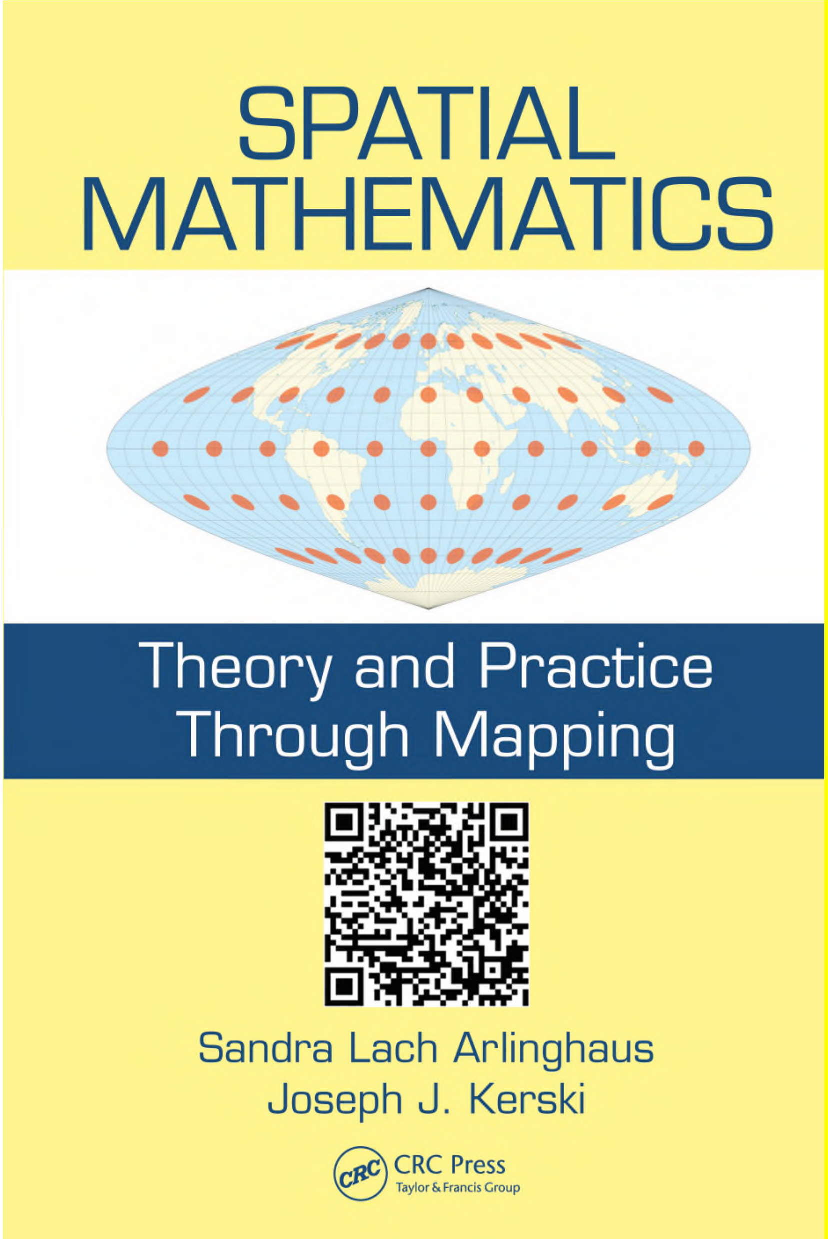


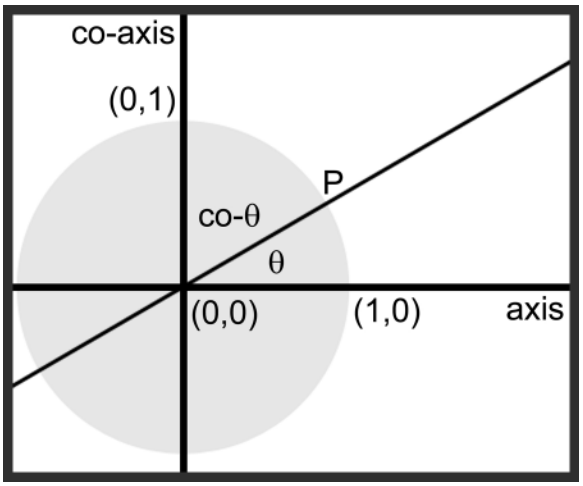

# Chapter 2

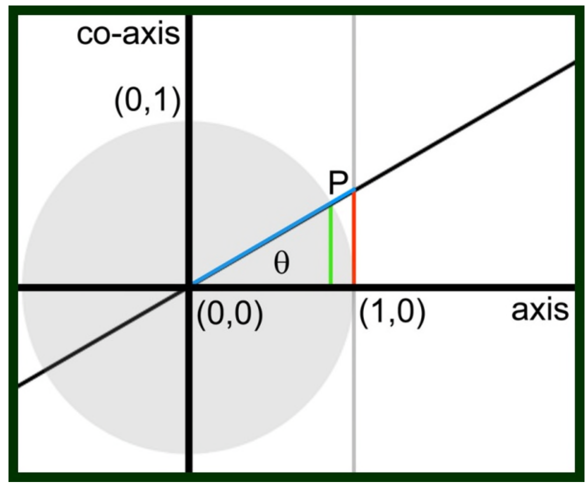
## Location, Trigonometry, and Measurement of the Sphere

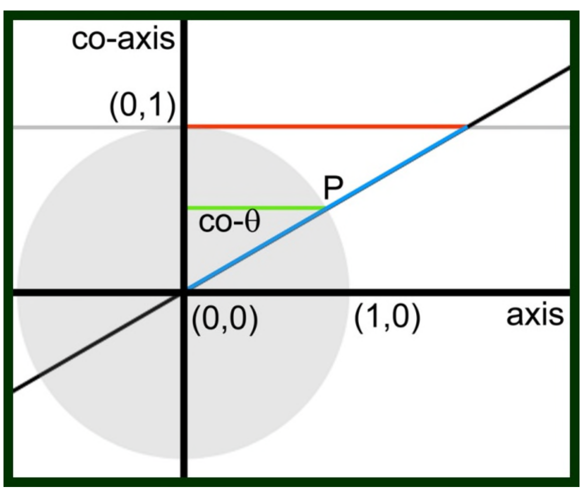
# Absolute vs. Relative Location

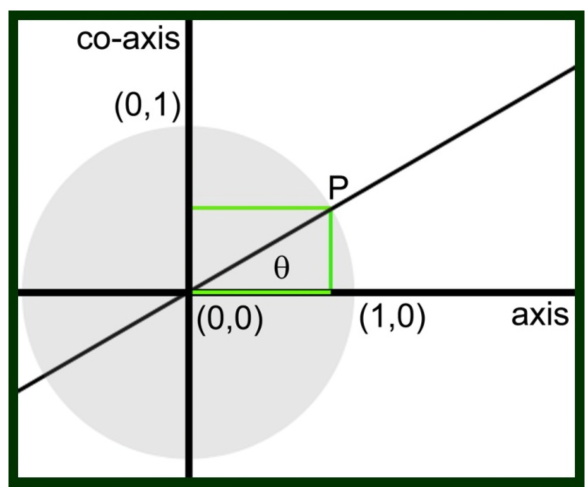
- **Absolute Location** uses a unique set of coordinates for an object on the Earth's surface, such as a set of latitude and longitude coordinates or a unique street address.
- **Relative Location** uses terms such as left, right, near, adjacent to, and other terms that refer to known objects to affix a desired location.
- Absolute and relative location are both useful concepts in spatial mathematics.

