

Chapter 2 Mechanical Properties of Polymers

- 2.1 Calculate and compare the values of engineering strain ε and the true strain ε_T as functions of $\delta L/L_0$ for $\delta L/L_0 = 0.0, 0.5, 1.0, 1.5$, and 2.0 . Do the same for the engineering stress and the true stress. Assume $\sigma_T = K \varepsilon_T^N$ and $K = 2000$ and $N = 0.2$.

Reiterating the definitions for stress and strain

$$\text{Engineering strain, } \varepsilon = \frac{\Delta L}{L_0}$$

$$\text{True strain, } \varepsilon_T = \ln(1 + \varepsilon)$$

$$\text{Engineering stress, } \sigma = \frac{P}{A_0}$$

$$\text{True stress, } \sigma_T = \frac{P}{A} = \sigma(1 + \varepsilon)$$

ε	ε_T	σ	σ_T
0	0.0000000	0	0
0.001	0.0009995	502	502
0.01	0.0099503	788	795
0.5	0.4054651	1113	1670
1.0	0.6931472	929	1859
1.5	0.9162907	786	1965
2.0	1.0986123	679	2038

- 2.2 Derive equation (2.8).

$$\tau = G\gamma$$

Consider Mohr's circle for pure shear, the 2-D principle stresses become $\sigma_1 = \tau$,

$$\sigma_2 = -\tau, \text{ similarly } \varepsilon_1 = \frac{\gamma}{2}, \varepsilon_2 = -\frac{\gamma}{2}.$$

From Hooke's Law

$$\varepsilon_1 = \frac{1}{E}(\sigma_1 - \nu\sigma_2)$$

Substituting for $\varepsilon_1, \sigma_1, \sigma_2$ yields

$$\frac{\gamma}{2} = \frac{1}{E}(\tau + \nu\tau)$$

Substituting for γ

$$\frac{\tau}{2G} = \frac{1}{E}(\tau + \nu\tau)$$

Rearranging and eliminating τ

$$G = \frac{E}{2(1+\nu)}$$

- 2.3 If a compression test is conducted on a brittle glassy amorphous polymer, at what angle would the fracture plane be oriented? (Review the "Mohr Coulomb" theory of failure).

$$\tau = C - \mu\sigma$$

where:

C = cohesion

μ = angle of friction

$$\frac{d\tau}{d\sigma} = -\mu = -\tan\left(\frac{\pi}{2} - 2\phi\right)$$

$$2\phi < \frac{\pi}{2}$$

$$\phi < \frac{\pi}{4} \text{ or } 45^\circ$$

- 2.4 Verify equations (2.19) - (2.23).

The complex modulus was defined as $E^* = E' + iE''$, since we know that

$$J^* = \frac{1}{E^*}$$

$$J^* = \frac{1}{E' + iE''}$$

$$J^* = \frac{1}{E' + iE''} \frac{E' - iE''}{E' - iE''}$$

$$J^* = \frac{E' - iE''}{E'^2 + E''^2}$$

$$J^* = \frac{E' - iE''}{E^2}$$

$$J^* = \frac{E'}{E^2} - \frac{iE''}{E^2}$$

$$J^* = J' - iJ''$$

$$\therefore J' = \frac{E'}{E^2} \text{ and } J'' = \frac{E''}{E^2}$$

Squaring the relationship between complex compliance and modulus

$$(J^*)^2 = \left(\frac{1}{E^*}\right)^2$$

or

$$J^2 = \frac{1}{E^2}$$

$$E' = J' E^2$$

$$\therefore E' = \frac{J'}{J^2}$$

As a side note

$$\frac{J''}{J'} = \frac{\frac{E''}{E^2}}{\frac{E'}{E^2}} = \frac{E''}{E'}$$

- 2.5 Show that the viscosity η of a linear viscous polymer element is equal to E''/ω where $E^* = E' + iE''$ and ω is the frequency of vibration.

$$E'' = 2M\alpha\omega^2$$

$$\alpha\omega = \frac{\eta}{2M}$$

$$E'' = 2M\left(\frac{\eta}{2M}\right)\omega$$

$$\eta = \frac{E''}{\omega}$$

- 2.6 A polymer with a complex modulus E^* is tested in a vibrating beam test. Two different polymers A and B were tested. The periods of resonant oscillation were determined to be 0.05 sec and 0.06 sec respectively. Which polymer has the greatest modulus A or B? What part of E^* is determined by the period of oscillation E' or E'' ?

E' determines the spring constant k of a spring, mass, damper system, and E'' determines the damping constant, typically c or h . The natural frequency of an undamped system is $\omega_n = \sqrt{\frac{k}{m}}$ and since $k \propto E'$, $\omega_n^2 \propto E'$. A damped system has a damped natural frequency $\omega_d = \omega_n \sqrt{1 - \xi^2}$, where $\xi = \frac{c}{2\sqrt{km}}$ and $c \propto E''$. For most systems, especially those vibrating at 0.05 and 0.06 sec, the damping is relatively low and ω_d is dominated by ω_n .

$$\tau = \frac{2\pi}{\omega}$$

$$\tau_A = \frac{2\pi}{\omega_A} = 0.05 \text{ s}$$

$$\tau_B = \frac{2\pi}{\omega_B} = 0.06 \text{ s}$$

$$\frac{\omega_B}{\omega_A} = \frac{0.05}{0.06}$$

$$\omega_n^2 \propto E'$$

$$\frac{E'_B}{E'_A} = \left(\frac{\omega_B}{\omega_A} \right)^2 = \left(\frac{0.05}{0.06} \right)^2$$

$$E'_A = \left(\frac{0.06}{0.05} \right)^2 = 1.44 E'_B$$

- 2.7 The creep data given below were measured for a polymer at three different stress levels. Does the polymer have a creep compliance $J(t)$? If so what is the $J(t)$ at time $t=100$ min.? If the polymer has a $J(t)$ what kind of polymer is it?

