

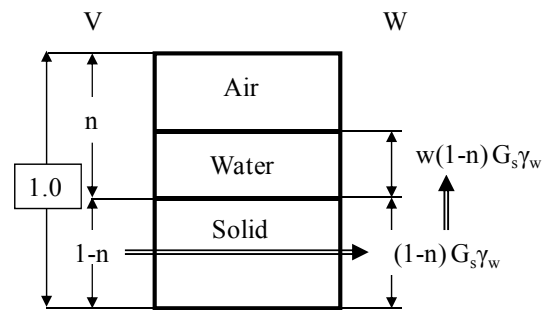
## 2. Physical Properties of Soils

2.1 For a given soil, derive the following relation by drawing the three phase diagram.

$$\gamma_t = G_s \gamma_w (1-n) (1+w)$$

**Solution:**

In three phase diagram, set  $V=1.0$  and find  $W_s$  and  $W_w$  as in the figure and thus,



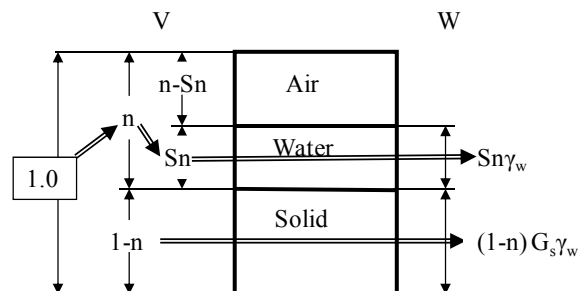
$$\gamma_t = W/V = [w(1-n)G_s\gamma_w + (1-n)G_s\gamma_w]/1.0 = G_s \gamma_w (1-n) (1+w) \text{ **proven**}$$

2.2 For a given soil, derive the following relation by drawing the three phase diagram.

$$\gamma_t = G_s \gamma_w (1-n) + n S \gamma_w$$

**Solution:**

In three phase diagram, set  $V=1.0$  and find  $V_w$ ,  $V_s$ ,  $W_w$ ,  $W_s$  as in the figure and thus,



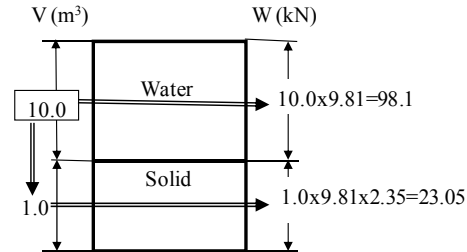
$$\gamma_t = W/V = [Sn\gamma_w + (1-n)G_s\gamma_w]/1.0 = G_s \gamma_w (1-n) + n S \gamma_w \text{ **proven**}$$

2.3 For an organic soil, the void ratio  $e$  is found to be 10.0 and  $G_s$  is 2.35. If this soil is fully saturated, find

- Total unit weight of the soil  $\gamma_t$
- Water content  $w$
- Whether this soil sink in water?

**Solution:**

Set  $V_w = 10.0 \text{ m}^3$  in the three phase diagram and determine  $V_s$ ,  $W_w$ ,  $W_s$  as in the figure.



(a)  $\gamma_t = W/V = (98.1 + 23.05)/(10.0 + 1.0) = \mathbf{11.01 \text{ kN/m}^3}$

(b)  $w = W_w/W_s = 98.1/23.05 = 4.26 = \mathbf{426 \%}$

(c)  $\gamma' = \gamma_t - \gamma_w = 11.01 - 9.81 = 1.2 \text{ kN/m}^3 > 0$ , and thus **it sinks in water**.