# Visual Trigonometry



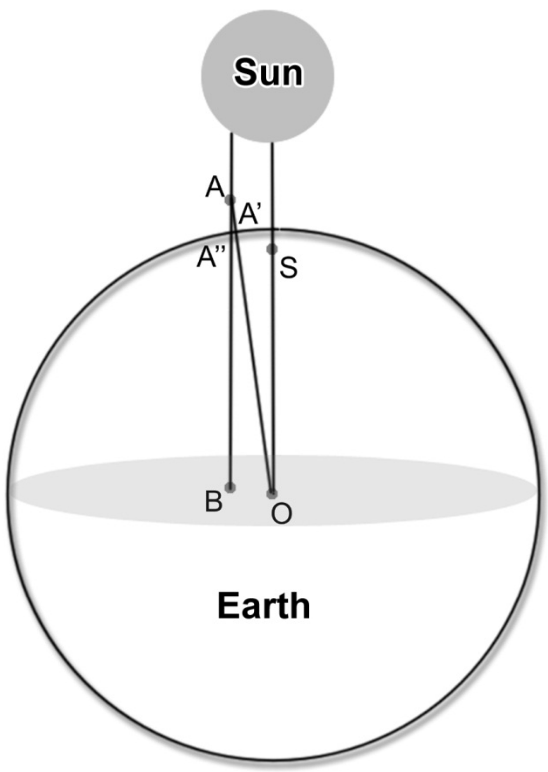



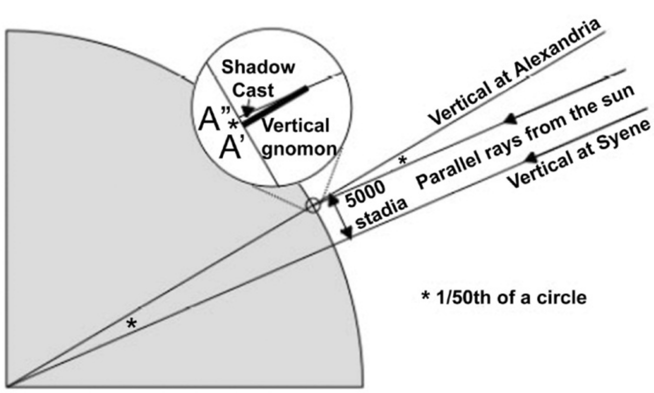



# Visual Trigonometry

- **Top left.** Unit circle, axis, and complementary (orthogonal) axis designated as co-axis. A secant line intersects the circle at P and forms an angle of θ with the axis and an angle of co- θ with the co-axis.
- **Top right.** Derivations of sine (green), tangent (red), and secant (blue) functions of angle θ.
- **Bottom left.** Derivations of co-sine (green), co-tangent (red), and co-secant (blue) functions of co-angle θ.
- **Bottom right.** Shows right triangle interpretation of cosine as adjacent side over hypotenuse.
- Source: Modified from Arlinghaus, S. L. and W. C. Arlinghaus, 2005. *Spatial Synthesis: Centrality and Hierarchy*. Volume I, Book 1. http://www.imagenet.org , Introduction. QR code links to animation from that book, http://www-personal.umich.edu/~copyrght/image/books/Spatial%20Synthesis/trig/anisandytrig.gif

# Eratosthenes’ Measurement of the Earth




Eratosthenes’s measurement of the circumference of the Earth, based on a Theorem of Euclid. In the top image, the sun is shown as a small gray ball far away from the Earth (large ball). The sun’s rays are assumed parallel to each other as they strike the Earth’s surface. The bottom image shows the detail of the configuration of obelisk (gnomon) and shadow in relation to the surface of the Earth.

Where does geometry come into play? (Look for alternate interior angles).

Where does trigonometry come into play?

# Detail of Eratosthenes' Measurement of the Sphere

- Assume the Earth is a sphere.
  - The circumference of the sphere is measured along a great circle on the sphere.
  - Find the circumference of the Earth by finding the length of intercepted arc of a small central angle.
  - Find two places on the surface of the Earth that lie on the same meridian (or close to it): Meridians are halves of great circles.
  - Eratosthenes chose Alexandria and Syene, near contemporary Aswan (A' and S, respectively).

# Detail of Eratosthenes' Measurement of the Sphere

- Assume that the rays of the Sun are parallel to each other.
  – The Sun's rays are directly overhead, on the Summer Solstice (c. June 21), at 23.5 degrees N. Latitude.
  – Syene is located at about 23.5 degrees N. Latitude. Hence, on the Summer Solstice, sunlight will pass to the bottom of a narrow well (and it will not do so on other days), S.
  – Alexandria is north of Syene. Thus, on June 21, objects at Alexandria will cast shadows whereas those at Syene will not.
  – Eratosthenes focused on an obelisk (AA') or post located in an open area (Figure 2.1). The base of the post is at A' in Alexandria and its tip is at A (assuming the obelisk is a straight extension of OA'—that is, it is not a "leaning" tower). He measured the shadow (A'A'') that the obelisk cast, functioning in the manner of a gnomon on a sundial, and then measured the height of the obelisk (AA') (perhaps using a string anchored to the tip of the obelisk).

