CHAPTER 2 –Solutions to Homework Problems

1.

Determine what the uniform flow rate will be in a circular channel with a diameter of 2 m, if the bottom slope is 0.0004, and its wall roughness is 0.0003 m, if the depth of flow is 1.3 m. The kinematic viscosity of the water flowing in this pipe is $\nu = 1.519 \times 10^{-6}$ m^2/s.

Given: A circular channel with D = 2 m, e=.0003 m, ν=1.519x10^{-6} m^2/s, S_o=.004, Y=1.3 m

Find: Q & C Using Chezy's Equation
Solution: The Chezy Equation and the Chezy C equation need to be solved simultaneously, or

$$Q=CA(R_hS_o)^{1/2} \quad (1)$$

$$C=-(32g)^{1/2}Log_{10}[e/(12R_h)+.884C/(R_eg^{1/2})] \quad (2)$$

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

One might do this by hand by: (1) guessing C, (2) solve Eq. 1, (3) compute Reynold's Number R_e and solve Eq. 2 (an implicit equation), and (4) repeat steps (2) through (3) until convergence has occurred. Program CHEZYCTC obtains such a solution:

Program CHEZYCTC.FOR

```
      REAL X(7)
      CHARACTER*19 FMT/'(1X,A2,3H = ,F10.4)'/
      CHARACTER*1 V(7)/'m','b','S','Y','e','Q','V'/
1     WRITE(*,*)' Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc'
      READ(*,*) II,IC,VISC
      IF(II.LT.1) STOP
      G=32.2
      IF(II.GT.1) G=9.81
      VISC2=.221*VISC/SQRT(G)
      G32=SQRT(32.*G)
      IF(IC.GT.0) THEN
      I2=2
      V(2)='D'
      ELSE
      I2=1
      V(2)='b'
      ENDIF
      WRITE(*,100)(I,V(I),I=I2,7)
100   FORMAT(' Give No. of Unknown',/(I2,' - ',A2))
      READ(*,*) IU
      IF(IU.GT.5) GO TO 10
      WRITE(*,*)' Give 1 if Q will be given or 2 if V is known'
      READ(*,*) IV
      GO TO 12
10    IV=IU-5
12    WRITE(*,*)' Give values to knowns & GUESS for unknown'
      I3=7
      IF(IV.EQ.1) I3=6
      DO 20 I=I2,I3
      IF(IV.EQ.2 .AND. I.EQ.6) GO TO 20
      WRITE(*,"(A2,' = ',\)") V(I)
      READ(*,*) X(I)
20    CONTINUE
50    M=0
52    XX=X(IU)
      X(IU)=1.005*X(IU)
      DX=X(IU)-XX
55    IF(IC.EQ.1) THEN
      COSB=1.-2.*X(4)/X(2)
      BETA=ACOS(COSB)
      A=.25*X(2)**2*(BETA-COSB*SIN(BETA))
      P=X(2)*BETA
      ELSE
      A=(X(2)+X(1)*X(4))*X(4)
```

```
      P=X(2)+2.*X(4)*SQRT(X(1)**2+1.)
      ENDIF
      ADL=G32*ALOG10(X(5)*P/(12.*A)+VISC2*(P/A)**1.5/SQRT(X(3)))
      IF(IV.EQ.1) THEN
      F=X(6)*SQRT(P/(A*X(3)))/A+ADL
      ELSE
      F=X(7)*SQRT(P/(A*X(3)))+ADL
      ENDIF
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 60
      X(IU)=XX
      F1=F
      GO TO 55
60    DIF=DX*F/(F1-F)
      X(IU)=XX-DIF
      IF(ABS(DIF).GT. .00001 .AND. M.LT.30) GO TO 52
      IF(IV.EQ.1) THEN
      X(7)=X(6)/A
      ELSE
      X(6)=A*X(7)
      ENDIF
      DO 70 I=1,7
      FMT(18:18)='3'
      IF(I.EQ.3 .OR. I.EQ.5) FMT(18:18)='6'
70    WRITE(*,FMT) V(I),X(I)
      WRITE(*,FMT) 'C',X(7)*SQRT(P/(A*X(3)))
      GO TO 1
      END
```

Input to CHEZYCTC:

1 1 1.41E-5
D= 5
S= .0004
Y= 1.3
e= .0003
Q= 2.5
v= .000001519

Output:

Q= 2.474 m^3/s
C= 75.384
A= 2.16 m^2
P=3.75 m
R_e=1,736,360

A TK-Solver Model (PRB2 1.TK taken from CHEZYSC.TK)

```
================ VARIABLE SHEET
St Input----- Name---- Output---- Unit------
              Q        2.4741131
              C        75.383706
   9.81       g
   .0003      e
              Rh       .57629524
              Re       1736909.7
              A        2.1616707
   .0004      S
   1.3        Y
   1.519E-6   v
   2          D
              beta     1.875489

===================== RULE SHEET ===================
S Rule------------------------------------
* C+sqrt(32*g)*log(e/12/Rh+.884*C/Re/sqrt(g))=0
* Q-A*C*sqrt(Rh*S)=0
* A=D*D/4*(beta-cos(beta)*sin(beta))
* Rh=A/(beta*D)
* Re=4*(Q/A)*Rh/v
* cos(beta)=1-2*Y/D
```

The MathCAD model given below might also be used to solve Problems such as this one using Chezy's equation. The three equations that need to be solved simultaneously for the three variables C, R_e and Q are: (1) The equation that a gives C for the transitional zone, (2) The definition of Reynolds Number and (3) Chezy's equation

PRB2_1.MCD

$g := 9.81 \quad So := .0004 \quad Y := 1.3 \quad v := 1.519 \cdot 10^{-6} \quad D := 2 \quad e := .0003$

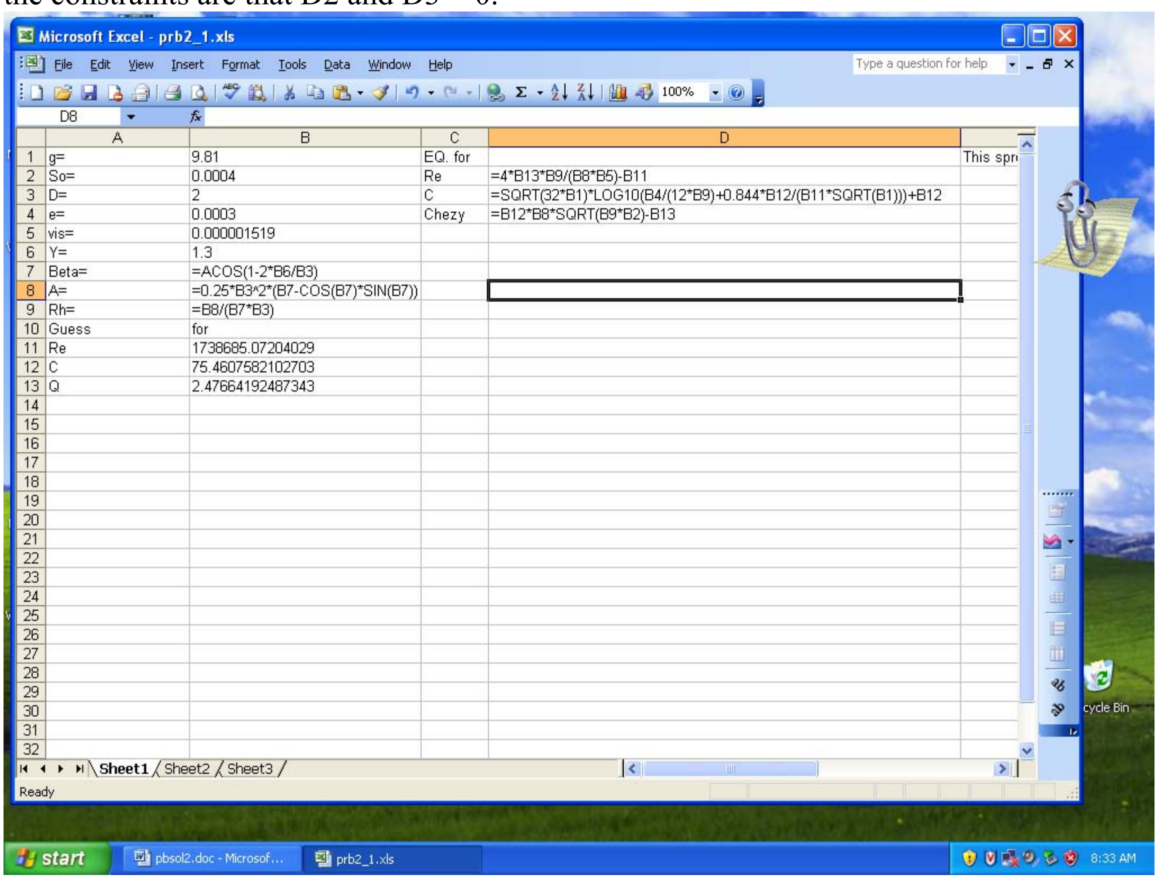
$Q := 2 \quad \beta := \mathrm{acos}\left(1 - 2\cdot\frac{Y}{D}\right) \quad A := .25\cdot D^2\cdot(\beta - \cos(\beta)\cdot\sin(\beta)) \quad Rh := \frac{A}{\beta\cdot D} \quad C := 75 \quad Re := 10^5$

Given

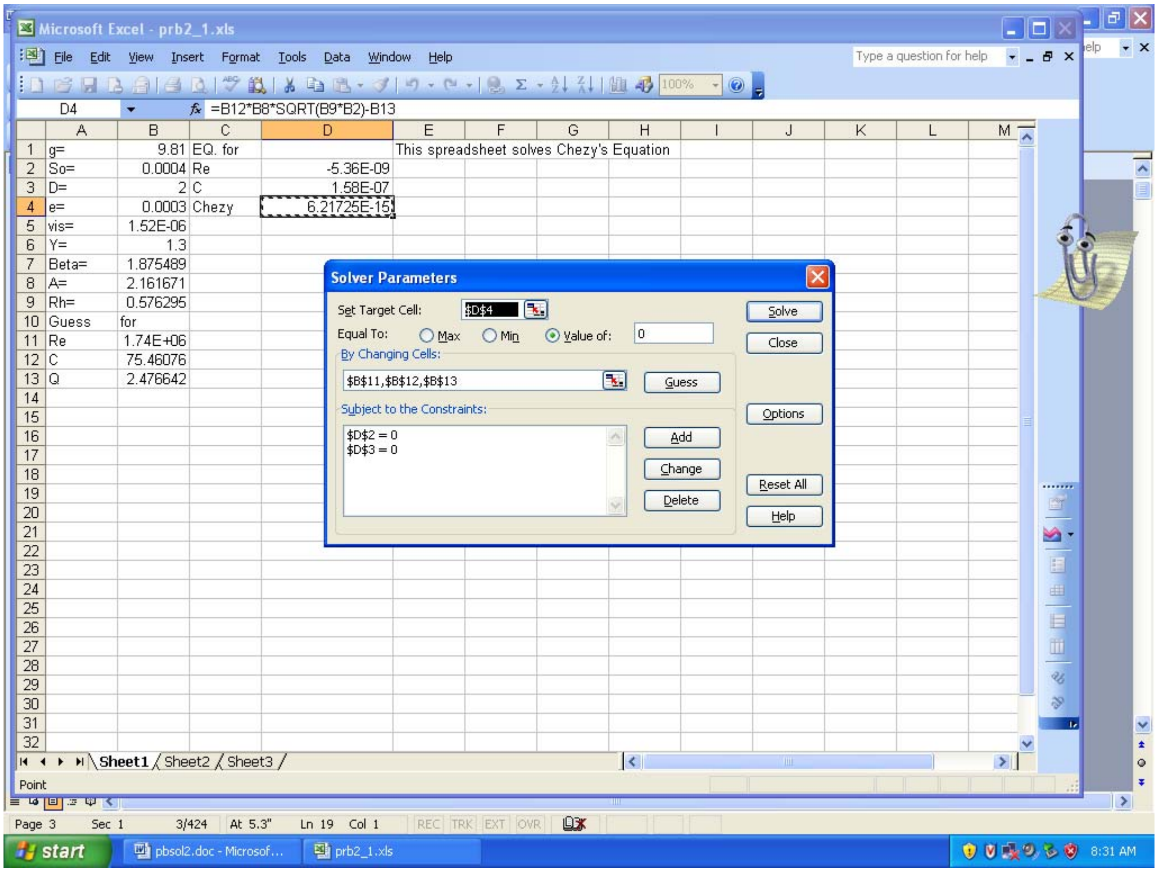
$$C + \sqrt{32\cdot g}\cdot\log\left(\frac{e}{12\cdot Rh} + \frac{.884\cdot C}{Re\cdot\sqrt{g}}\right) = 0 \qquad Re = \frac{4\cdot Q\cdot Rh}{A\cdot v} \qquad Q = A\cdot C\cdot\sqrt{Rh\cdot So}$$

$$\mathrm{Find}(C, Re, Q) = \begin{pmatrix} 75.384 \\ 1.737\cdot 10^{6} \\ 2.474 \end{pmatrix}$$

Problems based on Chezy equation might also be solved using spreadsheets such as EXCEL as shown below. With EXCEL the three equations for the three unknown are solved using the add-in SOLVER. In the spread sheet below PRB2_1.XLS Column B contains the variables as defined in Column A. The equations are given in Column D as defined in Column C. In using SOLVER the target cell D4 is to set to a value of 0 and the constraints are that D2 and D3 = 0.



	A	B	C	D	
1	g=	9.81	EQ. for		This spr
2	So=	0.0004	Re	=4*B13*B9/(B8*B5)-B11	
3	D=	2	C	=SQRT(32*B1)*LOG10(B4/(12*B9)+0.844*B12/(B11*SQRT(B1)))+B12	
4	e=	0.0003	Chezy	=B12*B8*SQRT(B9*B2)-B13	
5	vis=	0.000001519			
6	Y=	1.3			
7	Beta=	=ACOS(1-2*B6/B3)			
8	A=	=0.25*B3^2*(B7-COS(B7)*SIN(B7))			
9	Rh=	=B8/(B7*B3)			
10	Guess	for			
11	Re	1738685.07204029			
12	C	75.4607582102703			
13	Q	2.47664192487343			



g=	9.81	EQ. for	
So=	0.0004	Re	-5.36E-09
D=	2	C	1.58E-07
e=	0.0003	Chezy	6.21725E-15
vis=	1.52E-06		
Y=	1.3		
Beta=	1.875489		
A=	2.161671		
Rh=	0.576295		
Guess	for		
Re	1.74E+06		
C	75.46076		
Q	2.476642		

Notice that the equations in Column D are written so they will equal zero when the correct value of the variable R_e, C and Q are obtained at the end of column B. The equations are shown in the first EXCEL image above.

Using the MATLAB script chezy1.m to solve this problem produces the following in the command window: (See Problem 4 for listing of MATLAB script & function)

```
>> chezy1
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 2
Give k. viscosity 1.519e-6
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
D = 5
```

```
 Number = 2
Give value to known variables and guess for unknown
e = .0003
S = .0004
Y = 1.3
D = 2
Provide guess for unknown Q = 2.5
Unknown Q = 2.4741, C = 75.3837
e = 0.0003
Q = 2.4741
S = 0.0004
Y = 1.3
D = 2
```

2.

What depth of flow would be expected in a 5 ft diameter channel that has a bottom slope of 0.004, if the wall roughness is 0.0005 ft and 200 cfs is flowing in this channel under uniform conditions. ($v = 1.41X10^{-5}$ ft^2/sec)

Given: A circular channel with
D = 5 ft
S_o = .004
e = .0005 ft
Q = 200 cfs
$\nu = 1.41x10^{-5}$ ft^2/s

Find: Y and C

Solution: The Chezy Equation and the Chezy C equation need to be solved simultaneously, or

$$Q=CA(R_hS_o)^{1/2} \qquad (1)$$

$$C=-(32g)^{1/2}Log_{10}[e/(12R_h)+.884C/(R_eg^{1/2})] \qquad (2)$$

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

Using Program CHEZYCTS (given in previous problem)
Y = 4.256 ft ←
C = 144.225 ←

TK-Solver Model PRB2 2.TK

VARIABLE SHEET

St	Input	Name	Output	Unit
	200	Q		
		C	144.22489	
	32.2	g		
	.0005	e		
		Rh	1.5160132	
		Re	4830205.2	
		A	17.807718	
	.004	S		

```
             Y       4.2555064
   .0000141  v
   5         D
             beta    2.3492827

═══════════════ RULE SHEET ═════════════════
S Rule──────────────────────────────────────────
C+sqrt(32*g)*log(e/12/Rh+.884*C/Re/sqrt(g))=0
* Q-A*C*sqrt(Rh*S)=0
* A=D*D/4*(beta-cos(beta)*sin(beta))
* Rh=A/(beta*D)
* Re=4*(Q/A)*Rh/v
* cos(beta)=1-2*Y/D
```

When using MathCAD to solve Chezys Equation for the depth, as asked for in this problem, it is necessary to add the equations for the area (which means also the equation for β), and the equation giving the hydraulic radius to the list of equations being solved simultaneously for Y, C, R_e, A, R_h and β. Thus 6 equations are solved for 6 unknowns, as shown in the model below.

PRB2_2.MCD

$g := 32.2 \quad So := .004 \quad Y := 4 \quad v := 1.41\cdot10^{-5} \quad D := 5 \quad e := .0005 \quad C := 75 \quad Re := 5\cdot10^{6} \quad Q := 200$

$A := 20 \quad Rh := 1.5 \quad \beta := 2.5$

Given

$$C + \sqrt{32\cdot g}\cdot\log\left(\frac{e}{12\cdot Rh} + \frac{.884\cdot C}{Re\cdot\sqrt{g}}\right) = 0 \qquad Re = \frac{4\cdot Q\cdot Rh}{A\cdot v} \qquad Q = A\cdot C\cdot\sqrt{Rh\cdot So} \qquad \beta = \mathrm{acos}\left(1 - 2\cdot\frac{Y}{D}\right)$$

$$A = .25\cdot D^2\cdot(\beta - \cos(\beta)\cdot\sin(\beta)) \qquad Rh = \frac{A}{\beta\cdot D}$$

$$\mathrm{Find}(Y, C, Re, A, Rh, \beta) = \begin{bmatrix} 4.256 \\ 144.225 \\ 4.83\cdot10^{6} \\ 17.808 \\ 1.516 \\ 2.349 \end{bmatrix}$$

If the equivalent sand roughness is reduced by an order of magnitude to e = 0.00005 m, the the solution is as given below.

$g := 32.2 \quad So := .004 \quad Y := 4 \quad v := 1.41\cdot10^{-5} \quad D := 5 \quad e := .00005 \quad C := 75 \quad Re := 5\cdot10^{6} \quad Q := 200$

$A := 20 \quad Rh := 1.5 \quad \beta := 2.5$

Given

$$C + \sqrt{32\cdot g}\cdot\log\left(\frac{e}{12\cdot Rh} + \frac{.884\cdot C}{Re\cdot\sqrt{g}}\right) = 0 \qquad Re = \frac{4\cdot Q\cdot Rh}{A\cdot v} \qquad Q = A\cdot C\cdot\sqrt{Rh\cdot So}$$

$$\beta = \mathrm{acos}\left(1 - 2\cdot\frac{Y}{D}\right) \qquad A = .25\cdot D^2\cdot(\beta - \cos(\beta)\cdot\sin(\beta)) \qquad Rh = \frac{A}{\beta\cdot D}$$

$$\text{Find}(Y, C, Re, A, Rh, \beta) = \begin{bmatrix} 3.718 \\ 164.573 \\ 5.456 \cdot 10^{6} \\ 15.659 \\ 1.506 \\ 2.08 \end{bmatrix}$$

Using the script "chezy2.m" in the MATLAB folder for Chapter 2 produces the following in the dialog window:

```
>> chezy2
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 1
Give k. viscosity 1.41e-5
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
D = 5
 Number = 4
Give value to known variables and guess for unknown
e = .0005
Q = 200
S = .004
Y = 4
D = 5
Unknown Y = 4.2555, C = 144.2249
e = 0.0005
Q = 200
S = 0.004
Y = 4.2555
D = 5
```

3.

What is the depth of flow in problem # 2 if the flow rate is 100 cfs?

Given: Same as Problem 2 except Q=100 cfs.
Solution:
Input to Program CHEZYCTC

```
1 1 1.41E-5
4
D= 5
S= .004
Y= 2.5
e= .0005
Q= 100
```

Output: Y=2.529 ft ←, C=141.44 ←

TK-Solver Model CHEZYSC.TK

```
================ VARIABLE SHEET ================
St Input----- Name---- Output---- Unit------ Comment
              C        141.44185
   32.2       g
   .0005      e
              Rh       1.2591442
              Re       3585601.4
   100        Q
              A        9.962179
   .004       S
              Y        2.528941
   .0000141   v
   5          D
              beta     1.582373

================ RULE SHEET ================
S Rule-----------------------------------------
C+sqrt(32*g)*log(e/12/Rh+.884*C/Re/sqrt(g))=0
  Q-A*C*sqrt(Rh*S)=0
  A=D*D/4*(beta-cos(beta)*sin(beta))
  Rh=A/(beta*D)
  Re=4*(Q/A)*Rh/v
  cos(beta)=1-2*Y/D
```

To use the MathCAD model of the previous problem one only need to change the value assigned to the flow rate Q as shown below.

$g := 32.2$ $So := .004$ $Y := 4$ $v := 1.41\cdot 10^{-5}$ $D := 5$ $e := .0005$ $C := 75$ $Re := 5\cdot 10^{6}$ $Q := 100$

$A := 20$ $Rh := 1.5$ $\beta := 2.5$

Given

$$C + \sqrt{32\cdot g}\cdot \log\left(\frac{e}{12\cdot Rh} + \frac{.884\cdot C}{Re\cdot\sqrt{g}}\right) = 0 \qquad Re = \frac{4\cdot Q\cdot Rh}{A\cdot v} \qquad Q = A\cdot C\cdot\sqrt{Rh\cdot So} \qquad \beta = \operatorname{acos}\left(1 - 2\cdot\frac{Y}{D}\right)$$

$$A = .25\cdot D^2\cdot(\beta - \cos(\beta)\cdot\sin(\beta)) \qquad Rh = \frac{A}{\beta\cdot D}$$

$$\mathrm{Find}(Y, C, Re, A, Rh, \beta) = \begin{bmatrix} 2.529 \\ 141.442 \\ 3.586\cdot 10^{6} \\ 9.962 \\ 1.259 \\ 1.582 \end{bmatrix}$$

Using the MATLAB script chezy2 produces the following in the dialog window:

```
>> chezy2
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 1
Give k. viscosity 1.41e-5
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
D = 5
 Number = 4
Give value to known variables and guess for unknown
e = .0005
Q = 100
S = .004
Y = 2
D = 5
Unknown Y = 2.5289, C = 141.4418
e = 0.0005
Q = 100
S = 0.004
Y = 2.5289
D = 5
```

4.

Determine the depth of flow that would occur in a trapezoidal channel with b = 10 ft, m = 1.5, S_o = 0.0008 if it is carrying 400 cfs. The wall roughness for this channel is e = 0.001 ft. What is Chezy's C? What is the corresponding value of Manning's n? (ν = 1.41X10^{-5} ft^2/sec)

Given: b=10 ft, m=1.5 S_o=.0008, Q = 400 cfs, e=.001 ft, ν=1.41x10^{-5} ft^2/s

Wanted: Y, C and n (for Mannings Equation)

Solution:

A=62.0 ft^2 V=400/62 = 6.452 ft/sec ←
P=24.09 ft
R_h=2.57 C=142.17 ←
R_e=4,710,060 Y=3.909 ft ←

Mannings Equation

$n=1.486A^{5/3}S_o^{1/2}/(QP^{2/3}) = .01224$ ←

A MathCAD model designed to solve for the depth in a trapezoidal channel with the flow rate and other needed variables given is shown below

PRB2_4.MCD

g := 32.2 So := .0008 Y := 4 v := $1.41\cdot10^{-5}$ b := 10 m := 1.5 e := .001

C := 100 Re := $5\cdot10^6$ Q := 400 A := 50 Rh := 1.5

Given

$$C+\sqrt{32\cdot g}\cdot\log\left(\frac{e}{12\cdot Rh}+\frac{.884\cdot C}{Re\cdot\sqrt{g}}\right)=0 \qquad Re=\frac{4\cdot Q\cdot Rh}{A\cdot v} \qquad Q=A\cdot C\cdot\sqrt{Rh\cdot So} \qquad Rh=\frac{A}{b+2\cdot Y\cdot\sqrt{m^2+1}}$$

$$A=(b+m\cdot Y)\cdot Y$$

$$\text{Find}(Y,C,Re,A,Rh)=\begin{bmatrix}3.908\\142.226\\4.71\cdot10^6\\61.988\\2.573\end{bmatrix}$$

The following TK-Solver model also solves this problem.

CHEZYS.TK

```
════════════════════ VARIABLE SHEET ═════════════════════
St Input──── Name─── Output─── Unit───── Comment─────────
              C       142.46819
   32.2       g
   .001       e
              Rh      2.5711847
              Re      5492087.5
   400        Q
              A       61.905671
   .0008      S
   10         b
   1.5        m
              Y       3.9041772
   .0000121   v
════════════════════ RULE SHEET ════════════════════════
S Rule─────────────────────────────────────────────────
C+sqrt(32*g)*log(e/12/Rh+.884*C/Re/sqrt(g))=0
```

```
Q-A*C*sqrt(Rh*S)=0
A=(b+m*Y)*Y
Rh=A/(b+2*Y*sqrt(m*m+1))
Re=4*(Q/A)*Rh/v
```

The scipt chezy1.m listed below and given in the MATLAB folder (Subfolder CHAPTER2) is used below to solve the problem.

SCRIPT chezy1.m

```
% This script solve the Chezy Equation with the Chezy C equation
% simultaneously.
UNT='eQSYbme'; UNC='eQSYD';
IT=input('Give:1=trap.,2=cir. for channel type ');
KU=input('Give:1=ES or 2=SI units ');  C=60;  g=9.81;
vis=input('Give k. viscosity ');
if KU==1
    C=120;  g=32.2
end
if IT==1
  UN=UNT; I2=6;
else
    UN=UNC; I2=5;
end
disp('Give the number for the unknown:');
for i=1:I2
      disp([UN(i) ' = ' num2str(i)]);
 end
 IU=input(' Number = ');
 UNA=UN(IU:IU);  UNN=UN;
 UN(IU:IU)=[];
 disp('Give value to known variables and guess for unknown');
 j=0;
 for i=1:I2-1
      j=j+1;
      if i==IU
          j=j+1;
      end
       X(j)=input([UN(i) ' = ']);
 end
  e12=X(1)/12; s32g=sqrt(32*g); g884=0.884*vis/sqrt(g)/4;
  XU(1)=input([' Provide guess for unknown ' UNA ' = ']); XU(2)=C;
  opt=optimset('display','off');
  XX=fsolve('fchezy',XU,opt,IT,IU,e12,s32g,g884,X);
  X(IU)=XX(1);
  disp(['Unknown ' UNN(IU) ' = ' num2str(X(IU)) ', C = '
num2str(XX(2))]);
  for i=1:I2
        disp([UNN(i) ' = ' num2str(X(i))]);
   end
```

function fchezy.m

```
function FF=fchezy(XU,IT,IU,e12,s32g,g884,X)
X(IU)=XU(1); C=XU(2);
if IT==1
         A=(X(5)+X(6)*X(4))*X(4);
         P=X(5)+2*sqrt(X(6)^2+1)*X(4);
     else
         cosb=1-2*X(4)/X(5); beta=acos(cosb);
         A=X(5)^2/4*(beta-cosb*sin(beta));
```

```
        P=beta*X(5);
    end
     FF(1)=X(2)-C*A*sqrt(A/P*X(3));
     FF(2)=C+s32g*log10(X(1)*P/(12.*A)+g884*C*P/X(2));
```

Command Window dialog:

```
>> chezy1
Give:1=trap.,2=cir. for channel type 1
Give:1=ES or 2=SI units 1
Give k. viscosity 1.41e-5

Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
b = 5
m = 6
 Number = 4
Give value to known variables and guess for unknown
e = .001
Q = 400
S = .0008
b = 10
m = 1.5
 Provide guess for unknown Y = 5
Unknown Y = 3.9079, C = 142.2263
e = 0.001
Q = 400
S = 0.0008
Y = 3.9079
b = 10
m = 1.5
```

5.

What size pipe should be used to carry 6 m^3/s down a mild slope of .0015 if the pipe is not to flow more than 3/4 full. (e = .0015 m and temperature = 20°C)

Given: $Q=6\ m^3/s$, $S_o=.0015$, $Y/D=3/4=.75$, $e=.0015\ m$, $T=20°\ C$ $\nu=1.004\times10^{-6}\ m^2/s$

Solution:

$\beta=\cos^{-1}(1-1.5) = 2.094395$

$R_e = 4R_hV/\nu = 1201940.57DV = [1201940.57(4)Q]/[D(\beta-\cos\beta\sin\beta)]$

$R_h=A/P = [D^2/4(\beta-\cos\beta\sin\beta)]/(P\beta) = .301687D$

$$Q=CA(R_hS_o)^{1/2} \qquad (1)$$

$$C=-(32g)^{1/2}\mathrm{Log}_{10}[e/(12R_h)+.884C/(R_eg^{1/2})] \qquad (2)$$

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

$C=-17.717788\ \mathrm{Log}(.0041434/D+2.47285C\times10^{-8}DC)$

$A=D^2/4(\beta-\cos\beta\sin\beta) = .631852D^2$

$6=C(.631852D^2).021274D^{1/2}$

$C=446.3874/D^{5/2}$

$$446.3874/D^{5/2} = -17.717788\mathrm{Log}(.00041434/D+1.103852\times10^{-5}/D^{3/2})$$

$D=2.15218$ m ←, $C=65.69$, $R_h=.649285$ m, $A = 2.92666\ m^2$

The following TK-Solver model is designed to solve problems in which the depth is ¾ of the Pipe diameter. (Note this model is the same as CHEZYSC.TK with an added Rule that specifies that Y=.75D.)

CHEZYCR.TK

VARIABLE SHEET

St	Input	Name	Output	Unit	Comment
		C	117.85877		
	32.2	g			
	.004	e			
		Rh	1.6288628		
		Re	2198009.4		
	80	Q			
		A	16.818445		
	.001	S			
		Y	3.4485566		
	.0000141	v			
	6	D			
		beta	1.720878		

VARIABLE SHEET

St	Input	Name	Output	Unit	Comment
		C	65.692537		
	9.81	g			
	.0015	e			
		Rh	.64928456		
		Re	5308518.5		
	6	Q			
		A	2.9266585		
	.0015	S			

```
            Y       1.6141341
   1.003E-6  v
            D       2.1521788
            beta    2.0943951
```

One can eventually come close to the answer by using a program that solves for any of the variables in Chezy equation, as used for the previous problem by repeatedly providing a value of Y that is ¾ of the previously solved for diameter. Using the MATLAB script "chezy2.m" for this type of solution gives the following, after a few tries.

```
>> chezy2
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 2
Give k. viscosity 1.004e-6
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
D = 5
 Number = 5
Give value to known variables and guess for unknown
e = .0015
Q = 6
S = .0015
Y = 1.625
D = 2.
Unknown D = 2.1411, C = 65.6697
e = 0.0015
Q = 6
S = 0.0015
Y = 1.625
D = 2.1411
>> chezy2
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 2
Give k. viscosity 1.004e-6
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
D = 5
 Number = 5
Give value to known variables and guess for unknown
e = .0015
Q = 6
S = .0015
Y = 1.61
D = 2
Unknown D = 2.1566, C = 65.7008
e = 0.0015
Q = 6
S = 0.0015
Y = 1.61
D = 2.1566     (.75)2.1566 = 1.6175 which is close to the 1.61 specified.
```

6.

A 6 ft circular conduit has a bottom slope $S_o = 0.001$, and a wall roughness of e = 0.004 ft. What will the depth be for a flow rate of 80 cfs. (temperature = 50°F) Using this depth compute the corresponding value for Manning's coefficient.

<u>Given:</u> D=6 ft, S_o=.001, e=.004 ft, Q=80 cfs, $\nu=1.407\times10^{-5}$ ft²/sec (T=50°F)

<u>Solving:</u>

$Q=CA(R_hS_o)^{1/2}$ (1)

$C=-(32g)^{1/2}Log_{10}[e/(12R_h)+.884C/(R_eg^{1/2})]$ (2)

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

Gives: Y=3.449 ft ← Equivalent n = .0137 ←

The following TK-Solver Model, which is CHEZYSC.TK with variables changed for this problem.

```
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit----- Comment---------
             C       117.85877
   32.2      g
   .004      e
             Rh      1.6288628
             Re      2198009.4
   80        Q
             A       16.818445
   .001      S
             Y       3.4485566
   .0000141  v
   6         D
             beta    1.720878
==================== RULE SHEET ========================
S Rule------------------------------------------------
  C+sqrt(32*g)*log(e/12/Rh+.884*C/Re/sqrt(g))=0
  Q-A*C*sqrt(Rh*S)=0
  A=D*D/4*(beta-cos(beta)*sin(beta))
  Rh=A/(beta*D)
  Re=4*(Q/A)*Rh/v
  cos(beta)=1-2*Y/D
```

<u>MATLAB SOLUTION</u>

Using the script "chezy1.m" that solves Chezy's equation (and the Chezy C equation) results in the following solution in the command window.

```
>> chezy1
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 1
Give k. viscosity 1.407e-5
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
D = 5
 Number = 4
Give value to known variables and guess for unknown
e = .004
Q = 80
S = .001
D = 6
 Provide guess for unknown Y = 3
Unknown Y = 3.4485, C = 117.8599
e = 0.004
Q = 80
S = 0.001
Y = 3.4485
D = 6
```

7.

Use Chezy's formula to fill in the blank in the table below that applies for uniform flow in channels of trapezoidal shapes. ($v = 1.41X10^{-5}$ ft^2/sec)

Flow rate Q(cfs)	Rough. e(ft)	Bot. Slope S_o	Bot. Width b(ft)	Side Slope m	Depth Y(ft)
______	.005	0.0006	6.0	1.6	4.2
324.5	_____	0.0008	8.0	0.0	5.0
550.0	.003	______	10.0	1.3	4.8
400.0	.004	0.0015	____	1.5	4.0
450.0	.004	0.0009	7.0	_____	4.1
600.0	.005	0.00075	10.0	1.2	_____

Solve for blanks in table

Q(cfs)	e (ft)	S_o	b(ft)	m	Y (ft)	C
246.47	.005	.0006	6.0	1.6	4.2	120.497
234.5	.001087	.0008	8.0	0.0	5.0	139.041
550	.003	.000965	10.0	1.3	4.8	130.514
400	.004	.0015	7.33	1.5	4.0	123.74
450	.004	.0009	7.0	2.956	4.1	123.366
600	.005	.00075	10.0	1.2	5.637	125.300

The MathCAD model below is designed to solve Chezy's equation for any of the variables in a trapezoidal channel. Since the Reynolds Number, the Area, and the Hydraulic radius are unknown in addition to C and the unknown variable, the model solves 5 equations simultaneously for 5 unknowns. The model as given solves the first problem of the table. To solve the other problems one needs to changes the assigned values to those given for the problem and replace Q but the unknown in the **Find** statement.

PRB4_7.MCD

$g := 32.2 \quad So := .0006 \quad Y := 4.2 \quad \nu := 1.41 \cdot 10^{-5} \quad b := 6 \quad m := 1.6 \quad e := .005 \quad C := 100 \quad Q := 200$

$A := (b + m \cdot Y) \cdot Y \qquad Rh := \frac{A}{b + 2 \cdot Y \cdot \sqrt{m^2 + 1}} \qquad Re := \frac{4 \cdot Q \cdot Rh}{A \cdot \nu}$

Given

$C + \sqrt{32 \cdot g} \cdot \log\left(\frac{e}{12 \cdot Rh} + \frac{.884 C}{Re \cdot \sqrt{g}}\right) = 0 \quad Re = \frac{4 \cdot Q \cdot Rh}{A \cdot \nu} \quad Q = A \cdot C \cdot \sqrt{Rh \cdot So} \quad Rh = \frac{A}{b + 2 \cdot Y \cdot \sqrt{m^2 + 1}} \; 0$

$A = (b + m \cdot Y) \cdot Y$

$$\text{Find}(Q, C, A, Re, Rh) = \begin{bmatrix} 246.57 \\ 120.497 \\ 53.424 \\ 3.201 \cdot 10^{6} \\ 2.445 \end{bmatrix}$$

8.

The table below applies for uniform flow in circular channels. Fill in the mission blanks. (v = $1.317X10^{-6}$ m^2/sec)

Flow rate $Q(m^3/s)$	Rough. e(m)	Bot. Slope S_o	Diameter D(m)	Depth Y(m)
______	.005	0.00080	3.0	2.3
20.0	______	0.00068	5.0	2.52
25.0	.003	________	6.0	2.9
30.0	.0025	0.00040	______	3.0
60.0	.0045	0.00045	8.0	______

Solve for blanks in table. Solutions were obtained using Program CHEZYCTC which solves Chezy's Equation and Chezy's C equation simultaneously, or

$$Q=CA(R_hS_o)^{1/2} \qquad (1)$$

$$C=-(32g)^{1/2}Log_{10}[e/(12R_h)+.884C/(R_eg^{1/2})] \qquad (2)$$

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

Q (m^3/s)	e(m)	S_o	D(m)	Y(m)	C	$R_e x10^{-6}$
9.23	.005	.00080	3.0	2.3	59.07	3.8
20	.001876	.00068	5.0	2.52	69.06	6.67
25.0	.003	.0005218	6.0	2.9	67.23	6.39
30.0	.0025	.0004	7.91	3.0	68.80	7.53
60.0	.0045	.00045	8.0	4.492	66.48	11.66

A MathCAD model PRB2_8.MCD is given below to solve the above two (i.e. 6 equations) simultaneously. The values for the variables are for the first problem in the table. To solve the other problems, the given variables need only be changed. (What changes would be needed for the model to apply for a trapezoidal channel? (See Problem 8, or 10 below, for and example of such a model.)

PRB2_8.MCD

$D := 3 \quad e := .005 \quad So := .0008 \quad Y := 2.3 \quad \beta := acos\left(1 - 2\cdot\frac{Y}{D}\right) \quad A := \frac{D^2}{4}\cdot(\beta - cos(\beta)\cdot sin(\beta)) \quad P := D\cdot\beta$

$g := 9.81 \quad \nu := 1.317\cdot 10^{-6} \quad Q := 10 \quad C := 90 \quad K := (32\cdot g)^{.5} \quad Re := \frac{4\cdot Q}{\nu\cdot P}$

Given $\quad Re = \frac{4\cdot Q}{\nu\cdot P} \quad cos(\beta) = 1 - 2\cdot\frac{Y}{D} \quad A = \frac{D^2}{4}\cdot(\beta - cos(\beta)\cdot sin(\beta)) \quad P = D\cdot\beta \quad Q = C\cdot A\cdot\left(\frac{A}{P}\cdot So\right)^{.5}$

$C = -K\cdot log\left(\frac{e\cdot P}{12\cdot A} + \frac{.884C}{Re\cdot\sqrt{g}}\right) \qquad Find(Re, \beta, A, P, C, Q) = \begin{bmatrix} 4.396\cdot 10^6 \\ 2.133 \\ 5.815 \\ 6.4 \\ 59.089 \\ 9.264 \end{bmatrix}$

9.

Assume the ratio of Y/D (depth to diameter) is the same as in Problem 6 but the conduit is of 1 ft diameter. What flow rate is occurring? What is Manning's n for this flow? Why is n not the same as in Problem 6?

Given:
D=1 ft, S_o=.001, e=.004 ft

Find: Q and corresponding n

Using Program CHEZYCTC to solve problem using Chezy's Equation (the bolded values are in response to the Program's prompts)

```
  Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc
  1  1 .1410E-04
 Give No. of Unknown
 2 -  D
 3 -  S
 4 -  Y
 5 -  e
 6 -  Q
 7 -  V
          6
  Give values to knowns & GUESS for unknown
 D =     1.0000
 S =      .0010
 Y =      .5748
 e =      .0040
 Q =      .7000
```

Solution:

```
  m =       .000
  D =      1.000
  S =    .001000
  Y =       .575
  e =    .004000
  Q =       .709  <==
  V =      1.518
  C =     92.110
```

Solving Mannings Equation gives n = .0130 <==

10.

A flow rate of 40 m^3/s is flowing in a trapezoidal channel under uniform conditions at a depth of Y = 2 m. The channel has a bottom width of b = 3 m, and a side slope of m = 1.5. Its wall roughness e = 0.002 m. What is the bottom slope of this channel? ($\nu = 1.519 \times 10^{-6}$ m^2/s).

Given: Q=40 m^3/s, Y=2 m, b=3 m, m=1.5, e=.002 m, $\nu=1.519\times10^{-6}$ m^2/s

Need to solve Chezy's Equation and Chezy's C equation simultaneously, or

$$Q=CA(R_hS_o)^{1/2} \qquad (1)$$

$$C=-(32g)^{1/2}\mathrm{Log}_{10}[e/(12R_h)+.884C/(R_eg^{1/2})] \qquad (2)$$

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

Using Program CHEZYCTC to solve problem using Chezy's Equation (the bolded values are in response to the Program's prompts)

```
  Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc
  2  0 .1519E-05
 Give No. of Unknown
 1 -  m
 2 -  b
 3 -  S
 4 -  Y
 5 -  e
 6 -  Q
 7 -  V
          3
  Give 1 if Q will be given or 2 if V is known
          1
  Give values to knowns & GUESS for unknown
 m =     1.5000
 b =     3.0000
 S =      .0020
 Y =     2.0000
 e =      .0020
 Q =    40.0000
Solution:
  m =      1.500
  b =      3.000
  S =    .002040  <==
  Y =      2.000
  e =    .002000
  Q =     40.000
  V =      3.333
  C =     68.082  <==
```

The following MathCAD model, which is basically the same as that used to solve Problem 4, with the exception that So is specified as one of the variables to find.

PRB2_10.MCD

$g := 9.81 \quad So := .0008 \quad Y := 2 \quad v := 1.519 \cdot 10^{-6} \quad b := 3 \quad m := 1.5 \quad e := .002 \quad C := 65$

$Re := 5 \cdot 10^{6} \quad Q := 40 \quad A := 20 \quad Rh := 1.5$

Given

$$C + \sqrt{32 \cdot g} \cdot \log\left(\frac{e}{12 \cdot Rh} + \frac{.884 \cdot C}{Re \cdot \sqrt{g}}\right) = 0 \qquad Re = \frac{4 \cdot Q \cdot Rh}{A \cdot v} \qquad Q = A \cdot C \cdot \sqrt{Rh \cdot So} \qquad Rh = \frac{A}{b + 2 \cdot Y \cdot \sqrt{m^2 + 1}}$$

$$A = (b + m \cdot Y) \cdot Y$$

$$\text{Find}(C, Re, A, Rh, So) = \begin{bmatrix} 68.082 \\ 1.032 \cdot 10^{7} \\ 12 \\ 1.175 \\ 2.04 \cdot 10^{-3} \end{bmatrix}$$

11.

Water is to enter a circular channel from a reservoir so that the depth in the channel will be 3.5 ft. The channel will have a bottom slope of $S_o = 0.0009$, and a wall roughness of e = 0.002 ft. What size pipe must be used to carry a flow rate of 120 cfs?

Given: Y=3.5 ft, S_o=.0009, e=.002 ft, Q=120 cfs, ν=1.217x10^{-5} ft^2/sec

Find: D

Using Program CHEZYCTC to solve problem using Chezy's Equation (the bolded values are in response to the Program's prompts)

```
  Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc
  1  1 .1410E-04
 Give No. of Unknown
 2 -  D
 3 -  S
 4 -  Y
 5 -  e
 6 -  Q
 7 -  V
           2
  Give 1 if Q will be given or 2 if V is known
           1
  Give values to knowns & GUESS for unknown
 D =     9.0000
 S =      .0009
 Y =     3.5000
 e =      .0020
 Q =   120.0000

Solution:
  m =      1.500
  D =      8.842  <==
  S =    .000900
  Y =      3.500
  e =    .002000
  Q =    120.000
  V =      5.306
  C =    129.001   <==
```

The following MathCAD model, which is essentially that used in solving Problem 2, with the exception that the diameter D is one of the variables that is to be solved with the Find statement.

PRB2_11.MCD

$g := 32.2$ $So := .0009$ $Y := 3.5$ $\nu := 1.41\cdot 10^{-5}$ $D := 8$ $e := .002$ $C := 120$ $Re := 5\cdot 10^{6}$ $Q := 120$

$A := 20$ $Rh := 1.5$ $\beta := 2.5$

Given

$$C + \sqrt{32 \cdot g} \cdot \log\left(\frac{e}{12 \cdot Rh} + \frac{.884 \cdot C}{Re \cdot \sqrt{g}}\right) = 0 \qquad Re = \frac{4 \cdot Q \cdot Rh}{A \cdot v} \qquad Q = A \cdot C \cdot \sqrt{Rh \cdot So} \qquad \beta = acos\left(1 - 2 \cdot \frac{Y}{D}\right)$$

$$A = .25 \cdot D^2 \cdot (\beta - \cos(\beta) \cdot \sin(\beta)) \qquad Rh = \frac{A}{\beta \cdot D}$$

$$Find(D, A, Rh, Re, C, \beta) = \begin{bmatrix} 8.842 \\ 22.617 \\ 1.88 \\ 2.829 \cdot 10^6 \\ 129.001 \\ 1.361 \end{bmatrix}$$

<u>MATLAB SOLUTION</u>

```
Using script “chezy1.m” to solve this problem produce the following dialog, and solution
in the command window:
>> chezy1
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 1
Give k. viscosity 1.217e-5
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
D = 5
 Number = 5
Give value to known variables and guess for unknown
e = .002
Q = 120
S = .0009
Y = 3.5
 Provide guess for unknown D = 9
Unknown D = 8.83, C = 129.1381
e = 0.002
Q = 120
S = 0.0009
Y = 3.5
D = 8.83
```

12.

Use the following example of an open channel to show that the selection of an inappropriate n causes a much larger variation in the computed flow rate, than does a corresponding percent error in the selection of e. D = 3 m, Y = 2 m, S_o = 0.0008. Assume the correct value of n = 0.013, but you used n = 0.015 (a 15% error), and that the correct e = 0.00057 ft, but you used e = 0.00066 (also a 15% error). (v = 1.519X10^{-6} m^2/s)

Given: D=3 m, Y=2 m, S_O=.0008, ν=1.519x10^{-6} m^2/s

Wanted: Show that a 15 % error in n (for Mannings Equation) produces a larger different than a 15 % error in e (for Chezy's Equation)

Chezy's Equation: e=.00057 m to e=.00066 m (15% increase)
Using Program CHEZYCTC to solve problem using Chezy's Equation (the bolded values are in response to the Program's prompts)

```
  Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc
  2  1 .1519E-05
 Give No. of Unknown
 2 -  D
 3 -  S
 4 -  Y
 5 -  e
 6 -  Q
 7 -  V
          6
  Give values to knowns & GUESS for unknown
 D =     3.0000
 S =      .0008
 Y =     2.0000
 e =      .00057
 Q =    10.0000

Solution:
  m =       1.500
  D =       3.000
  S =     .000800
  Y =       2.000
  e =     .000570
  Q =       9.915 <==
  V =       1.981
  C =      74.927

  Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc
  2  1 .1519E-05
```

```
 Give No. of Unknown
 2 -  D
 3 -  S
 4 -  Y
 5 -  e
 6 -  Q
 7 -  V
          6
  Give values to knowns & GUESS for unknown
 D =     3.0000
 S =      .0008
 Y =     2.0000
 e =      .00066
 Q =    10.0000

Solution:
  m =      1.500
  D =      3.000
  S =    .000800
  Y =      2.000
  e =    .000660
  Q =      9.776  <==
  V =      1.953
  C =     73.882
```

Difference in Solution for Q = 9.915-9.776 =0.139 or 1.4 % Error ←

Using Mannings Equation:

n=.013 Q = 9.95 m^3/s

n=.015 Q = 8.62 m^3/s

Difference= 1.33 m^3/s or a 13.4 5 Error ←

13.

Determine the appropriate bottom width that should be used in the design of an open channel with a trapezoidal cross section and a side slope of m = 1.6, if the depth of flow is to be 5.8 m when the flow rate is 550 m^3/s. The slope of the channel bottom is S_o = 0.00113, and its wall roughness is e = 0.0015m.

Given: m=1.6, Y=5.8 m, Q=550 m^3/s, S_o=.00113, e=.0015 m, $\nu=1.519\times10^{-6}$ m^2/s

Find: bottom width b

Solution: obtained using Program CHEZYCTC (Bolded values are inputs to given prompts from Program) This program solves the following simultaneously:

$$Q=CA(R_hS_o)^{1/2} \qquad (1)$$

$$C=-(32g)^{1/2}Log_{10}[e/(12R_h)+.884C/(R_eg^{1/2})] \qquad (2)$$

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

```
  Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc
  2  0 .1519E-05
 Give No. of Unknown
 1 -  m
 2 -  b
 3 -  S
 4 -  Y
 5 -  e
 6 -  Q
 7 -  V
           2
  Give 1 if Q will be given or 2 if V is known
           1
  Give values to knowns & GUESS for unknown
 m =     1.6000
 b =    10.0000
 S =      .0011
 Y =     5.8000
 e =      .0015
 Q =   550.0000
Solution:
  m =      1.600
  b =      9.886  <==
  S =    .001130
  Y =      5.800
  e =    .001500
  Q =    550.000
  V =      4.948
  C =     78.687  <==  , Rh=3.50 m, Re=45,577,400
```

With changes in the assigned values of variables, and requesting that the Find statement solve for the bottom width b, rather than the depth Y, the MathCAD model used in Problem 4 can be used to solve this problem. This model is given below. (The slight difference in answers is due to using a different value for the kinematic viscosity.)

PRB2_13.MCD

$g := 9.81 \quad So := .00113 \quad Y := 5.8 \quad v := 1.007\cdot 10^{-6} \quad b := 10 \quad m := 1.6 \quad e := .0015 \quad C := 70 \quad Re := 5\cdot 10^{6}$

$Q := 550 \quad A := 50 \quad Rh := 1.5$

Given

$$C + \sqrt{32\cdot g}\cdot \log\left(\frac{e}{12\cdot Rh} + \frac{.884\cdot C}{Re\cdot\sqrt{g}}\right) = 0 \qquad Re = \frac{4\cdot Q\cdot Rh}{A\cdot v} \qquad Q = A\cdot C\cdot\sqrt{Rh\cdot So} \qquad Rh = \frac{A}{b + 2\cdot Y\cdot\sqrt{m^2 + 1}}$$

$$A = (b + m\cdot Y)\cdot Y$$

$$\mathrm{Find}(b, C, Re, A, Rh) = \begin{bmatrix} 9.8795 \\ 78.7206 \\ 6.8774\cdot 10^{7} \\ 111.1252 \\ 3.4982 \end{bmatrix}$$

MATLAB SOLUTION (Using script "chezy2.m"

```
>> chezy2
Give:1=trap.,2=cir. for channel type 1
Give:1=ES or 2=SI units 2
Give k. viscosity 1.519e-6
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
b = 5
m = 6
 Number = 5
Give value to known variables and guess for unknown
e = .0015
Q = 550
S = .00113
Y = 5.8
b = 10
m = 1.6
Unknown b = 9.8864, C = 78.6867
e = 0.0015
Q = 550
S = 0.00113
Y = 5.8
b = 9.8864
m = 1.6
```

14.

Water enters a trapezoidal canal from a reservoir at the uniform depth of 5.2 ft. The canal is to convey 450 cfs over a long distance at a slope of 0.0005, and stability considerations dictate that the side slope should be 2 to 1. If the wall roughness e = 0.012 ft, determine what the bottom width of the canal should be.

Given: Y=5.2 ft, Q=450 cfs, S_o=.0005, m=2.0, e=.012 ft.
Wanted: bottom width b=?

Solution obtained using Program CHEZYCTC (Bolded values are inputs to given prompts from Program). This program solves the following simultaneously:

$$Q=CA(R_hS_o)^{1/2} \quad (1)$$

$$C=-(32g)^{1/2}\text{Log}_{10}[e/(12R_h)+.884C/(R_eg^{1/2})] \quad (2)$$

With $R_e=4VR_h/\nu = 4Q/(\nu P)$

```
  Give 1=ES or 2=SI(or 0/=STOP),0=trap or 1=cir. & Visc
  1  0 .1218E-04
 Give No. of Unknown
 1 -  m
 2 -  b
 3 -  S
 4 -  Y
 5 -  e
 6 -  Q
 7 -  V
          2
  Give 1 if Q will be given or 2 if V is known
          1
  Give values to knowns & GUESS for unknown
 m =     2.0000
 b =    10.0000
 S =      .0005
 Y =     5.2000
 e =      .0120
 Q =   450.0000
Solution:
  m =      2.000
  b =      9.107  <==
  S =    .000500
  Y =      5.200
  e =    .012000
  Q =    450.000
  V =      4.436
  C =    112.060  <==    Re=3,943,580
```

15.

The following data defines the cross section of an earthen canal, in which x represents the distance in feet from the left bank when looking downstream, and y represents the corresponding vertical distance from this bank elevation to the elevation of the canal bottom.

x(ft)	0	3	5	7	10	12	14	18	21	25	27	29
y(ft)	0	1.4	2.3	2.8	3.4	3.5	3.3	2.8	1.5	0.9	0.2	0.0

Estimate the flow rate in the canal if its bottom slope is 0.001, and the depth of flow is 3.2 ft.

Assume n=.030

Solution:

Q=123.13 cfs

A=52.77 ft^2

P=29.14 ft

T=26.30 ft

16.

Use Manning's formula to fill in the blank in the table below that applies for uniform flow in channels of trapezoidal shapes.

Flow rate Q(cfs)	Coeff. n	Bot. Slope S_o	Bot. Width b(ft)	Side Slope m	Depth Y(ft)
_____	.013	0.0006	6.0	1.6	4.2
324.5	_____	0.0008	8.0	0.0	5.0
550.0	.014	_____	10.0	1.3	4.8
400.0	.013	0.0015	_____	1.5	4.0
450.0	.013	0.0009	7.0	_____	4.1
600.0	.013	0.00075	10.0	1.2	_____

Using Mannings Equation fill in blanks of table

Q(cfs)	n	S_o	b(ft)	m	Y(ft)
271.5	.013	.0006	6.0	1.6	4.2
324.5	.0086	.0008	8.0	0.0	5.0
550.0	.014	.00101	10.0	1.3	4.8
400.0	.013	.0015	6.608	1.5	4.0
450.0	.013	.0009	7.0	2.62	4.1
600.0	.013	.00075	10.0	1.2	5.312

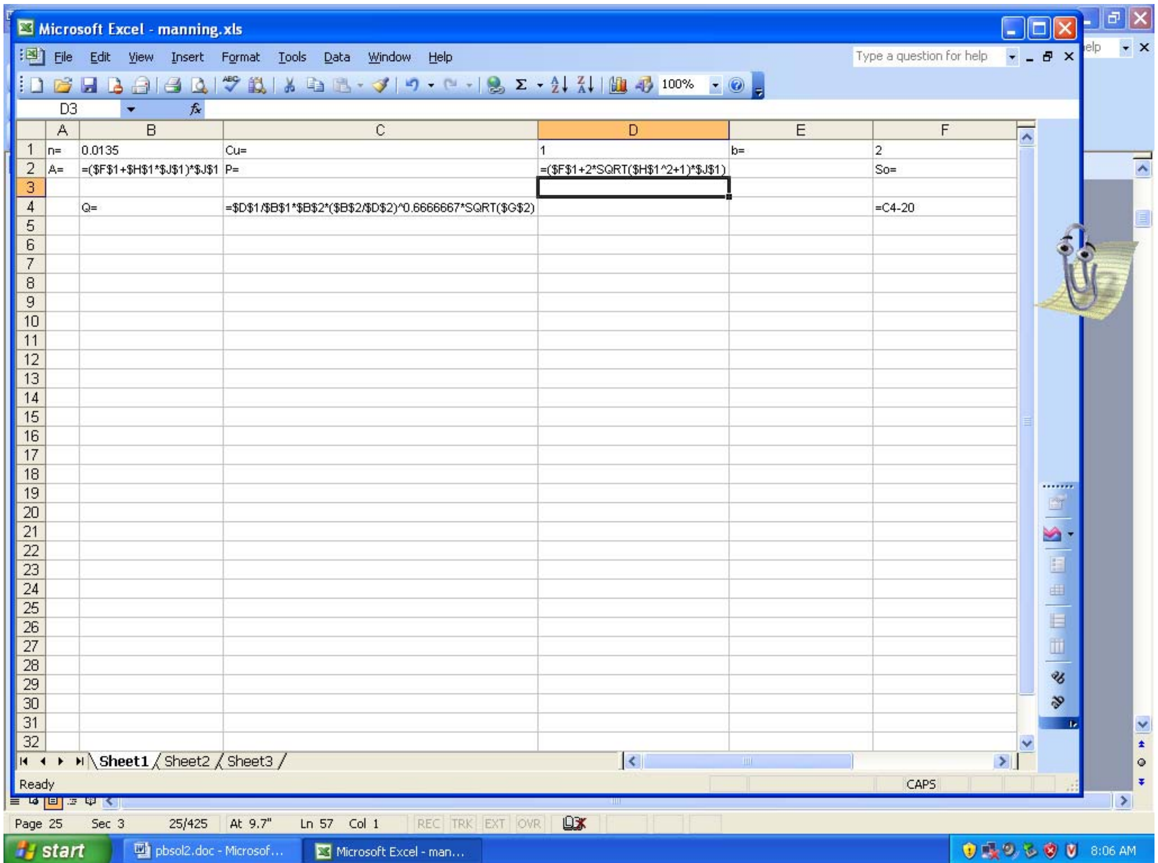
Simple equations, whether explicit or implicit, can readily be solved using a spreadsheet. For example the EXCEL spreadsheet below is designed to solve Manning's equation. It is arranged so that when the other values are given it solves for the flow rate in cell C4, which contains Manning's Equation. However, any of the variables might be solved by using the "**Goal Seek**" (an optimization procedure) in later versions of EXCEL. In the spreadsheet below the cell F4 contains C4-flow rate (in which flow rate is the desired vale for Q). When Goal Seek is selected with the F4 cell highlighted one will see its dialog box open with

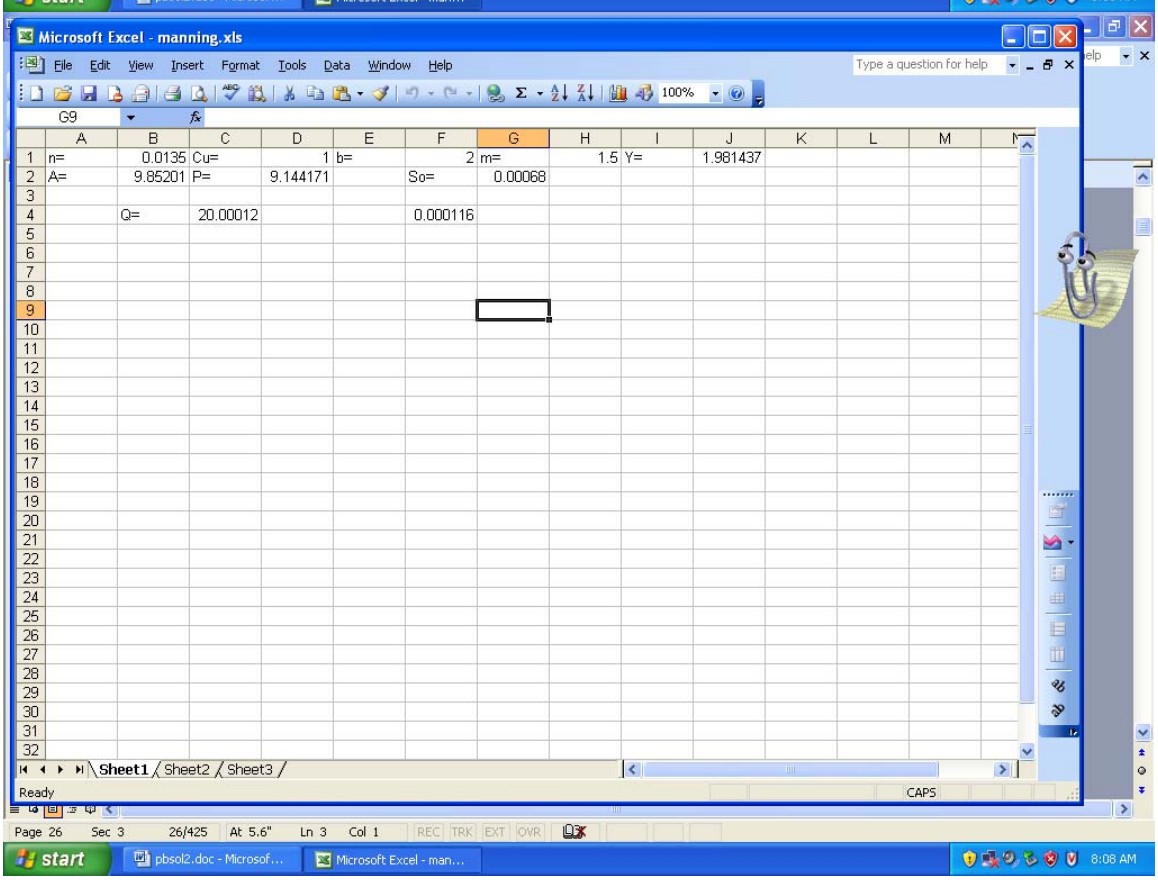
Set Cell: **F4**

To Value: **0**

By changing cell: **Cell that contains unknown** (which for the solution below is J1

The EXCEL model printed below solves the last problem in Problem 17. The first image displays the equations, i.e. in cell B2 the computation of the area, in D2 the computation of the wetted Perimeter, and in C4 Mannings equation.


Microsoft Excel - manning.xls
n=
0.0135
Cu=
1
b=
2
A=
=($F$1+$H$1*$J$1)*$J$1
P=
=($F$1+2*SQRT($H$1^2+1)*$J$1)
So=
Q=
=$D$1/$B$1*$B$2*($B$2/$D$2)^0.6666667*SQRT($G$2)
=C4-20
Sheet1
Sheet2
Sheet3
Ready
CAPS
8:06 AM


Microsoft Excel - manning.xls
n=
0.0135
Cu=
1
b=
2
m=
1.5
Y=
1.981437
A=
9.85201
P=
9.144171
So=
0.00068
Q=
20.00012
0.000116
Sheet1
Sheet2
Sheet3
Ready
CAPS
8:08 AM

n=	0.0135	Cu=	1	b=	2	m=	1.5	Y=	1.981437
A=	9.85201	P=	9.1442	So=	.00068				
	Q=	20.00012		0.000116					

The solutions to various parts of Problems 16 and 17 as obtained using this EXCEL model are:

Problem 16
1: Q=271.4915
2: n=.008823
3: S_o=.001009
4: b=6.607797
5: m=2.61726
6: Y=5.3122

Problem 17
1: Q=39.80559
2: n=.011248
3: S_o=.001137
4: b=5.77099
5: m=1.186969
6: Y=1.981437

Using the MathCAD model PRB2_17.MCD, developed for Problem 17, to solve the last problem in the table for this Problem 16 gives the following:

$Q := 600 \quad n := .013 \quad So := .00075 \quad b := 10 \quad m := 1.2 \quad Y := 6 \quad Cu := 1.486$

$$Y := \mathrm{root}\left[Q - \frac{Cu}{n}\cdot\frac{((b + m\cdot Y)\cdot Y)^{1.6666667}}{\left(b + 2\cdot Y\cdot\sqrt{m^2 + 1}\right)^{.66666667}}\cdot\sqrt{So}, Y\right] \qquad Y = 5.312$$

17.

Fill in the missing blank in the table below using Manning's formula. These are trapezoidal channel, and the flow in them is uniform.

Flow rate $Q(m^3/s)$	Coeff. n	Bot. Slope S_o	Bot. Width b(m)	Side Slope m	Depth Y(m)
______	.014	0.0015	3.0	1.0	2.3
50.0	______	0.0012	3.0	1.5	2.2
60.0	.013	______	2.5	2.0	2.5
100.0	.015	0.0008	______	2.0	3.0
80.0	.014	0.00075	6.0	______	2.9
20.0	.0135	0.00068	2.0	1.5	______

Using Mannings Equation fill in blanks of table

$Q(m^3/s)$	n	S_o	b(m)	m	Y(m)
39.81	.014	.0015	3.0	1.0	2.3
50.0	.01125	.0012	3.0	1.5	2.2
60.0	.014	.001318	2.5	2.0	2.5
100.0	.015	.0008	5.771	2.0	3.0
80.0	.014	.00075	6.0	1.187	2.9
20.0	.0135	.00068	2.0	1.5	1.981

The **root** function in MathCAD is a good means of solving Mannings Equation for any of the variables (or any other single equation) for any of the variables as the unknown. A model using the root function is given below. The use of the **root** function will be useful in solving subsequent problems by trial and error, when other equations are available and there are additional unknowns. The model as given solve the first problem of the table. To solve the other problems one needs to change the assigned values, and in the statement that uses the **root** function the variable on the left of the assign statement and the last argument of the **root** function need to be changed. (You might also use this model to solve the problem in the table of Problem 16.)

PRB2_17.MCD

$Q := 40 \quad n := .014 \quad So := .0015 \quad b := 3 \quad m := 1 \quad Y := 2.3 \quad Cu := 1$

$$Q := \mathrm{root}\left[Q - \frac{Cu}{n}\cdot\frac{((b + m\cdot Y)\cdot Y)^{1.6666667}}{\left(b + 2\cdot Y\cdot\sqrt{m^2+1}\right)^{.66666667}}\cdot\sqrt{So}, Q\right] \qquad Q = 39.806$$

18.

For each entry in the table for problem 17 compute the value of Chezy's C coefficient, and from this C determine what the wall roughness e is.

$\nu=1.317\text{x}10^{-6}$ m^2/s, Solve for e and C. Solutions obtained using CHEZYCTC

$Q (m^3/s)$	n	S_o	b(m)	m	Y(m)	e(m)	C
39.81	.014	.0015	3.0	1.0	2.3	.000940	74.460
50.0	.01125	.0012	3.0	1.5	2.2	.000063	92.489
60.0	.014	.001137	2.5	2.0	2.5	.000409	81.062
100.0	.015	.0008	5.771	2.0	3.0	.00148	73.801
80.0	.014	.00075	6.0	1.187	2.9	.000735	78.963
20.0	.0135	.00068	2.0	1.5	1.981	.000711	75/033

The first entry in the above table can be obtained using the MATLAB scipt "chezy1.m" as shown below.

```
>> chezy1
Give:1=trap.,2=cir. for channel type 1
Give:1=ES or 2=SI units 2
Give k. viscosity 1.317e-6
Give the number for the unknown:
e = 1
Q = 2
S = 3
Y = 4
b = 5
m = 6
 Number = 1
Give value to known variables and guess for unknown
Q = 39.81
S = .0015
Y = 2.3
b = 3
m = 1
 Provide guess for unknown e = .001
Unknown e = 0.00093981, C = 74.4605    <-
e = 0.00093981                          <-
Q = 39.81
S = 0.0015
Y = 2.3
b = 3
m = 1
```

19.

Write a computer program that is designed to solve Manning's formula completely for a trapezoidal section. It should display a menu of variables, allow the user to select the unknown, then request values for the knowns, and finally provide the solution to the unknown.

Program MANNING is such a program. Both a Fortran and Pascal version are listed below.