**2.4** For a given soil, the void ratio  $e$ , water content  $w$ , and specific gravity  $G_s$  are found to be 0.50, 15 %, and 2.65, respectively. Find

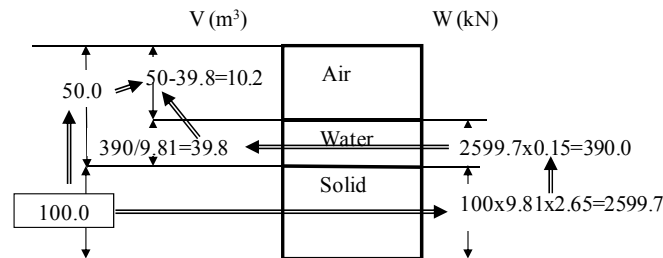
(a) Total unit weight of the soil  $\gamma_t$

(b) Degree of saturation  $S$

(c) Dry unit weight  $\gamma_d$  if the water in the void is removed

**Solution:**

Assume that  $V_s = 100 \text{ m}^3$  and find the rest of components in the three phase diagram as in the figure.



(a)  $\gamma_t = W/V = (390.0 + 2599.7)/(50 + 100) = \mathbf{19.93 \text{ kN/m}^3}$

(b)  $S = V_w/(V_a + V_w) = 39.8/50 = 0.796 = \mathbf{79.6 \%}$

(c)  $\gamma_d = W_s/V = 2599.7/(100 + 50) = \mathbf{17.33 \text{ kN/m}^3}$

**2.5** For a given soil,  $G_s = 2.70$ ,  $\gamma_t = 19.0 \text{ kN/m}^3$ ,  $w = 12.5 \%$  were measured. Determine

(a) Degree of saturation  $S$

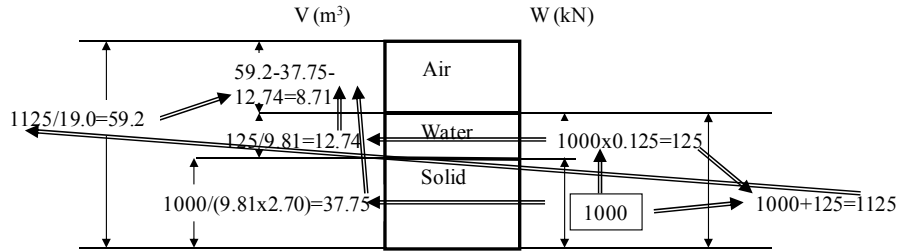
(b) Dry unit weight of the soil  $\gamma_d$

(c) Submerged unit weight of the soil  $\gamma'$  as is

(d) Total unit weight of the soil  $\gamma_t$  if the air void is filled with water

**Solution:**

Assume that  $W_s = 1000 \text{ kN}$  and find the rest of components in the three phase diagram as below.



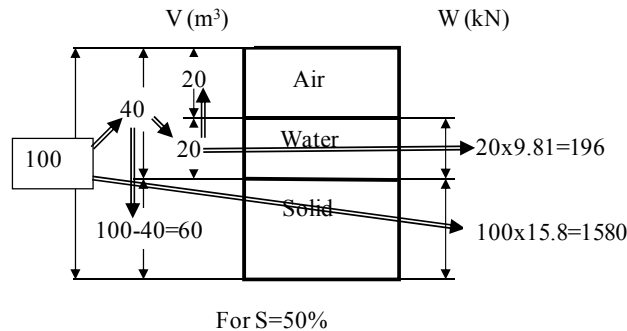
- (a)  $S = V_w / (V_a + V_w) = 12.74 / (8.71 + 12.74) = 0.594 = 59.4 \%$   
 (b)  $\gamma_d = W_s / V = 1000 / 59.2 = 16.89 \text{ kN/m}^3$   
 (c)  $\gamma' = \gamma_t - \gamma_w = 19.0 - 9.81 = 9.19 \text{ kN/m}^3$   
 (d)  $W_{\text{air filled with water}} = V_a \gamma_w = 8.71 \times 9.81 = 85.4 \text{ kN}$  thus,  
 $\gamma_t = W / V = (1000 + 125 + 85.4) / 59.2 = 20.44 \text{ kN/m}^3$

- 2.6** The dry unit weight of a soil is found to be  $15.8 \text{ kN/m}^3$  and its porosity  $n = 0.40$ . Determine  
 (a) The total unit weight of the soil  $\gamma_t$  when the soil's degree of saturation  $S$  is increased to 50 %  
 (b) The total unit weight of the soil  $\gamma_t$  when the soil is fully saturated  
 (c) The specific gravity  $G_s$  of this soil

**Solution:**

Assume  $V=100 \text{ m}^3$  and compute the rest of components in the three phase diagram as below in case of  $S=50 \%$ .

