## Chapter 2 (Exercise)

1) Let $A_n$ be the total value of an investment at the end of n-terms, earning a compound interest at a rate of r per term, with an additional deposit of d at the end of each term.
   ($i$) Construct an Annuity Saving Model, assuming $A_0$ to be the original investment.
   ($ii$) Find the total value after 3 years if the initial investment is Rs. 10000, each month the account gets 1% interest and Rs. 1000 is deposited each month.
   ($iii$) Find the total value after 7 years if the initial investment is Rs. 20000, the account gets 6% annual interest compounded monthly and a deposit of Rs. 3000 is made each month.
2) Let $L_n$ be the unpaid balance on a loan with an interest rate of r per term. A payment of P is made at the end of each year.
   ($i$) Assuming $L_0$ to be the original amount borrowed, construct a loan payment model for the unpaid balance at the end of each n years.
   ($ii$) Find the balance after 3 years when payments of Rs. 500 are made quarterly on an original loan of Rs. 9600 with an interest rate of 10.5% per quarter.
   ($iii$) Find the balance after 5 years when payment of Rs. 400 are made monthly on an original loan of Rs. 20000 with annual interest of 8%.
3) Let $P_n$ be the population of the n-th generation, growing linearly at a rate r and undergoes either immigration or migration at a constant level k.
   ($i$) Formulate a linear discrete immigration/migration model.
   ($ii$) Find the population of 4-th generation when the initial population is 3900, growth rate is 7% per generation and immigration is occurring at a constant rate of 190 per generation.
   ($iii$) How the population of 4-th generation changes if there is a migration of 190 per generator, instead of immigration.
4) Let $T_n$ be the amount of pollutant present in a contaminated lake, which is cleaned by filtering out a certain fraction $\alpha$ of all pollutants present at that time, but another $\beta$ tons of pollutants seep in $(0 < \alpha < 1, \beta > 0)$.
   ($i$) Formulate a mathematical model, assuming $T_0$ tons of pollutants are initially in that lake.
   ($ii$) If each week 10% of all pollutants present can be removed but another 2 tons seep in, find the values of $\alpha$ and $\beta$ and write the iterate equation for this process.
   ($iii$) How many tons of pollutant will be there in the lake after 2 years , if initially it is contaminated with 70 tons of pollutant ?
   ($iv$) How long will it take before the pollutant level falls below 2 tons ?
5) Let a pendulum swings in such a manner that the greatest negative (or positive) angle it makes on one side of the vertical is always a certain negative fraction $(-\alpha)$ of the greatest positive (or negative) angle it pre-

viously made on the other side of the vertical $(0 < \alpha < 1)$.
$(i)$ Formulate a mathematical model for this process if $\theta_n$ is the n-th greatest angle (positive or negative) and $\theta_0$ is the initial angle of the pendulum when released. Hence, write the iterative equation if $\theta_0 = 10^o$ and $\theta_2 = 8^o$.
$(ii)$ Suppose, the greatest negative (or positive) angle it makes on one side of the vertical is always 5% of the greatest positive (or negative) angle it previously made on the other side of the vertical, which always occurs 3 seconds earlier, how many times will the pendulum cross the vertical before the magnitude of the angle it achieves is $1^o$ or less ?
$(iii)$ Approximately, how long will it take for this to happen ?

6) Suppose a person is saving for retirement by depositing equal amounts every quarter into a retirement plan to earn 9% annual interest compounded quarterly. If the account initially had zero balance,
$(i)$ How much will be available at retirement time in 25 years if each deposit is Rs. 10000.00 ?
$(ii)$ How much should each deposit be to have Rs. 20,00,000.00 at retirement in 25 years ?

7) In a certain forest 3% of the tree are destroyed naturally each year. Also, 4000 tree are harvested for timber (cut down for industrial use) but 8000 new trees are either planted or sprout up on their own.
$(i)$ Formulate a discrete model for the yearly number of trees $T_n$ in that forest.
$(ii)$ What is the maximum number of trees the forest will have , if the currently estimated trees in that forest be 200000 ?

8) A credit card company charges 3% interest on any previous unpaid balance from a person and the person pays of 20% of that previous balance. Also, another Rs. 300 is charged on that credit card from the person for not paying the full amount.
$(i)$ Formulate a discrete model for the monthly unpaid balance.
$(ii)$ What level will the unpaid balance gradually approach ?

9) Let a town be effected with viral fever. Each day 10% of those have the viral fever in the town recover from it, while another 500 people are effected with the viral fever. If there are currently 2000 cases of viral fever,
$(i)$ Formulate a discrete mathematical model and find out how many cases will be there, two weeks from now.
$(ii)$ At that level, will the number of cases eventually stabilize?

10) It is known that as demand of a product increases, its prices increases and if the price of a product decreases, its supply increases. Let $D_n$ be the demand, $S_n$ be the supply and $P_n$ be the prices of a certain product. We assume that the demand force increases continuously from negative to positive as the current demand $D_n$ increases and same is the behavior for the supply $S_n$. Also, as the price of a product increases, its supply increases but demand from consumer decreases, that is, supply is directly

proportional to the price and demand is inversely proportional to the price.

($i$) Formulate a non-linear discrete price model for the price of a product at time n from the above information.

($ii$) What happens to the above model when the price is close to zero ? How that can be verified by using a different function?

11) Let $I_n$ be the number of infected population at any given time n and r be the fraction of the infected population who have recovered in time $(n+1)$. It is assumed that the numbers of newly infected population are directly proportional to the size of the infected population $I_n$ and to the size of the susceptible population $N - I_n$, where N is the population size.

($i$) Formulate a discrete contagious disease model for the infected population $I_{n+1}$.

($ii$) Write down the model when the number of cases is proportional to ($a$) $I_n^2$ and $(1 - \frac{I_n}{N})^2$, ($b$) $I_n^2$ and $(1 - \frac{I_n^2}{N^2})$, ($c$) $I_n$ and $\exp(\frac{-I_n}{N})$

($iii$) In a population of size $10^6$, each week 80% of those infected recover. If 1000 are infected in one week, then the following week 1500 are infected. Formulate a mathematical model.

($iv$) In a population of size $4 \times 10^6$, 40% infected are still sick in a week and the maximum number of new infected case possible in any week is 50000, formulate a model with this information.

($v$) In a population of size 10000, if 100 are infected, the number doubled in the following week. Formulate a model for assuming recovery is not possible.

12) A radioactive element is known to decay at the rate of 2% every 20 years.

(i) If initially you had 165 grams of this element, how much would you have in 60 years?

(ii) What is the half-life of this element?

($iii$) Suppose that the bones of a certain animal maintains a constant level of this element while the animal is living, but the element begins to decay as soon as the animal dies. If a bone of this animal is found and is determined to have only 10% of its original level of this element, how old is the bone?