Program MANNING.FOR

```
      CHARACTER*1 V(6)/'Q','n','S','Y','b','m'/
      CHARACTER*60 F1/"(' Solution'/,6(A3,' = ',E12.5,/),' Unknown',A3,'
     & = 'E12.5)"/
      REAL X(6)/100.,.012,.001,5.,10.,1./
1     WRITE(*,"(' Give: 1 = ES or 2 = SI units ')")
      READ(*,*) IES
      IF(IES.EQ.2) THEN
      Cu=1.
      G=9.81
      ELSE
      Cu=1.486
      G=32.2
      ENDIF
2     WRITE(*,"(' Is Channel 1=Trap. or 2=Circle',/,' Give 1 or 2 ')")
      READ(*,*) ITY
      IF(ITY.EQ.2) THEN
      V(5)='D'
      N=5
      F1(15:15)='5'
      ELSE
      V(5)='b'
      V(6)='m'
      F1(15:15)='6'
      N=6
      ENDIF
3     WRITE(*,"(' Give No. of Unknown'/,(I4,A3))")(I,V(I),I=1,N)
      READ(*,*) IU
      WRITE(*,"(' Give values to knowns')")
      DO 4 I=1,N
      IF(I.EQ.IU) GO TO 4
      WRITE(*,"(A5,' = ',\)") V(I)
      READ(*,*) X(I)
4     CONTINUE
      IF(ITY.EQ.2) THEN
      IF(IU.EQ.5) THEN
      X(5)=2.*X(4)
      ELSE IF(IU.EQ.4) THEN
      X(4)=.5*X(5)
      ENDIF
      ENDIF
      M=0
5     F=X(1)*X(2)-Cu*AR(ITY,A,P,T,X)**1.6666667/P**.6666667*SQRT(X(3))
      X1=X(IU)
      X(IU)=1.005*X1
      FF=X(1)*X(2)-Cu*AR(ITY,A,P,T,X)**1.6666667/P**.6666667*SQRT(X(3))
      DIF=F*(X(IU)-X1)/(FF-F)
      X(IU)=X1-DIF
      M=M+1
      IF(M.LT.30 .AND. ABS(DIF).GT.1.E-5) GO TO 5
      IF(M.EQ.30) WRITE(*,"(' Failed to converge',E12.3)") DIF
      WRITE(*,F1) (V(I),X(I),I=1,N),V(IU),X(IU)
      WRITE(*,"(' Specific Energy =',E12.5,/' Froude No. =',E12.5,//)")
     & X(4)+(X(1)/AR(ITY,A,P,T,X))**2/(2.*G),SQRT(X(1)**2*T/(G*A**3))
      WRITE(*,"(' Give 0=Stop, 1=New Prob., 2=Dif type, 3=Dif units ')")
      READ(*,*) M
      IF(M.EQ.0) STOP
      GO TO (3,2,1),M
      END
      FUNCTION AR(ITY,A,P,T,X)
```

```
      REAL X(6)
      IF(ITY.EQ.2) THEN
      BETA=ACOS(1.-2.*X(4)/X(5))
      A=.25*X(5)**2*(BETA-.5*SIN(2.*BETA))
      P=X(5)*BETA
      T=X(5)*SIN(BETA)
      ELSE
      A=(X(5)+X(6)*X(4))*X(4)
      P=X(5)+2.*X(4)*SQRT(X(6)**2+1.)
      T=X(5)+2.*X(4)*X(6)
      ENDIF

      AR=A
      RETURN
      END
```

Program MANNING.PAS

```
Program Manning;
Var X:array[1..6] of real;
 A,P,T:real;
Function Expn(a,b:real):real;   Begin
 if a>0. then Expn:=exp(b*Ln(a)) else if a=0 then Expn:=0 else if a<0 then
  begin Writeln('Bad Expn argument'); Expn:=0 end;
End; {Power function a**b }
Procedure AR(ITY:integer);
 Var ARG,BETA:real;
 Begin
  if ITY=2 then begin
   ARG:=1.-2.*X[4]/X[5]; if abs(ARG) < 0.000001 then BETA:=Pi/2. else if
   ARG>0. then BETA:=ArcTan(sqrt(1.-sqr(ARG))/ARG) else
   BETA:=Pi-ArcTan(sqrt(abs(1.-sqr(ARG)))/abs(ARG)); T:=SIN(BETA);
   A:=0.25*sqr(X[5])*(BETA-ARG*T); T:=T*X[5]; P:=BETA*X[5] end
  else begin A:=(X[5]+X[6]*X[4])*X[4]; T:=X[5]+2.*X[4]*X[6];
   P:=X[5]+2.*X[4]*sqrt(sqr(X[6])+1) end;
End; {Procedure AR}
Const V:array[1..6] of Char=('Q','n','S','Y','b','m');
      idg:array[1..6] of byte=(2,5,7,3,3,3);
Label L1,L2,L3,L5;
Var IES,IU,ITY,N,m,I:byte; Cu,g,F,FF,X1,DIF:real;
BEGIN
 X[1]:=100.;X[2]:=0.014;X[3]:=0.001;X[4]:=5;X[5]:=10;X[6]:=1;
L1:Write('Give: 1=ES or 2=SI '); Readln(IES);
 if IES=2 then begin Cu:=1;g:=9.81 end else
  begin Cu:=1.486;g:=32.2 end;
L2:Write(' Is channel 1=trap. or 2=circle?  Give 1 or 2 ');Readln(ITY);
 if ITY=2 then begin V[5]:='D';N:=5 end else
  begin V[5]:='b';V[6]:='m'; N:=6 end;
L3:Writeln('Give No. of Unknown'); For I:=1 to N do Writeln(I,' ',V[I]);
 Readln(IU); m:=0;
 Writeln('Give values to knowns');
 For I:=1 to N do if I<>IU then begin
  Write(V[I],' = ');Readln(X[I]) end;
  if ITY=2 then if IU=5 then X[5]:=2.*X[4] else if IU=4 then X[4]:=X[5]/2;

L5:AR(ITY);F:=X[1]*X[2]-Cu*Expn(A,1.666667)/Expn(P,0.666667)*sqrt(X[3]);
  X1:=X[IU]; X[IU]:=1.005*X1;
  AR(ITY);FF:=X[1]*X[2]-Cu*Expn(A,1.666667)/Expn(P,0.666667)*sqrt(X[3]);
 DIF:=F*(X[IU]-X1)/(FF-F); X[IU]:=X1-DIF; Inc(m);
 if (m<30) and (abs(DIF)>1.e-5) then goto L5;
 if m=30 then Writeln(' Failed to converge ',DIF);
 Writeln('Solution'); For I:=1 to N do Writeln(V[I],X[I]:12:IDG[I]);
 Writeln('Unknown ',V[IU],X[IU]:12:IDG[IU]); AR(ITY);
 Writeln('Specific Energy =',X[4]+sqr(X[1]/A)/(2.*g):12:3 );
 Writeln('Froude No. =',sqrt(sqr(X[1])*T/(g*A*sqr(A))):12:3);
 Writeln; Writeln; Writeln('Give 0=Stop, 1=New prob., 2=Dif type, 3=Dif units');
 Readln(m);
 if m=1 then goto L3 else if m=2 then goto L2 else if m=3 then goto L1 else exit;
END.
```

20.

The table below applies for uniform flow in circular channels. Fill in the mission blanks.

Flow rate Q(cfs)	Coeff. n	Bot. Slope S_o	Diameter D(ft)	Depth Y(ft)
_______	.0142	0.00055	10.0	6.8
300.0	_______	0.00065	10.0	7.2
250.0	.013	________	5.0	3.2
200.0	.013	0.03000	______	2.5
100.0	.013	0.00100	8.0	_____

Fill in the blanks of the table for a circular channel using Mannings equation (ES units) See next problem for an EXCEL (spreadsheet) and TK-Solver model to solve problem dealing with the solution of Mannings Equation in a circular channel. The MathCAD model given below for this problem can not only be used to solve the problems in the above table but also those in Problem 21. Since for circular channels the angle β is introduced to find the area and perimeter, it is necessary to use the **Given - Find** block rather than the **root** function which can be used to solve Mannings Equation in trapezoidal channels.

Q(cfs)	n	S_o	D(ft)	Y(ft)
286.0	.0142	.00055	10.0	6.8
300.0	.0146	.00065	10.0	7.2
250.0	.013	.01684	5.0	3.2
200.0	.013	.03000	4.4423	2.5
100.0	.013	.001	8.0	3.25026

PRB2 20.MCD

$Q := 300 \quad n := .0142 \quad So := .00055 \quad D := 10 \quad Y := 6.8 \quad Cu := 1.486$

$$\beta := \operatorname{acos}\left(1 - \frac{2 \cdot Y}{D}\right) \qquad A := .25 \cdot D^2 \cdot (\beta - \cos(\beta) \cdot \sin(\beta))$$

Given $\quad \cos(\beta) = 1 - 2 \cdot \frac{Y}{D} \qquad A = .25 \cdot D^2 \cdot (\beta - \cos(\beta) \cdot \sin(\beta)) \qquad Q = \frac{Cu}{n} \cdot \frac{A^{1.6666667}}{(D \cdot \beta)^{.6666667}} \cdot \sqrt{So}$

$$\operatorname{Find}(Q, \beta, A) = \begin{pmatrix} 285.998 \\ 1.939 \\ 56.873 \end{pmatrix}$$

To use this model to solve the other problems in the table, the appropriate values need to be assigned to the variables, and the first argument of the **Find** statement needs to be changed to the unknown variable.

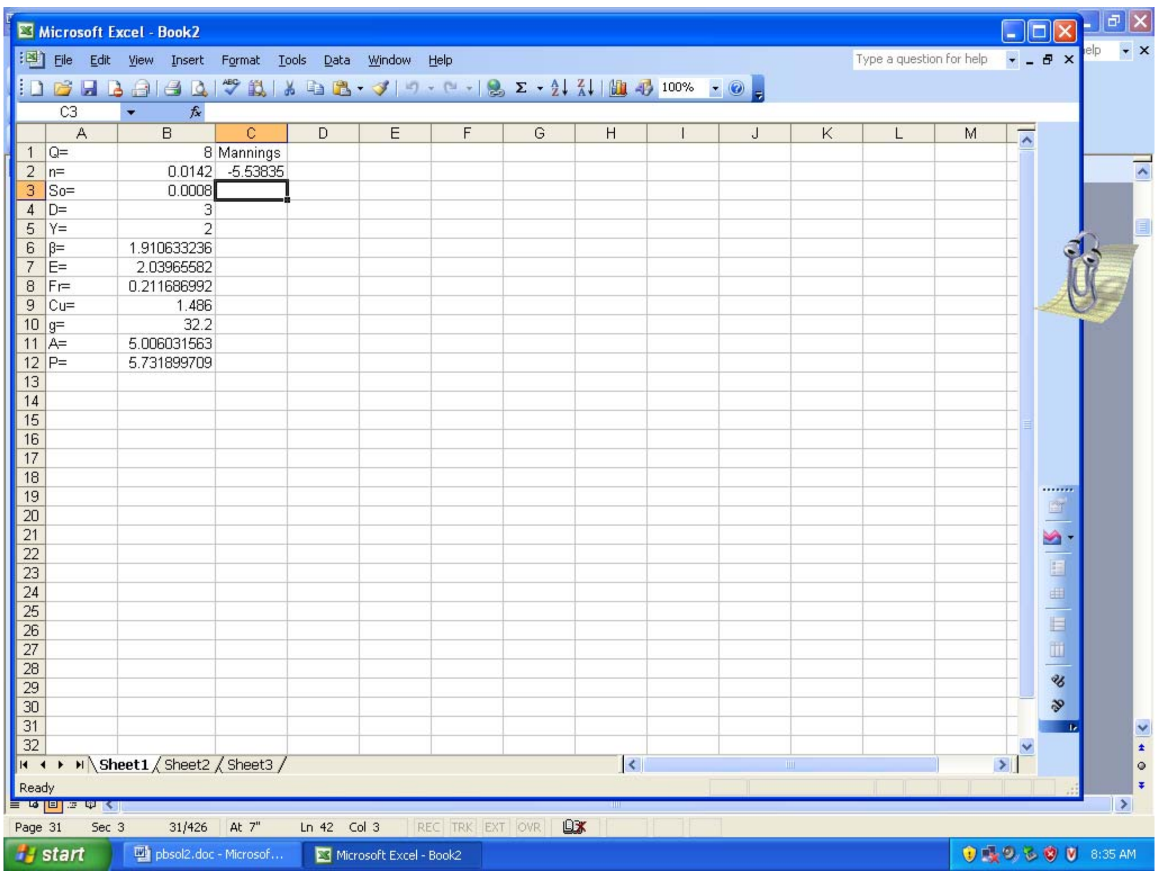
21.

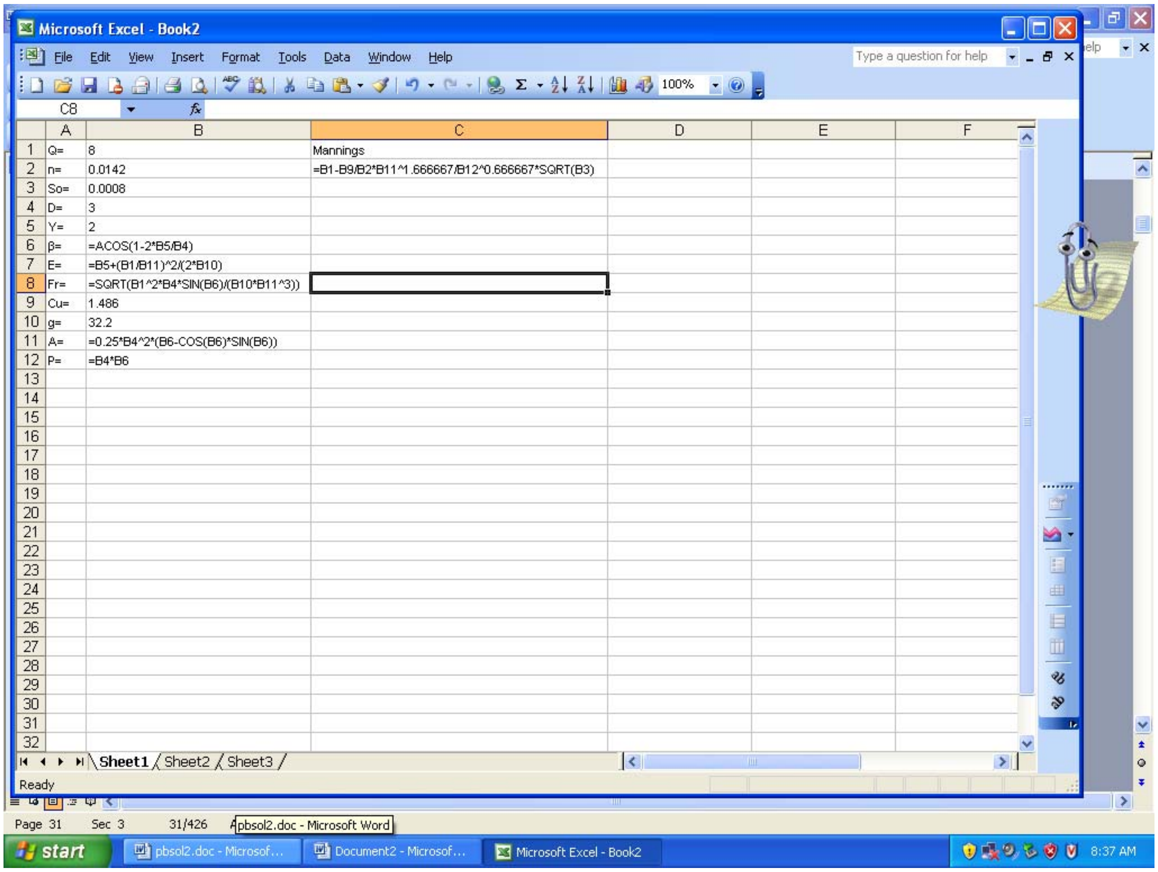
The table below applies for uniform flow in circular channels. Fill in the mission blanks.

Flow rate $Q(m^3/s)$	Coeff. n	Bot. Slope S_o	Diameter $D(m)$	Depth $Y(m)$
______	.015	0.00080	3.0	2.3
20.0	______	0.00068	5.0	2.52
25.0	.013	________	6.0	2.9
30.0	.0125	0.00040	______	3.0
60.0	.0145	0.00045	8.0	______

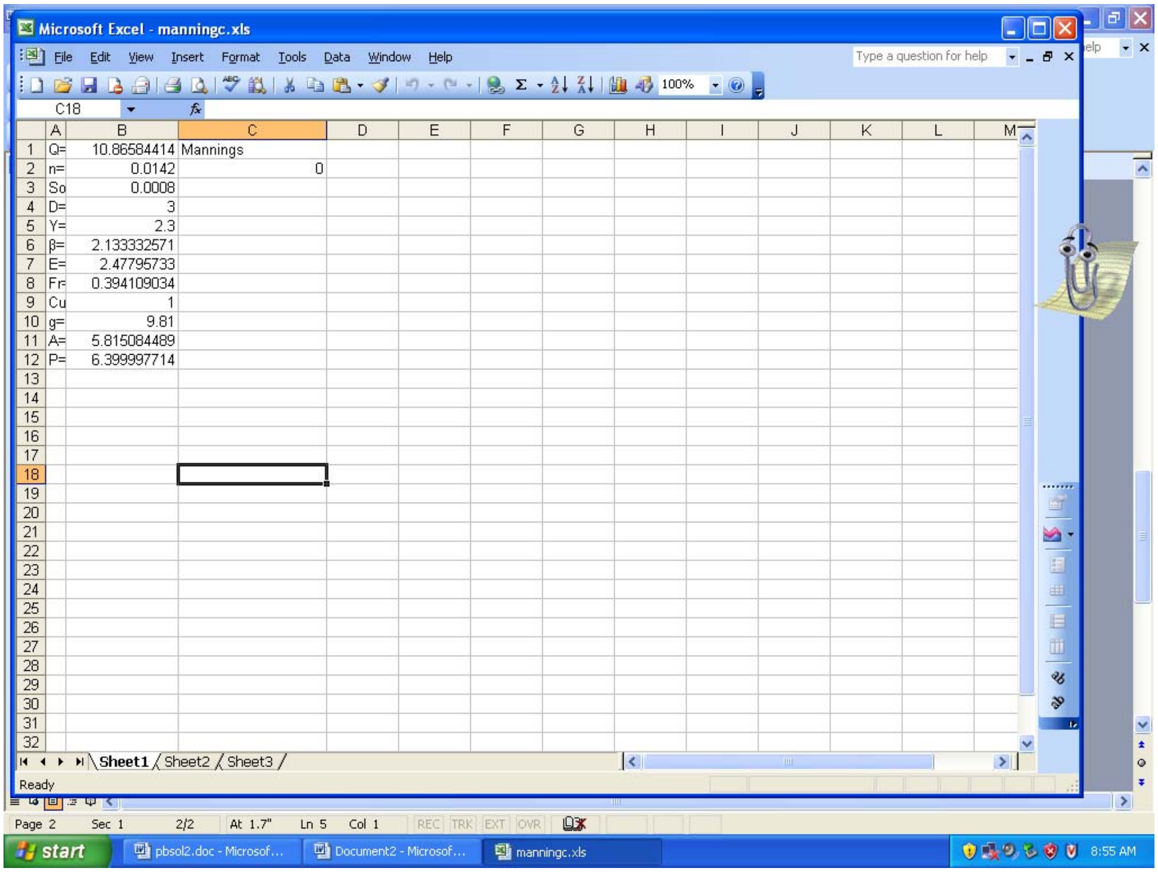
Fill in the blanks of the table for a circular channel using Mannings Equation (SI units) These problem can be solved using any number of methods and or programs. One such program is MANN_TC.

$Q(m^3/s)$	n	S_o	$D(m)$	$Y(m)$	$E(m)$	F_r
10.87	.0142	.0008	3.0	2.3	2.4595	.3731
20.0	.0151	.00068	5.0	2.52	2.7273	.4572
25.0	.013	.000344	6.0	2.9	3.0738	.3924
30.0	.0125	.0004	6.076	3.0	3.2254	.4381
60.0	.0145	.00045	8.0	4.066	4.3445	.4684





The individual problems can be solved by using the "built-in" **Solver** that EXCEL has. To have **Solver** obtain the solution to the first problem it is told that Cell C2 (which contains Mannings Equation) should be zero by changing the value of Cell B1. The solution is given in the image below.



To solve the second problem of the table above, **Solver** is again instructed to make the cell C2=0 by changing the value of Cell B2. The result is: (n=.0150555)

20	Mannings
0.015055517	2.67163E-07
0.00068	
5	
2.52	
1.578796412	
2.727280608	
0.457163774	
1	
9.81	
9.917475976	
7.893982061	

To solve the third problem for S_o it becomes necessary to also add the constraint that cell B3 (which is S_o) be >= 0; otherwise **Solver** may attempt using a negative value that results in a problem with the square root.

25	Mannings
0.013	-1.82567E-07
0.000345622	
6	
2.9	
1.537456818	
3.073827432	
0.392431085	

1
9.81
13.53727807
9.224740905

The solution to the 4th problem of the table follows: (Now Cell B4 is to be changed)

Q=	30	Mannings	
n=	0.0125		-9.56183E-07
So=	0.0004		
D=	6.075600128		
Y=	3		
β=	1.558352769		
E=	3.225391777		
Fr=	0.438137525		
Cu=	1		
g=	9.81		
A=	14.26601643		
P=	9.467928285		

The solution to the 5th problem of the table follows. Now Cell B5 (Y) is to be changed.

Q=	60	Mannings	
n=	0.0145		-8.63155E-08
So=	0.00045		
D=	8		
Y=	4.065798424		
β=	1.587246675		
E=	4.344487851		
Fr=	0.416840688		
Cu=	1		
g=	9.81		
A=	25.65910488		
P=	12.6979734		

The following is a TK-Solver model to solve the 5 parts of the above problem. When using this model it is necessary that the given values of the variables are given in the input column of the variables sheet, and the other variable values can be left blank, except for the last two problems in which D and Y are being solved. For these to last problems guesses are needed and a G must be given in the status column before pressing the F9 key to get a solutions.

TK-Solver model MANNINGC.TK

```
==================== RULE SHEET ========================
S Rule--------------------------------------------------
  Q=Cu/n*A^1.666667/P^.666667*sqrt(So)
  beta=acos(1.-2*Y/D)
  P=D*beta
  A=D^2/4*(beta-cos(beta)*sin(beta))
  E=Y+(Q/A)^2/(2.*g)
  Fr=sqrt(Q^2*D*sin(beta)/(g*A^3))
(a) Solve for the flow rate Q
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment--------
              Q        10.865844
   9.81       g
   1          Cu
   .0142      n
              A        5.8150845
              P        6.3999977
```

```
   .0008      So
              beta    2.1333326
   2.3        Y
   3          D
              E       2.4779573
              Fr      .39410903
```

(b) Solve for Mannings n

```
==================== VARIABLE SHEET ====================
St Input───── Name──── Output──── Unit────── Comment─────────
   20         Q
   9.81       g
   1          Cu
              n       .01505552
              A       9.917476
              P       7.8939821
   .00068     So
              beta    1.5787964
   2.52       Y
   5          D
              E       2.7272806
              Fr      .45716377
```

(c) Solve for the bottom slope S_o

```
==================== VARIABLE SHEET ====================
St Input───── Name──── Output──── Unit────── Comment──────────
   25         Q
   9.81       g
   1          Cu
   .013       n
              A       13.537278
              P       9.2247409
              So      .00034562
              beta    1.5374568
   2.9        Y
   6          D
              E       3.0738274
              Fr      .39243108
```

(d) Solve for the pipe diameter D

```
==================== VARIABLE SHEET ====================
St Input───── Name──── Output──── Unit────── Comment──────────
   30         Q
   9.81       g
   1          Cu
   .0125      n
              A       14.266016
              P       9.4679282
   .0004      So
              beta    1.5583528
   3          Y
              D       6.0755999
              E       3.2253918
              Fr      .43813753
```

(e) Solve for the depty Y of flow

```
==================== VARIABLE SHEET ====================
St Input───── Name──── Output──── Unit────── Comment─────────
   60         Q
   9.81       g
   1          Cu
   .0145      n
              A       25.659105
              P       12.697973
   .00045     So
              beta    1.5872467
              Y       4.0657984
   8          D
              E       4.3444878
              Fr      .41684069
```

Using the MathCAD model PRB2_20.MCD given in the previous problem to solve the last problem in the table for this problem results in:

$Q := 60 \quad n := .0145 \quad So := .00045 \quad D := 8 \quad Y := 2.5 \quad Cu := 1. \quad \beta := \mathrm{acos}\left(1 - \frac{2 \cdot Y}{D}\right)$

$A := .25 \cdot D^2 \cdot (\beta - \cos(\beta) \cdot \sin(\beta))$

Given $\cos(\beta) = 1 - 2\cdot\frac{Y}{D}$ $A = .25\cdot D^2\cdot(\beta - \cos(\beta)\cdot\sin(\beta))$ $Q = \frac{Cu}{n}\cdot\frac{A^{1.6666667}}{(D\cdot\beta)^{.6666667}}\cdot\sqrt{So}$

$$\text{Find}(Y,\beta,A) = \begin{pmatrix} 4.0658 \\ 1.58725 \\ 25.65911 \end{pmatrix}$$

Partial Solution using MATLAB and the script "manning.m"

```
>> manning
Give:1=trap.,2=cir. for channel type 2
Give:1=ES or 2=SI units 2
Give the number for the unknown:
n = 1
Q = 2
S = 3
Y = 4
D = 5
 Number = 2
Give value to known variables and guess for unknown
n = .015
S = .0008
Y = 2.3
D = 3
Unknown Q = 10.2863  <-
n = 0.015
Q = 10.2863    <-
S = 0.0008
Y = 2.3
D = 3
```

22.

Compute the normal depth in a trapezoidal channel with b = 10 ft, m = 1.5, S_o = .0008 if Q = 400 cfs is flowing in the channel and it has a bottom roughness coefficient for Mannings equation of n_b = .05, and a side roughness coefficient n_s = .013.

Given: b=10 ft, m=1.5, S_o=.0008, Q=400 cfs, n_b=.05, n_s=.013
Find: The uniform depth of flow
Solution: Since the equivalent Mannings n will depend upon the unknown depth, i.e. $n_{eq}=(b/P)n_b+[2(m^2+1)^{1/2}Y/P]n_s$ it is necessary to include this equation for n in Mannings Equation as it is being solved for the depth of flow. One procedure would be to repeatedly solve Mannings Equation for the Depth, and after each such solution recomputed n_{eq} and repeat this until there is a smaller change in value than the accuracy desired. Another approach is to substitute the above equation for n_{eq} into Mannings Equation and solve the more complex equation for Y. A third approach would be to solve the above equation for n_{eq} and Mannings Eqution simultaneously. This would likely be the approach taken when using such applications as MathCAD or TK-Solver. (See solution to Problem 24 for TK-Solver Model)

The solution is: Y = 5.670 ft 🡐

The Fortran program below used the first approach in solving problem.
Program MANCOMP.FOR

```
      CHARACTER*1 V(6)/'Q','n','S','Y','b','m'/
      REAL X(6)
1     WRITE(*,*)' Give Number of Unknown(0=STOP)'
      DO 10 I=1,6
      IF(I.EQ.2) GO TO 10
      WRITE(*,100) I,V(I)
100   FORMAT(I5,2X,A1)
10    CONTINUE
      READ(*,*) IU
      IF(IU.LT.1) STOP
      WRITE(*,*)' Give n for bottom and n for sides'
      READ(*,*) FNB,FNS
      WRITE(*,*)' Give value after variable (include guess for unknown)'
      DO 20 I=1,6
      IF(I.EQ.2) GO TO 20
      WRITE(*,101) V(I)
101   FORMAT(1X,A1,' =',\)
      READ(*,*) X(I)
20    CONTINUE
      WRITE(*,101)'g'
      READ(*,*) G
      Cu=1.486
      IF(G.LT.20.) Cu=1.
      NCT=0
30    PS=2.*SQRT(X(6)**2+1.)*X(4)
      P=X(5)+PS
      X(2)=(X(5)/P)*FNB+(PS/P)*FNS
      F=X(2)*X(1)-Cu*SQRT(X(3))*((X(5)+X(6)*X(4))*X(4))**1.666667/
     &P**.666667
      NCT=NCT+1
      IF(MOD(NCT,2).EQ.0) GO TO 40
      FF=F
      XX=X(IU)
      X(IU)=1.005*X(IU)
      GO TO 30
40    DIF=(X(IU)-XX)*FF/(F-FF)
      X(IU)=XX-DIF
      IF(ABS(DIF).GT. 1.E-5 .AND. NCT.LT.40) GO TO 30
      IF(NCT.EQ.40) WRITE(*,*)' Failed to converge'
      WRITE(*,*)' Solution including equivalent n'
      WRITE(*,120)(V(I),X(I),I=1,6)
120   FORMAT(1X,A1,' =',F12.6)
      GO TO 1
      END
```

23.

A rectangular laboratory flume has plexiglass sides (n_s = .010) and gravel on its bottom (n_b = .035). The flume is 3 feet wide has a bottom slope of S_o = .001. If the depth of flow is 1.5 feet estimate the flowrate in the flume.

Given: b=3 ft, Y=1.5 ft, S_o=.001, n_b=.035, n_s=.010

Solution: Since the depth is known we can compute the equivalent roughness first, i.e.,

n_{eq} = (3/6).035 + (3/6).010 = .0225

Now solve Mannings Equation for Q, or

$Q = (1.486/.0225)(4.5)^{5/3}/6^{2/3}(.001)^{1/2} = 7.76$ cfs ←

This problem can also be solved with Program MANCOMP, listed in the previous problem

24.

An 8 ft wide rectangular channel made of concrete (n = .013) is filled to a depth of 1 ft with gravel (n = .04). The slope of the channel is S_o = .0015. For a flowrate of Q = 90 cfs predict the height of water in the channel with and without the gravel.

Given: b=8 ft, Q=90.0 cfs S_o=.0015, n_n=.04 (with gravel), n_s=.013 (and bottom without 1 ft of gravel)

Find: Y without and with 1 ft of gravel in bottom of channel.

Solution:

(a) Without gravel: Solve Mannings Equation, i.e.,

$Q = (1.486/.013)A^{5/3}/P^{2/3}S_o^{1/2} = 90 \rightarrow Y_o = 2.067$ ft ←

(b) With 1 ft of gravel in bottom of channel:

$$Q = \frac{1.486}{(b/P).04+(2Y/P).013}\frac{A^{5/3}}{P^{2/3}}(.0015)^{1/2} = 90$$

Solution gives: Y_o = 3.52 ft and therefore depth with 1 ft of gravel is:

Y = 4.52 ft.

The program MANCOMP, listed in Problem 22, can be used to obtain 4.52 ft for the (b) part of the problem.
The TK-Solver Model MANCOMP.TK listed below can also be used to solve this problem.

VARIABLE SHEET

St	Input	Name	Output	Unit
	90	Q		
	1.486	Cu		
		neq	.02736075	
	8	b		
	0	m		
		Y	3.5204974	
		P	15.040995	

```
  .0015     So
  .04       nb
  .013      ns

                RULE SHEET
S Rule
  Q=Cu/neq*((b+m*Y)*Y)^1.666667/P^.6666667*sqrt(So)
  P=b+2*sqrt(m*m+1)*Y
  neq=(b/P)*nb+(2*Y*sqrt(m*m+1)/P)*ns
```

25.

Determine the alternate depth in a rectangular channel to a depth $Y_2 = 0.2$ m if the flow rate is $q = 3$ m^2/s.

Given: $q=3$ m^2/s, $Y_2=0.2$ m
Find: Alternate Depth

Solution:

$Y_1+q^2/(2gY_1^2) = Y_2+q^2/(2gY_2^2) = .2 + 9/(19.62(.04)) = 11.668$ m

$Y_1 = 11.667$ m ←

26.

Determine the alternate depth in a rectangular channel to a depth $Y_2 = 0.2$ m if the flow rate is $q = 3$ m^2/s.

Given: b=5 ft, m=1.2, Q=150 cfs, $Y_2=2.0$ ft

Find: Alternative Depth Y_1

Solution: $E_2 = 4.858$ ft, $F_{r1}= 2.380$, $E_1=4.858$ → $Y_1 = 4.720$ ft ←

27.

Determine the depth that will occur upstream of gate in a trapezoidal channel with b = 6 ft, and m = 1.2 if the flow rate is Q = 200 cfs, and the channel bottom rises in elevation by 1.5 feet across the gate, and the depth of flow downstream of the gate is $Y_2 = 2.0$ ft.

Given: Trapezoidal Channel with b= 6 ft, m=1.2 with Q=200 cfs. Bottom rises 1.5 ft across a gate with downstream depth $Y_2=2.0$ ft
Find: Depth Y_1 upstream from gate

Solution:

$E_2=2+200^2/[64.4((6+1.2(4))4)^2] = 4.201$ ft

$E_1=4.201+1.5 = 5.701$ → $Y_1 = 5.587$ ft ←

Solving problems asking that alternate depths be found are easy using math software such as MathCAD. First the specific energy is solved by giving the equation for it, Then this same equation is solved within a Given – Find block for the alternative depth. The guess provided causes the solution to converge on the correct depth. The model below accomplishes this task.

PRB4_27.MCD

Finds Alternate Depth in Trapezoidal Channel

Variables $b := 6 \quad m := 1.2 \quad Q := 200 \quad Y1 := 2 \quad g := 32.2 \quad E := Y1 + \dfrac{Q^2}{2 \cdot g \cdot ((b + m \cdot Y1) \cdot Y1)^2}$

$delz := 1.5 \qquad E = 4.201 \qquad Y2 := 4$

Given $Y2 + \dfrac{Q^2}{2 \cdot g \cdot ((b + m \cdot Y2) \cdot Y2)^2} - delz = E \qquad Find(Y2) = 5.577$

Using the MATLAB script enertwo.m, that is on the CD in folder CHAPTER under MATLAB results in the follow dialog in the command window to solve this problem.

```
>> enertwo
Give g = 32.2
Give type of channel (0=trap,1=cir) 0
Give number of the unknown
  1    Q1
  2    Q2
  3    KL
  4    dz
  5    Y1
  6    Y2
  7    b1
  8    m1
  9    b2
 10    m2
 5
Give value to variables
Q1(known) = 200
Q2(known) = 200
KL(known) = 0
dz(known) = 1.5
Y1(guess) = 5
Y2(known) = 2
b1(known) = 6
m1(known) = 1.2
b2(known) = 6
m2(known) = 1.2
Specific energy E1 =   5.7007,  E2 =   4.2007, E =   5.7007
Froude Numbers Fr1 =   0.2606,  Fr2 =   1.6821
  1 Q1  (known)   =   200.0000
  2 Q2  (known)   =   200.0000
  3 KL  (known)   =     0.0000
  4 dz  (known)   =     1.5000
  5 Y1  (unknown) =     5.5767      ←
  6 Y2  (known)   =     2.0000
  7 b1  (known)   =     6.0000
  8 m1  (known)   =     1.2000
  9 b2  (known)   =     6.0000
 10 m2  (known)   =     1.2000
```

28.

Determine the alternate depth to 4 m in a trapezoidal channel with b = 3 m, and m = 1.5 if the flow rate is Q = 30 m^3/s.

Given: Trapezoidal Channel with b=3 m, m=1.5, and Q=30 m^3/s.
Find: Alternative depth to Y=4 m

Solution:

Solve for E = 4.0287 m
Now solve for Y with E=4.0354 → Y_{alt} = 0.882 m, ←

Using the MathCAD model from the previous problem gives:
Finds Alternate Depth in Trapezoidal Channel

Variables $b := 3 \quad m := 1.5 \quad Q := 30 \quad Y1 := 4 \quad g := 9.81 \quad E := Y1 + \frac{Q^2}{2 \cdot g \cdot ((b + m \cdot Y1) \cdot Y1)^2} \quad delz := 0$

$E = 4.035 \quad Y2 := 1.5$

Given

$$Y2 + \frac{Q^2}{2 \cdot g \cdot ((b + m \cdot Y2) \cdot Y2)^2} - delz = E \qquad Find(Y2) = 0.882$$

Using MATLAB script "enertwo.m" which is on the CD gives the following:

```
>> enertwo
Give g = 9.81
Give type of channel (0=trap,1=cir) 0
Give number of the unknown
  1    Q1
  2    Q2
  3    KL
  4    dz
  5    Y1
  6    Y2
  7    b1
  8    m1
  9    b2
 10    m2
 6
Give value to variables
Q1(known) = 30
Q2(known) = 30
KL(known) = 0
dz(known) = 0
Y1(known) = 4
Y2(guess) = 1.5
b1(known) = 3
m1(known) = 1.5
b2(known) = 3
m2(known) = 1.5
Specific energy E1 =   4.0354,  E2 =   4.0354, E =   4.0354
Froude Numbers Fr1 =   0.1717,  Fr2 =   3.0555
  1 Q1  (known)   =    30.0000
  2 Q2  (known)   =    30.0000
  3 KL  (known)   =     0.0000
  4 dz  (known)   =     0.0000
  5 Y1  (known)   =     4.0000
  6 Y2  (unknown) =     0.8822
  7 b1  (known)   =     3.0000
  8 m1  (known)   =     1.5000
  9 b2  (known)   =     3.0000
 10 m2  (known)   =     1.5000
```

29.

The depth of flow downstream from a sluice gate in a rectangular channel is 1.2 ft. If the flow rate per unit width under the gate is q = 20 cfs/ft determine the depth upstream of the gate.

<u>Given:</u> Depth Downstream from sluice gate Y_2 = 1.2 ft, and q=20 cfs/ft
<u>Find:</u> Upstream depth Y_1

<u>Solution:</u>

$Y_1+q^2/(2gY_1^2) = Y_2+q^2/(2gY_2^2) = 1.2 + 400/(64.4x(1.2)^2) = 5.513$

Gives cubic equation

$Y^3-5.513Y^2+6.211 = 0$

Use synethic division to exact root

```
1  -5.513     0     6.211  | 1.2
     1.2    -5.176  -6.211  |
---------------------------
   1  -4.313  -5.175
```

$Y_1 = 4.313 + [4.313^2 + 4(5.176)]^{1/2}/2 = 5.291$ ft ←

Using the MATLAB "script enertwo.m," which is on the CD under MATLAB and CHAPTER2 in the back cover of the book results in the following:

```
>> enertwo
Give g = 32.2
Give type of channel (0=trap,1=cir) 0
Give number of the unknown
  1    Q1
  2    Q2
  3    KL
  4    dz
  5    Y1
  6    Y2
  7    b1
  8    m1
  9    b2
 10    m2
 5
Give value to variables
Q1(known) = 20
Q2(known) = 20
KL(known) = 0
dz(known) = 0
Y1(guess) = 5
Y2(known) = 1.2
b1(known) = 1
m1(known) = 0
b2(known) = 1
m2(known) = 0
Specific energy E1 =   5.5133,  E2 =   5.5133, E =   5.5133
Froude Numbers Fr1 =   0.2896,  Fr2 =   2.6812
  1 Q1  (known)   =    20.0000
  2 Q2  (known)   =    20.0000
  3 KL  (known)   =     0.0000
  4 dz  (known)   =     0.0000
  5 Y1  (unknown) =     5.2915
  6 Y2  (known)   =     1.2000
  7 b1  (known)   =     1.0000
  8 m1  (known)   =     0.0000
  9 b2  (known)   =     1.0000
 10 m2  (known)   =     0.0000
```

30.

The depths upstream and downstream from a sluice gate in a rectangular channel are measured to be 5.2 ft and 1.1 ft respectively. If the channel is 15 ft wide determine the flow rate passing under the gate.

Given: Upstream and downstream depth across a gate are 5.2 and 1.1 ft. b=15 ft, Find: Q

Solution:

$E_1=E_2$

$Y_1+Q^2/(2gA_1^2)=Y_2+Q^2/(2gA_2^2)$; $5.8+Q^2/(64.4(15x5.8)^2)=1.1+Q^2/(64.4(15x1.1)^2)$

Q = 274.c cfs ←

The MATLAB script "enertwo.m" can be used to solve problems of this type. It allows for two different flowrates; however if one of the flowrates is designated as the unknown, you are ask if both of these flowrates should be the same.

```
>> enertwo
Give g = 32.2
Give type of channel (0=trap,1=cir) 0
Give number of the unknown
  1    Q1
  2    Q2
  3    KL
  4    dz
  5    Y1
  6    Y2
  7    b1
  8    m1
  9    b2
 10    m2
 1
Give value to variables
Should both flows be computed? (0=no,1=yes) 1
Q1(guess) = 270
Q2(known) = 270
KL(known) = 0
dz(known) = 0
Y1(known) = 5.2
Y2(known) = 1.1
b1(known) = 15
m1(known) = 0
b2(known) = 15
m2(known) = 0
Specific energy E1 =   5.3921,  E2 =   5.3921, E =   5.2579
Froude Numbers Fr1 =   0.2718,  Fr2 =   2.7935
  1 Q1  (unknown) =   274.3216                          ←
  2 Q2  (known)   =   274.3216
  3 KL  (known)   =     0.0000
  4 dz  (known)   =     0.0000
  5 Y1  (known)   =     5.2000
  6 Y2  (known)   =     1.1000
  7 b1  (known)   =    15.0000
  8 m1  (known)   =     0.0000
  9 b2  (known)   =    15.0000
 10 m2  (known)   =     0.0000
```

31.

A specific energy diagram has the specific energy E along the abscissa and the depth Y along the ordinate with different curves for constant values of flowrate Q. Generate tables of values, and then plot these to make a Depth-Discharge diagram for constant specific energies, i.e. graph Q on the abscissa, Y on the ordinate and different curves apply for constant values of the specific. Make this graph specific for a trapezoidal channel with b = 10 ft, m = 1.25, and use three curves for E = 3 ft, E = 4 ft and E = 5 ft. Prove that the maximum flowrate for any constant E corresponds to the critical depth Y_c.

Given: b=10 ft, m=1.25

Wanted: Make specific Energy diagram for E=3 ft, 4 ft and 5 ft

Solution: The Program below is designed to generate data to plot the requested diagram with Q along abscissa and Y along ordinate.

Program Q DIAG.FOR

```
10    READ(*,*) E,B,FM,YC,G,N
      IF(E.LT.1.E-5) STOP
      A=(B+FM*YC)*YC
      QM=SQRT(G*A**3/(B+2.*YC*FM))
      DQ=QM/FLOAT(N)
      WRITE(*,*) QM,DQ
      WRITE(3,100) 0.,0.
100   FORMAT(2F12.5)
      Y1=0.
      G2=2.*G
      Y=.1
      DO 30 I=1,N
      NCT=0
      Q=DQ*FLOAT(I)
      QS=Q**2
20    A=(B+FM*Y)*Y
      F=G2*A**2*(E-Y)-QS
      YM=.995*Y
      AM=(B+FM*YM)*YM
      F1=G2*AM**2*(E-YM)-QS
      DIF=F*(Y-YM)/(F-F1)
      Y=Y-DIF
      NCT=NCT+1
      IF(ABS(DIF).GT.1.E-5 .AND. NCT.LT.20) GO TO 20
      WRITE(*,*) Q,Y,NCT
      WRITE(3,100) Q,Y
      YY=Y
      Y=2.*Y-Y1
30    Y1=YY
      Y=1.05*YY
      DO 50 I=N-1,1,-1
      NCT=0
      Q=DQ*FLOAT(I)
      QS=Q**2
40    A=(B+FM*Y)*Y
      F=G2*A**2*(E-Y)-QS
      YM=.995*Y
      AM=(B+FM*YM)*YM
      F1=G2*AM**2*(E-YM)-QS
      DIF=F*(Y-YM)/(F-F1)
      Y=Y-DIF
      NCT=NCT+1
      IF(ABS(DIF).GT. 1.E-5 .AND. NCT.LT.20) GO TO 40
      WRITE(*,*) Q,Y,NCT
      WRITE(3,100) Q,Y
      YY=Y
      Y=2.*Y-Y1
50    Y1=YY
      WRITE(3,101) 0.,E
```

```
101   FORMAT(2F12.5,/,'                    ')
      GO TO 10
      END
```

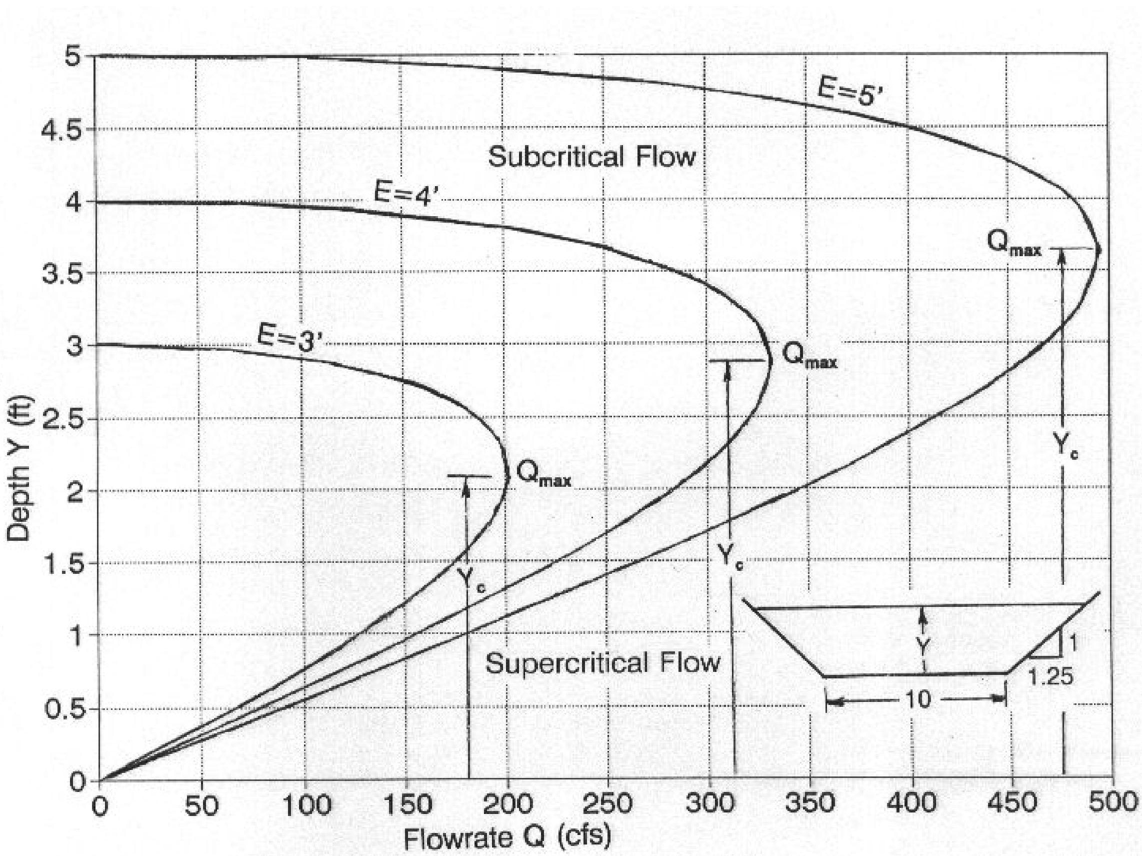
Input to generate diagram for trap. With b-10’,m=1.25 for E=3’,4’ & 5’

```
3  10 1.25 2.123 32.2 50
4  10 1.25 2.866 32.2 50
5  10 1.25 3.620 32.2 50
```

Output:

Q	Y	169.63240	2.63630	99.79554	.62792
.00000	.00000	165.59360	2.66158	106.44860	.67074
4.03887	.02909	161.55470	2.68481	113.10160	.71377
8.07774	.05826	157.51590	2.70629	119.75470	.75703
12.11660	.08751	153.47700	2.72625	126.40770	.80052
16.15547	.11685	149.43810	2.74486	133.06070	.84428
20.19434	.14629	145.39920	2.76227	139.71380	.88832
24.23321	.17582	141.36040	2.77860	146.36680	.93267
28.27208	.20547	137.32150	2.79395	153.01980	.97736
32.31094	.23523	133.28260	2.80840	159.67290	1.02241
36.34981	.26511	129.24380	2.82202	166.32590	1.06785
40.38868	.29513	125.20490	2.83487	172.97890	1.11372
44.42755	.32528	121.16600	2.84701	179.63200	1.16005
48.46642	.35559	117.12720	2.85849	186.28500	1.20689
52.50528	.38605	113.08830	2.86933	192.93800	1.25428
56.54415	.41668	109.04940	2.87959	199.59110	1.30227
60.58302	.44749	105.01060	2.88929	206.24410	1.35091
64.62189	.47850	100.97170	2.89846	212.89720	1.40027
68.66075	.50970	96.93283	2.90713	219.55020	1.45043
72.69962	.54112	92.89397	2.91531	226.20320	1.50145
76.73849	.57277	88.85509	2.92303	232.85630	1.55345
80.77736	.60467	84.81622	2.93031	239.50930	1.60653
84.81622	.63683	80.77736	2.93716	246.16230	1.66082
88.85509	.66927	76.73849	2.94360	252.81540	1.71648
92.89397	.70201	72.69962	2.94965	259.46840	1.77370
96.93283	.73506	68.66075	2.95531	266.12150	1.83272
100.97170	.76846	64.62189	2.96059	272.77450	1.89382
105.01060	.80222	60.58302	2.96551	279.42750	1.95739
109.04940	.83638	56.54415	2.97007	286.08060	2.02393
113.08830	.87097	52.50528	2.97429	292.73360	2.09413
117.12720	.90601	48.46642	2.97816	299.38660	2.16898
121.16600	.94155	44.42755	2.98171	306.03970	2.24999
125.20490	.97763	40.38868	2.98492	312.69270	2.33973
129.24380	1.01430	36.34981	2.98782	319.34570	2.44319
133.28260	1.05162	32.31094	2.99039	325.99880	2.57323
137.32150	1.08963	28.27208	2.99266	332.65180	2.85431
141.36040	1.12843	24.23321	2.99462	325.99880	3.12911
145.39920	1.16810	20.19434	2.99627	319.34570	3.22926
149.43810	1.20873	16.15547	2.99761	312.69270	3.30267
153.47700	1.25044	12.11660	2.99866	306.03970	3.36222
157.51590	1.29339	8.07774	2.99940	299.38660	3.41288
161.55470	1.33776	4.03887	2.99985	292.73360	3.45721
165.59360	1.38376	.00000	3.00000	286.08060	3.49674
169.63240	1.43170			279.42750	3.53243
173.67130	1.48195	.00000	.00000	272.77450	3.56500
177.71020	1.53506	6.65304	.04145	266.12150	3.59492
181.74910	1.59179	13.30607	.08292	259.46840	3.62258
185.78790	1.65331	19.95911	.12440	252.81540	3.64828
189.82680	1.72161	26.61214	.16592	246.16230	3.67223
193.86570	1.80056	33.26518	.20747	239.50930	3.69463
197.90450	1.90024	39.91822	.24908	232.85630	3.71564
201.94340	2.07313	46.57125	.29076	226.20320	3.73538
197.90450	2.32351	53.22429	.33250	219.55020	3.75395
193.86570	2.40123	59.87733	.37434	212.89720	3.77147
189.82680	2.45814	66.53036	.41627	206.24410	3.78799
185.78790	2.50429	73.18340	.45832	199.59110	3.80359
181.74910	2.54357	79.83643	.50050	192.93800	3.81834
177.71020	2.57795	86.48947	.54281	186.28500	3.83227
173.67130	2.60860	93.14250	.58528	179.63200	3.84545

172.97890	3.85791	148.70950	.82212	416.38660	4.43269
166.32590	3.86969	158.62350	.87729	406.47270	4.47227
159.67290	3.88082	168.53740	.93263	396.55870	4.50864
153.01980	3.89132	178.45140	.98815	386.64470	4.54225
146.36680	3.90124	188.36540	1.04390	376.73070	4.57346
139.71380	3.91058	198.27930	1.09989	366.81680	4.60255
133.06070	3.91938	208.19330	1.15616	356.90280	4.62976
126.40770	3.92765	218.10730	1.21273	346.98890	4.65526
119.75470	3.93540	228.02120	1.26965	337.07490	4.67921
113.10160	3.94266	237.93520	1.32694	327.16090	4.70176
106.44860	3.94944	247.84920	1.38465	317.24690	4.72300
99.79554	3.95575	257.76320	1.44282	307.33300	4.74305
93.14250	3.96161	267.67710	1.50150	297.41900	4.76198
86.48947	3.96701	277.59110	1.56073	287.50500	4.77986
79.83643	3.97199	287.50500	1.62058	277.59110	4.79676
73.18340	3.97653	297.41900	1.68110	267.67710	4.81274
66.53036	3.98066	307.33300	1.74237	257.76320	4.82784
59.87733	3.98437	317.24690	1.80447	247.84920	4.84212
53.22429	3.98768	327.16090	1.86748	237.93520	4.85561
46.57125	3.99059	337.07490	1.93150	228.02120	4.86835
39.91822	3.99310	346.98890	1.99667	218.10730	4.88036
33.26518	3.99521	356.90280	2.06310	208.19330	4.89168
26.61214	3.99694	366.81680	2.13097	198.27930	4.90234
19.95911	3.99828	376.73070	2.20047	188.36540	4.91236
13.30607	3.99924	386.64470	2.27183	178.45140	4.92176
6.65304	3.99981	396.55870	2.34534	168.53740	4.93055
.00000	4.00000	406.47270	2.42135	158.62350	4.93876
		416.38660	2.50035	148.70950	4.94641
.00000	.00000	426.30060	2.58293	138.79550	4.95350
9.91397	.05518	436.21450	2.66994	128.88160	4.96005
19.82793	.11022	446.12850	2.76260	118.96760	4.96608
29.74190	.16514	456.04250	2.86277	109.05360	4.97158
39.65587	.21997	465.95650	2.97358	99.13967	4.97658
49.56984	.27472	475.87040	3.10115	89.22571	4.98108
59.48380	.32942	485.78440	3.26128	79.31174	4.98508
69.39777	.38407	495.69840	3.62785	69.39777	4.98860
79.31174	.43870	485.78440	3.94108	59.48380	4.99164
89.22571	.49332	475.87040	4.06341	49.56984	4.99420
99.13967	.54796	465.95650	4.15295	39.65587	4.99629
109.05360	.60264	456.04250	4.22552	29.74190	4.99792
118.96760	.65738	446.12850	4.28722	19.82793	4.99907
128.88160	.71218	436.21450	4.34118	9.91397	4.99977
138.79550	.76709	426.30060	4.38927	.00000	5.00000


E=5'
E=4'
E=3'
Subcritical Flow
Supercritical Flow
Qmax
Qmax
Qmax
Yc
Yc
Yc
Y
1
1.25
10
Depth Y (ft)
Flowrate Q (cfs)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
50
100
150
200
250
300
350
400
450
500

32.

The depth upstream from a sluice gate in a rectangular channel 4.1 m. The gate is set at a distance 0.5 m above the bottom of the channel, and the contraction coefficient for the gate is 0.58. Determine the flow rate passing the gate per unit width.

Given: Depth upstream of gate, Y_1=4.1 m, Gate height Y_G=.5 m (C_c=.58)
Find: q (flow rate per unit width)

Solution:

Y_2=.58(.5) = 0.29 m

Energy $Y_1+q^2/(2gY_1^2) = Y_2+q^2/(2gY_2^2)$

$4.1+q^2/(19.62(4.1)^2) = .29+q^2/(19.62(.29)^2)$

$3.81 = (.606045-.003032)q^2$

q = 2.514 m^2/s ←

33.

A contraction reduces a rectangular channel from b = 4 m to b = 3 m. The downstream channel has a bottom slope S_{o2} = 0.0015, and n_2 = .013. For a flow rate Q = 10 m^3/s determine the upstream depth and the change in water surface elevation. The bottom elevation of the channel does not change through the contraction.

Given: Transition with Q=10 m^3/s with long channel downstream
b_1=4 m, b_2=3 m, S_{o2}=.0015, n_2=.013

Solution:
From Mannings Equation Y_2=1.391 m ←

$E_2=1.391+100/(19.62x4.173^2) = 1.6837$ m

$Y_1+q^2/(2gY_1^2) = 1.6837 = Y_1+100/(19.62x16Y_1^2)$, $Y_1+.31855/Y_1^2=1.6837$

Y_1=1.551 m ← Drop in water surface = 1.684-1.391 = 0.293 m ←

34.

In Problem 33 the bottom rises 0.2 meters. Now determine the upstream depth and change in water surface elevation.

Now Δz=0.2 m in previous problem

So E_1=1.6837-.2 = 1.4837 m

$Y_1+q^2/(2gY_1^2) = 1.4837$ $Y_1+.31855/Y_1^2=1.4837$ Y_1=1.784 m ←

Drop in Water Surface = 1.784 – 1.391 = .193 m ←

35.

In Problem 33 the bottom fall 0.2 meters. Now determine the upstream depth and the change in water surface elevation.

Now Δz=-0.2 m in previous problem

So E_1=1.6837+.2 = 1.8837 → Y_1= 1.293 m ←

Drop in Water Surface = 1.684 – 1.391 -.2 = .093 m ←

36.

How much must the bottom rise or fall in Problem 33 so that critical flow occurs at the smallest section of the contraction, but the water level upstream is the same as in Problem 33?

Critical flow occur at end of transition and Y_1=1.551 m (as in Problem 33)
So E_1=1.6837, q=10/3 = 3.333 m^2/s, $Y_c=(q^2/g)^{1/3}$ = 1.0434 m,
$E_c=1.5Y_c$=1.5636 m
Δz = 1.6836 – 1.5636 = 0.1212 m ←

37.

A transition from a trapezoidal channel with b_1 = 10 ft, m_1 = 1.5, and S_{o1} = 0.0001 to a circular channel with D_2 = 8 ft, n_2 = .012, and S_{o2} = 0.058. What are the depths at the upstream and downstream ends of this transition if the flow rate Q = 450 cfs?

Given: A transition between a trapezoidal and circular channel containing Q = 450 cfs.

b_1=10 ft, m_1=1.5, S_{o1}=.0001, n_1=.013, D_2=8 ft, S_{o2}=.058, n_2=.012

Solution:

Assume critical flow occurs at the beginning of the circular section.

$Q^2T/(gA^3)=1$ → Y_c = 5.402 ft, E_c=7.813 ft. ←

Uniform depth in upstream trapezoidal channel (Solve Y_o from Mannings Equation)

Y_{o1}= 6.975 ft ←, E_{o1}=7.130 ft ←

Solve Energy equation across the transition for the depth at the end of the trapezoidal channel

$Y_1+(Q/A_1)^2/(2g) = E_c$ → Y_1 = 7.699 ft ←

Since $Y_1>Y_{o1}$ assumption of critical flow is correct and the depth at the beginning of the transition will be Y_1 = 7.699 ft. ←

A final thing to check is that the circular channel is steep. In pipe solve Mannings Equation for Y_{o2} = 2.357 ft $<Y_{c2}$.

A TK-Solver model to solve this problem is given below. In this model guesses are provided for the first four variables, with a G in the status column, and values given to the known variables D,n,So,b,m and Q. If the depth Y1 is greater than Yo then critical flow governs as is assumed in solving this system of equations. Should Yo>Y1 then the upstream depth will be Yo and one would need to solve the energy equation Eo=E2 for Y2, i.e., one would not set the Froude Number to 1.

```
________________ VARIABLE SHEET ___________
St Input____ Name___ Output___ Unit_______
             Yc      5.4024632
             Yo      6.9761257
             Y1      7.6985968
             Yo2     2.3571226
   1         Fr2
   450       Q
   8         D
   32.2      g
   10        b
   1.5       m
   1.486     Cu
   .012      n
   .0001     So
   .058      So2
   .012      n2
             beta    1.9290249
             Ac      36.118134
             Ec      7.81286
             Ao      142.76075
             Po      35.152779
             bet2    1.1475534
             Ao2     12.369203

________________ RULE SHEET ________________
S Rule________________________________________
  beta=acos(1-2*Yc/D)
  Ac=.25*D^2*(beta-cos(beta)*sin(beta))
  Fr2=Q^2*D*sin(beta)/(g*Ac^3)
  Ec=Yc+(Q/Ac)^2/(2*g)
  Y1+(Q/((b+m*Y1)*Y1))^2/(2*g)=Ec
  Ao=(b+m*Yo)*Yo
  Po=b+2.*Yo*sqrt(m*m+1)
  Q=Cu/n*(Ao/Po)^.6666667*Ao*sqrt(So)
  bet2=acos(1-2*Yo2/D)
  Ao2=.25*D^2*(bet2-cos(bet2)*sin(bet2))
  Q=Cu/n2*Ao2*(Ao2/(D*bet2))^.6666667*sqrt(So2)
```

To solve the problem for a different flow rate all one needs to do is change the value given to Q, place G in the status columns for the first four variables and press F9, to get the following for Q = 550 cfs.

```
________________ VARIABLE SHEET ___________
St Input____ Name___ Output___ Unit_______
             Yc      5.9766863
             Yo      7.6914736
             Y1      8.7578383
             Yo2     2.6160232
   1         Fr2
   550       Q
   8         D
   32.2      g
   10        b
   1.5       m
   1.486     Cu
   .012      n
   .0001     So
   .058      So2
   .012      n2
             beta    2.087678
             Ac      40.276691
             Ec      8.8722422
             Ao      165.65289
```

```
             Po      37.732002
             bet2    1.2174981
             Ao2     14.285978

Or for Q=250 cfs
_______________ VARIABLE SHEET ___________
St Input____ Name___ Output___ Unit_______
             Yc      3.9769065
             Yo      5.2008788
             Y1      5.4365587
             Yo2     1.7511021
   1         Fr2
   250       Q
   8         D
   32.2      g
   10        b
   1.5       m
   1.486     Cu
   .012      n

   .0001     So
   .058      So2
   .012      n2
             beta    1.5650229
             Ac      24.947994
             Ec      5.5361821
             Ao      92.582498
             Po      28.752035
             bet2    .97372311
             Ao2     8.1403533
```

Another question that could be ask is what diameter D should the downstream channel have so that the flow here is critical, and the upstream depth is normal. To solve for this diameter, as well as the depth in the downstream channel and the normal depth in the upstream channel the following three equations need to be solved for Y2=Yc, Yo(in the upstream channel), and D: (1) The critical flow equation in the circular channel, (2) Mannings equation in the upstream channel, and (3) The energy equation Eo=Ec. The previous TK-Solver model can be modified as given below (PRB2_37A.TK) to give D=9.693 ft.

```
_______________ VARIABLE SHEET ___________
St Input____ Name___ Output___ Unit_______
             Y       5.0985179

             Yo      6.9761257
             D       9.6933895
   1         Fr2
   450       Q
   32.2      g
   10        b
   1.5       m
   1.486     Cu
   .012      n
   .0001     So
             beta    1.6227774
             Ec      7.1304099
             Ao      142.76075
             Po      35.152779
             A       39.338634

_______________ RULE SHEET ________________
S Rule_____________________________________
  beta=acos(1-2*Y/D)
  A=.25*D^2*(beta-cos(beta)*sin(beta))
  Fr2=Q^2*D*sin(beta)/(g*A^3)
  Ec=Y+(Q/A)^2/(2*g)
  Ao=(b+m*Yo)*Yo
  Po=b+2.*Yo*sqrt(m*m+1)
  Q=Cu/n*(Ao/Po)^.6666667*Ao*sqrt(So)
* Q=Cu/n2*Ao2*(Ao2/(D*bet2))^.6666667*sqrt(So2)
  Yo+(Q/Ao)^2/(2*g)=Ec
```

Problem 2:37 Solved using a spreadsheet, i.e., EXCEL (model PRB2_37.XLS)

Q	D	Y	Beta	g	A	T
450	8	5.403145	1.929207	32.2	36.12324	7.491645
Fr2 =	0.999508			Ec =	7.81286	
b	m	A	P	Y	Et	Ec-Et
10	1.5	165.9187	37.76097	7.699506	7.813728	-0.00087
n =	0.012	So1 =	0.0001	Q =	450.0007	0.000724
Yo =	6.976131	Ao =	142.7609	Po =	35.1528	

The above spreadsheet consists of four parts:

(1) The top 4 rows that solve the critical flow equation in the downstream circular section. The critical depth is solved in Cell C2 using "Goal Seek" by setting the Froude Number in Cell B4 to 1 and changing the value in Cell C2.

(2) The solution of E1=E2 in rows 6 and 7. In this part Cell G7 is optimized to 0 using "Goal Seek" by changing the value for the upstream depth Y in Cell E7. This gives the answer to the upstream depth under the assumption that critical depth (i.e. E_c) will control in the downstream circular section.

(3) The Mannings Equation is solved in rows 9 and 10 for the upstream trapezoidal channel. In this part Cell G9, which equates the Q solved in Cell F9 to the given Q in Cell A2, is minimized to 0 using "Goal Seek" by adjusting the normal depth in Cell B10.

(4) Mannings Equation is solved in the downstream circular section for Y_{o2}. Cell C13 contains Mannings Equation written to equal zero, and this cell is optimized to 0 using "Goal Seek" by changing the downstream normal depth Y_{o2} in Cell F11.

The following worksheet solves the problem when Q=550 cfs. To solve problems with different flow rates Q, only the value in Cell A2 needs to have its value changed, and then "Goal Seek" used four times as above to optimize (i.e., solve the equations) for the needed unknown variables. Below the solution for Q=250 cfs is given after the solution for Q = 550 cfs. Note since $E_c>E_o$ that critical depth still governs.

Q	D	Y	Beta	g	A	T
550	8	5.975911	2.087455	32.2	40.2713	6.955796

Fr2 =	1.000529			Ec =	8.872242	

b	m	A	P	Y	Et	Ec-Et
10	1.5	202.6278	41.57682	8.757833	8.872237	5.35E-06

n =	0.012	So1 =	0.0001	Q =	549.9991	-0.00095
Yo =	7.691467	Ao =	165.6527	Po =	37.73198	

Q	D	Y	Beta	g	A	T
250	8	3.976925	1.565028	32.2	24.94814	7.999867

Fr2 =	0.999982			Ec =	5.536182	

b	m	A	P	Y	Et	Ec-Et
10	1.5	98.71181	29.60343	5.437014	5.536613	-0.00043

n =	0.012	So1 =	0.0001	Q =	250	3.12E-05
Yo =	5.200879	Ao =	92.58251	Po =	28.75204	

38.

Water from a reservoir with a water surface elevation 5 m above the bottom of a 10 m wide and steep rectangular channel enters it unrestricted. Determine the flow rate.

Given: H= 5 m (reservoir). Steep rectangular channel with b =10 m
Find: Q

Solution:
$Y_c = (2/3)H = (2/3)5 = 3.333$ m, $q = (gY_c^3)^{1/2} = 5.218$ m^2/s

$Q = bq = 52.18$ m^3/s ←

39.

A gate exists in Problem 38 with a contraction coefficient of 0.8 to control the flow rate. If the gate is 0.3 meters above the channel bottom what is the flow rate?

Given: Gate with Y_G=.3 m and C_c = .8 in b=10 m channel of previous problem.
Find: Q

Solution:

$Y_2 = .8(.3) = 0.24$ m

Energy $5=Y_2+q^2/(2gY_2^2)$ → $q = 2.319$ m^2/s

$Q = bq = 23.19$ m^3/s ←

40.

A transition occurs between an upstream trapezoidal channel with a bottom width $b_1 = 3$ m, and a side slope $m_1 = 1.5$ to a rectangular section with a bottom width $b_2 = 2.5$ m. Simultaneously with the reduction of width the bottom drops by $\Delta z = 0.2$ m from the trapezoidal to the rectangular sections. If the bottom slope of the trapezoidal channel is $S_{o1} = .0006$ and has a Mannings roughness coefficient n = .013, determine the maximum flowrate that can pass through the transition without increasing the depth upstream therefrom above the uniform depth. What is the maximum flowrate if the bottom remains horizontal across the transition?

Given: A transition from b_1=3 m, m_1=1.5 to b_2=2.5 m, m_2=0, with rise of 0.2 m. S_{o1}=.0006, n=.013
Find: Maximum Q with increasing depth above normal upstream
Q_{max}=? if bottom remains horizontal

Solution: Unknowns: Q_{max} & Y_1

Equations:

$$E_1 = Y_1 + (Q/A_1)^2/(2g) = E_c = (3/2)Y_c = (3/2)[(Q/2.5)^2/g]^{1/3} - .2 \qquad (1)$$

$$Q = (1/n)A_1^{5/3}/P_1^{2/3}S_{o1}^{1/2} = (1/.013)A_1^{5/3}/P_1^{2/3}(.0006)^{1/2} \qquad (2)$$

Gives: $Q=Q_{max}$= 5.992 m^3/s and Y_1 = 0.952 m ←

E_1=1.055 m, $Y_c=Y_2 = [(Q/2.5)^2/g]^{1/3}$ = 0.837 m, E_c = 1.255 m

(c) Same two equations with Δz=0 gives:

Q = 1.013 m^3/s and Y_1 = 0.349 m ← $Y_c=Y_2$ = 0.256 m

41.

A steep and a mild rectangular channel with a bottom width of b = 10 feet take water from a reservoir whose water surface is 5 feet above the channel bottom. The mild channel has a bottom slope S_o = .00075, and a Mannings roughness coefficient n = .013. Determine the flowrate in both of these channel. Assume the entrance loss coefficient is 0.

Given: Flow from reservoir with H=5 ft comes into b= 10 ft wide (a) steep channel and (b) mild channel with n=.013 and S_o=.00075
Find: Q

Solution:

Steep

$Y_c = (2/3)H = 3.333$ ft

$q = (gY_c^3)^{1/2} = (32.2 \times 3.333^3)^{1/2} = 34.534$ cfs/ft

$Q = 345.39$ cfs ←

Mild

$5 = Y_o + (Q/A_o)^2/(2g)$ (1)

$Q = (1.486/.013) A_o^{5/3}/P_o^{2/3} (.00075)^{1/2}$ (2)

Solution:

$Y = 4.518$ ft, $Q = 251.7$ cfs ←

42.

A transition from a trapezoidal channel with b_1 = 4 m, m_1 = 1 to a rectangular channel with b_2 = 3 m occurs. The bottom of the channel remains at the same elevation, and both channel have a Mannings roughness coefficient, $n_1 = n_2$ = .014. The upstream channel has a bottom slope S_{o1} = .0005, and the downstream channel has a bottom slope S_{o2} = .009. If a flowrate of Q = 24 m^3/s is occurring what are the depths at the beginning of the transition and at its end?

What change in elevation of the bottom of the channel would be necessary so the upstream depth remains at uniform depth?

What should the width of the downstream rectangular channel be if uniform flow to exist throughout the upstream channel if the bottom elevation does not change?

Given: At transition from b_1=4 m, m_1=1 to rectangular channel with b_2=3 m, with n=.014. S_{o1}=.0005, S_{o2}=.009, and Q = 24 m^3/s.

Find: (a) Y_1=? (b) Δz so uniform flows exist upstream and downstream, (c) What bottom width b_2 if Δz=0

Solution:

Solving Mannings equation downstream Y_{o2}=1.447 m, E_{o2}=3.004 m, F_{r2}=1.467

$E_1 = E_2 = 3.004$ → $Y_1 = 2.933$ m, F_{r1}=.262

Y_{o1}=2.089 m <Y_1 so upstream depth is Y_1 = 2.933 m ←
E_{o1}=2.271 m

(b) $E_{o1} = E_{o2} + \Delta z$, $\Delta z = 2.271 - 3.004 = -0.733$ m drop ←

(c) $E_2 = E_{o1} = 2.271 = 1.447 + (24)^2/[19.62(1.447b)^2]$

$.824 = 14.021/b^2$ $b = 4.125$ m ←

43.

Generate the data needed to plot a dimensionless specific energy graph that has the subcritical dimensionless depth $Y_1' = mY_1/b$ as the abscissa and the supercritical depth $Y_2' = mY_2/b$ as the ordinate that applies for trapezoidal channels, and create this graph. The graph should have curves for $Q' = m^3Q^2/(gb^5)$ the same as Figure 4.

Wanted: A dimensionless specific energy graph for trapezoidal channels that has the subcritical depth along the abscissa and supercritical depth plotted on the ordinate.

For a trapezoidal section: $A=bY+mY^2$ and $T=b+2mY$, so critical depth is defined by,

$Q^2(b+2mY_c)/g = (bY_c+mY_c^2)^3$ Let $\mathbf{Y' = mY_c/b}$ and $\mathbf{Q' = m^3Q^2/(gb^5)}$

$(bQ^2/g)(1+2Y_c') = [(b^2/m)(mY_c/b+m^2Y_c^2/b^2)]^3$ or

$$\mathbf{Q'(1+2Y_c') = (Y_c'+Y_c')^3} \qquad (1)$$

Alternate Depths $E_1 = E_2$

$E=Y+Q^2/[(2g)(bY+mY^2)^2]$ multiple by m/b

$mE/b=mY/b+mQ^2\{2gb[(b^2/m)(mY/b+m^2Y^2/b^2)]^2\}$ or

$E' = Y' + mQ^2/[(2gb^5)(Y'+Y'^2)^2]$ or $\mathbf{E' = Y' + .5Q'/(Y'+Y'^2)^2}$

$E_1' = E_2'$ results in,

$$\mathbf{Y_1' + .5Q'/(Y_1'+Y_1'^2)^2 = Y_2' + .5Q'/(Y_2'+Y_2'^2)^2} \leftarrow \qquad (2)$$

The program given below is written to generate data to plot the requested graph. Notice that this program first solves the dimensionless critical flow equation (1) for Y_c', and thereafter for 30 increments of Y_2' solves the corresponding values of Y_1' from equation (2).