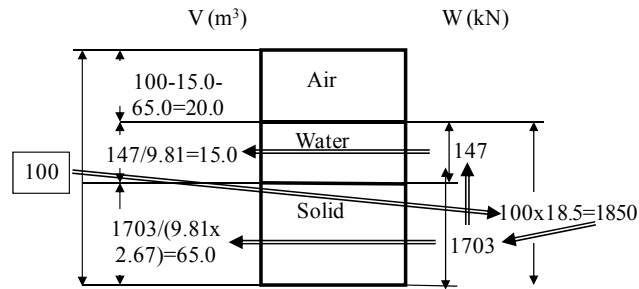
- (a)  $\gamma_t = W / V = (1580 + 196) / 100 = 17.76 \text{ kN/m}^3$   
 (b)  $W_{\text{air filled with water}} = V_a \gamma_w = 20 \times 9.81 = 196 \text{ kN}$  thus,  
 $\gamma_t = W / V = (1580 + 196 + 196) / 100 = 19.72 \text{ kN/m}^3$   
 (c)  $G_s = W_s / (V_s \gamma_w) = 1580 / (60 \times 9.81) = 2.68$



- 2.7** Soil collected from a site is found to have  $\gamma_t=18.5 \text{ kN/m}^3$ ,  $w=8.6 \%$  and  $G_s=2.67$  at a site. After a heavy rainfall overnight, 10 % increase in the degree of saturation  $S$  was observed. Determine  
 (a) The degree of saturation  $S$  of the soil before the rainfall  
 (b) The void ratio  $e$  of the soil before the rainfall  
 (c) The water content  $w$  after 10 % increase in  $S$   
 (d) The total unit weight  $\gamma_t$  after 10 % increase in  $S$

**Solution:**

Assume that  $V=100 \text{ m}^3$  and calculate all other components in the three phase diagram below.



Note that in the above figure, since  $W_s + wW_s = W$  ( $W_s + 0.086W_s = 1850$ ),  $W_s = 1703 \text{ kN}$

(a)  $S = V_w / (V_a + V_w) = 15 / (20 + 15) = 0.429 = \mathbf{42.9 \%}$

(b)  $e = (V_a + V_w) / V_s = (20 + 15) / 65 = \mathbf{0.538}$

(c) After the rainfall, S becomes  $0.429 + 0.1 = 0.529$

Thus,  $V_w = S(V_a + V_w) = 0.529 \times (20 + 15) = 18.5 \text{ m}^3$  and  $V_a = (20 + 15) - 18.5 = 16.5 \text{ m}^3$

$W_w = V_w \gamma_w = 18.5 \times 9.81 = 181.5 \text{ kN}$

Then,  $w = W_w / W_s = 181.5 / 1703 = 0.107 = \mathbf{10.7 \%}$

(d)  $\gamma_t = W / V = (181.5 + 1703) / 100 = \mathbf{18.85 \text{ kN/m}^3}$

**2.8** In a construction site,  $100 \text{ m}^3$  of the volume is excavated.  $\gamma_t$ ,  $G_s$ , and  $w$  of the excavated soil are  $18.5 \text{ kN/m}^3$ ,  $2.68$ , and  $8.2 \%$ , respectively.

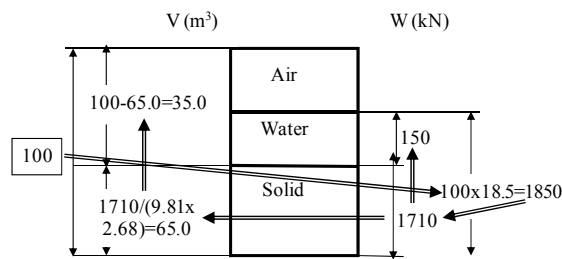
(a) How heavy is the whole excavated soil?

