

Problem 4 Similarly to *Problem 3* $f(t_p) = \begin{cases} \frac{2}{10} & , t_p \in [0..2] \\ \frac{1}{10} & , t_p \in [2..8] \\ 0 & , \text{elsewhere} \end{cases}$

From the graph $R(t_p) = \begin{cases} 0.6 & , t_p = 2 \\ 0.4 & , t_p = 4 \\ 0.2 & , t_p = 6 \\ 0 & , t_p = 8 \end{cases}$

$$M(t_p) \times (1 - R(t_p)) = \int_0^{t_p} z f(z)\, dz = \begin{cases} \int_0^{t_p} \frac{2 \times z}{10}\, dz & , t_p \in [0..2] \\ \int_0^2 \frac{2 \times z}{10}\, dz + \int_2^{t_p} \frac{z}{10}\, dz & , t_p \in [2..8] \end{cases} =$$

$$= \begin{cases} \frac{t_p^2}{10} & , t_p \in [0..2] \\ \frac{t_p^2 + 4}{20} & , t_p \in [2..8] \end{cases}$$

After substitutions, the suggested formula gives:

$$D(t_p) = \frac{T_p \times R(t_p) + T_f \times (1 - R(t_p))}{t_p \times R(t_p) + M(t_p) \times (1 - R(t_p))} = \begin{cases} 0.9375 & , t_p = 2 \\ 0.7692 & , t_p = 4 \\ 0.7813 & , t_p = 6 \\ 0.8824 & , t_p = 8 \end{cases} \frac{Days}{Month}$$

Clearly, preventive replacement after 4 months of operation is the most preferable.

Problem 5 For the uniform distribution over [0..20000]

$$f(t) = \begin{cases} \frac{1}{20000} & , t \in [0..20000] \\ 0 & , \text{elsewhere} \end{cases}$$

Similarly to the previous problems,

$$R(t_p) = \begin{cases} 1 & , t_p < 0 \\ \frac{20000 - t_p}{20000} & , t_p \in [0..20000] \\ 0 & , t_p > 20000 \end{cases}, M(t_p) \times (1 - R(t_p)) = \int_0^{t_p} z f(z)\, dz = \frac{t_p^2}{40000}$$

Substitution of the given values of D_p and D_f into the proposed equation gives:

$$D(t_p) = \frac{3 \times \frac{20000 - t_p}{20000} + 9 \times \frac{t_p}{20000}}{t_p \times \frac{20000 - t_p}{20000} + \frac{t_p^2}{40000}} = \frac{120000 + 12 \times t_p}{40000 \times t_p - t_p^2} = \begin{cases} 0.00103 & , t_p = 5000 \\ 0.0008 & , t_p = 10000 \\ 0.0008 & , t_p = 15000 \\ 0.0009 & , t_p = 20000 \end{cases}$$

Hence, there are two equally preferable replacement ages among the given four.

Problem 6 Weibull paper analysis (Figure 1) gives estimations $\mu = 49000$ km, $\eta = 55000$ km, $\beta = 1.7$

