Chapter 2

Problem 2.1
The equation for droplet spacing in a lattice configuration is

$$\frac{L}{D} = \left(\frac{\pi}{6\alpha_d}\right)^{1/3} = \left(\frac{\pi}{6 \times 0.4}\right)^{1/3} = 1.09$$

Problem 2.2
The data given are $\bar{\rho}_d = 0.5$ kg/m^3, $D = 100\mu$, $\bar{\rho}_c = 1.2$ kg/m^3 and $\rho_d =$ 1000 kg/m^3.
Number density:

$$n = \frac{\bar{\rho}_d}{V_d} = \frac{6\bar{\rho}_d}{\pi\rho_d D^3} = 9.5 \times 10^8 \ /\mathrm{m}^3$$

Disperse phase volume fraction:

$$\alpha_d = \bar{\rho}_d/\rho_d = 0.5/10^3 = 0.5 \times 10^{-3}$$

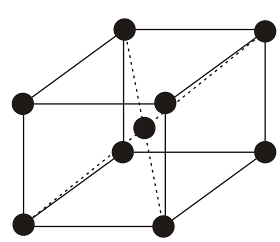
Continuous phase bulk density:

$$\bar{\rho}_c = (1 - \alpha_d)\rho_c = 1.2 \ \mathrm{kg/m}^3$$

Concentration:

$$C = \bar{\rho}_d/\bar{\rho}_c = 0.416$$

Problem 2.3



Consider a cube in which particles each corner and they are all touching a particle at the cube center. The particle locations are illustrated in the diagram. The total distance across the diagonal is $2D$. The length of a side, L, would be related to the distance across the diagonal by

$$3L^2 = (2D)^2$$

$$L = (\frac{4}{3})^{1/2} D$$

There are two whole particles within the volume so the solids volume fraction is

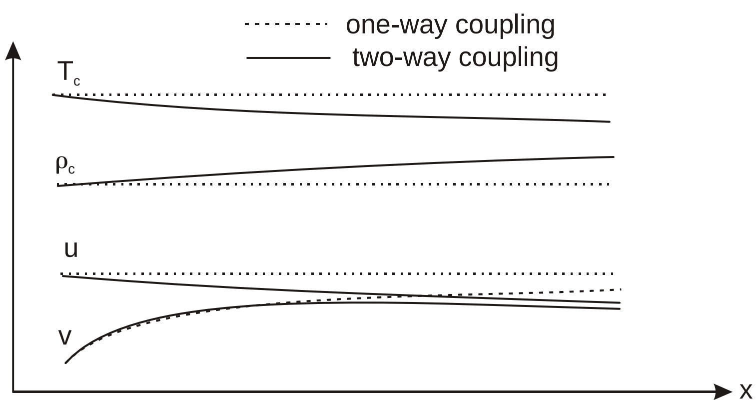
$$\alpha_d = \frac{\pi/3D^3}{L^3} = \frac{\pi}{3}(\frac{3}{4})^{3/2} = 0.680$$

The maximum solids volume fraction for the lattice configuration is $L = D$ and gives

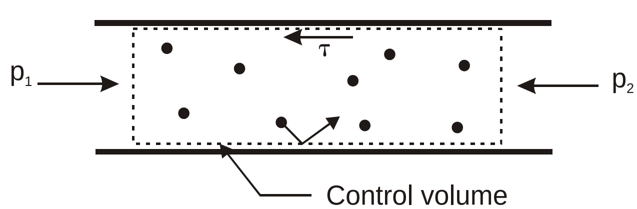
$$\alpha_d = \frac{\pi}{6} = 0.524$$

Problem 2.4

Droplets released in hot stream will cool the carrier gas, increase the density and reduce the velocity to satisfy the continuity equation. The reduced gas velocity will lead to less acceleration of the droplets as they proceed toward velocity equilibrium with the gas.



Problem 2.5



Consider the duct shown in the figure. The control volume is designated by the dashed lines. The momentum equation states that the rate of change of momentum in the control volume plus the net efflux of momentum through the control surface is equal to the forces acting on the control surface. When the particle bounce from the wall they enter the control volume and when they impinge on the wall the exit the control volume. The pressure force is given by $(p_1 - p_2)A$ where A is the cross-sectional area. The force due to skin friction is $-c_f P\Delta L\frac{1}{2}\rho_c u_c^2$. The net efflux of momentum due to the particles is $\dot{m}_d(v - v_0)$ where v_0 is the rebound velocity and $\dot{m}_d$ is the particle flow rate per unit area into the control volume through the control surface adjacent to the wall. The flow is steady and there is no momentum change of the continuous phase. Thus the momentum equation is

$$(p_1 - p_2)A - c_f PL\frac{1}{2}\rho_c u^2 = \dot{m}_d P\Delta L(v - v_0)$$

The value for v_0 is $0.5v$ so the momentum equation can be written as

$$\frac{\Delta p}{\Delta L} = c_f\frac{2}{D}\rho_c u^2 + \frac{\dot{m}_d P\Delta L}{A\Delta L}0.5v$$

The Darcy-Weisbach friction factor is defined as

$$\frac{\Delta p}{\Delta L} = \frac{f}{D}\frac{1}{2}\rho_c u^2$$

so the equation for pressure drop can be expressed as

$$\frac{f}{D}\frac{1}{2}\rho_c u^2 = c_f\frac{2}{D}\rho_c u^2 + \frac{\dot{m}_d P\Delta L}{A\Delta L}0.5v$$

