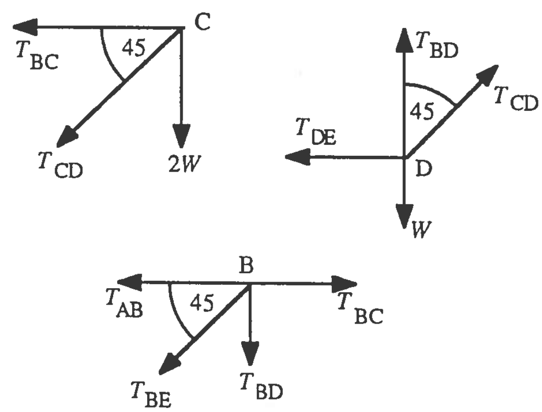
Chapter 2

Statically Determinate Systems

2.1



For equilibrium of forces at C in the vertical direction

$$-T_{\mathrm{CD}} \sin 45^{\circ} - 2W = 0 \text{ giving } T_{\mathrm{CD}} = \mathbf{-2.828}W$$

and in the horizontal direction

$$-T_{\mathrm{BC}} - T_{\mathrm{CD}} \cos 45^{\circ} = 0 \text{ giving } T_{\mathrm{BC}} = \mathbf{2}W$$

For forces at D in the vertical direction

$$T_{\mathrm{BD}} + T_{\mathrm{CD}} \cos 45^{\circ} - W = 0 \text{ giving } T_{\mathrm{BD}} = \mathbf{3}W$$

and in the horizontal direction

$$T_{\mathrm{CD}} \sin 45^{\circ} - T_{\mathrm{DE}} = 0 \text{ giving } T_{\mathrm{DE}} = \mathbf{-2}W$$

For forces at B in the vertical direction

$$-T_{\mathrm{BD}} - T_{\mathrm{BE}} \sin 45^\circ = 0 \text{ giving } T_{\mathrm{BE}} = -\mathbf{4.243}\boldsymbol{W}$$

and in the horizontal direction

$$T_{\mathrm{BC}} - T_{\mathrm{AB}} - T_{\mathrm{BE}} \cos 45^\circ = 0 \text{ giving } T_{\mathrm{AB}} = \mathbf{5}\boldsymbol{W}$$

2.2 In using program SDPINJ, the actual lengths of the members are not important, provided the correct angles between them are maintained. The following dimensions are assumed: AB = BC = BD = DE = 1. The load W is taken as 1.

The five nodes are numbered in the order A to E, and the six elements in the order: AB, BC, CD, DE, BE and BD. The origin of coordinates is at E (node 5).

The computed force factors (multiplying W) are identical to those obtained in Problem 2.1.

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STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE      X            Y              NODE      X            Y
  1  0.0000E+00  0.1000E+01             2  0.1000E+01  0.1000E+01
  3  0.2000E+01  0.1000E+01             4  0.1000E+01  0.0000E+00
  5  0.0000E+00  0.0000E+00

NODES CONNECTED BY THE ELEMENTS
   M    I    J        M    I    J        M    I    J        M    I    J
   1    1    2        2    2    3        3    3    4        4    4    5
   5    5    2        6    2    4

FORCE COMPONENTS AT THE LOADED NODES
  NODE             FX             FY
    3          0.0000E+00     -0.2000E+01
    4          0.0000E+00     -0.1000E+01

NODES FORMING PINNED SUPPORTS :    1   5

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE        ELEMENT     FORCE        ELEMENT     FORCE
       1      0.5000E+01        2      0.2000E+01        3     -0.2828E+01
       4     -0.2000E+01        5     -0.4243E+01        6      0.3000E+01

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE    R (X DIRN)     R (Y DIRN)
    1    -0.5000E+01     0.0000E+00
    5     0.5000E+01     0.3000E+01
```

2.3 Let the length of members AE, ED, DC, BE and BD in Fig. P2.3 be L, and let the upward vertical reactions at A and C be R_{A} and R_{C}, respectively. For moment equilibrium of the structure about C

$$R_{\mathrm{A}} \times 3L - P \times 2L - 5P \times 1.5L = 0 \text{ giving } R_{\mathrm{A}} = 3.167P$$

and for force equilibrium in the vertical direction

$$R_{\mathrm{A}} + R_{\mathrm{C}} = P + 5P \text{ giving } R_{\mathrm{C}} = 2.833P$$

Taking nodes in the order A, E, D, and C, and considering equilibrium of forces in the vertical and horizontal directions:

$$R_{\mathrm{A}} + T_{\mathrm{AB}} \sin 30^\circ = 0 \text{ giving } T_{\mathrm{AB}} = -\mathbf{6.333}P$$
$$T_{\mathrm{AB}} \cos 30^\circ + T_{\mathrm{AE}} = 0 \text{ giving } T_{\mathrm{AE}} = \mathbf{5.485}P$$
$$T_{\mathrm{BE}} \sin 60^\circ - P = 0 \text{ giving } T_{\mathrm{BE}} = \mathbf{1.155}P$$
$$T_{\mathrm{BE}} \cos 60^\circ + T_{\mathrm{DE}} - T_{\mathrm{AE}} = 0 \text{ giving } T_{\mathrm{DE}} = \mathbf{4.908}P$$
$$T_{\mathrm{BD}} \sin 60^\circ = 0 \text{ giving } T_{\mathrm{BD}} = \mathbf{0}$$
$$T_{\mathrm{CD}} - T_{\mathrm{BD}} \cos 60^\circ - T_{\mathrm{DE}} = 0 \text{ giving } T_{\mathrm{CD}} = \mathbf{4.908}P$$
$$R_{\mathrm{C}} + T_{\mathrm{BC}} \sin 30^\circ = 0 \text{ giving } T_{\mathrm{BC}} = -\mathbf{5.667}P$$

2.4 The actual lengths of the members are not important, provided the correct angles between them are maintained. All the members except AB and BC are assumed to be of length 1.5 (In Problem 4.5 the same structure is considered, and this common length is given as 1.5 m). The load P is taken as 1.

The five nodes are numbered in the order A to E, and the seven elements in the order: AB, BE, BD, BC, AE, DE, and CD. The origin of coordinates is at A (node 1).

The computed force factors (multiplying P) are identical to those obtained in Problem 2.3.

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STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE      X            Y              NODE      X            Y
  1  0.0000E+00   0.0000E+00            2  0.2250E+01   0.1299E+01
  3  0.4500E+01   0.0000E+00            4  0.3000E+01   0.0000E+00
  5  0.1500E+01   0.0000E+00

NODES CONNECTED BY THE ELEMENTS
   M    I    J         M    I    J         M    I    J         M    I    J
   1    1    2         2    2    5         3    2    4         4    2    3
   5    1    5         6    5    4         7    4    3

FORCE COMPONENTS AT THE LOADED NODES
  NODE           FX              FY
    2         0.0000E+00     -0.5000E+01
    5         0.0000E+00     -0.1000E+01

NODES FORMING PINNED SUPPORTS :     1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
    3              .00
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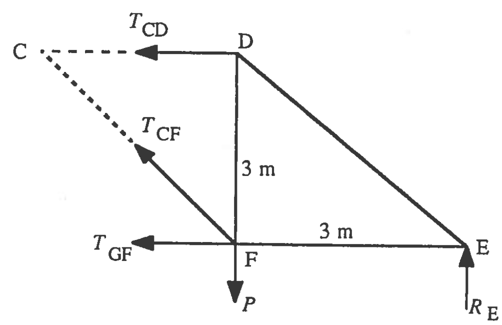
COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT    FORCE       ELEMENT    FORCE       ELEMENT    FORCE
       1    -0.6333E+01       2     0.1155E+01       3     0.3576E-06
       4    -0.5667E+01       5     0.5485E+01       6     0.4908E+01
       7     0.4908E+01

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE     R (X DIRN)      R (Y DIRN)
    1    -0.4768E-06     0.3167E+01

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE      REACTION
    3     0.2833E+01
```

2.5 Cutting the structure through members CD, CF, and GF, and treating the right hand part of it as a free body.



Let R_E be the vertical reaction at E, and P the downward vertical load at F (which is equal to 250 kN, zero or zero, depending on whether the actual load is applied at F, G or H, respectively). For moment equilibrium about F

$$-T_{CD} \times 3 - R_E \times 3 = 0 \text{ giving } T_{CD} = -R_E$$

For force equilibrium in the vertical direction

$$T_{CF} \sin 45^\circ + R_E - P = 0 \text{ giving } T_{CF} = \sqrt{2}(P - R_E)$$

For moment equilibrium about C

$$T_{GF} \times 3 + P \times 3 - R_E \times 6 = 0 \text{ giving } T_{GF} = 2R_E - P$$

For the three positions of the 250 kN load, the results are as follows

Load at	R_E (kN)	P (kN)	T_{CD} (kN)	T_{CF} (kN)	T_{GF} (kN)
F	187.5	250	−187.5	88.39	125
G	125.0	0	−125.0	−176.8	250
H	62.5	0	−62.5	−88.39	125

2.6 The eight nodes are numbered in the order A to H, and the thirteen elements in the order: AB, BC, CD, DE, EF, FG, GH, AH, BH, CH, CG, CF, and DF. The origin of coordinates is at A (node 1), lengths and forces are in m and kN, respectively, and the load of 250 kN is applied at G (node 7).