13) A population of weasels is growing at rate of 3% per year. Let $w_n$ be the number of weasels, $n$ years from now and suppose that there are currently 350 weasels.

(i) Formulate a linear discrete model which describes how the population changes from year to year.

(ii) Solve the difference equation of part (i). If the population growth continues at the rate of 3%, how many weasels will there be 15 years from now?

($iii$) Plot $w_n$ versus n for $n = 0, 1, 2, ..., 100$.

(iv) How many years will it take for the population to double?

(v) Find $lim_{n\to\infty} w_n$. What does this say about the long-term size of the

population? Will this really happen?
(vi) If the rate of growth of the weasel population was 5% instead of 3%, how many years would it take for the population to double?

14) A cup of coffee has an initial temperature of $165^o$F, but cools to $155^o$F in one minute when placed in a room with a temperature of $70^o$F. Let $T_n$ be the temperature of the coffee after $n$ minutes.
(i) Formulate a linear discrete model which describes the change in temperature of the coffee from minute to minute and solve it.
(ii) Find the temperature of the coffee after 25 minutes.
$(iii)$ Find $lim_{n\to\infty}T_n$.
(iv) Plot $T_n$ versus $n$ for $n = 0, 1, 2, ...120$.
(v) Does the temperature ever reach $70^o$F?

15) The forensics team from a local police department is called to the scene of an apparent murder. The body of the victim is found in a room where the thermostat was set at $72^oF$. The police investigators believe that the victim died in the room and that the body has been there ever since.
(i) Formulate a linear discrete system that describes how the temperature of the dead body changes from hour to hour and solve it.
(ii) At 5 A.M., when the investigators first arrived, they took the temperature of the body and found that it was $87.5^o$F. Just before the body was to be removed from the room an hour later, they measured the temperature to be $80.4^o$F. Find the time when the victim was murdered.
$(iii)$ Using the data from (ii), modify the difference equation to describe how the temperature of the dead body changes
(a) over each 15-minute interval
(b) from minute to minute.

16) An industry dumps a metal-based pollutant into a local lake. Currently there are 350 pounds of pollutant in the lake and 20 pounds are added each year, all at once at the end of the year. It is known that the amount of pollutant lost through evaporation and decomposed chemically in a year is 10% of the amount in the lake.
(i) Formulate a linear discrete system that models the amount of the pollutant in the lake after each year.
(ii) Show graphically, what will happen in the long run if the situation described above persists? How does the outcome depend on the initial amount of the pollutant in the lake?
$(iii)$ The maximum safe amount of pollutant in this lake is 125 pounds of pollutant. If we set a goal to achieve this safe level in 15 years, determine how much pollutant can be added to the lake each year.
(iv) What, if the amount of pollutant that is added each year is divided into equal parcels, are added each month throughout the year? How does this change the model in part (i) and the results obtained in parts (ii) and $(iii)$?

17) A person has signed a contract to write a book, where he has already written the first 90 pages. But, the Publisher wants the book to be 300

pages long. The author has started writing K pages of the book, each day, starting from today.

(i) Formulate a linear discrete system that models $w_n$, the total number of pages written after $n$ days and the unknown parameter K, taking w(0) = 90.

(ii) The publisher has given the author 21 days to finish the book. Find the approximate number of pages the author needs to write each day to complete the book.

18) You are counseling a recent graduate on retirement planning. You estimate he will earn 9% per year (or 0.75% per month) on a retirement account. With his high-paying job, he will be able to invest 5000 per month initially. To allow for inflation and pay raises, you suggest increasing this amount by 0.5% per month. Assuming that he makes his first deposit at the end of his first month of employment.

(i) Formulate a discrete dynamical system that models the amount in this person's retirement account and solve it.

(ii) Graphically, determine the balance in the account after 20 years and use the analytic solution to verify it.

$(iii)$ How does the analytic solution change if the monthly investment increases by 0.75% per month instead of 0.5% per month?

19) Let the kidney's filter out 45% of the vitamin A in the plasma each day and the liver absorbs 35% of the vitamin A in the plasma each day. Also, 2% of the vitamin A in the liver is absorbed back into the plasma each day. Let 15% of the vitamin A in the plasma converted to a chemical B and 5% of the chemical B is filtered out by the kidney's each day. Assume that the daily intake is 2 mg. of vitamin A and 1 mg. of the chemical B each day.

(i) Formulate a discrete dynamical model for vitamin A in the plasma, vitamin A in the liver and chemical B.

(ii) Find the equilibrium solution and check for its stability.

$(iii)$ Suppose a person ingests 1.4 mg. of vitamin A each day and wants the equilibrium for vitamin A in the plasma to be 3 mg. How much of the chemical B should you ingest each day to accomplish this? What will be the equilibrium amount of the chemical B in the plasma and what will be the equilibrium amount of vitamin A in the liver with this consumption ?

20) Consider a linear predator prey model. Let the prey's growth rate be 1.2 and that of predation's be 1.3; the prey population diminishes by 0.3 times that of predator ; the predator's population is increased by 0.4 times that of the prey. Construct a discrete model from the given information and discuss its stability.

21) Let the two species $X_n$ and $Y_n$ compete for food, water, habitat etc. at any time n. Both the species grow at the rate $r_1$ and $r_2$ respectively and they diminish by an amount directly proportional to the size of the other,

say at a rate $S_1$ and $S_2$ respectively.
(i) Formulate a linear competition model with the above information.
(ii) Suppose there is a constant migration or constant immigration, $K_1$ and $K_2$ respectively, how the model will change ?
$(iii)$ Let the growth rate $r_1$ of first species be 1.5 and that of second species be 1.25. Also, each is diminished by 0.4 times the population of the other. Construct a discrete model and comment on its stability.
(iv) In addition to the information in $(iii)$, if the first species undergo immigration of 2500 at each time step and the second one migrates at 1200 per time step, formulate a new model and comment on its stability.

22) A soccer player gets a contract for Rs. 50000.00 per game and a Rs. 3,00,000.00 signing bonus.
(i) Formulate a discrete dynamical system to model this player's earning over the season.
(ii) How much this player will earn if he plays 120 games and how many games he needs to play to earn at least Rs. 1000000.
23) A car holds 30 litres of petrol (full tank) and gets 10 km to a litre of petrol.
(i) Formulate a discrete dynamical system to model the amount of petrol left in the car's tank after driving for x km.
(ii) How much gas will be left in the tank after you have driven 120 km and how many km you can drive before running out of petrol?

24) Research evidence has shown that the number of chirps some crickets make depends on the temperature. At $70^oF$, one species of crickets makes about 124 chirps per minute. The number of chirps increases by 4 per minute for each degree the temperature rises. Formulate a discrete dynamical system to model the situation that relates the number of chirps per minute in terms of the temperature and use this expression to find approximate temperature if you count about 24 chirps in 10 seconds?