Program PRB2 41A.FOR to solve for dimensionless alternative depths, trapezoidal channels.

```
      REAL Qp(24)/.001,.0025,.005,.01,.015,.02,.03,.04,.05,.07,.08,.09,
     &.1,.12,.14,.16,.18,.2,.3,.4,.5,.6,.8,.9/
      YPc=.07
      DO 10 I=1,24
      M=0
1     F=Qp(I)*(1.+2.*YPc)-(YPc+YPc**2)**3
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 2
      YY=YPc
      FF=F
      YPc=1.005*YPc
      GO TO 1
2     DIF=(YPc-YY)*FF/(F-FF)
      YPc=YY-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 1
      WRITE(3,100) Qp(I),YPc,YPc,YPc+.5*QP(I)/(YPc+YPc**2)**2
100   FORMAT(' For Q''=',F8.3,/3F10.5)
      DY=(1.5-YPc)/30.
      IF(I.EQ.1) YP1=.9*YPc
      DO 10 J=1,30
      YP2=YPc+DY*FLOAT(J)
```

```
      E2=YP2+.5*QP(I)/(YP2+YP2**2)**2
      M=0
4     F=YP1+.5*QP(I)/(YP1+YP1**2)**2-E2
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 6
      YY=YP1
      FF=F
      YP1=1.005*YP1
      GO TO 4
6     DIF=(YP1-YY)*FF/(F-FF)
      YP1=YY-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 4
      IF(M.EQ.30) WRITE(*,*)' Failed',I,J,DIF
10    WRITE(3,'(3F10.5)') YP2,YP1,E2
      END
```

Output from above Program:

Y_2'	Y_1'	E'
For Q'=	.001	
.09672	.09672	.14116
.14349	.06838	.16206
.19027	.05576	.20002
.23705	.04836	.24286
.28382	.04337	.28759
.33060	.03970	.33318
.37737	.03686	.37922
.42415	.03457	.42552
.47093	.03267	.47197
.51770	.03106	.51851
.56448	.02968	.56512
.61125	.02847	.61177
.65803	.02740	.65845
.70481	.02644	.70515
.75158	.02558	.75187
.79836	.02480	.79860
.84513	.02409	.84534
.89191	.02344	.89209
.93869	.02284	.93884
.98546	.02229	.98559
1.03224	.02177	1.03235
1.07902	.02129	1.07911
1.12579	.02084	1.12588
1.17257	.02041	1.17264
1.21934	.02002	1.21941
1.26612	.01964	1.26618
1.31290	.01929	1.31295
1.35967	.01895	1.35972
1.40645	.01863	1.40649
1.45322	.01833	1.45326
1.50000	.01804	1.50004
For Q'=	.002	
.12974	.12974	.18792
.17541	.09883	.20482
.22109	.08282	.23824
.26676	.07280	.27771
.31244	.06581	.31987
.35811	.06058	.36340
.40379	.05647	.40768
.44947	.05313	.45241
.49514	.05034	.49742
.54082	.04797	.54262
.58649	.04591	.58794
.63217	.04411	.63334
.67784	.04252	.67881
.72352	.04108	.72432
.76919	.03979	.76987
.81487	.03862	.81544
.86054	.03755	.86103
.90622	.03657	.90664
.95189	.03566	.95226
.99757	.03481	.99789
1.04325	.03403	1.04352
1.08892	.03330	1.08916
1.13460	.03261	1.13481
1.18027	.03197	1.18046
1.22595	.03136	1.22612
1.27162	.03079	1.27177
1.31730	.03025	1.31743
1.36297	.02973	1.36309
1.40865	.02925	1.40876
1.45432	.02878	1.45442
1.50000	.02834	1.50009
For Q'=	.005	
.16161	.16161	.23255
.20622	.12923	.24663
.25084	.11063	.27623
.29545	.09837	.31252
.34006	.08956	.35210
.38468	.08285	.39349
.42929	.07751	.43593
.47390	.07313	.47903
.51852	.06946	.52255
.56313	.06631	.56635
.60774	.06358	.61036
.65235	.06117	.65451
.69697	.05903	.69875
.74158	.05711	.74308
.78619	.05537	.78746
.83081	.05379	.83189
.87542	.05234	.87635
.92003	.05101	.92083
.96464	.04978	.96534
1.00926	.04863	1.00987
1.05387	.04757	1.05440
1.09848	.04657	1.09895
1.14310	.04564	1.14351
1.18771	.04476	1.18808
1.23232	.04393	1.23265
1.27694	.04315	1.27723
1.32155	.04241	1.32181
1.36616	.04170	1.36640
1.41077	.04103	1.41099
1.45539	.04040	1.45558
1.50000	.03979	1.50018
For Q'=	.010	
.20079	.20079	.28680
.24410	.16741	.29831
.28740	.14639	.32393
.33071	.13179	.35653
.37402	.12095	.39295
.41732	.11252	.43162
.46063	.10571	.47168
.50394	.10007	.51264
.54725	.09530	.55422
.59055	.09119	.59622
.63386	.08759	.63852
.67717	.08442	.68104
.72047	.08158	.72373
.76378	.07903	.76654
.80709	.07672	.80944
.85039	.07461	.85241
.89370	.07267	.89545
.93701	.07089	.93853
.98032	.06923	.98164
1.02362	.06770	1.02479
1.06693	.06626	1.06796
1.11024	.06492	1.11115
1.15354	.06365	1.15435
1.19685	.06247	1.19757
1.24016	.06135	1.24081
1.28346	.06029	1.28405
1.32677	.05928	1.32730
1.37008	.05833	1.37055
1.41339	.05742	1.41382
1.45669	.05655	1.45708
1.50000	.05573	1.50036
For Q'=	.015	
.22765	.22765	.32367
.27006	.19395	.33381
.31248	.17166	.35707
.35489	.15570	.38733
.39730	.14362	.42163
.43971	.13409	.45843
.48212	.12632	.49681
.52453	.11984	.53626
.56695	.11432	.57645
.60936	.10955	.61716
.65177	.10537	.65824
.69418	.10166	.69960
.73659	.09834	.74118
.77900	.09535	.78291
.82141	.09263	.82477
.86383	.09015	.86672
.90624	.08786	.90875
.94865	.08576	.95084
.99106	.08380	.99299
1.03347	.08198	1.03517
1.07588	.08028	1.07739
1.11830	.07869	1.11963
1.16071	.07719	1.16190
1.20312	.07578	1.20419
1.24553	.07445	1.24649
1.28794	.07319	1.28881
1.33035	.07199	1.33113
1.37277	.07086	1.37347
1.41518	.06978	1.41582
1.45759	.06874	1.45817
1.50000	.06776	1.50053
For Q'=	.020	

.24869	.24869	.35239
.29040	.21488	.36161
.33211	.19178	.38321
.37382	.17489	.41174
.41553	.16192	.44444
.45725	.15158	.47977
.49896	.14310	.51683
.54067	.13598	.55508
.58238	.12989	.59415
.62409	.12461	.63382
.66580	.11997	.67393
.70751	.11584	.71436
.74922	.11214	.75504
.79093	.10880	.79591
.83264	.10576	.83693
.87435	.10298	.87807
.91606	.10042	.91930
.95777	.09806	.96061
.99948	.09586	1.00198
1.04119	.09382	1.04340
1.08290	.09191	1.08486
1.12461	.09011	1.12636
1.16632	.08843	1.16788
1.20803	.08684	1.20943
1.24974	.08534	1.25100
1.29145	.08392	1.29259
1.33316	.08257	1.33419
1.37487	.08128	1.37581
1.41658	.08006	1.41743
1.45829	.07890	1.45907
1.50000	.07778	1.50071

For Q'= .030

.28140	.28140	.39676
.32202	.24762	.40479
.36264	.22353	.42407
.40326	.20540	.45010
.44388	.19119	.48040
.48450	.17969	.51350
.52512	.17016	.54851
.56574	.16209	.58486
.60636	.15515	.62217
.64698	.14909	.66019
.68760	.14374	.69874
.72822	.13897	.73769
.76884	.13468	.77695
.80946	.13080	.81645
.85008	.12726	.85614
.89070	.12401	.89599
.93132	.12102	.93596
.97194	.11826	.97602
1.01256	.11568	1.01617
1.05318	.11328	1.05639
1.09380	.11104	1.09666
1.13442	.10893	1.13698
1.17504	.10694	1.17734
1.21566	.10507	1.21773
1.25628	.10330	1.25815
1.29690	.10162	1.29859
1.33752	.10002	1.33905
1.37814	.09851	1.37954
1.41876	.09706	1.42003
1.45938	.09568	1.46054
1.50000	.09436	1.50107

For Q'= .040

.30694	.30694	.43122
.34671	.27331	.43845
.38648	.24864	.45613
.42625	.22971	.48036
.46602	.21465	.50887
.50578	.20234	.54026
.54555	.19204	.57368
.58532	.18327	.60855
.62509	.17569	.64447
.66486	.16904	.68118
.70463	.16315	.71849
.74440	.15789	.75626
.78416	.15315	.79438
.82393	.14885	.83279
.86370	.14492	.87142
.90347	.14131	.91023
.94324	.13798	.94919
.98301	.13489	.98827
1.02278	.13202	1.02745
1.06254	.12934	1.06671
1.10231	.12683	1.10604
1.14208	.12447	1.14542
1.18185	.12225	1.18486
1.22162	.12015	1.22433
1.26139	.11816	1.26385
1.30116	.11628	1.30339
1.34093	.11449	1.34296
1.38069	.11278	1.38255
1.42046	.11116	1.42215
1.46023	.10960	1.46178
1.50000	.10812	1.50142

For Q'= .050

.32818	.32818	.45976
.36724	.29475	.46640
.40631	.26972	.48288
.44537	.25021	.50570
.48443	.23452	.53277
.52349	.22158	.56279
.56255	.21069	.59490
.60161	.20137	.62854
.64067	.19327	.66330
.67973	.18616	.69891
.71879	.17983	.73517
.75785	.17417	.77194
.79691	.16906	.80910
.83597	.16441	.84658
.87503	.16015	.88432
.91409	.15624	.92226
.95315	.15263	.96037
.99221	.14928	.99861
1.03127	.14616	1.03697
1.07033	.14325	1.07543
1.10939	.14051	1.11396
1.14846	.13794	1.15256
1.18752	.13552	1.19122
1.22658	.13323	1.22993
1.26564	.13106	1.26868
1.30470	.12901	1.30746
1.34376	.12705	1.34628
1.38282	.12519	1.38512
1.42188	.12341	1.42399
1.46094	.12172	1.46287
1.50000	.12009	1.50178

For Q'= .070

.36274	.36274	.50598
.40064	.32974	.51179
.43855	.30429	.52649
.47646	.28402	.54719
.51437	.26744	.57205
.55228	.25360	.59990
.59019	.24183	.62993
.62810	.23166	.66157
.66601	.22278	.69444
.70391	.21493	.72824
.74182	.20792	.76279
.77973	.20162	.79791
.81764	.19592	.83349
.85555	.19071	.86944
.89346	.18594	.90569
.93137	.18155	.94218
.96928	.17748	.97888
1.00719	.17370	1.01575
1.04509	.17018	1.05276
1.08300	.16688	1.08988
1.12091	.16379	1.12710
1.15882	.16087	1.16441
1.19673	.15812	1.20179
1.23464	.15552	1.23924
1.27255	.15306	1.27673
1.31046	.15072	1.31427
1.34836	.14849	1.35186
1.38627	.14637	1.38947
1.42418	.14435	1.42712
1.46209	.14241	1.46479
1.50000	.14055	1.50249

For Q'= .080

.37733	.37733	.52543
.41475	.34455	.53093
.45218	.31899	.54495
.48960	.29845	.56480
.52702	.28156	.58878
.56444	.26736	.61574
.60186	.25525	.64490
.63929	.24475	.67571
.67671	.23556	.70778
.71413	.22741	.74083
.75155	.22013	.77464
.78898	.21356	.80905
.82640	.20761	.84396
.86382	.20218	.87925
.90124	.19720	.91487
.93867	.19260	.95074
.97609	.18834	.98684
1.01351	.18439	1.02312
1.05093	.18069	1.05954
1.08835	.17723	1.09610
1.12578	.17398	1.13276
1.16320	.17093	1.16952
1.20062	.16804	1.20635
1.23804	.16531	1.24325
1.27547	.16272	1.28022
1.31289	.16026	1.31723
1.35031	.15791	1.35428
1.38773	.15568	1.39138
1.42516	.15355	1.42850
1.46258	.15151	1.46566
1.50000	.14956	1.50284

For Q'= .090

.39064	.39064	.54313
.42762	.35808	.54837
.46460	.33244	.56179
.50158	.31169	.58091
.53855	.29452	.60410
.57553	.28003	.63026
.61251	.26762	.65864
.64949	.25684	.68870
.68647	.24736	.72004
.72345	.23895	.75240
.76043	.23142	.78554
.79741	.22463	.81931
.83438	.21846	.85359
.87136	.21282	.88829
.90834	.20764	.92332
.94532	.20286	.95863
.98230	.19843	.99417
1.01928	.19431	1.02990
1.05626	.19046	1.06580
1.09323	.18685	1.10183
1.13021	.18347	1.13798

1.16719	.18028	1.17422
1.20417	.17726	1.21056
1.24115	.17441	1.24697
1.27813	.17171	1.28344
1.31511	.16914	1.31996
1.35209	.16669	1.35653
1.38906	.16435	1.39315
1.42604	.16212	1.42980
1.46302	.15999	1.46649
1.50000	.15795	1.50320
For Q'=	.100	
.40290	.40290	.55940
.43947	.37054	.56441
.47604	.34486	.57731
.51261	.32394	.59577
.54918	.30653	.61826
.58575	.29179	.64370
.62232	.27912	.67137
.65889	.26808	.70074
.69546	.25836	.73142
.73203	.24971	.76313
.76860	.24196	.79566
.80517	.23495	.82884
.84174	.22859	.86254
.87831	.22276	.89668
.91488	.21741	.93117
.95145	.21246	.96595
.98802	.20787	1.00098
1.02459	.20360	1.03621
1.06116	.19961	1.07161
1.09773	.19587	1.10716
1.13430	.19236	1.14283
1.17087	.18904	1.17861
1.20744	.18592	1.21448
1.24401	.18295	1.25043
1.28058	.18014	1.28644
1.31715	.17747	1.32252
1.35372	.17492	1.35864
1.39029	.17250	1.39482
1.42686	.17018	1.43103
1.46343	.16796	1.46728
1.50000	.16583	1.50356
For Q'=	.120	
.42493	.42493	.58859
.46076	.39297	.59321
.49660	.36725	.60522
.53243	.34607	.62256
.56827	.32830	.64381
.60411	.31314	.66800
.63994	.30003	.69442
.67578	.28855	.72256
.71161	.27841	.75206
.74745	.26935	.78262
.78328	.26121	.81404
.81912	.25384	.84614
.85496	.24712	.87881
.89079	.24097	.91194
.92663	.23530	.94545
.96246	.23006	.97928
.99830	.22519	1.01338
1.03413	.22066	1.04769
1.06997	.21642	1.08220
1.10581	.21244	1.11687
1.14164	.20869	1.15168
1.17748	.20516	1.18661
1.21331	.20183	1.22163
1.24915	.19866	1.25675
1.28499	.19566	1.29194
1.32082	.19281	1.32721
1.35666	.19009	1.36253
1.39249	.18749	1.39790
1.42833	.18501	1.43332
1.46416	.18263	1.46877
1.50000	.18036	1.50427
For Q'=	.140	
.44438	.44438	.61429
.47957	.41281	.61860
.51475	.38710	.62989
.54994	.36575	.64629
.58513	.34769	.66650
.62032	.33220	.68961
.65550	.31873	.71494
.69069	.30690	.74202
.72588	.29640	.77048
.76107	.28701	.80003
.79625	.27854	.83047
.83144	.27086	.86163
.86663	.26384	.89338
.90181	.25741	.92561
.93700	.25147	.95825
.97219	.24597	.99123
1.00738	.24086	1.02450
1.04256	.23609	1.05800
1.07775	.23163	1.09171
1.11294	.22744	1.12560
1.14813	.22350	1.15963
1.18331	.21978	1.19380
1.21850	.21626	1.22808
1.25369	.21292	1.26246
1.28888	.20975	1.29692
1.32406	.20674	1.33146
1.35925	.20386	1.36606
1.39444	.20112	1.40072
1.42963	.19849	1.43543
1.46481	.19598	1.47018
1.50000	.19357	1.50498
For Q'=	.160	
.46186	.46186	.63735
.49647	.43065	.64140
.53107	.40500	.65207
.56568	.38352	.66767
.60028	.36525	.68698
.63489	.34949	.70914
.66949	.33573	.73353
.70410	.32360	.75967
.73870	.31281	.78720
.77330	.30312	.81585
.80791	.29437	.84541
.84251	.28642	.87571
.87712	.27914	.90663
.91172	.27246	.93806
.94633	.26628	.96991
.98093	.26056	1.00212
1.01554	.25524	1.03463
1.05014	.25026	1.06740
1.08475	.24560	1.10039
1.11935	.24123	1.13357
1.15395	.23711	1.16690
1.18856	.23321	1.20038
1.22316	.22953	1.23398
1.25777	.22604	1.26769
1.29237	.22272	1.30149
1.32698	.21956	1.33537
1.36158	.21654	1.36932
1.39619	.21366	1.40333
1.43079	.21091	1.43740
1.46540	.20827	1.47152
1.50000	.20574	1.50569
For Q'=	.180	
.47779	.47779	.65832
.51186	.44692	.66215
.54594	.42134	.67229
.58001	.39978	.68718
.61408	.38134	.70569
.64816	.36536	.72702
.68223	.35136	.75056
.71631	.33898	.77585
.75038	.32792	.80255
.78445	.31799	.83038
.81853	.30899	.85915
.85260	.30079	.88867
.88667	.29329	.91883
.92075	.28638	.94952
.95482	.28000	.98065
.98889	.27408	1.01216
1.02297	.26856	1.04398
1.05704	.26340	1.07608
1.09112	.25857	1.10840
1.12519	.25403	1.14093
1.15926	.24974	1.17363
1.19334	.24570	1.20647
1.22741	.24186	1.23945
1.26148	.23823	1.27254
1.29556	.23477	1.30573
1.32963	.23148	1.33901
1.36371	.22834	1.37237
1.39778	.22534	1.40579
1.43185	.22247	1.43928
1.46593	.21972	1.47281
1.50000	.21708	1.50640
For Q'=	.200	
.49244	.49244	.67758
.52603	.46190	.68122
.55961	.43641	.69089
.59320	.41480	.70516
.62679	.39622	.72297
.66037	.38006	.74355
.69396	.36586	.76632
.72754	.35325	.79084
.76113	.34198	.81678
.79471	.33182	.84387
.82830	.32260	.87190
.86188	.31419	.90071
.89547	.30649	.93018
.92905	.29938	.96019
.96264	.29281	.99065
.99622	.28671	1.02151
1.02981	.28101	1.05269
1.06339	.27569	1.08416
1.09698	.27070	1.11588
1.13056	.26600	1.14780
1.16415	.26157	1.17990
1.19773	.25738	1.21217
1.23132	.25342	1.24457
1.26490	.24965	1.27709
1.29849	.24607	1.30972
1.33207	.24266	1.34244
1.36566	.23940	1.37524
1.39924	.23629	1.40812
1.43283	.23331	1.44106
1.46641	.23046	1.47406
1.50000	.22772	1.50711
For Q'=	.300	
.55260	.55260	.75637
.58418	.52347	.75932
.61576	.49854	.76730
.64734	.47693	.77925
.67892	.45802	.79437
.71050	.44132	.81206
.74208	.42643	.83183
.77366	.41308	.85332
.80524	.40101	.87623
.83682	.39005	.90031

.86840	.38003	.92538
.89998	.37083	.95128
.93156	.36235	.97789
.96314	.35449	1.00510
.99472	.34719	1.03282
1.02630	.34038	1.06098
1.05788	.33401	1.08953
1.08946	.32804	1.11841
1.12104	.32241	1.14757
1.15262	.31711	1.17699
1.18420	.31209	1.20662
1.21578	.30734	1.23645
1.24736	.30283	1.26645
1.27894	.29855	1.29660
1.31052	.29446	1.32688
1.34210	.29056	1.35728
1.37368	.28684	1.38779
1.40526	.28327	1.41839
1.43684	.27986	1.44908
1.46842	.27658	1.47984
1.50000	.27343	1.51067

For Q'= .400

.59910	.59910	.81701
.62913	.57115	.81952
.65916	.54681	.82637
.68919	.52542	.83676
.71922	.50647	.85003
.74925	.48954	.86568
.77928	.47431	.88331
.80931	.46054	.90259
.83934	.44801	.92325
.86937	.43655	.94509
.89940	.42601	.96793
.92943	.41630	.99162
.95946	.40730	1.01605
.98949	.39893	1.04110
1.01952	.39112	1.06670
1.04955	.38382	1.09277
1.07958	.37697	1.11926
1.10961	.37052	1.14611
1.13964	.36444	1.17328
1.16967	.35870	1.20072
1.19970	.35326	1.22842
1.22973	.34809	1.25633
1.25976	.34318	1.28444
1.28979	.33851	1.31272
1.31982	.33405	1.34116
1.34985	.32978	1.36973
1.37988	.32571	1.39843
1.40991	.32180	1.42723
1.43994	.31805	1.45614
1.46997	.31445	1.48514
1.50000	.31099	1.51422

For Q'= .500

.63749	.63749	.86691
.66624	.61055	.86910
.69499	.58681	.87515
.72374	.56571	.88437
.75249	.54684	.89625
.78124	.52984	.91034
.80999	.51445	.92630
.83874	.50043	.94385
.86749	.48761	.96275
.89624	.47582	.98280
.92499	.46494	1.00384
.95374	.45486	1.02574
.98249	.44548	1.04839
1.01124	.43674	1.07168
1.03999	.42857	1.09553
1.06874	.42089	1.11989
1.09749	.41368	1.14467
1.12624	.40688	1.16984
1.15499	.40045	1.19535
1.18375	.39436	1.22116
1.21250	.38859	1.24723
1.24125	.38310	1.27355
1.27000	.37787	1.30008
1.29875	.37289	1.32680
1.32750	.36813	1.35368
1.35625	.36358	1.38073
1.38500	.35922	1.40791
1.41375	.35503	1.43522
1.44250	.35102	1.46264
1.47125	.34716	1.49016
1.50000	.34344	1.51778

For Q'= .600

.67042	.67042	.90963
.69807	.64438	.91158
.72573	.62120	.91699
.75338	.60044	.92531
.78103	.58172	.93607
.80869	.56475	.94891
.83634	.54929	.96353
.86399	.53514	.97966
.89164	.52213	.99710
.91930	.51012	1.01566
.94695	.49899	1.03521
.97460	.48864	1.05561
1.00225	.47900	1.07675
1.02991	.46997	1.09855
1.05756	.46151	1.12092
1.08521	.45355	1.14380
1.11286	.44605	1.16713
1.14052	.43897	1.19085
1.16817	.43226	1.21493
1.19582	.42590	1.23933
1.22347	.41986	1.26401
1.25113	.41411	1.28895
1.27878	.40863	1.31411
1.30643	.40339	1.33947
1.33408	.39838	1.36502
1.36174	.39359	1.39074
1.38939	.38900	1.41661
1.41704	.38458	1.44262
1.44469	.38035	1.46874
1.47235	.37627	1.49499
1.50000	.37234	1.52133

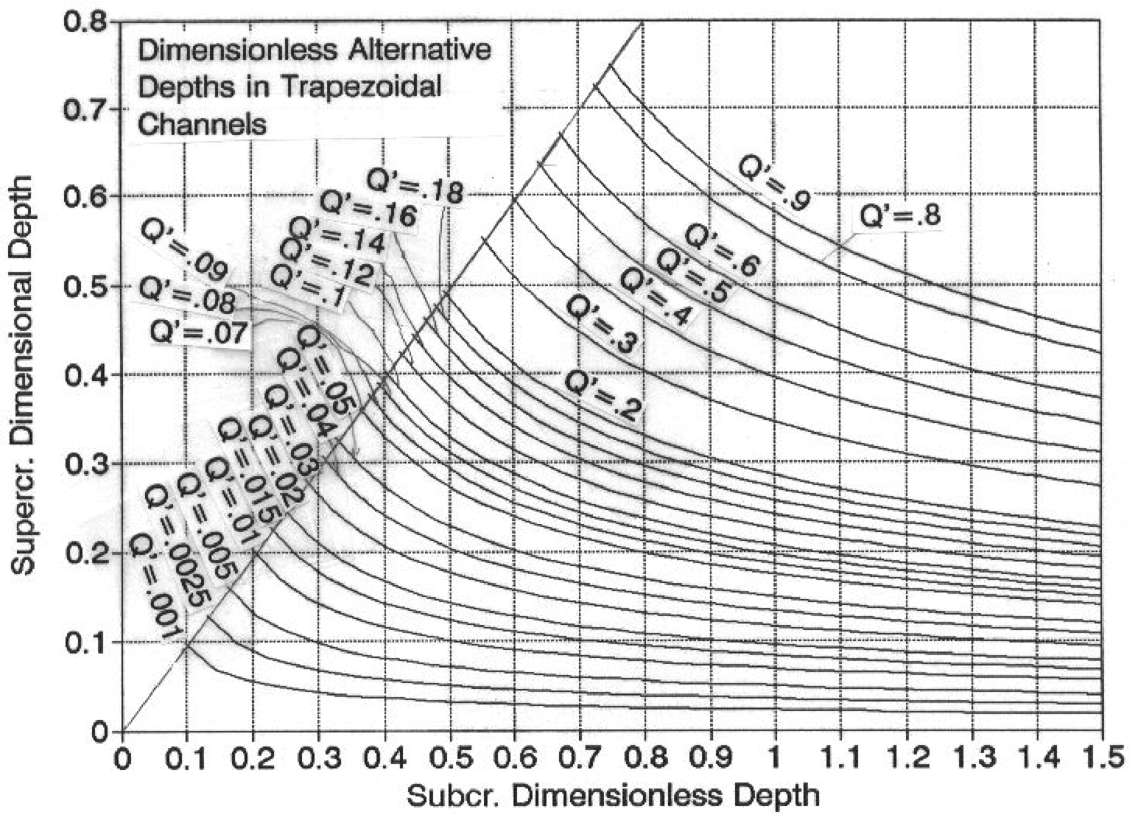
For Q'= .800

.72540	.72540	.98074
.75122	.70090	.98234
.77704	.67877	.98683
.80286	.65870	.99378
.82868	.64039	1.00286
.85450	.62362	1.01379
.88032	.60820	1.02631
.90614	.59396	1.04022
.93196	.58078	1.05535
.95778	.56852	1.07154
.98360	.55710	1.08868
1.00942	.54641	1.10664
1.03524	.53640	1.12534
1.06106	.52699	1.14470
1.08688	.51813	1.16463
1.11270	.50976	1.18508
1.13852	.50185	1.20600
1.16434	.49435	1.22733
1.19016	.48723	1.24903
1.21598	.48045	1.27107
1.24180	.47400	1.29341
1.26762	.46785	1.31603
1.29344	.46197	1.33890
1.31926	.45634	1.36199
1.34508	.45095	1.38528
1.37090	.44578	1.40876
1.39672	.44081	1.43241
1.42254	.43604	1.45622
1.44836	.43144	1.48017
1.47418	.42702	1.50425
1.50000	.42275	1.52844

For Q'= .900

.74901	.74901	1.01122
.77404	.72518	1.01269
.79908	.70355	1.01682
.82411	.68382	1.02324
.84914	.66574	1.03166
.87417	.64911	1.04182
.89921	.63377	1.05350
.92424	.61955	1.06651
.94927	.60634	1.08070
.97431	.59402	1.09592
.99934	.58251	1.11206
1.02437	.57173	1.12902
1.04941	.56159	1.14670
1.07444	.55205	1.16502
1.09947	.54305	1.18393
1.12450	.53454	1.20335
1.14954	.52647	1.22324
1.17457	.51881	1.24355
1.19960	.51154	1.26424
1.22464	.50461	1.28527
1.24967	.49800	1.30661
1.27470	.49168	1.32823
1.29974	.48565	1.35010
1.32477	.47986	1.37221
1.34980	.47432	1.39453
1.37483	.46900	1.41705
1.39987	.46388	1.43974
1.42490	.45896	1.46259
1.44993	.45422	1.48560
1.47497	.44965	1.50874
1.50000	.44524	1.53200

Plot of above data:



44. Repeat the graph of the previous problem except make it apply for circular channels.

Obtain the same type of graph as in the previous problem except for a circular channel.

Circular Channel $\cos\beta = 1-2Y/D$ Let $\mathbf{Y'=Y/D}$

$\cos\beta = 1-2Y'$, $A=(D^2/4)(\beta-\cos\beta\sin\beta)$, $T=D\sin\beta$

Critical Depth

$Q^2D\sin\beta/g = (D^2/64)(\beta-\cos\beta\sin\beta)^3$ or

$$\mathbf{64Q'\sin\beta = (\beta-\cos\beta\sin\beta)^3} \quad \leftarrow \quad \text{in which } \mathbf{Q' = Q^2/(gD^5)}$$

Specific Energy

$E = Y + Q^2/[2gD^4/16)(\beta-\cos\beta\sin\beta)^2]$ divide by D gives

$E/D = Y/D + Q^2/[gD^5/8)(\beta-\cos\beta\sin\beta)^2$ Let $E'=E/D$, $Y'=Y/D$ and $Q'=Q^2/(gD^5)$

$$\mathbf{E' = Y' + 8Q'/(\beta-\cos\beta\sin\beta)^2} \quad \leftarrow$$

$E_1' = E_2'$

$$\mathbf{Y_1'+8Q'/(\beta_1-\cos\beta_1\sin\beta_1)^2 = Y_2'+8Q'/(\beta_2-\cos\beta_2\sin\beta_2)^2}$$

Program PRB2_41B.FOR using the above equations to generate the data needed to plot the requested graph.

```
      REAL Qp(22)/.01,.02,.03,.04,.05,.06,.07,.08,.09,.1,.12,.14,.16,
     &.18,.2,.25,.3,.4,.5,.6,.7,.8/
      YPc=.3
      DO 10 I=1,22
      M=0
1     COSB=1.-2.*YPc
      BETA=ACOS(COSB)
      SINB=SIN(BETA)
      F=64.*Qp(I)*SINB-(BETA-COSB*SINB)**3
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 2
      YY=YPc
      FF=F
      YPc=1.005*YPc
      GO TO 1
2     DIF=(YPc-YY)*FF/(F-FF)
      YPc=YY-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 1
      IF(M.EQ.30) WRITE(*,*)' Failed to find critical',DIF
      WRITE(3,100) Qp(I),YPc,YPc,ACOS(1.-2.*YPc),ACOS(1.-2.*YPc)
100   FORMAT(' For Q''=',F8.3,/4F10.5)
      DY=(.99-YPc)/30.
      YP1=.9*YPc
      DO 10 J=1,30
      YP2=YPc+DY*FLOAT(J)
      COSB=1.-2.*YP2
      BETA=ACOS(COSB)
      SINB=SIN(BETA)
      E2=YP2+8.*Qp(I)/(BETA-SINB*COSB)**2
      M=0
4     COSB=1.-2.*YP1
      BETA=ACOS(COSB)
      SINB=SIN(BETA)
      F=YP1+8.*Qp(I)/(BETA-SINB*COSB)**2-E2
```

```
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 6
      YY=YP1
      FF=F
      YP1=1.005*YP1
      GO TO 4
6     DIF=(YP1-YY)*FF/(F-FF)
      YP1=YY-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 4
      IF(M.EQ.30) WRITE(*,*)' Did not converge',I,J,DIF
10    WRITE(3,'(4F10.5)') YP2,YP1,ACOS(1.-2.*YP1),ACOS(1.-2.*YP2)
      END
```

Output from above Program:

```
 For Q'=    .010
    .31294    .31294   1.18734   1.18734
    .33551    .29275   1.14340   1.23556
    .35807    .27640   1.10716   1.28299
    .38064    .26287   1.07666   1.32976
    .40321    .25146   1.05057   1.37599
    .42578    .24170   1.02792   1.42181
    .44835    .23323   1.00801   1.46731
    .47092    .22579    .99033   1.51260
    .49349    .21920    .97447   1.55777
    .51606    .21330    .96015   1.60291
    .53862    .20799    .94711   1.64812
    .56119    .20316    .93517   1.69349
    .58376    .19875    .92418   1.73911
    .60633    .19471    .91400   1.78509
    .62890    .19098    .90454   1.83154
    .65147    .18752    .89571   1.87857
    .67404    .18430    .88744   1.92631
    .69661    .18130    .87967   1.97492
    .71917    .17848    .87234   2.02456
    .74174    .17584    .86541   2.07543
    .76431    .17334    .85885   2.12778
    .78688    .17099    .85261   2.18189
    .80945    .16876    .84668   2.23814
    .83202    .16665    .84102   2.29700
    .85459    .16464    .83561   2.35912
    .87716    .16272    .83043   2.42540
    .89972    .16089    .82546   2.49718
    .92229    .15914    .82069   2.57659
    .94486    .15747    .81611   2.66754
    .96743    .15586    .81169   2.77867
    .99000    .15432    .80742   2.94126
 For Q'=    .020
    .37448    .37448   1.31705   1.31705
    .39500    .35564   1.27792   1.35922
    .41552    .33965   1.24433   1.40102
    .43603    .32590   1.21514   1.44251
    .45655    .31393   1.18947   1.48379
    .47707    .30340   1.16669   1.52492
    .49759    .29407   1.14630   1.56597
    .51810    .28572   1.12790   1.60701
    .53862    .27820   1.11119   1.64811
    .55914    .27138   1.09592   1.68935
    .57966    .26517   1.08189   1.73079
    .60017    .25947   1.06894   1.77251
    .62069    .25423   1.05693   1.81458
    .64121    .24937   1.04575   1.85711
    .66172    .24487   1.03530   1.90017
    .68224    .24067   1.02551   1.94387
    .70276    .23674   1.01629   1.98834
    .72328    .23306   1.00761   2.03370
    .74379    .22959    .99939   2.08012
    .76431    .22633    .99161   2.12777
    .78483    .22324    .98421   2.17688
    .80534    .22031    .97717   2.22773
    .82586    .21754    .97046   2.28065
    .84638    .21490    .96404   2.33610
    .86690    .21238    .95790   2.39468
    .88741    .20998    .95201   2.45724
    .90793    .20768    .94635   2.52501
    .92845    .20548    .94092   2.60002
    .94897    .20336    .93568   2.68584
    .96948    .20133    .93062   2.79041
    .99000    .19938    .92573   2.94126
 For Q'=    .030
    .41616    .41616   1.40231   1.40231
    .43528    .39836   1.36608   1.44100
    .45441    .38286   1.33432   1.47949
    .47354    .36925   1.30621   1.51785
    .49267    .35718   1.28113   1.55613
    .51180    .34641   1.25857   1.59439
    .53092    .33672   1.23814   1.63269
    .55005    .32796   1.21953   1.67107
    .56918    .31998   1.20249   1.70960
    .58831    .31269   1.18680   1.74835
    .60744    .30598   1.17229   1.78736
    .62657    .29979   1.15883   1.82671
    .64569    .29406   1.14628   1.86647
    .66482    .28873   1.13454   1.90672
    .68395    .28375   1.12354   1.94755
    .70308    .27909   1.11318   1.98904
    .72221    .27472   1.10341   2.03131
    .74133    .27061   1.09418   2.07450
    .76046    .26673   1.08542   2.11873
    .77959    .26306   1.07710   2.16419
    .79872    .25958   1.06918   2.21110
    .81785    .25628   1.06163   2.25970
    .83697    .25313   1.05442   2.31034
    .85610    .25014   1.04752   2.36343
    .87523    .24728   1.04090   2.41956
    .89436    .24454   1.03455   2.47952
    .91349    .24192   1.02844   2.54450
    .93262    .23941   1.02255   2.61641
    .95174    .23699   1.01688   2.69863
    .97087    .23466   1.01139   2.79857
    .99000    .23241   1.00608   2.94126
 For Q'=    .040
    .44860    .44860   1.46782   1.46782
    .46665    .43166   1.43368   1.50404
    .48469    .41666   1.40334   1.54018
    .50274    .40330   1.37616   1.57628
    .52079    .39130   1.35165   1.61238
    .53883    .38047   1.32939   1.64854
    .55688    .37063   1.30908   1.68480
    .57493    .36165   1.29045   1.72122
    .59297    .35342   1.27327   1.75783
    .61102    .34584   1.25738   1.79470
    .62907    .33883   1.24260   1.83189
    .64711    .33233   1.22883   1.86944
    .66516    .32627   1.21595   1.90744
    .68321    .32062   1.20385   1.94595
    .70125    .31532   1.19248   1.98505
```

.71930	.31034	1.18174	2.02484
.73735	.30566	1.17159	2.06541
.75539	.30123	1.16197	2.10690
.77344	.29705	1.15283	2.14943
.79149	.29308	1.14413	2.19318
.80953	.28931	1.13583	2.23835
.82758	.28572	1.12791	2.28519
.84563	.28231	1.12033	2.33402
.86367	.27904	1.11306	2.38524
.88172	.27592	1.10609	2.43942
.89977	.27293	1.09938	2.49731
.91781	.27005	1.09292	2.56007
.93586	.26730	1.08670	2.62950
.95391	.26464	1.08069	2.70884
.97195	.26208	1.07487	2.80506
.99000	.25960	1.06922	2.94126

For Q'= .050

.47555	.47555	1.52187	1.52187
.49270	.45934	1.48939	1.55619
.50984	.44483	1.46022	1.59049
.52699	.43174	1.43385	1.62481
.54414	.41989	1.40988	1.65919
.56129	.40909	1.38796	1.69368
.57844	.39921	1.36783	1.72832
.59559	.39014	1.34926	1.76315
.61273	.38176	1.33206	1.79822
.62988	.37401	1.31606	1.83358
.64703	.36680	1.30114	1.86927
.66418	.36008	1.28717	1.90536
.68133	.35380	1.27406	1.94191
.69848	.34791	1.26173	1.97899
.71563	.34238	1.25009	2.01667
.73277	.33717	1.23908	2.05505
.74992	.33224	1.22864	2.09422
.76707	.32758	1.21873	2.13429
.78422	.32316	1.20930	2.17540
.80137	.31896	1.20031	2.21772
.81852	.31497	1.19172	2.26144
.83566	.31116	1.18350	2.30680
.85281	.30752	1.17563	2.35410
.86996	.30404	1.16807	2.40375
.88711	.30070	1.16081	2.45628
.90426	.29750	1.15382	2.51242
.92141	.29443	1.14709	2.57328
.93855	.29147	1.14058	2.64061
.95570	.28861	1.13429	2.71749
.97285	.28586	1.12820	2.81055
.99000	.28318	1.12227	2.94126

For Q'= .060

.49879	.49879	1.56837	1.56837
.51516	.48324	1.53726	1.60112
.53153	.46917	1.50910	1.63391
.54791	.45639	1.48346	1.66676
.56428	.44471	1.45999	1.69972
.58066	.43400	1.43842	1.73282
.59703	.42414	1.41850	1.76610
.61340	.41503	1.40003	1.79959
.62978	.40658	1.38286	1.83336
.64615	.39872	1.36683	1.86743
.66252	.39139	1.35183	1.90186
.67890	.38452	1.33774	1.93670
.69527	.37808	1.32448	1.97202
.71165	.37203	1.31197	2.00787
.72802	.36632	1.30014	2.04434
.74439	.36092	1.28892	2.08150
.76077	.35581	1.27827	2.11945
.77714	.35097	1.26814	2.15830
.79351	.34637	1.25847	2.19818
.80989	.34198	1.24925	2.23926
.82626	.33780	1.24042	2.28171
.84264	.33381	1.23197	2.32577
.85901	.32999	1.22386	2.37175
.87538	.32633	1.21607	2.42002
.89176	.32282	1.20857	2.47110
.90813	.31945	1.20134	2.52571
.92450	.31620	1.19437	2.58491
.94088	.31307	1.18762	2.65037
.95725	.31004	1.18110	2.72508
.97363	.30712	1.17476	2.81535
.99000	.30428	1.16859	2.94126

For Q'= .070

.51933	.51933	1.60947	1.60947
.53502	.50437	1.57953	1.64089
.55071	.49073	1.55225	1.67239
.56640	.47824	1.52727	1.70399
.58209	.46677	1.50428	1.73572
.59778	.45618	1.48305	1.76762
.61347	.44638	1.46336	1.79972
.62915	.43728	1.44503	1.83207
.64484	.42881	1.42793	1.86470
.66053	.42089	1.41191	1.89765
.67622	.41347	1.39687	1.93098
.69191	.40651	1.38271	1.96473
.70760	.39996	1.36935	1.99896
.72329	.39378	1.35672	2.03373
.73898	.38793	1.34474	2.06912
.75467	.38240	1.33337	2.10520
.77035	.37715	1.32255	2.14208
.78604	.37215	1.31223	2.17984
.80173	.36740	1.30239	2.21864
.81742	.36287	1.29297	2.25860
.83311	.35854	1.28395	2.29993
.84880	.35439	1.27530	2.34284
.86449	.35042	1.26699	2.38762
.88018	.34661	1.25899	2.43466
.89587	.34295	1.25129	2.48444
.91156	.33943	1.24386	2.53766
.92724	.33603	1.23668	2.59536
.94293	.33276	1.22973	2.65915
.95862	.32959	1.22300	2.73190
.97431	.32651	1.21646	2.81965
.99000	.32352	1.21007	2.94126

For Q'= .080

.53781	.53781	1.64649	1.64649
.55289	.52339	1.61759	1.67676
.56796	.51015	1.59110	1.70713
.58303	.49796	1.56672	1.73763
.59810	.48670	1.54419	1.76829
.61318	.47625	1.52328	1.79913
.62825	.46654	1.50383	1.83020
.64332	.45748	1.48566	1.86152
.65840	.44901	1.46864	1.89314
.67347	.44107	1.45267	1.92510
.68854	.43361	1.43763	1.95744
.70361	.42658	1.42343	1.99021
.71869	.41995	1.41001	2.02347
.73376	.41368	1.39729	2.05728
.74883	.40774	1.38520	2.09170
.76391	.40209	1.37371	2.12682
.77898	.39673	1.36276	2.16272
.79405	.39162	1.35230	2.19951
.80912	.38674	1.34230	2.23731
.82420	.38209	1.33273	2.27627
.83927	.37763	1.32354	2.31657
.85434	.37336	1.31472	2.35843
.86942	.36926	1.30624	2.40213
.88449	.36532	1.29807	2.44804
.89956	.36153	1.29019	2.49663
.91464	.35788	1.28259	2.54860
.92971	.35436	1.27523	2.60492
.94478	.35095	1.26810	2.66718
.95985	.34765	1.26118	2.73813

.97493	.34445	1.25445	2.82357
.99000	.34133	1.24787	2.94126
For Q'=	.090		
.55465	.55465	1.68032	1.68032
.56917	.54073	1.65234	1.70957
.58368	.52787	1.62657	1.73894
.59819	.51597	1.60275	1.76846
.61270	.50492	1.58064	1.79815
.62721	.49463	1.56006	1.82805
.64172	.48503	1.54085	1.85818
.65623	.47603	1.52285	1.88859
.67075	.46759	1.50594	1.91930
.68526	.45966	1.49002	1.95036
.69977	.45218	1.47501	1.98181
.71428	.44511	1.46080	2.01370
.72879	.43843	1.44734	2.04607
.74330	.43209	1.43456	2.07900
.75782	.42607	1.42240	2.11254
.77233	.42035	1.41081	2.14677
.78684	.41490	1.39975	2.18178
.80135	.40969	1.38918	2.21768
.81586	.40472	1.37906	2.25457
.83037	.39996	1.36935	2.29261
.84488	.39539	1.36003	2.33197
.85940	.39102	1.35107	2.37286
.87391	.38681	1.34244	2.41556
.88842	.38276	1.33412	2.46043
.90293	.37886	1.32608	2.50793
.91744	.37510	1.31832	2.55872
.93195	.37146	1.31080	2.61377
.94647	.36794	1.30351	2.67461
.96098	.36453	1.29643	2.74389
.97549	.36121	1.28953	2.82718
.99000	.35797	1.28277	2.94126
For Q'=	.100		
.57016	.57016	1.71157	1.71157
.58415	.55669	1.68442	1.73990
.59814	.54420	1.65931	1.76837
.61214	.53258	1.63601	1.79700
.62613	.52175	1.61431	1.82582
.64013	.51163	1.59405	1.85486
.65412	.50214	1.57507	1.88415
.66812	.49322	1.55725	1.91372
.68211	.48484	1.54046	1.94360
.69611	.47692	1.52462	1.97384
.71010	.46944	1.50965	2.00447
.72410	.46236	1.49545	2.03554
.73809	.45565	1.48198	2.06711
.75209	.44927	1.46916	2.09922
.76608	.44320	1.45694	2.13195
.78008	.43741	1.44529	2.16537
.79407	.43189	1.43414	2.19956
.80807	.42661	1.42347	2.23462
.82206	.42155	1.41325	2.27067
.83606	.41671	1.40343	2.30786
.85005	.41206	1.39399	2.34634
.86405	.40759	1.38490	2.38633
.87804	.40329	1.37614	2.42810
.89204	.39914	1.36769	2.47200
.90603	.39515	1.35952	2.51847
.92003	.39129	1.35162	2.56817
.93402	.38755	1.34396	2.62204
.94802	.38393	1.33652	2.68154
.96201	.38042	1.32929	2.74926
.97601	.37699	1.32223	2.83053
.99000	.37365	1.31532	2.94126
For Q'=	.120		
.59797	.59797	1.76800	1.76800
.61103	.58534	1.74231	1.79473
.62410	.57353	1.71838	1.82162
.63717	.56246	1.69604	1.84870
.65024	.55206	1.67511	1.87599
.66331	.54228	1.65545	1.90351
.67637	.53305	1.63694	1.93130
.68944	.52433	1.61948	1.95938
.70251	.51608	1.60296	1.98779
.71558	.50826	1.58731	2.01657
.72864	.50083	1.57246	2.04574
.74171	.49377	1.55833	2.07536
.75478	.48704	1.54487	2.10547
.76785	.48062	1.53203	2.13613
.78092	.47449	1.51975	2.16739
.79398	.46862	1.50800	2.19934
.80705	.46301	1.49674	2.23204
.82012	.45762	1.48594	2.26560
.83319	.45245	1.47555	2.30013
.84625	.44748	1.46556	2.33576
.85932	.44270	1.45594	2.37265
.87239	.43809	1.44665	2.41100
.88546	.43364	1.43768	2.45107
.89853	.42934	1.42901	2.49319
.91159	.42519	1.42061	2.53780
.92466	.42117	1.41247	2.58550
.93773	.41726	1.40456	2.63718
.95080	.41348	1.39687	2.69423
.96386	.40979	1.38937	2.75908
.97693	.40618	1.38205	2.83665
.99000	.40265	1.37485	2.94126
For Q'=	.140		
.62246	.62246	1.81823	1.81823
.63471	.61058	1.79380	1.84359
.64696	.59939	1.77091	1.86913
.65921	.58884	1.74942	1.89487
.67147	.57887	1.72919	1.92083
.68372	.56943	1.71011	1.94704
.69597	.56049	1.69207	1.97353
.70822	.55199	1.67497	2.00032
.72047	.54392	1.65874	2.02744
.73272	.53623	1.64331	2.05493
.74497	.52889	1.62861	2.08282
.75722	.52189	1.61459	2.11116
.76948	.51520	1.60119	2.13999
.78173	.50879	1.58837	2.16936
.79398	.50265	1.57609	2.19933
.80623	.49675	1.56430	2.22996
.81848	.49109	1.55298	2.26135
.83073	.48565	1.54209	2.29357
.84298	.48041	1.53160	2.32673
.85524	.47535	1.52148	2.36096
.86749	.47048	1.51172	2.39642
.87974	.46577	1.50228	2.43330
.89199	.46122	1.49315	2.47185
.90424	.45681	1.48431	2.51236
.91649	.45254	1.47572	2.55528
.92874	.44839	1.46739	2.60116
.94099	.44436	1.45928	2.65086
.95325	.44043	1.45137	2.70570
.96550	.43660	1.44366	2.76792
.97775	.43285	1.43609	2.84214
.99000	.42917	1.42865	2.94126
For Q'=	.160		
.64439	.64439	1.86376	1.86376
.65592	.63319	1.84043	1.88792
.66744	.62258	1.81848	1.91226
.67896	.61252	1.79777	1.93683
.69048	.60296	1.77820	1.96162
.70200	.59387	1.75966	1.98667
.71352	.58522	1.74206	2.01200
.72504	.57696	1.72534	2.03764
.73656	.56908	1.70941	2.06362
.74808	.56155	1.69421	2.08996
.75960	.55434	1.67970	2.11670

.77112	.54743	1.66581	2.14389
.78264	.54081	1.65251	2.17156
.79416	.53445	1.63974	2.19977
.80568	.52833	1.62749	2.22857
.81720	.52244	1.61570	2.25802
.82872	.51677	1.60435	2.28821
.84024	.51131	1.59341	2.31921
.85176	.50603	1.58286	2.35113
.86328	.50093	1.57266	2.38409
.87480	.49600	1.56279	2.41825
.88632	.49122	1.55324	2.45378
.89784	.48660	1.54399	2.49092
.90936	.48211	1.53500	2.52997
.92088	.47774	1.52627	2.57133
.93240	.47350	1.51778	2.61554
.94392	.46937	1.50950	2.66342
.95544	.46534	1.50141	2.71621
.96696	.46139	1.49351	2.77602
.97848	.45752	1.48574	2.84713
.99000	.45371	1.47808	2.94126
For Q'=	.180		
.66428	.66428	1.90557	1.90557
.67514	.65369	1.88324	1.92866
.68599	.64362	1.86213	1.95195
.69685	.63402	1.84215	1.97545
.70771	.62486	1.82319	1.99920
.71857	.61612	1.80517	2.02321
.72942	.60776	1.78801	2.04749
.74028	.59975	1.77165	2.07209
.75114	.59208	1.75602	2.09703
.76200	.58473	1.74107	2.12233
.77285	.57766	1.72676	2.14803
.78371	.57087	1.71302	2.17417
.79457	.56434	1.69984	2.20079
.80543	.55805	1.68716	2.22793
.81628	.55199	1.67496	2.25566
.82714	.54614	1.66320	2.28403
.83800	.54049	1.65186	2.31311
.84885	.53502	1.64090	2.34299
.85971	.52974	1.63031	2.37377
.87057	.52462	1.62006	2.40556
.88143	.51966	1.61012	2.43851
.89228	.51484	1.60049	2.47280
.90314	.51017	1.59114	2.50864
.91400	.50562	1.58204	2.54632
.92486	.50120	1.57319	2.58624
.93571	.49688	1.56456	2.62890
.94657	.49267	1.55614	2.67508
.95743	.48856	1.54791	2.72595
.96829	.48452	1.53984	2.78351
.97914	.48055	1.53190	2.85174
.99000	.47663	1.52404	2.94126
For Q'=	.200		
.68247	.68247	1.94437	1.94437
.69272	.67245	1.92294	1.96649
.70298	.66288	1.90261	1.98882
.71323	.65372	1.88330	2.01136
.72348	.64495	1.86492	2.03415
.73373	.63654	1.84739	2.05721
.74398	.62847	1.83066	2.08054
.75423	.62072	1.81466	2.10419
.76448	.61328	1.79933	2.12817
.77473	.60611	1.78464	2.15252
.78498	.59920	1.77053	2.17726
.79523	.59255	1.75697	2.20243
.80548	.58613	1.74392	2.22808
.81573	.57993	1.73135	2.25424
.82599	.57394	1.71922	2.28098
.83624	.56814	1.70751	2.30834
.84649	.56253	1.69619	2.33640
.85674	.55710	1.68524	2.36524
.86699	.55183	1.67464	2.39496
.87724	.54671	1.66436	2.42566
.88749	.54174	1.65438	2.45748
.89774	.53691	1.64469	2.49060
.90799	.53221	1.63526	2.52523
.91824	.52763	1.62609	2.56164
.92849	.52316	1.61714	2.60020
.93875	.51880	1.60841	2.64140
.94900	.51453	1.59987	2.68598
.95925	.51035	1.59150	2.73505
.96950	.50625	1.58329	2.79050
.97975	.50220	1.57520	2.85601
.99000	.49819	1.56718	2.94126
For Q'=	.250		
.72214	.72214	2.03116	2.03116
.73106	.71337	2.01168	2.05119
.73999	.70492	1.99307	2.07143
.74892	.69676	1.97526	2.09191
.75785	.68889	1.95820	2.11262
.76678	.68129	1.94183	2.13360
.77571	.67394	1.92610	2.15486
.78464	.66682	1.91097	2.17642
.79357	.65994	1.89640	2.19831
.80249	.65327	1.88235	2.22055
.81142	.64680	1.86880	2.24317
.82035	.64053	1.85570	2.26621
.82928	.63445	1.84304	2.28970
.83821	.62854	1.83079	2.31369
.84714	.62279	1.81892	2.33821
.85607	.61720	1.80741	2.36333
.86500	.61177	1.79623	2.38911
.87393	.60647	1.78538	2.41561
.88285	.60131	1.77483	2.44294
.89178	.59628	1.76456	2.47118
.90071	.59136	1.75456	2.50047
.90964	.58656	1.74480	2.53095
.91857	.58187	1.73528	2.56283
.92750	.57728	1.72598	2.59634
.93643	.57278	1.71687	2.63182
.94536	.56836	1.70795	2.66970
.95428	.56403	1.69920	2.71064
.96321	.55976	1.69060	2.75560
.97214	.55554	1.68211	2.80621
.98107	.55137	1.67372	2.86555
.99000	.54722	1.66537	2.94126
For Q'=	.300		
.75542	.75542	2.10696	2.10696
.76324	.74772	2.08913	2.12525
.77106	.74024	2.07200	2.14375
.77888	.73298	2.05551	2.16248
.78670	.72592	2.03962	2.18144
.79452	.71906	2.02429	2.20066
.80234	.71238	2.00950	2.22015
.81016	.70589	1.99520	2.23994
.81798	.69957	1.98137	2.26004
.82579	.69341	1.96797	2.28047
.83361	.68741	1.95499	2.30128
.84143	.68155	1.94240	2.32248
.84925	.67584	1.93017	2.34410
.85707	.67027	1.91829	2.36620
.86489	.66483	1.90674	2.38880
.87271	.65951	1.89550	2.41196
.88053	.65431	1.88455	2.43574
.88835	.64923	1.87387	2.46020
.89617	.64425	1.86345	2.48543
.90399	.63937	1.85328	2.51150
.91181	.63459	1.84334	2.53855
.91963	.62990	1.83362	2.56670
.92745	.62530	1.82409	2.59614
.93526	.62077	1.81476	2.62707
.94308	.61632	1.80559	2.65981

.95090	.61194	1.79658	2.69473
.95872	.60761	1.78771	2.73241
.96654	.60333	1.77896	2.77369
.97436	.59910	1.77031	2.81997
.98218	.59488	1.76172	2.87382
.99000	.59067	1.75315	2.94126

For Q'= .400

.80845	.80845	2.23559	2.23559
.81450	.80246	2.22046	2.25106
.82055	.79658	2.20576	2.26673
.82660	.79081	2.19151	2.28261
.83265	.78515	2.17766	2.29870
.83871	.77959	2.16420	2.31504
.84476	.77414	2.15110	2.33162
.85081	.76878	2.13834	2.34846
.85686	.76352	2.12590	2.36560
.86291	.75835	2.11378	2.38303
.86897	.75326	2.10194	2.40079
.87502	.74826	2.09039	2.41891
.88107	.74335	2.07910	2.43740
.88712	.73851	2.06805	2.45631
.89317	.73375	2.05725	2.47567
.89922	.72906	2.04667	2.49551
.90528	.72444	2.03630	2.51589
.91133	.71988	2.02613	2.53686
.91738	.71539	2.01615	2.55849
.92343	.71096	2.00635	2.58085
.92948	.70658	1.99672	2.60404
.93553	.70226	1.98725	2.62817
.94159	.69798	1.97791	2.65338
.94764	.69375	1.96872	2.67985
.95369	.68956	1.95964	2.70780
.95974	.68540	1.95067	2.73756
.96579	.68127	1.94179	2.76955
.97184	.67716	1.93298	2.80441
.97790	.67306	1.92423	2.84314
.98395	.66895	1.91549	2.88752
.99000	.66482	1.90672	2.94126

For Q'= .500

.84847	.84847	2.34192	2.34192
.85319	.84384	2.32908	2.35517
.85791	.83915	2.31625	2.36859
.86263	.83457	2.30384	2.38219
.86734	.83004	2.29172	2.39600
.87206	.82556	2.27987	2.41001
.87678	.82114	2.26826	2.42425
.88150	.81676	2.25690	2.43872
.88621	.81243	2.24576	2.45345
.89093	.80815	2.23484	2.46844
.89565	.80392	2.22412	2.48372
.90037	.79972	2.21360	2.49931
.90508	.79557	2.20327	2.51523
.90980	.79146	2.19312	2.53151
.91452	.78739	2.18313	2.54818
.91924	.78336	2.17330	2.56527
.92395	.77936	2.16363	2.58282
.92867	.77539	2.15410	2.60088
.93339	.77146	2.14470	2.61950
.93811	.76755	2.13543	2.63874
.94282	.76367	2.12627	2.65868
.94754	.75982	2.11723	2.67941
.95226	.75599	2.10828	2.70105
.95698	.75217	2.09942	2.72372
.96169	.74837	2.09064	2.74761
.96641	.74458	2.08193	2.77297
.97113	.74080	2.07328	2.80011
.97585	.73701	2.06466	2.82950
.98056	.73322	2.05605	2.86186
.98528	.72940	2.04743	2.89836
.99000	.72554	2.03877	2.94126

For Q'= .600

.87906	.87906	2.43123	2.43123
.88276	.87605	2.42203	2.44265
.88646	.87172	2.40899	2.45422
.89016	.86807	2.39816	2.46596
.89385	.86445	2.38752	2.47788
.89755	.86086	2.37708	2.48998
.90125	.85729	2.36681	2.50227
.90495	.85373	2.35671	2.51477
.90865	.85020	2.34676	2.52749
.91234	.84669	2.33696	2.54045
.91604	.84320	2.32731	2.55365
.91974	.83972	2.31780	2.56712
.92344	.83626	2.30842	2.58088
.92714	.83282	2.29916	2.59494
.93083	.82940	2.29002	2.60934
.93453	.82599	2.28099	2.62410
.93823	.82259	2.27207	2.63925
.94193	.81921	2.26324	2.65484
.94563	.81584	2.25451	2.67089
.94932	.81248	2.24586	2.68747
.95302	.80912	2.23730	2.70463
.95672	.80577	2.22881	2.72245
.96042	.80243	2.22038	2.74101
.96411	.79908	2.21201	2.76042
.96781	.79574	2.20368	2.78082
.97151	.79239	2.19540	2.80239
.97521	.78903	2.18714	2.82537
.97891	.78565	2.17889	2.85009
.98260	.78226	2.17064	2.87704
.98630	.77883	2.16236	2.90698
.99000	.77536	2.15403	2.94126

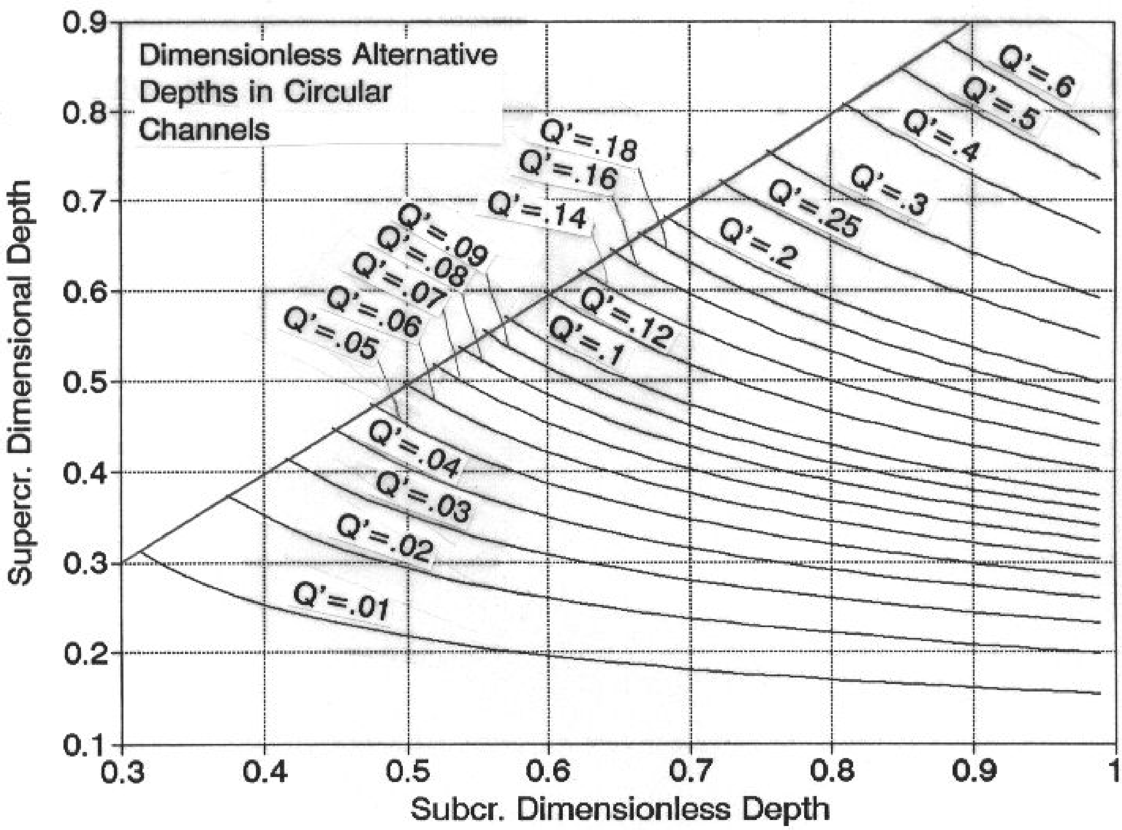
For Q'= .700

.90254	.90254	2.50660	2.50660
.90545	.90546	2.51652	2.51649
.90837	.90837	2.52652	2.52653
.91128	.91128	2.53670	2.53671
.91420	.91420	2.54703	2.54704
.91711	.91711	2.55753	2.55753
.92003	.92003	2.56819	2.56819
.92295	.92294	2.57903	2.57903
.92586	.92586	2.59005	2.59006
.92878	.92878	2.60129	2.60129
.93169	.93169	2.61273	2.61273
.93461	.93461	2.62440	2.62441
.93752	.93752	2.63632	2.63632
.94044	.94044	2.64850	2.64850
.94335	.94335	2.66097	2.66097
.94627	.94627	2.67373	2.67374
.94918	.94918	2.68684	2.68684
.95210	.95210	2.70030	2.70030
.95501	.95501	2.71415	2.71415
.95793	.95793	2.72844	2.72844
.96085	.96085	2.74321	2.74321
.96376	.96376	2.75852	2.75852
.96668	.96668	2.77444	2.77444
.96959	.96959	2.79104	2.79104
.97251	.97251	2.80844	2.80844
.97542	.97542	2.82675	2.82675
.97834	.97834	2.84616	2.84616
.98125	.98125	2.86690	2.86690
.98417	.98417	2.88928	2.88928
.98708	.98708	2.91381	2.91381
.99000	.99000	2.94125	2.94126

For Q'= .800

.92060	.92060	2.57030	2.57030
.92291	.92291	2.57890	2.57891
.92523	.92523	2.58764	2.58765
.92754	.92754	2.59650	2.59651
.92985	.92985	2.60549	2.60550
.93217	.93217	2.61462	2.61462
.93448	.93448	2.62390	2.62390

.93679	.93679	2.63332	2.63332	.96455	.96455	2.76279	2.76279
.93911	.93911	2.64291	2.64291	.96687	.96687	2.77550	2.77550
.94142	.94142	2.65267	2.65267	.96918	.96918	2.78865	2.78865
.94373	.94373	2.66262	2.66262	.97149	.97149	2.80229	2.80229
.94605	.94605	2.67275	2.67276	.97381	.97381	2.81648	2.81648
.94836	.94836	2.68310	2.68310	.97612	.97612	2.83129	2.83129
.95067	.95067	2.69367	2.69367	.97843	.97843	2.84682	2.84682
.95299	.95299	2.70447	2.70447	.98075	.98075	2.86318	2.86318
.95530	.95530	2.71553	2.71553	.98306	.98306	2.88054	2.88055
.95761	.95761	2.72687	2.72687	.98537	.98537	2.89912	2.89912
.95993	.95993	2.73850	2.73851	.98769	.98769	2.91920	2.91920
.96224	.96224	2.75047	2.75047	.99000	.99000	2.94126	2.94126



45.

Develop a relationship for the diameter D in a circular channel with the bottom width b and the side slope m in a trapezoidal channel so the dimensionless flowrates Q' are the same in these to sections. Assume that a circular section exists downstream from a gate with a diameter that satisfied the above relationship, and that a trapezoidal section with b and m exists upstream from the gate. Develop the data and plot 3 graphs that apply for m = 0.5, m = 1.0 and m = 1.5 that provide the relationship between the downstream supercritical depth in the circular section to the upstream subcritical depth in the trapezoidal section, assuming that the above relationship of D to b and m applies so the dimensionless depths are the same in the two sections.

Wanted: A dimensionless graph that relates D in a circular channel to b and m of a trapezoidal channel so the dimensionless flow rates Q′ are the same.

Trap. Circular

$Q' = Q'$

$m^3Q^2/(gb^5)=Q^2/(gD^5)$ or $m^3/b^5 = 1/D^5$ or $\mathbf{D = b/m^{3/5}}$ ←

$E_{trap} = E_{cir}$

$(b/m)E'_{trap} = DE'_{cir} = b/m^{.6}E'_{cir}$ or $\mathbf{E'_{trap} = m^{.4}E'_{cir}}$ ←

The program PRB2_45 generates data to make 3 requested plots. Notice this program adds to that used for Problem 44 the solution of Y_1' from

$Y_1' + .5Q'/(Y_1'+Y_1'^2)^2 = E'_{trap} = m^{.4}E'_{cir}$

The graphs follow, as well as the output from the program.