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STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE      X           Y             NODE      X           Y
 1  0.0000E+00  0.0000E+00           2  0.3000E+01  0.3000E+01
 3  0.6000E+01  0.3000E+01           4  0.9000E+01  0.3000E+01
 5  0.1200E+02  0.0000E+00           6  0.9000E+01  0.0000E+00
 7  0.6000E+01  0.0000E+00           8  0.3000E+01  0.0000E+00

NODES CONNECTED BY THE ELEMENTS
  M    I    J       M    I    J       M    I    J       M    I    J
  1    1    2       2    2    3       3    3    4       4    4    5
  5    5    6       6    6    7       7    7    8       8    8    1
  9    2    8      10    8    3      11    3    7      12    3    6
 13    4    6

FORCE COMPONENTS AT THE LOADED NODES
  NODE            FX             FY
    7         0.0000E+00    -0.2500E+03

NODES FORMING PINNED SUPPORTS :     1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
    5              .00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE       ELEMENT     FORCE       ELEMENT     FORCE
       1     -0.1768E+03       2     -0.1250E+03       3     -0.1250E+03
       4     -0.1768E+03       5      0.1250E+03       6      0.2500E+03
       7      0.2500E+03       8      0.1250E+03       9      0.1250E+03
      10     -0.1768E+03      11      0.2500E+03      12     -0.1768E+03
      13      0.1250E+03

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE     R (X DIRN)      R (Y DIRN)
    1      0.0000E+00      0.1250E+03

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE      REACTION
    5      0.1250E+03
```

The forces in the members are: $-$**176.8 kN** in AB, DE, CF and CH (elements 1, 4, 12 and 10); $-$**125.0 kN** in BC and CD (elements 2 and 3); **+125.0 kN** in EF, AH, BH and DF (elements 5, 8, 9 and 13); **+250.0 kN** in FG, GH and CG (elements 6, 7 and 11).

2.7 Since the structure and loading are symmetrical, the upward vertical reactions at A and E are $R_{\mathrm{A}} = R_{\mathrm{E}} = \mathbf{2.5}F$, and it is only necessary to find the forces in one half of the structure - the left hand half, say. The force in GH can be found by sectioning the structure through, say, BC, CJ and GH, and considering moment equilibrium about C for the part of the structure to the left of this

$$R_{\mathrm{A}} \times 1.5L - F \times 1.5L - F \times 0.75L - T_{\mathrm{GH}} \times L \sin 60^\circ = 0$$
$$\text{giving } T_{\mathrm{GH}} = \mathbf{1.732}F$$

where L is the distance between G and H, also A and H, and C and H. Considering nodes A and I, for equilibrium of forces in the vertical and horizontal directions:

$$R_{\mathrm{A}} - F + T_{\mathrm{AB}} \sin 30^\circ = 0 \text{ giving } T_{\mathrm{AB}} = -\mathbf{3}F$$
$$T_{\mathrm{AB}} \cos 30^\circ + T_{\mathrm{AI}} = 0 \text{ giving } T_{\mathrm{AI}} = \mathbf{2.598}F$$
$$T_{\mathrm{BI}} \sin 60^\circ = 0 \text{ giving } T_{\mathrm{BI}} = 0$$
$$T_{\mathrm{HI}} + T_{\mathrm{BI}} \cos 60^\circ - T_{\mathrm{AI}} = 0 \text{ giving } T_{\mathrm{HI}} = \mathbf{2.598}F$$

For equilibrium of forces at H in the direction perpendicular to HJ

$$T_{\mathrm{HI}} \cos 30^\circ + T_{\mathrm{BH}} \cos 30^\circ - T_{\mathrm{GH}} \cos 30^\circ = 0$$
$$\text{giving } T_{\mathrm{BH}} = -\mathbf{0.8660}F$$

and in the vertical direction

$$T_{\mathrm{BH}} \sin 60^\circ + T_{\mathrm{HJ}} \sin 60^\circ = 0 \text{ giving } T_{\mathrm{HJ}} = \mathbf{0.8660}F$$

For equilibrium of forces at J in the directions perpendicular and parallel to the line CJH

$$T_{\mathrm{BJ}} \sin 60^\circ = 0 \text{ giving } T_{\mathrm{BJ}} = 0$$
$$T_{\mathrm{CJ}} - T_{\mathrm{BJ}} \cos 60^\circ - T_{\mathrm{HJ}} = 0 \text{ giving } T_{\mathrm{CJ}} = \mathbf{0.8660}F$$

Finally, for equilibrium of forces at B in the direction along ABC

$$T_{\mathrm{BC}} + T_{\mathrm{BJ}} \cos 30^\circ - T_{\mathrm{BI}} \cos 30^\circ - T_{\mathrm{AB}} - F \cos 60^\circ = 0$$
$$\text{giving } T_{\mathrm{BC}} = -\mathbf{2.5}F$$

2.8 Member AI is taken to be of unit length, and the load F is taken as 1.

The eleven nodes are numbered in the order A to K, and the nineteen elements in the order: AB, BC, CD, DE, EF, FG, GH, HI, AI, BI, BH, BJ, HJ, CJ, CK, GK, DK, DG and DF. The origin of coordinates is at A (node 1).

The computed force factors (multiplying F) are identical to those obtained in Problem 2.7.

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STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE      X          Y             NODE      X          Y
  1  0.0000E+00  0.0000E+00          2  0.1500E+01  0.8660E+00
  3  0.3000E+01  0.1732E+01          4  0.4500E+01  0.8660E+00
  5  0.6000E+01  0.0000E+00          6  0.5000E+01  0.0000E+00
  7  0.4000E+01  0.0000E+00          8  0.2000E+01  0.0000E+00
  9  0.1000E+01  0.0000E+00         10  0.2500E+01  0.8660E+00
 11  0.3500E+01  0.8660E+00

NODES CONNECTED BY THE ELEMENTS
  M    I    J        M    I    J        M    I    J        M    I    J
  1    1    2        2    2    3        3    3    4        4    4    5
  5    5    6        6    6    7        7    7    8        8    8    9
  9    9    1       10    9    2       11    2    8       12    2   10
 13    8   10       14   10    3       15    3   11       16   11    7
 17   11    4       18    7    4       19    4    6

FORCE COMPONENTS AT THE LOADED NODES
  NODE            FX            FY
   1          0.0000E+00    -0.1000E+01
   2          0.0000E+00    -0.1000E+01
   3          0.0000E+00    -0.1000E+01
   4          0.0000E+00    -0.1000E+01
   5          0.0000E+00    -0.1000E+01

NODES FORMING PINNED SUPPORTS :     1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE           ANG (DEG)
   5                .00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE         ELEMENT     FORCE         ELEMENT     FORCE
       1    -0.3000E+01          2    -0.2500E+01          3    -0.2500E+01
       4    -0.3000E+01          5     0.2598E+01          6     0.2598E+01
       7     0.1732E+01          8     0.2598E+01          9     0.2598E+01
      10     0.0000E+00         11    -0.8660E+00         12     0.5960E-07
      13     0.8660E+00         14     0.8660E+00         15     0.8660E+00
      16     0.8660E+00         17     0.0000E+00         18    -0.8660E+00
      19     0.0000E+00

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE     R (X DIRN)      R (Y DIRN)
   1      0.0000E+00      0.2500E+01

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE      REACTION
   5      0.2500E+01
```