(b) What is the porosity of the soil?

(c) If the excavated soil is dried out to have  $5 \%$  water content at the site, how heavy does it become?

**Solution:**

$V = 100 \text{ m}^3$  is given and determine the other components in the three phase diagram below.



Note that in the above figure, since  $W_s + wW_s = W$  ( $W_s + 0.082W_s = 1850$ ),  $W_s = 1710 \text{ kN}$

(a)  $W = \mathbf{1850 \text{ kN}}$

(b)  $n = V_{\text{air+water}} / W = 35.0 / 100 = \mathbf{0.350}$

(c)  $W_w = wW_s = 0.05 \times 1710 = 85.5 \text{ kN}$ , thus  $W = W_s + W_w = 1710 + 85.5 = \mathbf{1795.5 \text{ kN}}$

2.9 Table below shows a data set from a sieve analysis.

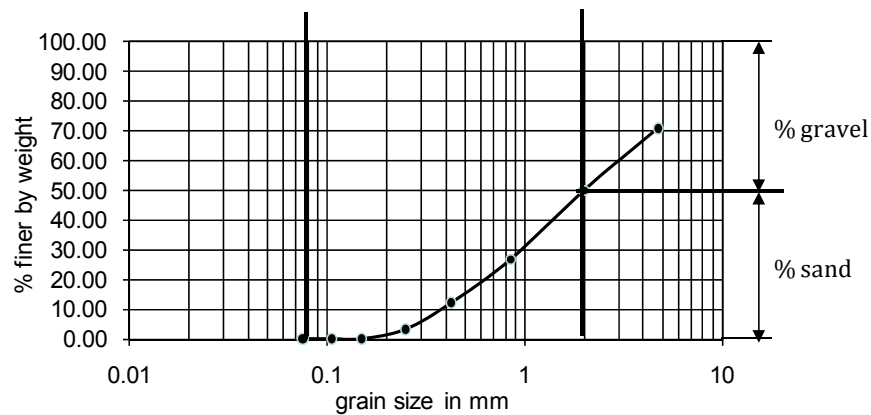
- Complete the rest of the table using a spreadsheet as in Table 2.2
- Plot the grain size distribution curve
- Determine  $D_{10}$ ,  $D_{30}$ ,  $D_{50}$  and  $D_{60}$
- Compute  $C_u$  and  $C_g$
- Report the % gravel, % sand, % silt and % clay according to AASHTO

**Solution:**

(a)

U.S. Sieve No.	D, mm	Weight retained gf	% retained	% cumulative	% Finer
4	4.75	135.9	29.19	29.19	70.81
10	2	97.5	20.94	50.13	49.87
20	0.85	108	23.20	73.32	26.68
40	0.425	67.8	14.56	87.89	12.11
60	0.25	41.4	8.89	96.78	3.22
100	0.15	15	3.22	100.00	0.00
140	0.106	0	0.00	100.00	0.00
200	0.075	0	0.00	100.00	0.00
Pan		0	0.00	100.00	0.00
total		465.6	100		

(b)



From the gradation curve above,

- $D_{10} = 0.38 \text{ mm}$ ,  $D_{30} = 0.97 \text{ mm}$ ,  $D_{50} = 2.0 \text{ mm}$ ,  $D_{60} = 3.0 \text{ mm}$
- $C_u = D_{60}/D_{10} = 3.0/0.38 = 7.89$ ,  $C_g = (D_{30})^2/(D_{60}D_{10}) = 0.97^2/(3.0 \times 0.38) = 0.825$
- % gravel = 50 %, and % sand = 50 %