25) The pavement on a bridge is 1000 m long at $70^oF$. The length of the pavement grows by 0.012 m for each degree rise in temperature. Formulate a discrete dynamical system to model the length of the bridge in terms of the temperature. What will be the length of the bridge if the temperature reaches $105^oF$?

26) The air pressure at sea level is 1000 g/cm$^2$. The pressure increases by 2.08 g/cm$^2$ for each cm. of depth in salt water. Write a discrete mathematical model for this discrete dynamical system.
27) Most of us keeps the water tap open while shaving. In this problem, we are going to study how water is wasted during shaving. Let $w(n)$ be the amount of water that is wasted per day by $n$ number of people that let the water run while shaving.

(i) Formulate a discrete dynamical system to model this situation.
(ii) Assuming you shave 5 times a week, estimate how much water would be wasted per day if you leave the water running while shaving.

28) Each year, a person who smokes single pack of cigarettes a day, will absorb about 2.7 mg of cadmium (an extremely dangerous heavy-metal pollutant and is most toxic when inhaled). For simplicity it is assumed that the cadmium is absorbed at the end of the year. Some people eliminate about 8% of the cadmium from their body each year. Formulate a dynamical system for $u(n)$, the amount of cadmium in such a person's body after n years as a result of smoking and hence find the amount of cadmium in such a person's body as a result of smoking for 20 years.
29) A boy absorbs 0.4 mg of lead into his plasma each day, starting from today. Each day, 35% of the lead in his plasma is absorbed into his bones and 9% of the lead in his plasma is eliminated in the urine (approximately). Also, 0.118% of the lead in his bones is absorbed back into his plasma each day. Formulate a discrete mathematical model for this situation, assuming p(n) to be the amount of lead in this boy's plasma and b(n) to be the amount of lead in this boy's bones, at the beginning of day n. Hence, find p(4)and b(4), where p(0)=0 and b(0)=0 .
30) Suppose a person's body burns 130 kcal per week for each pound it weighs. Suppose this person presently weighs 165 pounds and consumes about 21,000 kcal per week. Suppose this person decides to eat 200 kcal less each week than the week before. Let c(n) represent the number of kilocalories this person consumes during the n-th week of this diet and w(n) be the weight of this person after n weeks of this diet. Develop a dynamical system of two equations, one for c(n) and one for w(n), assuming c(1)=20,800 and w(0)=165 (To work this problem, you need to know that burning 3600 kcal would reduce a person's weight by 1 pound and consuming 3600 kcal would result in the gaining 1 pound).

**Solutions to Chapter 2**

**1.**

**(i)** Interest rate per term-$r$, Original investment -$A_0$, Additional investment after each term = $d$.

$$
\begin{aligned}
A_1 &= rA_0 + d + A_0\\
A_2 &= (1+r)A_1 + d = (1+r)^2A_0 + (1+r)d + d\\
\ldots\ldots \ldots &\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\\
A_n &= (1+r)A_{n-1} + d\\
&= (1+r)^nA_0 + d[1 + (1+r) + (1+r)^2 + \ldots.. + (1+r)^{n-1}]\\
&= (1+r)^nA_0 + \frac{[(1+r)^n - 1]}{r}d
\end{aligned}
$$

**(ii)** $A_0 = Rs.10,000$, $d = Rs1,000$, $r = 1\%$ per month. We want the amount after 36 month, i.e. 3 years.

$$
\begin{aligned}
A_{36} = (1+0.01)^{36} \times 10,000 + \frac{(1+0.01)^{36} - 1}{0.01} \times 1000\\
A_{36} = (1+0.01)^{36} \times 1,10,000 - 1,00,000\\
A_{36} = \ Rs.\ 57,384.57
\end{aligned}
$$

**(iii)** Rs. 3,42,629.17

**2.**

**(i)** $L_n$ = unpaid balance on loan after $n$ terms, $r$= rate of interest, $L_0$ = Original amount borrowed and $P$ = Payment made each term.

$$
\begin{aligned}
L_1 &= L_0 + rL_0 - P\\
L_2 &= (1+r)L_1 - P = (1+r)^2L_0 - (1+r)P - P\\
L_3 &= (1+r)L_2 - P = (1+r)^3L_0 - [(1+r)^2 + (1+r) + 1]P\\
L_n &= (1+r)L_{n-1} - P\\
L_n &= (1+r)^nL_0 - \left[\frac{(1+r)^n - 1}{r}\right]P
\end{aligned}
$$

**(ii)** $\mathrm{L}_0 = Rs\ 9,600$, r $= 10.5$ % per quarter, P $= Rs.500$.
We want the balance after 3 years, that is, 12 quarters.

$$
L_{12} = (1+0.105)^{12} \times 9600 - \left[\frac{(1+0.105)^{12} - 1}{0.105}\right] \times 500 = 20795.17
$$

**(iii)** Loan terms = 5 year ie. 60 months, Original loan = Rs. 20,000, Interest=8% annual = $\frac{2}{3}$% per month, Payment = $Rs.400$.

$$L_{60} = \left(1 + \frac{2}{300}\right)^{60} \times 20,000 - \left[\frac{\left(1 + \frac{2}{300}\right)^{60} - 1}{\frac{2}{300}}\right] \times 400 = 406.17$$

**3.**

**(i)** $P_n$ = Population after $n$ generations, $R$ = Growth rate, $k$ = Immigration/migration (according to whether positive or negative), $P_0$ = Initial population.

$$\begin{aligned}
P_1 &= P_0 + RP_0 + k \\
P_2 &= (1+R)P_1 + k = (1+R)^2 P_0 + [(1+R)+1]k \\
P_3 &= (1+R)P_2 + k = (1+R)^3 P_0 + [1 + (1+R) + (1+R)^2]k
\end{aligned}$$

$$- - - - - - - - - - - - - - - - - - - - - - - - - - - - - - -$$

$$P_n = (1+R)^n P_0 + \frac{(1+R)^n - 1}{R} k = (1+R)^n \left[P_0 + \frac{k}{R}\right] - \frac{k}{R}$$

**(ii)** $P_0 = 3900, R = 7\%, k = 190.$

$$P_4 = (1.07)^4 \left[3900 + \frac{190}{0.07}\right] - \frac{190}{0.07} = 5956.$$

**(iii)** $k = -190 \ \ \text{(migration)}, P_4 = 4269.$

**4.**

**(i)** $T_0$ = Initial contamination, $\alpha$ = Cleaning fraction ($0 < \alpha < 1$), $\beta$ = Additional pollutant($\beta > 0$) and $T_n$ = Contamination after $n$ periods.

$$\begin{aligned}
T_1 &= (1-\alpha)T_0 + \beta \\
T_2 &= (1-\alpha)T_1 + \beta = (1-\alpha)^2 T_0 + \beta[(1-\alpha)+1] \\
T_3 &= (1-\alpha)T_2 + \beta = (1-\alpha)^3 T_0 + \beta[1 + (1-\alpha) + (1-\alpha)] \\
T_n &= (1-\alpha)^n T_0 + \frac{\beta[(1-\alpha)^n - 1]}{(1-\alpha) - 1} \\
T_n &= (1-\alpha)^n T_0 + \frac{\beta}{\alpha} - \frac{\beta}{\alpha}(1-\alpha)^n
\end{aligned}$$