Program PRB2_45.FOR

```
      REAL Qp(22)/.01,.02,.03,.04,.05,.06,.07,.08,.09,.1,.12,.14,.16,
     &.18,.2,.25,.3,.4,.5,.6,.7,.8/,FM(3)/.5,1.,1.5/,EPtrap(3),YPr(3)
      YPc=.3
      DO 10 I=1,22
      M=0
1     COSB=1.-2.*YPc
      BETA=ACOS(COSB)
      SINB=SIN(BETA)
      F=64.*Qp(I)*SINB-(BETA-COSB*SINB)**3
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 2
      YY=YPc
      FF=F
      YPc=1.005*YPc
      GO TO 1
2     DIF=(YPc-YY)*FF/(F-FF)
      YPc=YY-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 1
      IF(M.EQ.30) WRITE(*,*)' Failed to find critical',DIF
      WRITE(3,100) QP(I)
100   FORMAT(' For Q''=',F8.3)
      BETA=ACOS(1.-2.*YPc)
      Ec=YPc+8.*QP(I)/(BETA-SIN(BETA)*COS(BETA))**2
      Yr=1.2*YPc
      DO 5 K=1,3
      EPtrap(K)=FM(K)**.4*Ec
      M=0
3     F=Yr+.5*QP(I)/(Yr+Yr**2)**2-EPtrap(K)
```

```
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 4
      YY=Yr
      FF=F
      Yr=1.005*Yr
      GO TO 3
4     DIF=(Yr-YY)*FF/(F-FF)
      Yr=YY-DIF
      IF(ABS(DIF).GT. 1.E-5 .AND. M.LT.30) GO TO 3
      IF(M.EQ.30) WRITE(*,*)' Yr failed',I,J,K,DIF
      YPr(K)=Yr
5     Yr=1.05*Yr
      WRITE(3,110) YPc,YPc,BETA,BETA,Ec,(EPtrap(K),YPr(K),K=1,3)
      DY=(.99-YPc)/30.
      YP1=.9*YPc
      DO 10 J=1,30
      YP2=YPc+DY*FLOAT(J)
      COSB=1.-2.*YP2
      BETA=ACOS(COSB)
      SINB=SIN(BETA)
      E2=YP2+8.*Qp(I)/(BETA-SINB*COSB)**2
      M=0
6     COSB=1.-2.*YP1
      BETA=ACOS(COSB)
      SINB=SIN(BETA)
      F=YP1+8.*Qp(I)/(BETA-SINB*COSB)**2-E2
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 7
      YY=YP1
      FF=F
      YP1=1.005*YP1
      GO TO 6
7     DIF=(YP1-YY)*FF/(F-FF)
      YP1=YY-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 6
      IF(M.EQ.30) WRITE(*,*)' Did not converge',I,J,DIF
      IF(J.EQ.1) Yr=YP1

      DO 9 K=1,3
      EPtrap(K)=FM(K)**.4*E2
      M=0
14    F=Yr+.5*QP(I)/(Yr+Yr**2)**2-EPtrap(K)
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 16
      YY=Yr
      FF=F
      Yr=1.005*Yr
      GO TO 14
16    DIF=(Yr-YY)*FF/(F-FF)
      Yr=YY-DIF
      IF(ABS(DIF).GT. 1.E-5 .AND. M.LT.30) GO TO 14
      IF(M.EQ.30) WRITE(*,*)' Yr failed',I,J,K,DIF
9     YPr(K)=Yr
10    WRITE(3,110) YP2,YP1,ACOS(1.-2.*YP2),ACOS(1.-2.*YP1),E2,(EPtrap(K)
     &,YPr(K),K=1,3)
110   FORMAT(11F8.5)
      END
```

Data generated by above Program:

For Q'= .010

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.31294	.31294	1.18734	1.18734	.42621	.32301	.28607	.42621	.41138	.50126	.49198
.33551	.29275	1.23556	1.14340	.42903	.32514	.28916	.42903	.41448	.50457	.49546
.35807	.27640	1.28299	1.10716	.43637	.33071	.29702	.43637	.42253	.51321	.50453
.38064	.26287	1.32976	1.07666	.44701	.33877	.30795	.44701	.43411	.52572	.51761
.40321	.25146	1.37599	1.05057	.46008	.34868	.32083	.46008	.44821	.54109	.53362
.42578	.24170	1.42181	1.02792	.47499	.35998	.33497	.47499	.46417	.55863	.55181
.44835	.23323	1.46731	1.00801	.49131	.37235	.34994	.49131	.48149	.57782	.57163
.47092	.22579	1.51260	.99033	.50873	.38555	.36547	.50873	.49983	.59831	.59270
.49349	.21920	1.55777	.97447	.52701	.39940	.38139	.52701	.51897	.61981	.61473

.51606	.21330	1.60291	.96015	.54598	.41378	.39758	.54598	.53870	.64212	.63753
.53862	.20799	1.64812	.94711	.56551	.42857	.41398	.56551	.55892	.66508	.66093
.56119	.20316	1.69349	.93517	.58548	.44371	.43053	.58548	.57951	.68857	.68481
.58376	.19875	1.73911	.92418	.60582	.45913	.44719	.60582	.60041	.71250	.70909
.60633	.19471	1.78509	.91400	.62647	.47478	.46394	.62647	.62155	.73678	.73369
.62890	.19098	1.83154	.90454	.64738	.49062	.48076	.64738	.64290	.76137	.75856
.65147	.18752	1.87857	.89571	.66850	.50663	.49763	.66850	.66441	.78621	.78365
.67404	.18430	1.92631	.88744	.68980	.52277	.51454	.68980	.68607	.81126	.80893
.69661	.18130	1.97492	.87967	.71126	.53903	.53149	.71126	.70784	.83650	.83436
.71917	.17848	2.02456	.87234	.73285	.55540	.54847	.73285	.72971	.86189	.85994
.74174	.17584	2.07543	.86541	.75456	.57185	.56547	.75456	.75167	.88742	.88563
.76431	.17334	2.12778	.85885	.77636	.58837	.58249	.77636	.77371	.91306	.91142
.78688	.17099	2.18189	.85261	.79826	.60497	.59953	.79826	.79581	.93881	.93730
.80945	.16876	2.23814	.84668	.82023	.62162	.61658	.82023	.81797	.96465	.96326
.83202	.16665	2.29700	.84102	.84227	.63832	.63366	.84227	.84018	.99058	.98929
.85459	.16464	2.35912	.83561	.86437	.65507	.65074	.86437	.86244	1.01657	1.01538
.87716	.16272	2.42540	.83043	.88653	.67187	.66784	.88653	.88474	1.04263	1.04153
.89972	.16089	2.49718	.82546	.90875	.68870	.68495	.90875	.90708	1.06876	1.06773
.92229	.15914	2.57659	.82069	.93101	.70558	.70208	.93101	.92946	1.09495	1.09399
.94486	.15747	2.66754	.81611	.95333	.72249	.71922	.95333	.95188	1.12119	1.12031
.96743	.15586	2.77867	.81169	.97570	.73944	.73638	.97570	.97435	1.14750	1.14667
.99000	.15432	2.94126	.80742	.99813	.75644	.75358	.99813	.99687	1.17388	1.17311

For Q'= .020

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.37448	.37448	1.31705	1.31705	.51318	.38892	.34115	.51318	.49491	.60354	.59230
.39500	.35564	1.35922	1.27792	.51517	.39042	.34345	.51517	.49711	.60588	.59476
.41552	.33965	1.40102	1.24433	.52051	.39447	.34953	.52051	.50301	.61216	.60138
.43603	.32590	1.44251	1.21514	.52847	.40051	.35828	.52847	.51177	.62153	.61121
.45655	.31393	1.48379	1.18947	.53851	.40811	.36890	.53851	.52273	.63333	.62357
.47707	.30340	1.52492	1.16669	.55020	.41698	.38081	.55020	.53541	.64708	.63792
.49759	.29407	1.56597	1.14630	.56324	.42685	.39362	.56324	.54944	.66241	.65386
.51810	.28572	1.60701	1.12790	.57736	.43756	.40708	.57736	.56454	.67902	.67107
.53862	.27820	1.64811	1.11119	.59238	.44894	.42100	.59238	.58050	.69669	.68931
.55914	.27138	1.68935	1.09592	.60815	.46089	.43527	.60815	.59716	.71523	.70840
.57966	.26517	1.73079	1.08189	.62454	.47331	.44980	.62454	.61437	.73451	.72819
.60017	.25947	1.77251	1.06894	.64145	.48613	.46452	.64145	.63205	.75440	.74856
.62069	.25423	1.81458	1.05693	.65881	.49928	.47940	.65881	.65012	.77481	.76941
.64121	.24937	1.85711	1.04575	.67654	.51272	.49440	.67654	.66850	.79566	.79067
.66172	.24487	1.90017	1.03530	.69460	.52641	.50950	.69460	.68716	.81690	.81228
.68224	.24067	1.94387	1.02551	.71293	.54030	.52467	.71293	.70604	.83846	.83419
.70276	.23674	1.98834	1.01629	.73151	.55438	.53991	.73151	.72512	.86031	.85636
.72328	.23306	2.03370	1.00761	.75030	.56862	.55521	.75030	.74437	.88241	.87874
.74379	.22959	2.08012	.99939	.76927	.58300	.57055	.76927	.76376	.90473	.90132
.76431	.22633	2.12777	.99161	.78841	.59751	.58592	.78841	.78329	.92724	.92407
.78483	.22324	2.17688	.98421	.80770	.61212	.60133	.80770	.80292	.94991	.94697
.80534	.22031	2.22773	.97717	.82711	.62683	.61678	.82711	.82266	.97275	.97001
.82586	.21754	2.28065	.97046	.84664	.64163	.63224	.84664	.84249	.99572	.99316
.84638	.21490	2.33610	.96404	.86628	.65652	.64774	.86628	.86240	1.01881	1.01643
.86690	.21238	2.39468	.95790	.88601	.67147	.66325	.88601	.88239	1.04202	1.03980
.88741	.20998	2.45724	.95201	.90584	.68650	.67879	.90584	.90244	1.06533	1.06326
.90793	.20768	2.52501	.94635	.92575	.70158	.69436	.92575	.92257	1.08875	1.08681
.92845	.20548	2.60002	.94092	.94574	.71674	.70995	.94574	.94276	1.11227	1.11045
.94897	.20336	2.68584	.93568	.96582	.73195	.72558	.96582	.96302	1.13588	1.13417
.96948	.20133	2.79041	.93062	.98599	.74724	.74124	.98599	.98336	1.15960	1.15800
.99000	.19938	2.94126	.92573	1.00627	.76261	.75695	1.00627	1.00379	1.18345	1.18195

For Q'= .030

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.41616	.41616	1.40231	1.40231	.57300	.43425	.37954	.57300	.55263	.67390	.66148
.43528	.39836	1.44100	1.36608	.57457	.43544	.38141	.57457	.55437	.67574	.66342
.45441	.38286	1.47949	1.33432	.57886	.43870	.38644	.57886	.55912	.68079	.66874
.47354	.36925	1.51785	1.30621	.58536	.44362	.39385	.58536	.56630	.68843	.67679
.49267	.35718	1.55613	1.28113	.59367	.44992	.40300	.59367	.57542	.69821	.68704
.51180	.34641	1.59439	1.25857	.60347	.45735	.41342	.60347	.58612	.70973	.69910
.53092	.33672	1.63269	1.23814	.61452	.46572	.42476	.61452	.59810	.72272	.71265
.55005	.32796	1.67107	1.21953	.62660	.47487	.43679	.62660	.61113	.73693	.72743
.56918	.31998	1.70960	1.20249	.63956	.48470	.44933	.63956	.62502	.75217	.74324
.58831	.31269	1.74835	1.18680	.65327	.49508	.46225	.65327	.63963	.76829	.75990
.60744	.30598	1.78736	1.17229	.66760	.50595	.47547	.66760	.65483	.78516	.77730
.62657	.29979	1.82671	1.15883	.68249	.51723	.48893	.68249	.67054	.80266	.79530

.64569	.29406	1.86647	1.14628	.69785	.52887	.50256	.69785	.68666	.82072	.81384
.66482	.28873	1.90672	1.13454	.71361	.54081	.51635	.71361	.70315	.83926	.83282
.68395	.28375	1.94755	1.12354	.72973	.55303	.53025	.72973	.71995	.85822	.85220
.70308	.27909	1.98904	1.11318	.74616	.56549	.54425	.74616	.73701	.87755	.87192
.72221	.27472	2.03131	1.10341	.76287	.57815	.55833	.76287	.75430	.89719	.89193
.74133	.27061	2.07450	1.09418	.77982	.59099	.57248	.77982	.77180	.91713	.91220
.76046	.26673	2.11873	1.08542	.79699	.60400	.58669	.79699	.78947	.93732	.93270
.77959	.26306	2.16419	1.07710	.81435	.61716	.60095	.81435	.80730	.95774	.95341
.79872	.25958	2.21110	1.06918	.83188	.63045	.61526	.83188	.82527	.97836	.97430
.81785	.25628	2.25970	1.06163	.84957	.64386	.62961	.84957	.84337	.99916	.99536
.83697	.25313	2.31034	1.05442	.86741	.65737	.64399	.86741	.86158	1.02014	1.01657
.85610	.25014	2.36343	1.04752	.88538	.67099	.65841	.88538	.87989	1.04127	1.03792
.87523	.24728	2.41956	1.04090	.90347	.68470	.67286	.90347	.89831	1.06255	1.05940
.89436	.24454	2.47952	1.03455	.92167	.69850	.68734	.92167	.91681	1.08396	1.08099
.91349	.24192	2.54450	1.02844	.93999	.71238	.70186	.93999	.93541	1.10550	1.10271
.93262	.23941	2.61641	1.02255	.95841	.72634	.71642	.95841	.95410	1.12717	1.12454
.95174	.23699	2.69863	1.01688	.97695	.74039	.73102	.97695	.97288	1.14897	1.14649
.97087	.23466	2.79857	1.01139	.99560	.75452	.74567	.99560	.99176	1.17091	1.16857
.99000	.23241	2.94126	1.00608	1.01440	.76877	.76040	1.01440	1.01077	1.19301	1.19081

For Q'= .040

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.44860	.44860	1.46782	1.46782	.62020	.47003	.41029	.62020	.59834	.72941	.71617
.46665	.43166	1.50404	1.43368	.62151	.47102	.41187	.62151	.59979	.73094	.71779
.48469	.41666	1.54018	1.40334	.62512	.47375	.41617	.62512	.60379	.73519	.72226
.50274	.40330	1.57628	1.37616	.63064	.47794	.42260	.63064	.60989	.74168	.72910
.52079	.39130	1.61238	1.35165	.63777	.48334	.43065	.63777	.61774	.75007	.73791
.53883	.38047	1.64854	1.32939	.64625	.48977	.43993	.64625	.62704	.76004	.74836
.55688	.37063	1.68480	1.30908	.65588	.49707	.45013	.65588	.63753	.77137	.76020
.57493	.36165	1.72122	1.29045	.66650	.50511	.46103	.66650	.64904	.78385	.77321
.59297	.35342	1.75783	1.27327	.67795	.51379	.47247	.67795	.66139	.79732	.78722
.61102	.34584	1.79470	1.25738	.69013	.52302	.48432	.69013	.67445	.81165	.80208
.62907	.33883	1.83189	1.24260	.70294	.53273	.49650	.70294	.68812	.82671	.81766
.64711	.33233	1.86944	1.22883	.71630	.54285	.50894	.71630	.70231	.84242	.83387
.66516	.32627	1.90744	1.21595	.73014	.55334	.52159	.73014	.71694	.85870	.85063
.68321	.32062	1.94595	1.20385	.74439	.56415	.53440	.74439	.73195	.87547	.86786
.70125	.31532	1.98505	1.19248	.75903	.57523	.54735	.75903	.74730	.89267	.88550
.71930	.31034	2.02484	1.18174	.77399	.58657	.56042	.77399	.76293	.91027	.90351
.73735	.30566	2.06541	1.17159	.78924	.59813	.57358	.78924	.77882	.92821	.92184
.75539	.30123	2.10690	1.16197	.80476	.60989	.58683	.80476	.79494	.94646	.94045
.77344	.29705	2.14943	1.15283	.82051	.62183	.60015	.82051	.81125	.96499	.95933
.79149	.29308	2.19318	1.14413	.83648	.63393	.61353	.83648	.82774	.98377	.97843
.80953	.28931	2.23835	1.13583	.85264	.64618	.62696	.85264	.84440	1.00277	.99774
.82758	.28572	2.28519	1.12791	.86898	.65856	.64045	.86898	.86120	1.02199	1.01724
.84563	.28231	2.33402	1.12033	.88548	.67107	.65398	.88548	.87813	1.04140	1.03691
.86367	.27904	2.38524	1.11306	.90214	.68369	.66755	.90214	.89519	1.06098	1.05675
.88172	.27592	2.43942	1.10609	.91893	.69642	.68117	.91893	.91236	1.08074	1.07674
.89977	.27293	2.49731	1.09938	.93586	.70925	.69483	.93586	.92965	1.10065	1.09687
.91781	.27005	2.56007	1.09292	.95292	.72218	.70853	.95292	.94704	1.12071	1.11714
.93586	.26730	2.62950	1.08670	.97011	.73521	.72228	.97011	.96454	1.14093	1.13754
.95391	.26464	2.70884	1.08069	.98743	.74833	.73609	.98743	.98215	1.16130	1.15810
.97195	.26208	2.80506	1.07487	1.00489	.76157	.74996	1.00489	.99989	1.18184	1.17880
.99000	.25960	2.94126	1.06922	1.02253	.77493	.76392	1.02253	1.01779	1.20258	1.19971

For Q'= .050

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.47555	.47555	1.52187	1.52187	.65990	.50011	.43654	.65990	.63689	.77609	.76223
.49270	.45934	1.55619	1.48939	.66101	.50095	.43790	.66101	.63813	.77740	.76362
.50984	.44483	1.59049	1.46022	.66413	.50331	.44164	.66413	.64159	.78107	.76748
.52699	.43174	1.62481	1.43385	.66893	.50696	.44731	.66893	.64691	.78672	.77343
.54414	.41989	1.65919	1.40988	.67518	.51169	.45448	.67518	.65380	.79407	.78115
.56129	.40909	1.69368	1.38796	.68267	.51737	.46283	.68267	.66202	.80287	.79039
.57844	.39921	1.72832	1.36783	.69122	.52385	.47208	.69122	.67136	.81293	.80091
.59559	.39014	1.76315	1.34926	.70069	.53103	.48204	.70069	.68167	.82407	.81255
.61273	.38176	1.79822	1.33206	.71097	.53881	.49256	.71097	.69279	.83616	.82513
.62988	.37401	1.83358	1.31606	.72195	.54713	.50351	.72195	.70462	.84907	.83855
.64703	.36680	1.86927	1.30114	.73354	.55592	.51481	.73354	.71705	.86270	.85268
.66418	.36008	1.90536	1.28717	.74567	.56511	.52639	.74567	.73000	.87697	.86744
.68133	.35380	1.94191	1.27406	.75828	.57467	.53819	.75828	.74340	.89180	.88275
.69848	.34791	1.97899	1.26173	.77132	.58455	.55018	.77132	.75720	.90713	.89854
.71563	.34238	2.01667	1.25009	.78474	.59472	.56233	.78474	.77135	.92291	.91476

.73277	.33717	2.05505	1.23908	.79849	.60514	.57461	.79849	.78580	.93909	.93137
.74992	.33224	2.09422	1.22864	.81255	.61580	.58699	.81255	.80052	.95563	.94831
.76707	.32758	2.13429	1.21873	.82689	.62667	.59947	.82689	.81549	.97249	.96555
.78422	.32316	2.17540	1.20930	.84148	.63772	.61204	.84148	.83067	.98964	.98307
.80137	.31896	2.21772	1.20031	.85629	.64895	.62468	.85629	.84604	1.00707	1.00083
.81852	.31497	2.26144	1.19172	.87131	.66033	.63738	.87131	.86160	1.02473	1.01882
.83566	.31116	2.30680	1.18350	.88653	.67186	.65014	.88653	.87731	1.04262	1.03702
.85281	.30752	2.35410	1.17563	.90192	.68353	.66296	.90192	.89317	1.06073	1.05541
.86996	.30404	2.40375	1.16807	.91747	.69532	.67583	.91747	.90918	1.07902	1.07398
.88711	.30070	2.45628	1.16081	.93319	.70723	.68875	.93319	.92531	1.09750	1.09272
.90426	.29750	2.51242	1.15382	.94905	.71925	.70172	.94905	.94157	1.11616	1.11162
.92141	.29443	2.57328	1.14709	.96506	.73138	.71474	.96506	.95796	1.13499	1.13068
.93855	.29147	2.64061	1.14058	.98122	.74363	.72782	.98122	.97447	1.15399	1.14990
.95570	.28861	2.71749	1.13429	.99753	.75598	.74096	.99753	.99111	1.17317	1.16929
.97285	.28586	2.81055	1.12820	1.01400	.76847	.75418	1.01400	1.00789	1.19254	1.18885
.99000	.28318	2.94126	1.12227	1.03067	.78110	.76751	1.03067	1.02486	1.21214	1.20864

For Q'= .060

					m=.5		**m=1.0**		**m=1.5**	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.49879	.49879	1.56837	1.56837	.69453	.52636	.45975	.69453	.67063	.81682	.80248
.51516	.48324	1.60112	1.53726	.69551	.52709	.46094	.69551	.67171	.81797	.80369
.53153	.46917	1.63391	1.50910	.69824	.52917	.46425	.69824	.67475	.82119	.80708
.54791	.45639	1.66676	1.48346	.70249	.53239	.46929	.70249	.67945	.82619	.81234
.56428	.44471	1.69972	1.45999	.70805	.53660	.47574	.70805	.68559	.83272	.81922
.58066	.43400	1.73282	1.43842	.71474	.54168	.48330	.71474	.69295	.84060	.82748
.59703	.42414	1.76610	1.41850	.72243	.54750	.49175	.72243	.70136	.84963	.83694
.61340	.41503	1.79959	1.40003	.73098	.55398	.50090	.73098	.71069	.85969	.84745
.62978	.40658	1.83336	1.38286	.74030	.56104	.51062	.74030	.72080	.87065	.85888
.64615	.39872	1.86743	1.36683	.75028	.56861	.52078	.75028	.73159	.88239	.87110
.66252	.39139	1.90186	1.35183	.76087	.57663	.53131	.76087	.74298	.89484	.88403
.67890	.38452	1.93670	1.33774	.77198	.58505	.54213	.77198	.75489	.90791	.89757
.69527	.37808	1.97202	1.32448	.78357	.59384	.55320	.78357	.76725	.92154	.91166
.71165	.37203	2.00787	1.31197	.79558	.60294	.56447	.79558	.78002	.93567	.92624
.72802	.36632	2.04434	1.30014	.80797	.61233	.57591	.80797	.79314	.95024	.94126
.74439	.36092	2.08150	1.28892	.82071	.62198	.58749	.82071	.80658	.96522	.95666
.76077	.35581	2.11945	1.27827	.83376	.63187	.59920	.83376	.82030	.98056	.97241
.77714	.35097	2.15830	1.26814	.84709	.64197	.61101	.84709	.83428	.99624	.98848
.79351	.34637	2.19818	1.25847	.86067	.65227	.62291	.86067	.84848	1.01222	1.00483
.80989	.34198	2.23926	1.24925	.87450	.66275	.63490	.87450	.86289	1.02848	1.02144
.82626	.33780	2.28171	1.24042	.88854	.67339	.64697	.88854	.87749	1.04500	1.03830
.84264	.33381	2.32577	1.23197	.90279	.68419	.65910	.90279	.89226	1.06175	1.05537
.85901	.32999	2.37175	1.22386	.91722	.69513	.67129	.91722	.90720	1.07873	1.07266
.87538	.32633	2.42002	1.21607	.93184	.70620	.68355	.93184	.92229	1.09591	1.09013
.89176	.32282	2.47110	1.20857	.94662	.71740	.69586	.94662	.93753	1.11330	1.10780
.90813	.31945	2.52571	1.20134	.96156	.72873	.70823	.96156	.95290	1.13087	1.12563
.92450	.31620	2.58491	1.19437	.97667	.74018	.72066	.97667	.96841	1.14864	1.14365
.94088	.31307	2.65037	1.18762	.99193	.75174	.73316	.99193	.98406	1.16659	1.16183
.95725	.31004	2.72508	1.18110	1.00736	.76344	.74574	1.00736	.99986	1.18473	1.18020
.97363	.30712	2.81535	1.17476	1.02297	.77527	.75840	1.02297	1.01582	1.20309	1.19878
.99000	.30428	2.94126	1.16859	1.03880	.78726	.77118	1.03880	1.03198	1.22171	1.21760

For Q'= .070

					m=.5		**m=1.0**		**m=1.5**	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.51933	.51933	1.60947	1.60947	.72550	.54983	.48076	.72550	.70087	.85324	.83852
.53502	.50437	1.64089	1.57953	.72636	.55048	.48182	.72636	.70183	.85426	.83959
.55071	.49073	1.67239	1.55225	.72880	.55232	.48476	.72880	.70453	.85712	.84260
.56640	.47824	1.70399	1.52727	.73260	.55520	.48929	.73260	.70873	.86159	.84731
.58209	.46677	1.73572	1.50428	.73759	.55899	.49512	.73759	.71425	.86747	.85348
.59778	.45618	1.76762	1.48305	.74364	.56357	.50201	.74364	.72090	.87458	.86094
.61347	.44638	1.79972	1.46336	.75061	.56885	.50976	.75061	.72854	.88277	.86953
.62915	.43728	1.83207	1.44503	.75839	.57475	.51821	.75839	.73704	.89193	.87910
.64484	.42881	1.86470	1.42793	.76690	.58120	.52721	.76690	.74629	.90194	.88955
.66053	.42089	1.89765	1.41191	.77605	.58814	.53668	.77605	.75621	.91270	.90076
.67622	.41347	1.93098	1.39687	.78578	.59551	.54652	.78578	.76671	.92414	.91266
.69191	.40651	1.96473	1.38271	.79603	.60328	.55667	.79603	.77772	.93619	.92516
.70760	.39996	1.99896	1.36935	.80674	.61139	.56707	.80674	.78918	.94879	.93820
.72329	.39378	2.03373	1.35672	.81787	.61983	.57769	.81787	.80105	.96188	.95173
.73898	.38793	2.06912	1.34474	.82937	.62855	.58850	.82937	.81328	.97541	.96570
.75467	.38240	2.10520	1.33337	.84123	.63753	.59946	.84123	.82583	.98935	.98005
.77035	.37715	2.14208	1.32255	.85339	.64675	.61055	.85339	.83867	1.00366	.99477
.78604	.37215	2.17984	1.31223	.86584	.65619	.62176	.86584	.85178	1.01830	1.00980

.80173	.36740	2.21864	1.30239	.87856	.66582	.63308	.87856	.86512	1.03326	1.02513
.81742	.36287	2.25860	1.29297	.89152	.67565	.64449	.89152	.87867	1.04850	1.04074
.83311	.35854	2.29993	1.28395	.90470	.68564	.65598	.90470	.89243	1.06400	1.05659
.84880	.35439	2.34284	1.27530	.91810	.69579	.66754	.91810	.90638	1.07976	1.07268
.86449	.35042	2.38762	1.26699	.93169	.70609	.67918	.93169	.92049	1.09575	1.08898
.88018	.34661	2.43466	1.25899	.94547	.71654	.69089	.94547	.93477	1.11195	1.10549
.89587	.34295	2.48444	1.25129	.95943	.72712	.70266	.95943	.94921	1.12837	1.12220
.91156	.33943	2.53766	1.24386	.97357	.73783	.71450	.97357	.96380	1.14499	1.13910
.92724	.33603	2.59536	1.23668	.98787	.74867	.72641	.98787	.97853	1.16181	1.15618
.94293	.33276	2.65915	1.22973	1.00234	.75964	.73839	1.00234	.99342	1.17884	1.17346
.95862	.32959	2.73190	1.22300	1.01700	.77074	.75046	1.01700	1.00847	1.19607	1.19093
.97431	.32651	2.81965	1.21646	1.03185	.78199	.76262	1.03185	1.02369	1.21353	1.20862
.99000	.32352	2.94126	1.21007	1.04693	.79343	.77493	1.04693	1.03914	1.23128	1.22658

For Q'= .080

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.53781	.53781	1.64649	1.64649	.75367	.57117	.50010	.75367	.72843	.88637	.87133
.55289	.52339	1.67676	1.61759	.75444	.57176	.50104	.75444	.72929	.88728	.87228
.56796	.51015	1.70713	1.59110	.75663	.57342	.50368	.75663	.73171	.88985	.87499
.58303	.49796	1.73763	1.56672	.76006	.57602	.50778	.76006	.73551	.89389	.87924
.59810	.48670	1.76829	1.54419	.76459	.57945	.51308	.76459	.74051	.89921	.88483
.61318	.47625	1.79913	1.52328	.77009	.58362	.51938	.77009	.74656	.90568	.89162
.62825	.46654	1.83020	1.50383	.77645	.58844	.52652	.77645	.75354	.91316	.89946
.64332	.45748	1.86152	1.48566	.78358	.59384	.53433	.78358	.76133	.92155	.90823
.65840	.44901	1.89314	1.46864	.79140	.59977	.54270	.79140	.76985	.93075	.91784
.67347	.44107	1.92510	1.45267	.79983	.60616	.55154	.79983	.77901	.94067	.92818
.68854	.43361	1.95744	1.43763	.80882	.61297	.56075	.80882	.78873	.95124	.93918
.70361	.42658	1.99021	1.42343	.81832	.62017	.57029	.81832	.79895	.96240	.95078
.71869	.41995	2.02347	1.41001	.82826	.62770	.58009	.82826	.80963	.97410	.96290
.73376	.41368	2.05728	1.39729	.83862	.63555	.59013	.83862	.82070	.98628	.97551
.74883	.40774	2.09170	1.38520	.84935	.64368	.60035	.84935	.83214	.99890	.98855
.76391	.40209	2.12682	1.37371	.86042	.65208	.61074	.86042	.84390	1.01192	1.00198
.77898	.39673	2.16272	1.36276	.87181	.66071	.62128	.87181	.85596	1.02531	1.01577
.79405	.39162	2.19951	1.35230	.88348	.66955	.63194	.88348	.86828	1.03904	1.02989
.80912	.38674	2.23731	1.34230	.89542	.67860	.64272	.89542	.88085	1.05309	1.04431
.82420	.38209	2.27627	1.33273	.90761	.68784	.65360	.90761	.89365	1.06743	1.05901
.83927	.37763	2.31657	1.32354	.92004	.69726	.66457	.92004	.90665	1.08203	1.07397
.85434	.37336	2.35843	1.31472	.93267	.70683	.67562	.93267	.91985	1.09690	1.08917
.86942	.36926	2.40213	1.30624	.94552	.71657	.68676	.94552	.93323	1.11200	1.10460
.88449	.36532	2.44804	1.29807	.95855	.72645	.69797	.95855	.94678	1.12733	1.12024
.89956	.36153	2.49663	1.29019	.97177	.73647	.70925	.97177	.96049	1.14288	1.13609
.91464	.35788	2.54860	1.28259	.98518	.74663	.72061	.98518	.97437	1.15865	1.15214
.92971	.35436	2.60492	1.27523	.99876	.75692	.73204	.99876	.98841	1.17463	1.16839
.94478	.35095	2.66718	1.26810	1.01253	.76735	.74355	1.01253	1.00261	1.19082	1.18485
.95985	.34765	2.73813	1.26118	1.02649	.77793	.75516	1.02649	1.01698	1.20723	1.20151
.97493	.34445	2.82357	1.25445	1.04065	.78866	.76688	1.04065	1.03154	1.22389	1.21841
.99000	.34133	2.94126	1.24787	1.05507	.79959	.77874	1.05507	1.04634	1.24084	1.23560

For Q'= .090

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.55465	.55465	1.68032	1.68032	.77962	.59084	.51810	.77962	.75388	.91690	.90159
.56917	.54073	1.70957	1.65234	.78032	.59137	.51895	.78032	.75466	.91772	.90245
.58368	.52787	1.73894	1.62657	.78230	.59287	.52134	.78230	.75685	.92005	.90490
.59819	.51597	1.76846	1.60275	.78542	.59523	.52505	.78542	.76029	.92371	.90876
.61270	.50492	1.79815	1.58064	.78955	.59836	.52989	.78955	.76485	.92857	.91386
.62721	.49463	1.82805	1.56006	.79458	.60218	.53568	.79458	.77039	.93449	.92007
.64172	.48503	1.85818	1.54085	.80042	.60661	.54227	.80042	.77680	.94136	.92727
.65623	.47603	1.88859	1.52285	.80699	.61158	.54952	.80699	.78398	.94908	.93535
.67075	.46759	1.91930	1.50594	.81421	.61705	.55731	.81421	.79186	.95757	.94422
.68526	.45966	1.95036	1.49002	.82202	.62297	.56558	.82202	.80034	.96676	.95380
.69977	.45218	1.98181	1.47501	.83036	.62929	.57422	.83036	.80938	.97657	.96401
.71428	.44511	2.01370	1.46080	.83919	.63598	.58320	.83919	.81890	.98695	.97481
.72879	.43843	2.04607	1.44734	.84845	.64301	.59245	.84845	.82887	.99785	.98612
.74330	.43209	2.07900	1.43456	.85813	.65034	.60194	.85813	.83924	1.00922	.99790
.75782	.42607	2.11254	1.42240	.86817	.65795	.61163	.86817	.84997	1.02103	1.01012
.77233	.42035	2.14677	1.41081	.87855	.66581	.62150	.87855	.86102	1.03324	1.02272
.78684	.41490	2.18178	1.39975	.88924	.67392	.63153	.88924	.87237	1.04581	1.03569
.80135	.40969	2.21768	1.38918	.90022	.68224	.64169	.90022	.88399	1.05873	1.04899
.81586	.40472	2.25457	1.37906	.91147	.69076	.65197	.91147	.89587	1.07196	1.06259
.83037	.39996	2.29261	1.36935	.92296	.69948	.66236	.92296	.90797	1.08548	1.07647
.84488	.39539	2.33197	1.36003	.93470	.70837	.67285	.93470	.92029	1.09928	1.09062

.85940	.39102	2.37286	1.35107	.94665	.71743	.68343	.94665	.93281	1.11334	1.10502
.87391	.38681	2.41556	1.34244	.95881	.72664	.69410	.95881	.94551	1.12764	1.11965
.88842	.38276	2.46043	1.33412	.97118	.73601	.70485	.97118	.95840	1.14218	1.13451
.90293	.37886	2.50793	1.32608	.98373	.74553	.71568	.98373	.97146	1.15695	1.14958
.91744	.37510	2.55872	1.31832	.99648	.75519	.72660	.99648	.98470	1.17194	1.16486
.93195	.37146	2.61377	1.31080	1.00941	.76499	.73759	1.00941	.99809	1.18714	1.18035
.94647	.36794	2.67461	1.30351	1.02253	.77493	.74868	1.02253	1.01167	1.20258	1.19605
.96098	.36453	2.74389	1.29643	1.03585	.78503	.75987	1.03585	1.02542	1.21824	1.21198
.97549	.36121	2.82718	1.28953	1.04939	.79529	.77117	1.04939	1.03938	1.23417	1.22816
.99000	.35797	2.94126	1.28277	1.06320	.80575	.78263	1.06320	1.05359	1.25041	1.24464

For Q'= .100

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.57016	.57016	1.71157	1.71157	.80378	.60915	.53502	.80378	.77761	.94531	.92978
.58415	.55669	1.73990	1.68442	.80441	.60963	.53578	.80441	.77831	.94605	.93056
.59814	.54420	1.76837	1.65931	.80621	.61100	.53795	.80621	.78030	.94817	.93279
.61214	.53258	1.79700	1.63601	.80906	.61315	.54134	.80906	.78345	.95152	.93631
.62613	.52175	1.82582	1.61431	.81285	.61602	.54577	.81285	.78763	.95597	.94099
.64013	.51163	1.85486	1.59405	.81748	.61953	.55111	.81748	.79272	.96142	.94670
.65412	.50214	1.88415	1.57507	.82286	.62361	.55720	.82286	.79863	.96775	.95334
.66812	.49322	1.91372	1.55725	.82894	.62822	.56394	.82894	.80528	.97489	.96081
.68211	.48484	1.94360	1.54046	.83563	.63329	.57122	.83563	.81258	.98277	.96903
.69611	.47692	1.97384	1.52462	.84288	.63879	.57895	.84288	.82047	.99130	.97794
.71010	.46944	2.00447	1.50965	.85065	.64467	.58708	.85065	.82890	1.00044	.98745
.72410	.46236	2.03554	1.49545	.85889	.65092	.59554	.85889	.83780	1.01012	.99753
.73809	.45565	2.06711	1.48198	.86756	.65748	.60428	.86756	.84713	1.02031	1.00811
.75209	.44927	2.09922	1.46916	.87661	.66435	.61327	.87661	.85686	1.03097	1.01916
.76608	.44320	2.13195	1.45694	.88603	.67149	.62247	.88603	.86695	1.04205	1.03063
.78008	.43741	2.16537	1.44529	.89579	.67888	.63185	.89579	.87736	1.05352	1.04249
.79407	.43189	2.19956	1.43414	.90585	.68651	.64140	.90585	.88807	1.06536	1.05471
.80807	.42661	2.23462	1.42347	.91621	.69435	.65109	.91621	.89905	1.07753	1.06726
.82206	.42155	2.27067	1.41325	.92683	.70240	.66091	.92683	.91029	1.09002	1.08011
.83606	.41671	2.30786	1.40343	.93770	.71064	.67084	.93770	.92176	1.10281	1.09326
.85005	.41206	2.34634	1.39399	.94880	.71906	.68089	.94880	.93345	1.11587	1.10667
.86405	.40759	2.38633	1.38490	.96013	.72765	.69103	.96013	.94535	1.12919	1.12033
.87804	.40329	2.42810	1.37614	.97168	.73639	.70127	.97168	.95744	1.14277	1.13424
.89204	.39914	2.47200	1.36769	.98343	.74530	.71159	.98343	.96972	1.15659	1.14837
.90603	.39515	2.51847	1.35952	.99537	.75435	.72201	.99537	.98218	1.17064	1.16273
.92003	.39129	2.56817	1.35162	1.00751	.76355	.73251	1.00751	.99482	1.18492	1.17731
.93402	.38755	2.62204	1.34396	1.01985	.77290	.74310	1.01985	1.00763	1.19942	1.19210
.94802	.38393	2.68154	1.33652	1.03238	.78240	.75379	1.03238	1.02062	1.21416	1.20712
.96201	.38042	2.74926	1.32929	1.04512	.79205	.76458	1.04512	1.03381	1.22914	1.22237
.97601	.37699	2.83053	1.32223	1.05809	.80188	.77551	1.05809	1.04721	1.24439	1.23788
.99000	.37365	2.94126	1.31532	1.07133	.81192	.78660	1.07133	1.06087	1.25997	1.25371

For Q'= .120

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.59797	.59797	1.76800	1.76800	.84783	.64253	.56625	.84783	.82098	.99711	.98124
.61103	.58534	1.79473	1.74231	.84836	.64293	.56689	.84836	.82157	.99773	.98189
.62410	.57353	1.82162	1.71838	.84987	.64408	.56869	.84987	.82324	.99952	.98376
.63717	.56246	1.84870	1.69604	.85229	.64591	.57154	.85229	.82590	1.00236	.98674
.65024	.55206	1.87599	1.67511	.85551	.64835	.57530	.85551	.82945	1.00615	.99072
.66331	.54228	1.90351	1.65545	.85947	.65136	.57986	.85947	.83381	1.01080	.99560
.67637	.53305	1.93130	1.63694	.86410	.65487	.58512	.86410	.83889	1.01625	1.00131
.68944	.52433	1.95938	1.61948	.86934	.65884	.59097	.86934	.84463	1.02242	1.00776
.70251	.51608	1.98779	1.60296	.87515	.66324	.59733	.87515	.85097	1.02925	1.01490
.71558	.50826	2.01657	1.58731	.88147	.66803	.60415	.88147	.85785	1.03668	1.02265
.72864	.50083	2.04574	1.57246	.88826	.67318	.61135	.88826	.86523	1.04467	1.03098
.74171	.49377	2.07536	1.55833	.89549	.67866	.61888	.89549	.87305	1.05317	1.03983
.75478	.48704	2.10547	1.54487	.90312	.68444	.62671	.90312	.88130	1.06214	1.04916
.76785	.48062	2.13613	1.53203	.91113	.69051	.63479	.91113	.88992	1.07156	1.05894
.78092	.47449	2.16739	1.51975	.91948	.69683	.64310	.91948	.89888	1.08138	1.06912
.79398	.46862	2.19934	1.50800	.92815	.70341	.65160	.92815	.90817	1.09158	1.07968
.80705	.46301	2.23204	1.49674	.93712	.71021	.66028	.93712	.91775	1.10213	1.09059
.82012	.45762	2.26560	1.48594	.94638	.71722	.66912	.94638	.92761	1.11302	1.10183
.83319	.45245	2.30013	1.47555	.95590	.72444	.67810	.95590	.93773	1.12421	1.11338
.84625	.44748	2.33576	1.46556	.96567	.73184	.68721	.96567	.94808	1.13570	1.12521
.85932	.44270	2.37265	1.45594	.97568	.73943	.69644	.97568	.95866	1.14747	1.13732
.87239	.43809	2.41100	1.44665	.98591	.74718	.70579	.98591	.96945	1.15951	1.14969
.88546	.43364	2.45107	1.43768	.99636	.75510	.71524	.99636	.98045	1.17180	1.16230
.89853	.42934	2.49319	1.42901	1.00703	.76318	.72479	1.00703	.99164	1.18434	1.17516

.91159	.42519	2.53780	1.42061	1.01789	.77142	.73444	1.01789	1.00303	1.19712	1.18825
.92466	.42117	2.58550	1.41247	1.02896	.77981	.74420	1.02896	1.01460	1.21014	1.20157
.93773	.41726	2.63718	1.40456	1.04023	.78835	.75405	1.04023	1.02636	1.22340	1.21512
.95080	.41348	2.69423	1.39687	1.05172	.79705	.76402	1.05172	1.03832	1.23690	1.22890
.96386	.40979	2.75908	1.38937	1.06342	.80592	.77411	1.06342	1.05048	1.25066	1.24294
.97693	.40618	2.83665	1.38205	1.07536	.81497	.78434	1.07536	1.06288	1.26471	1.25726
.99000	.40265	2.94126	1.37485	1.08760	.82425	.79476	1.08760	1.07556	1.27910	1.27192

For Q'= .140

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.62246	.62246	1.81823	1.81823	.88749	.67259	.59480	.88749	.86015	1.04376	1.02764
.63471	.61058	1.84359	1.79380	.88794	.67294	.59533	.88794	.86065	1.04429	1.02820
.64696	.59939	1.86913	1.77091	.88924	.67392	.59686	.88924	.86207	1.04582	1.02980
.65921	.58884	1.89487	1.74942	.89131	.67549	.59928	.89131	.86435	1.04825	1.03235
.67147	.57887	1.92083	1.72919	.89409	.67759	.60250	.89409	.86741	1.05152	1.03578
.68372	.56943	1.94704	1.71011	.89752	.68019	.60643	.89752	.87118	1.05555	1.04000
.69597	.56049	1.97353	1.69207	.90155	.68324	.61099	.90155	.87559	1.06029	1.04496
.70822	.55199	2.00032	1.67497	.90612	.68671	.61611	.90612	.88060	1.06567	1.05059
.72047	.54392	2.02744	1.65874	.91121	.69057	.62170	.91121	.88615	1.07165	1.05684
.73272	.53623	2.05493	1.64331	.91677	.69478	.62773	.91677	.89220	1.07819	1.06366
.74497	.52889	2.08282	1.62861	.92276	.69932	.63413	.92276	.89872	1.08524	1.07101
.75722	.52189	2.11116	1.61459	.92915	.70417	.64086	.92915	.90565	1.09276	1.07884
.76948	.51520	2.13999	1.60119	.93593	.70930	.64789	.93593	.91298	1.10073	1.08713
.78173	.50879	2.16936	1.58837	.94305	.71470	.65518	.94305	.92067	1.10911	1.09583
.79398	.50265	2.19933	1.57609	.95051	.72035	.66269	.95051	.92869	1.11787	1.10493
.80623	.49675	2.22996	1.56430	.95827	.72623	.67042	.95827	.93702	1.12700	1.11439
.81848	.49109	2.26135	1.55298	.96632	.73234	.67833	.96632	.94565	1.13647	1.12420
.83073	.48565	2.29357	1.54209	.97465	.73865	.68641	.97465	.95454	1.14627	1.13432
.84298	.48041	2.32673	1.53160	.98324	.74516	.69464	.98324	.96369	1.15637	1.14475
.85524	.47535	2.36096	1.52148	.99207	.75185	.70301	.99207	.97308	1.16675	1.15547
.86749	.47048	2.39642	1.51172	1.00114	.75872	.71152	1.00114	.98270	1.17742	1.16646
.87974	.46577	2.43330	1.50228	1.01043	.76577	.72015	1.01043	.99254	1.18835	1.17771
.89199	.46122	2.47185	1.49315	1.01995	.77298	.72890	1.01995	1.00258	1.19954	1.18921
.90424	.45681	2.51236	1.48431	1.02967	.78035	.73776	1.02967	1.01283	1.21098	1.20096
.91649	.45254	2.55528	1.47572	1.03961	.78787	.74673	1.03961	1.02328	1.22266	1.21294
.92874	.44839	2.60116	1.46739	1.04975	.79556	.75581	1.04975	1.03392	1.23459	1.22517
.94099	.44436	2.65086	1.45928	1.06010	.80340	.76501	1.06010	1.04476	1.24676	1.23763
.95325	.44043	2.70570	1.45137	1.07067	.81141	.77433	1.07067	1.05581	1.25919	1.25035
.96550	.43660	2.76792	1.44366	1.08146	.81959	.78378	1.08146	1.06707	1.27188	1.26332
.97775	.43285	2.84214	1.43609	1.09251	.82797	.79339	1.09251	1.07858	1.28488	1.27659
.99000	.42917	2.94126	1.42865	1.10386	.83657	.80320	1.10386	1.09039	1.29823	1.29022

For Q'= .160

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.64439	.64439	1.86376	1.86376	.92383	.70013	.62129	.92383	.89612	1.08650	1.07020
.65592	.63319	1.88792	1.84043	.92422	.70043	.62174	.92422	.89655	1.08695	1.07068
.66744	.62258	1.91226	1.81848	.92534	.70128	.62304	.92534	.89778	1.08827	1.07206
.67896	.61252	1.93683	1.79777	.92713	.70264	.62512	.92713	.89975	1.09038	1.07427
.69048	.60296	1.96162	1.77820	.92955	.70447	.62790	.92955	.90241	1.09323	1.07725
.70200	.59387	1.98667	1.75966	.93255	.70674	.63132	.93255	.90570	1.09675	1.08094
.71352	.58522	2.01200	1.74206	.93608	.70942	.63530	.93608	.90956	1.10091	1.08529
.72504	.57696	2.03764	1.72534	.94011	.71247	.63979	.94011	.91397	1.10564	1.09024
.73656	.56908	2.06362	1.70941	.94460	.71587	.64473	.94460	.91887	1.11093	1.09576
.74808	.56155	2.08996	1.69421	.94952	.71960	.65007	.94952	.92423	1.11671	1.10179
.75960	.55434	2.11670	1.67970	.95484	.72364	.65578	.95484	.93001	1.12297	1.10832
.77112	.54743	2.14389	1.66581	.96054	.72795	.66182	.96054	.93619	1.12967	1.11530
.78264	.54081	2.17156	1.65251	.96659	.73254	.66814	.96659	.94274	1.13679	1.12270
.79416	.53445	2.19977	1.63974	.97297	.73737	.67472	.97297	.94963	1.14429	1.13050
.80568	.52833	2.22857	1.62749	.97966	.74245	.68153	.97966	.95684	1.15216	1.13867
.81720	.52244	2.25802	1.61570	.98665	.74774	.68856	.98665	.96436	1.16038	1.14719
.82872	.51677	2.28821	1.60435	.99391	.75325	.69578	.99391	.97215	1.16892	1.15604
.84024	.51131	2.31921	1.59341	1.00144	.75895	.70318	1.00144	.98021	1.17778	1.16521
.85176	.50603	2.35113	1.58286	1.00922	.76485	.71073	1.00922	.98852	1.18693	1.17467
.86328	.50093	2.38409	1.57266	1.01724	.77093	.71844	1.01724	.99707	1.19636	1.18441
.87480	.49600	2.41825	1.56279	1.02550	.77718	.72629	1.02550	1.00584	1.20607	1.19442
.88632	.49122	2.45378	1.55324	1.03397	.78361	.73427	1.03397	1.01484	1.21604	1.20469
.89784	.48660	2.49092	1.54399	1.04267	.79019	.74238	1.04267	1.02405	1.22626	1.21522
.90936	.48211	2.52997	1.53500	1.05157	.79694	.75061	1.05157	1.03346	1.23673	1.22599
.92088	.47774	2.57133	1.52627	1.06069	.80385	.75896	1.06069	1.04308	1.24746	1.23701
.93240	.47350	2.61554	1.51778	1.07002	.81092	.76744	1.07002	1.05289	1.25842	1.24827
.94392	.46937	2.66342	1.50950	1.07956	.81815	.77604	1.07956	1.06292	1.26964	1.25977

.95544	.46534	2.71621	1.50141	1.08932	.82555	.78477	1.08932	1.07315	1.28112	1.27153
.96696	.46139	2.77602	1.49351	1.09931	.83312	.79364	1.09931	1.08362	1.29287	1.28356
.97848	.45752	2.84713	1.48574	1.10956	.84089	.80268	1.10956	1.09433	1.30493	1.29590
.99000	.45371	2.94126	1.47808	1.12013	.84890	.81194	1.12013	1.10536	1.31736	1.30860

For Q'= .180

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.66428	.66428	1.90557	1.90557	.95755	.72569	.64613	.95755	.92957	1.12615	1.10973
.67514	.65369	1.92866	1.88324	.95788	.72594	.64652	.95788	.92994	1.12655	1.11015
.68599	.64362	1.95195	1.86213	.95886	.72668	.64764	.95886	.93101	1.12770	1.11135
.69685	.63402	1.97545	1.84215	.96043	.72787	.64944	.96043	.93274	1.12954	1.11328
.70771	.62486	1.99920	1.82319	.96255	.72948	.65186	.96255	.93506	1.13204	1.11590
.71857	.61612	2.02321	1.80517	.96519	.73148	.65484	.96519	.93795	1.13514	1.11914
.72942	.60776	2.04749	1.78801	.96831	.73384	.65833	.96831	.94136	1.13881	1.12298
.74028	.59975	2.07209	1.77165	.97188	.73655	.66229	.97188	.94526	1.14301	1.12736
.75114	.59208	2.09703	1.75602	.97587	.73957	.66667	.97587	.94961	1.14770	1.13226
.76200	.58473	2.12233	1.74107	.98025	.74289	.67143	.98025	.95438	1.15285	1.13763
.77285	.57766	2.14803	1.72676	.98500	.74649	.67653	.98500	.95955	1.15844	1.14346
.78371	.57087	2.17417	1.71302	.99010	.75036	.68195	.99010	.96508	1.16444	1.14971
.79457	.56434	2.20079	1.69984	.99553	.75447	.68765	.99553	.97096	1.17083	1.15635
.80543	.55805	2.22793	1.68716	1.00127	.75882	.69360	1.00127	.97716	1.17758	1.16337
.81628	.55199	2.25566	1.67496	1.00731	.76340	.69979	1.00731	.98367	1.18467	1.17074
.82714	.54614	2.28403	1.66320	1.01362	.76818	.70619	1.01362	.99047	1.19210	1.17844
.83800	.54049	2.31311	1.65186	1.02020	.77317	.71279	1.02020	.99754	1.19984	1.18647
.84885	.53502	2.34299	1.64090	1.02704	.77835	.71956	1.02704	1.00486	1.20788	1.19479
.85971	.52974	2.37377	1.63031	1.03411	.78371	.72651	1.03411	1.01243	1.21620	1.20340
.87057	.52462	2.40556	1.62006	1.04143	.78925	.73361	1.04143	1.02024	1.22480	1.21229
.88143	.51966	2.43851	1.61012	1.04896	.79497	.74086	1.04896	1.02827	1.23366	1.22144
.89228	.51484	2.47280	1.60049	1.05672	.80084	.74825	1.05672	1.03652	1.24279	1.23085
.90314	.51017	2.50864	1.59114	1.06469	.80688	.75577	1.06469	1.04498	1.25216	1.24051
.91400	.50562	2.54632	1.58204	1.07287	.81309	.76343	1.07287	1.05365	1.26178	1.25042
.92486	.50120	2.58624	1.57319	1.08126	.81945	.77121	1.08126	1.06253	1.27165	1.26057
.93571	.49688	2.62890	1.56456	1.08987	.82596	.77913	1.08987	1.07160	1.28177	1.27097
.94657	.49267	2.67508	1.55614	1.09868	.83265	.78717	1.09868	1.08089	1.29214	1.28161
.95743	.48856	2.72595	1.54791	1.10772	.83950	.79536	1.10772	1.09040	1.30277	1.29252
.96829	.48452	2.78351	1.53984	1.11700	.84653	.80370	1.11700	1.10014	1.31368	1.30370
.97914	.48055	2.85174	1.53190	1.12654	.85376	.81222	1.12654	1.11014	1.32490	1.31519
.99000	.47663	2.94126	1.52404	1.13640	.86123	.82096	1.13640	1.12045	1.33649	1.32706

For Q'= .200

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.68247	.68247	1.94437	1.94437	.98914	.74963	.66963	.98914	.96098	1.16331	1.14681
.69272	.67245	1.96649	1.92294	.98944	.74985	.66997	.98944	.96131	1.16366	1.14718
.70298	.66288	1.98882	1.90261	.99030	.75050	.67094	.99030	.96225	1.16467	1.14823
.71323	.65372	2.01136	1.88330	.99168	.75155	.67251	.99168	.96376	1.16629	1.14993
.72348	.64495	2.03415	1.86492	.99355	.75297	.67462	.99355	.96581	1.16850	1.15224
.73373	.63654	2.05721	1.84739	.99589	.75474	.67724	.99589	.96837	1.17124	1.15511
.74398	.62847	2.08054	1.83066	.99866	.75684	.68032	.99866	.97139	1.17450	1.15851
.75423	.62072	2.10419	1.81466	1.00184	.75925	.68382	1.00184	.97486	1.17824	1.16241
.76448	.61328	2.12817	1.79933	1.00540	.76195	.68772	1.00540	.97873	1.18243	1.16678
.77473	.60611	2.15252	1.78464	1.00932	.76492	.69197	1.00932	.98300	1.18704	1.17159
.78498	.59920	2.17726	1.77053	1.01358	.76815	.69654	1.01358	.98763	1.19205	1.17682
.79523	.59255	2.20243	1.75697	1.01817	.77163	.70141	1.01817	.99261	1.19745	1.18243
.80548	.58613	2.22808	1.74392	1.02306	.77534	.70656	1.02306	.99791	1.20320	1.18842
.81573	.57993	2.25424	1.73135	1.02825	.77927	.71195	1.02825	1.00351	1.20930	1.19476
.82599	.57394	2.28098	1.71922	1.03371	.78341	.71758	1.03371	1.00940	1.21573	1.20143
.83624	.56814	2.30834	1.70751	1.03944	.78775	.72341	1.03944	1.01557	1.22246	1.20842
.84649	.56253	2.33640	1.69619	1.04542	.79228	.72944	1.04542	1.02200	1.22950	1.21571
.85674	.55710	2.36524	1.68524	1.05164	.79700	.73566	1.05164	1.02868	1.23682	1.22330
.86699	.55183	2.39496	1.67464	1.05810	.80189	.74205	1.05810	1.03560	1.24441	1.23116
.87724	.54671	2.42566	1.66436	1.06478	.80696	.74859	1.06478	1.04274	1.25227	1.23929
.88749	.54174	2.45748	1.65438	1.07169	.81219	.75529	1.07169	1.05011	1.26039	1.24767
.89774	.53691	2.49060	1.64469	1.07881	.81758	.76214	1.07881	1.05770	1.26876	1.25632
.90799	.53221	2.52523	1.63526	1.08614	.82314	.76912	1.08614	1.06549	1.27738	1.26521
.91824	.52763	2.56164	1.62609	1.09367	.82885	.77625	1.09367	1.07349	1.28624	1.27434
.92849	.52316	2.60020	1.61714	1.10142	.83472	.78351	1.10142	1.08169	1.29535	1.28372
.93875	.51880	2.64140	1.60841	1.10937	.84075	.79090	1.10937	1.09011	1.30471	1.29334
.94900	.51453	2.68598	1.59987	1.11754	.84694	.79844	1.11754	1.09873	1.31431	1.30321
.95925	.51035	2.73505	1.59150	1.12593	.85329	.80612	1.12593	1.10758	1.32418	1.31335
.96950	.50625	2.79050	1.58329	1.13456	.85983	.81396	1.13456	1.11666	1.33433	1.32376
.97975	.50220	2.85601	1.57520	1.14345	.86657	.82199	1.14345	1.12600	1.34479	1.33449

.99000	.49819	2.94126	1.56718	1.15266	.87356	.83025	1.15266	1.13566	1.35562	1.34559

For Q'= .250

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.72214	.72214	2.03116	2.03116	1.06107	.80414	.72386	1.06107	1.03270	1.24790	1.23134
.73106	.71337	2.05119	2.01168	1.06128	.80430	.72410	1.06128	1.03294	1.24815	1.23161
.73999	.70492	2.07143	1.99307	1.06192	.80478	.72480	1.06192	1.03363	1.24890	1.23238
.74892	.69676	2.09191	1.97526	1.06295	.80556	.72594	1.06295	1.03475	1.25011	1.23365
.75785	.68889	2.11262	1.95820	1.06435	.80663	.72748	1.06435	1.03628	1.25176	1.23537
.76678	.68129	2.13360	1.94183	1.06611	.80796	.72941	1.06611	1.03820	1.25383	1.23753
.77571	.67394	2.15486	1.92610	1.06821	.80955	.73169	1.06821	1.04048	1.25630	1.24010
.78464	.66682	2.17642	1.91097	1.07063	.81139	.73432	1.07063	1.04311	1.25915	1.24307
.79357	.65994	2.19831	1.89640	1.07336	.81346	.73726	1.07336	1.04608	1.26236	1.24641
.80249	.65327	2.22055	1.88235	1.07638	.81575	.74049	1.07638	1.04935	1.26591	1.25011
.81142	.64680	2.24317	1.86880	1.07969	.81825	.74400	1.07969	1.05293	1.26980	1.25416
.82035	.64053	2.26621	1.85570	1.08326	.82096	.74777	1.08326	1.05680	1.27400	1.25852
.82928	.63445	2.28970	1.84304	1.08709	.82386	.75179	1.08709	1.06094	1.27850	1.26320
.83821	.62854	2.31369	1.83079	1.09116	.82695	.75602	1.09116	1.06534	1.28329	1.26818
.84714	.62279	2.33821	1.81892	1.09547	.83021	.76047	1.09547	1.06999	1.28836	1.27345
.85607	.61720	2.36333	1.80741	1.10002	.83366	.76513	1.10002	1.07489	1.29371	1.27900
.86500	.61177	2.38911	1.79623	1.10478	.83727	.76997	1.10478	1.08002	1.29931	1.28481
.87393	.60647	2.41561	1.78538	1.10977	.84105	.77499	1.10977	1.08537	1.30517	1.29088
.88285	.60131	2.44294	1.77483	1.11496	.84498	.78018	1.11496	1.09094	1.31128	1.29720
.89178	.59628	2.47118	1.76456	1.12036	.84907	.78553	1.12036	1.09672	1.31763	1.30377
.90071	.59136	2.50047	1.75456	1.12596	.85332	.79104	1.12596	1.10271	1.32421	1.31058
.90964	.58656	2.53095	1.74480	1.13176	.85771	.79671	1.13176	1.10890	1.33103	1.31763
.91857	.58187	2.56283	1.73528	1.13775	.86225	.80252	1.13775	1.11529	1.33809	1.32491
.92750	.57728	2.59634	1.72598	1.14394	.86695	.80847	1.14394	1.12189	1.34537	1.33243
.93643	.57278	2.63182	1.71687	1.15033	.87179	.81458	1.15033	1.12868	1.35288	1.34018
.94536	.56836	2.66970	1.70795	1.15693	.87679	.82083	1.15693	1.13568	1.36064	1.34816
.95428	.56403	2.71064	1.69920	1.16373	.88194	.82723	1.16373	1.14289	1.36863	1.35640
.96321	.55976	2.75560	1.69060	1.17074	.88726	.83379	1.17074	1.15031	1.37689	1.36489
.97214	.55554	2.80621	1.68211	1.17799	.89275	.84052	1.17799	1.15798	1.38541	1.37366
.98107	.55137	2.86555	1.67372	1.18551	.89845	.84745	1.18551	1.16590	1.39425	1.38273
.99000	.54722	2.94126	1.66537	1.19333	.90438	.85462	1.19333	1.17415	1.40345	1.39218

For Q'= .300

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.75542	.75542	2.10696	2.10696	1.12564	.85307	.77331	1.12564	1.09732	1.32384	1.30735
.76324	.74772	2.12525	2.08913	1.12580	.85320	.77348	1.12580	1.09749	1.32403	1.30755
.77106	.74024	2.14375	2.07200	1.12628	.85356	.77400	1.12628	1.09802	1.32460	1.30814
.77888	.73298	2.16248	2.05551	1.12707	.85416	.77484	1.12707	1.09887	1.32552	1.30910
.78670	.72592	2.18144	2.03962	1.12814	.85497	.77600	1.12814	1.10004	1.32679	1.31042
.79452	.71906	2.20066	2.02429	1.12950	.85600	.77745	1.12950	1.10151	1.32838	1.31208
.80234	.71238	2.22015	2.00950	1.13113	.85723	.77918	1.13113	1.10327	1.33029	1.31407
.81016	.70589	2.23994	1.99520	1.13301	.85866	.78119	1.13301	1.10531	1.33251	1.31638
.81798	.69957	2.26004	1.98137	1.13514	.86028	.78344	1.13514	1.10762	1.33502	1.31898
.82579	.69341	2.28047	1.96797	1.13751	.86207	.78594	1.13751	1.11018	1.33781	1.32188
.83361	.68741	2.30128	1.95499	1.14012	.86405	.78867	1.14012	1.11300	1.34087	1.32506
.84143	.68155	2.32248	1.94240	1.14294	.86619	.79162	1.14294	1.11605	1.34419	1.32852
.84925	.67584	2.34410	1.93017	1.14599	.86849	.79477	1.14599	1.11933	1.34777	1.33223
.85707	.67027	2.36620	1.91829	1.14924	.87096	.79813	1.14924	1.12284	1.35159	1.33620
.86489	.66483	2.38880	1.90674	1.15269	.87358	.80168	1.15269	1.12656	1.35566	1.34042
.87271	.65951	2.41196	1.89550	1.15635	.87635	.80541	1.15635	1.13049	1.35996	1.34488
.88053	.65431	2.43574	1.88455	1.16020	.87927	.80931	1.16020	1.13463	1.36449	1.34957
.88835	.64923	2.46020	1.87387	1.16424	.88233	.81338	1.16424	1.13897	1.36924	1.35449
.89617	.64425	2.48543	1.86345	1.16847	.88553	.81761	1.16847	1.14350	1.37421	1.35964
.90399	.63937	2.51150	1.85328	1.17288	.88887	.82200	1.17288	1.14822	1.37940	1.36500
.91181	.63459	2.53855	1.84334	1.17747	.89236	.82654	1.17747	1.15314	1.38480	1.37059
.91963	.62990	2.56670	1.83362	1.18224	.89597	.83124	1.18224	1.15824	1.39041	1.37639
.92745	.62530	2.59614	1.82409	1.18720	.89973	.83608	1.18720	1.16353	1.39624	1.38241
.93526	.62077	2.62707	1.81476	1.19234	.90362	.84106	1.19234	1.16901	1.40228	1.38865
.94308	.61632	2.65981	1.80559	1.19766	.90766	.84620	1.19766	1.17467	1.40854	1.39511
.95090	.61194	2.69473	1.79658	1.20317	.91183	.85148	1.20317	1.18053	1.41502	1.40179
.95872	.60761	2.73241	1.78771	1.20888	.91616	.85692	1.20888	1.18660	1.42174	1.40871
.96654	.60333	2.77369	1.77896	1.21479	.92064	.86252	1.21479	1.19287	1.42869	1.41587
.97436	.59910	2.81997	1.77031	1.22093	.92529	.86829	1.22093	1.19937	1.43591	1.42330
.98218	.59488	2.87382	1.76172	1.22732	.93013	.87427	1.22732	1.20613	1.44342	1.43103
.99000	.59067	2.94126	1.75315	1.23400	.93519	.88048	1.23400	1.21319	1.45128	1.43910

For Q'= .400

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.80845	.80845	2.23559	2.23559	1.24062	.94021	.86279	1.24062	1.21285	1.45907	1.44297
.81450	.80246	2.25106	2.22046	1.24072	.94029	.86289	1.24072	1.21296	1.45918	1.44309
.82055	.79658	2.26673	2.20576	1.24101	.94051	.86319	1.24101	1.21328	1.45953	1.44345
.82660	.79081	2.28261	2.19151	1.24150	.94088	.86368	1.24150	1.21380	1.46010	1.44404
.83265	.78515	2.29870	2.17766	1.24216	.94138	.86437	1.24216	1.21451	1.46088	1.44485
.83871	.77959	2.31504	2.16420	1.24301	.94202	.86524	1.24301	1.21542	1.46187	1.44588
.84476	.77414	2.33162	2.15110	1.24403	.94280	.86628	1.24403	1.21652	1.46307	1.44713
.85081	.76878	2.34846	2.13834	1.24522	.94370	.86750	1.24522	1.21780	1.46448	1.44858
.85686	.76352	2.36560	2.12590	1.24658	.94473	.86888	1.24658	1.21926	1.46607	1.45023
.86291	.75835	2.38303	2.11378	1.24810	.94588	.87043	1.24810	1.22089	1.46786	1.45209
.86897	.75326	2.40079	2.10194	1.24978	.94715	.87213	1.24978	1.22270	1.46984	1.45413
.87502	.74826	2.41891	2.09039	1.25162	.94855	.87399	1.25162	1.22467	1.47200	1.45637
.88107	.74335	2.43740	2.07910	1.25361	.95006	.87600	1.25361	1.22681	1.47434	1.45880
.88712	.73851	2.45631	2.06805	1.25575	.95168	.87816	1.25575	1.22911	1.47686	1.46140
.89317	.73375	2.47567	2.05725	1.25804	.95342	.88046	1.25804	1.23156	1.47956	1.46419
.89922	.72906	2.49551	2.04667	1.26048	.95527	.88290	1.26048	1.23418	1.48243	1.46716
.90528	.72444	2.51589	2.03630	1.26307	.95723	.88548	1.26307	1.23695	1.48547	1.47031
.91133	.71988	2.53686	2.02613	1.26580	.95930	.88819	1.26580	1.23987	1.48868	1.47363
.91738	.71539	2.55849	2.01615	1.26868	.96148	.89103	1.26868	1.24294	1.49207	1.47713
.92343	.71096	2.58085	2.00635	1.27170	.96377	.89401	1.27170	1.24617	1.49562	1.48080
.92948	.70658	2.60404	1.99672	1.27487	.96617	.89712	1.27487	1.24955	1.49934	1.48464
.93553	.70226	2.62817	1.98725	1.27818	.96868	.90036	1.27818	1.25309	1.50324	1.48867
.94159	.69798	2.65338	1.97791	1.28164	.97130	.90373	1.28164	1.25677	1.50731	1.49287
.94764	.69375	2.67985	1.96872	1.28525	.97404	.90723	1.28525	1.26062	1.51155	1.49725
.95369	.68956	2.70780	1.95964	1.28901	.97689	.91087	1.28901	1.26463	1.51598	1.50181
.95974	.68540	2.73756	1.95067	1.29294	.97986	.91465	1.29294	1.26880	1.52059	1.50657
.96579	.68127	2.76955	1.94179	1.29703	.98296	.91857	1.29703	1.27315	1.52540	1.51153
.97184	.67716	2.80441	1.93298	1.30129	.98619	.92264	1.30129	1.27768	1.53042	1.51669
.97790	.67306	2.84314	1.92423	1.30575	.98957	.92687	1.30575	1.28240	1.53566	1.52209
.98395	.66895	2.88752	1.91549	1.31042	.99311	.93128	1.31042	1.28735	1.54115	1.52774
.99000	.66482	2.94126	1.90672	1.31533	.99683	.93591	1.31533	1.29255	1.54693	1.53368

For Q'= .500

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}
.84847	.84847	2.34192	2.34192	1.34381	1.01841	.94423	1.34381	1.31695	1.58042	1.56490
.85319	.84384	2.35517	2.32908	1.34387	1.01846	.94430	1.34387	1.31702	1.58050	1.56498
.85791	.83915	2.36859	2.31625	1.34406	1.01861	.94448	1.34406	1.31722	1.58072	1.56521
.86263	.83457	2.38219	2.30384	1.34437	1.01884	.94479	1.34437	1.31756	1.58109	1.56559
.86734	.83004	2.39600	2.29172	1.34480	1.01917	.94522	1.34480	1.31802	1.58160	1.56612
.87206	.82556	2.41001	2.27987	1.34536	1.01959	.94577	1.34536	1.31861	1.58225	1.56679
.87678	.82114	2.42425	2.26826	1.34603	1.02010	.94643	1.34603	1.31933	1.58304	1.56761
.88150	.81676	2.43872	2.25690	1.34682	1.02070	.94721	1.34682	1.32017	1.58396	1.56856
.88621	.81243	2.45345	2.24576	1.34772	1.02138	.94810	1.34772	1.32113	1.58503	1.56966
.89093	.80815	2.46844	2.23484	1.34874	1.02215	.94910	1.34874	1.32222	1.58622	1.57089
.89565	.80392	2.48372	2.22412	1.34987	1.02301	.95021	1.34987	1.32343	1.58755	1.57227
.90037	.79972	2.49931	2.21360	1.35111	1.02395	.95142	1.35111	1.32475	1.58901	1.57377
.90508	.79557	2.51523	2.20327	1.35246	1.02497	.95275	1.35246	1.32619	1.59060	1.57541
.90980	.79146	2.53151	2.19312	1.35392	1.02608	.95418	1.35392	1.32775	1.59232	1.57719
.91452	.78739	2.54818	2.18313	1.35550	1.02727	.95571	1.35550	1.32943	1.59417	1.57910
.91924	.78336	2.56527	2.17330	1.35718	1.02855	.95735	1.35718	1.33122	1.59615	1.58114
.92395	.77936	2.58282	2.16363	1.35897	1.02991	.95909	1.35897	1.33313	1.59826	1.58331
.92867	.77539	2.60088	2.15410	1.36087	1.03135	.96094	1.36087	1.33515	1.60049	1.58562
.93339	.77146	2.61950	2.14470	1.36289	1.03287	.96289	1.36289	1.33730	1.60286	1.58806
.93811	.76755	2.63874	2.13543	1.36501	1.03448	.96494	1.36501	1.33956	1.60536	1.59064
.94282	.76367	2.65868	2.12627	1.36725	1.03618	.96710	1.36725	1.34194	1.60799	1.59335
.94754	.75982	2.67941	2.11723	1.36960	1.03796	.96936	1.36960	1.34444	1.61076	1.59620
.95226	.75599	2.70105	2.10828	1.37207	1.03983	.97173	1.37207	1.34706	1.61366	1.59919
.95698	.75217	2.72372	2.09942	1.37466	1.04180	.97421	1.37466	1.34981	1.61671	1.60233
.96169	.74837	2.74761	2.09064	1.37737	1.04385	.97680	1.37737	1.35269	1.61990	1.60561
.96641	.74458	2.77297	2.08193	1.38021	1.04601	.97951	1.38021	1.35570	1.62324	1.60905
.97113	.74080	2.80011	2.07328	1.38319	1.04826	.98233	1.38319	1.35886	1.62674	1.61266
.97585	.73701	2.82950	2.06466	1.38631	1.05063	.98529	1.38631	1.36216	1.63041	1.61643
.98056	.73322	2.86186	2.05605	1.38958	1.05311	.98838	1.38958	1.36563	1.63426	1.62039
.98528	.72940	2.89836	2.04743	1.39302	1.05571	.99162	1.39302	1.36927	1.63831	1.62455
.99000	.72554	2.94126	2.03877	1.39666	1.05847	.99503	1.39666	1.37312	1.64258	1.62895

For Q'= .600

					m=.5		m=1.0		m=1.5	
Y_2'	Y_1'	β_1	β_2	E_{cir}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}	E_{trap}	Y_{trap}