2.9 Since the structure and loading are symmetrical, the upward vertical reactions at A and E are $R_{\mathrm{A}} = R_{\mathrm{E}} = 25$ kN, and it is only necessary to find the forces in, say, the left hand half of the structure. At A and H, for equilibrium of forces in the vertical and horizontal directions:

$$R_{\mathrm{A}} + T_{\mathrm{AB}} \sin 45^\circ = 0 \text{ giving } T_{\mathrm{AB}} = \mathbf{-35.36\ kN}$$
$$T_{\mathrm{AB}} \cos 45^\circ + T_{\mathrm{AH}} = 0 \text{ giving } T_{\mathrm{AH}} = \mathbf{25\ kN}$$
$$T_{\mathrm{BH}} - 10 = 0 \text{ giving } T_{\mathrm{BH}} = \mathbf{10\ kN}$$
$$T_{\mathrm{GH}} - T_{\mathrm{AH}} = 0 \text{ giving } T_{\mathrm{GH}} = \mathbf{25\ kN}$$

For equilibrium of forces at B in the directions along and perpendicular to AB

$$T_{\mathrm{BC}} \cos 15^\circ - T_{\mathrm{AB}} - T_{\mathrm{BH}} \cos 45^\circ = 0$$
$$\text{giving } T_{\mathrm{BC}} = \mathbf{-29.28\ kN}$$
$$T_{\mathrm{BC}} \sin 15^\circ + T_{\mathrm{BG}} + T_{\mathrm{BH}} \cos 45^\circ = 0$$
$$\text{giving } T_{\mathrm{BG}} = \mathbf{0.5077\ kN}$$

Due to symmetry, $T_{\mathrm{BC}} = T_{\mathrm{CD}}$, and for equilibrium of forces at C in the vertical direction

$$-2T_{\mathrm{BC}} \cos 60^\circ - T_{\mathrm{CG}} = 0 \text{ giving } T_{\mathrm{CG}} = \mathbf{29.28\ kN}$$

2.10 Members AH, GH, FG and EF are taken to be of unit length.

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STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE      X            Y          NODE      X            Y
  1  0.0000E+00  0.0000E+00        2  0.1000E+01  0.1000E+01
  3  0.2000E+01  0.1577E+01        4  0.3000E+01  0.1000E+01
  5  0.4000E+01  0.0000E+00        6  0.3000E+01  0.0000E+00
  7  0.2000E+01  0.0000E+00        8  0.1000E+01  0.0000E+00

NODES CONNECTED BY THE ELEMENTS
   M    I    J       M    I    J       M    I    J       M    I    J
   1    1    2       2    2    3       3    3    4       4    4    5
   5    5    6       6    6    7       7    7    8       8    8    1
   9    8    2      10    2    7      11    7    3      12    7    4
  13    4    6

FORCE COMPONENTS AT THE LOADED NODES
  NODE            FX           FY
    8         0.0000E+00   -0.1000E+02
    7         0.0000E+00   -0.3000E+02
    6         0.0000E+00   -0.1000E+02

NODES FORMING PINNED SUPPORTS :     1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
    5              .00
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COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT    FORCE        ELEMENT    FORCE        ELEMENT    FORCE
       1    -0.3536E+02        2    -0.2928E+02        3    -0.2928E+02
       4    -0.3536E+02        5     0.2500E+02        6     0.2500E+02
       7     0.2500E+02        8     0.2500E+02        9     0.1000E+02
      10     0.5077E+00       11     0.2928E+02       12     0.5077E+00
      13     0.1000E+02

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE     R (X DIRN)     R (Y DIRN)
    1    -0.1715E-05     0.2500E+02

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE     REACTION
    5     0.2500E+02
```

The eight nodes are numbered in the order A to H, and the thirteen elements in the order: AB, BC, CD, DE, EF, FG, GH, AH, BH, BG, CG, DG and DF. The origin of coordinates is at A (node 1).

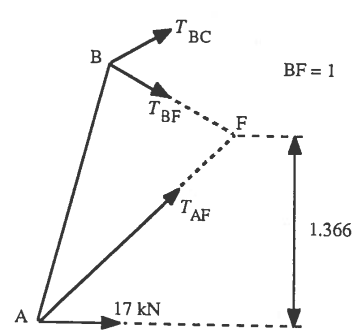
The computed forces are identical to those obtained in Problem 2.9.

2.11 Cutting the structure through members BC, BF and AF, and treating the left hand part of it as a free body. If members BC, BF and CF (Fig. P2.11) are of length 1, then $AB = AF = 0.5/\cos 75° = 1.932$, and the vertical height of F above A is $1.932 \sin 45° = 1.366$. Also, the length of the perpendicular from F onto BC is $1 \sin 60° = 0.8660$. Hence, for moment equilibrium about F

$$T_{\mathrm{BC}} \times 0.8660 - 17 \times 1.366 = 0 \text{ giving } T_{\mathrm{BC}} = \mathbf{26.8\ kN}$$

Also, for force equilibrium in the direction perpendicular to AF

$$T_{\mathrm{BF}} \sin 75° + T_{\mathrm{BC}} \cos 75° + 17 \sin 45° = 0$$
$$\text{giving } T_{\mathrm{BF}} = \mathbf{-19.6\ kN}$$



2.12 Members BC, BF, CF, CD, and DF are taken to be of unit length. The six nodes are numbered in the order A to F, and the nine elements in the order: AB, BC, CD, DE, EF, AF, BF, CF and DF. The origin of coordinates is midway between points A and E.

A is treated as a pinned support, and E as a roller support in the horizontal direction (in order to prevent the program treating the structure as a mechanism).

The computed forces in the members are **+24.04 kN** in AB and DE (elements 1 and 4), **+26.81 kN** in BC and CD (elements 2 and 3), **-32.84 kN** in AF and EF (elements 6 and 5), **-19.63 kN** in BF and DF (elements 7 and 9) **-26.82 kN** in CF (element 8).

Note that the computed reactions at the assumed supports are negligibly small.

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STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE     X           Y            NODE     X            Y
  1 -0.1366E+01  0.0000E+00        2 -0.8660E+00  0.1866E+01
  3  0.0000E+00  0.2366E+01        4  0.8660E+00  0.1866E+01
  5  0.1366E+01  0.0000E+00        6  0.0000E+00  0.1366E+01

NODES CONNECTED BY THE ELEMENTS
   M    I    J       M    I    J       M    I    J       M    I    J
   1    1    2       2    2    3       3    3    4       4    4    5
   5    5    6       6    6    1       7    2    6       8    3    6
   9    4    6

FORCE COMPONENTS AT THE LOADED NODES
  NODE           FX             FY
   1          0.1700E+02     0.0000E+00
   5         -0.1700E+02     0.0000E+00

NODES FORMING PINNED SUPPORTS :   1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
   5              .00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE       ELEMENT     FORCE       ELEMENT     FORCE
       1      0.2404E+02       2      0.2681E+02       3      0.2681E+02
       4      0.2404E+02       5     -0.3284E+02       6     -0.3284E+02
       7     -0.1963E+02       8     -0.2682E+02       9     -0.1963E+02

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE    R (X DIRN)      R (Y DIRN)
   1    -0.8259E-06     -0.5641E-06

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE     REACTION
   5     0.2471E-05
```

2.13 All members are taken to be of unit length. The nine nodes are numbered in the order A to I, and the fifteen elements in the order: AB, BC, CD, DE, EF, FG, GH, HI, AI, BI, CI, CH, DH, DG and EG. The origin of coordinates is at A (node 1), with the X axis lying along the line AF (for convenience in calculating the coordinates of the nodes).

With a 100 kN applied vertically downwards at G, the results are as follows.