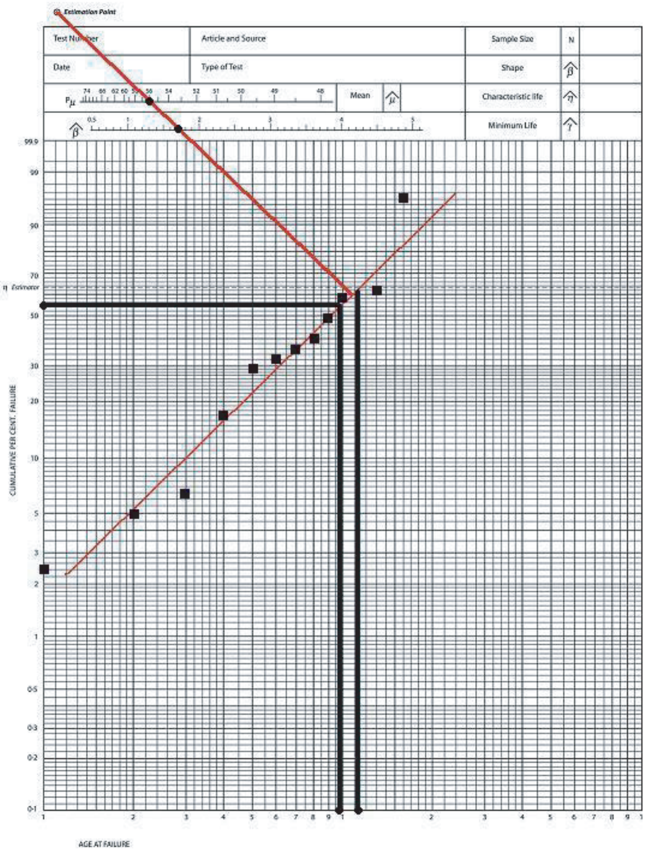

Figure 1: Problem 6 Weibull plot

Using the Γ function table (see Appendix 7) and the formula

$$\sigma^2 = \eta^2 \times [\Gamma(1 + \frac{2}{\beta}) - \Gamma^2(1 + \frac{1}{\beta})]$$

gives

$$\sigma^2 = (55000)^2 \times [1.1765\Gamma(1.1765) - \Gamma^2(1.5882)] = 3025 \times 10^6 \times (1.0883 - 0.796) =$$
$$= 885.2 \times 10^6$$

$\sigma = 29753$ km and therefore $\frac{\mu}{\sigma} = \frac{49000}{29753} = 1.65$. Now we can proceed to Glasser's paper analysis (Figure 2)

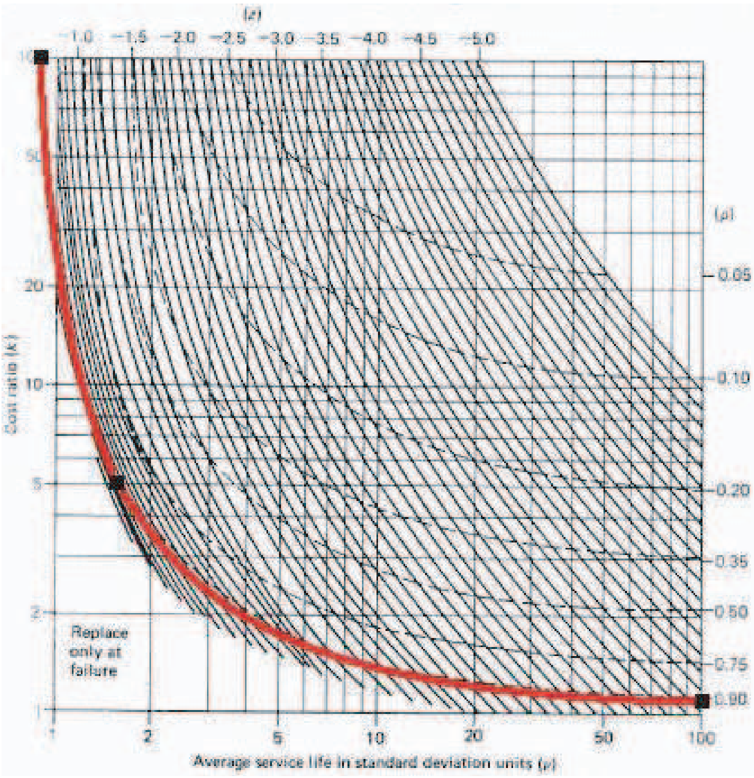

Figure 2: Problem 6 Glasser's graph

According to the graph $\rho = 0.92$, which means 8% of expected improvement, and $Z = -0.6$. Furthermore,