**2.10** Table below shows a data set from a sieve analysis.

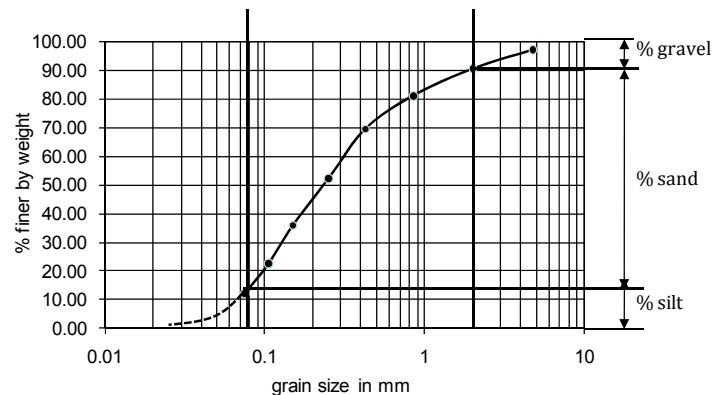
- Complete the rest of the table using a spreadsheet as in Table 2.2
- Plot the grain size distribution curve.
- Determine  $D_{10}$ ,  $D_{30}$ ,  $D_{50}$  and  $D_{60}$ .
- Compute  $C_u$  and  $C_g$ .
- Report the % gravel, % sand, % silt and % clay according to AASHTO.

**Solution:**

(a)

U.S. Sieve No.	D, mm	Weight retained, gf	% retained	% cumulative	% Finer
4	4.75	16.8	2.88	2.88	97.12
10	2	38.4	6.58	9.46	90.54
20	0.85	54.9	9.41	18.87	81.13
40	0.425	67.8	11.62	30.49	69.51
60	0.25	101.7	17.43	47.92	52.08
100	0.15	94.2	16.14	64.06	35.94
140	0.106	77.4	13.26	77.33	22.67
200	0.075	61.8	10.59	87.92	12.08
Pan		70.5	12.08	100.00	0.00
total		583.5	100.00		

(b)



From the gradation curve above,

(c)  $D_{10} = 0.069 \text{ mm}$ ,  $D_{30} = 0.13 \text{ mm}$ ,  $D_{50} = 0.23 \text{ mm}$ ,  $D_{60} = 0.305 \text{ mm}$

(d)  $C_u = D_{60}/D_{10} = 0.305/0.069 = 4.42$ ,  $C_g = (D_{30})^2/(D_{60}D_{10}) = 0.13^2/(0.305 \times 0.069) = 0.803$

(e) % gravel = 10 %, % sand = 77 %, and % silt = 13 %

**2.11** Table below shows a data set from a sieve analysis.

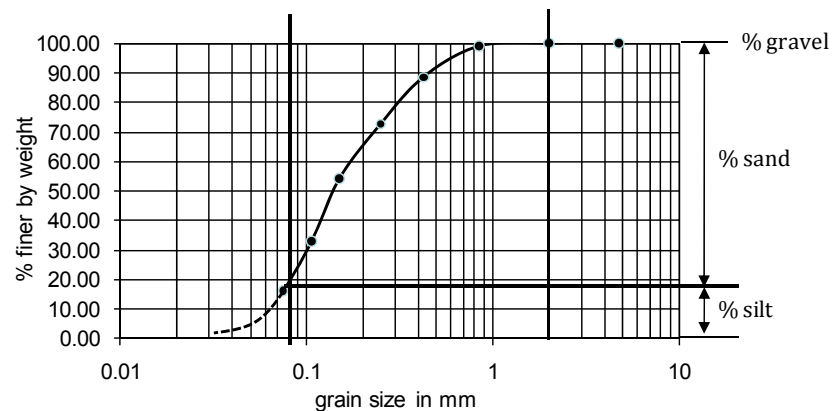
- Complete the rest of the table using a spreadsheet as in Table 2.2
- Plot the grain size distribution curve.
- Determine  $D_{10}$ ,  $D_{30}$ ,  $D_{50}$  and  $D_{60}$ .
- Compute  $C_u$  and  $C_g$ .
- Report the % gravel, % sand, % silt and % clay according to AASHTO.