But 10% of the particle mass flow impinges on the wall per diameter length of duct so

$$\dot{m}_d P\Delta L = 0.1\dot{M}_d\frac{\Delta L}{D}$$

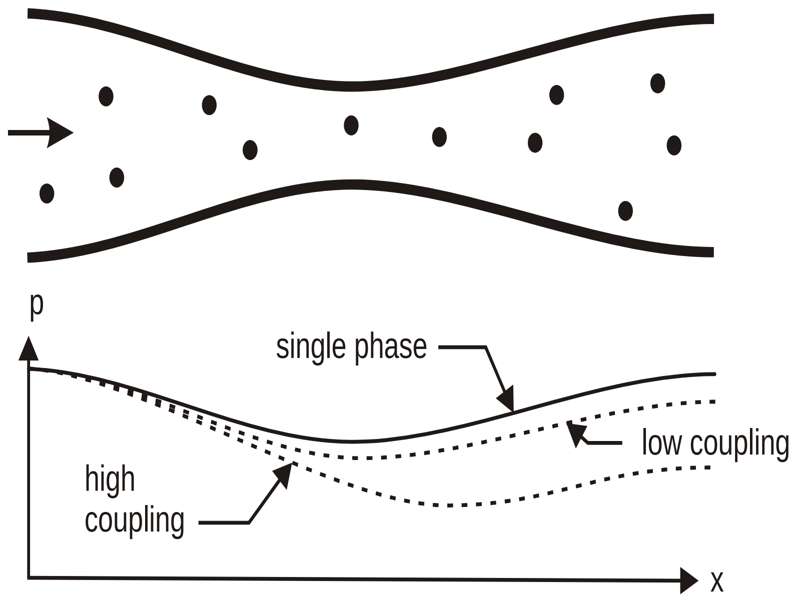
so the equation for the friction factor is

$$f = 4c_f + 0.1\frac{\dot{M}_d}{\rho_c A u_c}\frac{v}{u}$$

For $\alpha_c \sim 1$ one has

$$f = 4c_f + 0.1Z\frac{v}{u}$$

Problem 2.6



With small momentum coupling, the pressure variation varies little from the single phase. However, with a large momentum coupling there is a greater pressure drop since the gas has to accelerate the particles. Also the minimum pressure drop occurs after the throat because the particles are still being accelerated beyond the throat.

Problem 2.7

The data provided are $\dot{m}_c = 0.1$ kg/s, $\dot{m}_d = 0.01$ kg/s, $u_d = 30$ m/s, $D = 100\mu$m, $D_{pipe} = 5$ cm and $\lambda = 0.02$ cm^2/s. For standard conditions, $\rho_c = 1.2$ kg/m^3.

Evaporation time is $D^2/\lambda = 0.005$ s. Carrier phase velocity is $u = \dot{m}_c/\rho_c A = 42.4$ m/s. The mass transfer Stokes number is

$$St_{mass} = \frac{\tau_m u}{L} = 4.24$$

The concentration is

$$C = z\frac{u}{v} = 0.141$$

The mass coupling parameter is

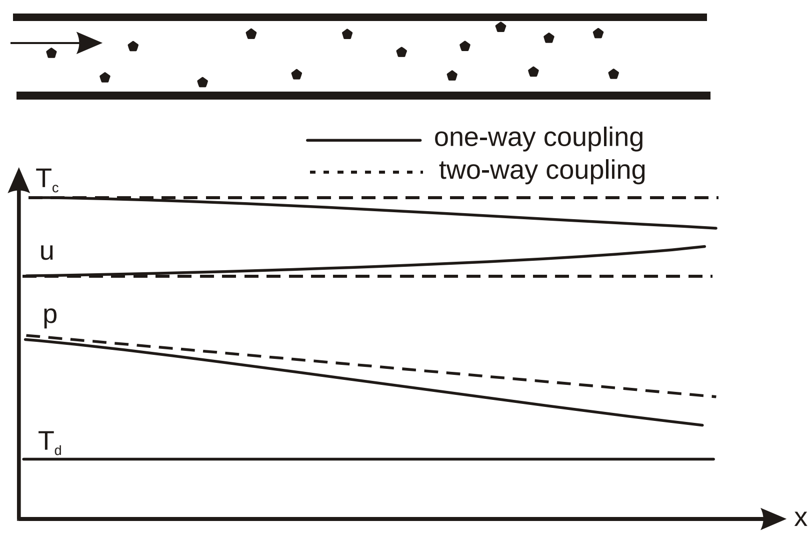
$$\Pi_{mass} = C/St_{mass} = 0.141/4.24 = 0.03$$

The latent heat coupling parameter is

$$\Pi_L = \Pi_{mass} \frac{h_L}{c_c T_c} = 0.278$$

In this case the mass coupling is negligible but the heat transfer due to effect evaporation should be considered although the effect may not be significant.

Problem 2.8



The temperature of the ice particle does not change because of the change in phase. With two-way coupling the gas phase temperature drops due to heat transfer to the ice particles. The density increases due to lower temperature so the velocity increases to satisfy continuity. The pressure decreases more due to the acceleration of the gas.