.87906	.87906	2.43123	2.43123	1.43986	1.09121	1.02069	1.43986	1.41412	1.69339	1.67855
.88276	.87605	2.44265	2.42203	1.43991	1.09124	1.02073	1.43991	1.41417	1.69344	1.67860
.88646	.87172	2.45422	2.40899	1.44003	1.09134	1.02085	1.44003	1.41430	1.69359	1.67876
.89016	.86807	2.46596	2.39816	1.44024	1.09150	1.02105	1.44024	1.41453	1.69384	1.67901
.89385	.86445	2.47788	2.38752	1.44054	1.09172	1.02133	1.44054	1.41484	1.69419	1.67937
.89755	.86086	2.48998	2.37708	1.44091	1.09201	1.02169	1.44091	1.41524	1.69463	1.67983
.90125	.85729	2.50227	2.36681	1.44137	1.09236	1.02213	1.44137	1.41572	1.69517	1.68038
.90495	.85373	2.51477	2.35671	1.44191	1.09277	1.02265	1.44191	1.41630	1.69581	1.68104
.90865	.85020	2.52749	2.34676	1.44254	1.09324	1.02324	1.44254	1.41696	1.69654	1.68179
.91234	.84669	2.54045	2.33696	1.44324	1.09377	1.02391	1.44324	1.41770	1.69736	1.68264
.91604	.84320	2.55365	2.32731	1.44402	1.09437	1.02466	1.44402	1.41854	1.69829	1.68359
.91974	.83972	2.56712	2.31780	1.44489	1.09502	1.02549	1.44489	1.41945	1.69930	1.68464
.92344	.83626	2.58088	2.30842	1.44583	1.09574	1.02639	1.44583	1.42046	1.70042	1.68578
.92714	.83282	2.59494	2.29916	1.44686	1.09652	1.02736	1.44686	1.42155	1.70162	1.68702
.93083	.82940	2.60934	2.29002	1.44797	1.09736	1.02842	1.44797	1.42272	1.70293	1.68837
.93453	.82599	2.62410	2.28099	1.44916	1.09826	1.02955	1.44916	1.42398	1.70433	1.68981
.93823	.82259	2.63925	2.27207	1.45043	1.09922	1.03075	1.45043	1.42533	1.70583	1.69135
.94193	.81921	2.65484	2.26324	1.45179	1.10025	1.03204	1.45179	1.42677	1.70742	1.69299
.94563	.81584	2.67089	2.25451	1.45323	1.10134	1.03340	1.45323	1.42829	1.70912	1.69473
.94932	.81248	2.68747	2.24586	1.45476	1.10250	1.03484	1.45476	1.42991	1.71091	1.69658
.95302	.80912	2.70463	2.23730	1.45637	1.10372	1.03637	1.45637	1.43162	1.71281	1.69853
.95672	.80577	2.72245	2.22881	1.45807	1.10501	1.03797	1.45807	1.43342	1.71481	1.70059
.96042	.80243	2.74101	2.22038	1.45987	1.10637	1.03966	1.45987	1.43531	1.71692	1.70275
.96411	.79908	2.76042	2.21201	1.46175	1.10780	1.04143	1.46175	1.43731	1.71914	1.70503
.96781	.79574	2.78082	2.20368	1.46374	1.10930	1.04329	1.46374	1.43940	1.72147	1.70743
.97151	.79239	2.80239	2.19540	1.46582	1.11088	1.04524	1.46582	1.44161	1.72392	1.70995
.97521	.78903	2.82537	2.18714	1.46801	1.11254	1.04729	1.46801	1.44392	1.72650	1.71260
.97891	.78565	2.85009	2.17889	1.47031	1.11429	1.04943	1.47031	1.44635	1.72921	1.71538
.98260	.78226	2.87704	2.17064	1.47274	1.11613	1.05169	1.47274	1.44891	1.73205	1.71830
.98630	.77883	2.90698	2.16236	1.47529	1.11806	1.05406	1.47529	1.45160	1.73506	1.72139
.99000	.77536	2.94126	2.15403	1.47799	1.12011	1.05657	1.47799	1.45445	1.73824	1.72465

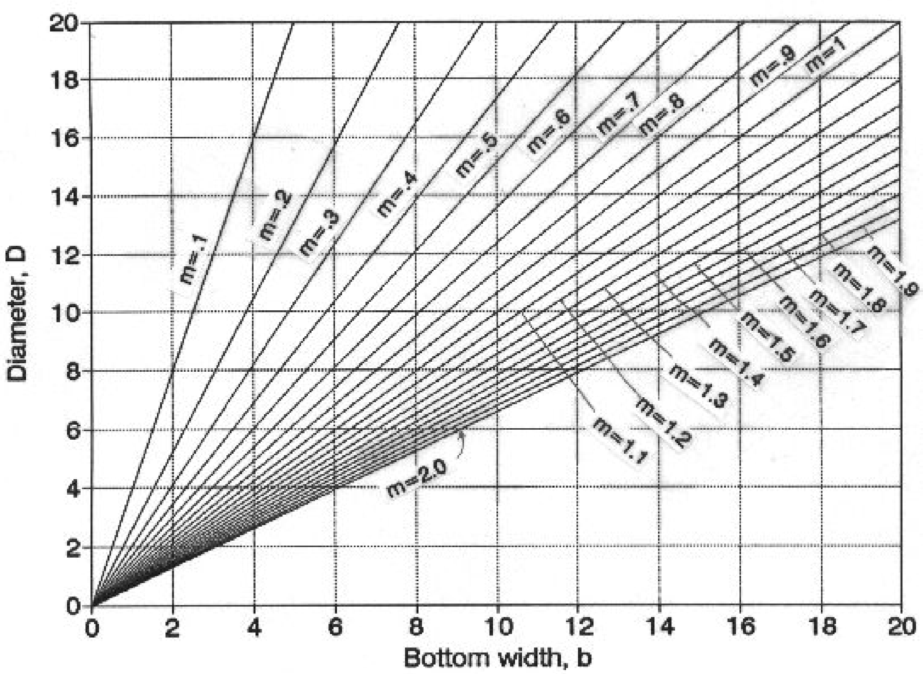
For Q'= .700

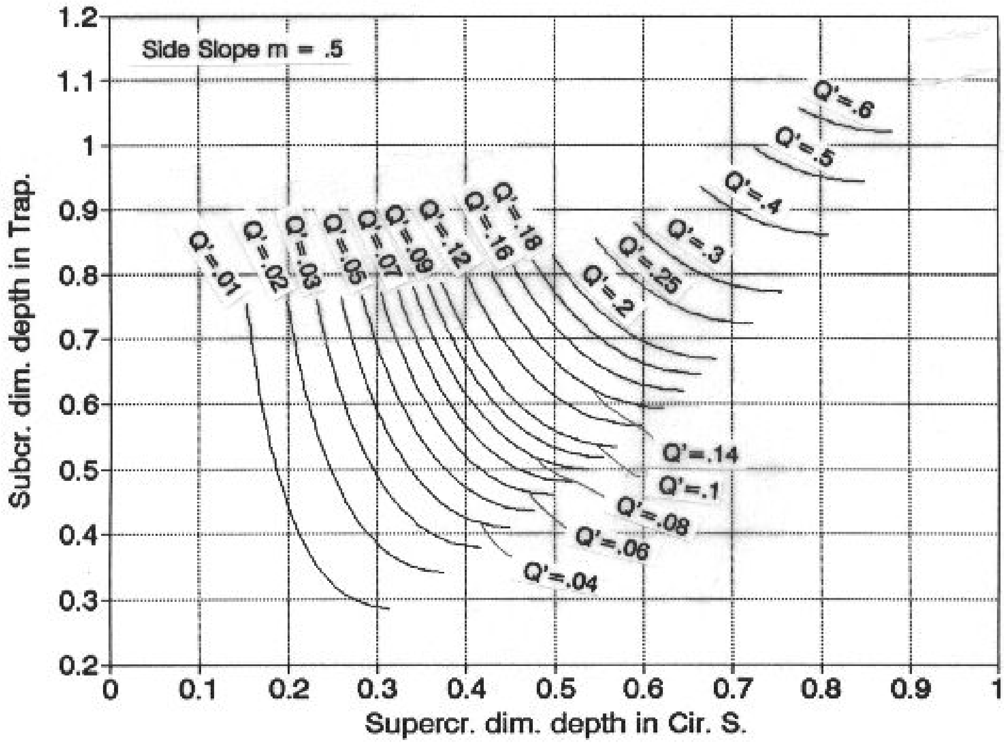
					m=.5		m=1.0		m=1.5	
Y_2'	**Y_1'**	**β_1**	**β_2**	**E_{cir}**	**E_{trap}**	**Y_{trap}**	**E_{trap}**	**Y_{trap}**	**E_{trap}**	**Y_{trap}**
.90254	.90254	2.50660	2.50660	1.53139	1.16058	1.09386	1.53139	1.50686	1.80103	1.78692
.90545	.90546	2.51649	2.51652	1.53142	1.16060	1.09388	1.53142	1.50689	1.80107	1.78696
.90837	.90837	2.52653	2.52652	1.53151	1.16066	1.09397	1.53151	1.50698	1.80117	1.78706
.91128	.91128	2.53671	2.53670	1.53165	1.16078	1.09410	1.53165	1.50714	1.80135	1.78724
.91420	.91420	2.54704	2.54703	1.53186	1.16093	1.09429	1.53186	1.50736	1.80159	1.78749
.91711	.91711	2.55753	2.55753	1.53212	1.16113	1.09454	1.53212	1.50764	1.80190	1.78781
.92003	.92003	2.56819	2.56819	1.53245	1.16138	1.09484	1.53245	1.50798	1.80228	1.78820
.92295	.92294	2.57903	2.57903	1.53283	1.16167	1.09520	1.53283	1.50838	1.80273	1.78866
.92586	.92586	2.59006	2.59005	1.53327	1.16200	1.09561	1.53327	1.50885	1.80325	1.78919
.92878	.92878	2.60129	2.60129	1.53377	1.16238	1.09607	1.53377	1.50937	1.80384	1.78980
.93169	.93169	2.61273	2.61273	1.53433	1.16281	1.09659	1.53433	1.50996	1.80449	1.79047
.93461	.93461	2.62441	2.62440	1.53495	1.16327	1.09717	1.53495	1.51062	1.80522	1.79122
.93752	.93752	2.63632	2.63632	1.53563	1.16379	1.09780	1.53563	1.51133	1.80602	1.79204
.94044	.94044	2.64850	2.64850	1.53637	1.16435	1.09848	1.53637	1.51211	1.80689	1.79293
.94335	.94335	2.66097	2.66097	1.53717	1.16496	1.09922	1.53717	1.51295	1.80783	1.79390
.94627	.94627	2.67374	2.67373	1.53803	1.16561	1.10002	1.53803	1.51386	1.80884	1.79494
.94918	.94918	2.68684	2.68684	1.53895	1.16631	1.10088	1.53895	1.51484	1.80993	1.79605
.95210	.95210	2.70030	2.70030	1.53994	1.16705	1.10179	1.53994	1.51587	1.81109	1.79724
.95501	.95501	2.71415	2.71415	1.54099	1.16785	1.10276	1.54099	1.51698	1.81232	1.79851
.95793	.95793	2.72844	2.72844	1.54210	1.16870	1.10379	1.54210	1.51816	1.81364	1.79985
.96085	.96085	2.74321	2.74321	1.54329	1.16959	1.10488	1.54329	1.51940	1.81503	1.80128
.96376	.96376	2.75852	2.75852	1.54454	1.17054	1.10603	1.54454	1.52072	1.81650	1.80279
.96668	.96668	2.77444	2.77444	1.54586	1.17154	1.10725	1.54586	1.52211	1.81805	1.80438
.96959	.96959	2.79104	2.79104	1.54725	1.17260	1.10853	1.54725	1.52357	1.81969	1.80606
.97251	.97251	2.80844	2.80844	1.54872	1.17371	1.10988	1.54872	1.52512	1.82141	1.80783
.97542	.97542	2.82675	2.82675	1.55026	1.17488	1.11130	1.55026	1.52674	1.82323	1.80969
.97834	.97834	2.84616	2.84616	1.55189	1.17611	1.11279	1.55189	1.52845	1.82514	1.81165
.98125	.98125	2.86690	2.86690	1.55360	1.17741	1.11436	1.55360	1.53026	1.82716	1.81372
.98417	.98417	2.88928	2.88928	1.55541	1.17878	1.11602	1.55541	1.53215	1.82928	1.81589
.98708	.98708	2.91381	2.91381	1.55731	1.18022	1.11776	1.55731	1.53415	1.83152	1.81819
.99000	.99000	2.94126	2.94125	1.55932	1.18175	1.11960	1.55932	1.53627	1.83389	1.82062

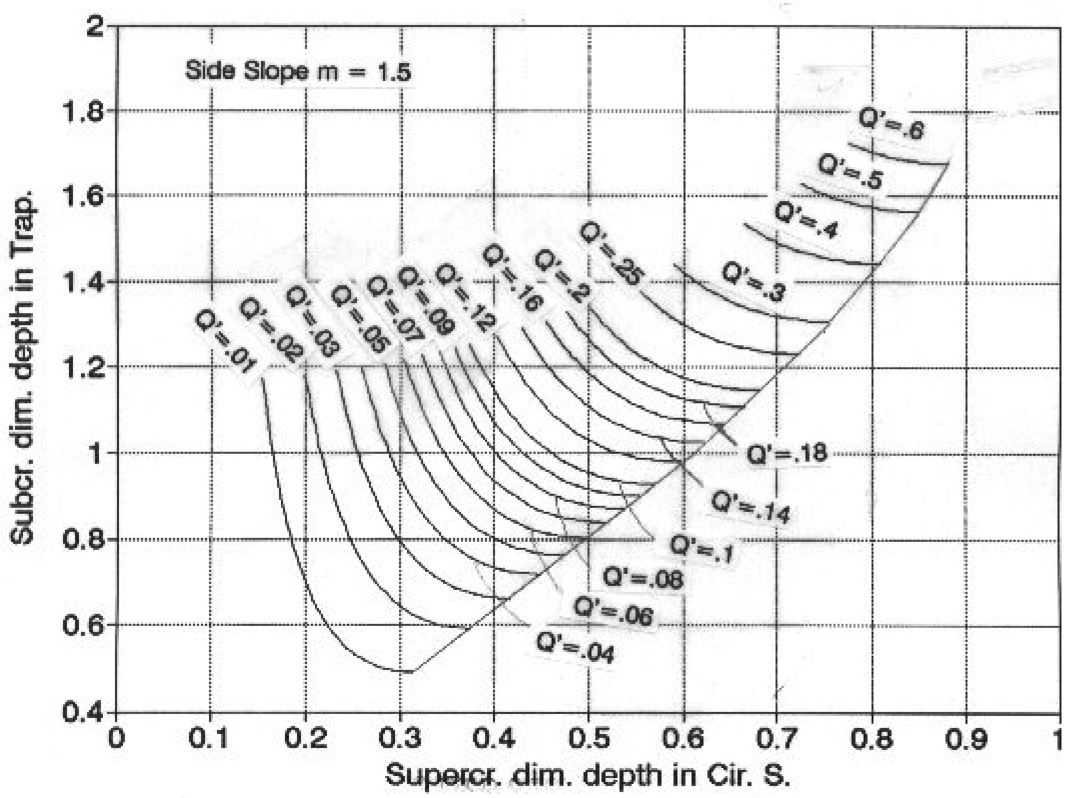
For Q'= .800

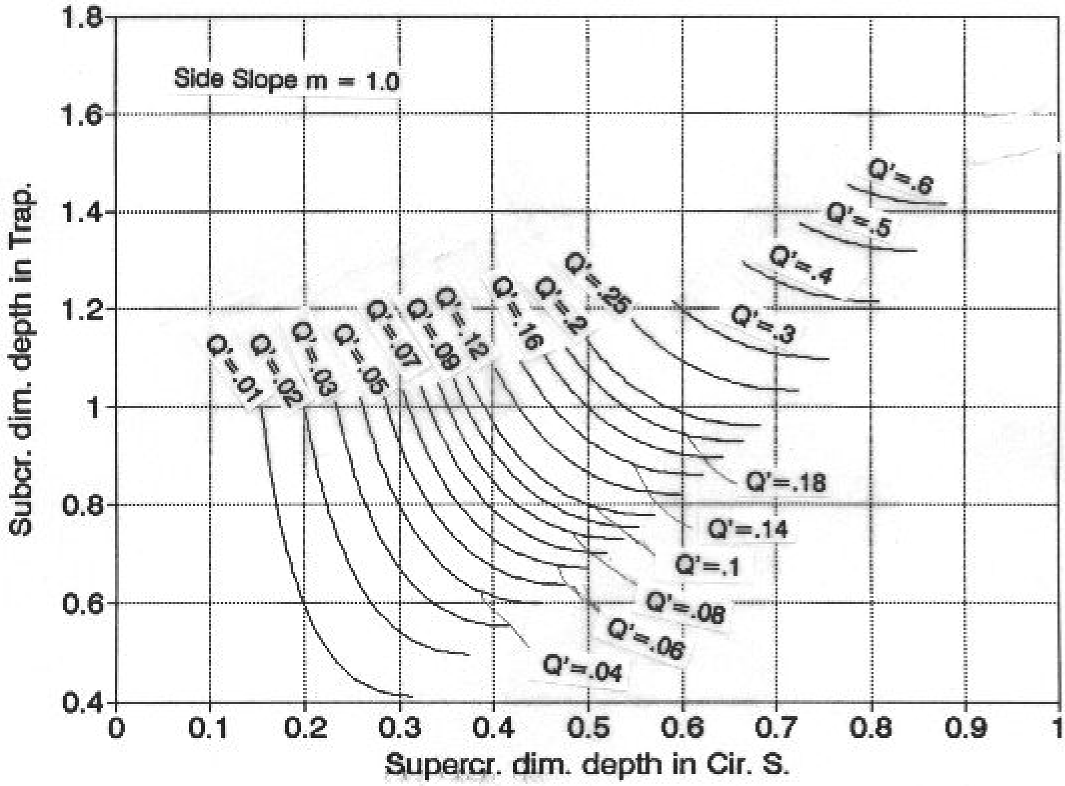
					m=.5		m=1.0		m=1.5	
Y_2'	**Y_1'**	**β_1**	**β_2**	**E_{cir}**	**E_{trap}**	**Y_{trap}**	**E_{trap}**	**Y_{trap}**	**E_{trap}**	**Y_{trap}**
.92060	.92060	2.57030	2.57030	1.61994	1.22768	1.16477	1.61994	1.59667	1.90517	1.89181
.92291	.92291	2.57891	2.57890	1.61996	1.22770	1.16478	1.61996	1.59669	1.90520	1.89183
.92523	.92523	2.58765	2.58764	1.62002	1.22775	1.16484	1.62002	1.59675	1.90527	1.89191

.92754	.92754	2.59651	2.59650	1.62013	1.22783	1.16494	1.62013	1.59687	1.90540	1.89204
.92985	.92985	2.60550	2.60549	1.62027	1.22794	1.16507	1.62027	1.59702	1.90557	1.89222
.93217	.93217	2.61462	2.61462	1.62047	1.22808	1.16525	1.62047	1.59722	1.90580	1.89245
.93448	.93448	2.62390	2.62390	1.62070	1.22826	1.16546	1.62070	1.59747	1.90607	1.89273
.93679	.93679	2.63332	2.63332	1.62098	1.22847	1.16571	1.62098	1.59776	1.90640	1.89306
.93911	.93911	2.64291	2.64291	1.62130	1.22871	1.16600	1.62130	1.59809	1.90677	1.89345
.94142	.94142	2.65267	2.65267	1.62166	1.22899	1.16633	1.62166	1.59848	1.90720	1.89388
.94373	.94373	2.66262	2.66262	1.62207	1.22930	1.16670	1.62207	1.59890	1.90768	1.89438
.94605	.94605	2.67276	2.67275	1.62252	1.22964	1.16711	1.62252	1.59938	1.90821	1.89492
.94836	.94836	2.68310	2.68310	1.62302	1.23002	1.16756	1.62302	1.59990	1.90880	1.89552
.95067	.95067	2.69367	2.69367	1.62356	1.23043	1.16805	1.62356	1.60047	1.90943	1.89617
.95299	.95299	2.70447	2.70447	1.62415	1.23087	1.16859	1.62415	1.60108	1.91012	1.89688
.95530	.95530	2.71553	2.71553	1.62478	1.23135	1.16916	1.62478	1.60175	1.91087	1.89764
.95761	.95761	2.72687	2.72687	1.62546	1.23187	1.16978	1.62546	1.60246	1.91167	1.89846
.95993	.95993	2.73851	2.73850	1.62619	1.23242	1.17044	1.62619	1.60323	1.91253	1.89934
.96224	.96224	2.75047	2.75047	1.62697	1.23301	1.17114	1.62697	1.60404	1.91344	1.90027
.96455	.96455	2.76279	2.76279	1.62779	1.23364	1.17189	1.62779	1.60491	1.91441	1.90127
.96687	.96687	2.77550	2.77550	1.62867	1.23430	1.17268	1.62867	1.60583	1.91545	1.90232
.96918	.96918	2.78865	2.78865	1.62960	1.23501	1.17352	1.62960	1.60680	1.91654	1.90344
.97149	.97149	2.80229	2.80229	1.63058	1.23575	1.17441	1.63058	1.60783	1.91770	1.90463
.97381	.97381	2.81648	2.81648	1.63162	1.23654	1.17535	1.63162	1.60892	1.91892	1.90588
.97612	.97612	2.83129	2.83129	1.63272	1.23737	1.17634	1.63272	1.61007	1.92021	1.90719
.97843	.97843	2.84682	2.84682	1.63387	1.23824	1.17738	1.63387	1.61128	1.92156	1.90858
.98075	.98075	2.86318	2.86318	1.63509	1.23917	1.17848	1.63509	1.61255	1.92300	1.91005
.98306	.98306	2.88055	2.88054	1.63637	1.24014	1.17963	1.63637	1.61390	1.92450	1.91159
.98537	.98537	2.89912	2.89912	1.63772	1.24116	1.18085	1.63772	1.61531	1.92609	1.91322
.98769	.98769	2.91920	2.91920	1.63915	1.24224	1.18213	1.63915	1.61680	1.92777	1.91493
.99000	.99000	2.94126	2.94126	1.64066	1.24338	1.18348	1.64066	1.61838	1.92954	1.91674




Side Slope m = .5
Q'=.6
Q'=.5
Q'=.4
Q'=.3
Q'=.25
Q'=.2
Q'=.18
Q'=.16
Q'=.12
Q'=.09
Q'=.07
Q'=.05
Q'=.03
Q'=.02
Q'=.01
Q'=.14
Q'=.1
Q'=.08
Q'=.06
Q'=.04
Subcr. dim. depth in Trap.
Supercr. dim. depth in Cir. S.


Side Slope m = 1.5
Q'=.6
Q'=.5
Q'=.4
Q'=.3
Q'=.25
Q'=.2
Q'=.16
Q'=.12
Q'=.09
Q'=.07
Q'=.05
Q'=.03
Q'=.02
Q'=.01
Q'=.18
Q'=.14
Q'=.1
Q'=.08
Q'=.06
Q'=.04
Subcr. dim. depth in Trap.
Supercr. dim. depth in Cir. S.


Side Slope m = 1.0
Subcr. dim. depth in Trap.
Supercr. dim. depth in Cir. S.
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1
Q'=.01
Q'=.02
Q'=.03
Q'=.04
Q'=.05
Q'=.06
Q'=.07
Q'=.08
Q'=.09
Q'=.1
Q'=.12
Q'=.14
Q'=.16
Q'=.18
Q'=.2
Q'=.25
Q'=.3
Q'=.4
Q'=.5
Q'=.6

46.

Develop a graphical solution that provides the depth downstream from a gate if the depth upstream is known, or provides the depth upstream from the gate if the depth downstream therefrom is known. Have this graphical solution apply for flowrates per unit width of rectangular channel from q = 1 m^2/s to 10 m^2/s. Note that if a log-log plot is used that the curves for the different unit flowrates are not nearly as curved as when the plot uses a linear horizontal and vertical axis. The graphical solution can be developed rather easily using a spreadsheet since the equations that provide the alternative depths are explicit for a rectangular channel. Also write a computer program that provides a table of this solutions for different flowrates, q. (You should also generate the above tables using CHANNEL.)

Develop a graphical solution across a gate, etc.
q-known

$$Y_1+q^2/(2gY_1^2) = Y_2+q^2/(2gY_2^2)$$

$$Y_1-Y_2=(q^2/2g)(1/Y_2^2 - 1/Y_1^2) =[q^2(Y_1^2-Y_2^2)]/[2gY_1^2Y_2^2]= [q^2(Y_1-Y_2)(Y_1+Y_2)]/[2gY_1^2Y_2^2]$$

or $1 = [q^2(1+Y_2/Y_1)]/[2gY_1Y_2^2]$ or

$$Y_2^2 = (q^2/g)[(1+Y_2/Y_1)/Y_1] \quad \leftarrow$$

One might plot the graph in a number of alternative ways. The one selected here is to plot curves for different q's on a graph with Y_1 as the abscissa and Y_2 as the ordinate.

Program PRB2 41.FOR

```
      REAL Q(10)/3.,4.,5.,6.,7.5,10.,12.5,15.,20.,25./,G/9.81/
      DATA B1,B2,FM1/3.,2.5,1.5/
      DO 10 I=1,10
      Yc=((Q(I)/B2)**2/G)**.3333333
      Ec=1.5*Yc
      DY2=(Yc-.15-.05*FLOAT(I))/20.
      Q2G=Q(I)**2/(2.*G)
      WRITE(3,100) Q(I),Yc,Ec
100   FORMAT(/' For Q =',F8.3,' Yc=',F8.3,' Ec=',F8.3)
      IF(I.EQ.1) Y1=1.25*Yc
      DO 10 J=0,20
      M=0
      Y2=Yc-DY2*FLOAT(J)
      E2=Y2+Q2G/(B2*Y2)**2
2     F=Y1+Q2G/((B1+FM1*Y1)*Y1)**2-E2
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 4
      FF=F
      YY=Y1
      Y1=1.005*Y1
      GO TO 2
4     DIF=(Y1-YY)*FF/(F-FF)
      Y1=YY-DIF
      IF(ABS(DIF).GT. 1.E-5 .AND. M.LT.30) GO TO 2
      IF(M.EQ.30) WRITE(*,*) ' Failed',I,J,DIF
10    WRITE(3,'(3F8.4)') Y1,Y2,E2
      END
```

Output: (b_1=3 m, m_1=1.5, b_2=2.5 m (rectangular))

```
        Y2      Y1      E
For Q =   3.000 Yc=    .528 Ec=    .791          .7705   .4456   .8152
  .7420   .5275   .7913                          .7849   .4293   .8276
  .7430   .5111   .7921                          .8032   .4129   .8434
  .7460   .4948   .7946                          .8259   .3965   .8633
  .7514   .4784   .7991                          .8537   .3801   .8881
  .7594   .4620   .8059                          .8874   .3638   .9184
```

```
  .9280   .3474   .9556
  .9768   .3310  1.0009
 1.0354   .3146  1.0561
 1.1061   .2983  1.1233
 1.1915   .2819  1.2056
 1.2955   .2655  1.3067
 1.4231   .2491  1.4317
 1.5812   .2328  1.5876
 1.7795   .2164  1.7840
 2.0318   .2000  2.0349

For Q =   4.000 Yc=    .639 Ec=    .959
  .9063   .6390   .9586
  .9074   .6196   .9595
  .9109   .6001   .9624
  .9170   .5807   .9676
  .9261   .5612   .9755
  .9387   .5418   .9863
  .9551   .5223  1.0006
  .9759   .5029  1.0188
 1.0017   .4834  1.0418
 1.0332   .4640  1.0701
 1.0714   .4445  1.1049
 1.1173   .4251  1.1472
 1.1725   .4056  1.1987
 1.2386   .3862  1.2612
 1.3180   .3667  1.3370
 1.4137   .3473  1.4293
 1.5296   .3278  1.5420
 1.6710   .3084  1.6806
 1.8450   .2889  1.8522
 2.0614   .2695  2.0666
 2.3341   .2500  2.3377

For Q =   5.000 Yc=    .742 Ec=   1.112
 1.0582   .7415  1.1123
 1.0594   .7195  1.1133
 1.0632   .6974  1.1166
 1.0699   .6753  1.1224
 1.0798   .6532  1.1310
 1.0934   .6311  1.1429
 1.1113   .6091  1.1586
 1.1338   .5870  1.1787
 1.1618   .5649  1.2038
 1.1959   .5428  1.2347
 1.2372   .5208  1.2725
 1.2868   .4987  1.3185
 1.3462   .4766  1.3741
 1.4172   .4545  1.4413
 1.5021   .4325  1.5226
 1.6040   .4104  1.6209
 1.7267   .3883  1.7404
 1.8755   .3662  1.8863
 2.0573   .3442  2.0655
 2.2815   .3221  2.2874
 2.5611   .3000  2.5653

For Q =   6.000 Yc=    .837 Ec=   1.256
 1.2009   .8374  1.2561
 1.2021   .8130  1.2572
 1.2061   .7886  1.2607
 1.2132   .7643  1.2669
 1.2238   .7399  1.2762
 1.2382   .7155  1.2889
 1.2571   .6912  1.3057
 1.2809   .6668  1.3271
 1.3104   .6424  1.3538
 1.3464   .6181  1.3866
 1.3899   .5937  1.4266
 1.4420   .5693  1.4751
 1.5042   .5449  1.5335
 1.5783   .5206  1.6039
 1.6667   .4962  1.6885
 1.7722   .4718  1.7905
 1.8988   .4475  1.9137
 2.0512   .4231  2.0630
 2.2362   .3987  2.2452
 2.4623   .3744  2.4691
 2.7417   .3500  2.7466

For Q =   7.500 Yc=    .972 Ec=   1.458
 1.4015   .9717  1.4575
 1.4029   .9431  1.4588
 1.4076   .9145  1.4630
 1.4159   .8859  1.4704
 1.4283   .8573  1.4814
 1.4452   .8288  1.4966
 1.4674   .8002  1.5166
 1.4954   .7716  1.5421
 1.5303   .7430  1.5739
 1.5728   .7144  1.6132
 1.6243   .6858  1.6610
 1.6861   .6573  1.7191
 1.7602   .6287  1.7893
 1.8488   .6001  1.8739
 1.9546   .5715  1.9759
 2.0815   .5429  2.0991
 2.2341   .5143  2.2483
 2.4187   .4858  2.4298
 2.6435   .4572  2.6519
 2.9197   .4286  2.9259
 3.2627   .4000  3.2670

For Q =  10.000 Yc=   1.177 Ec=   1.766
 1.7093  1.1771  1.7657
 1.7113  1.1408  1.7674
 1.7174  1.1044  1.7730
 1.7284  1.0680  1.7829
 1.7448  1.0317  1.7979
 1.7674   .9953  1.8185
 1.7971   .9590  1.8457
 1.8349   .9226  1.8806
 1.8821   .8863  1.9245
 1.9401   .8499  1.9789
 2.0108   .8136  2.0457
 2.0966   .7772  2.1273
 2.2001   .7408  2.2267
 2.3252   .7045  2.3476
 2.4765   .6681  2.4950
 2.6602   .6318  2.6749
 2.8842   .5954  2.8957
 3.1597   .5591  3.1682
 3.5013   .5227  3.5074
 3.9298   .4864  3.9339
 4.4744   .4500  4.4771

For Q =  12.500 Yc=   1.366 Ec=   2.049
 1.9930  1.3659  2.0489
 1.9953  1.3226  2.0510
 2.0028  1.2793  2.0579
 2.0161  1.2360  2.0701
 2.0360  1.1927  2.0884
 2.0635  1.1494  2.1139
 2.0998  1.1061  2.1475
 2.1461  1.0628  2.1908
 2.2041  1.0195  2.2454
 2.2758   .9763  2.3132
 2.3636   .9330  2.3969
 2.4705   .8897  2.4995
 2.6004   .8464  2.6252
 2.7583   .8031  2.7788
 2.9505   .7598  2.9671
```

3.1857	.7165	3.1987
3.4751	.6732	3.4849
3.8344	.6299	3.8414
4.2848	.5866	4.2897
4.8570	.5433	4.8602
5.5949	.5000	5.5968

For Q = 15.000 Yc= 1.542 Ec= 2.314

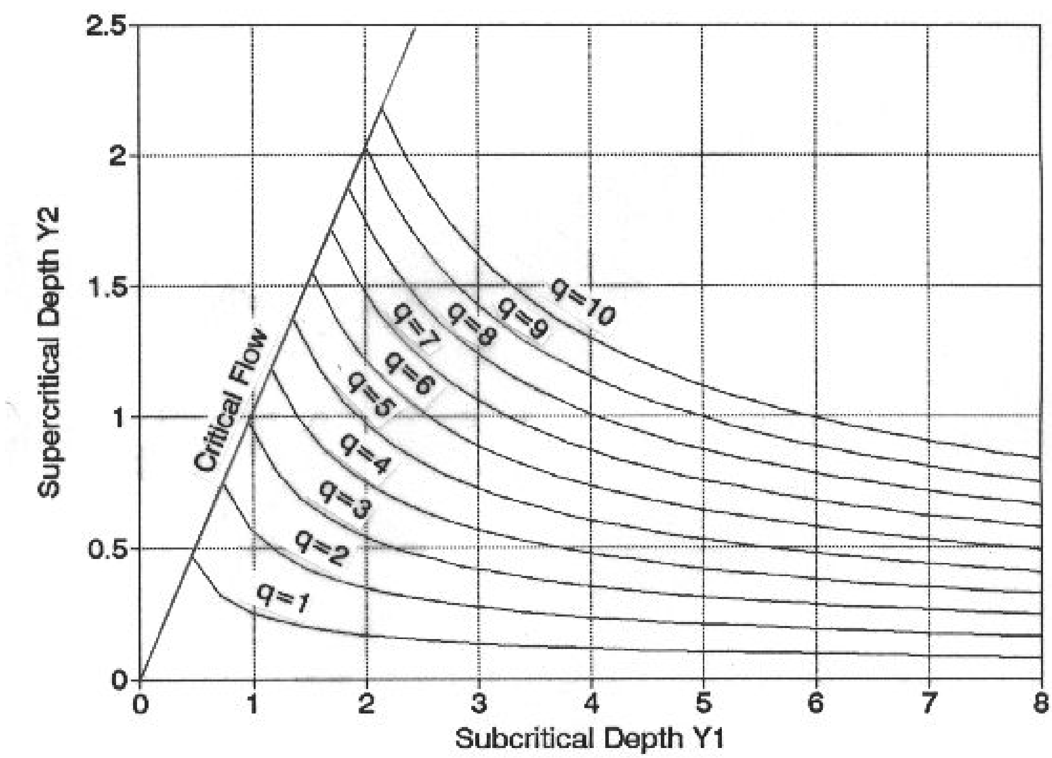
2.2586	1.5425	2.3137
2.2613	1.4928	2.3162
2.2699	1.4432	2.3241
2.2852	1.3936	2.3384
2.3083	1.3440	2.3598
2.3402	1.2943	2.3896
2.3823	1.2447	2.4290
2.4361	1.1951	2.4798
2.5038	1.1455	2.5439
2.5876	1.0958	2.6238
2.6905	1.0462	2.7225
2.8163	.9966	2.8440
2.9697	.9470	2.9931
3.1567	.8974	3.1760
3.3856	.8477	3.4009
3.6669	.7981	3.6787
4.0149	.7485	4.0236
4.4494	.6989	4.4556
4.9981	.6492	5.0022
5.7002	.5996	5.7029
6.6141	.5500	6.6157

For Q = 20.000 Yc= 1.869 Ec= 2.803

2.7497	1.8685	2.8028
2.7533	1.8051	2.8062
2.7648	1.7417	2.8170
2.7854	1.6783	2.8364
2.8165	1.6148	2.8657
2.8598	1.5514	2.9067
2.9172	1.4880	2.9613
2.9913	1.4246	3.0320
3.0850	1.3611	3.1218
3.2021	1.2977	3.2347
3.3472	1.2343	3.3755
3.5265	1.1708	3.5503
3.7477	1.1074	3.7673
4.0214	1.0440	4.0369
4.3614	.9806	4.3731
4.7866	.9171	4.7952
5.3234	.8537	5.3294
6.0093	.7903	6.0132
6.8987	.7269	6.9011
8.0734	.6634	8.0747
9.6603	.6000	9.6610

For Q = 25.000 Yc= 2.168 Ec= 3.252

3.2013	2.1683	3.2524
3.2057	2.0923	3.2566
3.2198	2.0164	3.2700
3.2452	1.9405	3.2940
3.2835	1.8646	3.3306
3.3371	1.7887	3.3817
3.4085	1.7128	3.4502
3.5010	1.6369	3.5391
3.6185	1.5610	3.6528
3.7661	1.4850	3.7962
3.9504	1.4091	3.9760
4.1795	1.3332	4.2007
4.4645	1.2573	4.4815
4.8202	1.1814	4.8333
5.2664	1.1055	5.2761
5.8311	1.0296	5.8379
6.5535	.9537	6.5580
7.4906	.8777	7.4934
8.7278	.8018	8.7294
10.3974	.7259	10.3983
12.7131	.6500	12.7135



The method used above to generate the data and plot the graph starts with critical depth, i.e. the line from which all the curves originate is a 45 degree line (it may not look like one, but notice approximately twice as much distant exists along the ordinate for a given increment as along the abscissa). An alternative graph might just solve the energy equation across the gate $E_1=E_2$. To accomplish this task the following is done:

$Y_1 + V_1^2/(2g) = Y_2 + V_2^2/(2g) = Y_2 + VH$ in which for convenience in writing VH is used for the known downstream velocity head. Since Y_1 represents the unknown depth, the above equation is written as,

$Y + q^2/(2gY^2) = Y_2+VH$ which can be written as the following cubic equation

$$Y^3 -(Y_2+VH)Y^2 + q^2/(2g) = 0$$

Dividing out the known root $Y=Y_2$

$$Y^2 - (VH)Y - Y_2(VH)$$

$$Y-Y_2 \mid Y^3 -(Y_2+VH)Y^2 + 0Y + q^2/(2g)$$

$$Y^3 -(Y_2 \quad)Y^2$$

$$-VHY^2$$

$$-VHY^2+Y_2(VH)Y$$

$$-Y_2(VH)Y+q^2/(2g)$$

$$-Y_2(VH)Y+Y_2^2(VH)$$

$$-Y_2^2(VH)+ q^2/(2g)=0$$

(this is zero since if the the depth squared is multiplied by the velocity head $q^2/(2g)$ results

Now the remaining quadratic equation

$$Y^2-(VH)Y-Y_2(VH) = 0$$

can be solved using the quadratic formula, or

$$\mathbf{Y = .5[VH+\{(VH)^2+4Y_2VH\}^{1/2}]} \leftarrow$$

One might have used E_2 rather than VH as shown below.

$Y_1 + V_1^2/(2g) = Y_2 + V_2^2/(2g) = E_2$ and the cubic equation becomes

$$Y^3 -E_2Y^2 + q^2/(2g) = 0$$

Dividing out the known root $Y=Y_2$

$$Y^2 +(Y_2-E_2)Y +(Y_2^2-Y_2E_2)$$

$$Y-Y_2 \mid Y^3 - E_2Y^2 + 0Y + q^2/(2g)$$

$$Y^3 - Y_2Y^2$$

$$(Y_2-E_2)Y^2$$

$$(Y_2-E_2)Y^2-(Y_2^2-Y_2E_2)Y$$

$(Y_2^2-Y_2E_2)Y+q^2/(2g)$

$(Y_2^2-Y_2E_2)Y-Y_2^3+Y_2^2E_2$

$Y_2^3-Y_2^2E_2+q^2/(2g)=0$

Notice that the remaining quadratic equation
$Y^2 + (Y_2-E_2)Y + (Y_2^2-Y_2E_2) = 0$ becomes the same as above since $(Y_2-E_2) = -VH$

and $Y_2^2-Y_2E_2 = Y_2^2 - Y_2(Y_2+VH) = -(VH)Y_2$

So the quadratic equation becomes:

$Y^2-(VH)Y-Y_2(VH) = 0$

And can be solved using the quadratic formula, or

$Y = .5[VH+\{(VH)^2+4Y_2VH\}^{1/2}]$ ←

The programs below (in Fortran, C and Pascal) generate a table of values for plotting Y_1 versus Y_2 across a gate with no energy loss.

<u>Program GATE.FOR</u>

```
      REAL Y1(6),Y2(6),q(6),Yc(6),Yb(6),DY(6)
      DATA q/1,2,4,6,8,10/,Yb/.15,.25,.35,.5,.6,.7/
      DO 10 I=1,6
      Yc(I)=(q(I)**2/9.81)**.33333333
10    DY(I)=(Yc(I)-Yb(I))/19.
      WRITE(3,100) Yc
100   FORMAT(' Critical Depths',/,6F8.5)
      DO 20 I=1,20
      DO 15 J=1,6
      Y2(J)=Yb(J)+FLOAT(I-1)*DY(J)
      VH=(q(J)/Y2(J))**2/19.62
15    Y1(J)=.5*(VH+SQRT(VH*VH+4.*Y2(J)*VH))
20    WRITE(3,110)(Y2(J),Y1(J),J=1,6)
110   FORMAT(6(1X,2F8.5))
      END
```

<u>Program GATE.C</u>

```
#include <math.h>
#include <stdio.h>
#include <stdlib.h>
FILE *fil;
 float y1[6],y2[6],q[6]={1.,2.,4.,6.,8.,10.},yc[6],dy[6],yb[6]={.15,.25,.35,.5,.6,.7},vh;
 int i,j;
main(void) {
 fil=fopen("gate.out","w");
 for(i=0;i<6;i++){yc[i]=pow(q[i]*q[i]/9.81,.3333333);dy[i]=(yc[i]-yb[i])/19.;}
 for(i=0;i<6;i++) fprintf(fil,"%8.4f",yc[i]);fprintf(fil,"\n");
 for(i=0;i<20;i++){ for(j=0;j<6;j++){y2[j]=yb[j]+dy[j]*i;vh=(q[j]/y2[j])*(q[j]/y2[j])/19.62;
   y1[j]=.5*(vh+sqrt(vh*vh+4.*y2[j]*vh));} fprintf(fil,"\n");
  for(j=0;j<6;j++) fprintf(fil,"%8.4f %8.4f",y2[j],y1[j]);} fclose(fil);
}
```

<u>Program GATE.PAS</u>

```
Program Gate;
uses crt;
Const
  q:array[1..6] of real=(1,2,4,6,8,10); Yb:array[1..6] of real=(0.15,0.25,0.35,0.5,0.6,0.7);
Var
 I,J:Integer;
 Y1,Y2,Yc,DY:array[1..6] of real;  VH:real;
IOUT :Text;
BEGIN
```

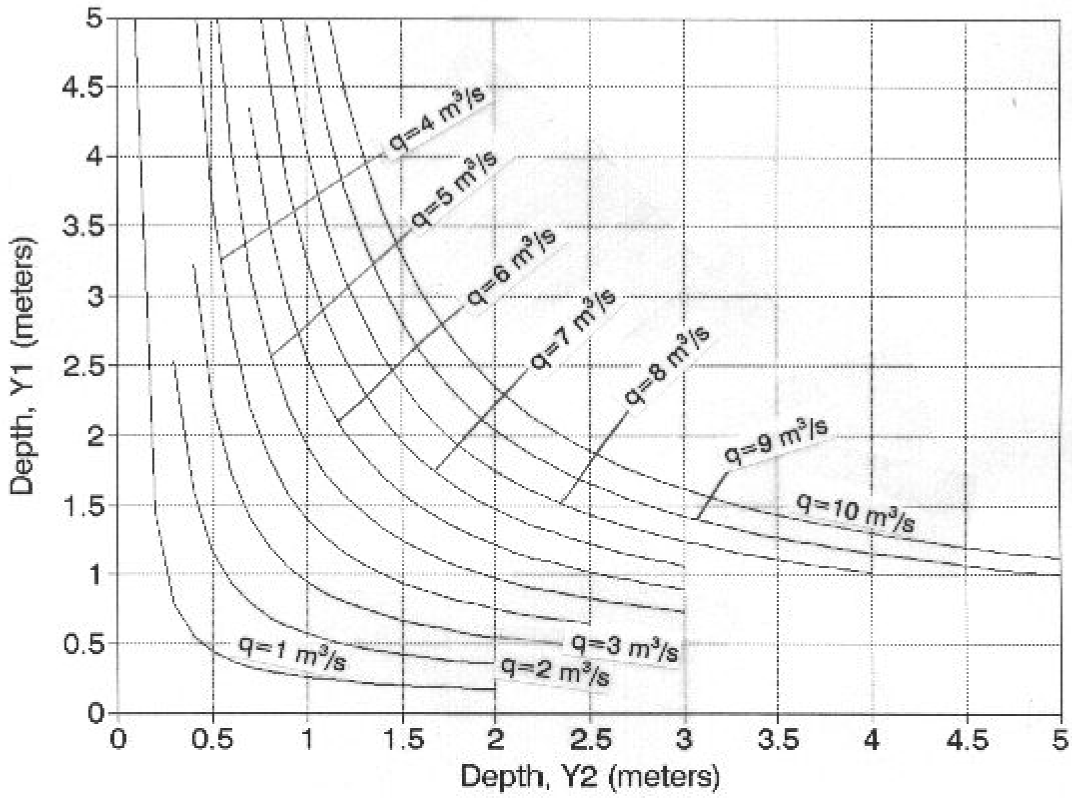
```
 Assign(IOUT,'gate.out');Rewrite(IOUT);
 For I:=1 to 6 do begin Yc[i]:=Exp(0.33333333*Ln(sqr(q[I])/9.81));
   DY[I]:=(Yc[I]-Yb[I])/19. end;
 For I:=1 to 20 do Begin
   For J:=1 to 6 do begin
    Y2[J]:=Yb[J]+(I-1)*DY[J];
    VH:=sqr(q[J]/Y2[J])/19.62;
    Y1[J]:=0.5*(VH+sqrt(VH*VH+4.0*Y2[J]*VH)) end;
  For J:=1 to 6 do begin Write(IOUT,' ',Y2[J]:8:4,Y1[J]:8:4) end;
  Writeln(IOUT); End;  close(IOUT);
END.
```

<u>Output from above Programs:</u>

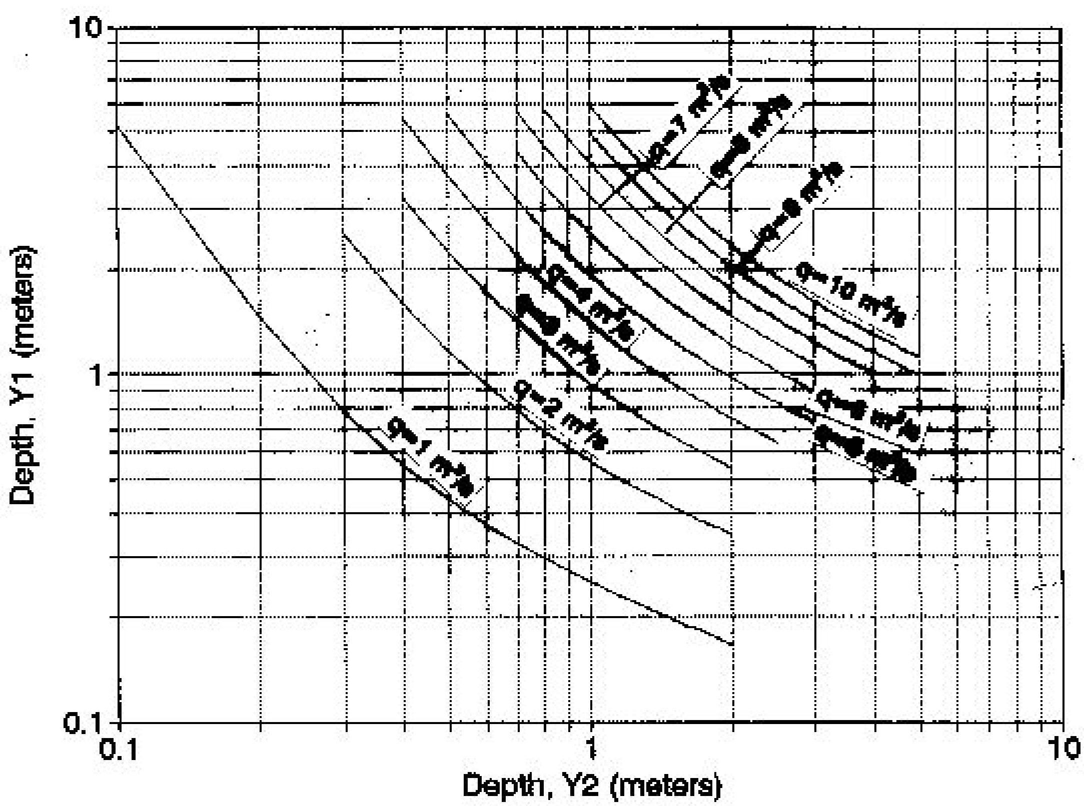
Critical Depths

.46714 .74153 1.17711 1.54245 1.86855 2.16825

Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1	Y_2	Y_1
.15000	2.40646	.25000	3.49529	.35000	6.99041	.50000	7.80936	.60000	9.62584	.70000	11.06005
.16669	1.98811	.27587	2.93101	.39353	5.63359	.55487	6.47079	.66677	7.95246	.77728	9.15269
.18338	1.68094	.30174	2.50855	.43706	4.66869	.60973	5.48417	.73353	6.72377	.85455	7.74915
.20007	1.44907	.32761	2.18441	.48060	3.95927	.66460	4.73702	.80030	5.79627	.93183	6.68773
.21677	1.26989	.35348	1.93043	.52413	3.42309	.71946	4.15809	.86706	5.07955	1.00911	5.86626
.23346	1.12861	.37935	1.72776	.56766	3.00826	.77433	3.70056	.93383	4.51442	1.08638	5.21769
.25015	1.01522	.40522	1.56340	.61119	2.68077	.82919	3.33263	1.00059	4.06090	1.16366	4.69659
.26684	.92280	.43109	1.42818	.65472	2.41761	.88406	3.03217	1.06736	3.69120	1.24094	4.27138
.28353	.84640	.45696	1.31549	.69826	2.20279	.93893	2.78342	1.13412	3.38560	1.31821	3.91958
.30022	.78245	.48283	1.22049	.74179	2.02495	.99379	2.57490	1.20089	3.12979	1.39549	3.62486
.31691	.72830	.50870	1.13953	.78532	1.87586	1.04866	2.39815	1.26766	2.91322	1.47277	3.37517
.33361	.68199	.53457	1.06989	.82885	1.74944	1.10352	2.24679	1.33442	2.72796	1.55004	3.16145
.35030	.64200	.56044	1.00945	.87239	1.64113	1.15839	2.11598	1.40119	2.56800	1.62732	2.97681
.36699	.60718	.58631	.95657	.91592	1.54746	1.21326	2.00195	1.46795	2.42870	1.70460	2.81595
.38368	.57661	.61218	.90998	.95945	1.46576	1.26812	1.90180	1.53472	2.30645	1.78187	2.67469
.40037	.54959	.63805	.86863	1.00298	1.39394	1.32299	1.81321	1.60148	2.19837	1.85915	2.54978
.41706	.52555	.66392	.83172	1.04651	1.33036	1.37785	1.73433	1.66825	2.10221	1.93643	2.43860
.43375	.50403	.68979	.79858	1.09005	1.27370	1.43272	1.66368	1.73501	2.01613	2.01370	2.33903
.45044	.48466	.71566	.76866	1.13358	1.22290	1.48758	1.60005	1.80178	1.93865	2.09098	2.24939
.46714	.46714	.74153	.74153	1.17711	1.17711	1.54245	1.54245	1.86855	1.86855	2.16825	2.16825



q=1 m³/s
q=2 m³/s
q=3 m³/s
q=4 m³/s
q=5 m³/s
q=6 m³/s
q=7 m³/s
q=8 m³/s
q=9 m³/s
q=10 m³/s
Depth, Y1 (meters)
Depth, Y2 (meters)



q=1 m³/s
q=2 m³/s
q=4 m³/s
q=7 m³/s
q=10 m³/s
Depth, Y1 (meters)
Depth, Y2 (meters)

The above graph is plotted from the same data as the previous graph, except that it is a Log-Log plot of this data, rather than a linear plot.

To use Program CHANNEL to generate the data, the critical depths should first be computed corresponding to the unit flow rates that will be used so that the downstream depth can begin just below these values. The "guesses" given to CHANNEL for the upstream depth will be slightly above these critical values so that the upstream subcritical depth is computed. In using CHANNEL T will be pressed when we are ready to specify the problem telling CHANNEL that a series of solutions is to be obtained, then give the file name, and after the other input give the final value for the downstream depth, and the number of table entries. This process will be repeated for each new unit flow rate. The results of these solution is given below with the first showing the table as produced by CHANNEL, but the rest only giving the first two column for the downstream and the corresponding upstream depths.

Critical Depths

q (m**2/s)	1	2	3	4	5	6	7	8	9	10
Yc (m)	0.467	0.742	0.972	1.177	1.366	1.542	1.709	1.869	2.021	2.168

Output Tables from CHANNEL

For a flow rate per unit width of q = 1 m**2/sec

Y	Y	b	Q	b	Q	z	K
0.460	0.474	1.000	1.000	1.000	1.000	0.000	0.000
0.450	0.485	1.000	1.000	1.000	1.000	0.000	0.000
0.440	0.497	1.000	1.000	1.000	1.000	0.000	0.000
0.430	0.509	1.000	1.000	1.000	1.000	0.000	0.000
0.420	0.522	1.000	1.000	1.000	1.000	0.000	0.000
0.410	0.535	1.000	1.000	1.000	1.000	0.000	0.000
0.400	0.550	1.000	1.000	1.000	1.000	0.000	0.000
0.390	0.566	1.000	1.000	1.000	1.000	0.000	0.000
0.380	0.583	1.000	1.000	1.000	1.000	0.000	0.000
0.370	0.601	1.000	1.000	1.000	1.000	0.000	0.000
0.360	0.621	1.000	1.000	1.000	1.000	0.000	0.000
0.350	0.643	1.000	1.000	1.000	1.000	0.000	0.000
0.340	0.666	1.000	1.000	1.000	1.000	0.000	0.000
0.330	0.691	1.000	1.000	1.000	1.000	0.000	0.000
0.320	0.719	1.000	1.000	1.000	1.000	0.000	0.000
0.310	0.750	1.000	1.000	1.000	1.000	0.000	0.000
0.300	0.783	1.000	1.000	1.000	1.000	0.000	0.000
0.290	0.820	1.000	1.000	1.000	1.000	0.000	0.000
0.280	0.861	1.000	1.000	1.000	1.000	0.000	0.000
0.270	0.907	1.000	1.000	1.000	1.000	0.000	0.000
0.260	0.958	1.000	1.000	1.000	1.000	0.000	0.000
0.250	1.016	1.000	1.000	1.000	1.000	0.000	0.000
0.240	1.081	1.000	1.000	1.000	1.000	0.000	0.000
0.230	1.155	1.000	1.000	1.000	1.000	0.000	0.000
0.220	1.240	1.000	1.000	1.000	1.000	0.000	0.000
0.210	1.337	1.000	1.000	1.000	1.000	0.000	0.000

For a flow rate per unit width of q = 2 m**2/sec

Y	Y
0.700	0.786
0.650	0.851
0.600	0.931
0.550	1.033
0.500	1.165
0.450	1.344
0.400	1.594
0.350	1.961
0.300	2.533
0.250	3.495

For a flow rate per unit width of q = 3 m**2/sec

Y	Y
0.900	1.051
0.850	1.118
0.800	1.196
0.750	1.290
0.700	1.403

0.650	1.543
0.600	1.719
0.550	1.945
0.500	2.244
0.450	2.650
0.400	3.223
0.350	4.067
0.300	5.381
0.250	7.581

For a flow rate per unit width of q = 4 m**2/sec

Y	Y
1.100	1.262
1.050	1.326
1.000	1.399
0.950	1.483
0.900	1.580
0.850	1.695
0.800	1.831
0.750	1.995
0.700	2.195
0.650	2.444
0.600	2.758
0.550	3.164
0.500	3.702
0.450	4.436
0.400	5.470
0.350	6.990
0.300	9.352
0.250	13.293

For a flow rate per unit width of q = 5 m**2/sec

Y	Y
1.300	1.436
1.258	1.487
1.215	1.542
1.173	1.603
1.131	1.671
1.088	1.746
1.046	1.830
1.004	1.924
0.962	2.031
0.919	2.152
0.877	2.291
0.835	2.452
0.792	2.639
0.750	2.859
0.708	3.121
0.665	3.435
0.623	3.818
0.581	4.289
0.538	4.880
0.496	5.632
0.454	6.611
0.412	7.915
0.369	9.702
0.327	12.240
0.285	16.009
0.242	21.942
0.200	32.054

For a flow rate per unit width of q = 6 m**2/sec

Y	Y
1.500	1.587
1.450	1.643
1.400	1.705
1.350	1.773
1.300	1.849
1.250	1.933
1.200	2.028
1.150	2.135
1.100	2.256
1.050	2.394
1.000	2.553
0.950	2.738
0.900	2.955
0.850	3.212
0.800	3.519
0.750	3.891
0.700	4.348
0.650	4.917
0.600	5.639
0.550	6.573
0.500	7.809
0.450	9.491
0.400	11.855
0.350	15.321
0.300	20.683
0.250	29.606
0.200	46.071

For a flow rate per unit width of q = 7 m**2/sec

Y	Y
1.650	1.772
1.600	1.829
1.550	1.891
1.500	1.960
1.450	2.034
1.400	2.117
1.350	2.208
1.300	2.310
1.250	2.423
1.200	2.550
1.150	2.694
1.100	2.858
1.050	3.046
1.000	3.263
0.950	3.515
0.900	3.811
0.850	4.163
0.800	4.583
0.750	5.094
0.700	5.721
0.650	6.502
0.600	7.493
0.550	8.774
0.500	10.467
0.450	12.768
0.400	15.999
0.350	20.732
0.300	28.046
0.250	40.208
0.200	62.636

For a flow rate per unit width of q = 8 m**2/sec

Y	Y
1.800	1.941
1.750	1.998
1.700	2.060
1.650	2.127
1.600	2.201
1.550	2.281
1.500	2.368
1.450	2.464
1.400	2.571
1.350	2.689

1.300	2.820
1.250	2.967
1.200	3.133
1.150	3.321
1.100	3.535
1.050	3.780
1.000	4.065
0.950	4.396
0.900	4.785
0.850	5.246
0.800	5.800
0.750	6.471
0.700	7.296
0.650	8.324
0.600	9.626

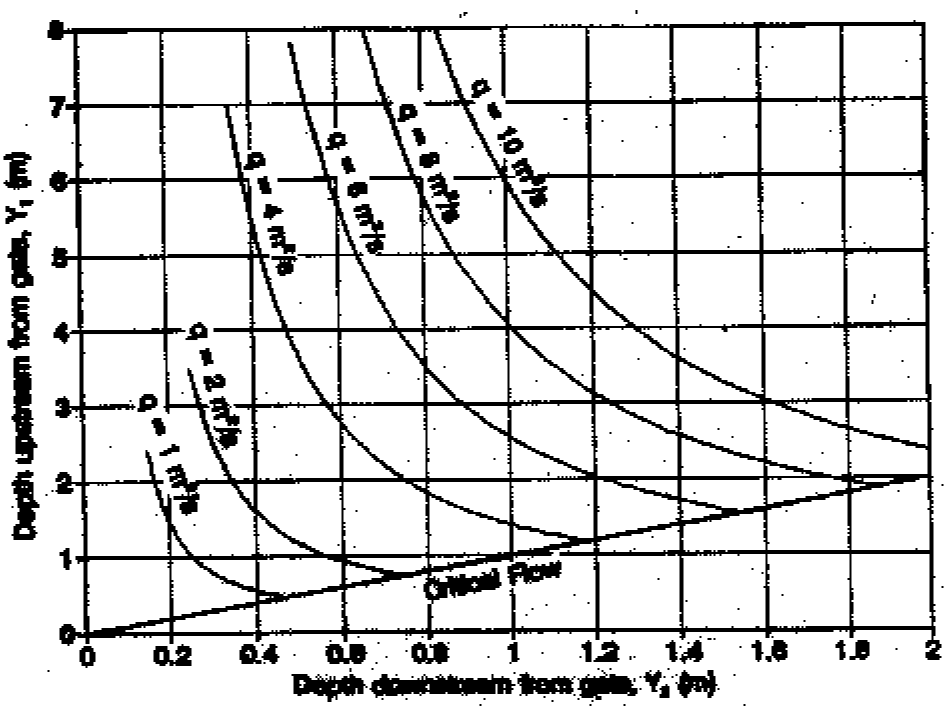
For a flow rate per unit width of q = 9 m**2/sec

Y	Y
1.950	2.096
1.900	2.153
1.850	2.214
1.800	2.280
1.750	2.351
1.700	2.429
1.650	2.512
1.600	2.604
1.550	2.704
1.500	2.813
1.450	2.934
1.400	3.068
1.350	3.216
1.300	3.382
1.250	3.568
1.200	3.778
1.150	4.016
1.100	4.287
1.050	4.599
1.000	4.961
0.950	5.382
0.900	5.877
0.850	6.465
0.800	7.170

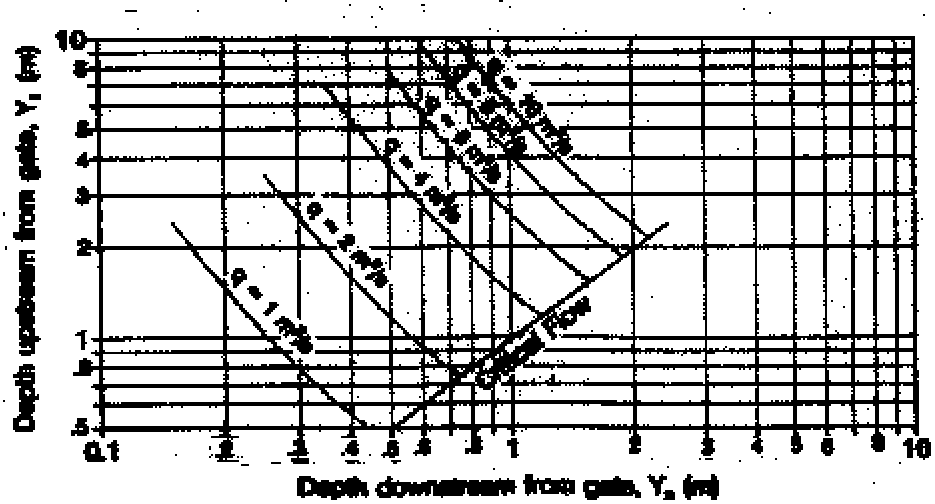
For a flow rate per unit width of q = 10 m**2/sec

Y	Y
2.050	2.296
2.000	2.356
1.950	2.420
1.900	2.489
1.850	2.564
1.800	2.644
1.750	2.731
1.700	2.825
1.650	2.927
1.600	3.039
1.550	3.162
1.500	3.296
1.450	3.445
1.400	3.609
1.350	3.792
1.300	3.997
1.250	4.227
1.200	4.486
1.150	4.781
1.100	5.118
1.050	5.505
1.000	5.953
0.950	6.476
0.900	7.091
0.850	7.821
0.800	8.696

An alternative way of plotting the data is to start with the critical depth as a line through the graph. Along this line the values for Y_2 on the abscissa and Y_1 on the ordinate are the same, and the larger upstream depths Y_1 for smaller values of Y_2 will be in the upper portion of the graph. Such a linear graph as well as a log-log graph are given below.



Depth upstream from gate, Y1 (m)
Depth downstream from gate, Y2 (m)
q = 10 m³/s
q = 8 m³/s
q = 6 m³/s
q = 4 m³/s
q = 2 m³/s
q = 1 m³/s
Critical Flow



Depth upstream from gate, Y1 (m)
Depth downstream from gate, Y2 (m)
q = 2 m³/s
q = 1 m³/s
Critical Flow

Heading put on table of data is given below for use in solving problems of flow past a gate in a rectangular channel.

Critical Depths

q=1 m^2/s	q=2 m^2/s	q=4 m^2/s	q=6 m^2/s	q=8 m^2/s	q=10 m^2/s
.46714	.74153	1.17711	1.54245	1.86855	2.16825

Alternative Depths

Flow rate per unit width, q (m^2/s)											
1		2		4		6		8		10	
Y_2 (m)	Y_1 (m)	Y_2 (m)	Y_1 (m)	Y_2 (m)	Y_1 (m)	Y_2 (m)	Y_1 (m)	Y_2 (m)	Y_1 (m)	Y_2 (m)	Y_1 (m)
.15000	2.40646	.25000	3.49529	.35000	6.9904	.50000	7.80936	.60000	9.62584	.70000	11.0601
.16669	1.98811	.27587	2.93101	.39353	5.63359	.55487	6.47079	.66677	7.95246	.77728	9.15269
.18338	1.68094	.30174	2.50855	.43706	4.66869	.60973	5.48417	.73353	6.72377	.85455	7.74915
.20007	1.44907	.32761	2.18441	.48060	3.95927	.66460	4.73702	.80030	5.79627	.93183	6.68773
.21677	1.26989	.35348	1.93043	.52413	3.42309	.71946	4.15809	.86706	5.07955	1.00911	5.86626
.23346	1.12861	.37935	1.72776	.56766	3.00826	.77433	3.70056	.93383	4.51442	1.08638	5.21769
.25015	1.01522	.40522	1.56340	.61119	2.68077	.82919	3.33263	1.00059	4.06090	1.16366	4.69659
.26684	.92280	.43109	1.42818	.65472	2.41761	.88406	3.03217	1.06736	3.69120	1.24094	4.27138
.28353	.84640	.45696	1.31549	.69826	2.20279	.93893	2.78342	1.13412	3.38560	1.31821	3.91958
.30022	.78245	.48283	1.22049	.74179	2.02495	.99379	2.57490	1.20089	3.12979	1.39549	3.62486
.31691	.72830	.50870	1.13953	.78532	1.87586	1.04866	2.39815	1.26766	2.91322	1.47277	3.37517
.33361	.68199	.53457	1.06989	.82885	1.74944	1.10352	2.24679	1.33442	2.72796	1.55004	3.16145
.35030	.64200	.56044	1.00945	.87239	1.64113	1.15839	2.11598	1.40119	2.56800	1.62732	2.97681
.36699	.60718	.58631	.95657	.91592	1.54746	1.21326	2.00195	1.46795	2.42870	1.70460	2.81595
.38368	.57661	.61218	.90998	.95945	1.46576	1.26812	1.90180	1.53472	2.30645	1.78187	2.67469
.40037	.54959	.63805	.86863	1.00298	1.39394	1.32299	1.81321	1.60148	2.19837	1.85915	2.54978
.41706	.52555	.66392	.83172	1.04651	1.33036	1.37785	1.73433	1.66825	2.10221	1.93643	2.43860
.43375	.50403	.68979	.79858	1.09005	1.27370	1.43272	1.66368	1.73501	2.01613	2.01370	2.33903
.45044	.48466	.71566	.76866	1.13358	1.22290	1.48758	1.60005	1.80178	1.93865	2.09098	2.24939
.46714	.46714	.74153	.74153	1.17711	1.17711	1.54245	1.54245	1.86855	1.86855	2.16825	2.16825

47.

Repeat the previous problem for an upstream trapezoidal channel with a bottom width $b_1 = 3$ m and a side slope m = 1.5 and a rectangular channel at the position of the gate with a bottom width $b_2 = 2.5$ m. Use flowrates of Q = 3 m^3/s to 25 m^3/s.

Wanted: Repeat the previous problem for an upstream trapezoidal channel with b_1=3 m, m_1=1.5. Downstream the channel is rectangular with b_2=2.5 m, Use Q=3 m^3/s to 25 m^3/s

Solution:

$Y_1 + Q^2/(2gA_1^2) = Y_2 + Q^2/(2gA_2^2)$

$Y_1 + Q^2/[2g(bY_1+mY_1^2)^2] = Y_2 + q^2/(2gY_2^2) = E_1$

Or cubic equation for Y_2

$f(Y_2) = Y_2^3 - E_1Y_2^2 + q^2/(2g) = 0$ $\qquad$ $df/dY_2 = 2Y_2^2 - 2E_1Y_2$

The program below generates data that provides values of Y_2 corresponding to a series of values for Y_1. Notice this program using the subroutine LAGU that extracts all roots and then prints out only the real positive root. (Below another program that solves the upstream depth, using the Newton Method, from the energy equation is given. In this subsequent program the critical depth is first computed at the downstream section.)