**Solution:**

(a)

U.S. Sieve No.	D, mm	Weight retained, gf	% retained	% cumulative	% Finer
4	4.75	0	0.00	0.00	100.00
10	2	0	0.00	0.00	100.00
20	0.85	6.9	1.00	1.00	99.00
40	0.425	71.7	10.41	11.41	88.59
60	0.25	109.2	15.85	27.26	72.74
100	0.15	126.9	18.42	45.69	54.31
140	0.106	147.6	21.43	67.12	32.88
200	0.075	115.8	16.81	83.93	16.07
Pan		110.7	16.07	100.00	0.00
total		688.8	100		

(b)



From the gradation curve above,

- $D_{10} = 0.062 \text{ mm}$  (based on an extended curve for  $D < 0.075 \text{ mm}$ )  
 $D_{30} = 0.10 \text{ mm}$ ,  $D_{50} = 0.14 \text{ mm}$ ,  $D_{60} = 0.18 \text{ mm}$
- $C_u = D_{60}/D_{10} = 0.18/0.062 = 2.90$ ,  $C_g = (D_{30})^2/(D_{60}D_{10}) = 0.10^2/(0.18 \times 0.062) = 0.896$
- % gravel = 0 %, % sand = 82 %, and % silt = 18 %

**2.12** Table below shows a data set from a sieve analysis.

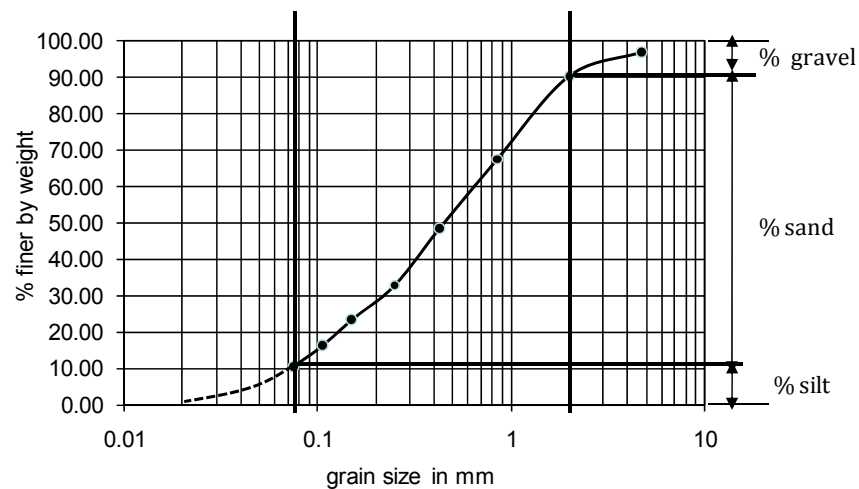
- Complete the rest of the table using a spreadsheet as in Table 2.2
- Plot the grain size distribution curve.
- Determine  $D_{10}$ ,  $D_{30}$ ,  $D_{50}$  and  $D_{60}$ .
- Compute  $C_u$  and  $C_g$ .
- Report the % gravel, % sand, % silt and % clay according to AASHTO.

**Solution:**

(a)

U.S. Sieve No.	D, mm	Weight retained, gf	% retained	% cumulative	% Finer
4	4.75	15.6	2.92	2.92	97.08
10	2	35.4	6.64	9.56	90.44
20	0.85	121.8	22.83	32.40	67.60
40	0.425	102.3	19.18	51.57	48.43
60	0.25	82.8	15.52	67.10	32.90
100	0.15	50.4	9.45	76.55	23.45
140	0.106	37.8	7.09	83.63	16.37
200	0.075	30.6	5.74	89.37	10.63
Pan		56.7	10.63	100.00	0.00
total		533.4	100		

(b)



From the gradation curve above,

- $D_{10} = 0.070 \text{ mm}$  (based on an extended curve for  $D < 0.075 \text{ mm}$ )  
 $D_{30} = 0.205 \text{ mm}$ ,  $D_{50} = 0.42 \text{ mm}$ ,  $D_{60} = 0.63 \text{ mm}$
- $C_u = D_{60}/D_{10} = 0.63/0.070 = 9.0$ ,  $C_g = (D_{30})^2/(D_{60}D_{10}) = 0.205^2/(0.63 \times 0.070) = 0.953$
- % gravel = 10 %, % sand = 78 %, and % silt = 12 %



**2.13** The table below shows the sieve analysis data on the left and a hydrometer test data on the right for the minus #200 sieve material for a given soil.