Yes it does.

Check for linearity of compliance at $t=100$ minutes. If the polymer is a linear viscoelastic material the compliance will be the same for all stress levels.

$$\text{For } \sigma = 1000 \text{ psi} \quad J(t=100) = \frac{\varepsilon}{\sigma} = \frac{0.01}{1000} = 1.0(10^{-5})$$

$$\text{For } \sigma = 1500 \text{ psi} \quad J(t=100) = \frac{\varepsilon}{\sigma} = \frac{0.015}{1500} = 1.0(10^{-5})$$

$$\text{For } \sigma = 2000 \text{ psi} \quad J(t=100) = \frac{\varepsilon}{\sigma} = \frac{0.02}{2000} = 1.0(10^{-5})$$

The same calculations at $t=300$ minutes yields the same conclusion of linearity with a creep compliance of $1.5(10^{-5})$ 1/psi.

Linearly elastic, almost an ideal fluid as evidenced by the initial strain and strain as a function of time.

- 2.8 The figure below gives some dimensions and forces for the DMA test, Figure 2.7.

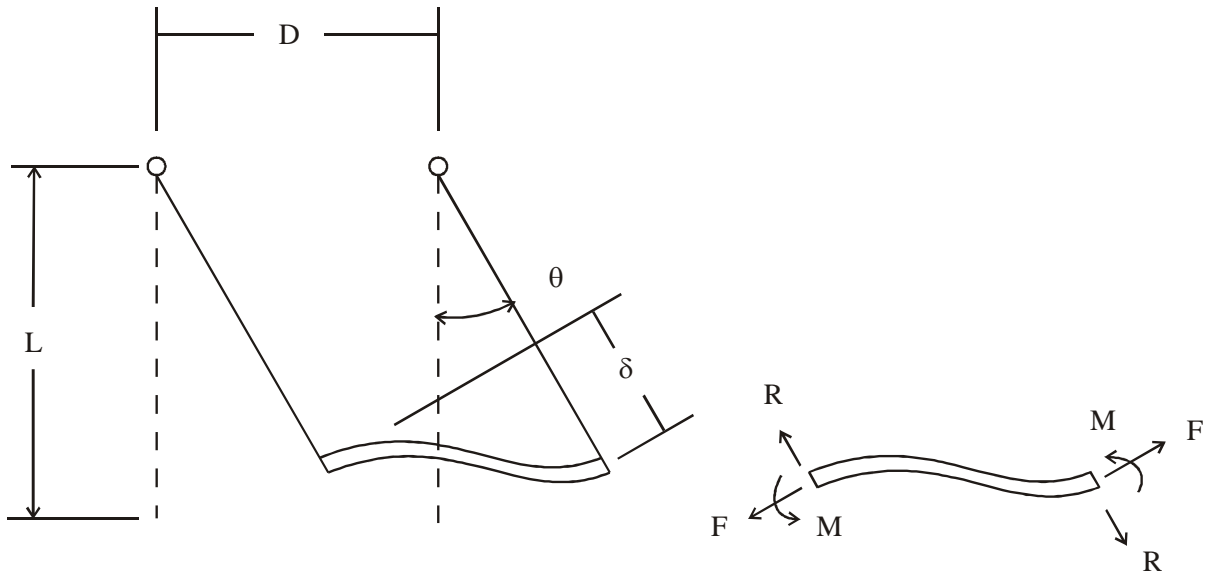
- Show that the force F is approximately zero for small deflections.
- Show that the deflection δ of the beam specimen is related to the moment by

$$\delta = \frac{ml^2}{6EI}$$

where:

$$\delta = D \sin \Theta$$

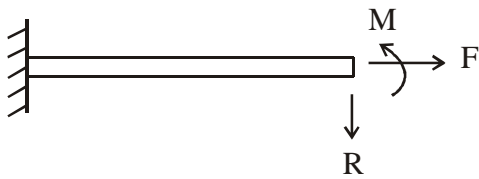
- Write the equation of motion of the system to control the motion. Use dynamics of rotation equations for rigid bars.



$$\sum M_{lhs} = F\delta + 2M - RD \cos \theta$$

$$\delta = D \sin \theta$$

Cantilever Beam Analogy



$$EI\delta = \frac{Rl^3}{3} - \frac{Ml^2}{2}$$

$$EIy' = -\frac{Rl^3}{2} + Ml + M = \frac{Rl}{2}$$

Now

$$F = \frac{RD \cos \theta - Rl}{\delta} = \frac{R}{\delta} (D \cos \theta - l) \approx 0 \text{ for small deflection}$$

$$EI\delta = EI(D \sin \theta) = \frac{2Ml^2}{3} - \frac{Ml^2}{2} = \frac{Ml^2}{6}$$

Thus,

$$\delta = \frac{Ml^2}{6} \text{ and } M = \frac{6EID \sin \theta}{D^2}$$

Where $l \approx D$