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STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE     X            Y              NODE     X            Y
  1  0.0000E+00   0.0000E+00           2  0.5000E+00   0.8660E+00
  3  0.1500E+01   0.8660E+00           4  0.2500E+01   0.8660E+00
  5  0.3500E+01   0.8660E+00           6  0.4000E+01   0.0000E+00
  7  0.3000E+01   0.0000E+00           8  0.2000E+01   0.0000E+00
  9  0.1000E+01   0.0000E+00

NODES CONNECTED BY THE ELEMENTS
   M    I    J        M    I    J        M    I    J        M    I    J
   1    1    2        2    2    3        3    3    4        4    4    5
   5    5    6        6    6    7        7    7    8        8    8    9
   9    9    1       10    2    9       11    9    3       12    3    8
  13    8    4       14    4    7       15    7    5

FORCE COMPONENTS AT THE LOADED NODES
  NODE           FX             FY
   7         -0.2588E+02    -0.9659E+02

NODES FORMING PINNED SUPPORTS :    1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
   6             15.00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
   ELEMENT     FORCE       ELEMENT     FORCE       ELEMENT     FORCE
      1     -0.2788E+02       2     -0.2788E+02       3     -0.5577E+02
      4     -0.8365E+02       5     -0.8365E+02       6      0.2242E+02
      7      0.2442E+02       8     -0.3465E+01       9     -0.3135E+02
     10      0.2788E+02      11     -0.2788E+02      12      0.2788E+02
     13     -0.2788E+02      14      0.2788E+02      15      0.8365E+02

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE    R (X DIRN)     R (Y DIRN)
   1      0.4529E+02     0.2415E+02

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE     REACTION
   6      0.7500E+02
```

With the load at H, the forces in the members are

```
COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
   ELEMENT     FORCE       ELEMENT     FORCE       ELEMENT     FORCE
      1     -0.5577E+02       2     -0.5577E+02       3     -0.1115E+03
      4     -0.5577E+02       5     -0.5577E+02       6      0.1494E+02
      7      0.7071E+02       8      0.4483E+02       9     -0.1094E+02
     10      0.5577E+02      11     -0.5577E+02      12      0.5577E+02
     13      0.5577E+02      14     -0.5577E+02      15      0.5577E+02
```

With the load at I, the forces in the members are

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)

ELEMENT	FORCE	ELEMENT	FORCE	ELEMENT	FORCE
1	-0.8365E+02	2	-0.8365E+02	3	-0.5577E+02
4	-0.2788E+02	5	-0.2788E+02	6	0.7472E+01
7	0.3536E+02	8	0.6324E+02	9	0.9476E+01
10	0.8365E+02	11	0.2788E+02	12	-0.2788E+02
13	0.2788E+02	14	-0.2788E+02	15	0.2788E+02

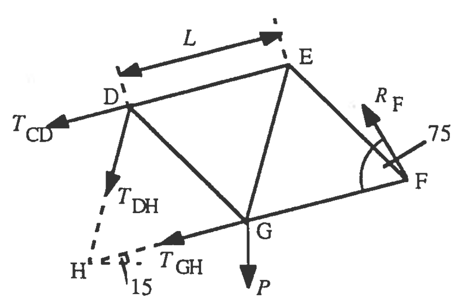
The force of greatest magnitude in each of the members for the three loading conditions is: $-$**83.65 kN** in AB, BC, DE and EF (elements 1, 2, 4 and 5), **+83.65 kN** in BI and GE (elements 10 and 15), $-$**111.5 kN** in CD (element 3), **+22.42 kN** in FG (element 6), **+70.71 kN** in GH (element 7), **+63.24 kN** in HI (element 8)) $-$**31.35 kN** in AI (element 9) $-$**55.77 kN** in CI and DG (elements 11 and 14), **+55.77 kN** in CH and DH (elements 12 and 13)

2.14 Let R_F be the reaction at F, which acts at right angles to the slope on which the roller support rests, and therefore at 60° to the horizontal and at 75° to the line AF. Therefore, for moment equilibrium of the entire structure (Fig. P2.13) about A, when the 100 kN applied load is at G

$$100 \cos 15^\circ \times 3L - R_F \sin 75^\circ \times 4L = 0$$

where L is the length of each of the members, and from which $R_F = 75$ kN. Similarly, when the load is at H and I, $R_F = 50$ and 25 kN, respectively.

Cutting the structure through members CD, DH, and GH, and treating the right hand part of it as a free body:



Let P be the downward vertical load at G (which is equal to 100 kN, zero or zero, depending on whether the actual load is applied at G, H or I, respectively). For moment equilibrium about H

$$P \cos 15^\circ \times L - R_F \sin 75^\circ \times 2L - T_{CD} \times L \sin 60^\circ = 0$$
$$\text{giving } T_{CD} = 1.115(P - 2R_F)$$

For moment equilibrium about G

$$-T_{\mathrm{CD}} \times L \sin 60^\circ - R_{\mathrm{F}} \sin 75^\circ \times L - T_{\mathrm{DH}} \times L \sin 60^\circ = 0$$
$$\text{giving } T_{\mathrm{DH}} = 1.115(R_{\mathrm{F}} - P)$$

For force equilibrium in the direction parallel to FG

$$T_{\mathrm{CD}} + T_{\mathrm{GH}} + T_{\mathrm{DH}} \cos 60^\circ + P \cos 75^\circ + R_{\mathrm{F}} \cos 75^\circ = 0$$
$$\text{giving } T_{\mathrm{GH}} = 1.414 R_{\mathrm{F}} - 0.8165 P$$

For the three positions of the 100 kN load, the results are as follows

Load at	R_{F} (kN)	P (kN)	T_{CD} (kN)	T_{DH} (kN)	T_{GH} (kN)
G	75	100	−55.77	−27.88	+24.42
H	50	0	−111.5	+55.77	+70.71
I	25	0	−55.77	+27.88	+35.36

The forces of greatest magnitude in each of the members CD, DH and GH for the three positions of the load are **−111.5 kN**, **+55.77 kN** and **+70.71 kN**, respectively, in exact agreement with the results of Problem 2.13.

2.15 The numbers of members, unknown reaction components and joints are: $N_{\mathrm{m}} = 7$, $N_{\mathrm{r}} = 6$ and $N_{\mathrm{j}} = 6$. Since $N_{\mathrm{m}} + N_{\mathrm{r}} > 2N_{\mathrm{j}}$, the structure is **statically indeterminate** (one redundancy).

2.16 The numbers of members, unknown reaction components and joints are: $N_{\mathrm{m}} = 8$, $N_{\mathrm{r}} = 4$ and $N_{\mathrm{j}} = 6$. Since $N_{\mathrm{m}} + N_{\mathrm{r}} = 2N_{\mathrm{j}}$, the structure may be statically determinate according to this test. In fact the members are not well arranged and the structure is a **mechanism**.

2.17 The numbers of members, unknown reaction components and joints are: $N_{\mathrm{m}} = 10$, $N_{\mathrm{r}} = 4$ and $N_{\mathrm{j}} = 6$. Since $N_{\mathrm{m}} + N_{\mathrm{r}} > 2N_{\mathrm{j}}$, the structure is **statically indeterminate** (two redundancies).

2.18 The numbers of members, unknown reaction components and joints are: $N_{\mathrm{m}} = 13$, $N_{\mathrm{r}} = 3$ and $N_{\mathrm{j}} = 8$. Since $N_{\mathrm{m}} + N_{\mathrm{r}} = 2N_{\mathrm{j}}$ and the members are well arranged, the structure is **statically determinate**.

2.19 The numbers of members, unknown reaction components and joints are: $N_{\rm m} = 6$, $N_{\rm r} = 6$ and $N_{\rm j} = 6$. Since $N_{\rm m} + N_{\rm r} = 2N_{\rm j}$ and the members are well arranged, the structure is **statically determinate**.

2.20 The numbers of members, unknown reaction components and joints are: $N_{\rm m} = 9$, $N_{\rm r} = 2$ and $N_{\rm j} = 6$. Since $N_{\rm m} + N_{\rm r} < 2N_{\rm j}$, the structure is a **mechanism** (free to move horizontally as a rigid body).

2.21 To classify the structure shown in Fig. P2.15a using program SDPINJ, it is not necessary to use precise dimensions for the structure (which are not given), provided the element interconnections and support arrangements are correct.