Program PB2 41.FOR

```
      PARAMETER (EPS=1.E-6)
      LOGICAL FOUND,FOUN1
      REAL Q(11),YS(11)
      COMPLEX C(11),ROOTS(10),AD(51),Z1,Z2,Z3
      DATA Q/3.,4.,5.,6.,8.,10.,11.,12.5,15.,20.,25./
      DATA YS/.097,.128,.158,.188,.246,.3,.356,.369,.434,.558,.676/
      EPS1=2.*EPS*EPS
      ND=3
      DO 90 K=1,11
      FOUN1=.TRUE.
      WRITE(3,69) Q(K)
69    FORMAT('                        ',F8.2)
      Q2=Q(K)**2/19.62
      QL=(Q(K)/2.5)**2/19.62
      YC=((Q(K)/2.5)**2/9.81)**.33333333
      DY1=(5.-YS(K))/50.
      DO 80 II=1,51
      FOUND=.FALSE.
      Y1=YS(K)+FLOAT(II-1)*DY1
      E1=Y1+Q2/((3.+1.5*Y1)*Y1)**2
      C(1)=CMPLX(QL,0.)
      C(2)=CMPLX(0.,0.)
      C(3)=CMPLX(-E1,0.)
      C(4)=CMPLX(1.,0.)
      DO 20 J=1,ND+1
20    AD(J)=C(J)
      DO 30 J=ND,1,-1
      Z1=CMPLX(0.,0.)
      CALL LAGU(AD,J,Z1,EPS)
      IF(ABS(AIMAG(Z1)).LE.EPS1*ABS(REAL(Z1))) Z1=CMPLX(REAL(Z1),0.)
      ROOTS(J)=Z1
      Z2=AD(J+1)
      DO 30 JJ=J,1,-1
      Z3=AD(JJ)
      AD(JJ)=Z2
30    Z2=Z1*Z2+Z3
```

```
      DO 50 J=2,ND
      Z1=ROOTS(J)
      DO 40 I=J-1,1,-1
      IF(REAL(ROOTS(I)).LE.REAL(Z1)) GO TO 50
40    ROOTS(I+1)=ROOTS(I)
      I=0
50    ROOTS(I+1)=Z1
      DO 60 I=1,ND
      IF(AIMAG(ROOTS(I)).NE. 0.) GO TO 60
      IF(REAL(ROOTS(I)).LT. 0.) GO TO 60
      YP=REAL(ROOTS(I))
      IF(Y1.LT.YC) THEN
      IF(YP.GT.YC) THEN
      Y2=YP
      FOUND=.TRUE.
      ENDIF
      ELSE
      IF(YP.LT.YC) THEN
      Y2=YP
      FOUND=.TRUE.
      ENDIF
      ENDIF
C60    WRITE(6,70) I,ROOTS(I)
70    FORMAT(I5,2F14.8)
60    IF(FOUND) GO TO 79
      IF(FOUN1) WRITE(3,78)
78    FORMAT('                    ')
      FOUN1=.FALSE.
      GO TO 80
79    WRITE(3,71) Y1,Y2
80    CONTINUE
71    FORMAT(2F10.4)
90    CONTINUE
      END
      SUBROUTINE LAGU(C,ND,Z1,EPS)
      COMPLEX C(ND+1),Z1,DX,ZO,Z2,Z3,Z4,Z5,DZ,SS,Z6,Z7,Z8,ZERO,XX,FF
      ZERO=CMPLX(0.,0.)
      DO 20 ITER=1,50
      Z2=C(ND+1)
      Z3=ZERO
      Z4=ZERO
      DO 10 J=ND,1,-1
      Z4=Z1*Z4+Z3
      Z3=Z1*Z3+Z2
10    Z2=Z1*Z2+C(J)
      IF(CABS(Z2).LE.1.E-8) THEN
      DX=ZERO
      ELSE IF(CABS(Z3).LE.1.E-8.AND.CABS(Z4).LE.1.E-8) THEN
      DX=CMPLX(CABS(Z2/C(ND+1))**(1./FLOAT(ND)),0.)
      ELSE
      Z5=Z3/Z2
      Z8=Z5*Z5
      DZ=Z8-2.*Z4/Z2
      XX=(ND-1)*(ND*DZ-Z8)
      YY=ABS(REAL(XX))
      ZZ=ABS(AIMAG(XX))
      IF(YY.LT.1.E-12 .AND. ZZ .LT.1.E-12) THEN
      SS=ZERO
      ELSE IF (YY.GE.ZZ) THEN
      FF=(1./YY)*XX
      SS=SQRT(YY)*CSQRT(FF)
      ELSE
      FF=(1./ZZ)*XX
      SS=SQRT(ZZ)*CSQRT(FF)
      ENDIF
      Z6=Z5+SS
      Z7=Z5-SS
      IF(CABS(Z6).LT.CABS(Z7)) Z6=Z7
      DX=FLOAT(ND)/Z6
      ENDIF
      ZO=Z1-DX
```

```
      IF(Z1.EQ.ZO) RETURN
      Z1=ZO
      IF(CABS(DX).LE.EPS*CABS(Z1)) RETURN
20    CONTINUE
      WRITE(6,*)' FAILED TO CONVERGE'
      RETURN
      END
```

<u>Output from above Program</u>

```
                         3.00
   .0970   5.0215
   .1951   1.2610

   .7834    .4308
   .8815    .3664
   .9795    .3302
  1.0776    .3045
  1.1757    .2847
  1.2737    .2687
  1.3718    .2553
  1.4698    .2439
  1.5679    .2340
  1.6660    .2253
  1.7640    .2175
  1.8621    .2106
  1.9601    .2043
  2.0582    .1985
  2.1563    .1932
  2.2543    .1884
  2.3524    .1839
  2.4504    .1797
  2.5485    .1758
  2.6466    .1722
  2.7446    .1688
  2.8427    .1655
  2.9407    .1625
  3.0388    .1596
  3.1369    .1569
  3.2349    .1543
  3.3330    .1519
  3.4310    .1495
  3.5291    .1473
  3.6272    .1452
  3.7252    .1431
  3.8233    .1412
  3.9213    .1393
  4.0194    .1375
  4.1175    .1358
  4.2155    .1341
  4.3136    .1325
  4.4116    .1309
  4.5097    .1294
  4.6078    .1280
  4.7058    .1266
  4.8039    .1252
  4.9019    .1239
  5.0000    .1227
                         4.00
   .1280   5.0079
   .2254   1.6154
   .3229    .7037

   .9075    .6186
  1.0050    .4812
  1.1024    .4310
  1.1998    .3971
  1.2973    .3715
  1.3947    .3508
  1.4922    .3337
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  1.7845    .2952
  1.8819    .2853
  1.9794    .2763
  2.0768    .2682
  2.1742    .2608
  2.2717    .2541
  2.3691    .2478
  2.4666    .2420
  2.5640    .2366
  2.6614    .2316
  2.7589    .2269
  2.8563    .2225
  2.9538    .2183
  3.0512    .2144
  3.1486    .2107
  3.2461    .2072
  3.3435    .2038
  3.4410    .2006
  3.5384    .1976
  3.6358    .1947
  3.7333    .1919
  3.8307    .1893
  3.9282    .1867
  4.0256    .1843
  4.1230    .1819
  4.2205    .1797
  4.3179    .1775
  4.4154    .1754
  4.5128    .1734
  4.6102    .1714
  4.7077    .1696
  4.8051    .1677
  4.9026    .1660
  5.0000    .1643
                         5.00
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   .2548   1.9143
   .3517    .9571

  1.1264    .5937
  1.2232    .5278
  1.3201    .4858
  1.4169    .4546
  1.5138    .4297
  1.6106    .4091
  1.7074    .3915
  1.8043    .3763
  1.9011    .3628
  1.9980    .3509
  2.0948    .3401
  2.1916    .3304
  2.2885    .3215
  2.3853    .3133
  2.4822    .3057
  2.5790    .2987
  2.6758    .2922
  2.7727    .2862
  2.8695    .2805
  2.9664    .2751
  3.0632    .2701
  3.1600    .2653
  3.2569    .2608
```

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2.7865	.3465	
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		8.00
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1.5771	.7933	
1.6722	.7293	

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1.8624	.6482	
1.9574	.6189	
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2.1476	.5723	
2.2427	.5532	
2.3378	.5361	
2.4328	.5208	
2.5279	.5068	
2.6230	.4940	
2.7181	.4822	
2.8132	.4713	
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3.4787	.4123	
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3.8590	.3876	
3.9541	.3821	
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4.1443	.3718	
4.2394	.3670	
4.3344	.3624	
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4.5246	.3537	
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4.8098	.3417	
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1.9920	.8226	
2.0860	.7813	
2.1800	.7474	
2.2740	.7186	
2.3680	.6935	
2.4620	.6713	
2.5560	.6515	
2.6500	.6336	
2.7440	.6173	
2.8380	.6023	
2.9320	.5885	
3.0260	.5758	
3.1200	.5639	
3.2140	.5527	
3.3080	.5423	
3.4020	.5325	
3.4960	.5232	
3.5900	.5145	
3.6840	.5062	
3.7780	.4983	
3.8720	.4908	
3.9660	.4836	
4.0600	.4768	
4.1540	.4703	
4.2480	.4640	
4.3420	.4580	
4.4360	.4523	
4.5300	.4468	

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4.8120	.4314	
4.9060	.4266	
5.0000	.4220	
		11.00
.3560	4.1963	
.4489	2.5675	
.5418	1.6035	
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2.0278	.9314	
2.1207	.8787	
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2.4922	.7473	
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2.9566	.6526	
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3.2353	.6122	
3.3282	.6005	
3.4210	.5895	
3.5139	.5791	
3.6068	.5693	
3.6997	.5601	
3.7926	.5513	
3.8854	.5429	
3.9783	.5349	
4.0712	.5273	
4.1641	.5201	
4.2570	.5131	
4.3498	.5065	
4.4427	.5001	
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4.7214	.4823	
4.8142	.4768	
4.9071	.4715	
5.0000	.4664	
		12.50
.3690	4.9488	
.4616	3.0673	
.5542	2.0026	
2.0362	1.1924	
2.1288	1.0779	
2.2214	1.0083	
2.3140	.9563	
2.4066	.9145	
2.4993	.8793	
2.5919	.8490	
2.6845	.8223	
2.7771	.7984	
2.8697	.7769	
2.9624	.7574	
3.0550	.7394	
3.1476	.7229	
3.2402	.7076	
3.3328	.6933	
3.4255	.6800	
3.5181	.6675	
3.6107	.6557	
3.7033	.6446	
3.7959	.6341	
3.8886	.6241	

3.9812	.6146	
4.0738	.6056	
4.1664	.5970	
4.2590	.5888	
4.3517	.5810	
4.4443	.5734	
4.5369	.5662	
4.6295	.5593	
4.7221	.5526	
4.8148	.5461	
4.9074	.5400	
5.0000	.5340	
		15.00
.4340	4.9259	
.5253	3.2475	
.6166	2.1927	
2.2604	1.5016	
2.3517	1.2794	
2.4430	1.1895	
2.5344	1.1262	
2.6257	1.0763	
2.7170	1.0349	
2.8083	.9995	
2.8996	.9684	
2.9910	.9408	
3.0823	.9159	
3.1736	.8933	
3.2649	.8726	
3.3562	.8536	
3.4476	.8359	
3.5389	.8194	
3.6302	.8040	
3.7215	.7896	
3.8128	.7760	
3.9042	.7631	
3.9955	.7510	
4.0868	.7395	
4.1781	.7285	
4.2694	.7181	
4.3608	.7081	
4.4521	.6986	
4.5434	.6895	
4.6347	.6808	
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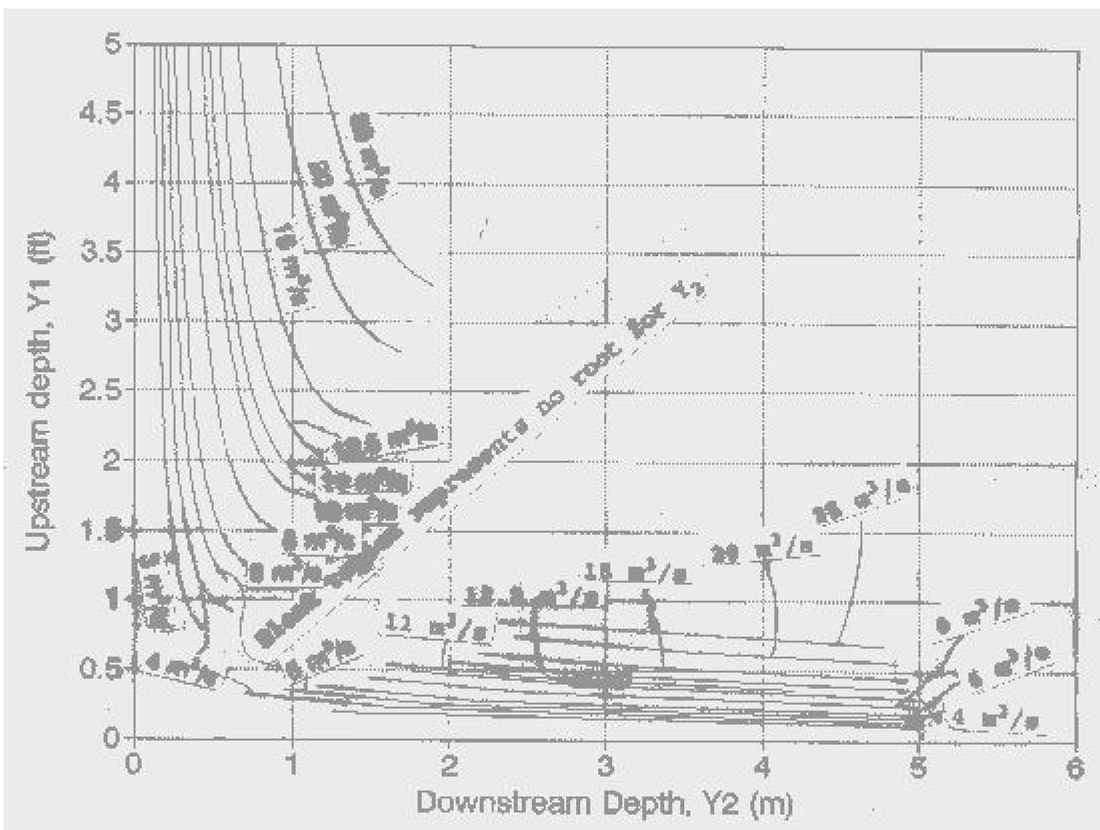
4.8174	.6643	
4.9087	.6566	
5.0000	.6491	
		20.00
.5580	4.8678	
.6468	3.4666	
.7357	2.4120	
2.7790	1.6950	
2.8678	1.5414	
2.9567	1.4523	
3.0455	1.3861	
3.1344	1.3327	
3.2232	1.2876	
3.3120	1.2484	
3.4009	1.2138	
3.4897	1.1828	
3.5786	1.1547	
3.6674	1.1289	
3.7562	1.1052	
3.8451	1.0833	
3.9339	1.0628	
4.0228	1.0437	
4.1116	1.0258	
4.2004	1.0089	
4.2893	.9929	

```
    4.3781     .9778
    4.4670     .9634
    4.5558     .9498
    4.6446     .9367
    4.7335     .9243
    4.8223     .9124
    4.9112     .9010
    5.0000     .8901
                           25.00
     .6760    4.7793
     .7625    3.5489
     .8490    2.3317

    3.2704    1.8876
    3.3569    1.7655
    3.4434    1.6819
    3.5298    1.6165
    3.6163    1.5622
    3.7028    1.5156
    3.7893    1.4745
    3.8758    1.4379
    3.9622    1.4048
    4.0487    1.3745
    4.1352    1.3467
    4.2217    1.3209
    4.3082    1.2969
    4.3946    1.2744
    4.4811    1.2534
    4.5676    1.2335
    4.6541    1.2147
    4.7406    1.1970
    4.8270    1.1801
    4.9135    1.1640
    5.0000    1.1487
```

The following graph plots the above output data with Y_2 as the abscissa and Y_1 as the ordinate.



Below another program PB2_47.FOR is given that first solves for Y_c in the rectangular section, and then starts with depth just below this for Y_2, first computes E_2 and then solves for Y_1 using the Newton Method so E_1=E_2.

Program PB2_47.FOR

```
      REAL Q(11)/3.,4.,5.,7.5,10.,12.5,15.,17.5,20.,22.5,25./
```

```
      WRITE(*,*)' Give: b1,m1,b2,g'
      READ(*,*) B1,FM1,B2,G
      G2=2.*G
      DO 50 I=1,11
      QS=Q(I)/B2
      YC=(QS*QS/G)**.33333333
      WRITE(3,100) Q(I),QS,YC
100   FORMAT(' For Q=',F5.1,' q=',F8.3,' Yc=',F8.3,/1X,27('-'),/,
     &'    Y2        Y1       E1',/,1X,27('-'))
      IYB=10.*(YC-.02)
      YB=FLOAT(IYB)/10.
      Y1=1.05*YC
      DO 10 J=0,30
      Y2=YB-.05*FLOAT(J)
      E2=Y2+(QS/Y2)**2/G2
      NCT=0

2     F=Y1+(Q(I)/((B1+FM1*Y1)*Y1))**2/G2-E2
      NCT=NCT+1
      IF(MOD(NCT,2).EQ.0) GO TO 4
      YY=Y1
      FF=F
      Y1=1.005*Y1
      GO TO 2
4     DIF=(Y1-YY)*FF/(F-FF)
      Y1=YY-DIF
      IF(ABS(DIF).GT.1.E-6 .AND. NCT.LT.40) GO TO 2
      IF(NCT.EQ.40) WRITE(*,*)' Did not converge',q(i),y2
      WRITE(3,110) Y2,Y1,E2
      IF(Y1.GT.15.) GO TO 50
10    Y1=1.01*Y1
110   FORMAT(3F9.3)
50    CONTINUE
      END
```

<u>Output</u>

```
 For Q=  3.0 q=   1.200 Yc=
.528
 ---------------------------
    Y2        Y1       E1
 ---------------------------
     .500     .745     .794
     .450     .767     .812
     .400     .821     .859
     .350     .921     .949
     .300    1.098    1.115
     .250    1.416    1.424
     .200    2.032    2.035
     .150    3.411    3.412
     .100    7.439    7.439
     .050   29.408   29.408
 For Q=  4.0 q=   1.600 Yc=
.639
 ---------------------------
    Y2        Y1       E1
 ---------------------------
     .600     .911     .962
     .550     .933     .981
     .500     .979    1.022
     .450    1.060    1.094
     .400    1.190    1.215
     .350    1.399    1.415
     .300    1.741    1.750
     .250    2.334    2.338
     .200    3.461    3.462
     .150    5.949    5.949
     .100   13.148   13.148
     .050   52.242   52.242
 For Q=  5.0 q=   2.000 Yc=
.742
 ---------------------------
    Y2        Y1       E1
 ---------------------------
     .700    1.063    1.116
     .650    1.082    1.133
     .600    1.120    1.166
     .550    1.184    1.224
     .500    1.284    1.315
     .450    1.433    1.457
     .400    1.659    1.674
     .350    2.005    2.014
     .300    2.561    2.565
     .250    3.510    3.512
     .200    5.296    5.297
     .150    9.211    9.211
     .100   20.487   20.487
 For Q=  7.5 q=   3.000 Yc=
.972
 ---------------------------
    Y2        Y1       E1
 ---------------------------
     .900    1.411    1.466
     .850    1.432    1.485
     .800    1.468    1.517
     .750    1.521    1.565
     .700    1.598    1.636
     .650    1.704    1.736
     .600    1.849    1.874
     .550    2.048    2.066
     .500    2.322    2.335
     .450    2.707    2.715
     .400    3.263    3.267
     .350    4.093    4.095
     .300    5.396    5.397
     .250    7.589    7.589
     .200   11.668   11.668
     .150   20.537   20.537
 For Q= 10.0 q=   4.000 Yc=
1.177
 ---------------------------
    Y2        Y1       E1
 ---------------------------
    1.100    1.718    1.774
    1.050    1.736    1.790
    1.000    1.764    1.815
     .950    1.806    1.854
     .900    1.863    1.907
     .850    1.940    1.979
     .800    2.041    2.074
     .750    2.172    2.200
     .700    2.342    2.364
     .650    2.564    2.580
     .600    2.853    2.865
     .550    3.238    3.246
     .500    3.757    3.762
     .450    4.474    4.477
     .400    5.496    5.497
     .350    7.007    7.007
     .300    9.361    9.361
     .250   13.298   13.298
     .200   20.587   20.587
 For Q= 12.5 q=   5.000 Yc=
1.366
 ---------------------------
    Y2        Y1       E1
 ---------------------------
    1.300    1.999    2.054
    1.250    2.011    2.065
    1.200    2.032    2.085
    1.150    2.063    2.113
    1.100    2.106    2.153
```

1.050	2.162	2.206
1.000	2.235	2.274
.950	2.327	2.362
.900	2.443	2.473
.850	2.589	2.614
.800	2.771	2.791
.750	3.000	3.015
.700	3.289	3.300
.650	3.658	3.666
.600	4.134	4.139
.550	4.759	4.762
.500	5.595	5.597
.450	6.741	6.742
.400	8.363	8.364
.350	10.752	10.752
.300	14.458	14.458
.250	20.637	20.637

For Q= 15.0 q= 6.000 Yc= 1.542

Y2	Y1	E1
1.500	2.261	2.315
1.450	2.268	2.323
1.400	2.283	2.336
1.350	2.305	2.357
1.300	2.336	2.386
1.250	2.377	2.424
1.200	2.430	2.474
1.150	2.497	2.537
1.100	2.580	2.616
1.050	2.682	2.714
1.000	2.807	2.835
.950	2.959	2.983
.900	3.146	3.165
.850	3.374	3.390
.800	3.655	3.667
.750	4.003	4.012
.700	4.438	4.445
.650	4.989	4.993
.600	5.694	5.697
.550	6.614	6.616
.500	7.839	7.839
.450	9.511	9.511
.400	11.868	11.868
.350	15.328	15.328

For Q= 17.5 q= 7.000 Yc= 1.709

Y2	Y1	E1
1.600	2.522	2.576
1.550	2.537	2.590
1.500	2.559	2.610
1.450	2.589	2.638
1.400	2.627	2.674
1.350	2.676	2.720
1.300	2.736	2.778
1.250	2.810	2.848
1.200	2.900	2.934
1.150	3.008	3.038
1.100	3.137	3.164
1.050	3.292	3.315
1.000	3.478	3.497
.950	3.702	3.717
.900	3.971	3.983
.850	4.297	4.307
.800	4.695	4.702
.750	5.185	5.190
.700	5.793	5.797
.650	6.559	6.561
.600	7.536	7.537
.550	8.805	8.806
.500	10.489	10.490
.450	12.783	12.783
.400	16.009	16.009

For Q= 20.0 q= 8.000 Yc= 1.869

Y2	Y1	E1
1.800	2.754	2.807
1.750	2.763	2.815
1.700	2.777	2.829
1.650	2.798	2.848
1.600	2.825	2.874
1.550	2.861	2.908
1.500	2.905	2.950
1.450	2.959	3.001
1.400	3.025	3.064
1.350	3.104	3.140
1.300	3.197	3.230
1.250	3.308	3.338
1.200	3.439	3.465
1.150	3.594	3.617
1.100	3.777	3.796
1.050	3.993	4.009
1.000	4.249	4.262
.950	4.554	4.564
.900	4.919	4.927
.850	5.359	5.365
.800	5.893	5.897
.750	6.546	6.549
.700	7.355	7.357
.650	8.369	8.371
.600	9.660	9.661
.550	11.333	11.333
.500	13.548	13.548
.450	16.558	16.559

For Q= 22.5 q= 9.000 Yc= 2.021

Y2	Y1	E1
2.000	2.980	3.032
1.950	2.984	3.036
1.900	2.992	3.044
1.850	3.006	3.056
1.800	3.025	3.074
1.750	3.050	3.098
1.700	3.082	3.129
1.650	3.122	3.166
1.600	3.170	3.213
1.550	3.228	3.268
1.500	3.297	3.335
1.450	3.379	3.414
1.400	3.475	3.506
1.350	3.587	3.615
1.300	3.717	3.743
1.250	3.870	3.892
1.200	4.048	4.067
1.150	4.256	4.272
1.100	4.499	4.512
1.050	4.784	4.795
1.000	5.120	5.128
.950	5.518	5.524
.900	5.992	5.997
.850	6.560	6.564
.800	7.248	7.251
.750	8.088	8.089
.700	9.124	9.125
.650	10.421	10.421
.600	12.067	12.068
.550	14.198	14.198
.500	17.014	17.014

For Q= 25.0 q= 10.000 Yc= 2.168

Y2	Y1	E1
2.100	3.205	3.256
2.050	3.212	3.263
2.000	3.224	3.274
1.950	3.241	3.290
1.900	3.264	3.312
1.850	3.293	3.339
1.800	3.328	3.373
1.750	3.371	3.414
1.700	3.423	3.464
1.650	3.483	3.522
1.600	3.555	3.591
1.550	3.638	3.671
1.500	3.734	3.765
1.450	3.846	3.874
1.400	3.975	4.000
1.350	4.124	4.147
1.300	4.297	4.316
1.250	4.495	4.512
1.200	4.725	4.739
1.150	4.992	5.004
1.100	5.303	5.312
1.050	5.665	5.673
1.000	6.091	6.097
.950	6.593	6.597
.900	7.189	7.192
.850	7.902	7.904
.800	8.762	8.764
.750	9.810	9.811
.700	11.101	11.102
.650	12.713	12.714
.600	14.758	14.758

The same result are obtained by the program GATET.FOR that uses the Newton Method to obtain the solution. The table of data generated by this program is given before the table listing. This table could be used to solve problem involving alternative depths across a gate with an upstream trapezoidal channel with a bottom width of 3 m and a side slope of 1.5, and a rectangular channel with a bottom width of 2.5 m in which the gate is placed. Below the program listing the data is plotted as in the latter portion of the previous problem starting with the critical depth line in the lower portion of the graph. Now this critical depth line does not have equal values of Y_2 and Y_1, since the upstream channel is trapezoidal.

<u>Program GATET.FOR</u>

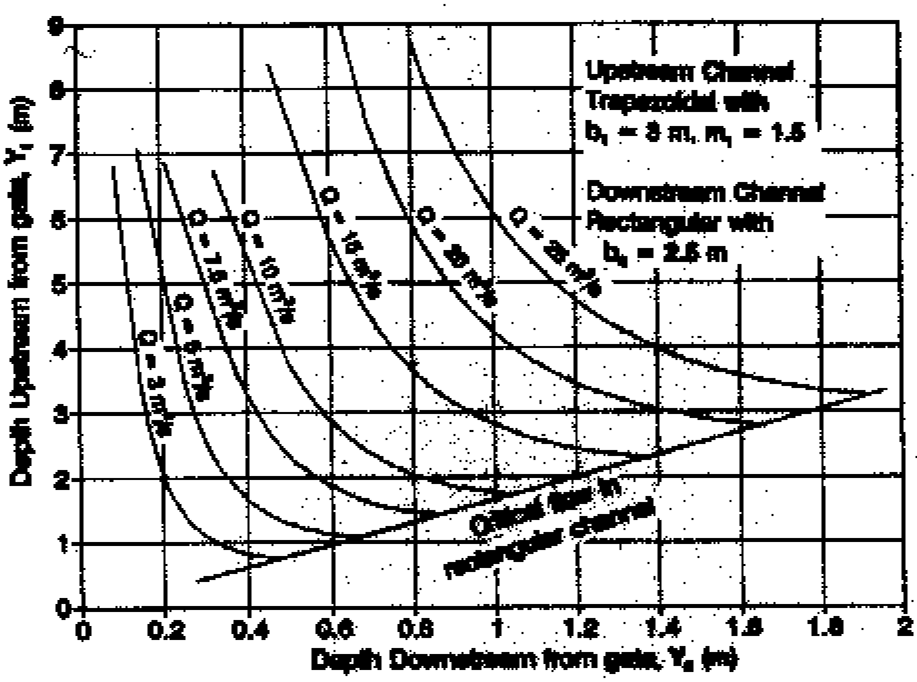
Upstream channel trapezoidal with b_1=3 m, m_1=1.5; Downstream channel rectangular with b_2=2.5 m

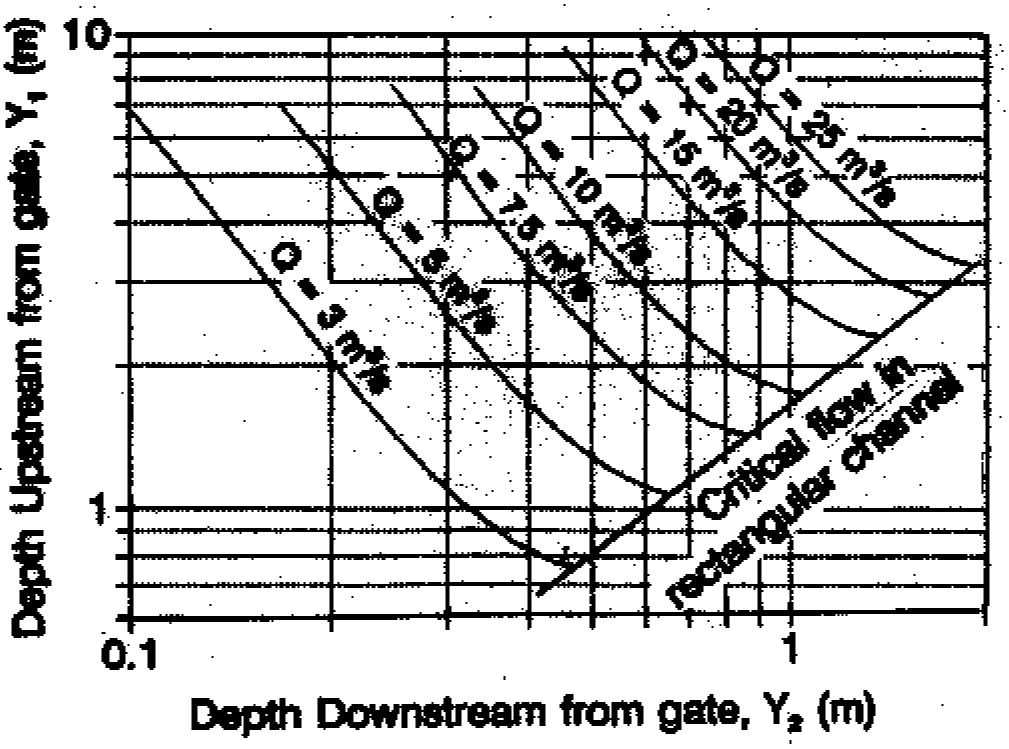
Critical Depths						
Q=3 m^3/s	Q=5 m^3/s	Q=7.5 m^3/s	Q=10 m^3/s	Q=15 m^3/s	Q=20 m^3/s	Q=25 m^3/s
.46714	.65666	.86047	1.04239	1.36591	1.65469	1.92010

Flow rates Q (m**3/s)													
3		5		7.5		10		15		20		25	
Y_2(m)	Y_1(m)	Y_2(m)	Y_1(m)	Y_2(m)	Y_1(m)	Y_2(m	Y_1(m)	Y_2(m)	Y_1(m)	Y_2(m)	Y_1(m)	Y_2(m)	Y_1(m)
.15000	3.41138	.25000	3.51046	.35000	4.09257	.50000	3.75714	.60000	5.69418	.70000	7.35518	.80000	8.76222
.16669	2.80699	.27140	3.03676	.37687	3.60348	.52855	3.44122	.64031	5.11174	.75025	6.54260	.85895	7.76466
.18338	2.36393	.29281	2.66708	.40373	3.21338	.55709	3.17635	.68062	4.63611	.80049	5.88685	.91790	6.96358
.20007	2.03053	.31421	2.37396	.43060	2.89824	.58564	2.95276	.72093	4.24398	.85074	5.35188	.97686	6.31292
.21677	1.77423	.33561	2.13840	.45747	2.64089	.61419	2.76293	.76125	3.91808	.90099	4.91144	1.03581	5.77932
.23346	1.57365	.35702	1.94693	.48433	2.42880	.64273	2.60097	.80156	3.64537	.95123	4.54601	1.09476	5.33820
.25015	1.41433	.37842	1.78979	.51120	2.25265	.67128	2.46224	.84187	3.41588	1.00148	4.24089	1.15371	4.97109
.26684	1.28620	.39982	1.65981	.53807	2.10543	.69983	2.34305	.88218	3.22188	1.05173	3.98479	1.21267	4.66393
.28353	1.18210	.42123	1.55160	.56494	1.98177	.72837	2.24041	.92249	3.05732	1.10197	3.76901	1.27162	4.40585
.30022	1.09684	.44263	1.46107	.59180	1.87751	.75692	2.15192	.96280	2.91742	1.15222	3.58668	1.33057	4.18838
.31691	1.02656	.46403	1.38508	.61867	1.78942	.78547	2.07561	1.00311	2.79833	1.20247	3.43238	1.38952	4.00483
.33361	.96841	.48544	1.32118	.64554	1.71493	.81401	2.00985	1.04342	2.69698	1.25271	3.30177	1.44848	3.84985
.35030	.92019	.50684	1.26745	.67240	1.65197	.84256	1.95330	1.08374	2.61084	1.30296	3.19135	1.50743	3.71913
.36699	.88022	.52824	1.22236	.69927	1.59890	.87111	1.90484	1.12405	2.53784	1.35321	3.09825	1.56638	3.60917
.38368	.84721	.54965	1.18469	.72614	1.55436	.89965	1.86351	1.16436	2.47629	1.40345	3.02013	1.62533	3.51710
.40037	.82012	.57105	1.15343	.75300	1.51726	.92820	1.82851	1.20467	2.42474	1.45370	2.95502	1.68429	3.44055
.41706	.79814	.59245	1.12777	.77987	1.48667	.95675	1.79916	1.24498	2.38200	1.50395	2.90130	1.74324	3.37753
.43375	.78058	.61386	1.10702	.80674	1.46180	.98529	1.77485	1.28529	2.34704	1.55419	2.85759	1.80219	3.32636
.45044	.76689	.63526	1.09061	.83361	1.44200	1.0138	1.75507	1.32560	2.31898	1.60444	2.82271	1.86114	3.28564
.46714	.75661	.65666	1.07803	.86047	1.42670	1.0424	1.73936	1.36591	2.29706	1.65469	2.79565	1.92010	3.25413

<u>Listing of Program GATET.FOR</u>

```
      REAL Y1(7),Y2(7),Q(7),Yc(7),Yb(7),DY(7),YS(7)
      DATA Q/3,5,7.5,10,15,20,25/,Yb/.15,.25,.35,.5,.6,.7,.8/
      DATA b1,fm1,b2/3.,1.5,2.5/YS/3.411,3.51,4.093,3.757,5.694,7.355,
     &8.762/
      DO 10 I=1,7
      Yc(I)=((Q(I)/b1)**2/9.81)**.33333333  /  should change b1 to b2
10    DY(I)=(Yc(I)-Yb(I))/19.
      WRITE(3,100) Yc
100   FORMAT(' Critical Depths',/,7F10.5)
      DO 20 I=1,20
      DO 15 J=1,7
      Y2(J)=Yb(J)+FLOAT(I-1)*DY(J)
      E2=Y2(J)+(Q(J)/(b2*Y2(J)))**2/19.62
      NCT=0
      YY=YS(J)
12    A1=(b1+fm1*YY)*YY
      VH=(Q(J)/A1)**2/19.62
      F=YY+VH-E2
      DF=1.-2.*VH/A1*(b1+2.*fm1*YY)
      DIF=F/DF
      YY=YY-DIF
      NCT=NCT+1
      IF(ABS(DIF).GT..000001 .AND. NCT.LT.20) GO TO 12
      IF(NCT.GE.20) WRITE(*,*)' DIF=',DIF,YY,F,I,J
      YS(J)=.95*YY
15    Y1(J)=YY
20    WRITE(3,110)(Y2(J),Y1(J),J=1,7)
110   FORMAT(7(1X,2F8.5))
      END
```


Upstream Channel
Trapezoidal with
b_1 = 3 m, m_1 = 1.5
Downstream Channel
Rectangular with
b_2 = 2.5 m
Q = 3 m³/s
Q = 5 m³/s
Q = 7.5 m³/s
Q = 10 m³/s
Q = 15 m³/s
Q = 20 m³/s
Q = 25 m³/s
Critical flow in
rectangular channel
Depth Upstream from gate, Y_1 (m)
Depth Downstream from gate, Y_2 (m)


Q = 3 m³/s
Q = 5 m³/s
Q = 7.5 m³/s
Q = 10 m³/s
Q = 15 m³/s
Q = 20 m³/s
Q = 25 m³/s
Critical flow in
rectangular channel
Depth Upstream from gate, Y_1 (m)
Depth Downstream from gate, Y_2 (m)

48.

A pipe with a diameter D = 6 m and on a steep slope takes water from a reservoir with a water surface elevation 5 m above the bottom of the pipe. What flow rate might be expected to enter the pipe?

Given: A steep circular channel with D=6 m. If H=5 m what is Q?

Solution: The following Energy and Critical Flow Equations need to be solve for Y_c and Q.

$H=Y_c+(Q/A_c)^2/(2g)$ Energy (1)

$Q^2T_c = gA_c^3$ Critical Flow (2)

Combining (1) and (2) gives $5 = Y_c + A_c/(2T_c)$

$Y_c = 3.533$ m ← $\beta=\cos^{-1}(1-2(3.533)/6) = 1.74941$ rad, $A=36/4(\beta-\cos\beta\sin\beta)=17.31825$ m^2, $T = D\sin\beta = 5.84991$, $Q = [gA^3/T]^{1/2} =$
and $Q = 93.3$ m^3/s ←

The MATLAB script "e_cr1.m", given in the folder MATLAB (subfolder CHAPTER2), is designed to solve problems such as this one. The dialog in the command window to solve this problems is as follows:

```
>> e_cr1
Give g 9.81
Give type of channel (0=trap, 1=cir.) 1
Give: D,H,Ke enclose in [ ] [6 5 0]
Solution: Y =    3.533, Q =      92.91
```

49.

A trapezoidal channel with b = 7 ft, and m = 1.5 and a bottom slope of $S_o = 0.08$ get water directly from a reservoir with its water surface 4.5 ft above the channel bottom. What flow rate would you predict will enter this channel?

Given: Trapezoidal with b=7 ft, m=1.5, S_o= .08, and H = 4.5 ft.
Find: Q

Solution: The slope suggests the flow will be supercritical so the following Energy and Critical Flow Equations need to be solve for Y_c and Q.

$H=Y_c+(Q/A_c)^2/(2g)$ Energy (1)

$Q^2T_c = gA_c^3$ Critical Flow (2)

Combining (1) and (2) gives $4.5 = Y_c + A_c/(2T_c)$, with $A=(7+1.5Y)Y$ & $T=7+3Y$

Solving gives: $Y_c = 3.326$ ft ← and Q = 345.5 cfs ←

Using the MATLAB script to solve this problem gives following in command window:

```
>> e_cr1
Give g 32.2
Give type of channel (0=trap, 1=cir.) 0
Give: b,m,H,Ke enclosed in [ ] [7 1.5 4.5 0]
Solution: Y =    3.326, Q =     346.72
```

50.

A transition takes a trapezoidal channel with b = 10 ft, and m = 1.5 to a rectangular section with b = 8 ft. Through the transition the bottom of the channel rises 0.2 ft. The slope of the very long upstream trapezoidal channel is S_o = 0.00015, and its n = 0.014. For a flow rate of Q = 330 cfs determine the following: (a) the depth upstream from the transition, (b) the depth immediately downstream from the transition, (c) the change in water surface through the transition, and (d) the Froude Numbers associated with both the upstream flow and that immediate downstream from the transition. If the depth in the rectangular channel is not to change downstream from the transition what should its slope be? Its n = 0.014 also. If its slope is less than this amount what will happen?

Given: Q=330 cfs, b_1=10 ft, m_1=1.5, S_{o1}=.00015, n=.014, b_2=8 ft (rect), Δz=.2 ft

Wanted: (1) Y_1, (b) Y_2, (c) Δws, (d) F_{r1}, F_{r2} and S_{o2}

Solution:

(a) $Q = (1.486/.014)A^{5/3}/P^{2/3}S_o^{1/2} = 330$ → Y_{o1} = 5.840 ft & E_{o1} = 5.98 ft

$Q^2T_2/(gA_2^3) = 1$ → Y_{c2} = 3.753 ft, E_{c2} = 5.63 ft.

(b) Since $E_{o1} > E_{c2}+\Delta z$, the upstream depth will be uniform to the transition at Y_1 = 5.840 ft, and

$E_2 = 5.98 - .2 = 5.78 = Y_2 + (1+.04)(Q/A_2)^2/(2g)$ → Y_2 = 4.28 ft ←

(c) Δws = 5.84 – 4.28 -.2 = 1.36 ft drop ←

(d) F_{r1} = 0.27 ← F_{r2} = 0.82 ← and S_{o2} = .006405 ←
If the slope is less than this amount the depth in the upstream channel will be larger than the uniform flow depth.

Using MATLAB to solve this problem produces the following for the upstream normal depth for part (a).

```
>> manning
Give:1=trap.,2=cir. for channel type 1
Give:1=ES or 2=SI units 1
Give the number for the unknown:
n = 1
Q = 2
S = 3
Y = 4
b = 5
m = 6
 Number = 4
Give value to known variables and guess for unknown
n = .014
Q = 330
S = .00015
b = 10
m = 1.5
 Provide guess for unknown Y = 4
Unknown Y = 5.8396                      ←
n = 0.014
Q = 330
S = 0.00015
Y = 5.8396
b = 10
m = 1.5
```

Part (b)

```
>> enertwo
Give g = 32.2
Give type of channel (0=trap,1=cir) 0
Give number of the unknown
  1    Q1
  2    Q2
  3    KL
  4    dz
  5    Y1
  6    Y2
  7    b1
  8    m1
  9    b2
 10    m2
 6
Give value to variables
Q1(known) = 330
Q2(known) = 330
KL(known) = .04
dz(known) = .2
Y1(known) = 5.8396
Y2(guess) = 5
b1(known) = 10
m1(known) = 1.5
b2(known) = 8
m2(known) = 0
Specific energy E1 =   5.9805,  E2 =   5.7229, E =   5.9805
Froude Numbers Fr1 =   0.2661,  Fr2 =   0.8205
  1 Q1 (known)   =   330.0000
  2 Q2 (known)   =   330.0000
  3 KL (known)   =     0.0400
  4 dz (known)   =     0.2000
  5 Y1 (known)   =     5.8396
  6 Y2 (unknown) =     4.2815         ←

  7 b1 (known)   =    10.0000
  8 m1 (known)   =     1.5000
  9 b2 (known)   =     8.0000
 10 m2 (known)   =     0.0000
```

51.

If the water surface is not to change through a transition from a trapezoidal to a rectangular channel with the dimensions in previous problem how much must the bottom rise or fall as a tabular function of the upstream depth. Plot this function?

Given: The transition of the previous problem from a trapezoidal to a rectangular channel (b_1=10 ft, m_1=1.5, and b_2=8 ft)

Find: Δz as a function of the upstream depth so the water surface does not change.

Solution: If the water surface is to remain horizontal, then the velocities heads must be the same, which requires that the area be the same, i.e. $V_2=V_1 \rightarrow A_2 = A^1$.

$Y_1(b_1+m_1Y_1) = Y_2b_2$ also $E_1 = E_2 + \Delta z$ or

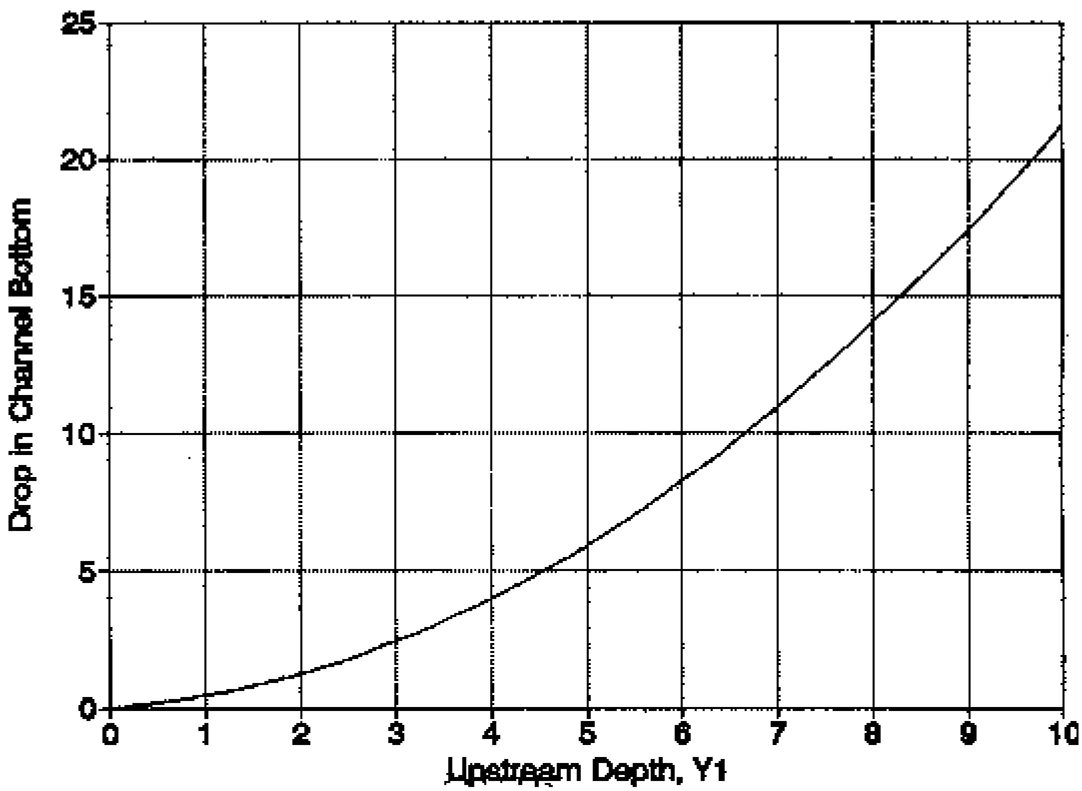
$Y_1 = Y_2 + \Delta z$ or $Y_2 = Y_1 - \Delta z$

$Y_1(b_1+m_1Y_1) = (Y_1-\Delta z)b_2$ or for the transition of this problem

$Y_1(10+1.5Y_1) = (Y_1-\Delta z)8$ or $10Y_1+1.5Y_1^2-8Y_1+8\Delta z = 0$

$1.5Y_1^2+2Y_1+8\Delta z = 0$ Since Δz must drop (be negative)

$|\Delta z| = .1875Y_1^2 + .25Y_1 = (.1875Y_1 + .25)Y_1 \leftarrow$ is mathematical relationship



52.

What is the maximum rise that can take place in the bottom elevation through the transition in Problem 50 for the upstream flow to be possible?

Find: What is Maximum height of rise in Problem 50?

Flow will be critical at end of transition in rectangular channel so,

$Y_2 = Y_c = [(330/8)^2/32.2]^{1/3} = 3.753$ ft, $E_c = 1.5Y_c = 5.629$ ft ←

From Problem 50 $E_{o1} = 5.98$ ft,

$\Delta z = E_{o1} - E_{c2} = 5.98 - 5.629 = 0.35$ ft ←

53.

Everything is the same as in Problem 50 except that the bottom rises by 0.8 ft through the transition. What is the depth upstream?

Given: Same as Problem 50 except $\Delta z = 0.8$ ft
Find: Y_1

Solution: From Problem 52 $Y_{c2} = 3.753$ ft, $E_{c2} = 5.629$ ft

$E_1 = E_{c2}+\Delta z = 5.629+.8 = 6.429$ ft

$Y_1 + (Q/A_1)^2/(2g) = 6.429$ → $Y_1 = 6.317$ ft ←

54.

A broad crested weir consists of a rounded rectangular hump placed in the bottom of a channel that is sufficiently high to cause critical flow over its top, and is a common flow measurement device. If the depth Y_1 upstream from a broad crested weir, that is 0.6 m high, is 2 m, and the rectangular channel containing the weir is b = 4 m wide, what is the flowrate Q? What is the depth over the crest of the weir?

Given: a broad crested weir with height $\Delta z = 0.6$ m. b=4m Y_1=2 m
Find: Q

Solution: Depth will be critical over crest of weir if upstream depth of 2 m is equal to or larger then the normal depth in the upstream channel which must be assumed to be true. Thus

$Y_1 + q^2/(2gY_1^2) = E_{c2}+\Delta z = 1.5Y_{c2} + .6 = 1.5[q^2/g]^{1/3}+0.6$

$2+q^2/(19.62x4) = 1.5[q^2/9.81]^{1/3} + 0.6$

Solving $q = 3.238$ m^2/s and $Q = bq = 12.952$ m^3/s ←
(2nd root of above equation is $q = 14.971$ m^2/s)

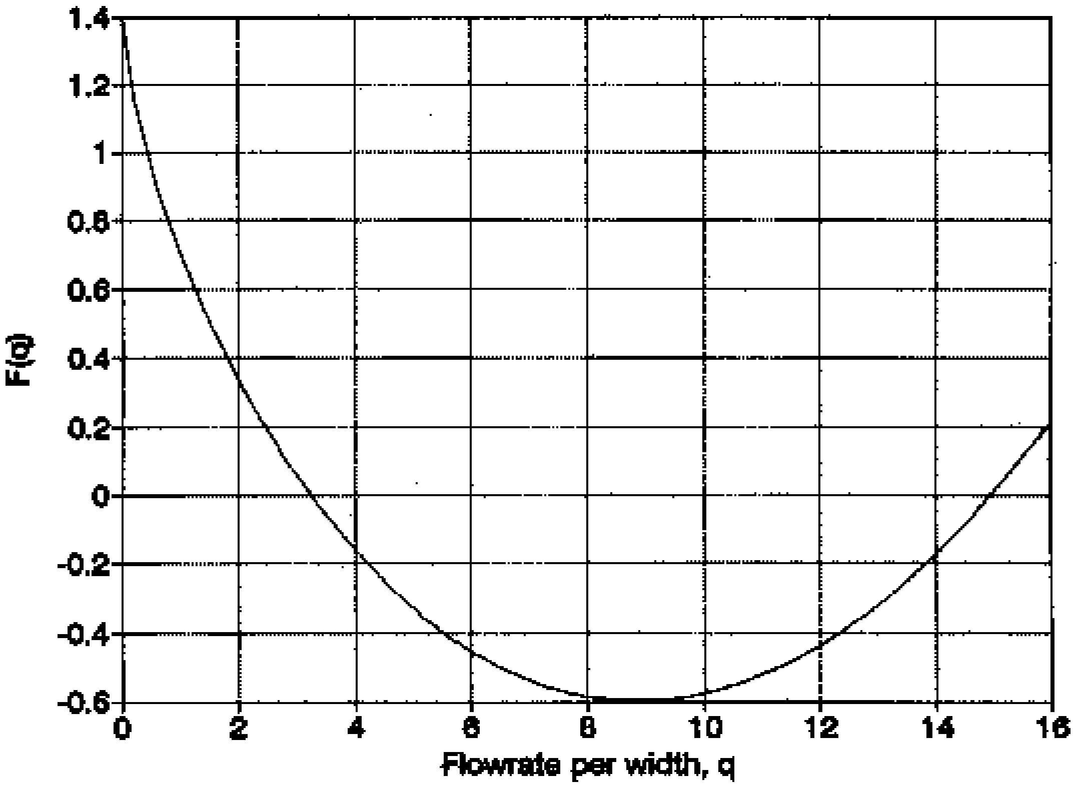
$Y_{c2} = [q^2/g]^{1/3} = 1.0224$ m, $E_{c2} = 1.5336$ m, $E_1 = E_{c2}+\Delta z = 2.1336$ m ←

55.

For the flow over the broad crested weir of the previous problem plot the function F(q) on the ordinate, and q on the abscissa of a graph and show that two solutions exist, and demonstrate that one of these is associated with an upstream supercritical flow, and the other subcritical. However, since physically the supercritical condition cannot exist that the only physically viable solution is the smaller of the two roots. (Note that if the upstream flow were supercritical that a hydraulic jump would form upstream so that the flow is subcritical in front of the weir so that the flow depth decreases as the velocity head increases to pass the reduced flow section.)

Find: Show that two roots exist for flow per unit width across a broad crested weir by plotting F(q) versus q.

Quadratic equation from previous problem:
$2+q^2/(19.62\times4) = 1.5[q^2/9.81]^{1/3} + 0.6$



The two roots are where the curve cross the F(q) = 0 horizontal line and are:

First root: q = 3.238 m
Y_1=2 m, F_{r1}=.366 (subcritical)

Alternate depth Y_1= 0.588, F_{r1} = 2.293

Second root: q = 14.971 m

$Y_c=[q^2/g]^{1/3}$ = 2.838 m, $E_c=1.5Y_c$ = 4.256 m

Y_1= 2 m, $F_{r1} =[q^2/(gY_1^3)]^{1/2}$ = 1.690 supercritical

E_1=4.856 m alternate depth Y_1=4.212 m (F_{r1}=.553), but Y_1=2 m is upstream depth

56.

Water enters a trapezoidal channel with b = 8 ft, m = 2.0, and a bottom slope of S_o = 0.000115 from a reservoir whose water surface is 8 ft above the channel bottom. If the entrance loss coefficient is K_L = 0.04 determine the flow rate and depth of flow in this channel. The channel is very long, and n = .013.

Given: Water enters a trapezoidal channel with b= 8 ft, m=2,n=.013 and S_o=.000115 from a reservoir with H= 8 ft and loss coefficient K_e = .04.
Find: Depth and Flow rate (Y_o & Q)

Solution: Mannings and Energy equations need to be solved simultaneously

$$Q = (C_u/n)A^{5/3}/P^{2/3}S_o^{1/2} \qquad (1)$$

$$H = Y + (1+K_e)(Q/A)^2/(2g) \qquad (2)$$

These two equations can be solved simultaneously, or reduced to one equation that gives Y by substitution (1) into (2).

Solving gives: Y_o = 7.830 ft, Q = 601.11 cfs ←

Using the MATLAB script "e_un.m" given on the CD in the back cover MATLAB, subfolder CHAPTER2 produces the following in the command window:

```
>> e_un
Give: g,entrance loss coef,Ke & 0=trap,or 1=circle in [ ] [32.2 .04 0]
  1  b
  2  m
  3  S
  4  n
  5  Q
  6  H
  7  Y
Give two numbers for 2 unknown variables in [ ]: [5 7]
Give values to variables
1  b (known) = 8
2  m (known) = 2
3  S (known) = .000115
4  n (known) = .013
5  Q (guess) = 700
6  H (known) = 8
7  Y (guess) = 7.8
Solution:
  1 b =    8.0000
  2 m =    2.0000
  3 S = 0.0001150
  4 n =    0.0130
  5 Q =  601.1137
  6 H =    8.0000
  7 Y =    7.8300
```

57.

Gates control flow in a trapezoidal channel with b = 6 m, m = 1.5, n = .014, and S_o =0.00025. There are 3 rectangular gates each 1.5 m wide with contraction coefficients of C_c = 0.8. Two gates are set 0.5 m above the channel bottom and the third gate is set 0.2 m above the channel bottom. The depth upstream from the gates is 4.2 meters. Determine the flow rate in the channel. How much is the upstream depth above or below the uniform (normal) depth in this channel?

Given: H = 4.2 m reservoir head. Three gates with width of 1.5 m each, C_c = .8, with two gates set at Y_G=0.5 m, and one at Y_G=0.2 m. Downstream channel b= 6 m, m=2, n=.013 and S_o=.000115.

Solution:
Assume gates will operate submerged and velocity head under gate is dissipated.
Depth downstream from two gates Y= .8(.5) = 0.4 m and from one gate Y=.8(.2)=.16 m
Unknown Variable(4): Q, Q_1 (flow rate past two gates), Q_2(flow past other gate), $Y=Y_o$

Equations:

$2Q_1 + Q_2 = Q$ Continuity (1)

$Q=(1/n)A^{5/3}/P^{2/3}S_o^{1/2}$ Uniform Flow downstream (2)

$4.2=Y+(Q/A)^2/(2g)+(Q_1/A_1)^2/(2g)$ Energy, first gates (3)
$A_1 = 1.5(.4) = 0.6\ m^2$

$4.2=Y+(Q/A)^2/(2g)+(Q_2/A_2)^2/(2g)$ Energy, second gate (4)
$A_1 = 1.5(.16) = 0.24\ m^2$

Notice the last terms in (3) and (4) are the head losses past the submerged gates.
Solving these four equations gives:
$Q = 10.821\ m^3/s$ ←, $Q_1=4.509\ m^3/s$ ←, $Q_2=1.804\ m^3/s$ ← and Y=1.262 m ←
Since $Y>Y_G$ assumption of submerged flow will be accepted. Later after the momentum principle and GVF's are covered in Chapters 3 and 4, this assumption may be modified.

TK-Solver solution (PB2 57.TK)

```
--------------- VARIABLE SHEET---------------
St Input---- Name ---Output ---Unit ---------
             Q1      4.5087906
             Q2      1.8035163
             Q       10.821098
             A       9.9572457
             Y       1.2616198

---------------RULE SHEET---------------
S Rule-----------------------------------
  2*Q1+Q2=Q
  A=(6+1.5*Y)*Y
  Q=A^1.666667/(6+2*Y*sqrt(1.5^2+1))^.6666667*sqrt(.00025)/.014
  4.2=Y+(Q/A)^2/19.62+(Q1/.6)^2/19.62
  4.2=Y+(Q/A)^2/19.62+(Q2/.24)^2/19.62
```

58.

The land slopes at a rate of 0.0016 ft per foot of length and you are to design a trapezoidal channel that will carry 400 cfs of water when the depth of water in the reservoir is 5 ft above its bottom. The channel is to have a side slope of m = 1.4, and a Manning's n = 0.013. What should its bottom width be?
What is the Froude Number associated with this flow?

Given: S_o=.0016, m=1.4, n=.013, Q=400 cfs, H =5 ft,
Wanted: bottom width b

Solve Energy and Mannings Equations simultaneously

$Q = (C_u/n)A^{5/3}/P^{2/3}S_o^{1/2}$ (1)

$H = Y + (1+K_e)(Q/A)^2/(2g)$ (2)

Using CHANNEL to solve problem gives the following solution: (Ke is taken = 0)
b = 6.800 ft ← and Y_o = 3.961 ft ← (F_r =.872)

The following MathCAD model used the Given – Find block capability to solve the above two equations for b and Y_o. (Notice in this model that after b and Y are solved that the variables Yo and bo are given the solved values and the Froude Number is computed.

PRB2_58.MCD

$Q := 400$ $So := .0016$ $m := 1.4$ $n := .013$ $H := 5$ $Ke := 0$ $Cu := 1.486$ $g := 32.2$ $b := 10$ $Y := 4.8$

Given

$$Q = \frac{Cu}{n} \cdot \frac{((b + m \cdot Y) \cdot Y)^{1.666667}}{\left(b + 2 \cdot Y \cdot \sqrt{m^2 + 1}\right)^{.66667}} \cdot \sqrt{So} \qquad H = Y + (1 + Ke) \cdot \frac{Q^2}{2 \cdot g \cdot ((b + m \cdot Y) \cdot Y)^2}$$

$$\mathrm{Find}(b, Y) = \begin{pmatrix} 6.7996 \\ 3.9607 \end{pmatrix} \qquad Yo := 3.9607 \qquad bo := 6.7996 \qquad Fr := \sqrt{\frac{Q^2 \cdot (bo + 2 \cdot m \cdot Yo)}{g \cdot ((bo + m \cdot Yo) \cdot Yo)^3}}$$

$Fr = 0.872$

Solving this problem using the MATLAB script e_un produces the following in the command window:

```
>> e_un
Give: g,entrance loss coef,Ke & 0=trap,or 1=circle in [ ] [32.2 0 0]
  1  b
  2  m
  3  S
  4  n
  5  Q
  6  H
  7  Y
Give two numbers for 2 unknown variables in [ ]: [1 7]
Give values to variables
1  b (guess) = 7
2  m (known) = 1.4
3  S (known) = .0016
4  n (known) = .013
5  Q (known) = 400
6  H (known) = 5
7  Y (guess) = 4.8
Solution:
  1 b =    6.7995
  2 m =    1.4000
  3 S = 0.0016000
  4 n =    0.0130
  5 Q =  400.0000
  6 H =    5.0000
  7 Y =    3.9607
```

59.

Develop the stage discharge curve for the channel you designed in the previous problem with the reservoir water surface elevation varying from 0 to 5.5 feet above the channel bottom.

Given: Channel of previous problem, b=6.8 ft, H varies from 0 to 5.5 ft.

Wanted: Stage-Discharge relationship.

Solution: Now solve Energy and Mannings Equations for Q and Y for different H's (This can be done by repeatedly solving these two equations by modifying the above MathCAD model, or the CHANNEL program (which gives the results below.) The Program STAGEQ (listed below is also used.) Thereafter a TK-Solver program to generate a stage discharge relationship is also given.

H(ft)	Q (cfs)	Y (ft)
0.5	6.9	0.41
1.0	21.9	0.81
1.5	43.8	1.20
2.0	72.4	1.60
2.5	107.9	1.99
3.0	150.5	2.38
3.5	200.5	2.78

4.0	258.1	3.17
4.5	323.6	3.57
5.0	397.2	3.97
5.5	479.2	4.35
6.0	570.0	4.74

Program STAGEQ.FOR (Designed to generate stage-discharge data

```
      WRITE(*,*)' Give: b,m,n,So,g,Ke,Start H, DelH & N'
      READ(*,*) B,FM,FN,So,G,FKE,H1,DH,N
      G2=2.*G
      Cu=1.486
      IF(G.LT.20.) Cu=1.
      FMS=2.*SQRT(FM*FM+1.)
      Y=.95*H1
      Cu=Cu*SQRT(So)
      FKE=FKE+1.
      DO 30 I=1,N
      H=H1+FLOAT(I-1)*DH
      NCT=0
10    A=(B+FM*Y)*Y
      Q=Cu/FN*A**1.6666667/(B+FMS*Y)**.6666667
      F=H-Y-FKE*(Q/A)**2/G2
      NCT=NCT+1
      IF(MOD(NCT,2).EQ.0) GO TO 15
      FF=F
      YY=Y
      Y=1.005*Y
      GO TO 10
15    DIF=(Y-YY)*FF/(F-FF)
      Y=YY-DIF
      IF(ABS(DIF).GT. 1.E-6 .AND. NCT.LT.40) GO TO 10
      IF(NCT.EQ.40) WRITE(*,*)' Did not converge H=',H,DIF
      A=(B+FM*Y)*Y
      Q=Cu/FN*A**1.6666667/(B+FMS*Y)**.6666667
30    WRITE(3,100) H,Q,Y,Y+(Q/A)**2/G2
100   FORMAT(4F9.3)
      END
```

Input: 6.8 1.4 .013 .0016 32.2 0. .5 .5 12

Output:

H	Q	Y	E
.500	7.202	.413	.500
1.000	22.490	.810	1.000
1.500	44.612	1.204	1.500
2.000	73.484	1.598	2.000
2.500	109.248	1.992	2.500
3.000	152.129	2.386	3.000
3.500	202.380	2.781	3.500
4.000	260.267	3.174	4.000
4.500	326.057	3.568	4.500
5.000	400.016	3.961	5.000
5.500	482.404	4.353	5.500
6.000	573.476	4.745	6.000

These values are more accurate than those obtained above. Notice that since the entrance loss coefficient was given as 0 that the specific energy E is the same as the reservoir head H.

Using Program CHANNEL to obtain this stage-discharge relationship gives the following:

H	Y	b	m	n	S	K	Q
0.500	0.413	6.800	1.400	0.01300	0.001600	0.000	7.178
1.000	0.810	6.800	1.400	0.01300	0.001600	0.000	22.413
1.500	1.204	6.800	1.400	0.01300	0.001600	0.000	44.456
2.000	1.598	6.800	1.400	0.01300	0.001600	0.000	73.222
2.500	1.992	6.800	1.400	0.01300	0.001600	0.000	108.852
3.000	2.386	6.800	1.400	0.01300	0.001600	0.000	151.566

3.500	2.781	6.800	1.400	0.01300	0.001600	0.000	201.618
4.000	3.174	6.800	1.400	0.01300	0.001600	0.000	259.271
4.500	3.568	6.800	1.400	0.01300	0.001600	0.000	324.791
5.000	3.961	6.800	1.400	0.01300	0.001600	0.000	398.441
5.500	4.353	6.800	1.400	0.01300	0.001600	0.000	480.480
6.000	4.745	6.800	1.400	0.01300	0.001600	0.000	571.162

TK-Solver model STAGEQ.TK

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment---------
   6.8        b
   1.4        m
LG 0          Q
   1.486      Cu
   .013       n
   .0016      So
L  .5         H
   0          Ke
   32.2       g
LG 0          Y
LG 0          P
LG 0          A

==================== RULE SHEET ========================
S Rule-------------------------------------------------------
  A=(b+m*Y)*Y
  P=b+2*Y*sqrt(m^2+1)
  Q=Cu/n*A^1.666667/P^.6666667*sqrt(So)
  H=Y+(1+Ke)*(Q/A)^2/(2*g)
```

==================== TABLE: ====================

Element	H	Q	Y	A	P
1	.5	7.20249959	.41345271	3.05079882	8.22266188
2	1	22.4896432	.809649058	6.42335783	9.58594582
3	1.5	44.6119571	1.20382244	10.2148565	10.9422689
4	2	73.4836952	1.59787883	14.4400795	12.2981894
5	2.5	109.248342	1.99209262	19.102036	13.6546515
6	3	152.129051	2.38636216	24.1998768	15.0113054
7	3.5	202.380053	2.78051024	29.7312017	16.3675414
8	4	260.266817	3.1743673	35.6929485	17.722776
9	4.5	326.056929	3.56779246	42.0817889	19.0765245
10	5	400.015644	3.96067523	48.8943191	20.4284066
11	5.5	482.403674	4.35293171	56.1271559	21.7781338
12	6	573.47612	4.74449968	63.776986	23.1254918

60.

Determine the size of pipe that should be used to convey a flow rate of Q =16 m^3/s from a reservoir whose water surface elevation is 3 m above the pipe's bottom. The pipe is to have a bottom slope of S_o = 0.00035, and a Manning's n = 0.014. The entrance loss coefficient K_L = 0.1. What will the depth of flow be in this channel? What is the Froude Number associated with this flow.

Given: Q = 16 m^3/s, S_o=.00035, n=.014, K_e=.1, H = 3 m

Find: Diameter, D of pipe

Solution: Requires the solution of the Energy and Mannings Equations for Y and D

$$Q = (C_u/n)A^{5/3}/P^{2/3}S_o^{1/2} \qquad (1)$$

$$H = Y + (1+K_e)(Q/A)^2/(2g) = 3 \qquad (2)$$

Solving gives:

D = 4.311 m ← and Y = 2.865 m ←

The following MathCAD model is designed to solve the above two equations, but since the angle β, the area A and the perimeter P are also variables that depend upon the depth Y and diameter D, actual 5 equation need to be solved for 5 unknowns.

PRB2 60.MCD

Q := 16 So := .00035 β := 2 n := .014 H := 3 Ke := .1 Cu := 1. g := 9.81 D := 4.5 Y := 2.8 A := 10 P := 5

Given

$$\cos(\beta) = 1 - 2\cdot\frac{Y}{D} \qquad A = .25\cdot D^2\cdot(\beta - \cos(\beta)\cdot\sin(\beta)) \qquad P = \beta\cdot D$$

$$Q = \frac{Cu}{n}\cdot\left(\frac{A}{P}\right)^{.66666667}\cdot A\cdot\sqrt{So} \qquad H = Y + (1 + Ke)\cdot\frac{\left(\frac{Q}{A}\right)^2}{2\cdot g}$$

$$\text{Find}(\beta, A, P, Y, D) = \begin{bmatrix} 1.9061 \\ 10.2995 \\ 8.217 \\ 2.8647 \\ 4.311 \end{bmatrix}$$

Using the MATLAB script "e_un.m" (On the CD) produces the following in the command window:

```
>> e_un
Give: g,entrance loss coef,Ke & 0=trap,or 1=circle in [ ] [9.81 .1 1]
  1  D
  3  S
  4  n
  5  Q
  6  H
  7  Y
Give two numbers for 2 unknown variables in [ ]: [1 7]
Give values to variables
1  D (guess) = 4
3  S (known) = .00035
4  n (known) = .014
5  Q (known) = 16
6  H (known) = 3
```

```
7  Y (guess) = 2.8
Solution:
  1 D =    4.3110
  3 S = 0.0003500
  4 n =    0.0140
  5 Q =   16.0000
  6 H =    3.0000
  7 Y =    2.8647
```

If one desired a stage-discharge relationship for this channel, the program of the previous problem could be modified, as given below, to hand a circular channel. Now Q and Y are obtained from the above two equations, with (1) substituted into (2) to give a single equation that can be solved for the normal depth Y_o.

If we want to generate a stage discharge relationship for this circular channel as was done in the previous problem then the same two equations are solved, but now for Y and Q. Program STAGEQC.FOR obtains the needed series of solutions to provide a stage-discharge relationship. The input to this program to obtain the data given below its listing is:
4.311 .014 .00035 9.81 .1 .5 .25 11

Program STAGEQC.FOR

```
      WRITE(*,*)' Give: D,n,So,g,Ke,Start H, DelH & N'
      READ(*,*) D,FN,So,G,FKE,H1,DH,N
      RAD=.5*D
      RAD2=RAD*RAD
      G2=2.*G
      Cu=1.486
      IF(G.LT.20.) Cu=1.
      Y=.95*H1
      Cu=Cu*SQRT(So)
      FKE=FKE+1.
      DO 30 I=1,N
      H=H1+FLOAT(I-1)*DH
      NCT=0
10    COSB=1.-Y/RAD
      BETA=ACOS(COSB)
      A=RAD2*(BETA-SIN(BETA)*COSB)
      Q=Cu/FN*A**1.6666667/(D*BETA)**.6666667
      F=H-Y-FKE*(Q/A)**2/G2
      NCT=NCT+1
      IF(MOD(NCT,2).EQ.0) GO TO 15
      FF=F
      YY=Y
      Y=1.005*Y
      GO TO 10
15    DIF=(Y-YY)*FF/(F-FF)
      Y=YY-DIF
      IF(ABS(DIF).GT. 1.E-6 .AND. NCT.LT.40) GO TO 10
      IF(NCT.EQ.40) WRITE(*,*)' Did not converge H=',H,DIF
      BETA=ACOS(1.-Y/RAD)
      A=RAD2*(BETA-SIN(BETA)*COSB)
      Q=Cu/FN*A**1.6666667/(D*BETA)**.6666667
30    WRITE(3,100) H,Q,Y,Y+(Q/A)**2/G2
100   FORMAT(4F9.3)
      END
```

Output:

H(m)	Q(m**3/s)	Y(m)	E(m)
.500	.539	.480	.498
.750	1.238	.717	.747
1.000	2.210	.953	.996
1.250	3.430	1.189	1.245

1.500	4.871	1.426	1.494
1.750	6.501	1.663	1.742
2.000	8.285	1.901	1.991
2.250	10.181	2.140	2.240
2.500	12.146	2.380	2.490
2.750	14.130	2.622	2.739
3.000	16.076	2.865	2.988

61.

A 3 m diameter pipe with a steep bottom slope is taking water from a reservoir with a depth of 2.8 m above the pipes bottom. What flow rate would you estimate is entering the pipe, and at what depth?

Given: D=3 m diameter pipe on a steep slope is supplied by a reservoir with H = 2.8 m
Wanted: Q and Y

Solution: The energy and critical flow equations need to be solved simultaneously for Q and Y (with this depth being Y_c) or,

$$H = Y + (1+K_e)(Q/A)^2/(2g) = 2.8 \qquad (1)$$

$$Q^2T - gA^3 = 0 \qquad (2)$$

The solution can be obtained by solving the two equations simultaneously, or by solving (2) for $(Q^2/(gA^2) = A/T$ and substituting into (1) to give,

$$H - Y - (1+K_e)A/(2T) = 0$$

Solving gives: Y = 1.950 m 🡐 and Q = 19.78 m^3/s

Using the MATLAB script "e_cr1.m" (on the CD in the back cover) produces the following in the command window:

```
>> e_cr1
Give g 9.81
Give type of channel (0=trap, 1=cir.) 1
Give: D,H,Ke enclose in [ ] [3 2.8 0]
Solution: Y =    1.950, Q =      19.86
```

62.

You are to size a pipe that will take 30 m^3/s from a reservoir whose water surface elevation is 3 meters above the pipe's bottom, and the depth at its entrance in the pipe is not to exceed 2 m. Determine the size of pipe to use. After a very short distance a transition occurs to a trapezoidal section with b = 6 m, and m = 1.2. What is the depth in this trapezoidal section? What must the bottom slope of this trapezoidal channel be if this depth is not to change, and its Manning's roughness coefficient n = 0.013?

Given: Q = 30 m^3/s, H=3 m, Y_1= 2 m, transition from pipe to trapezoidal channel with b_2=6 m, m_2=1.2, n_2=.013
Wanted: D=? so Y_1=2 m. What is depth in trapezoidal channel? What is S_{o2}?

Solution:
Energy $F = H - Y_1 - (Q/A)^2/(2g) = 0$

Solve for the diameter D =4.446 m

However in checking the Froude Number F_r = 1.143 (supercritical). Therefore this is not possible. Solving the Energy and Critical flow equations for Y and D with H=3 m and Q=30 m^3/s gives:

Y = 2.142 m 🡐 and D= 4.391 m 🡐

Therefore the depth Y_1= 2 m is less than what the hydraulics will allow. The depth must be 2.142 m and this will produce critical flow at the entrance if D = 4.391 m.

$F_2 = H - Y_2 - (Q/A_2)^2/(2g) = 0$ in the trapezoidal section with b_2= 6 m
Solving gives: Y_2=2.942 m ← and Mannings equation gives S_{o2}=.000085 ←

63.

Water is to be taken from a reservoir with H = 6 ft using a trapezoidal channel with b = 8 m and m = 1.5. After a very short distance the section changes to that of a pipe with a diameter D = 6 m. The entrance loss coefficient including the transition is $K_L = 0.15$. What will the flow rate be if the roughness of the pipe wall is n = .013 and its slope is $S_o = .0008$? What will the depth be at the entrance of the pipe? What will the depth be at the entrance of the trapezoidal channel?

Given: transition from trapezoidal to circular channel takes water from reservoir with H = 6 m. b_1=8, m_1=1.5, D_2=6 m, S_{o2} = .0008 and entrance loss coefficient K_e=.15

Wanted: Q and Y_{o2}

Solution: Since the upstream trapezoidal channel is very short its losses will be ignored, or stated another way, its slope equal the slope of its energy line. Thus in the downstream circular channel the energy and uniform flow equations can be solved simultaneously.

$$Q = (C_u/n)A^{5/3}/P^{2/3}S_o^{1/2} \qquad (1)$$

$$H = Y + (1+K_e)(Q/A)^2/(2g) = 6 \qquad (2)$$

Giving: Q = 85.826 m^3/s ← and Y_{o2} = 5.398 m ←
With E_{o2} = 5.921 m and F_{ro2} = .375

In the trapezoidal channel
$E_1=5.921 = Y_1 + (Q/A_1)^2/(2g)$ gives: Y_1=5.883 m ← F_{r1} = .141

64.

Repeat the previous problem except the pipe is made of corrugated steel with a roughness of n = .035.
Wanted: Same as previous problem except n=.035

Q= 31.403 m^3/s ← Y_{o2} = 5.928 m ← with F_{ro2}=.077 and E_{o2}=5.991 m

So Y_1= 5.986 m ←

65.

Water enters a trapezoidal channel from a reservoirs whose bottom has a Mannings roughness coefficient of $n_b = 0.035$, and whose sides have a Mannings roughness coefficient $n_s = .012$. The channels bottom width is b = 5 m, and its side slope m = 1.5 and its bottom slope is $S_o = .00085$. If the depth of water in the reservoir is 3.2 m above the channel bottom and the entrance loss coefficient is $K_e = .08$ determine the discharge into the cannel. (Solve this problem by using an equivalent n_{eq} based on weighting according to the fraction of the perimeter each roughness applies for.)

Given: Trapezoidal channel with different n on bottom than sides takes water from a reservoir with H = 3.2 m. b= 5m, m=1.5, S_o=.00085, K_e=.08, n_b=.035, n_s=.012.

Wanted: Flow rate Q and depth Y_o.

Solution: The Energy and Mannings equations need to be solved simultaneously. Now Mannings equation will contain an equivalent $n_{eq} = (bn_b+2Yn_s(m^2+1)^{1/2})/P$

$$Q = (C_u/n_{eq})A^{5/3}/P^{2/3}S_o^{1/2} \quad (1)$$

$$H = Y + (1+K_e)(Q/A)^2/(2g) = 3.2 \quad (2)$$

Solving gives: $Y_o = 2.951$ m ←, $Q = 61.51$ m^3/s ←, n_{eq}=.01935

A TK-Solver Model PRB1_57.TK provides one means for solving these 3 equations.