**(ii)** Period =Weekly, $\alpha = 10\% = 0.1$, $\beta = 2$ tons.

$$T_n = (0.9)^n \left[T_0 - \frac{2}{0.1}\right] + \frac{2}{0.1} = (0.9)^n[T_0 - 20] - 20$$

**(iii)** $T_0$ = 70 tons, time = 2 years =104 weeks.

$$T_{104} = (0.9)^{104}\,[70 - 20] + 20 = 20.00087 \text{ tons}$$

**(iv)** $T_n = 50 \times (0.9)^n + 20 > 20$ for all $n\epsilon[0, 1, 2, ....]$ The total contamination will always be more than 20 tons. Thus, it will never fall below 2 tons.

**5.**

**(i)** $\theta_0$ = initial angle and $\alpha$ = fraction ($\alpha < 0$)
$\theta_1 = \alpha\theta_0$
$\theta_n = \alpha\theta_{n-1}$
$\theta_n = \alpha^n\theta_0$

$\theta_0 = 10^0$
$\theta_2 = 8^0$
$\alpha^2\theta_0 = \theta_2$
$\alpha^2 = 0.8$
$\alpha = \sqrt{0.8} = 0.894$
$\theta_n = (0.8)^{n/2} \times (-1)^n\theta_0 = (-1)^n(0.8)^{n/2} \times 10^0$

**(ii)** $\theta_n = 0.05\theta_{n-1}$
$t = 3sec.$
$\theta_n = 0.05^n\theta_0$
$\theta_n < 1^0$
$(0.05)^n 10^0 < 1^0$
$n > 0.768$
Hence after 1st turn, angle will be less than $1^0$

**(iii)** $\theta_n = \alpha^n\theta_0$
$\theta_n < 1^0$
$(0.894)^n 10 < 1$
$n > 20.55$
Therefore, it will take 20 turns before angle will be less than $1^0$, that is, 60 seconds.

**6.**

**(i)** Initial Deposit $= D_0 = 0$, Quarterly Deposit $= M =$ Rs. 10,000, Interest Rate = 9% $= r =$ 2.25% per quarter.

$$
\begin{aligned}
&D_0 = 0\\
&D_1 = (1+r)D_0 + M\\
&D_2 = (1+r)D_1 + M = M[(1+r)+1]\\
&D_3 = (1+r)D_2 + M = M[(1+r)^2+(1+r)+1]\\
&D_n = (1+r)D_{n-1} + M\\
&D_n = \frac{M[(1+r)^n-1]}{r}
\end{aligned}
$$

Time duration = 25 years=100 quarters.

$$D_{100} = \frac{10,000}{2.25\times10^{-2}}[(1.0225)^{100}-1] = 36,68,465$$

**(ii)** Given $D_{100}$=Rs. 20,00,000, then

$$20,00,000 = \frac{M[(1.0225)^{100}-1]}{2.25\times10^{-2}} \Rightarrow M = 5452$$

**7.**

**(i)** $T_0$ = initial no. of Trees, natural death rate = $r\%$, harvested Trees = $H$, planted Trees = $P$

$$
\begin{aligned}
T_1 &= (1-r)T_0 - H + P\\
T_2 &= (1-r)T_1 - H + P = (1-r)^2T_0 + (P-H)[1+(1-r)]\\
T_3 &= (1-r)T_2 + (P-H) = (1-r)^3T_0 + (P-H)[1+(1-r)+(1-r)^2]
\end{aligned}
$$

$$---------------------------------$$

$$T_n = (1-r)^nT_0 + \frac{(P-H)[-(1-r)^n+1]}{r}$$

$r = 3\%, H = 4,000, P = 8,000$

$$
\begin{aligned}
T_n &= (0.97)^nT_0 + \frac{(8,000-4,000)[1-(0.97)^n]}{0.03}\\
&= (0.97)^nT_0 + \frac{4\times10^5}{3}[1-(0.97)^n]
\end{aligned}
$$

**(ii)** Do yourself.

**8.**

**(i)** $T_0$ = Initial unpaid balance, Interest $=I$, Repayment rate$=R$, Additional charge $=A$.

$$T_1 = T_0(1+I)(1-R) + A$$

Let $(1+I)(1-R) = k$, then

$$\begin{aligned}
T_1 &= kT_0 + A \\
T_2 &= kT_1 + A = k^2T_0 + A(1+k) \\
T_3 &= kT_2 + A = k^3T_0 + A(1+k+k^2) \\
T_n &= k^nT_0 + \frac{A(1-k^n)}{(1-k)}
\end{aligned}$$

$I = 3\%, \quad R = 20\%, \quad A = 300., \; k = 1.03 \times 0.8 = 0.824$

$$\begin{aligned}
T_n &= (0.824)^nT_0 + 300\left[\frac{(0.824)^n - 1}{-0.176}\right] \\
&= \left[T_0 - \frac{300}{0.176}\right] \times (0.824)^n + 1704.55 \\
&= [T_0 - 1704.55](0.824)^n + 1704.55
\end{aligned}$$

**(ii)** $T_\infty = 1704.55$, therefore the unpaid balance will approach $Rs.\ 1,704.55$.

**9.**

**(i)** initial infected person $= I_0$, recovery rate $= R$, new infected person $= N$

$$\begin{aligned}
I_1 &= (1-R)I_0 + N \\
I_2 &= (1-R)I_1 + N = (1-R)^2I_0 + N[1+(1-R)] \\
I_3 &= (1-R)I_2 + N = (1-R)^3I_0 + N[1+(1-R)+(1-R)^2]
\end{aligned}$$

$-------------------------$

$$I_n = (1-R)^nI_0 + \frac{N[1-(1-R)^n]}{R}$$

$R = 10\%, \quad N = 500, \quad I_0 = 2,000$

$$\begin{aligned}
I_n &= (0.9)^n(2,000) + \frac{500}{0.1}[1-(0.9)^n] \\
I_n &= 5,000 - 3,000(0.9)^n \Rightarrow I_{14} = 4314
\end{aligned}$$