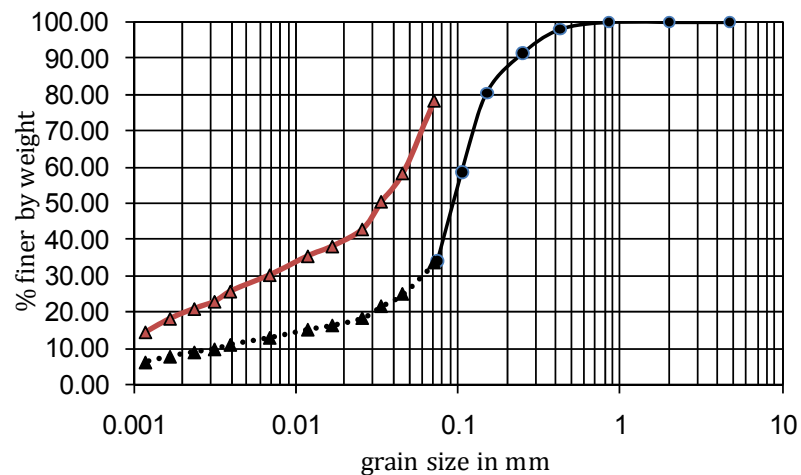
- (a) Plot grain size distribution curves for both tests individually on a graph
- (b) Combine two curves into a combined grain size distribution curve for the soil.

**Solution:**

(a)

Sieve analysis						Hydrometer analysis		
U.S. Sieve No.	D, mm	Weight retained, gf	% retained	% cum	% Finer	D, mm	% finer	combined
4	4.75	0	0.00	0.00	100.00	0.072	78.2	33.5
10	2	0	0.00	0.00	100.00	0.046	58.2	25.0
20	0.85	0	0.00	0.00	100.00	0.034	50.4	21.6
40	0.425	13.5	1.98	1.98	98.02	0.026	42.8	18.4
60	0.25	45.3	6.66	8.64	91.36	0.017	38.1	16.3
100	0.15	75.4	11.08	19.72	80.28	0.012	35.4	15.2
140	0.106	147.6	21.69	41.41	58.59	0.007	30.2	12.9
200	0.075	168.2	24.72	66.13	33.87	0.004	25.7	11.0
Pan		230.5	33.87	100.00	0.00	0.0032	22.9	9.8
total		680.5	100			0.0024	20.9	9.0
						0.0017	18.2	7.8
						0.0012	14.5	6.2

(b)



**2.14** The table below shows the sieve analysis data on the left and a hydrometer test data on the right for the minus #200 sieve material for a given soil.

- (a) Plot grain size distribution curves for both tests individually on a graph
- (b) Combine two curves into a combined grain size distribution curve for the soil.

**Solution:**

(a)

Sieve analysis						Hydrometer analysis		
U.S. Sieve No.	D, mm	Weight retained, gf	% retained	% cum	% Finer	D, mm	% finer	combined
4	4.75	0	0.00	0.00	100.00	0.071	67.8	35.8
10	2	0	0.00	0.00	100.00	0.05	57.2	30.2
20	0.85	11.2	1.43	1.43	98.57	0.03	48.2	25.5
40	0.425	14.5	1.86	3.29	96.71	0.024	43.5	23.0
60	0.25	51.8	6.64	9.93	90.07	0.015	39.2	20.7
100	0.15	81.3	10.42	20.35	79.65	0.011	37.1	19.6
140	0.106	189.3	24.25	44.60	55.40	0.0072	35.2	18.6
200	0.075	152.1	19.49	64.09	35.91	0.0046	31.5	16.6
Pan		280.3	35.91	100.00	0.00	0.0035	30.2	15.9
total		780.5	100			0.0025	29.1	15.4
						0.0016	27.2	14.4
						0.0012	26.2	13.8

(b)