$$t_p = \mu + Z \times \sigma = 49000 - 0.6 \times 29753 = 31148$$

Problem 7 Firstly, sort the data in increasing order

Hours	80	100	115	130	150	170	200
Days	3.33	4.17	4.79	5.42	6.25	7.08	8.33

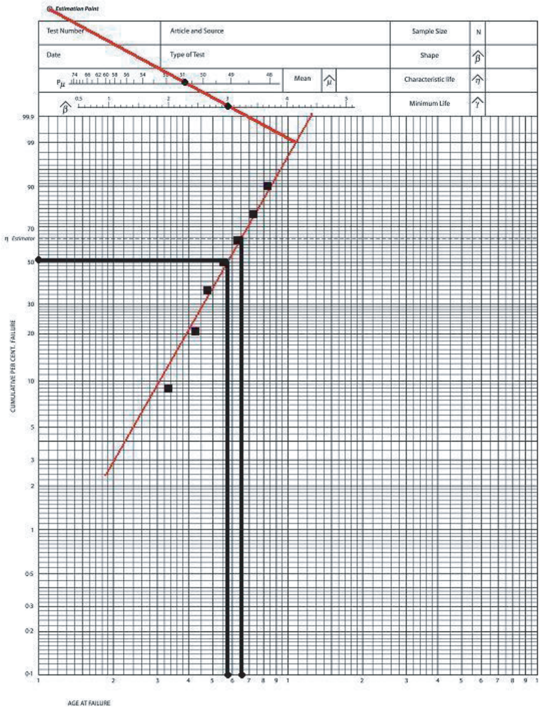

Figure 3: Problem 7 Weibull plot

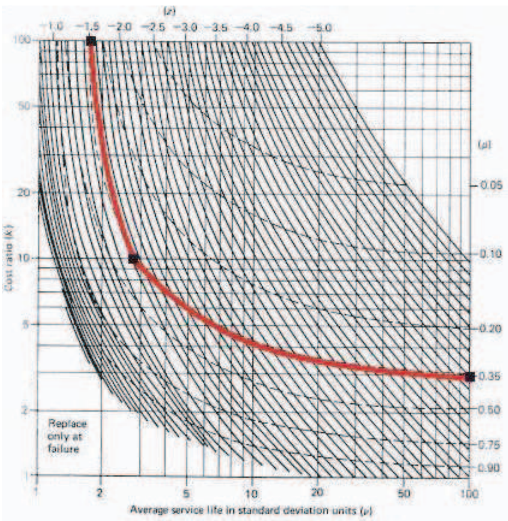

Figure 4: Problem 7 Glasser's Block graph

Secondly, it is reasonable to assume that the data belongs to the Weibull distribution. Then using the median ranks table (see Appendix 8) we get Weibull graph (Figure 3). From Figure 3 $\beta = 3$, $\mu = 5.7 \times 24 = 136.8$, $\eta = 6.5 \times 24 = 156$.

Next

$$\sigma^2 = \eta^2 \times [\Gamma(1+\frac{2}{\beta}) - \Gamma^2(1+\frac{1}{\beta})]$$

and therefore $\sigma^2 = (156)^2 \times [\Gamma(1.67) - \Gamma^2(1.33)] = 24336 \times 0.1059 = 2576$ $\sigma = 50.75$. Using the obtained result $\frac{\mu}{\sigma} = \frac{136.8}{50.75} = 2.7$ and $\frac{C_f}{C_p} = 10$

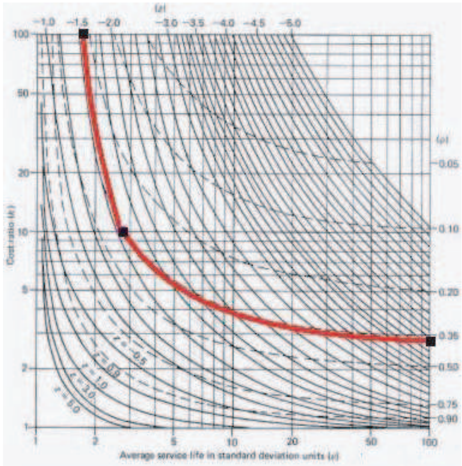

Figure 5: Problem 7 Glasser's Age Graph

From Figures 4 and 5: $Z = -1.5$, $\rho = 0.36$, $t_p = 136.8 - 1.5 \times 50.75 = 60.68$ for Block replacement policy and $Z = -1.55$, $\rho = 0.38$, $t_p = 58.14$ for age-based replacement policy.