# Detail of Eratosthenes' Measurement of the Sphere

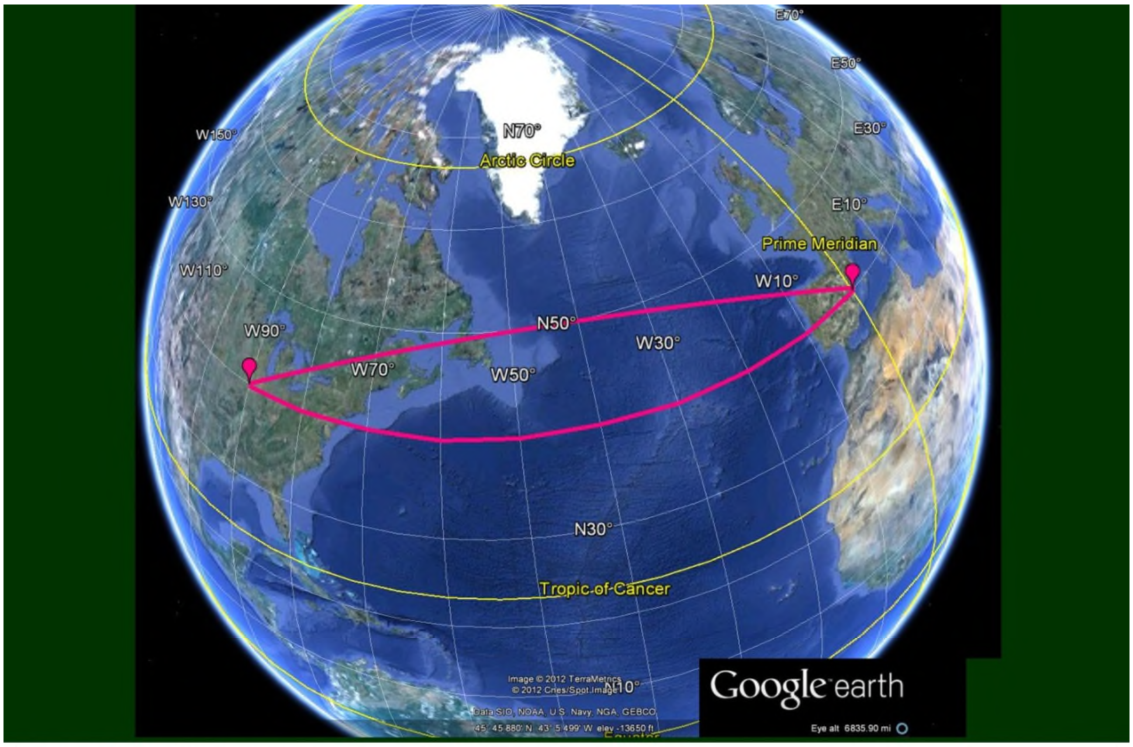
– According to Euclid, two parallel lines cut by a transversal have alternate interior angles that are equal. The Sun's rays are the parallel lines. One ray, at Alexandria, touches the tip of the obelisk and extends earthward toward the tip of the shadow of the obelisk, AA''. It is extended to AB in Figure 2.1. The other ray, SO, at Syene, goes into the well and extends abstractly to the center of the Earth, O. The obelisk, AA', also extends abstractly to the center of the Earth, O; thus, the line, AO, determined by the tip of the obelisk and the center of the Earth is a transversal cutting the two parallel rays, SO and AB, of the sun.

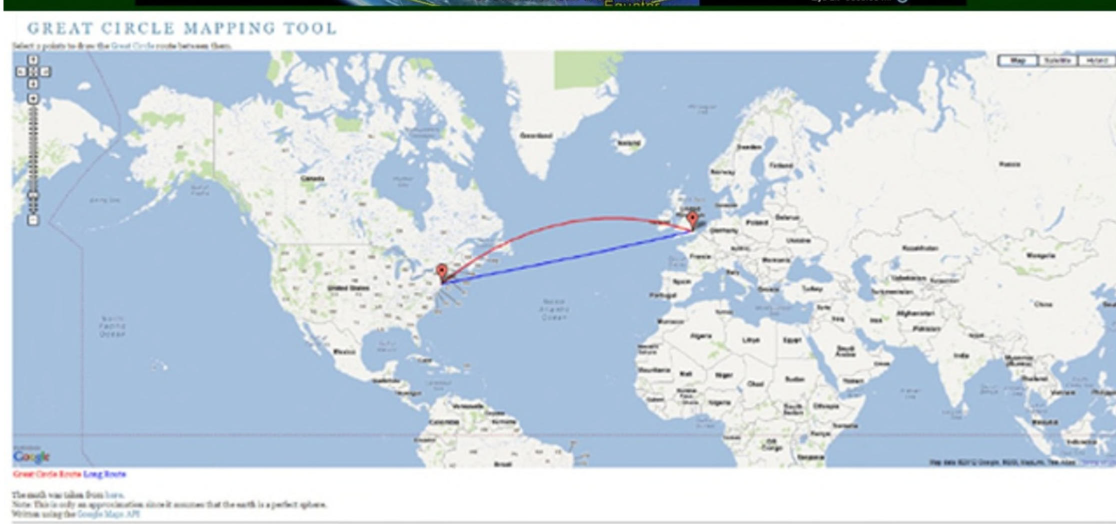
– Angles (BAO) and (SOA) are thus alternate interior angles in the geometric configuration described above; therefore, they are equal.