```
STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE     X           Y              NODE     X           Y
  1  0.0000E+00  0.0000E+00           2  0.0000E+00  0.1000E+01
  3  0.1000E+01  0.2000E+01           4  0.2000E+01  0.1000E+01
  5  0.2000E+01  0.0000E+00           6  0.1000E+01  0.0000E+00

NODES CONNECTED BY THE ELEMENTS
  M    I    J       M    I    J        M    I    J        M    I    J
  1    1    2       2    2    3        3    3    4        4    4    5
  5    2    6       6    3    6        7    4    6

NODES FORMING PINNED SUPPORTS :    1   6   5

THE STRUCTURE IS STATICALLY INDETERMINATE - STOP
```

2.22

```
STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE     X           Y              NODE     X           Y
  1  0.0000E+00  0.0000E+00           2  0.0000E+00  0.1000E+01
  3  0.0000E+00  0.2000E+01           4  0.1000E+01  0.2000E+01
  5  0.1000E+01  0.1000E+01           6  0.1000E+01  0.0000E+00

NODES CONNECTED BY THE ELEMENTS
  M    I    J       M    I    J        M    I    J        M    I    J
  1    1    2       2    2    3        3    3    4        4    4    5
  5    5    6       6    2    5        7    2    4        8    3    5

NODES FORMING PINNED SUPPORTS :    1   6

STRUCTURE IS NOT STATICALLY DETERMINATE - STOP
```

The program detects that the structure is not statically determinate from the fact that the linear equations generated are ill-conditioned.

2.23

```
STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE     X            Y                NODE     X            Y
  1  0.0000E+00  0.0000E+00              2  0.1000E+01  0.1000E+01
  3  0.2000E+01  0.1000E+01              4  0.3000E+01  0.0000E+00
  5  0.2000E+01  0.0000E+00              6  0.1000E+01  0.0000E+00

NODES CONNECTED BY THE ELEMENTS
   M    I    J       M    I    J       M    I    J       M    I    J
   1    1    2       2    2    3       3    3    4       4    4    5
   5    5    6       6    6    1       7    2    6       8    2    5
   9    6    3      10    3    5

NODES FORMING PINNED SUPPORTS :     1   4

THE STRUCTURE IS STATICALLY INDETERMINATE - STOP
```

2.24

```
STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE     X            Y                NODE     X            Y
  1  0.0000E+00  0.0000E+00              2 -0.5000E+00  0.8660E+00
  3  0.0000E+00  0.1732E+01              4  0.5000E+00  0.2000E+01
  5  0.1000E+01  0.1732E+01              6  0.1500E+01  0.8660E+00
  7  0.1000E+01  0.0000E+00              8  0.5000E+00  0.8660E+00

NODES CONNECTED BY THE ELEMENTS
   M    I    J       M    I    J       M    I    J       M    I    J
   1    1    2       2    2    3       3    3    4       4    4    5
   5    5    6       6    6    7       7    7    1       8    1    8
   9    2    8      10    3    8      11    5    8      12    6    8
  13    7    8

NODES FORMING PINNED SUPPORTS :     1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
   7              .00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE       ELEMENT     FORCE       ELEMENT     FORCE
       1     0.0000E+00        2     0.0000E+00        3     0.0000E+00
       4     0.0000E+00        5     0.0000E+00        6     0.0000E+00
       7     0.0000E+00        8     0.0000E+00        9     0.0000E+00
      10     0.0000E+00       11     0.0000E+00       12     0.0000E+00
      13     0.0000E+00

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE    R (X DIRN)     R (Y DIRN)
   1      0.0000E+00     0.0000E+00
```

Note that the computed forces are zero, since the statically determinate structure is not loaded.

2.25

```
STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE      X            Y              NODE      X            Y
  1  0.0000E+00   0.2000E+01            2  0.1000E+01   0.1000E+01
  3  0.2000E+01   0.0000E+00            4  0.1000E+01   0.0000E+00
  5  0.0000E+00   0.0000E+00            6  0.0000E+00   0.1000E+01

NODES CONNECTED BY THE ELEMENTS
   M    I    J        M    I    J        M    I    J        M    I    J
   1    1    2        2    2    3        3    3    4        4    4    5
   5    2    4        6    2    6

NODES FORMING PINNED SUPPORTS :    1   6   5

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE          ELEMENT     FORCE          ELEMENT     FORCE
       1     0.0000E+00           2     0.0000E+00           3     0.0000E+00
       4     0.0000E+00           5     0.0000E+00           6     0.0000E+00

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE      R (X DIRN)       R (Y DIRN)
    1       0.0000E+00       0.0000E+00
    6       0.0000E+00       0.0000E+00
    5       0.0000E+00       0.0000E+00
```

Note that the computed forces are zero, since the statically determinate structure is not loaded.

2.26

```
STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE      X            Y              NODE      X            Y
  1  0.0000E+00   0.0000E+00            2  0.0000E+00   0.1000E+01
  3  0.1000E+01   0.1000E+01            4  0.2000E+01   0.1000E+01
  5  0.2000E+01   0.0000E+00            6  0.1000E+01   0.0000E+00

NODES CONNECTED BY THE ELEMENTS
   M    I    J        M    I    J        M    I    J        M    I    J
   1    1    2        2    2    3        3    3    4        4    4    5
   5    5    6        6    6    1        7    1    3        8    3    6
   9    3    5

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
   1               .00
   5               .00

THE STRUCTURE IS A MECHANISM - STOP
```

2.27 The twenty one nodes are numbered in the order A to U, and the thirty nine elements in the order: AB, BC, CD, DE, EF, FG, GH, HI, IJ, AJ, AI, BI, CI, CH, CG, DG, EG, EK, FK, KN, FL, LM, MN, NO, OP, PQ, QR, RS, ST, TU, FU, LU, MU, MT, NT, NS, OS, OR and PR. The origin of coordinates is at J (node 10), lengths and forces are in m and kN, respectively.

```
STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES
NODE       X              Y            NODE       X              Y
  1 -0.3000E+01   0.0000E+00           2 -0.3000E+01   0.5000E+01
  3 -0.3000E+01   0.1000E+02           4 -0.3000E+01   0.1500E+02
  5 -0.3000E+01   0.2000E+02           6  0.0000E+00   0.2000E+02
  7  0.0000E+00   0.1500E+02           8  0.0000E+00   0.1000E+02
  9  0.0000E+00   0.5000E+01          10  0.0000E+00   0.0000E+00
 11 -0.1500E+01   0.2600E+02          12  0.1250E+01   0.2216E+02
 13  0.3750E+01   0.2216E+02          14  0.6250E+01   0.2216E+02
 15  0.8750E+01   0.2216E+02          16  0.1125E+02   0.2216E+02
 17  0.1250E+02   0.2000E+02          18  0.1000E+02   0.2000E+02
 19  0.7500E+01   0.2000E+02          20  0.5000E+01   0.2000E+02
 21  0.2500E+01   0.2000E+02

NODES CONNECTED BY THE ELEMENTS
   M    I    J        M    I    J        M    I    J        M    I    J
   1    1    2        2    2    3        3    3    4        4    4    5
   5    5    6        6    6    7        7    7    8        8    8    9
   9    9   10       10   10    1       11    1    9       12    9    2
  13    9    3       14    3    8       15    3    7       16    7    4
  17    7    5       18    5   11       19   11    6       20   11   14
  21    6   12       22   12   13       23   13   14       24   14   15
  25   15   16       26   16   17       27   17   18       28   18   19
  29   19   20       30   20   21       31   21    6       32   12   21
  33   21   13       34   13   20       35   20   14       36   14   19
  37   19   15       38   15   18       39   18   16

FORCE COMPONENTS AT THE LOADED NODES
  NODE            FX              FY
   17          0.0000E+00     -0.1200E+02

NODES FORMING PINNED SUPPORTS :     1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
   10              .00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE        ELEMENT     FORCE        ELEMENT     FORCE
       1      0.5000E+02        2      0.5000E+02        3      0.5000E+02
       4      0.5000E+02        5     -0.1250E+02        6     -0.6200E+02
       7     -0.6200E+02        8     -0.6200E+02        9     -0.6200E+02
      10      0.7629E-06       11      0.1335E-04       12      0.0000E+00
      13     -0.1112E-04       14      0.0000E+00       15      0.1236E-04
      16      0.0000E+00       17     -0.1251E-04       18      0.5154E+02
      19     -0.6609E+02       20      0.3183E+02       21      0.2445E+01
      22      0.2445E+01       23      0.4890E+01       24      0.2771E+02
      25      0.1386E+02       26      0.1386E+02       27     -0.6928E+01
      28     -0.2079E+02       29     -0.3464E+02       30     -0.3220E+02
      31     -0.2975E+02       32     -0.2445E+01       33      0.2445E+01
      34     -0.2445E+01       35      0.2445E+01       36      0.1386E+02
      37     -0.1386E+02       38      0.1386E+02       39     -0.1386E+02

