

## Chapter 2

# Sampling and Reconstruction

### Problem Solutions

1. In theory, the highest frequency that can be sampled in lowpass sampling (as opposed to bandpass sampling) without aliasing is  $0.5F_s$ . Thus, in this example, 24 kHz would be the highest frequency that could be sampled without aliasing. Note that real-world constraints, such as non-ideal reconstruction filters, would reduce this by at least 10%.
2. Since 30 kHz is 6 kHz above the “wrap-around” frequency of 24 kHz, the resulting aliased signal would appear at  $24 - 6 = 18$  kHz.
3. For a dynamic range  $D$  and  $b$ -bit samples, the approximate resolution (or LSB voltage) is

$$\frac{D}{2^b} = \frac{2 \text{ V}}{2^{16}} = 30.52 \text{ } \mu\text{V}$$

4. In general, the SQNR (in decibels) is estimated by:

$$\text{SQNR}_{\text{dB}} = 10 \log_{10} \frac{\sigma_x^2}{\sigma_e^2}$$

where  $\sigma_x^2$  is the input signal variance representing the signal power and  $\sigma_e^2$  is the quantization noise variance representing quantization noise power. For an ADC with dynamic range  $D$  and  $b$ -bit samples, and using common assumptions about the quantization error, this becomes

$$\text{SQNR}_{\text{dB}} = 10 \log_{10} \left( \frac{2^{2b} \cdot 12 \cdot \sigma_x^2}{D^2} \right) = 6.02b + 10.79 - 20 \log_{10} \left( \frac{D}{\sigma_x} \right).$$

This shows the well known relation that SQNR increases by approximately 6 dB for each 1-bit increase in the number of bits per sample. To calculate the third term of the equation above, an assumption must be made regarding the input signal type, since this term depends upon the standard deviation  $\sigma_x$  (which is equivalent to the RMS value). If we assume a sinusoid with an amplitude of  $A$ , the RMS value is  $A/\sqrt{2} = 0.7071A$ . According to the problem, the signal has an amplitude of  $A = 1$  V, and the dynamic range of the ADC is  $D = 2$  V, and the number of bits is 16. Thus, for the sinusoidal assumption,

$$\text{SQNR}_{\text{dB}} = (6.02)(16) + 10.79 - 20 \log_{10} \left( \frac{2}{0.7071} \right) = 98.08 \text{ dB}.$$

Noise shaping methods, such as those commonly employed in sigma-delta ADCs, can produce a higher SQNR than this.

5. A unit-amplitude sinusoidal signal of frequency  $f_0$  that is sampled at a sample frequency  $F_s$  such that  $f_0 = 0.5F_s$ , will produce samples having the pattern  $+1, -1, +1, -1, \dots$ . Point-by-point multiplication