**(ii)** The number of cases will eventually stabilize to 5000 for large n (n $\to \infty$.

**10.**

**(i)** Demand force= $a_1 D_n - b_1$, Supply force= $a_2 S_n - b_2$
Demand, $D_n \propto \frac{1}{P_n} \Rightarrow \; D_n = \frac{c_1}{P_n}$
Supply, $S_n \propto P_n \Rightarrow \; S_n = c_2 P_n$

$$\begin{aligned}
P_{n+1} &= P_n + \text{Demand force} - \text{Supply force} \\
P_{n+1} &= P_n + a_1 D_n - b_1 - a_2 S_n + b_2 \\
P_{n+1} &= P_n + \frac{a_1 c_1}{P_n} - b_1 - a_2 c_2 P_n + b_2 \\
P_{n+1} &= P_n(1 - a_2 c_2) + \frac{a_1 c_1}{P_n} + (b_2 - b_1) \\
P_{n+1} &= a P_n + b + c/P_n
\end{aligned}$$

where $a = 1 - a_2 c_2, b = b_2 - b_1, c = a_1 c_1$

**(ii)** Left to the readers.

**11.**

**(i)** $r$ - recovery rate, $I_n$ - infected people at time $t$

$$I_{n+1} = I_n - r I_n + \alpha I_n (N - I_n)$$

**(ii)**

$$\begin{aligned}
&\text{(a)} \quad I_{n+1} = I_n - r I_n + \alpha_1 I_n^2 \left(1 - \frac{I_n}{N}\right)^2 \\
&\text{(b)} \quad I_{n+1} = (1 - r) I_n + \alpha_2 I_2^2 \left(1 - \frac{I_n^2}{N^2}\right) \\
&\text{(c)} \quad I_{n+1} = (1 - r) I_n + \alpha_3 I_n e^{-I_n/N}
\end{aligned}$$

**(iii)** $r = 8\%, \quad N = 10^6, \quad I_n = 1000 \quad I_{n+1} = 1500$

$$\begin{aligned}
I_{n+1} &= I_n(1 - 0.8) + k I_n (10^6 - I_n) \\
\Rightarrow \; 1500 &= 0.2 \times 1000 + k \times 10^3 (10^6 - 10^3) \\
\Rightarrow \; k &= 1.3/(10^6 - 10^3) \\
\Rightarrow \; I_{n+1} &= 0.2 I_n + 1.3013 I_n - 1.3013 \times 10^{-6} I_n^2.
\end{aligned}$$

**(iv)** and **(v)** Do yourself.

**12.**

**(i)** Radioactive decay follow exponential model.

$$\begin{aligned}
P_t &= P_0(1-d)^t \\
\frac{P_t}{P_0} &= (1-d)^t
\end{aligned}$$

$$\begin{aligned}
For \quad t &= 20 \quad years. \\
\frac{P_t}{P_0} &= 0.98 = (1-d)^{20} \\
\therefore \frac{P_t}{P_0} &= (0.98)^{t/20} \\
P(t) &= P_0(0.98)^{t/20} \\
P(60) &= P_0(0.98)^3 \\
P_0 &= 1659 \\
P_{60} &= 155.39
\end{aligned}$$

**(ii)** Half Life ($\lambda$)- Time after which concentration reduces to half of initial concentration.
$\frac{1}{2}P_0 = P_0(0.98)^{\lambda/20}$
$0.5 = (0.98)^{\lambda/20}$
Ln$0.5 = \frac{\lambda}{20}\ln 0.98$
$\lambda = 20\frac{\ln 0.5}{\ln 0.98} = 686.2$ years Half life = 686.2 years.

**(iii)** $P_t = 0.1P_0$
$0.1P_0 = P_0(0.98)^{t/20}$ $0.1 = (0.98)^{t/20}$
$t/20 = \frac{ln0.1}{ln0.98}$
$t = 2280$ years.

**13.**

**(i) and (ii)** $P_0$ : initial population = 350, $r$: growth rate = 3%
$P_1 = P_0(1+r) = P_0 + rP_0 = 1.03P_0$

$$\begin{aligned}
P_2 &= P_1(1+r) = (1+r)^2P_0 \\
P_3 &= P_2(1+r) = (1+r)^3P_0 \\
P_n &= P_0(1+r)^n \\
P_n &= 350(1+0.3)^n = 350(1.03)^n
\end{aligned}$$

**(iii)** Do yourself.

**(iv)** $P_n = (1.03)^n P_0 = 2P_0 \Rightarrow\ (1.03)^n = 2 \Rightarrow nlog1.03 = log2$
$\Rightarrow n = 23.45 \approx 24$ years

**(v)** As $n \to \infty, P_n \to \infty$. The population size will increase unboundedly for many years. No, it will not happen because the resources necessary for growth are limited.

**(vi)** $r = 5\%$, therefore, $P_n = (1.05)^n P_0 = 2P_0 \Rightarrow (1.05)^n = 2$
$\Rightarrow n \log 1.05 = \log 2 \Rightarrow n = \frac{\log 2}{\log 1.05} = 14.2 \Rightarrow n = 15$ years.

**14.**

**(i)** Newton's Low of cooling :-
$T_{n+1} - T_n = k(T_n - T_s)$
where $T_s$: Surrounding temperature.
$T_n$ = temp at time $n$.

$$\begin{aligned}
T_{n+1} &= (k+1)T_n - kT_s \\
T_1 &= (k+1)T_0 - kT_s \\
T_2 &= (k+1)T_1 - kT_s = (k+1)^2 T_0 - kT_s[1 + (1+k)] \\
T_3 &= (k+1)T_2 - kT_s = (k+1)^3 T_0 - kT_s[1 + (1+k) + (k+1)^2] \\
T_n &= (k+1)^n T_0 - kT_s \frac{[(k+1)^n - 1]}{k} \\
T_n &= (k+1)^n (T_0 - T_s) + T_s
\end{aligned}$$

$T_0$ = initial temp. $= 165^0 F$ $T_1 = 155^0 F$
$T_s = 70^0 F$
$\therefore 155 = (k+1)165 - k(70)$
95 k = -10
$k = -10/95$
$T_n = \left(\frac{17}{19}\right)^n \times 95 + 70$

**(ii)** $T_{25} = 75.89^0 F$

**(iii)** As $n \to \infty$, $T_n \to T_s$ ie $70^0 F$

**(iv)** For large time, yes.

**15.**

**(i)** Newton's Law of cooling:

$$T_{n+1} - T_n = k(T_n - T_s)$$

where $T_s$ is the surrounding temperature and $T_n$ = Body temperature at time. Suppose at time $n = 0$, the person is murdered. Then, $T_0 = 98.6\,°\mathrm{F}$ and $T_s = 72\,°\mathrm{F}$.

$$\begin{aligned}
T_1 &= (k+1)T_0 - kT_s \\
T_2 &= (k+1)T_1 - kT_s = (k+1)^2T_0 - kT_s[1 + (1+k)] \\
T_3 &= (k+1)T_2 - kT_s = (k+1)^3T_0 - kT_s[1 + (1+k) + (k+1)^2]
\end{aligned}$$

— — — — — — — — — — — — — — — — — — — — — — — — — — — — — — —

$$\begin{aligned}
T_n &= (k+1)^nT_0 - kT_s\frac{[(k+1)^n - 1]}{1 + k - 1} \\
T_n &= (k+1)^n(T_0 - T_s) + T_s
\end{aligned}$$

Let us assume that after the murder, the forensics arrive after $P$ minutes.

$$\begin{aligned}
T_P &= (1+k)^p(26.6) + 72 = 87.5 \\
T_{P+60} &= (1+k)^{p+60}(26.6) + 72 = 80.4, \text{which implies} \\
(1+k)^P &= 0.5827 \text{ and } (1+k)^{P+60} = 0.3158 \\
\Rightarrow\ (1+k)^{60} &= 0.542 \\
\Rightarrow\ k &= -0.0101
\end{aligned}$$

Now, $(0.9898)^P = 0.5827 \Rightarrow\ P = 53.2 \approx 53$ minutes.
Therefore, the victim was murdered at $4:07$ AM.