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE     R (X DIRN)      R (Y DIRN)
    1    -0.7629E-05     -0.5000E+02

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE      REACTION
   10     0.6200E+02
```

2.28 Cutting the structure through members KN, FL, and FU, the jib of the crane to the right (Fig. P2.27) can be treated as a free body. The horizontal distance between points K and N is 7.75 m, and the vertical distance between them is 3.835 m. Therefore, member KN is at an angle of $\tan^{-1}(3.835/7.75) = 26.33°$ to the horizontal. For moment equilibrium about F

$$\begin{aligned}12 \times 12.5 - T_{\text{KN}} \sin 26.33° \times 6.25 \\ -T_{\text{KN}} \cos 26.33° \times 2.5 \sin 60° = 0 \\ \text{giving } T_{\text{KN}} = \mathbf{31.83\ kN}\end{aligned}$$

For force equilibrium in the vertical direction

$$\begin{aligned}T_{\text{KN}} \sin 26.33° - T_{\text{FL}} \sin 60° - 12 = 0 \\ \text{giving } T_{\text{FL}} = \mathbf{2.445\ kN}\end{aligned}$$

For force equilibrium in the horizontal direction

$$\begin{aligned}-T_{\text{KN}} \cos 26.33° - T_{\text{FL}} \cos 60° - T_{\text{FU}} = 0 \\ \text{giving } T_{\text{FU}} = -\mathbf{29.75\ kN}\end{aligned}$$

These results are identical to those obtained in Problem 2.27 (members KN, FL and FU are elements 20, 21 and 31, respectively).

2.29 The sixteen nodes are numbered in the order A to P, and the twenty nine elements in the order: AB, BC, CD, DE, EF, FG, GH, HI, IJ, JK, KL, LM, MN, NO, OP, AP, BP, BO, CO, CN, DN, DM, EM, FM, FL, GL, GK, HK and HJ. The origin of coordinates is at A (node 1), lengths and forces are in m and kN, respectively.

STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES

NODE	X	Y	NODE	X	Y
1	0.0000E+00	0.0000E+00	2	0.5000E+01	0.5000E+01
3	0.1000E+02	0.8000E+01	4	0.1500E+02	0.1000E+02
5	0.2000E+02	0.1100E+02	6	0.2500E+02	0.1000E+02
7	0.3000E+02	0.8000E+01	8	0.3500E+02	0.5000E+01
9	0.4000E+02	0.0000E+00	10	0.3500E+02	0.0000E+00
11	0.3000E+02	0.0000E+00	12	0.2500E+02	0.0000E+00
13	0.2000E+02	0.0000E+00	14	0.1500E+02	0.0000E+00
15	0.1000E+02	0.0000E+00	16	0.5000E+01	0.0000E+00

```
NODES CONNECTED BY THE ELEMENTS
  M    I    J       M    I    J       M    I    J       M    I    J
  1    1    2       2    2    3       3    3    4       4    4    5
  5    5    6       6    6    7       7    7    8       8    8    9
  9    9   10      10   10   11      11   11   12      12   12   13
 13   13   14      14   14   15      15   15   16      16   16    1
 17   16    2      18    2   15      19   15    3      20    3   14
 21   14    4      22    4   13      23   13    5      24   13    6
 25    6   12      26   12    7      27    7   11      28   11    8
 29    8   10

FORCE COMPONENTS AT THE LOADED NODES
  NODE           FX            FY
   14         0.0000E+00    -0.1000E+04

NODES FORMING PINNED SUPPORTS :    1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE          ANG (DEG)
   9               .00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
    ELEMENT     FORCE        ELEMENT     FORCE        ELEMENT     FORCE
       1     -0.8839E+03        2     -0.9111E+03        3     -0.1010E+04
       4     -0.6953E+03        5     -0.6953E+03        6     -0.6058E+03
       7     -0.5467E+03        8     -0.5303E+03        9      0.3750E+03
      10      0.3750E+03       11      0.4688E+03       12      0.5625E+03
      13      0.9375E+03       14      0.7813E+03       15      0.6250E+03
      16      0.6250E+03       17      0.1526E-04       18      0.2210E+03
      19     -0.1563E+03       20      0.2948E+03       21      0.7500E+03
      22     -0.5717E+03       23      0.2727E+03       24      0.2668E+03
      25     -0.1500E+03       26      0.1769E+03       27     -0.9375E+02
      28      0.1326E+03       29      0.0000E+00

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE     R (X DIRN)      R (Y DIRN)
   1     -0.3155E-04      0.6250E+03

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE      REACTION
   9      0.3750E+03
```

2.30 For the structure as a whole to be in equilibrium, the vertical support reactions at A and I are $R_{\mathrm{A}} = 625$ and $R_{\mathrm{I}} = 375$ kN, respectively. Cutting the structure through members DE, DM, and NM, the part of the structure to the left (Fig. P2.29) can be treated as a free body. Member DE is at an angle of $\tan^{-1}(1/5) = 11.31°$ to the horizontal, and member DM is at an angle of $\tan^{-1}(10/5) = 63.43°$ to the horizontal. For moment equilibrium about M

$$R_{\mathrm{A}} \times 20 - 1000 \times 5 + T_{\mathrm{DE}}(\sin 11.31° \times 5 + \cos 11.31° \times 10) = 0$$
$$\text{giving } T_{\mathrm{DE}} = \mathbf{-695.3\ kN}$$

For force equilibrium in the vertical direction

$$T_{\mathrm{DE}} \sin 11.31° - T_{\mathrm{DM}} \sin 63.43° + R_{\mathrm{A}} - 1000 = 0$$
$$\text{giving } T_{\mathrm{DM}} = \mathbf{-571.7\ kN}$$

For force equilibrium in the horizontal direction

$$
\begin{aligned}
T_{\mathrm{DE}} \cos 11.31^\circ + T_{\mathrm{DM}} \cos 63.43^\circ + T_{\mathrm{NM}} &= 0 \\
\text{giving } T_{\mathrm{NM}} &= \mathbf{937.5\ kN}
\end{aligned}
$$

These results are identical to those obtained in Problem 2.29 (members DE, DM and NM are elements 4, 22 and 13, respectively).

2.31 Let R_{I} be the reaction force at I, at right angles to the slope on which the roller rests. For moment equilibrium of the structure about A

$$
\begin{aligned}
&10 \times (5 + 10 + 18.66 + 27.32 + 35.98 + 44.64 + 49.64 + 54.64) \\
&+5 \times (8.660 + 17.32 + 22.32 + 27.32) - R_I \sin 80^\circ \times 54.64 = 0
\end{aligned}
$$

from which $R_{\mathrm{I}} = 52.72$ kip. For force equilibrium at I in the direction normal to member IJ

$$
T_{\mathrm{HI}} \sin 30^\circ + R_{\mathrm{I}} \sin 50^\circ - 10 \sin 60^\circ = 0
$$

which gives $T_{HI} = -\mathbf{63.45\ kip}$, in exact agreement with the result of Problem 2.32.

2.32 The fifteen nodes are numbered in the order A to O, and the twenty seven elements in the order: AB, BC, CD, DE, EF, FG, GH, HI, IJ, JK, KL, LM, AM, BM, CM, CL, LN, CN, DN, EN, EO, FO, GO, KO, GK, GJ and HJ. The origin of coordinates is at the mid point of KL, with the X axis horizontal. The results are shown below; the force of greatest magnitude is $-\mathbf{63.45\ kip}$ in HI (element 8).

STATICALLY DETERMINATE PLANE PIN-JOINTED STRUCTURE

COORDINATES OF THE NODES

NODE	X	Y	NODE	X	Y
1	-0.2732E+02	-0.1000E+02	2	-0.2232E+02	-0.1340E+01
3	-0.1732E+02	0.7320E+01	4	-0.8660E+01	0.1232E+02
5	0.0000E+00	0.1732E+02	6	0.8660E+01	0.1232E+02
7	0.1732E+02	0.7320E+01	8	0.2232E+02	-0.1340E+01
9	0.2732E+02	-0.1000E+02	10	0.1866E+02	-0.5000E+01
11	0.1000E+02	0.0000E+00	12	-0.1000E+02	0.0000E+00
13	-0.1866E+02	-0.5000E+01	14	-0.5000E+01	0.8660E+01
15	0.5000E+01	0.8660E+01			

NODES CONNECTED BY THE ELEMENTS

M	I	J	M	I	J	M	I	J	M	I	J
1	1	2	2	2	3	3	3	4	4	4	5
5	5	6	6	6	7	7	7	8	8	8	9
9	9	10	10	10	11	11	11	12	12	12	13
13	13	1	14	2	13	15	13	3	16	3	12
17	12	14	18	14	3	19	14	4	20	14	5
21	5	15	22	15	6	23	15	7	24	15	11
25	11	7	26	7	10	27	10	8			