Problem 8 Using $\frac{\mu}{\sigma} = \frac{20000}{1000} = 20$ and $\frac{C_f}{C_p} = 2$ and the Glasser's graph we get the following approximations: $Z = -2.1$, $\rho = 0.6$ (40% of expected improvement).

$$t_p = \mu + Z \times \sigma = 20000 - 2100 = 17900$$

Problem 9 $\frac{\mu}{\sigma} = \frac{150000}{10000} = 15$ and $\frac{C_f}{C_p} = 10$ Using the Glasser's graph for block replacement (Figure 6) we get: $\rho = 0.14$, $Z = -3.3$.

(a)

$$t_p = \mu + Z \times \sigma = 150000 - 33000 = 117000$$

(b)

$$1 - \rho = 0.86 \text{ or } 86\% \text{ of expected improvement.}$$

(c)

$$\text{R-o-o-F cost} = \frac{2000}{150000} = 0.0133 \ \frac{\$}{km}$$

$$\text{Optimal policy cost} = \rho \times \text{R-o-o-F} = 0.14 \times 0.0133 = 0.0018 \ \frac{\$}{km}$$

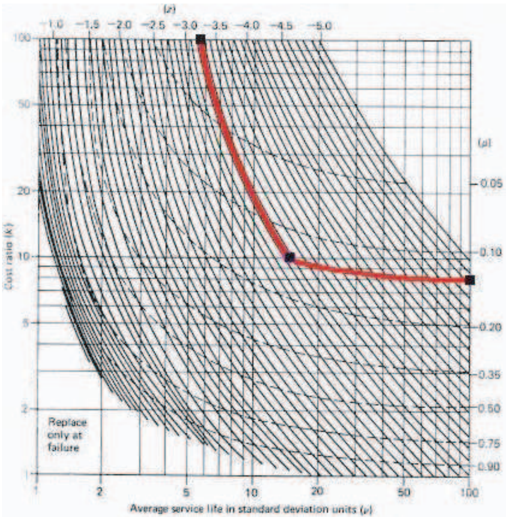

Figure 6: Problem 9 Glasser's graph

Problem 10

(a) One of the appropriate models is described in section 2.4.2

$$C(t_p) = \frac{C_p + C_f \times H(t_p)}{t_p}$$

(b) The most convenient way of solving the problem with the provided information is to use Glasser's graph for block replacement (Figure 7).

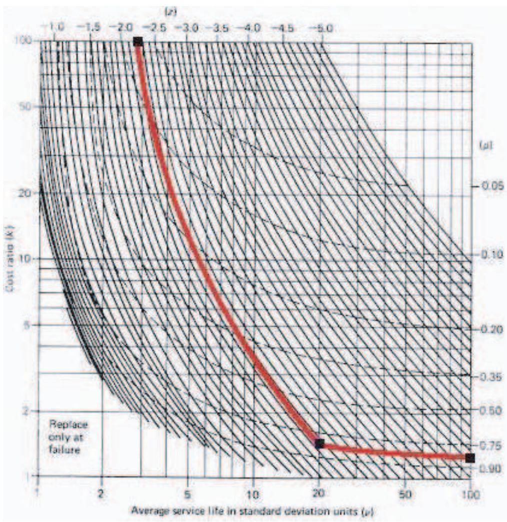

Figure 7: Problem 10 Glasser's Block graph

$$\frac{C_f}{C_p} = \frac{150}{100} = 1.5, \ \frac{\mu}{\sigma} = \frac{200}{10} = 20$$