**16.**

**(i)** $A_n$ = Amount of pollutant after $n$ years, $A_0$ = Amount initially present, $r$ = Rate of decomposition, $A$ = Amount of pollutant added each year.

$$\begin{aligned}
A_n &= A_{n-1} - rA_{n-1} + A \\
A_{n-1} &= (1-r)A_{n-1} + A
\end{aligned}$$

$$\begin{aligned}
A_1 &= (1-r)A_0 + A \\
A_2 &= (1-r)A_1 + A = (1-r)^2A_0 + A(1-r+1) \\
A_3 &= (1-r)A_2 + A = (1-r)^3A_0 + A(1+(1-r)+(1-r)^2) \\
\therefore A_n &= (1-r)^nA_0 + A[1+(1-r)+(1-r)^2+....(1-r)^{n-1}] \\
A_n &= (1-r)^nA_0 + \frac{A[1-(1-r)^n]}{1-(1-r)} \\
A_n &= \left(A_0 - \frac{A}{r}\right)(1-r)^n + \frac{A}{r}
\end{aligned}$$

$$A_n = 150(0.9)^n + 200$$

After 10 years, P=252.3 pounds
After 20 years, P = 218.24 pounds
After 30 years, P = 206.36 pounds
After 40 years, P = 202.22 pounds
After a long time (*ie* $n \to \infty$), amount of pollutant becomes constant at 200 pounds. The outcome is independent of initial amount of pollutant.

**(ii)** $A_n = 0.9A_{n-1} + X$
$A_n = (0.9)^nA_0 + 10 \times [1-(0.9)^n]$
$125 = (0.9)^{15} \times 350 + 10 \times [1-(0.9)^{15}]$
$x = 6.67$ pounds per years.

**(iii)** If pollutant added monthly.
Monthly decomposition rate $= \frac{10}{12}\% = r_m = \frac{5}{6}\%$
Pollutant after $n$ month.

$$\begin{aligned}
A_n &= (1-r_m)^n\left[A_0 - \frac{A_m}{r_m}\right] + \frac{A_m}{r_m} \quad \text{Amount added after end of each month} \\
A_n &= \left(\frac{1190}{1200}\right)^n\left[350 - \frac{200/12}{10/1200}\right] + \frac{20/12}{10/1200} \\
A_n &= 150\left(\frac{119}{120}\right)^n + 200
\end{aligned}$$

In this case amount after end of each year is more than amount after end of each year in previous case.
For equilibrium solution,
$A_n = A_{n-1} = A^*$
$0.9A^* + 20 = A^*$
$A^* = 200$

**17.**

**(i)**

$$\begin{aligned}
w_n &= w_{n-1} + K, \quad w_0 = 90\\
w_{n-1} &= w_{n-2} + K\\
w_{n-2} &= w_{n-3} + K\\
&\ldots\ldots\ldots\ldots\ldots\ldots\\
w_1 &= w_0 + K\\
\text{Adding we get, } w_n &= w_0 + nK
\end{aligned}$$

**(ii)** Here, $w_0 = 90, n = 21$ days $\Rightarrow\ 90 + 21K = 300\ \Rightarrow\ K = 10$, that is, the author has to write 10 pages each day to complete the book.

**18.**

**(i)** $B_n =$ Balance after $n$ months, $i =$ interest rate, $r =$ inflation rate, $A_0 =$ initial amount for retirement, $A_n =$ Monthly amount for $n$th month.

$$\begin{aligned}
A_n &= (1+r)A_{n-1}\\
\therefore A_n &= (1+r)^n A_0\\
B_n &= B_{n-1}(1+i) + A_{n-1}\\
B_1 &= B_0(1+i) + A_0 = A_0 \qquad [\because B_0 = 0]\\
B_2 &= B_1(1+i) + A_1 = A_0(1+i) + A_0(1+r)\\
B_3 &= B_2(1+i) + A_2 = A_0[(1+i)^2 + (1+i)(1+r) + (1+r)^2]\\
B_n &= A_0\frac{[(1+i)^n - (1+r)^n]}{[(1+i) - (1+r)]} = A_0\frac{[(1+i)^n - (1+r)^n]}{i-r}
\end{aligned}$$

$A = 5000, i = 0.0075, r = 0.005$
$B_n = \frac{5000}{0.0025}[(1.0075)^n - (1.005)^n] = 2 \times 10^6[1.0075^n - 1.005^n]$

$$\begin{aligned}
B_{240} &= 2 \times 10^6[1.0075^{20\times12} - 1.005^{20\times12}]\\
B_{240} &= Rs.53,97,894
\end{aligned}$$

**(ii)** From graph, $B_{20} \approx Rs.53,00,000$

**(iii)** If $r = 0.0075$, then $(1+r) = (1+i)$

$$\begin{aligned}
B_n &= A_0[(1+i)^{n-1} + (1+i)^{n-2}(1+r) + ... + (1+r)^{n-2}(1+i) + (1+r)^{n-1}]\\
B_n &= A_0(1+r)^{n-1} \times n\\
B_{240} &= 5000(1.0075)^n \times 20 \times 12\\
B_{240} &= 5000 \times 240(1.0075)^{239}\\
B_{240} &= Rs71,57,302
\end{aligned}$$

**19.**

**(i)** Let $p_n$ be vitamin A in the plasma, $l_n$ be vitamin A in the liver and $b_n$ be chemical B at time n. See Figure (1) for schematic diagram.

$$
\begin{aligned}
p_n &= p_{n-1} - 0.45p_{n-1} - 0.35p_{n-1} - 0.15p_{n-1} + 0.02l_{n-1} + 2 \\
l_n &= l_{n-1} + 0.35p_{n-1} - 0.02l_{n-1} \\
b_n &= b_{n-1} + 0.15p_{n-1} - 0.05b_{n-1} + 1
\end{aligned}
$$