```
FORCE COMPONENTS AT THE LOADED NODES
  NODE          FX            FY
   1         0.5000E+01    -0.1000E+02
   2         0.5000E+01    -0.1000E+02
   3         0.5000E+01    -0.1000E+02
   4         0.5000E+01    -0.1000E+02
   5         0.5000E+01    -0.1000E+02
   6         0.0000E+00    -0.1000E+02
   7         0.0000E+00    -0.1000E+02
   8         0.0000E+00    -0.1000E+02
   9         0.0000E+00    -0.1000E+02

NODES FORMING PINNED SUPPORTS :    1

NODES AND ANGLES OF ROLLER SUPPORTS
  NODE        ANG (DEG)
   9           10.00

COMPUTED FORCES IN THE ELEMENTS (TENSILE POSITIVE)
  ELEMENT    FORCE         ELEMENT    FORCE         ELEMENT    FORCE
     1    -0.5948E+02        2    -0.5582E+02        3    -0.5399E+02
     4    -0.5033E+02        5    -0.4515E+02        6    -0.5247E+02
     7    -0.5613E+02        8    -0.6345E+02        9     0.2607E+02
    10     0.2106E+02       11     0.2380E+02       12     0.3754E+02
    13     0.4687E+02       14    -0.9659E+01       15     0.1156E+02
    16     0.1905E+01       17     0.2012E+02       18     0.1383E+02
    19    -0.1155E+02       20     0.3128E+02       21     0.1231E+02
    22    -0.8966E+01       23     0.1073E+02       24     0.3646E+01
    25     0.1043E+02       26     0.6197E+01       27    -0.5176E+01

COMPUTED REACTION COMPONENTS AT THE PINNED SUPPORTS
  NODE    R (X DIRN)       R (Y DIRN)
   1    -0.1585E+02      0.3808E+02