# Detail of Eratosthenes' Measurement of the Sphere

- Trigonometry permits angle measurement (see review in subsequent section). Use the length of the obelisk shadow and the height of the obelisk to determine angle BAO; triangle AA'A'' is a right triangle with the right angle at A'. Thus, *tan* (A'AA'') = (length of shadow, AA'')/(height of obelisk, AA'). Eratosthenes's measurements of these values led him to conclude that the measure of angle (A'AA'') was 7 degrees and 12 minutes and so therefore it was also of angle (BAO).
  - The value of 7 degrees and 12 minutes is approximately 1/50$^{th}$ of the degree measure of a circle. Since he assumed that Alexandria and Syene both lay on a meridian (half of a great circle), it followed that the distance between these two locations was 1/50$^{th}$ of the circumference of the Earth.
  - Eratosthenes calculated the distance between Alexandria and Syene using records involving camel caravans. The distance he used was 5000 stadia. Thus, the circumference of the Earth is 250,000 stadia, which translates to somewhat more (depending on how ancient units convert to modern units) than current accepted values although it is remarkably close.

# Measurement along Great Circle Routes



# Measurement along Great Circle Routes

- The shortest distance between two points on a sphere is measured along an arc of a great circle.
- Two different views of the same paths, on the sphere and in the plane.
- Source of base maps: Based on Google Earth™ mapping service (top) © 2012 Google and Image © 2012 TerraMetrics, Data SIO NOAA U.S. Navy NGA GEBCO, ©2012 Cnes/Spot Image; and, Great Circle Mapping Tool (bottom)

# Associated Activity

Many of the assumptions made by Eratosthenes were not accurate; apparently, however, underfit and overfit of error balanced out to produce a good result. For example, Syene and Alexandria are not on the same meridian, and Syene is not at exactly 23.5 degrees N. Latitude. To gain a deeper understanding of the principles involved in this measurement, you have the opportunity to participate in the activity or “practice” in the following section. The activity involves using a GPS receiver to measure the distance between two lines of latitude, determining the distance between the two, and calculating the circumference of the Earth from those two readings.

# Types of Latitude-Longitude Coordinates

- Degrees-Minutes-Seconds (DMS): For example, 42 degrees, 31 minutes, 47 seconds.
- Decimal Degrees (DD): 42.529756.
- Decimal Minutes (DM):  42 degrees, 31.7833 minutes.
- Each of the above coordinates is equivalent, and each type is used in everyday decision-making.

# Other Coordinate Systems

- Local street addresses in the USA typically are referenced from a baseline and meridian.
- State Plane Coordinate System:  Used extensively in the USA; for example:  310,200 feet north, 88,410 feet west, Wisconsin Central Zone.
- Universal Transverse Mercator (UTM):  Global except for Polar regions; for example:  3,446,250 meters northing, 529,781 meters easting, Zone 13.
- Public Land Survey System (PLSS):  Many states in the USA; for example:  Northwest ¼ of the southeast ¼, Section 6, Township 2 South, Range 55 West, Sixth Principal Meridian.
- Military Grid Reference System (MGRS):  Global, derived from UTM, given in meters with a grid zone designator, a 100,000 square meter identifier, and a number that corresponds to the easting and northing in that cell.

# Putting Theory into Practice

- Find the length of one degree on the Earth-sphere.
- Determine sun angles at different seasons of the year at different locations on the Earth.
- Work with online GIS tools to measure latitude and longitude while considering the influence of map projections and other factors.