From the graph $Z = -2.3$, $\rho = 0.83$ (17% of expected savings)

$$t_p = \mu + Z \times \sigma = 200 - 23 = 177$$

Problem11

$$\frac{C_f}{C_p} = \frac{200 + 700}{200 + 100} = \frac{900}{300} = 3$$

and

$$\frac{\mu}{\sigma} = \frac{150}{15} = 10$$

Using the Glasser's Block replacement graph:

$$Z = -2.2, \ \rho = 0.45 \text{ (55\% of expected improvement) } t_p = 150 - 33 = 117$$

Problems 12-16 OREST is a user-friendly piece of software designed especially for solving optimal replacement decision problems. The following screen shots are meant to give the reader some idea on how to input data and use it for analysis. At any point the user can access the help file by pressing "Help" button in the top right corner of the window.

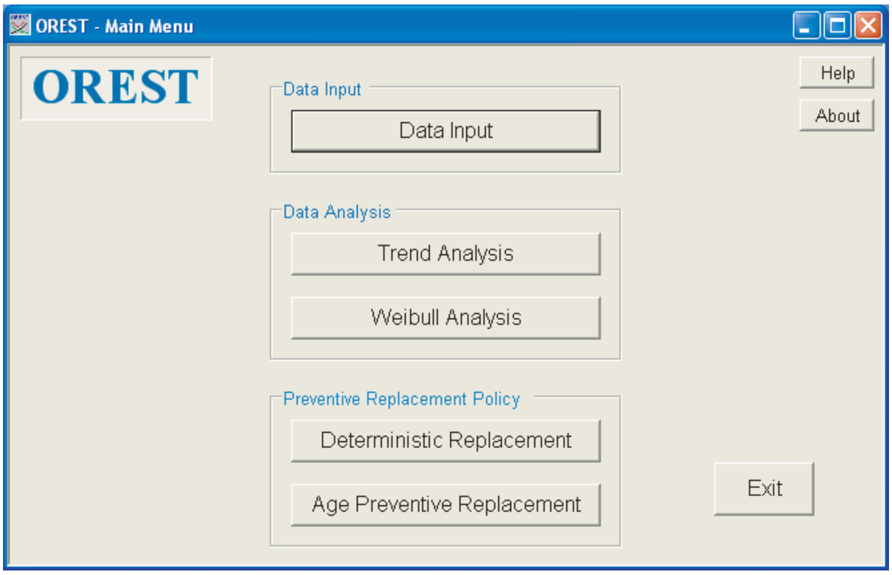

Figure 8: OREST screen shot

Figure 8 illustrates the main selection window of the program, where you can choose one of the available options:

- Data Input (see Figure 9), where one can create/change components
- Trend Analysis
- Weibull Analysis, where statistical analysis can be performed and saved/printed

- Deterministic Replacement
- Age Preventive Replacement, where replacement decisions are made.

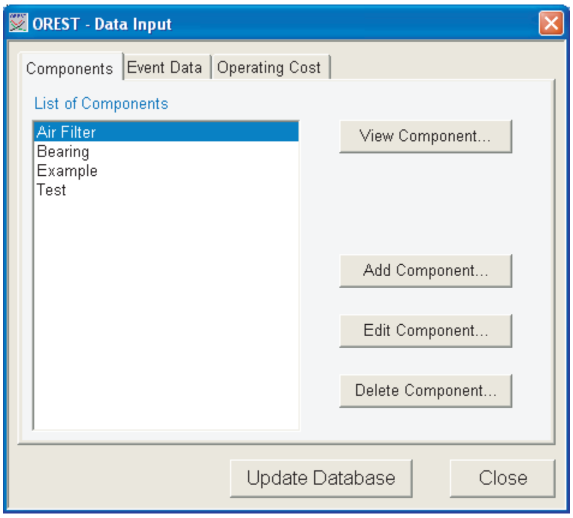

Figure 9: OREST screen shot

The main questions addressed in problems 12-16 are:

- Prepare the data for OREST
- Fit the Weibul model to data, determine the key parameters and perform a certain analysis
- Given the ratio between preventive and failure replacement costs, find an optimal replacement time
- Estimate a number of failures during a specified period of time under the optimal policy
- Given a replacement policy, find associated expected costs

Here is how OREST can help the user to answer the above questions.