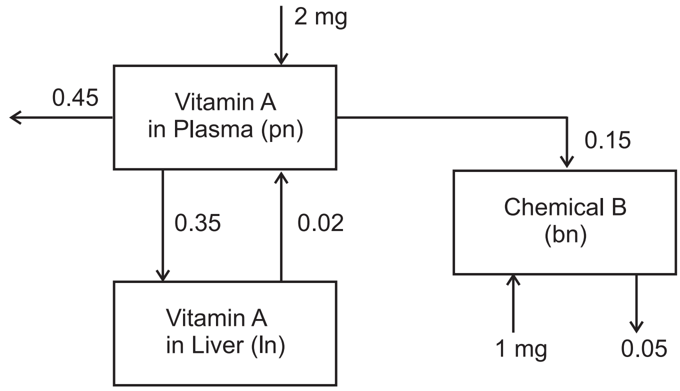


Fig. 1. *The schematic diagram of the problem.*

**(ii)** Do yourself.

**(iii)** Do yourself.

**20.**

Prey population after $n$ years = $D_n$.
Predator population after $n$ years = $T_n$.

$$
\begin{aligned}
D_n &= 1.2D_{n-1} - 0.3T_{n-1} \Rightarrow T_{n-1} = 4D_{n-1} - \frac{D_n}{0.3} \\
T_n &= 1.3T_{n-1} + 0.4D_{n-1} \Rightarrow D_{n-1} = 2.5T_n - 3.25T_{n-1}
\end{aligned}
$$

$$
\begin{aligned}
\therefore\quad 4D_n - \frac{D_{n+1}}{0.3} &= 5.2D_{n-1} - \frac{13}{3}D_n + 0.4D_{n-1} \\
\frac{D_{n+1}}{0.3} + \frac{25}{3}D_n + 5.6D_{n-1} &= 0 \\
D_{n+1} + 2.5D_n + 1.68D_{n-1} &= 0
\end{aligned}
$$

$$
\begin{aligned}
2.5T_{n+1} - 3.25T_n &= 3T_n - 3.9T_{n-1} - 0.3T_{n-1} \\
2.5T_{n+1} - 6.25T_n &+ 4.2T_{n-1} = 0 \\
T_{n+1} + 2.5T_n &+ 1.68T_{n-1} = 0
\end{aligned}
$$

Let $T_n = ck^n$

$$
\begin{aligned}
ck^{n+1} + 2.5ck^n + 1.68ck^{n-1} &= 0 \\
k^2 + 2.5k + 1.68 &= 0 \\
\therefore k &= 1.25 + 0.34i, 1.25 - 0.34i \\
T_n &= (\sqrt{1.25^2 + 0.34^2})^n [C_1 \cos 15.22n + C_2 \sin 15.22n] \\
T_n &= 1.3^n [C_1 \cos(15.22n) + C_2 \sin(15.22n)]
\end{aligned}
$$

Also,

$$
D_n = 1.3^n [C_3 \cos(15.22n) + C_4 \sin(15.22n)]
$$

we have

$$
W(n) = \begin{pmatrix} D_n \\ T_n \end{pmatrix}
$$

$$
A = \begin{bmatrix} 1.2 & -0.3 \\ 1.3 & 0.4 \end{bmatrix}
$$

Finding eigenvalues

$$
A = \begin{vmatrix} 1.2 - \lambda & -0.3 \\ 1.3 & 0.4 - \lambda \end{vmatrix} = 0
$$

$$
\lambda^2 - 1.6\lambda + 0.77 = 0
$$

$$
\lambda = 0.8 \pm 0.48i
$$

$$
|\lambda| = \sqrt{0.8^2 + 0.48^2} = 0.93 < 1
$$

$\therefore$ System is stable.

**21.**

**(i)** Two species $X_n$, $Y_n$.
For $X_n$, growth rate = $r_1$, dim. rate =$S_1$
For $Y_n$, growth rate = $r_2$, dim. rate =$S_2$

$$
\begin{aligned}
X_{n+1} &= r_1 X_n - S_1 Y_n \\
Y_{n+1} &= r_2 Y_n - S_2 X_n
\end{aligned}
$$

**(ii)** If there is constant immigration or migration of species

$$X_{n+1} = r_1 X_n - S_1 Y_n \mp K_1$$
$$Y_{n+1} = r_2 Y_n - S_2 X_n \mp K_2$$

We consider $+K_1$ if immigration is taking place and $-K_1$ if migration is taking place.

**(iii)** Do yourself.

**(iv)** $K_1 = 2500, \quad K_2 = -1200$

$$X_{n+1} = 1.5X_n - 0.4Y_n + 2500$$
$$Y_{n+1} = 1.25Y_n - 0.4X_n - 1200$$

For equilibrium solution

$$0.5X^* + 2500 = 0.4Y^*$$
$$0.4X^* + 1200 = 0.25Y^*$$

$$X^* = \frac{580}{0.14} \approx 4143, \qquad Y^* \approx 10429$$

Finding eigenvalues:

$$Ax = \lambda x$$

$$[A - \lambda I]x = 0$$

$$|A - \lambda I|x = 0$$

$$\begin{vmatrix} 1.5 - \lambda & 0.4 \\ 0.4 & 1.25 - \lambda \end{vmatrix} = 0$$

$$\lambda^2 - 2.75\lambda + 1.875 - 0.16 = 0$$

$$\lambda^2 - 2.75\lambda + 1.715 = 0$$

$$\lambda_1 = 0.955928, \quad \lambda_2 = 1.79408$$

One of the eigenvalues has modulus greater than 1, hence the system is unstable.

**22.**

**(i)** $P_n$ = earning after $n$ games, $W$ = signing bonus, $E$ earning per game.

$$
\begin{aligned}
P_n &= P_{n-1} + E \\
P_1 &= P_0 + E \\
P_2 &= P_0 + 2E \\
P_n &= P_0 + nE \\
P_0 &= W \\
\therefore\ & W + nE \\
W &= 300000, \quad E = 50000
\end{aligned}
$$

**(ii)** $n = 120$

$$
\begin{aligned}
P_n &= 3 \times 10^5 + 5 \times 10^4 \times 120 \\
P_n &= Rs.63,00,000
\end{aligned}
$$

**(iii)** $P_n = 10,00,000$

$$
\begin{aligned}
10 \times 10^5 &= 3 \times 10^5 + 5 \times 10^4 x \\
x &= 14 grams
\end{aligned}
$$

So, he needs to play 14 games to earn a million rupees.

**23.**

**(i)** $A_0$ = initial amount of petrol, $A_n$ = amount of petrol after $n$ km, $r$ = petrol used for 1 km.

$$
\begin{aligned}
A_n &= A_{n-1} - r \\
A_1 &= A_0 - r \\
A_2 &= A_1 - r = A_0 - 2r \\
\Rightarrow A_n &= A_0 - nr
\end{aligned}
$$

**(iia)** $A_0 = 30,\ r = 0.1 \Rightarrow A_{120} = 30 - 120 \times 0.1 = 18$. Therefore, after 120 km, 18 L petrol will be left.