COMPUTED REACTION FORCES AT THE ROLLER SUPPORTS
  NODE     REACTION
   9     0.5272E+02
```

2.33 Using equations (2.16) and (2.18)

$$\begin{aligned}\sigma_\theta &= \frac{\rho R}{t} = \frac{1.5 \times 250}{3} = \mathbf{125 MN/m^2} \\ \sigma_z &= \frac{\sigma_\theta}{2} = \mathbf{62.5\ MN/m^2}\end{aligned}$$

2.34 With pressure acting on the outer surface of the cylinder, the hoop and axial stresses due to this pressure are

$$\begin{aligned}\sigma_\theta &= -\frac{pR_\mathrm{o}}{t} = -\frac{0.5 \times 44}{3} = -7.333\ \mathrm{MN/m^2} \\ \sigma_z &= -\frac{pR_\mathrm{o}^2}{2tR_m} = -\frac{0.5 \times 44^2}{2 \times 3 \times 42.5}\mathrm{N/m^2} = -3.796\ \mathrm{MN/m^2}\end{aligned}$$

Rotation of the cylinder affects only the hoop stress, which is given by Eq. (2.22)

$$\sigma_\theta = \rho R_\mathrm{m}^2 \omega^2 = 8410 \times 0.0425^2 \times 1047^2 = 16.66\ \mathrm{MN/m^2}$$

where $\rho = 8410$ kg/m^3 is the density of brass, and $\omega = 1047$ rad/sec is the angular velocity. The total stresses due to pressure and rotation are

$$\sigma_\theta = 16.66 - 7.333 = \mathbf{9.33\ MN/m^2}\,, \quad \sigma_z = -\mathbf{3.80\ MN/m^2}$$

2.35 Using equation (2.21), the maximum pressure is

$$p = \frac{2t\sigma_c}{R} = \frac{2 \times 15 \times 180}{350} = \mathbf{15.4\ MN/m^2}$$

2.36 Using equation (2.22)

$$\sigma_\theta = \rho R_m^2 \omega^2 = pV^2$$

where V is the mean peripheral velocity, which is approximately equal to the ground speed. Hence

$$V = \sqrt{\frac{\sigma_\theta}{\rho}} = \sqrt{\frac{50 \times 10^6}{300}} = \mathbf{408\ m/s}$$

Because the hoop stress depends only on the density and ground speed, there is no optimum wheel diameter.

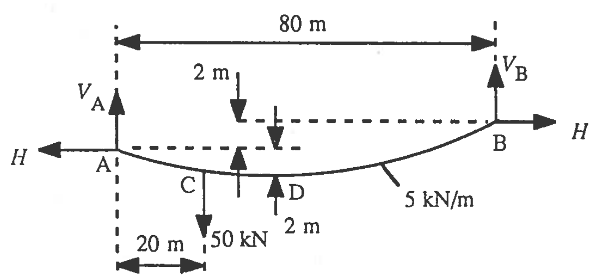
2.37 Using equation (2.16) for the hoop stress

$$\sigma_\theta = \frac{pR_i}{t} = \frac{360 \times 3.25}{0.15}\ \text{psi} = \mathbf{7.80\ ksi}$$

The cross-sectional area of the four rods is $4 \times \pi \times 0.75^2/4 = 1.767$ in^2. Since the tensile stress in them is 10 ksi, the total force in them is $10 \times 1.767 = 17.67$ kip. The internal cross-sectional area of the cylinder is $\pi \times 6.5^2/4 = 33.18$ in^2, and the pressure force acting on each end plate is 360×33.18 lb $= 11.94$ kip. For each end plate to be in equilibrium, the axial (tensile) force in the cylinder wall must be the difference between this pressure force and the opposing force in the rods: $11.94 - 17.67 = -5.73$ kip. Since the cross-sectional area of the cylinder wall is $\pi \times (6.8^2 - 6.5^2)/4 = 3.134$ in^2, the axial stress is

$$\sigma_z = -\frac{5.73}{3.134} = -\mathbf{1.83\ ksi}$$

2.38 Let the horizontal component of tension in the cable be H and the vertical reaction components at the ends A and B of the cable be V_{A} and V_{B}.



For force equilibrium in the vertical direction

$$V_{\mathrm{A}} + V_{\mathrm{B}} = 50 + 5 \times 80 = 450 \text{ kN} \tag{1}$$

and for moment equilibrium about A

$$50 \times 20 + 5 \times 80 \times 40 + H \times 2 - V_{\mathrm{B}} \times 80 = 0$$

$$17000 + 2H = 80V_{\mathrm{B}} \tag{2}$$

Let the horizontal distance between D and B be x. For force equilibrium of section DB in the vertical direction

$$V_{\mathrm{B}} = 5x \tag{3}$$

and for moment equilibrium about D

$$H \times 4 + 5x \times \frac{x}{2} - V_{\mathrm{B}} \times x = 0 \tag{4}$$

Using (3) to eliminate V_{B} from (4)

$$H = 0.625x^2 \tag{5}$$

and substituting (3) and (5) into (2)

$$1.25x^2 - 400x + 17000 = 0$$

a quadratic equation equation with roots $x = 269.5$ m and 50.46 m, only the second of which lies in the required range $0 < x < 60$. Using this value in (3) and (5)

$$V_{\mathrm{B}} = \mathbf{252\ kN} \quad \text{and} \quad H = \mathbf{1590\ kN}$$

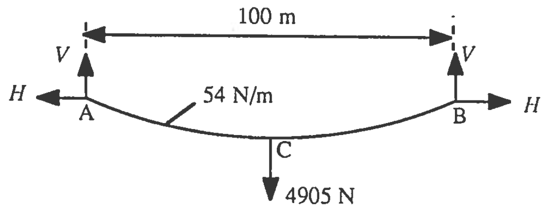
Hence, from (1)

$$V_{\mathrm{A}} = 450 - 252 = \mathbf{198\ kN}$$

and the maximum tension in the cable, which is at B (where the vertical component is greatest), is

$$\text{Maximum tension} = \sqrt{1590^2 + 252^2} = \mathbf{1610\ kN}$$

2.39 The weight of the cable car is $500 \times 9.81 = 4905$ N, and acts at the mid point C of the cable. Let the horizontal and vertical reaction components at the ends A and B of the cable be H and V.



For force equilibrium in the vertical direction

$$2V - 4905 - 54 \times 100 = 0 \text{ giving } V = 5152 \text{ N}$$

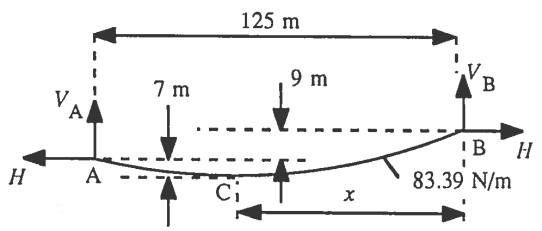
The mass per unit length of the steel cable is $54/9.81 = 5.505$ kg/m. With a density of 7850 kg/m^3, the cross sectional area is $5.505/7850 = 7.013 \times 10^{-4}$ m^2. Given the maximum tensile stress in the cable is 120 MN/m^2, the maximum tension is $120 \times 10^6 \times 7.013 \times 10^{-4} = 84160$ N, which occurs at both A and B. Hence

$$\sqrt{H^2 + 5152^2} = 84160$$
$$\text{and } H = \sqrt{84160^2 - 5152^2} = 84000 \text{ N}$$

If d is the dip at mid point C of the cable, then for moment equilibrium of section AC of the cable about C

$$V \times 50 - H \times d - 54 \times \frac{50^2}{2} = 0$$
$$d = \frac{5152 \times 50 - 54 \times 50^2/2}{84000} = \mathbf{2.26\ m}$$

2.40 With inner and outer diameters of 32 mm and 88 mm, the layer of ice has a cross-sectional area of $\pi(88^2 - 32^2)/4 = 5278$ mm^2. With a density of 900 kg/m^3, its mass per unit length of cable is therefore $5278 \times 10^{-6} \times 900 = 4.750$ kg/m. The total weight of the cable and ice is $(3.75 + 4.75) \times 9.81 = 83.39$ N/m.



Let the horizontal component of tension in the cable be H and the vertical reaction components at the ends A and B of the cable be V_A and V_B. Let the lowest point, C, of the cable be at a horizontal distance x from B. For moment equilibrium about A

$$H \times 9 + 83.39 \times \frac{125^2}{2} - V_B \times 125 = 0 \tag{1}$$

For force equilibrium of section CB in the vertical direction

$$V_B - 83.39x = 0 \tag{2}$$

and for moment equilibrium of the same section about B

$$H \times 16 - 83.39\frac{x^2}{2} = 0 \tag{3}$$

Substituting the expressions for V_B and H given by (2) and (3) into (1)

$$23.45x^2 - 10420x + 651500 = 0$$

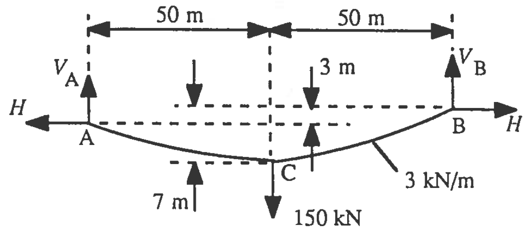
which is a quadratic equation for x with roots $x = 369.1$ m and 75.28 m, only the second of which satisfies the required condition $0 < x < 125$. Using (2) and (3)

$$V_B = 6278 \text{ N} \quad \text{and} \quad H = 14750 \text{ N}$$

and the maximum tension, at B, is

$$\text{Maximum tension} = \sqrt{14750^2 + 6278^2} \text{ N} = \mathbf{16.0\ kN}$$

2.41 Let the horizontal component of tension in the cable be H and the vertical reaction components at the ends A and B of the cable be V_A and V_B.



For force equilibrium in the vertical direction

$$V_A + V_B - 150 - 3 \times 100 = 0 \tag{1}$$

and for moment equilibrium about A

$$H \times 3 + 150 \times 50 + 3 \times 100 \times 50 - V_B \times 100 = 0 \tag{2}$$

For moment equilibrium of section CB about C

$$H \times 7 + 3 \times 50 \times 25 - V_B \times 50 = 0 \tag{3}$$

Using (3) to eliminate V_B from (2)

$$3H + 7500 + 15000 - 14H - 7500 = 0$$
$$H = 15000/11 = 1364\text{kN}$$

Using (3) to find V_B and then (1) to find V_A

$$V_B = 265.9 \text{ kN} \quad \text{and} \quad V_A = 184.1 \text{ kN}$$

The maximum tension, at B, is

$$\text{Maximum tension} = \sqrt{1364^2 + 265.9^2} = \mathbf{1390\ kN}$$

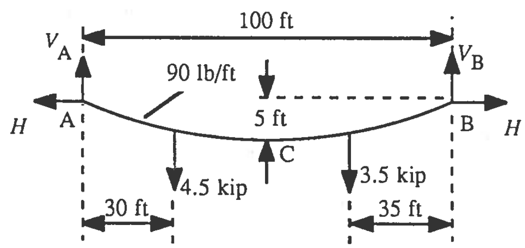
2.42 Let the horizontal component of tension in the cable be H and the vertical reaction components at the ends A and B of the cable be V_A and V_B.

For moment equilibrium of the whole cable about B

$$V_A \times 100 - 4.5 \times 70 - 3.5 \times 35 - 0.090 \times \frac{100^2}{2} = 0 \quad \Rightarrow \quad V_A = 8.875 \text{ kip}$$

and for force equilibrium in the vertical direction

$$V_A + V_B - 4.5 - 3.5 - 0.090 \times 100 = 0 \quad \Rightarrow \quad V_A + V_B = 17 \text{ kip}$$
$$\text{and} \quad V_B = 8.125 \text{ kip}$$



Let C be the point of maximum dip, at a horizontal distance of x from A ($30 < x < 65$). For force equilibrium of section AC of the cable

$$V_{\mathrm{A}} - 4.5 - 0.090 \times x = 0 \text{ giving } x = 48.61 \text{ ft}$$

and for moment equilibrium of the same section about A

$$4.5 \times 30 + 0.090 \times \frac{x^2}{2} - 5H = 0$$
$$\text{giving } H = 48.27 \text{ kip}$$

The maximum tension in the cable, which occurs at A, is $\sqrt{48.27^2 + 8.875^2} =$ **49.1 kip**

2.43 Let the horizontal component of tension in the cable be H and the vertical reaction components acting on the cable at the end A and at the pulley at C (where the cable first makes contact) be V_{A} and V_{c}, respectively. Now, the tension in the cable at C is $6W$, and

$$\sqrt{H^2 + V_{\mathrm{C}}^2} = 6W \qquad (1)$$

For force equilibrium of the cable in the vertical direction

$$V_{\mathrm{A}} + V_{\mathrm{C}} = W + 2W = 3W \qquad (2)$$

and moment equilibrium about C

$$2W \times \frac{9L}{10} + W \times \frac{L}{2} - V_{\mathrm{A}} \times L = 0$$

giving $V_{\mathrm{A}} = 2.3W$. Hence, from (2), $V_{\mathrm{C}} = 0.7W$, and from (1) $H = 5.959W$. If E is the lowest point of the cable, where there is no vertical component of tension, and E is a horizontal distance x from C ($0 < x < 0.9L$) and a vertical distance d below C, then for force equilibrium in the vertical direction of section CE of the cable

$$V_{\mathrm{C}} - \frac{x}{L}W = 0 \text{ giving } x = 0.7L$$

and for moment equilibrium of the same section about C

$$\frac{W}{L} \times \frac{x^2}{2} - H \times d = 0 \text{ giving } d = 0.0411L$$

(i) The lowest point of the cable is **0.7**L horizontally from C and **0.0411**L vertically below C.

(ii) The greatest tension in the cable, at A, is $\sqrt{H^2 + V_A^2}$= **6.39** W

(iii) The horizontal component of force acting on the pulley C is H, and the vertical component is the sum of V_C and the $6W$ weight supported at D, both in the downward direction. The resultant force is $\sqrt{(6W + V_\mathrm{C})^2 + H^2}$ = **8.97**W.