```
═══════════════ VARIABLE SHEET ══════════════
St Input──── Name─── Output─── Unit───── Comment
             neq     .01935336
   5         b
   .035      nb
             Y       2.9507607
   1.5       m
   .012      ns
             P       15.639119
             A       27.814286
   3.2       H
             Q       61.507165
   19.62     g2
   .00085    So

═══════════════ RULE SHEET ════════════════
S Rule─────────────────────────────────────
* neq=(b*nb+2*Y*sqrt(m^2+1)*ns)/P
* A=(b+m*Y)*Y
* P=b+2*Y*sqrt(m^2+1)
* H=Y+(Q/A)^2/g2
* Q=A^1.6666667*sqrt(So)/(neq*P^.666666)
```

66.

Repeat the previous problem except assume n_b = .012 and n_s = .035. Also solve Problem 65 assuming that $nP^{2/3}$ in Mannings equation should be replace by $(nP^{2/3})_b$ plus $(nP^{2/3})_s$. Why is the depth obtain by determine an equivalent Mannings n different.

Wanted: Repeat previous problem except n_b=.012 and n_s=.035

Solution: Q = 46.399 m^3/s ←, Y_o=3.074 m ←, and n_{eq}=.02785

(b) Use $nP = n_b b+n_s(2Y[m^2+1]^{1/2})$ in Mannings Eq. Now n's are also raised to the 2/3 power - not correct.

```
═══════════════ VARIABLE SHEET ════════════════
St Input───── Name──── Output──── Unit────── Comment
              neq      .02785033
   5          b
   .012       nb
              Y        3.0743294
   1.5        m
   .035       ns
              P        16.084652
              A        29.548898
   3.2        H
              Q        46.398904
   19.62      g2
   .00085     So

═══════════════ RULE SHEET ═════════════════
S Rule──────────────────────────────────────────
  neq=(b*nb+2*Y*sqrt(m^2+1)*ns)/P
  A=(b+m*Y)*Y
  P=b+2*Y*sqrt(m^2+1)
  H=Y+(Q/A)^2/g2
  Q=A^1.6666667*sqrt(So)/(neq*P^.666666)
```

(b) Now both the equivalent n and P are raised to the 2/3 power

```
═══════════════ VARIABLE SHEET ════════════════
St Input───── Name──── Output──── Unit────── Comment
              neq      .02802814
   5          b
   .012       nb
              Y        3.1881035
   1.5        m
   .035       ns
              P        16.494871
              A        31.186524
   3.2        H
              Q        15.066955
   19.62      g2
   .00085     So
═══════════════ RULE SHEET ═══════════════════
S Rule──────────────────────────────────────────
  neq=(b*nb+2*Y*sqrt(m^2+1)*ns)/P
  A=(b+m*Y)*Y
  P=b+2*Y*sqrt(m^2+1)
  H=Y+(Q/A)^2/g2
  Q=A^1.6666667*sqrt(So)/(neq*P)^.666666
```

Reversing the two n's

```
═══════════════ VARIABLE SHEET ═══════════════
St Input───── Name──── Output──── Unit────── Comment
              neq      .01898415
   5          b
   .035       nb
              Y        3.1800537
   1.5        m
   .012       ns
              P        16.465847
```

```
           A       31.069381
   3.2     H
           Q       19.436287
   19.62   g2
   .00085  So
```

67.

Develop the delivery diagrams for a trapezoidal channel with b = 3 m, m = 1.4, n = .013 for bottom slopes varying from S_o = .001 to S_o = .003. The entrance loss coefficient is K_e = .12 and the reservoir head varies from H = 0.2 m to H = 2.2 m.

Given: Trapezoidal with b=3 m, m=1.4, n=.013 and S_o varying from .001 to .003

Wanted: Delivery diagrams with reservoir head varying from 0.2 m to 2.2 m, K_e=.12

Solution: Program CHANNEL was used to obtain the solutions given in the table below for the 5 different bottom slopes, as well as the solution for critical flow at the entrance, and then the following program was used to take these solution results and combine them into the table given below.

Program DELIV.FOR

```
      CHARACTER*10 CH
      REAL Y(51,5),Q(51,5),YC(51),QC(51)
      WRITE(*,*)' Give number of table entries'
      READ(*,*) N
      READ(8,'(A10)') CH
      READ(8,'(A10)') CH
      DO 5 I=1,N
5     READ(8,120) YC(I),QC(I)
120   FORMAT(10X,F10.3,31X,F10.3)
      DO 10 J=1,5
      READ(J+9,'(A10)') CH
      READ(J+9,'(A10)') CH
      DO 10 I=1,N
10    READ(J+9,100) Y(I,J),Q(I,J)
100   FORMAT(10X,F10.3,51X,F10.3)
      H=.2
      DO 20 I=1,N
      WRITE(9,110) H,(Y(I,J),Q(I,J),Q(I,J)**2*(3.+3.*Y(I,J))/(32.2*((
     &3.+1.5*Y(I,J))*Y(I,J))**3),J=1,5),YC(I),QC(I)
110   FORMAT(F5.2,5(F7.3,F8.1,F7.2),F7.3,F8.1)
20    H=H+.1
      END
```

	So=.001			So=.002			So=.0025			So=.0028			So=.003			Critical	
H	Y	Q	Fr2	Y	Q	Fr	Y	Q	Fr2	Y	Q	Fr2	Y	Q	Fr2	Yc	Qc
.20	.172	.4	.09	.152	.4	.18	.139	.4	.24	.139	.4	.24	.136	.5	.27	.131	.4
.30	.255	.8	.11	.223	.9	.20	.204	.9	.28	.204	.9	.28	.199	.9	.30	.198	.9
.40	.338	1.2	.11	.294	1.4	.22	.267	1.4	.30	.267	1.4	.30	.261	1.4	.32	.266	1.4
.50	.420	1.8	.12	.365	2.0	.23	.330	2.0	.32	.330	2.0	.32	.323	2.0	.34	.336	2.0
.60	.503	2.5	.12	.435	2.7	.24	.393	2.7	.33	.393	2.7	.33	.384	2.7	.35	.406	2.7
.70	.585	3.2	.13	.505	3.5	.25	.456	3.5	.34	.456	3.5	.34	.445	3.4	.36	.476	3.5
.80	.668	4.1	.13	.575	4.4	.26	.519	4.3	.35	.519	4.3	.35	.507	4.3	.37	.548	4.4
.90	.750	5.0	.14	.646	5.4	.26	.582	5.3	.36	.582	5.3	.36	.568	5.3	.38	.619	5.4
1.00	.833	6.0	.14	.716	6.5	.27	.644	6.3	.37	.644	6.3	.37	.629	6.3	.39	.692	6.5
1.10	.915	7.2	.14	.786	7.7	.28	.707	7.5	.38	.707	7.5	.38	.690	7.4	.40	.765	7.7
1.20	.998	8.4	.15	.856	9.0	.28	.770	8.7	.38	.770	8.7	.38	.751	8.7	.41	.838	8.9
1.30	1.080	9.8	.15	.926	10.4	.29	.833	10.1	.39	.833	10.1	.39	.812	10.0	.42	.911	10.3
1.40	1.163	11.2	.15	.997	11.9	.29	.895	11.5	.40	.895	11.5	.40	.873	11.4	.43	.985	11.9
1.50	1.245	12.8	.15	1.067	13.5	.29	.958	13.1	.40	.958	13.1	.40	.934	12.9	.43	1.059	13.4
1.60	1.328	14.4	.15	1.137	15.2	.30	1.021	14.7	.41	1.021	14.7	.41	.995	14.5	.44	1.134	15.2
1.70	1.410	16.2	.16	1.207	17.0	.30	1.083	16.4	.42	1.083	16.4	.42	1.057	16.2	.44	1.208	17.0
1.80	1.492	18.1	.16	1.277	18.9	.31	1.146	18.2	.42	1.146	18.2	.42	1.118	18.0	.45	1.283	18.9
1.90	1.574	20.1	.16	1.347	21.0	.31	1.209	20.2	.42	1.209	20.2	.42	1.179	19.9	.45	1.358	21.0
2.00	1.657	22.2	.16	1.417	23.1	.31	1.271	22.2	.43	1.271	22.2	.43	1.239	21.9	.46	1.434	23.2
2.10	1.739	24.4	.16	1.487	25.4	.32	1.334	24.3	.43	1.334	24.3	.43	1.300	24.0	.46	1.509	25.5
2.20	1.821	26.8	.17	1.557	27.8	.32	1.396	26.6	.44	1.396	26.6	.44	1.361	26.2	.47	1.585	27.9

To use MathCAD, for example, to create a delivery diagram the **Given-Find** block might be utilized as shown in the model below in which: (1) the flow rate Q, (2) the depth Y, and (3) the area A are solved for any reservoir head H but assigning the function xx_j to **Find.** Thus each xx contains Q, Y and A, and since it is any array with subscripts 0 to 10 the delivery diagram is printed as shown. In the solution table $(xx_j)_0$ is the flow rate Q_j , $(xx_j)_1$ is the depth of flow in the channel Y_j and $(xx_j)_2$ is the area A_j associated with this normal depth.

PRB2_67.MCD

$b := 3 \quad m := 1.4 \quad Ke := .12 \quad g := 9.81 \quad g2 := 2 \cdot g \quad Ke1 := 1 + Ke \quad n := .013 \quad Cu := 1 \quad Cun := \frac{Cu}{n}$

$So := .001 \quad SoS := \sqrt{So} \quad sm2 := 2 \cdot \sqrt{m^2 + 1}$

$i := 0..10 \quad H_i := .2 \cdot (i + 1) \quad Y_i := .95 \cdot H_i \quad A_i := (b + m \cdot Y_i) \cdot Y_i \quad Q_i := A_i \cdot \sqrt{g2 \cdot (H_i - Y_i)}$

$Qa := Q_0 \quad Ya := Y_0 \quad Aa := A_0$

$$\text{Given} \quad Aa = (b + m \cdot Ya) \cdot Ya \qquad Ha = Ya + Ke1 \cdot \frac{\left(\frac{Qa}{Aa}\right)^2}{g2} \qquad Qa = Cun \cdot \frac{Aa^{1.6666667}}{(b + sm2 \cdot Ya)^{.6666667}} \cdot SoS$$

$x(Ha) := \text{Find}(Qa, Ya, Aa)$

$j := 0..10 \quad xx_j := x(H_j)$

H_j	$(xx_j)_0$	$(xx_j)_1$	$(xx_j)_2$
0.2	0.391	0.172	0.557
0.4	1.226	0.338	1.172
0.6	2.432	0.503	1.861
0.8	4.002	0.667	2.626
1	5.941	0.832	3.467
1.2	8.261	0.997	4.385
1.4	10.975	1.162	5.378
1.6	14.096	1.327	6.447
1.8	17.637	1.492	7.592
2	21.614	1.656	8.811
2.2	26.041	1.821	10.104

The above solution applies for $S_o = .001$ since this is the value of the bottom slope that is assigned. To solve the problem for a different bottom slope one only needs to change its assigned value. For example if S_o=.002 then the following is obtained.

$b := 3 \quad m := 1.4 \quad Ke := .12 \quad g := 9.81 \quad g2 := 2 \cdot g \quad Ke1 := 1 + Ke \quad n := .013 \quad Cu := 1 \quad Cun := \frac{Cu}{n}$

$So := .002 \quad SoS := \sqrt{So} \quad sm2 := 2 \cdot \sqrt{m^2 + 1}$

$i := 0..10 \quad H_i := .2 \cdot (i + 1) \quad Y_i := .95 \cdot H_i \quad A_i := (b + m \cdot Y_i) \cdot Y_i \quad Q_i := A_i \cdot \sqrt{g2 \cdot (H_i - Y_i)} \quad Qa := Q_0 \quad Ya := Y_0 \quad Aa := A_0$

$$\text{Given} \quad Aa = (b + m \cdot Ya) \cdot Ya \qquad Ha = Ya + Ke1 \cdot \frac{\left(\frac{Qa}{Aa}\right)^2}{g2} \qquad Qa = Cun \cdot \frac{Aa^{1.6666667}}{(b + sm2 \cdot Ya)^{.6666667}} \cdot SoS$$

$x(Ha) := \text{Find}(Qa, Ya, Aa)$

$j := 0..10 \quad xx_j := x(H_j)$

H_j	$(xx_j)_0$	$(xx_j)_1$	$(xx_j)_2$
0.2	0.448	0.152	0.487
0.4	1.367	0.294	1.002
0.6	2.67	0.434	1.567
0.8	4.344	0.574	2.186
1	6.392	0.715	2.86
1.2	8.822	0.855	3.589
1.4	11.643	0.995	4.374
1.6	14.867	1.136	5.214
1.8	18.504	1.276	6.108
2	22.566	1.416	7.057
2.2	27.064	1.556	8.059

Obtaining solutions over a range is relatively easy with TK-Solver. About all that is required is that the variable be given the status of L (for list), and those variables that needed to be solved from implicit equations, or a system of equations, be also given the status G (for guess). In solving this problem using TK-Solver the variable H is filled in using the **Constant Increment** option, and guess are provided for the unknowns: Y, Q and A. To obtain the solution the F10 key is pressed for list solve. The model that is given is designed obtain a solution for S_o =.001, but all that is needed to solve for a different bottom slope is to change the value of So and press the F10 key again.

PRB2_67.MCD

```
==================== VARIABLE SHEET ====================
St Input───── Name──── Output──── Unit────── Comment─────────
   3          b
   1          Cu
   .013       n
   1.4        m
   .001       So
   .12        Ke
   9.81       g
              g2
              Cun
              sm2
              SoS
              Ke1
L  0          H
LG 0          Y
LG 0          Q
LG 0          A
L             E
L             Fr

==================== RULE SHEET ========================
S Rule─────────────────────────────────────────────────
  Cun=Cu/n
  sm2=2*sqrt(m^2+1)
  Ke1=Ke+1
  g2=2*g
  SoS=sqrt(So)
  A=(b+m*Y)*Y
  H=Y+Ke1*(Q/A)^2/g2
  Q=Cun*A^1.666667/(b+sm2*Y)^.6666667*SoS
  E=Y+(Q/A)^2/g2
  Fr=sqrt(Q^2*(b+2*m*Y)/(g*A))
==================== TABLE:  ===========================
Title:      Delivery Diagram-Prob. 2-67
Element H────────── Q────────── Y────────── A────────── E────────── Fr─────────
1       .2         .391031388 .171858338 .556924419 .196984822 .312135872
```

2	.4	1.2256238	.337618723	1.17243713	.393316292	.71782685
3	.6	2.43164745	.502575526	1.8613416	.589561663	1.19463438
4	.8	4.00154624	.667453693	2.62605328	.78579861	1.73962514
5	1	5.94119014	.832381938	3.46714938	.982040922	2.35203374
6	1.2	8.26136087	.997342216	4.38459474	1.17828667	3.03170935
7	1.4	10.974911	1.16228154	5.37810235	1.37453017	3.77871722
8	1.6	14.0955708	1.32714481	6.44727309	1.57076551	4.59320313
9	1.8	17.6373825	1.49188421	7.59165853	1.76698759	5.47534026
10	2	21.6144114	1.65646105	8.81079165	1.96319226	6.4253066
11	2.2	26.0405945	1.82084506	10.1042026	2.15937626	7.44327513

After changing So to .002 and pressing F10 the following are the values in the Table Sheet, i.e. apply for S_o = .002.

═══════════ TABLE: ═══════════

Title: Delivery Diagram-Prob. 2-67

Element	H	Q	Y	A	E	Fr
1	.2	.448305802	.151666841	.487204484	.194821447	.379483941
2	.4	1.36713342	.293712449	1.00191115	.388612048	.852569055
3	.6	2.66973227	.434249187	1.56674886	.582240984	1.39822978
4	.8	4.34388741	.57449985	2.18556966	.77583927	2.01393857
5	1	6.39201823	.714770205	2.85956564	.969439665	2.69896576
6	1.2	8.82180841	.855107439	3.58901454	1.16304723	3.45305178
7	1.4	11.6431514	.995476893	4.37379462	1.35665824	4.27608342
8	1.6	14.8668844	1.13582133	5.21359009	1.55026657	5.16799147
9	1.8	18.5041932	1.27608231	6.10798739	1.74386596	6.12871474
10	2	22.5663104	1.41620767	7.05652487	1.93745082	7.15818749
11	2.2	27.0643569	1.55615354	8.05871998	2.13101645	8.25633564

After changing So to .003 and pressing F10 the following are the values in the Table Sheet, i.e. apply for S_o = .003.

═══════════ TABLE: ═══════════

Title: Delivery Diagram-Prob. 2-67

Element	H	Q	Y	A	E	Fr
1	.2	.459415655	.13637388	.43515861	.193182916	.408906925
2	.4	1.37067337	.261031497	.87848691	.385110518	.90185754
3	.6	2.64179001	.383635299	1.35695236	.576818068	1.46150914
4	.8	4.25766098	.505717969	1.87520484	.768469782	2.08606221
5	1	6.2178868	.627732135	2.43486309	.960114157	2.77502741
6	1.2	8.52735498	.749802729	3.03649397	1.15176458	3.52821625
7	1.4	11.1931926	.871936188	3.68019036	1.34342173	4.34550755
8	1.6	14.2235074	.994095583	4.36580319	1.53508167	5.22678068
9	1.8	17.626796	1.11623083	5.09305223	1.72673902	6.17189714
10	2	21.4116449	1.23829125	5.86158504	1.91838835	7.18069798
11	2.2	25.5865727	1.36023066	6.67101043	2.11002471	8.25300628

The maximum flow rate that can be delivered to a channel from a reservoir is limited to creating critical flow at the entrance of the channel. If the entrance loss is ignored the the relationship between the head H in the reservoir and the critical depth at the beginning of the reservoir, and the flow rate into the channel are governed by the critical flow and energy equations, or
$Q^2T/(gA^3) = 1$ and $E = Y + Q^2/(2gA^2)$, in which E is the reservoir H, but to be more general we will use E. From the first of these equations the velocity head is given by $Q^2/(2gA^2) = A/(2T)$, and substituting this result into the second equation results in:

$$E_c = Y_c + A/(2T)$$

in which the c subscript has been added to denote that these are critical values, e.g. the limiting values that result in critical flow. It is important to note that the last equation does not involve the critical flow rate. Q can be determined later by the above equation that notes that the velocity head equals A/(2T). Furthermore, since the last equation does not include a term for the acceleration of gravity, it applies equally well for SI or ES units, or in other words if E_c is fixed and the equation is solved for Y_c this solution will give the critical depth in meter if E_c is given in meters, or in feet if E_c is given in feet. Solving for Y_c with E_c given result in an implicit equation, whereas E_c can be solved directly given a value of Y_c. The former approach would only be used if for some reason the table of value for Y_c versus E_c needed to have a fixed increment for E_c.

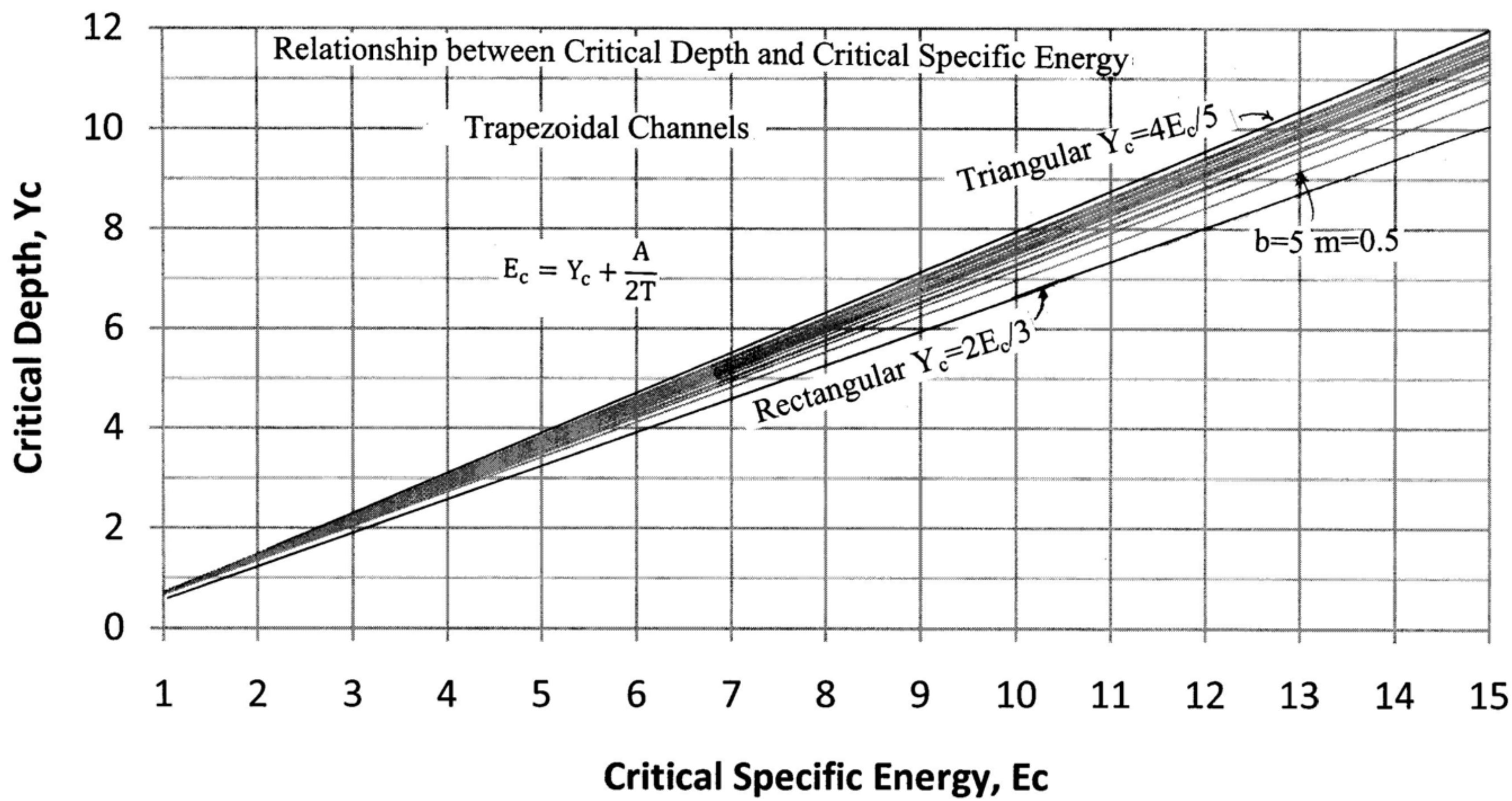
The following MATLAB script (YcversusE.m) obtains this type of a solution defining the relationship between critical depth and critical specific energy for a trapezoidal channel.

```
mm=[.5,1,1.5,2.,2.5];
bb=[0,1,5,10,15,20];
opt=optimset('display','off');
fid=fopen('Critdepth.txt','w');
   for i=1:5
       m=mm(i);
    for j=1:6
        b=bb(j);
        for E=1:15
        Yc=.667*E;
        Yc=fsolve('critdepth1',Yc,opt,E,m,b);
        fprintf(fid,'%4.1f %4.1f %4.1f %9.4f\n',m,b,E,Yc);
        end
        fprintf(fid,'\n');
    end
   end
fclose(fid);
 function f=critdepth1(Yc,E,m,b)
f=E-Yc-((b+m*Yc)*Yc)/(2*(b+2*m*Yc));
```

m	b	E_c	Y_c
0.5	5.0	1.0	0.6802
0.5	5.0	2.0	1.3833
0.5	5.0	3.0	2.1038
0.5	5.0	4.0	2.8379
0.5	5.0	5.0	3.5826
0.5	5.0	6.0	4.3356
0.5	5.0	7.0	5.0953
0.5	5.0	8.0	5.8604
0.5	5.0	9.0	6.6299
0.5	5.0	10.0	7.4031
0.5	5.0	11.0	8.1794
0.5	5.0	12.0	8.9582
0.5	5.0	13.0	9.7392
0.5	5.0	14.0	10.5221
0.5	5.0	15.0	11.3066
0.5	10.0	1.0	0.6738
0.5	10.0	2.0	1.3605
0.5	10.0	3.0	2.0586
0.5	10.0	4.0	2.7666
0.5	10.0	5.0	3.4833
0.5	10.0	6.0	4.2077
0.5	10.0	7.0	4.9388
0.5	10.0	8.0	5.6758
0.5	10.0	9.0	6.4182
0.5	10.0	10.0	7.1652
0.5	10.0	11.0	7.9163
0.5	10.0	12.0	8.6712
0.5	10.0	13.0	9.4294
0.5	10.0	14.0	10.1906
0.5	10.0	15.0	10.9545
0.5	20.0	1.0	0.6703
0.5	20.0	2.0	1.3475
0.5	20.0	3.0	2.0312
0.5	20.0	4.0	2.7210
0.5	20.0	5.0	3.4164
0.5	20.0	6.0	4.1171
0.5	20.0	7.0	4.8228
0.5	20.0	8.0	5.5332
0.5	20.0	9.0	6.2479
0.5	20.0	10.0	6.9666
0.5	20.0	11.0	7.6892
0.5	20.0	12.0	8.4154
0.5	20.0	13.0	9.1449
0.5	20.0	14.0	9.8776
0.5	20.0	15.0	10.6132
1.0	5.0	1.0	0.6916
1.0	5.0	2.0	1.4190
1.0	5.0	3.0	2.1678
1.0	5.0	4.0	2.9302
1.0	5.0	5.0	3.7016
1.0	5.0	6.0	4.4791
1.0	5.0	7.0	5.2611
1.0	5.0	8.0	6.0463
1.0	5.0	9.0	6.8339
1.0	5.0	10.0	7.6235
1.0	5.0	11.0	8.4145
1.0	5.0	12.0	9.2068
1.0	5.0	13.0	10.0000
1.0	5.0	14.0	10.7940
1.0	5.0	15.0	11.5887
1.0	10.0	1.0	0.6802
1.0	10.0	2.0	1.3833
1.0	10.0	3.0	2.1038
1.0	10.0	4.0	2.8379
1.0	10.0	5.0	3.5826
1.0	10.0	6.0	4.3356
1.0	10.0	7.0	5.0953
1.0	10.0	8.0	5.8604
1.0	10.0	9.0	6.6299
1.0	10.0	10.0	7.4031
1.0	10.0	11.0	8.1794
1.0	10.0	12.0	8.9582
1.0	10.0	13.0	9.7392
1.0	10.0	14.0	10.5221
1.0	10.0	15.0	11.3066
1.0	15.0	1.0	0.6760
1.0	15.0	2.0	1.3685
1.0	15.0	3.0	2.0749
1.0	15.0	4.0	2.7930
1.0	15.0	5.0	3.5208
1.0	15.0	6.0	4.2569
1.0	15.0	7.0	5.0000
1.0	15.0	8.0	5.7491
1.0	15.0	9.0	6.5034
1.0	15.0	10.0	7.2621
1.0	15.0	11.0	8.0247
1.0	15.0	12.0	8.7906
1.0	15.0	13.0	9.5595
1.0	15.0	14.0	10.3309
1.0	15.0	15.0	11.1047
1.0	20.0	1.0	0.6738
1.0	20.0	2.0	1.3605
1.0	20.0	3.0	2.0586
1.0	20.0	4.0	2.7666
1.0	20.0	5.0	3.4833
1.0	20.0	6.0	4.2077
1.0	20.0	7.0	4.9388
1.0	20.0	8.0	5.6758
1.0	20.0	9.0	6.4182
1.0	20.0	10.0	7.1652
1.0	20.0	11.0	7.9163
1.0	20.0	12.0	8.6712
1.0	20.0	13.0	9.4294
1.0	20.0	14.0	10.1906
1.0	20.0	15.0	10.9545
1.5	5.0	1.0	0.7013
1.5	5.0	2.0	1.4452
1.5	5.0	3.0	2.2100
1.5	5.0	4.0	2.9861
1.5	5.0	5.0	3.7689
1.5	5.0	6.0	4.5559
1.5	5.0	7.0	5.3459
1.5	5.0	8.0	6.1379
1.5	5.0	9.0	6.9313
1.5	5.0	10.0	7.7258
1.5	5.0	11.0	8.5212
1.5	5.0	12.0	9.3172
1.5	5.0	13.0	10.1138
1.5	5.0	14.0	10.9108
1.5	5.0	15.0	11.7082
1.5	10.0	1.0	0.6862
1.5	10.0	2.0	1.4026
1.5	10.0	3.0	2.1394
1.5	10.0	4.0	2.8904
1.5	10.0	5.0	3.6515
1.5	10.0	6.0	4.4200
1.5	10.0	7.0	5.1939
1.5	10.0	8.0	5.9721
1.5	10.0	9.0	6.7536
1.5	10.0	10.0	7.5377
1.5	10.0	11.0	8.3240
1.5	10.0	12.0	9.1119
1.5	10.0	13.0	9.9012
1.5	10.0	14.0	10.6918
1.5	10.0	15.0	11.4833
1.5	15.0	1.0	0.6802
1.5	15.0	2.0	1.3833
1.5	15.0	3.0	2.1038
1.5	15.0	4.0	2.8379
1.5	15.0	5.0	3.5826
1.5	15.0	6.0	4.3356
1.5	15.0	7.0	5.0953
1.5	15.0	8.0	5.8604
1.5	15.0	9.0	6.6299
1.5	15.0	10.0	7.4031
1.5	15.0	11.0	8.1794

1.5	15.0	12.0	8.9582
1.5	15.0	13.0	9.7392
1.5	15.0	14.0	10.5221
1.5	15.0	15.0	11.3066
1.5	20.0	1.0	0.6771
1.5	20.0	2.0	1.3724
1.5	20.0	3.0	2.0826
1.5	20.0	4.0	2.8051
1.5	20.0	5.0	3.5377
1.5	20.0	6.0	4.2788
1.5	20.0	7.0	5.0268
1.5	20.0	8.0	5.7808
1.5	20.0	9.0	6.5397
1.5	20.0	10.0	7.3030
1.5	20.0	11.0	8.0699
1.5	20.0	12.0	8.8399
1.5	20.0	13.0	9.6127
1.5	20.0	14.0	10.3879
1.5	20.0	15.0	11.1652
2.0	5.0	1.0	0.7095
2.0	5.0	2.0	1.4651
2.0	5.0	3.0	2.2396
2.0	5.0	4.0	3.0231
2.0	5.0	5.0	3.8117
2.0	5.0	6.0	4.6034
2.0	5.0	7.0	5.3970
2.0	5.0	8.0	6.1920
2.0	5.0	9.0	6.9879
2.0	5.0	10.0	7.7846
2.0	5.0	11.0	8.5818
2.0	5.0	12.0	9.3794
2.0	5.0	13.0	10.1773
2.0	5.0	14.0	10.9756
2.0	5.0	15.0	11.7740
2.0	10.0	1.0	0.6916
2.0	10.0	2.0	1.4190
2.0	10.0	3.0	2.1678
2.0	10.0	4.0	2.9302
2.0	10.0	5.0	3.7016
2.0	10.0	6.0	4.4791
2.0	10.0	7.0	5.2611
2.0	10.0	8.0	6.0463
2.0	10.0	9.0	6.8339
2.0	10.0	10.0	7.6235
2.0	10.0	11.0	8.4145
2.0	10.0	12.0	9.2068
2.0	10.0	13.0	10.0000
2.0	10.0	14.0	10.7940
2.0	10.0	15.0	11.5887
2.0	15.0	1.0	0.6843
2.0	15.0	2.0	1.3965
2.0	15.0	3.0	2.1284
2.0	15.0	4.0	2.8746
2.0	15.0	5.0	3.6310
2.0	15.0	6.0	4.3953
2.0	15.0	7.0	5.1655
2.0	15.0	8.0	5.9402
2.0	15.0	9.0	6.7187
2.0	15.0	10.0	7.5000
2.0	15.0	11.0	8.2837
2.0	15.0	12.0	9.0694
2.0	15.0	13.0	9.8567
2.0	15.0	14.0	10.6454
2.0	15.0	15.0	11.4352
2.0	20.0	1.0	0.6802
2.0	20.0	2.0	1.3833
2.0	20.0	3.0	2.1038
2.0	20.0	4.0	2.8379
2.0	20.0	5.0	3.5826
2.0	20.0	6.0	4.3356
2.0	20.0	7.0	5.0953
2.0	20.0	8.0	5.8604
2.0	20.0	9.0	6.6299
2.0	20.0	10.0	7.4031
2.0	20.0	11.0	8.1794
2.0	20.0	12.0	8.9582
2.0	20.0	13.0	9.7392
2.0	20.0	14.0	10.5221
2.0	20.0	15.0	11.3066
2.5	5.0	1.0	0.7165
2.5	5.0	2.0	1.4806
2.5	5.0	3.0	2.2613
2.5	5.0	4.0	3.0494
2.5	5.0	5.0	3.8413
2.5	5.0	6.0	4.6355
2.5	5.0	7.0	5.4311
2.5	5.0	8.0	6.2277
2.5	5.0	9.0	7.0249
2.5	5.0	10.0	7.8227
2.5	5.0	11.0	8.6208
2.5	5.0	12.0	9.4192
2.5	5.0	13.0	10.2178
2.5	5.0	14.0	11.0166
2.5	5.0	15.0	11.8156
2.5	10.0	1.0	0.6967
2.5	10.0	2.0	1.4330
2.5	10.0	3.0	2.1909
2.5	10.0	4.0	2.9612
2.5	10.0	5.0	3.7394
2.5	10.0	6.0	4.5226
2.5	10.0	7.0	5.3094
2.5	10.0	8.0	6.0988
2.5	10.0	9.0	6.8900
2.5	10.0	10.0	7.6826
2.5	10.0	11.0	8.4764
2.5	10.0	12.0	9.2710
2.5	10.0	13.0	10.0663
2.5	10.0	14.0	10.8622
2.5	10.0	15.0	11.6586
2.5	15.0	1.0	0.6881
2.5	15.0	2.0	1.4083
2.5	15.0	3.0	2.1495
2.5	15.0	4.0	2.9048
2.5	15.0	5.0	3.6699
2.5	15.0	6.0	4.4419
2.5	15.0	7.0	5.2190
2.5	15.0	8.0	6.0000
2.5	15.0	9.0	6.7840
2.5	15.0	10.0	7.5703
2.5	15.0	11.0	8.3585
2.5	15.0	12.0	9.1482
2.5	15.0	13.0	9.9391
2.5	15.0	14.0	10.7311
2.5	15.0	15.0	11.5239
2.5	20.0	1.0	0.6833
2.5	20.0	2.0	1.3933
2.5	20.0	3.0	2.1226
2.5	20.0	4.0	2.8661
2.5	20.0	5.0	3.6200
2.5	20.0	6.0	4.3818
2.5	20.0	7.0	5.1497
2.5	20.0	8.0	5.9225
2.5	20.0	9.0	6.6991
2.5	20.0	10.0	7.4788
2.5	20.0	11.0	8.2610
2.5	20.0	12.0	9.0453
2.5	20.0	13.0	9.8314
2.5	20.0	14.0	10.6189
2.5	20.0	15.0	11.4077

Rectangular and triangular channels represent special trapezoidal channels; the former with $m = 0$ and the later with $b = 0$. For a rectangular channel $A/(2T) = (b+mY)Y/[2(b+2mY)] = bY/2b = Y/2$. Thus the critical specific energy $E_c = Y_c + Y_c/2 = 3Y_c/2$, or $Y_c = (2/3)E_c$. For a triangular channel with $b = 0$, $A/(2T) = mY^2/(4mY) = Y/4$.


Relationship between Critical Depth and Critical Specific Energy
Trapezoidal Channels
$E_c = Y_c + \frac{A}{2T}$
Triangular $Y_c=4E_c/5$
b=5 m=0.5
Rectangular $Y_c=2E_c/3$
Critical Depth, Yc
Critical Specific Energy, Ec
12
10
8
6
4
2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15

The critical specific energy then becomes $E_c = Y_c + Y_c/4 = (5/4)Y_c$, Or $Y_c = (4/5)E_c$. The results from the above solution, as well as the special relationships for Rectangular and Triangular Channels have been plotted on the graph above. Note how the relationship for various bottom widths b, and side slopes m, are confined between the simple straight line relationship for a rectangular and triangular channel. This certainly provides a good guide in giving starting values to software that solves for Y_c or a given critical value of specific energy.

It is interesting to non-dimensionlize the critical equation $E_c = Y_c + A/(2T)$ for trapezoidal channel by dividing by the bottom width b. Let $e = E_c/b$ and $y_c = Y_c/b$, then

$$e=y_c+ [(1+my_c)y_c]/[2(1+2my_c)]$$

A plot of this equation is provided below.

Let's also investigate the relationship between the critical depth and critical specific energy for a circular channel. The basic equation does not change, e.g. $E_c = Y_c + A/(2T)$. As you well know for a circular channel $A = D_2/4(\beta\text{-}\sin\beta\cos\beta)$ and $T = D\sin\beta$, in which $\beta=\cos^{-1}(1\text{-}2Y_c/D)$. Let's non-dimensionalize the critical relationship between Y_c and E_c by dividing by the diameter D. Letting $e = E_c/D$, and $y_c = Y_c/D$, the critical relationship becomes:

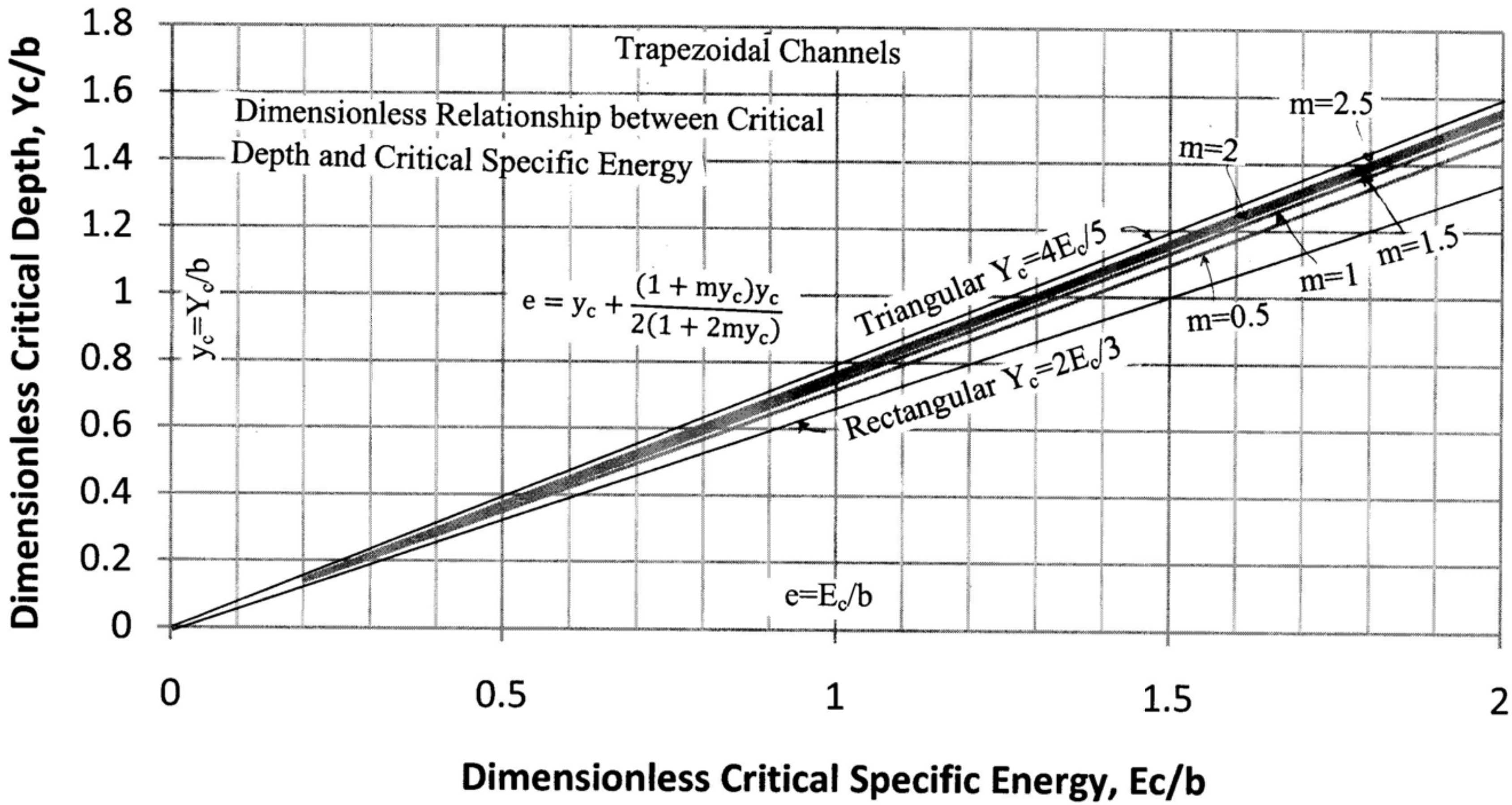
$$e = y_c + [(\beta\text{-}\sin\beta\cos\beta)]/[8(\sin\beta)], \text{ and as given above } \beta = \cos^{-1}(1\text{-}y_c)$$

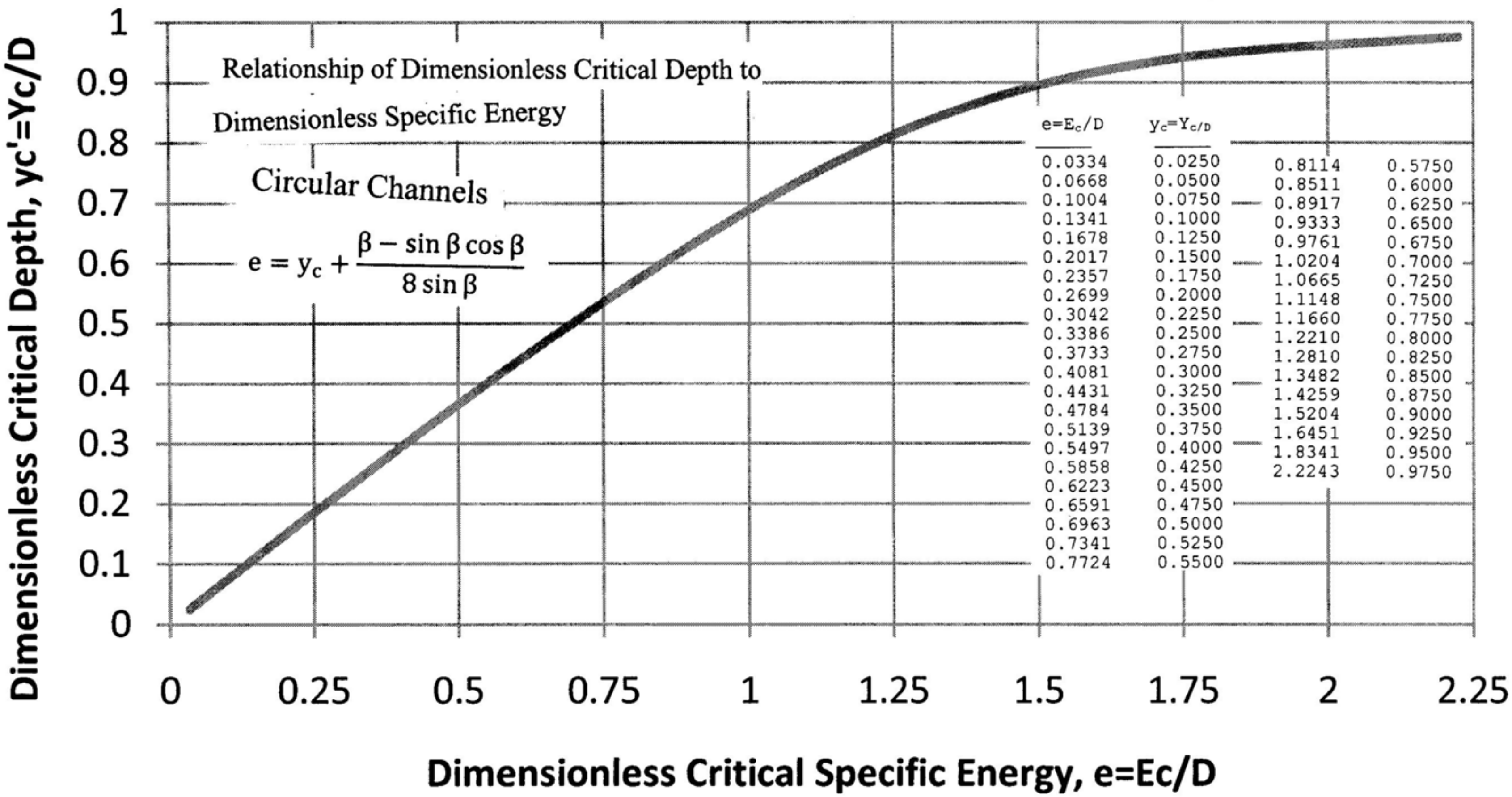
This equation shows that if the dimensionless critical depth y_c is plotted against the dimensionless specific energy e, that a single line will occur regardless of the size of the circular channel. This plot is given below the dimensionless graph for trapezoidal channels.

In obtaining the data to plot the dimensionless figure for circular channel values for e were solved from the above explicit equation given values for the dimensionless depth $y_c = Y_c/D$. The MATLAB script below solves for y_c given e. Note that as e becomes larger than 1.45 that y_c suddenly becomes larger than 1. Of course this is an incorrect root that the MATLAB built-in function "fsolve" is finding. On the graph the data used to plot it are given that were obtained by solving the above equation explicitly for e with given values of y_c.

```
opt=optimset('display','off');
fid=fopen('Critdepthd.txt','w');
   for E=.05:.05:1.75
        Yc=.667*E;
        Yc=fsolve('critdepthh',Yc,opt,E);
        fprintf(fid,'%5.2f %9.4f\n',E,Yc);
   end
        fprintf(fid,'\n');
   fclose(fid);
function f=critdepthh(Yc,E)
cosb=1-2*Yc; beta=acos(cosb);
f=E-Yc-(beta-sin(beta)*cosb)/(8*sin(beta));
```

e	y_c	e	y_c
0.05	0.0374	1.00	0.6886
0.10	0.0747	1.05	0.7162
0.15	0.1118	1.10	0.7425
0.20	0.1487	1.15	0.7674
0.25	0.1855	1.20	0.7907
0.30	0.2220	1.25	0.8124
0.35	0.2582	1.30	0.8324
0.40	0.2942	1.35	0.8506
0.45	0.3299	1.40	0.8671
0.50	0.3652	1.45	0.8819
0.55	0.4002	1.50	1.4403
0.60	0.4348	1.55	1.4097
0.65	0.4689	1.60	1.5567
0.70	0.5024	1.65	1.5635
0.75	0.5354	1.70	1.5133
0.80	0.5677	1.75	1.5345
0.85	0.5993		
0.90	0.6301		
0.95	0.6599		


Trapezoidal Channels
Dimensionless Relationship between Critical
Depth and Critical Specific Energy
$e = y_c + \frac{(1 + my_c)y_c}{2(1 + 2my_c)}$
$y_c = Y_c/b$
$e = E_c/b$
Triangular $Y_c = 4E_c/5$
Rectangular $Y_c = 2E_c/3$
m=2.5
m=2
m=1.5
m=1
m=0.5
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
0.5
1
1.5
2
Dimensionless Critical Depth, Yc/b
Dimensionless Critical Specific Energy, Ec/b



$e=E_c/D$	$y_c=Y_{c/D}$		
0.0334	0.0250	0.8114	0.5750
0.0668	0.0500	0.8511	0.6000
0.1004	0.0750	0.8917	0.6250
0.1341	0.1000	0.9333	0.6500
0.1678	0.1250	0.9761	0.6750
0.2017	0.1500	1.0204	0.7000
0.2357	0.1750	1.0665	0.7250
0.2699	0.2000	1.1148	0.7500
0.3042	0.2250	1.1660	0.7750
0.3386	0.2500	1.2210	0.8000
0.3733	0.2750	1.2810	0.8250
0.4081	0.3000	1.3482	0.8500
0.4431	0.3250	1.4259	0.8750
0.4784	0.3500	1.5204	0.9000
0.5139	0.3750	1.6451	0.9250
0.5497	0.4000	1.8341	0.9500
0.5858	0.4250	2.2243	0.9750
0.6223	0.4500		
0.6591	0.4750		
0.6963	0.5000		
0.7341	0.5250		
0.7724	0.5500		

68.

A 10 ft diameter pipe that has a Mannings n = .012 takes water from a reservoir whose head varies between H = 1 ft and H = 10 ft. The entrance loss coefficient is K_e = .12. Obtain delivery diagrams for this long channel for bottom slopes S_o = .001, .002, .0025, .0028 and .003.

Solution: Program CHANNEL was used to obtain the solutions given in the tble below for the 5 different bottom slopes, as well as the solution for critical flow at the entrance, and then the following program was used to take these solution results and generate the table given below.
Program DELIVC.FOR

```
      CHARACTER*10 CH
      REAL Y(51,5),Q(51,5),YC(51),QC(51),FR2(5)
      WRITE(*,*)' Give number of table entries'
      READ(*,*) N
      READ(8,'(A10)') CH
      READ(8,'(A10)') CH
      DO 5 I=1,N
5     READ(8,120) YC(I),QC(I)
120   FORMAT(10X,F10.3,21X,F10.3)
      DO 10 J=1,5
      READ(J+9,'(A10)') CH
      READ(J+9,'(A10)') CH
      DO 10 I=1,N
10    READ(J+9,100) Y(I,J),Q(I,J)
100   FORMAT(10X,F10.3,41X,F11.3)
      H=1.
      DO 20 I=1,N
      DO 18 J=1,5
      BETA=ACOS(1.-2.*Y(I,J)/10.)
18    FR2(J)=Q(I,J)**2*(10.*SIN(BETA))/(32.2*(25.*(BETA-COS(BETA)*SIN(
     &BETA)))**3)
      WRITE(9,110) H,(Y(I,J),Q(I,J),FR2(J),J=1,5),YC(I),QC(I)
110   FORMAT(F4.1,5(F6.3,F6.1,F5.2),F6.3,F6.1)
20    H=H+.2
      END
```

	So=.001			So=.002			So=.0025			So=.0028			So=.003			Critical	
H	Y	Q	Fr2	Y	Q	Fr	Y	Q	Fr2	Y	Q	Fr2	Y	Q	Fr2	Yc	Qc
1.0	.893	8.6	.31	.810	9.9	.61	.776	10.1	.75	.757	10.1	.84	.745	10.1	.89	.725	10.1
1.2	1.066	12.4	.33	.963	14.2	.64	.921	14.4	.79	.897	14.5	.88	.882	14.5	.94	.869	14.4
1.4	1.238	17.0	.34	1.115	19.3	.66	1.064	19.6	.82	1.036	19.6	.91	1.019	19.6	.97	1.013	19.6
1.6	1.409	22.3	.35	1.266	25.2	.68	1.207	25.5	.84	1.175	25.5	.94	1.154	25.4	1.00	1.157	25.4
1.8	1.580	28.2	.36	1.416	31.8	.70	1.349	32.1	.86	1.312	32.1	.96	1.288	32.0	1.03	1.300	32.0
2.0	1.751	34.8	.36	1.566	39.1	.71	1.490	39.5	.88	1.448	39.4	.98	1.422	39.2	1.05	1.443	39.4
2.2	1.922	42.1	.37	1.715	47.2	.72	1.630	47.5	.90	1.584	47.4	1.00	1.555	47.2	1.06	1.586	47.4
2.4	2.092	50.0	.37	1.864	55.9	.73	1.770	56.2	.91	1.719	56.1	1.01	1.687	55.8	1.08	1.728	56.1
2.6	2.263	58.5	.37	2.012	65.3	.74	1.910	65.7	.92	1.854	65.4	1.02	1.819	65.1	1.09	1.871	65.5
2.8	2.433	67.6	.38	2.161	75.4	.75	2.049	75.8	.93	1.989	75.5	1.03	1.951	75.1	1.10	2.012	75.6
3.0	2.604	77.3	.38	2.309	86.1	.75	2.189	86.5	.93	2.123	86.1	1.04	2.082	85.7	1.11	2.153	86.4
3.2	2.774	87.6	.38	2.458	97.5	.76	2.328	97.9	.94	2.258	97.4	1.05	2.214	96.9	1.12	2.294	97.8
3.4	2.945	98.4	.38	2.606	109.6	.76	2.468	109.9	.94	2.392	109.4	1.06	2.345	108.8	1.13	2.435	109.8
3.6	3.116	109.8	.38	2.755	122.2	.76	2.607	122.6	.95	2.527	122.0	1.06	2.477	121.3	1.13	2.575	122.4
3.8	3.287	121.6	.38	2.904	135.4	.76	2.747	135.9	.95	2.662	135.1	1.06	2.608	134.4	1.14	2.715	135.6
4.0	3.459	134.0	.38	3.053	149.2	.76	2.887	149.7	.95	2.797	148.9	1.06	2.740	148.1	1.14	2.854	149.5
4.2	3.631	146.8	.38	3.203	163.6	.76	3.027	164.1	.95	2.932	163.3	1.07	2.872	162.3	1.14	2.993	163.9
4.4	3.803	160.0	.37	3.353	178.6	.76	3.168	179.2	.95	3.067	178.2	1.07	3.004	177.2	1.14	3.131	178.9
4.6	3.976	173.7	.37	3.503	194.1	.76	3.309	194.7	.95	3.203	193.7	1.06	3.137	192.6	1.14	3.268	194.4
4.8	4.150	187.7	.37	3.654	210.1	.75	3.451	210.8	.95	3.340	209.8	1.06	3.270	208.6	1.14	3.406	210.5
5.0	4.323	202.1	.37	3.806	226.6	.75	3.593	227.5	.94	3.477	226.3	1.06	3.404	225.1	1.14	3.542	227.1

5.2	4.498	216.9	.36	3.958	243.5	.74	3.735	244.7	.94	3.614	243.5	1.06	3.538	242.1	1.13	3.678	244.2
5.4	4.673	231.9	.36	4.111	261.0	.74	3.878	262.3	.93	3.752	261.1	1.05	3.672	259.7	1.13	3.814	261.8
5.6	4.849	247.3	.35	4.264	278.9	.73	4.022	280.5	.93	3.891	279.2	1.04	3.808	277.7	1.12	3.948	279.9
5.8	5.026	262.9	.35	4.419	297.2	.73	4.167	299.1	.92	4.030	297.8	1.04	3.944	296.3	1.12	4.083	298.4
6.0	5.203	278.6	.34	4.574	315.9	.72	4.313	318.2	.91	4.170	316.9	1.03	4.080	315.4	1.11	4.216	317.4
6.2	5.381	294.6	.34	4.730	335.0	.71	4.459	337.7	.91	4.311	336.5	1.02	4.218	334.9	1.10	4.349	336.9
6.4	5.560	310.7	.33	4.888	354.5	.70	4.606	357.6	.90	4.453	356.5	1.01	4.356	354.8	1.09	4.481	356.7
6.6	5.740	326.9	.32	5.046	374.3	.69	4.755	377.9	.89	4.595	376.9	1.01	4.495	375.2	1.08	4.612	377.0
6.8	5.921	343.2	.32	5.205	394.4	.68	4.904	398.6	.88	4.739	397.7	1.00	4.635	396.1	1.07	4.742	397.6
7.0	6.103	359.5	.31	5.366	414.7	.67	5.055	419.7	.87	4.884	418.9	.98	4.777	417.3	1.06	4.872	418.6
7.2	6.286	375.7	.30	5.528	435.3	.66	5.206	441.1	.86	5.030	440.5	.97	4.919	439.0	1.05	5.001	440.0
7.4	6.471	391.9	.29	5.691	456.1	.65	5.359	462.7	.84	5.177	462.4	.96	5.063	461.0	1.04	5.128	461.7
7.6	6.656	407.9	.28	5.856	477.1	.64	5.514	484.7	.83	5.326	484.7	.95	5.207	483.3	1.03	5.255	483.7
7.8	6.843	423.7	.28	6.022	498.1	.63	5.670	506.9	.82	5.476	507.2	.93	5.353	506.0	1.01	5.381	506.1
8.0	7.031	439.2	.27	6.190	519.3	.61	5.827	529.3	.80	5.627	530.1	.92	5.501	529.0	1.00	5.506	528.7
8.2	7.221	454.4	.26	6.360	540.5	.60	5.986	551.9	.79	5.780	553.1	.90	5.650	552.2	.98	5.629	551.6
8.4	7.413	469.2	.25	6.531	561.7	.58	6.148	574.6	.77	5.935	576.4	.88	5.801	575.8	.96	5.752	574.8
8.6	7.606	483.5	.24	6.705	582.8	.57	6.310	597.4	.75	6.092	599.8	.87	5.954	599.5	.95	5.873	598.2
8.8	7.801	497.1	.22	6.881	603.7	.55	6.476	620.3	.73	6.251	623.4	.85	6.108	623.4	.93	5.993	621.8
9.0	7.998	510.1	.21	7.060	624.4	.53	6.643	643.1	.71	6.411	647.1	.83	6.265	647.5	.91	6.111	645.7
9.2	8.198	522.1	.20	7.241	644.8	.51	6.813	665.9	.69	6.575	670.8	.81	6.424	671.6	.89	6.229	669.7
9.4	8.400	533.1	.19	7.424	664.8	.49	6.985	688.5	.67	6.740	694.5	.79	6.585	695.9	.86	6.345	693.9
9.6	8.606	542.9	.17	7.612	684.3	.47	7.161	711.0	.65	6.909	718.1	.76	6.749	720.1	.84	6.459	718.2
9.8	8.814	551.2	.15	7.802	703.2	.45	7.339	733.1	.63	7.080	741.6	.74	6.915	744.3	.82	6.572	742.7
10.	9.027	557.6	.14	7.997	721.2	.42	7.522	754.8	.60	7.255	764.8	.71	7.085	768.3	.79	6.683	767.3

Whenever the Froude Number squared in the 3rd columns of the above table is greater than unity, then critical flow at the entrance will govern, and for these the value of Yc and Qc in the last two columns will govern. Rather than using the above program to combine the separate tables created by CHANNEL, the following program oculd be used that substitutes Yc and Qc in place of Y and Q should the Froude Number squared be larger than 1.

```
Program DELIVDC.FOR
      CHARACTER*10 CH
      REAL Y(51,5),Q(51,5),YC(51),QC(51)
      WRITE(*,*)' Give number of table entries'
      READ(*,*) N
      READ(8,'(A10)') CH
      READ(8,'(A10)') CH
      DO 5 I=1,N
5     READ(8,120) YC(I),QC(I)
120   FORMAT(10X,F10.3,21X,F10.3)
      DO 10 J=1,5
      READ(J+9,'(A10)') CH
      READ(J+9,'(A10)') CH
      DO 10 I=1,N
      READ(J+9,100) Y(I,J),Q(I,J)
100   FORMAT(10X,F10.3,41X,F11.3)
      BETA=ACOS(1.-2.*Y(I,J)/10.)
      FR2=Q(I,J)**2*(10.*SIN(BETA))/(32.2*(25.*(BETA-COS(BETA)*SIN(
     &BETA)))**3)
      IF(FR2.LT.1.) GO TO 10
      Y(I,J)=YC(I)
      Q(I,J)=QC(I)
10    CONTINUE
      H=1.
      DO 20 I=1,N
      WRITE(9,110) H,(Y(I,J),Q(I,J),J=1,5)
110   FORMAT(F5.2,5(F7.3,F8.1))
20    H=H+.2
      END
```

The solution to this problem is obtained below using TK-Solver. The model is very similar to that used in the previous problem with the exception that D replaces b and m, and the angle β (called beta in the model) is an additional unknown list variable. The variable sheet shows the bottom slope S_o = .001, but the subsequent table sheets that are given thereafter were obtained after changing So=.002, etc. as requested in the problem statement.

PRB2_68A.TK

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment---------
   10         D
   1.486      Cu
   .012       n
   .001       So
   .12        Ke
   32.2       g
              g2
              Cun
              SoS
              D24
              Ke1
L  0          H
LG 0          Y
LG 0          beta
LG 0          Q
LG 0          A
L             E
L             Fr

==================== RULE SHEET ====================
S Rule--------------------------------------------------
  Cun=Cu/n
  Ke1=Ke+1
  D24=D^2/4
  g2=2*g
  SoS=sqrt(So)
  beta=acos(1-2*Y/D)
  A=D24*(beta-cos(beta)*sin(beta))
  H=Y+Ke1*(Q/A)^2/g2
  Q=Cun*A^1.666667/(beta*D)^.6666667*SoS
  E=Y+(Q/A)^2/g2
  Fr=sqrt(Q^2*(D*sin(beta))/(g*A))

==================== TABLE: ====================
Title:      Delivery Diagram-Prob. 2-68
```

Element	H	Q	Y	beta	A	E
1	1	8.96750319	.876790686	.601227556	3.36911509	.986799002
2	2	36.286687	1.7153254	.85405018	8.968897	1.96949915
3	3	80.478298	2.5463338	1.05786524	15.7571131	2.95139291
4	4	139.458821	3.38010961	1.24086498	23.3590198	3.93358317
5	5	210.72351	4.22351047	1.41486732	31.5363383	4.91680469
6	6	291.219331	5.08254344	1.58730577	40.0953051	5.90170108
7	7	377.07671	5.96356921	1.76472341	48.8456208	6.88895384
8	8	463.09732	6.87450678	1.9550867	57.5661128	7.87941144
9	9	541.611838	7.82715617	2.17175232	65.9529357	8.87433816
10	10	598.895245	8.84506837	2.44809306	73.4917865	9.87625733

After changing So to .002, and then pressing the F10 key (for list solve) the following is the table sheet.

```
==================== TABLE: ====================
Title:      Delivery Diagram-Prob. 2-68
```

Element	H	Q	Y	beta	A	E
1	1	10.0609535	.785793844	.568255222	2.86674621	.97704934
2	2	39.5875581	1.511932	.798735009	7.47282175	1.947707
3	3	86.9084083	2.22402968	.982200051	13.0108395	2.91686032
4	4	150.428518	2.93555536	1.14517261	19.2280266	3.88595236
5	5	228.507179	3.65494248	1.29842977	25.9833712	4.85588669
6	6	319.295675	4.38931536	1.44835368	33.178279	5.82742665
7	7	420.532367	5.14605478	1.60001144	40.7302483	6.80136301
8	8	529.216779	5.93412362	1.75872529	48.556516	7.7786561
9	9	640.985148	6.76610306	1.93180772	56.5564937	8.7606539
10	10	748.634882	7.66294869	2.13245376	64.5805906	9.74960164

After changing So to .0025, and then pressing the F10 key (for list solve) the following is the table sheet.

TABLE:

Title: Delivery Diagram-Prob. 2-68

Element	H	Q	Y	beta	A	E
1	1	10.1467982	.748454642	.554224039	2.66800689	.973048712
2	2	39.460181	1.43024757	.775672252	6.89417107	1.9389551
3	3	86.2029122	2.09555139	.950975015	11.9535271	2.90309479
4	4	148.956695	2.75842947	1.10591793	17.6295167	3.86697459
5	5	226.38661	3.42739468	1.25084426	23.8071245	4.83150657
6	6	317.079671	4.10942701	1.39172625	30.4114937	5.79743861
7	7	419.382059	4.81156783	1.53310097	37.3860326	6.76552512
8	8	531.168338	5.54214529	1.67943898	44.680719	7.73665842
9	9	649.430647	6.31246574	1.83640138	52.2422475	8.7120499
10	10	769.38797	7.14031487	2.01314469	60.0001534	9.69360516

After changing So to .0028, and then pressing the F10 key (for list solve) the following is the table sheet.

TABLE:

Title: Delivery Diagram-Prob. 2-68

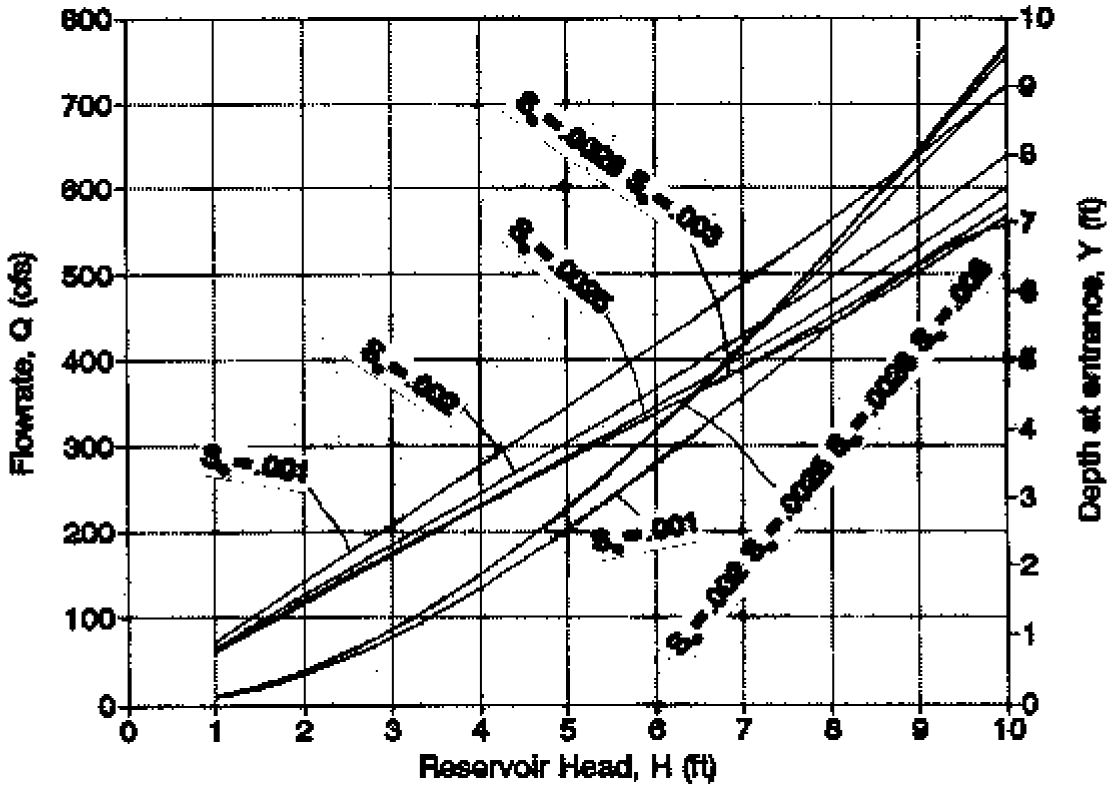
Element	H	Q	Y	beta	A	E
1	1	10.1284085	.728083659	.546433485	2.56147211	.970866106
2	2	39.1360167	1.386125	.76298696	6.58722844	1.93422768
3	3	85.2526087	2.02644096	.933889201	11.3944301	2.8956901
4	4	147.143331	2.66329203	1.08451493	16.7837265	3.85678129
5	5	223.615614	3.30514309	1.22497299	22.651746	4.81840819
6	6	313.458607	3.95880444	1.36102207	28.9336997	5.78130048
7	7	415.300908	4.63102291	1.49693377	35.5834889	6.74618103
8	8	527.424553	5.32963562	1.6367713	42.5638749	7.71388953
9	9	647.450276	6.06506139	1.78545329	49.8394209	8.68554229
10	10	771.687096	6.853192	1.95049261	57.3683281	9.662842

After changing So to .003, and then pressing the F10 key (for list solve) the following is the table sheet.