OREST deals with sets of data which are expected to contain independent samples from the Weibull distribution (see Appendix 2). Essentially, in order to qualify for that, elements of a set have to be independent. For this reason, for example, cumulative times to failures will not do. However, the set of inter-failure times is the one we can use (under the assumption of independence).

Fitting the data set to the Weibull model is done automatically upon pressing the correspondent button in the "Weibull Analysis" window. Further analysis can be performed using graphs of p.d.f., C.d.f, Reliability and failure rate functions.

At time of creating a new component, values of preventive and failure costs are fixed. Using this data, the optimal replacement policy can be obtained in "Age Preventive Replacement" window. Note that every time an analysis is performed the user has an opportunity to print the report.

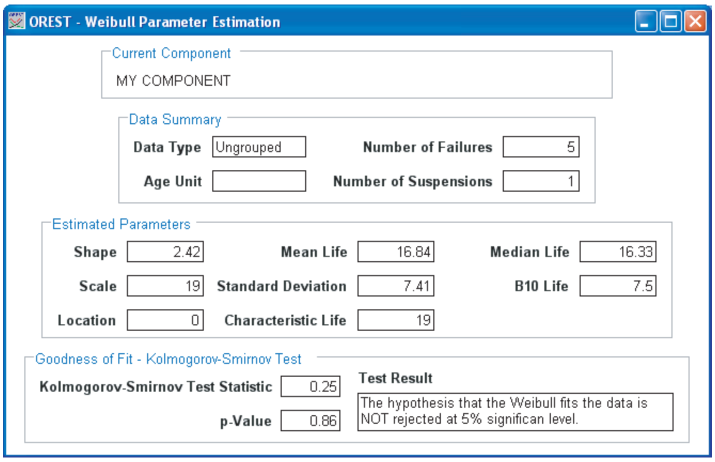

Figure 10: OREST screen shot

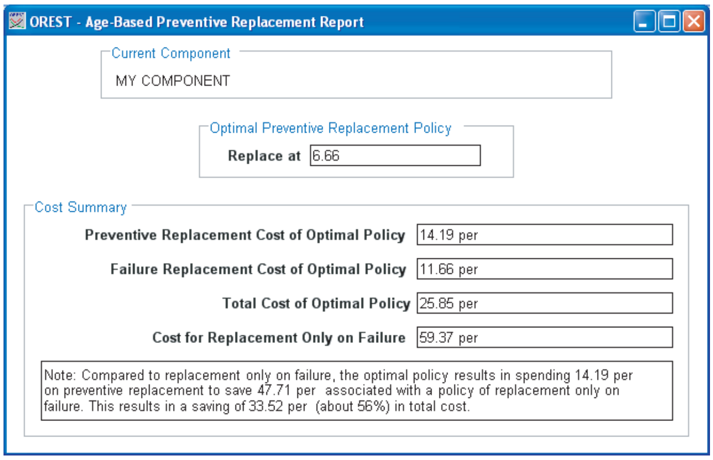

Figure 11: OREST screen shot

Along with the optimal policy, the report contains optimal costs, and more generally, a cost graph can be plotted illustrating total costs per unit of time associated with different replacement policies. Using these data one can estimate correspondent costs during some period of time by simply multiplying the cost per unit time by the length of a period. Furthermore, one can find the expected number of failure/preventive replacements by dividing the result by the cost of failure/preventive replacement.