**(iib)** $A_n = 0 \Rightarrow 30 - 0.1n = 0 \Rightarrow n = 300$. We can drive $300 - 120 = 180$ km before running out of fuel.

**24.**

$C_0$ = No. of chirps per minute at $70^0F$, $C_n$ = No. of chirps per minute at $(70 + n)^0F$, $C$ = Increase in chips per minute after temperature increase.

$$
\begin{aligned}
C_n &= C_{n-1} + C \\
C_1 &= C_0 + C \\
C_2 &= C_1 + C = C_0 + 2C \\
C_3 &= C_2 + C = C_0 + 3C \\
C_n &= C_0 + nC
\end{aligned}
$$

$C_0 = 124, \quad C = 4$

$$
\begin{aligned}
C_n &= 124 + 4n \\
C_n &= 144 \quad (6 \times 24) \\
124 + 4n &= 144
\end{aligned}
$$

$n = 5$
$\therefore$ temperature $= n + 70 = 75^0 F$
So, at temperature $75^0 F$, there are 24 chirps per 10 seconds.

**25.**

Initial Temp = 75 °F, $A_0$ = Length at 75 °C, $A_n$ = Length at $[70 + n]$ °F, $r \equiv$ increase in length per degree in temp.

$$
\begin{aligned}
A_n &= A_{n-1} + r \\
A_1 &= A_0 + r \\
A_2 &= A_1 + r = A_0 + 2r \\
A_3 &= A_2 + r = A_0 + 3r \\
\therefore\ A_n &= A_0 + nr
\end{aligned}
$$

$A_0 = 1000$ m, $\quad r = 0.012$ m, Temperature $T = 105$ °F
$\Rightarrow 105 = (70 + n) \ \Rightarrow\ n = 35$, therefore, $A_{35} = 1000 + 35 \times 0.012 =$100.42 m.

**26.**

$P_0$ = Pressure at sea level, $P_n$ = Pressure of air below $n$ am of sea level, $r$ = rate of pressure increase per am of depth.

$$
\begin{aligned}
P_n &= P_{n-1} + r \\
P_1 &= P_0 + r \\
P_2 &= P_1 + r = P_0 + 2r \\
P_3 &= P_2 + r = P_0 + 3r \\
\therefore P_n &= P_0 + nr
\end{aligned}
$$

$P_0 = 1000 g/cm^2, \quad r = 2.08 g/cm^2$

$P_n = 1000 + 2.08n$

**27.**

**(i)** Let "a" be water wasted per person while shaving, $W_n$ = water wasted until $n$ people have shaved.

$$
\begin{aligned}
W_n &= W_{n-1} + a \\
W_1 &= W_0 + a \\
W_0 &= 0 \quad \text{[As no water wasted when none of them shaved]} \\
W_1 &= a \\
W_2 &= W_1 + a = 2a \\
W_3 &= W_2 + a = 3a \\
\therefore W_n &= n\ a
\end{aligned}
$$

**(ii)** Water wasted during 1 shave = a $\Rightarrow$ water wasted in 5 shaves = 5 a. Water wasted per day in shaving $=\frac{5a}{7}$.

**28.**

$C_n \equiv$ Amount of Cadmium in body at the end of $n$ years, $r \equiv$ removal rate of Cadmium, $C_n = C_{n-1} - rC_{n-1} + C$, $C \equiv$ yearly addition.

$$
\begin{aligned}
C_1 &= C_0 + C \quad [C_0 = 0] \\
C_1 &= C \\
C_2 &= (1-r)C_1 + C = [(1-r)+1]C \\
C_3 &= (1-r)C_2 + C = [1 + (1-r) + (1-r)^2]C
\end{aligned}
$$

$$
\begin{aligned}
C_n &= [1 + (1-r) + (1-r)^2 + ... + (1-r)^{n-1}] \\
C_n &= C\left[\frac{1-(1-r)^n}{1-(1-r)}\right] \\
C_n &= \frac{C}{r}[1-(1-r)^n]
\end{aligned}
$$

$C = 2.7mg, \quad r = 8\%$

$$
\begin{aligned}
C_n &= \frac{2.7}{0.08}[1-(0.92)^n] \\
C_n &= 33.75[1-(0.92)^n] \\
C_{20} &= 33.75[1-(0.92)^{20}] = 27.38 \text{ mg}
\end{aligned}
$$

**29.**

Let $P_n, B_n$ be the amount of lead on the n-th day in plasma and bones.

$$
\begin{aligned}
P_n &= P_{n-1} - 0.35P_{n-1} - 0.09P_{n-1} + 0.4 + 1.18 \times 10^{-3}P_{n-1} \\
B_n &= B_{n-1} + 0.35P_{n-1} - 1.18 \times 10^{-3}B_{n-1}
\end{aligned}
$$

$P_0 = 0, \quad B_0 = 0, \; P_1 = 0.4\text{g}, \; B_1 = 0$

$$
\begin{aligned}
P_2 &= 0.4 + 0.56P_1 = 0.624g \\
B_2 &= 0.35P_1 + (1 - 1.18 \times 10^{-3})B_1 = 0.14g \\
P_3 &= 0.4 + 0.56 \times 0.624 = 0.7496g \\
B_3 &= 0.35P_2 + (1 - 1.18 \times 10^{-3})B_2 = 0.1888g \\
P_4 &= 0.4 + 0.56 \times 0.7496 = 0.82g \\
B_4 &= 0.35 \times 0.7496 + (1 - 1.18 \times 10^{-3}) \times 0.1888 = 0.45g.
\end{aligned}
$$

**30.**

$C_n \equiv$ calorie consumed during nth week, $W_n \equiv$ weight of person after nth week, $C_(n) = 21000 - 200n$.

$$
\begin{aligned}
W_n &= W_{n-1} + \frac{1}{3600}[C_n - 130W_{n-1}] \\
W_n &= \frac{347}{360}W_{n-1} + \frac{C_n}{3600} \\
W_0 &= \text{initial weight} \\
W_1 &= \frac{347}{360}W_0 + \frac{21000-200}{3600} \\
W_2 &= \frac{347}{360}W_1 + \frac{21 \times 10^3 - 200 \times 2}{3600}
\end{aligned}
$$

$$W_n = \left(\frac{347}{360}\right)^n W_0 + \sum_{i=0}^{n} \left(\frac{347}{360}\right)^{n-1} \frac{21000 - 200(n-1)}{3600}$$

$$W_n = \left(\frac{347}{360}\right)^n W_0 + \frac{21000}{3600\left(\frac{130}{3600}\right)} + \frac{200\left(\frac{3470}{3600}\right)\left[1 - \left(\frac{3470}{3600}\right)^{n-1}\right]}{\left(\frac{130}{3600}\right)^2}$$

$$- \frac{\left[\frac{210}{36} + (n-1)200\right]\left(\frac{3470}{3600}\right)^n}{\frac{130}{3600}} \quad \text{(Applying summation of AG series)}$$