TABLE:

Title: Delivery Diagram-Prob. 2-68

Element	H	Q	Y	beta	A	E
1	1	10.0959165	.715249216	.541473528	2.49505062	.969490987
2	2	38.852146	1.35848679	.754954618	6.39702853	1.93126644
3	3	84.4790679	1.98326186	.923104083	11.048686	2.89106377
4	4	145.689648	2.60391949	1.07103496	16.2607113	3.85041995
5	5	221.371891	3.22886832	1.2087097	21.9362873	4.81023589
6	6	310.428717	3.86478782	1.3417566	28.0160851	5.77122727
7	7	411.647217	4.51821262	1.47428912	34.4595003	6.73409421
8	8	523.537444	5.1966383	1.61013413	41.2357842	7.69963982
9	9	644.069666	5.90988918	1.75379383	48.3183278	8.6689167
10	10	770.148466	6.67266016	1.91190494	55.6790674	9.6434993



Delivery Diagrams for 10 ft diameter pipe ($n=.012$, $K_e=.19$)

69.

For the 12 ft diameter pipe with n = .012, that has a trapezoidal entrance section through which water from the reservoir passes before entering the pipe, and that was solved in the text with Program E_UNTC, develop the delivery diagrams for H varying from 1 ft to 8 ft. (b=10', m=1.2, K_e = .1.)

Solution:

Delivery diagram for a trapezoidal channel to a circular one.
b=10 ft, m=1.2, D=12 ft, n=.014 S_o=.001, K_e=.1,K=.1, Del z=0

Program E_UNTCD.FOR generates the data to obtain the requested delivery diagram. It is a modification of Program E_UNTC.FOR with a DO loop to obtain the needed series of solutions.

Program E_UNTCD.FOR

```
C Solves flow in circular channel that has a trapezoidal entrance at the reservoir.
C Equations are: Mannings, Energy at entrance, & Energy between 2 shapes.
      CHARACTER*17 FMT/'(1X,A2,'' ='',F9.3)'/
      CHARACTER*2 CX(12)/'b ','m ','D ','So','n ','Q ','H ','Y1','Y2',
     &'Ke','KL','Dz'/
      CHARACTER C1(12)/'2','2','2','5','3','1','2','2','2','3','3','2'/
      CHARACTER*5 CH(0:1)/'value','guess'/
      REAL F(3),F1(3),D(3,3)
      INTEGER*2 ID(12),INDX(3)
      COMMON X(12),G,G2,Cu
      WRITE(*,*)' Give: g, Hend & Del H'
      READ(*,*) G,H2,DELH
      G2=2.*G
      Cu=1.
      IF(G.GT.15.) Cu=1.486
1     DO 10 I=1,12
      WRITE(*,'(I3,2X,A2)') I,CX(I)
10    ID(I)=0
2     WRITE(*,*)' Give three numbers for 3 unknown variables'
      READ(*,*) I1,I2,I3
      IF(I1.LT.1.OR.I1.GT.12.OR.I2.LT.1.OR.I2.GT.12.OR.I3.LT.1.OR.I3.GT.
     &12) GO TO 2
      ID(I1)=1
      ID(I2)=1
      ID(I3)=1
      DO 20 I=1,12
      WRITE(*,100) CH(ID(I)),CX(I)
100   FORMAT(' Give ',A5,' for ',A2,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
      NUM=(H2-X(7))/DELH+1.5
      DO 70 K=1,NUM
      NCT=0
30    SUM=0.
      CALL FUN(F)
      I1=0
      DO 40 I=1,12
      IF(ID(I).EQ.0) GO TO 40
      XX=X(I)
      I1=I1+1
      X(I)=1.005*X(I)
      CALL FUN(F1)
      DO 35 J=1,3
35    D(J,I1)=(F1(J)-F(J))/(X(I)-XX)
      X(I)=XX
40    CONTINUE
      CALL SOLVEQ(3,1,3,D,F,1,DD,INDX)
      I1=0
      DO 50 I=1,12
      IF(ID(I).EQ.0) GO TO 50
      I1=I1+1
      SUM=SUM+ABS(F(I1))
      X(I)=X(I)-F(I1)
50    CONTINUE
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,' SUM=',SUM
```

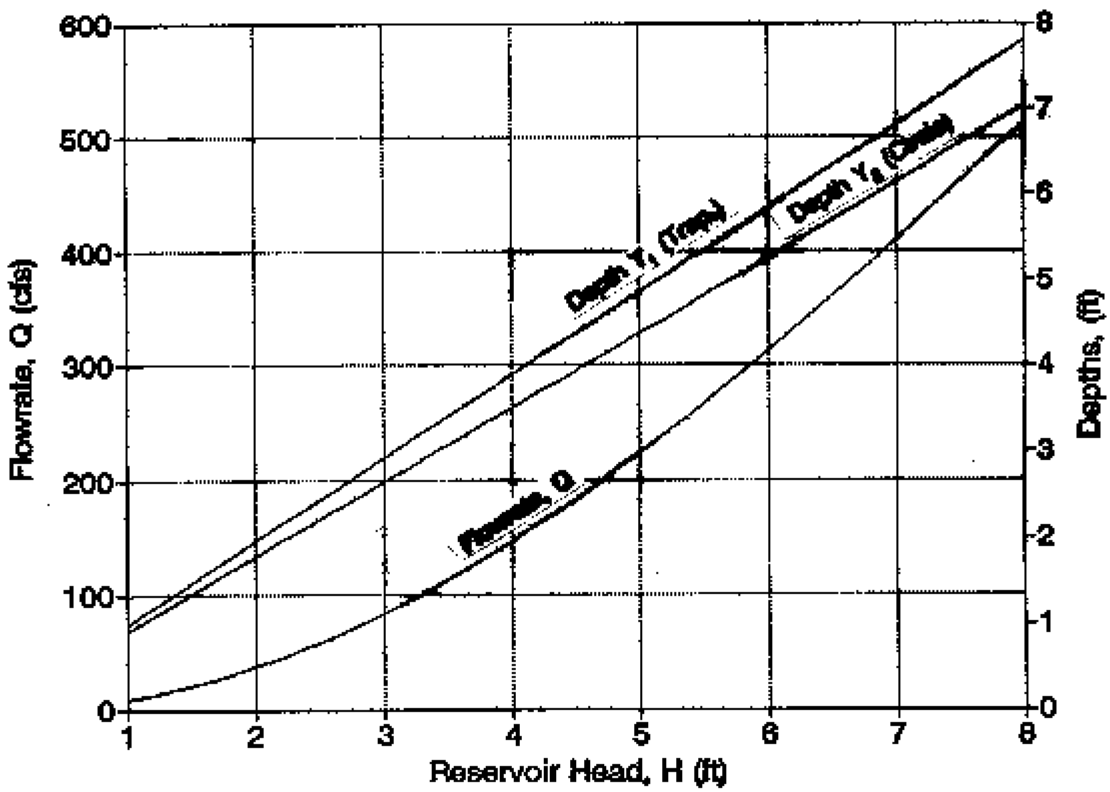
```
      IF(NCT.LT.30 .AND. SUM.GT.3.E-5) GO TO 30
      IF(K.EQ.1) THEN
      WRITE(*,*)' Solution:'
      DO 60 I=1,12
      FMT(16:16)=C1(I)
      WRITE(*,FMT) CX(I),X(I)
60    CONTINUE
      ENDIF
      FR21=X(6)**2*(X(1)+2.*X(8)*SQRT(X(2)**2+1.))/(G*((X(1)+X(2)*X(8))*
     &X(8))**3)
      BETA=ACOS(1.-2.*X(9)/X(3))
      A2=.25*X(3)**2*(BETA-COS(BETA)*SIN(BETA))
      FR22=X(6)**2*X(3)*BETA/(G*A2**3)
      WRITE(3,200) X(7),X(8),X(9),X(6),FR21,FR22
200   FORMAT(F6.2,2F8.3,F8.1,2F7.2)
70    X(7)=X(7)+DELH
      END
      SUBROUTINE FUN(F)
      REAL F(3)
      COMMON X(12),G,G2,Cu
      BETA=ACOS(1.-2.*X(9)/X(3))
      A2=.25*X(3)**2*(BETA-COS(BETA)*SIN(BETA))
      P2=X(3)*BETA
      A1=(X(1)+X(2)*X(8))*X(8)
      FKe=(1.+X(10))/G2
      FKL=(1.+X(11))/G2
      F(1)=X(5)*X(6)*P2**.6666667-Cu*A2**1.6666667*SQRT(X(4))
      F(2)=X(7)-X(8)-FKE*(X(6)/A1)**2
      F(3)=X(8)+(X(6)/A1)**2/G2-X(9)-FKL*(X(6)/A2)**2-X(12)
      RETURN
      END
```

Input to above Program:

```
32.2 8 .2
6 8 9
b=10
m=1.2
D=12
So=.001
n=.014
Q=9  (guess)
H=1
Y1=.1  (guess)
Y2=.2  (guess)
Ke=.1
KL=.1
Dz=0
```

H(ft)	Y_1(ft)	Y_2(ft)	Q(cfs)	F_{r1}^2	F_{r2}^2
1.00	.988	.905	9.1	.02	.29
1.20	1.184	1.081	13.2	.03	.31
1.40	1.378	1.256	18.1	.04	.32
1.60	1.572	1.430	23.8	.04	.34
1.80	1.766	1.604	30.2	.04	.35
2.00	1.959	1.777	37.4	.05	.36
2.20	2.152	1.950	45.3	.05	.37
2.40	2.345	2.123	53.9	.06	.38
2.60	2.538	2.296	63.2	.06	.39
2.80	2.730	2.468	73.2	.06	.40
3.00	2.923	2.640	83.8	.07	.41
3.20	3.116	2.813	95.2	.07	.41
3.40	3.308	2.985	107.1	.07	.42
3.60	3.501	3.158	119.7	.08	.43
3.80	3.694	3.330	132.9	.08	.44
4.00	3.887	3.503	146.7	.08	.44
4.20	4.080	3.676	161.0	.08	.45
4.40	4.274	3.849	175.9	.08	.45
4.60	4.467	4.023	191.3	.08	.46
4.80	4.661	4.196	207.3	.09	.46
5.00	4.855	4.370	223.7	.09	.47

5.20	5.050	4.545	240.6	.09	.47
5.40	5.244	4.719	258.0	.09	.48
5.60	5.439	4.895	275.7	.09	.48
5.80	5.635	5.070	293.9	.09	.49
6.00	5.830	5.246	312.5	.09	.49
6.20	6.026	5.423	331.4	.09	.49
6.40	6.222	5.600	350.6	.09	.50
6.60	6.419	5.778	370.1	.09	.50
6.80	6.616	5.956	389.9	.09	.50
7.00	6.813	6.135	409.9	.09	.51
7.20	7.010	6.314	430.1	.09	.51
7.40	7.208	6.494	450.5	.08	.51
7.60	7.406	6.675	471.0	.08	.52
7.80	7.605	6.857	491.7	.08	.52
8.00	7.804	7.039	512.3	.08	.52



Delivery Diagram for Problem 2-69c (Trap. entrance-Circular Channel)

Below this problem is solved using TK-Solver. For each reservoir head H the following three equations must be solved: (1) the Mannings equation in the downstream circular channel, (2) the energy equation at the entrance where the channel is trapezoidal, and (3) the energy equation across the transition from the trapezoidal to the circular channels. Note that the three variables Q, Y1 and Y2 are given the status of L (for list) and G (for guess), and H is given just the status of L. All four of these list variables are given values; H's list is filled in with the **constant increment** function, whereas the other 3 lists are filled in with the **linear** function.

PRB2_69.TK

```
═══════════════════ VARIABLE SHEET ═══════════════════
St Input──── Name─── Output─── Unit───── Comment────────
   32.2      g
```

```
   1.486     Cu
   12        D
   10        b
   1.2       m
   .014      n
   .001      So
L  1         H
   .1        KE
   .1        KL
LG 10.447717 Q
LG .99976909 Y1
LG .89864605 Y2
             beta    .55438251
             A2      3.845091
             DS2     36
             KE1     .01708075
             KL1     .01708075
             Cun     123.83333
             SoS     .03162278
             g2      64.4
             A1      11.197137
L            Fr1
L            Fr2
```

```
==================== RULE SHEET =========================
S Rule--------------------------------------------------
  g2=2*g
  KE1=(1+KE)/g2
  KL1=(1+KL)/g2
  Cun=Cu/n
  SoS=sqrt(So)
  beta=acos(1-2*Y2/D)
  DS2=D*D/4
  A2=DS2*(beta-cos(beta)*sin(beta))
  A1=(b+m*Y1)*Y1
  Q=Cun*A2^1.6666667/(D*beta)^.6666667*SoS
  H=Y1+KE1*(Q/A1)^2
  Y1+(Q/A1)^2/g2=Y2+KL1*(Q/A2)^2
  Fr1=sqrt(Q^2*(b+2*m*Y1)/(g*A1^3))
  Fr2=sqrt(Q^2*(D*sin(beta))/(g*A2^3))
```

The following Table sheet is obtained using n=.012 as shown above in the variable sheet. The Table sheet that is given thereafter is for n=.014 as solved above using the Fortran program.

==================== TABLE: ====================

Title: Delivery diagram for circular Ch. with trap. entrance

Element	H	Y1	Y2	Q	Fr1	Fr2
1	1	.986204177	.876600508	9.91203544	.167707117	.611706337
2	1.2	1.180556	1.04521809	14.3802147	.183471364	.626002234
3	1.4	1.37421899	1.21280992	19.6672441	.197328762	.637703233
4	1.6	1.56730302	1.37957823	25.7630776	.209607653	.647438671
5	1.8	1.75990912	1.54568316	32.6566471	.220545352	.655623984
6	2	1.95212951	1.71125578	40.3361311	.230321431	.662548373
7	2.2	2.14404799	1.87640617	48.7891139	.239076638	.668421489
8	2.4	2.33574042	2.04122869	58.0026812	.246924408	.673400241
9	2.6	2.52727528	2.20580569	67.9634765	.253958238	.677605102
10	2.8	2.71871422	2.37021004	78.6577325	.260256609	.681130529
11	3	2.91011265	2.53450712	90.0712853	.265886412	.684051867
12	3.2	3.10152022	2.69875619	102.189577	.270905376	.686430079
13	3.4	3.29298132	2.86301159	114.99765	.275363857	.688315082
14	3.6	3.48453559	3.02732353	128.480135	.279306171	.689748146
15	3.8	3.67621827	3.19173886	142.621235	.282771612	.690763669
16	4	3.86806067	3.35630163	157.404705	.285795248	.691390491
17	4.2	4.06009048	3.52105355	172.813824	.288408539	.691652911
18	4.4	4.25233217	3.6860344	188.831374	.290639839	.691571453
19	4.6	4.44480723	3.85128239	205.439605	.292514801	.691163478
20	4.8	4.63753452	4.01683443	222.620204	.294056706	.690443653
21	5	4.8305305	4.18272638	240.354259	.295286738	.689424332
22	5.2	5.02380948	4.34899334	258.622218	.296224215	.68811585
23	5.4	5.21738387	4.51566979	277.403849	.29688678	.686526767

24	5.6	5.41126432	4.68278981	296.678194	.297290565	.684664057
25	5.8	5.60545998	4.85038728	316.423516	.297450333	.68253326
26	6	5.79997864	5.01849601	336.617246	.297379598	.680138598
27	6.2	5.9948269	5.18714993	357.235925	.29709073	.677483071
28	6.4	6.19001029	5.35638325	378.255134	.296595042	.674568525
29	6.6	6.38553344	5.5262306	399.649425	.295902873	.671395693
30	6.8	6.58140018	5.69672721	421.392243	.295023654	.667964234
31	7	6.77761367	5.86790909	443.455833	.293965967	.664272732
32	7.2	6.97417651	6.03981317	465.811148	.292737596	.660318699
33	7.4	7.17109081	6.21247752	488.427736	.291345572	.656098544
34	7.6	7.3683583	6.38594152	511.27362	.289796209	.651607532
35	7.8	7.56598046	6.56024612	534.315159	.288095133	.646839725
36	8	7.76395851	6.73543406	557.516895	.28624731	.641787893

For n=.014

TABLE:

Title: Delivery diagram for circular Ch. with trap. entrance

Element	H	Y1	Y2	Q	Fr1	Fr2
1	1	.988433936	.905403577	9.09843158	.153398837	.526595831
2	1.2	1.18362171	1.08114414	13.2364951	.168190321	.538883532
3	1.4	1.37820161	1.25606047	18.1443201	.181213767	.548908179
4	1.6	1.57226892	1.43031748	23.8132662	.192765312	.557215629
5	1.8	1.7659124	1.60404678	30.2332038	.203060108	.564166469
6	2	1.95921389	1.77735686	37.3928627	.212261738	.570011884
7	2.2	2.15224836	1.95033937	45.2800507	.220498992	.574934063
8	2.4	2.34508414	2.12307334	53.8817948	.227876095	.579069423
9	2.6	2.53778327	2.29562804	63.184434	.234479276	.582522741
10	2.8	2.7304019	2.46806507	73.1736793	.240381175	.585376185
11	3	2.92299066	2.64043986	83.8346514	.245643899	.587695301
12	3.2	3.11559512	2.81280285	95.1519031	.250321213	.589533121
13	3.4	3.30825616	2.98520035	107.10943	.25446014	.590933054
14	3.6	3.50101035	3.15767531	119.69067	.258102172	.591930972
15	3.8	3.69389032	3.33026783	132.878502	.261284186	.592556746
16	4	3.88692507	3.50301565	146.655232	.264039162	.592835393
17	4.2	4.08014031	3.67595454	161.002578	.266396751	.592787955
18	4.4	4.27355877	3.84911865	175.901651	.268383727	.592432164
19	4.6	4.46720041	4.02254074	191.33293	.27002436	.591782973
20	4.8	4.66108274	4.19625246	207.276237	.271340711	.590852965
21	5	4.85522104	4.37028455	223.710704	.272352889	.589652676
22	5.2	5.04962854	4.54466706	240.614737	.273079261	.588190855
23	5.4	5.24431665	4.71942946	257.965982	.273536625	.586474668
24	5.6	5.43929515	4.89460086	275.741279	.273740365	.584509862
25	5.8	5.63457235	5.07021014	293.916615	.273704579	.582300892
26	6	5.83015525	5.24628609	312.467073	.273442187	.579851019
27	6.2	6.02604966	5.42285754	331.366775	.272965033	.577162388
28	6.4	6.22226039	5.59995349	350.588817	.272283964	.574236073
29	6.6	6.4187913	5.77760327	370.105201	.271408902	.571072119
30	6.8	6.61564548	5.95583666	389.886753	.270348908	.567669552
31	7	6.81282532	6.13468404	409.903041	.269112237	.564026376
32	7.2	7.01033261	6.31417652	430.122274	.267706381	.560139558
33	7.4	7.20816862	6.49434614	450.511193	.266138112	.556004987
34	7.6	7.40633423	6.675226	471.034949	.264413513	.551617419
35	7.8	7.60482996	6.85685048	491.656962	.262538005	.546970396
36	8	7.80365607	7.03925545	512.338762	.260516365	.542056148

70.

Program E_UNTC assumes uniform flow occurs in the downstream circular channel. Modify this program so critical depth may occur in either the upstream entrance (trapezoidal section), or at the beginning of the circular channel. Use this program to solve the problem solved in the text except that the channel is steep.

Wanted: Write a program to handle critical flow into a pipe with an entrance that is trapezoidal in section.

In handling critical flow when the section changes from a trapezoidal to a circle, there are two cases that must be considered: (1) Critical flow in the beginning of the circular section controls. In this case the flow in the upstream trapezoidal section will be subcritical, i.e. the trapezoidal section has a larger capacity than the downstream circular section. (2) Critical flow occurs in the upstream trapezoidal entrance, and the flow in the circular section will be supercritical, i.e. the trapezoidal entrance section has less capacity than the pipe.

The program E_CRTC.FOR is designed to solve which of these cases occurs. If ICASE=1 then it uses the critical flow equation in the trapezoidal section as Eq. 1. If ICASE=2 then it uses the critical flow equation in the circular section as Eq. 1. The other two equations used are: F(2)= energy at the entrance to the reservoir, and F(3) = energy across sections. This program is a modified version of E_UNTC and as such it allows any three of the ten variables available to be selected (n and S_o have been eliminated from E_UNTC.)

Program E_CRTC.FOR

```
C Solves flow in circular channel that has a trapezoidal entrance at the reservoir.
C Equations are: Critical Flow, Energy at entrance, & Energy between 2 shapes.
C Critical flow may occur: (1) in trapezoidal entrance, or (2) at beginning of
C circular channel.
      CHARACTER*4  SECT(2)/'Trap','Pipe'/
      CHARACTER*17 FMT/'(1X,A2,'' ='',F9.3)'/
      CHARACTER*2 CX(10)/'b ','m ','D ','Q ','H ','Y1','Y2','Ke','KL',
     &'Dz'/
      CHARACTER C1(10)/'2','2','2','1','2','2','2','3','3','2'/
      CHARACTER*5 CH(0:1)/'value','guess'/
      REAL F(3),F1(3),D(3,3)
      INTEGER*2 ID(12),INDX(3)
      COMMON X(12),G,G2,ICASE
1     WRITE(*,*)' Give:IOUT,g & ICASE (1=CRIT in trap,2=CRIT in pipe)'
      READ(*,*) IOUT,G,ICASE
      G2=2.*G
      DO 10 I=1,10
      WRITE(*,'(I3,2X,A2)') I,CX(I)
10    ID(I)=0
2     WRITE(*,*)' Give three numbers for 3 unknown variables'
      READ(*,*) I1,I2,I3
      IF(I1.LT.1.OR.I1.GT.12.OR.I2.LT.1.OR.I2.GT.12.OR.I3.LT.1.OR.I3.GT.
     &12) GO TO 2
      ID(I1)=1
      ID(I2)=1
      ID(I3)=1
      DO 20 I=1,10
      WRITE(*,100) CH(ID(I)),CX(I)
100   FORMAT(' Give ',A5,' for ',A2,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
25    NCT=0
30    SUM=0.
      CALL FUN(F)
      I1=0
      DO 40 I=1,10
      IF(ID(I).EQ.0) GO TO 40
      XX=X(I)
      I1=I1+1
      X(I)=1.005*X(I)
      CALL FUN(F1)
      DO 35 J=1,3
35    D(J,I1)=(F1(J)-F(J))/(X(I)-XX)
      X(I)=XX
40    CONTINUE
```

```
      CALL SOLVEQ(3,1,3,D,F,1,DD,INDX)
      I1=0
      DO 50 I=1,10
      IF(ID(I).EQ.0) GO TO 50
      I1=I1+1
      SUM=SUM+ABS(F(I1))
      X(I)=X(I)-F(I1)
50    CONTINUE
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,' SUM=',SUM
      IF(NCT.LT.30 .AND. SUM.GT.3.E-5) GO TO 30
      WRITE(*,*)' Solution Critical depth in ',SECT(ICASE)
      DO 60 I=1,10
      FMT(16:16)=C1(I)
      IF(IOUT.NE.6) WRITE(IOUT,FMT) CX(I),X(I)
60    WRITE(*,FMT) CX(I),X(I)
      WRITE(*,*)' Press enter to continue'
      READ(*,*)
      FR12=X(4)**2*(X(1)+2.*X(6)*X(2))/(G*((X(1)+X(2)*X(6))*X(6))**3)
      BETA=ACOS(1.-2.*X(7)/X(3))
      A2=.25*X(3)**2*(BETA-SIN(BETA)*COS(BETA))
      FR22=X(4)**2*X(3)*SIN(BETA)/(G*A2**3)
      IF(IOUT.NE.6) WRITE(IOUT,201) SECT(ICASE),X(4),X(6),X(7),FR12,FR22
      WRITE(*,201) SECT(ICASE),X(4),X(6),X(7),FR12,FR22
201   FORMAT(' Critical Flow in',A5,' controls',/,' Q =',F8.1,' Y1 =',F8
     &.3,' Y2 =',F8.3,' Froude N. (squared)=',2F7.3)
      WRITE(*,*)' Give 1 to solve another prob. (0=STOP)'
      READ(*,*) I2
      IF(I2.EQ.1) GO TO 1
      END
      SUBROUTINE FUN(F)
      REAL F(3)
      COMMON X(12),G,G2,ICASE
      BETA=ACOS(1.-2.*X(7)/X(3))
      A2=.25*X(3)**2*(BETA-COS(BETA)*SIN(BETA))
      P2=X(3)*BETA
      A1=(X(1)+X(2)*X(6))*X(6)
      FKe=(1.+X(8))/G2
      FKL=(1.+X(9))/G2
      IF(ICASE.EQ.1) THEN
      F(1)=X(4)**2*(X(1)+2.*X(2)*X(6))-G*A1**3
      ELSE
      F(1)=X(4)**2*X(3)*SIN(BETA)-G*A2**3
      ENDIF
      F(2)=X(5)-X(6)-FKE*(X(4)/A1)**2
      F(3)=X(6)+(X(4)/A1)**2/G2-X(7)-FKL*(X(4)/A2)**2-X(10)
      RETURN
      END
```

Two solutions from E_CRTC are given below. The first is for the specified problem. In this case it is clear that critical flow in the pipe controls, and this solution gives Q = 256.5 cfs, with Y_1=4.804 ft (F_{r1}^2=0.101)and Y_2=Y_c= 3.569 ft.

The second solution reduces the bottom width of the trapezoidal section to b= 3 ft, so that its capacity is less than that of the pipe. Now critical flow is in the trapezoidal entrance section and is Q_c=240.4 cfs with $Y_1 = Y_c$ = 3.720 ft and Y_2 = 3.200 ft (with F_{r2} = 1.342).

```
Solution # 1
b =   10.00
m =    1.20
D =   12.00
Q =  256.5
H =    5.00
Y1=    4.80
Y2=    3.57
Ke=     .100
KL=     .100
Dz=     .00
```

```
Critical Flow in Pipe controls
Q =  256.5  Y1 =4.804   Y2 =  3.569  Froude N. (squared)=  .101   1.000
```

Solution # 2

```
b =    3.00
m =    1.20
D =   12.00
Q =  240.4
H =    5.00
Y1=    3.72
Y2=    3.20
Ke=     .100
KL=     .100
Dz=     .00
Critical Flow in Trap controls
Q =  240.4  Y1 =3.720   Y2 =  3.200  Froude N. (squared)=  1.000  1.342
```

Below the TK-Solver model used in the previous problem to generate data for a deliver diagram has been modified slightly so that Mannings Equation is replaced by the Critical flow equation in the downstream circular channel. Thus the data generated by this model assumes that the downstream circular channel is steep, so that critical flow occur at its entrance. This delivery diagram is for an upstream trapezoidal channel with b = 10 ft, m=1.2 and a downstream circular channel with D = 12 ft. Both the entrance loss coefficient $K_e = 0.1$ and the loss coefficient through the transition $K_L = .1$. Notice from the solution table that all the downstream Froude Numbers are unity, as must be the case since the critical flow equation in the downstream channel is one of the equations being solved, and all of the Froude numbers F_{r1} for the upstream trapezoidal channel are less than unity; thus it is critical flow in the circular channel that governs.

PRB2_70.TK

```
==================== VARIABLE SHEET ====================
St Input───── Name──── Output──── Unit────── Comment──────────
   32.2       g
   12         D
   10         b
   1.2        m
L  1          H
   .1         KE
   .1         KL
LG 10.447717  Q
LG .99976909  Y1
LG .89864605  Y2
              beta    .55438251
              A2      3.845091
              DS2     36
              KE1     .01708075
              KL1     .01708075
              g2      64.4
              A1      11.197137
L             Fr1
L             Fr2

==================== RULE SHEET ========================
S Rule──────────────────────────────────────────────────
  g2=2*g
  KE1=(1+KE)/g2
  KL1=(1+KL)/g2
  beta=acos(1-2*Y2/D)
  DS2=D*D/4
  A2=DS2*(beta-cos(beta)*sin(beta))
  A1=(b+m*Y1)*Y1
  Q*Q*(sin(beta)*D)=g*A2^3
  H=Y1+KE1*(Q/A1)^2
```

```
Y1+(Q/A1)^2/g2=Y2+KL1*(Q/A2)^2
Fr1=sqrt(Q^2*(b+2*m*Y1)/(g*A1^3))
Fr2=sqrt(Q^2*(D*sin(beta))/(g*A2^3))
```

TABLE:

Title: Delivery diag.-cir. Ch. with trap. entr. & crit. flow

Element	H	Y1	Y2	Q	Fr1	Fr2
1	1	.982212567	.728023836	11.2046495	.190783033	1
2	1.2	1.17550011	.872785802	16.0642262	.206347625	1
3	1.4	1.3680872	1.01727636	21.7700479	.219983796	1
4	1.6	1.56009207	1.16149693	28.3109445	.232043415	1
5	1.8	1.75162017	1.30544778	35.6758105	.242776649	1
6	2	1.94276561	1.44912816	43.8535799	.252370314	1
7	2.2	2.13361236	1.59253633	52.8332043	.260969303	1
8	2.4	2.32423529	1.73566967	62.603633	.2686894	1
9	2.6	2.51470115	1.87852467	73.153796	.275625388	1
10	2.8	2.7050693	2.02109704	84.4725889	.281856408	1
11	3	2.89539251	2.16338168	96.5488592	.287449628	1
12	3.2	3.08571757	2.30537275	109.371395	.292462841	1
13	3.4	3.27608586	2.44706369	122.928912	.29694635	1
14	3.6	3.46653388	2.58844723	137.210048	.300944364	1
15	3.8	3.65709373	2.7295154	152.203352	.304496068	1
16	4	3.84779351	2.87025956	167.897275	.307636433	1
17	4.2	4.0386577	3.0106704	184.280168	.310396866	1
18	4.4	4.22970755	3.15073794	201.34027	.31280572	1
19	4.6	4.42096135	3.29045151	219.06571	.314888706	1
20	4.8	4.61243473	3.42979981	237.444494	.316669232	1
21	5	4.80414096	3.56877084	256.464508	.318168682	1
22	5.2	4.99609112	3.70735193	276.113511	.319406651	1
23	5.4	5.18829438	3.84552972	296.37913	.320401138	1
24	5.6	5.38075816	3.98329017	317.24886	.321168717	1
25	5.8	5.57348834	4.12061849	338.710062	.321724678	1
26	6	5.7664894	4.25749919	360.749958	.322083153	1
27	6.2	5.95976456	4.39391602	383.355629	.322257224	1
28	6.4	6.15331596	4.52985197	406.514017	.322259019	1
29	6.6	6.34714473	4.66528924	430.21192	.322099794	1
30	6.8	6.54125114	4.80020924	454.435994	.321790007	1
31	7	6.73563469	4.93459251	479.172747	.321339389	1
32	7.2	6.93029416	5.06841876	504.408547	.320756996	1
33	7.4	7.12522779	5.2016668	530.129612	.32005127	1
34	7.6	7.32043323	5.33431453	556.322018	.319230081	1
35	7.8	7.51590771	5.46633892	582.971697	.318300778	1
36	8	7.71164804	5.59771595	610.064434	.31727022	1

71.

In the text the program E_UNTC is designed to solve flowrate into a circular channel that has a trapezoidal entrance where it receives water from a reservoir. Write a computer program, or develop a computer model, to solve problems in which a trapezoidal channel has a circular section at its entrance to the supply reservoir. Use this program (or model) to solve for Q, Y_1 and Y_2 from H = 3 m, D = 5 m, K_e = .1, b = 3 m, m = 1.4, n = .013, S_o = .001, K_L = .1 and Δz = 0.8 m.

Wanted: Write a program that solves uniform flow in a trapezoidal channel that has a circular section at its entrance.

Solution: Program U_ENTC.FOR has been modified to U_UNCT.FOR to handle this case. The changes needed are to define A1 for the circular section and A2 for the trapezoidal channel in the subroutine FUN. Now the perimeter P2 is needed for the trapezoidal section rather than the circular section.

Program E_UNCT.FOR

```
C Solves flow in trapezoidal channel that has a circular entrance at the reservoir.
C Equations are: Mannings, Energy at entrance, & Energy between 2 shapes.
      CHARACTER*17 FMT/'(1X,A2,'' ='',F9.3)'/
      CHARACTER*2 CX(12)/'b ','m ','D ','So','n ','Q ','H ','Y1','Y2',
     &'Ke','KL','Dz'/
      CHARACTER C1(12)/'2','2','2','5','3','1','2','2','2','3','3','2'/
      CHARACTER*5 CH(0:1)/'value','guess'/
      REAL F(3),F1(3),D(3,3)
      INTEGER*2 ID(12),INDX(3)
      COMMON X(12),G,G2,Cu
      WRITE(*,*)' Give: g'
      READ(*,*) G
      G2=2.*G
      Cu=1.
      IF(G.GT.15.) Cu=1.486
1     DO 10 I=1,12
      WRITE(*,'(I3,2X,A2)') I,CX(I)
10    ID(I)=0
2     WRITE(*,*)' Give three numbers for 3 unknown variables'
      READ(*,*) I1,I2,I3
      IF(I1.LT.1.OR.I1.GT.12.OR.I2.LT.1.OR.I2.GT.12.OR.I3.LT.1.OR.I3.GT.
     &12) GO TO 2
      ID(I1)=1
      ID(I2)=1
      ID(I3)=1
      DO 20 I=1,12
      WRITE(*,100) CH(ID(I)),CX(I)
100   FORMAT(' Give ',A5,' for ',A2,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
      NCT=0
30    SUM=0.
      CALL FUN(F)
      I1=0
      DO 40 I=1,12
      IF(ID(I).EQ.0) GO TO 40
      XX=X(I)
      I1=I1+1
      X(I)=1.005*X(I)
      CALL FUN(F1)
      DO 35 J=1,3
35    D(J,I1)=(F1(J)-F(J))/(X(I)-XX)
      X(I)=XX
40    CONTINUE
      CALL SOLVEQ(3,1,3,D,F,1,DD,INDX)
      I1=0
      DO 50 I=1,12
      IF(ID(I).EQ.0) GO TO 50
      I1=I1+1
      SUM=SUM+ABS(F(I1))
```

```
      X(I)=X(I)-F(I1)
50    CONTINUE
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,' SUM=',SUM
      IF(NCT.LT.30 .AND. SUM.GT.1.E-6) GO TO 30
      WRITE(*,*)' Solution:'
      DO 60 I=1,12
      FMT(16:16)=C1(I)
      WRITE(*,FMT) CX(I),X(I)
60    CONTINUE
      WRITE(*,*)' Give 1 to solve another prob. (0=STOP)'
      READ(*,*) I2
      IF(I2.EQ.1) GO TO 1
      END
      SUBROUTINE FUN(F)
      REAL F(3)
      COMMON X(12),G,G2,Cu
      BETA=ACOS(1.-2.*X(8)/X(3))
      A1=.25*X(3)**2*(BETA-COS(BETA)*SIN(BETA))
      A2=(X(1)+X(2)*X(9))*X(9)
      P2=X(1)+2.*X(9)*SQRT(1.+X(2)**2)
      FKe=(1.+X(10))/G2
      FKL=(1.+X(11))/G2
      F(1)=X(5)*X(6)*P2**.6666667-Cu*A2**1.6666667*SQRT(X(4))
      F(2)=X(7)-X(8)-FKE*(X(6)/A1)**2
      F(3)=X(8)+(X(6)/A1)**2/G2-X(9)-FKL*(X(6)/A2)**2-X(12)
      RETURN
      END
```

The solution obtained consists of:

```
b =  3 m
m = 1.4
D = 5 m
So= .001
n=  .013
Q = 25.5 m^3/s
H = 3 m
Y1= 2.68 m
Y2= 1.80 m
Ke= .1
KL= .1
Dz= .8
```

If a problem with a trapezoidal channel (b=10 ft, m=1.2) that is 1.5 ft above the upstream circular channel with D=12 ft (S_o=.001, n=.014 K_e=.1 & K_L =.1)is solved the solution is:

```
b =  10 ft
m = 1.2
D = 12 ft
So= .001
n = .014
Q = 217.1 cfs
H = 5 ft
Y1= 4.45 ft
Y2= 2.94 ft
Ke= .1
KL= .1
Dz= 1.5
```

A TK-Solver model to solve this problem is given below. Before pressing F9 the status for Q, Y1 and Y2 is given as G (for guess).

PRB2_71.TK

```
===================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment---------
   9.81       g
   1          Cu
   5          D
   3          b
   1.4        m
   .013       n
   .001       So
   3          H
   .1         KE
   .1         KL
   .8         Delz
              Q       25.527868
              Y1      2.6826585
              Y2      1.8026504
              beta    1.6439249
              g2      19.62
              KE1     .05606524
              KL1     .05606524
              Cun     76.923077
              SoS     .03162278
              DS2     6.25
              A1      10.729956
              A2      9.9573194
              Fr1     .5178297
              Fr2     .73585924

===================== RULE SHEET ========================
S Rule---------------------------------------------------
  g2=2*g
  KE1=(1+KE)/g2
  KL1=(1+KL)/g2
  Cun=Cu/n
  SoS=sqrt(So)
  DS2=D*D/4
  beta=acos(1-2*Y1/D)
  A1=DS2*(beta-cos(beta)*sin(beta))
  A2=(b+m*Y2)*Y2
  Q=Cun*A2^1.666667/(b+2*Y2*sqrt(m*m+1))^.6666667*SoS
  H=Y1+KE1*(Q/A1)^2
  Y1+(Q/A1)^2/g2=Y2+KL1*(Q/A2)^2+Delz
  Fr1=sqrt(Q^2*(D*sin(beta))/(g*A1^3))
  Fr2=sqrt(Q^2*(b+2*m*Y2)/(g*A2^3))
```

72.

For the channel with the transitional entrance of the previous problem develop the delivery diagrams with H varying from 0.25 m to 5 m for bottom slopes varying from S_o = .0002 to S_o = .0015

Wanted: For the channel with the transitional entrance of the previous problem develop the delivery diagrams with H varying from 0.25 m to 5 m for bottom slopes varying between S_o = .0003 and .0015.

Solution:

Program E UNCT.FOR

```
C Solves flow in trapezoidal channel that has a circular entrance at the reservoir.
C Equations are: Mannings, Energy at entrance, & Energy between 2 shapes.
C  Repeats solution to develop values for delivery diagram.
      CHARACTER*17 FMT/'(1X,A2,'' ='',F9.3)'/
      CHARACTER*2 CX(12)/'b ','m ','D ','So','n ','Q ','H ','Y1','Y2',
     &'Ke','KL','Dz'/
      CHARACTER C1(12)/'2','2','2','5','3','1','2','2','2','3','3','2'/
      CHARACTER*5 CH(0:1)/'value','guess'/
      REAL F(3),F1(3),D(3,3)
      INTEGER*2 ID(12),INDX(3)
      COMMON X(12),G,G2,Cu
      WRITE(*,*)' Give: g, Hend & Del H'
      READ(*,*) G,H2,DELH
      G2=2.*G
      Cu=1.
      IF(G.GT.15.) Cu=1.486
1     DO 10 I=1,12
      WRITE(*,'(I3,2X,A2)') I,CX(I)
10    ID(I)=0
2     WRITE(*,*)' Give three numbers for 3 unknown variables'
      READ(*,*) I1,I2,I3
      IF(I1.LT.1.OR.I1.GT.12.OR.I2.LT.1.OR.I2.GT.12.OR.I3.LT.1.OR.I3.GT.
     &12) GO TO 2
      ID(I1)=1
      ID(I2)=1
      ID(I3)=1
      DO 20 I=1,12
      WRITE(*,100) CH(ID(I)),CX(I)
100   FORMAT(' Give ',A5,' for ',A2,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
      H1=X(7)
      NUM=(H2-X(7))/DELH+1.5
24    WRITE(3,300) X(4)
300   FORMAT(' So =',F9.6)
      DO 70 K=1,NUM
      NCT=0
30    SUM=0.
      CALL FUN(F)
      I1=0
      DO 40 I=1,12
      IF(ID(I).EQ.0) GO TO 40
      XX=X(I)
      I1=I1+1
      X(I)=1.005*X(I)
      CALL FUN(F1)
      DO 35 J=1,3
35    D(J,I1)=(F1(J)-F(J))/(X(I)-XX)
      X(I)=XX
40    CONTINUE
      CALL SOLVEQ(3,1,3,D,F,1,DD,INDX)
      I1=0
      DO 50 I=1,12
      IF(ID(I).EQ.0) GO TO 50
      I1=I1+1
      SUM=SUM+ABS(F(I1))
```

```
      X(I)=X(I)-F(I1)
50    CONTINUE
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,' SUM=',SUM
      IF(NCT.LT.30 .AND. SUM.GT.3.E-5) GO TO 30
      IF(K.EQ.1) THEN
      WRITE(*,*)' Solution:'
      DO 60 I=1,12
      FMT(16:16)=C1(I)
      WRITE(*,FMT) CX(I),X(I)
60    CONTINUE
      WRITE(*,*)' Press Enter to continue'
      READ(*,*)
      QQ=X(6)
      YY1=X(8)
      YY2=X(9)
      ENDIF
      FR22=X(6)**2*(X(1)+2.*X(9)*X(2))/(G*((X(1)+X(2)*X(9))*
     &X(9))**3)
      BETA=ACOS(1.-2.*X(8)/X(3))
      A2=.25*X(3)**2*(BETA-COS(BETA)*SIN(BETA))
      FR21=X(6)**2*X(3)*BETA/(G*A2**3)
      WRITE(3,200) X(7),X(8),X(9),X(6),FR21,FR22
200   FORMAT(F6.2,2F8.3,F8.1,2F7.2)
      IF(FR21.GT.1. .OR. FR22.GT.1.) THEN
      WRITE(*,203) X(7)
203   FORMAT(' Critical flow for H=',F8.2,' Cannot inc. further')
      WRITE(3,203) X(7)
      GO TO 81
      ENDIF
70    X(7)=X(7)+DELH
81    WRITE(*,*)' Give new So (negative=STOP)'
      READ(*,*) X(4)
      IF(X(4).LT. 0.) STOP
      X(7)=H1
      X(6)=QQ
      X(8)=YY1
      X(9)=YY2
      GO TO 24
      END
      SUBROUTINE FUN(F)
      REAL F(3)
      COMMON X(12),G,G2,Cu
      BETA=ACOS(1.-2.*X(8)/X(3))
      A1=.25*X(3)**2*(BETA-COS(BETA)*SIN(BETA))
      A2=(X(1)+X(2)*X(9))*X(9)
      P2=X(1)+2.*X(9)*SQRT(1.+X(2)**2)
      FKe=(1.+X(10))/G2
      FKL=(1.+X(11))/G2
      F(1)=X(5)*X(6)*P2**.6666667-Cu*A2**1.6666667*SQRT(X(4))
      F(2)=X(7)-X(8)-FKE*(X(6)/A1)**2
      F(3)=X(8)+(X(6)/A1)**2/G2-X(9)-FKL*(X(6)/A2)**2-X(12)
      RETURN
      END
```

```
Input:
9.81 5 .25
6 8 9
3
1.4
5
.25
.2
.25
.1
.1
0.8
and different So
```

<u>Output:</u>

```
 So =  .000200
```

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.049	.0	.00	.04
1.10	1.099	.290	.4	.00	.07
1.35	1.346	.529	1.2	.01	.08
1.60	1.590	.769	2.3	.02	.09
1.85	1.831	1.008	3.8	.03	.10
2.10	2.070	1.247	5.6	.05	.10
2.35	2.306	1.486	7.8	.07	.10
2.60	2.539	1.724	10.5	.09	.11
2.85	2.768	1.962	13.5	.11	.11
3.10	2.992	2.199	17.0	.14	.11
3.35	3.211	2.436	21.0	.18	.12
3.60	3.424	2.672	25.4	.22	.12
3.85	3.628	2.908	30.4	.27	.12
4.10	3.823	3.142	35.8	.33	.12
4.35	4.007	3.375	41.7	.41	.12
4.60	4.176	3.607	48.2	.51	.13
4.85	4.327	3.837	55.2	.63	.13
5.10	4.456	4.065	62.6	.78	.13

So = .000300

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.048	.0	.00	.06
1.10	1.099	.285	.5	.00	.11
1.35	1.344	.520	1.4	.01	.12
1.60	1.585	.754	2.7	.03	.14
1.85	1.824	.988	4.5	.05	.14
2.10	2.058	1.222	6.6	.07	.15
2.35	2.288	1.456	9.2	.10	.16
2.60	2.513	1.689	12.3	.13	.16
2.85	2.733	1.921	15.9	.16	.16
3.10	2.946	2.153	20.0	.20	.17
3.35	3.151	2.384	24.6	.25	.17
3.60	3.346	2.613	29.7	.32	.18
3.85	3.529	2.842	35.4	.39	.18
4.10	3.698	3.069	41.7	.49	.18
4.35	3.848	3.295	48.5	.60	.19
4.60	3.975	3.518	55.9	.75	.19
4.85	4.072	3.738	63.8	.93	.19
5.10	4.128	3.955	72.2	1.16	.19

Critical flow for H= 5.10 Cannot inc. further

So = .000400

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.048	.0	.00	.09
1.10	1.098	.280	.6	.00	.14
1.35	1.342	.510	1.6	.02	.17
1.60	1.582	.740	3.0	.04	.18
1.85	1.817	.970	5.0	.06	.19
2.10	2.047	1.199	7.4	.09	.20
2.35	2.272	1.427	10.2	.12	.21
2.60	2.491	1.655	13.6	.16	.21
2.85	2.702	1.882	17.6	.21	.22
3.10	2.905	2.109	22.1	.26	.22
3.35	3.097	2.334	27.2	.33	.23
3.60	3.275	2.557	32.8	.41	.23
3.85	3.438	2.780	39.0	.51	.24
4.10	3.579	3.000	45.9	.64	.24
4.35	3.692	3.218	53.2	.80	.25
4.60	3.767	3.432	61.2	1.00	.25

Critical flow for H= 4.60 Cannot inc. further

So = .000500

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.047	.0	.00	.11
1.10	1.098	.276	.6	.01	.18
1.35	1.341	.501	1.7	.02	.21
1.60	1.579	.727	3.3	.04	.22
1.85	1.811	.952	5.4	.07	.24
2.10	2.038	1.176	7.9	.10	.25
2.35	2.258	1.400	11.0	.14	.26
2.60	2.471	1.623	14.7	.19	.26

2.85	2.675	1.845	18.9	.24	.27
3.10	2.868	2.066	23.7	.31	.28
3.35	3.048	2.286	29.1	.39	.29
3.60	3.211	2.504	35.1	.49	.29
3.85	3.352	2.720	41.7	.62	.30
4.10	3.463	2.934	48.9	.78	.30
4.35	3.532	3.143	56.6	1.00	.31

Critical flow for H= 4.35 Cannot inc. further

So = .000600

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.047	.0	.00	.13
1.10	1.098	.271	.7	.01	.21
1.35	1.340	.493	1.8	.02	.25
1.60	1.576	.714	3.5	.05	.27
1.85	1.806	.934	5.7	.08	.28
2.10	2.029	1.154	8.4	.12	.30
2.35	2.246	1.374	11.7	.16	.31
2.60	2.453	1.592	15.5	.22	.32
2.85	2.651	1.810	19.9	.28	.33
3.10	2.835	2.026	25.0	.36	.33
3.35	3.004	2.241	30.6	.45	.34
3.60	3.152	2.453	36.9	.57	.35
3.85	3.271	2.664	43.7	.73	.35
4.10	3.350	2.870	51.2	.94	.36
4.35	3.354	3.071	59.0	1.24	.37

Critical flow for H= 4.35 Cannot inc. further

So = .000700

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.046	.0	.00	.15
1.10	1.097	.267	.7	.01	.25
1.35	1.339	.485	1.9	.03	.29
1.60	1.573	.701	3.7	.05	.31
1.85	1.801	.918	5.9	.09	.33
2.10	2.022	1.134	8.8	.13	.34
2.35	2.235	1.349	12.2	.18	.36
2.60	2.438	1.563	16.1	.24	.37
2.85	2.630	1.776	20.7	.31	.38
3.10	2.807	1.988	25.9	.40	.39
3.35	2.965	2.198	31.8	.51	.40
3.60	3.099	2.405	38.2	.65	.40
3.85	3.196	2.610	45.3	.83	.41
4.10	3.234	2.809	52.8	1.10	.42

Critical flow for H= 4.10 Cannot inc. further

So = .000800

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.046	.0	.00	.17
1.10	1.097	.263	.7	.01	.28
1.35	1.338	.477	2.0	.03	.33
1.60	1.571	.690	3.8	.06	.35
1.85	1.797	.902	6.1	.10	.37
2.10	2.016	1.114	9.1	.14	.39
2.35	2.226	1.325	12.6	.19	.41
2.60	2.425	1.535	16.7	.26	.42
2.85	2.612	1.744	21.4	.34	.43
3.10	2.782	1.951	26.7	.43	.44
3.35	2.931	2.156	32.7	.55	.45
3.60	3.051	2.359	39.3	.71	.46
3.85	3.126	2.558	46.4	.93	.47
4.10	3.108	2.749	54.0	1.28	.48

Critical flow for H= 4.10 Cannot inc. further

So = .000900

H(m)	Y1(m)	Y2(m)	Q(m^3/s)	F_{r1}^2	F_{r2}^2
.85	.850	.045	.0	.00	.19
1.10	1.097	.259	.7	.01	.32
1.35	1.337	.469	2.0	.03	.36
1.60	1.569	.678	3.9	.06	.40
1.85	1.794	.887	6.3	.10	.42
2.10	2.011	1.095	9.3	.15	.44
2.35	2.218	1.302	12.9	.21	.45
2.60	2.415	1.508	17.1	.28	.47
2.85	2.597	1.713	21.9	.36	.48

```
  3.10   2.761   1.916    27.3    .46    .49
  3.35   2.902   2.117    33.4    .60    .50
  3.60   3.009   2.315    40.1    .77    .51
  3.85   3.060   2.509    47.3   1.02    .52
 Critical flow for H=    3.85 Cannot inc. further
 So =  .001000
   H(m)   Y1(m)  Y2(m)  Q(m^3/s)     Fr1^2   Fr2^2
   .85    .850    .045      .0    .00    .21
  1.10   1.097    .255      .8    .01    .35
  1.35   1.336    .462     2.1    .03    .40
  1.60   1.568    .667     4.0    .07    .44
  1.85   1.791    .872     6.5    .11    .46
  2.10   2.007   1.076     9.5    .16    .49
  2.35   2.212   1.279    13.2    .22    .50
  2.60   2.405   1.482    17.4    .29    .52
  2.85   2.584   1.683    22.3    .38    .53
  3.10   2.744   1.882    27.8    .49    .55
  3.35   2.878   2.079    34.0    .63    .56
  3.60   2.973   2.273    40.7    .82    .57
  3.85   2.998   2.461    47.9   1.12    .58
 Critical flow for H=    3.85 Cannot inc. further
 So =  .001500
   H(m)   Y1(m)  Y2(m)  Q(m^3/s)     Fr1^2   Fr2^2
   .85    .850    .043      .0    .00    .31
  1.10   1.096    .238      .8    .01    .51
  1.35   1.334    .428     2.3    .04    .60
  1.60   1.563    .617     4.3    .08    .65
  1.85   1.783    .805     6.9    .12    .69
  2.10   1.994    .993    10.0    .18    .72
  2.35   2.194   1.180    13.8    .25    .74
  2.60   2.381   1.366    18.2    .33    .77
  2.85   2.551   1.550    23.3    .43    .79
  3.10   2.698   1.733    28.9    .56    .81
  3.35   2.815   1.913    35.2    .73    .82
  3.60   2.878   2.089    42.0    .97    .84
  3.85   2.783   2.254    49.0   1.46    .85
 Critical flow for H=    3.85 Cannot inc. further
```

Whenever the Froude Number squared in the upstream circular section is larger than unity the solution obtained above is not valid because critical flow in the upstream circular section will limit the flow rate to Qc. Thus in the above table the solution is not valid for the last entries when the program stopped incrementing H because $F_{r1}^2 > 1$. What should occur for these last invalid solutions is that the downstream Mannings equation should be replaced by the critical flow equation in the upstream circular channel. The program E_CRCT.FOR, listed below, is written to solve critical flow conditions in the upstream circular section.

One might wonder why critical flow in the upstream circular channel limits the flow rate as the head in the upstream reservoir increases. In fact the upstream channel would be the limiting control even more so if the downstream trapezoidal channel did not have its bottom elevation $\Delta z = 0.8$ m above that of the circular channel. One can appreciate why the upstream circular channel limits the flow rate by noting that the area of the trapezoidal channel increases more rapidly with increasing depths than does the area of the circular section, especially after the depth becomes equal to one-half the diameter. The table below, which was generated by TK-Solver, shows how the area of the circular section Ac and of the trapezoidal section At, (and the ratio two areas) vary with the depth of flow.

```
==================== RULE SHEET ========================
S Rule--------------------------------------------------
  beta=acos(1-2*Y/D)
  Ac=.25*D^2*(beta-cos(beta)*sin(beta))
  At=(b+m*Y)*Y
  Ratio=At/Ac
==================== VARIABLE SHEET ====================
St Input---- Name--- Output--- Unit----- Comment--------
   3         b
   1.4       m
   5         D
             beta
L  .2        Y
L            Ac
L            At
L            Ratio
==================== TABLE:  ============================
Title:
Element Areas rela Ac-------- At-------- Ratio-----
```

1	.2	.263443447	.656	2.490098
2	.4	.73587831	1.424	1.93510256
3	.6	1.33463548	2.304	1.72631406
4	.8	2.0278091	3.296	1.62539955
5	1	2.79559511	4.4	1.5739046
6	1.2	3.62361826	5.616	1.54983213
7	1.4	4.50049146	6.944	1.5429426
8	1.6	5.41666003	8.384	1.54781728
9	1.8	6.36376386	9.936	1.56134015
10	2	7.33424517	11.6	1.58162152
11	2.2	8.32108486	13.376	1.60748271
12	2.4	9.31761041	15.264	1.63818826
13	2.6	10.3173437	17.264	1.67329892
14	2.8	11.3138692	19.376	1.71258829
15	3	12.3007089	21.6	1.75599635
16	3.2	13.2711902	23.936	1.80360613
17	3.4	14.2182941	26.384	1.85563753
18	3.6	15.1344626	28.944	1.91245641
19	3.8	16.0113358	31.616	1.97460102
20	4	16.839359	34.4	2.04283311
21	4.2	17.607145	37.296	2.11823098
22	4.4	18.3003186	40.304	2.20236603
23	4.6	18.8990758	43.424	2.2976785
24	4.8	19.3715106	46.656	2.40848537
25	5	19.6349541	50	2.54647909

Program E CRCT.FOR

```
C Solves flow in trapezoidal channel that has a circular entrance at the reservoir.
C Equations are: Critical Flow, Energy at entrance, & Energy between 2 shapes.
C Critical flow may occur only in circular entrance
      CHARACTER*2 CX(10)/'b ','m ','D ','Q ','H ','Y1','Y2','Ke','KL',
     &'Dz'/
      CHARACTER*5 CH(0:1)/'value','guess'/
      REAL F(3),F1(3),D(3,3)
      INTEGER*2 ID(12),INDX(3)
      COMMON X(12),G,G2,ICASE
1     WRITE(*,*)' Give:g, Hend & del H'
      READ(*,*) G,H2,DELH
      G2=2.*G
      DO 10 I=1,10
      WRITE(*,'(I3,2X,A2)') I,CX(I)
10    ID(I)=0
2     WRITE(*,*)' Give three numbers for 3 unknown variables'
      READ(*,*) I1,I2,I3
      IF(I1.LT.1.OR.I1.GT.12.OR.I2.LT.1.OR.I2.GT.12.OR.I3.LT.1.OR.I3.GT.
     &12) GO TO 2
      ID(I1)=1
      ID(I2)=1
      ID(I3)=1
      DO 20 I=1,10
      WRITE(*,100) CH(ID(I)),CX(I)
100   FORMAT(' Give ',A5,' for ',A2,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
25    NCT=0
30    SUM=0.
      CALL FUN(F)
      I1=0
      DO 40 I=1,10
      IF(ID(I).EQ.0) GO TO 40
      XX=X(I)
      I1=I1+1
      X(I)=1.005*X(I)
      CALL FUN(F1)
      DO 35 J=1,3
35    D(J,I1)=(F1(J)-F(J))/(X(I)-XX)
      X(I)=XX
40    CONTINUE
      CALL SOLVEQ(3,1,3,D,F,1,DD,INDX)
      I1=0
      DO 50 I=1,10
```

```
      IF(ID(I).EQ.0) GO TO 50
      I1=I1+1
      SUM=SUM+ABS(F(I1))
      X(I)=X(I)-F(I1)
50    CONTINUE
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,' SUM=',SUM
      IF(NCT.LT.30 .AND. SUM.GT.3.E-5) GO TO 30
      FR22=X(4)**2*(X(1)+2.*X(7)*X(2))/(G*((X(1)+X(2)*X(7))*X(7))**3)
      WRITE(3,201) X(5),X(4),X(6),X(7),FR22
201   FORMAT(F5.2,F8.2,2F8.3,F7.2)
      X(5)=X(5)+DELH
      IF(X(5).LE.H2) GO TO 25
      END
      SUBROUTINE FUN(F)
      REAL F(3)
      COMMON X(12),G,G2,ICASE
      BETA=ACOS(1.-2.*X(6)/X(3))
      A1=.25*X(3)**2*(BETA-COS(BETA)*SIN(BETA))
      A2=(X(1)+X(2)*X(7))*X(7)
      FKe=(1.+X(8))/G2
      FKL=(1.+X(9))/G2
      F(1)=X(4)**2*X(3)*SIN(BETA)-G*A1**3
      F(2)=X(5)-X(6)-FKE*(X(4)/A1)**2
      F(3)=X(6)+(X(4)/A1)**2/G2-X(7)-FKL*(X(4)/A2)**2-X(10)
      RETURN
      END
```

Solving the same circular to trapezoidal channel, except with no rise in the bottom, i.e. $\Delta z=0$, but assuming that critical flow occurs at the end of the circular portion, using the above program E_CRCT.FOR.

```
Input:
9.81 5 .25
4 6 7
3
1.4
5
.25
.25
.18
.04
.1
.1
0
```

```
Output:
   H      Q       Y1      Y2  (Fr2)**2
-------------------------------------
  .25     .25    .183    .044   8.52
  .50    1.00    .364    .123   5.66
  .75    2.22    .545    .222   4.56
 1.00    3.89    .725    .332   4.00
 1.25    6.01    .904    .447   3.68
 1.50    8.54   1.082    .563   3.51
 1.75   11.48   1.259    .679   3.41
 2.00   14.79   1.434    .792   3.37
 2.25   18.46   1.608    .903   3.37
 2.50   22.46   1.780   1.010   3.39
 2.75   26.78   1.950   1.112   3.43
 3.00   31.39   2.119   1.211   3.49
 3.25   36.28   2.285   1.306   3.56
 3.50   41.40   2.448   1.396   3.65
 3.75   46.75   2.609   1.483   3.74
 4.00   52.29   2.767   1.565   3.84
 4.25   58.00   2.921   1.643   3.94
 4.50   63.85   3.071   1.718   4.06
 4.75   69.82   3.217   1.788   4.18
 5.00   75.88   3.358   1.855   4.30
```

When critical flow occurs in the upstream circular section the flow must be supercritical in the larger downstream trapezoidal section if the channel is steep, i.e. supercritcal flow will take place in the downstream trapezoidal channel. Therefore, in providing the input it is necessary that a guess be provided that will cause the Newton Method to converge to the supercritical downstream depth Y2. In this case the Energy and Critical Flow Equations were solved to get Yc=.183 and Qc=.25 for the circular section, and using this Energy the supercritical root Y2 = .043 m was solved and provided as the guesses.

To develop the Deliver Diagrams, the following Program DELIVG.FOR was written to use the results from E_UNCTD if Fr1 is not greater than unity, and the results from E_CRCT whenever critical flow in the beginning circular section controls.

Program DELIVG.FOR

```
      REAL Q(20),Y1(20),Y2(20),FR2(20)
      CHARACTER*30 CHAR
      DO 10 I=1,20
10    READ(2,100) Q(I),Y1(I),Y2(I),FR2(I)
100   FORMAT(5X,F8.2,2F8.3,F7.2)
15    READ(3,'(A30)',END=99) CHAR
      WRITE(4,'(A30)') CHAR
      DO 20 I=1,20
      READ(3,110) H,YY1,YY2,QQ,F1,F2
110   FORMAT(F6.2,2F8.3,F8.1,2F7.2)
      IF(F1.LE. 1.) GO TO 20
      YY1=Y1(I)
      YY2=Y2(I)
      QQ=Q(I)
      F2=FR2(I)
      F1=1.
20    WRITE(4,110) H,YY1,YY2,QQ,SQRT(F1),SQRT(F2)
      GO TO 15
99    STOP
      END
```

The output from this program provides the data for the delivery diagrams. It is important to note that whenever the Froude Number for the flow at the beginning of the trapezoidal channel is greater than 1 (whenever critical flow in the upstream circular section governs) that the slope of the bottom must be equal to whatever Mannings Equation would compute it to be, or larger than this amount, or else there will be a hydraulic jump if the channel's actual So is mild (or produces subcritical depth) for this flow rate. In other words some non smooth flow may occur in the transition from the circular to the trapezoidal sections. (Results for $\Delta z = 0$)

Output:

So = .000200

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.201	.189	.2	.85	.26
.50	.438	.428	.9	.64	.30
.75	.673	.667	1.8	.57	.32
1.00	.908	.906	3.1	.55	.32
1.25	1.140	1.144	4.7	.54	.33
1.50	1.371	1.383	6.7	.54	.35
1.75	1.600	1.621	9.0	.55	.35
2.00	1.826	1.859	11.7	.56	.35
2.25	2.050	2.098	14.8	.57	.36
2.50	2.271	2.335	18.2	.59	.36
2.75	2.490	2.573	21.9	.61	.36
3.00	2.705	2.810	26.1	.63	.37
3.25	2.917	3.047	30.6	.66	.37
3.50	3.125	3.284	35.4	.69	.37
3.75	3.329	3.520	40.5	.72	.39
4.00	3.528	3.756	45.9	.75	.39
4.25	3.722	3.992	51.6	.79	.39
4.50	3.910	4.228	57.5	.84	.39
4.75	4.090	4.463	63.5	.89	.39
5.00	4.261	4.697	69.5	.96	.40

So = .000300

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.187	.185	.2	.97	.32
.50	.416	.419	1.0	.75	.36

.75	.645	.653	2.1	.69	.37
1.00	.873	.887	3.5	.66	.39
1.25	1.098	1.120	5.4	.64	.40
1.50	1.322	1.354	7.7	.64	.41
1.75	1.543	1.587	10.3	.65	.42
2.00	1.761	1.820	13.3	.66	.42
2.25	1.976	2.053	16.8	.68	.44
2.50	2.188	2.286	20.6	.69	.45
2.75	2.397	2.519	24.9	.71	.45
3.00	2.603	2.751	29.5	.74	.45
3.25	2.804	2.983	34.4	.77	.46
3.50	3.002	3.215	39.8	.79	.46
3.75	3.195	3.447	45.4	.82	.47
4.00	3.383	3.678	51.4	.87	.47
4.25	3.566	3.909	57.5	.91	.47
4.50	3.742	4.140	63.9	.95	.48
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .000400

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.399	.411	1.0	.84	.41
.75	.621	.640	2.2	.77	.44
1.00	.842	.869	3.8	.73	.45
1.25	1.061	1.098	5.9	.72	.47
1.50	1.278	1.327	8.3	.72	.48
1.75	1.493	1.555	11.2	.73	.49
2.00	1.704	1.784	14.4	.74	.49
2.25	1.912	2.013	18.2	.75	.50
2.50	2.117	2.241	22.3	.77	.51
2.75	2.319	2.469	26.8	.79	.51
3.00	2.516	2.697	31.7	.82	.52
3.25	2.710	2.924	37.0	.85	.53
3.50	2.900	3.152	42.7	.88	.53
3.75	3.084	3.379	48.7	.91	.54
4.00	3.264	3.606	54.9	.94	.54
4.25	3.439	3.833	61.4	.98	.54
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .000500

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.383	.404	1.1	.91	.45
.75	.599	.628	2.3	.84	.48
1.00	.815	.853	4.0	.80	.50
1.25	1.028	1.077	6.2	.79	.52
1.50	1.240	1.302	8.8	.79	.53
1.75	1.448	1.526	11.8	.79	.54
2.00	1.653	1.750	15.2	.81	.55
2.25	1.856	1.974	19.1	.82	.56
2.50	2.054	2.198	23.4	.84	.57
2.75	2.250	2.422	28.2	.86	.57
3.00	2.441	2.646	33.3	.89	.58
3.25	2.628	2.869	38.8	.91	.58
3.50	2.811	3.093	44.7	.94	.59
3.75	2.990	3.316	50.9	.97	.59
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .000600

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.370	.397	1.1	.96	.49
.75	.580	.617	2.4	.89	.53
1.00	.790	.837	4.2	.86	.55

1.25	.999	1.058	6.4	.85	.57
1.50	1.205	1.278	9.1	.85	.57
1.75	1.408	1.498	12.2	.85	.59
2.00	1.608	1.718	15.8	.87	.60
2.25	1.805	1.938	19.8	.88	.61
2.50	1.998	2.158	24.3	.90	.62
2.75	2.188	2.378	29.2	.92	.62
3.00	2.374	2.598	34.4	.94	.63
3.25	2.556	2.818	40.1	.97	.64
3.50	2.733	3.037	46.2	1.00	.64
3.75	2.609	1.483	46.8	1.00	1.93
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .000700

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.364	.123	1.0	1.00	2.38
.75	.563	.607	2.5	.94	.57
1.00	.768	.823	4.3	.91	.59
1.25	.971	1.039	6.6	.90	.61
1.50	1.172	1.255	9.3	.90	.62
1.75	1.371	1.472	12.5	.91	.63
2.00	1.566	1.688	16.2	.92	.65
2.25	1.758	1.904	20.3	.93	.66
2.50	1.947	2.120	24.9	.95	.66
2.75	2.132	2.336	29.9	.97	.67
3.00	2.313	2.552	35.3	.99	.68
3.25	2.285	1.306	36.3	1.00	1.89
3.50	2.448	1.396	41.4	1.00	1.91
3.75	2.609	1.483	46.8	1.00	1.93
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .000800

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.364	.123	1.0	1.00	2.38
.75	.547	.598	2.5	.99	.60
1.00	.747	.810	4.4	.96	.62
1.25	.946	1.022	6.7	.95	.65
1.50	1.143	1.234	9.5	.95	.66
1.75	1.337	1.446	12.8	.95	.68
2.00	1.528	1.659	16.5	.96	.69
2.25	1.716	1.871	20.7	.97	.70
2.50	1.900	2.084	25.3	.99	.71
2.75	1.950	1.112	26.8	1.00	1.85
3.00	2.119	1.211	31.4	1.00	1.87
3.25	2.285	1.306	36.3	1.00	1.89
3.50	2.448	1.396	41.4	1.00	1.91
3.75	2.609	1.483	46.8	1.00	1.93
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .000900

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.364	.123	1.0	1.00	2.38
.75	.545	.222	2.2	1.00	2.14
1.00	.725	.332	3.9	1.00	2.00
1.25	.923	1.005	6.8	.99	.69
1.50	1.116	1.214	9.6	.99	.70

1.75	1.305	1.423	13.0	.99	.71
2.00	1.434	.792	14.8	1.00	1.84
2.25	1.608	.903	18.5	1.00	1.84
2.50	1.780	1.010	22.5	1.00	1.84
2.75	1.950	1.112	26.8	1.00	1.85
3.00	2.119	1.211	31.4	1.00	1.87
3.25	2.285	1.306	36.3	1.00	1.89
3.50	2.448	1.396	41.4	1.00	1.91
3.75	2.609	1.483	46.8	1.00	1.93
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .001000

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.364	.123	1.0	1.00	2.38
.75	.545	.222	2.2	1.00	2.14
1.00	.725	.332	3.9	1.00	2.00
1.25	.904	.447	6.0	1.00	1.92
1.50	1.082	.563	8.5	1.00	1.87
1.75	1.259	.679	11.5	1.00	1.85
2.00	1.434	.792	14.8	1.00	1.84
2.25	1.608	.903	18.5	1.00	1.84
2.50	1.780	1.010	22.5	1.00	1.84
2.75	1.950	1.112	26.8	1.00	1.85
3.00	2.119	1.211	31.4	1.00	1.87
3.25	2.285	1.306	36.3	1.00	1.89
3.50	2.448	1.396	41.4	1.00	1.91
3.75	2.609	1.483	46.8	1.00	1.93
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .001250

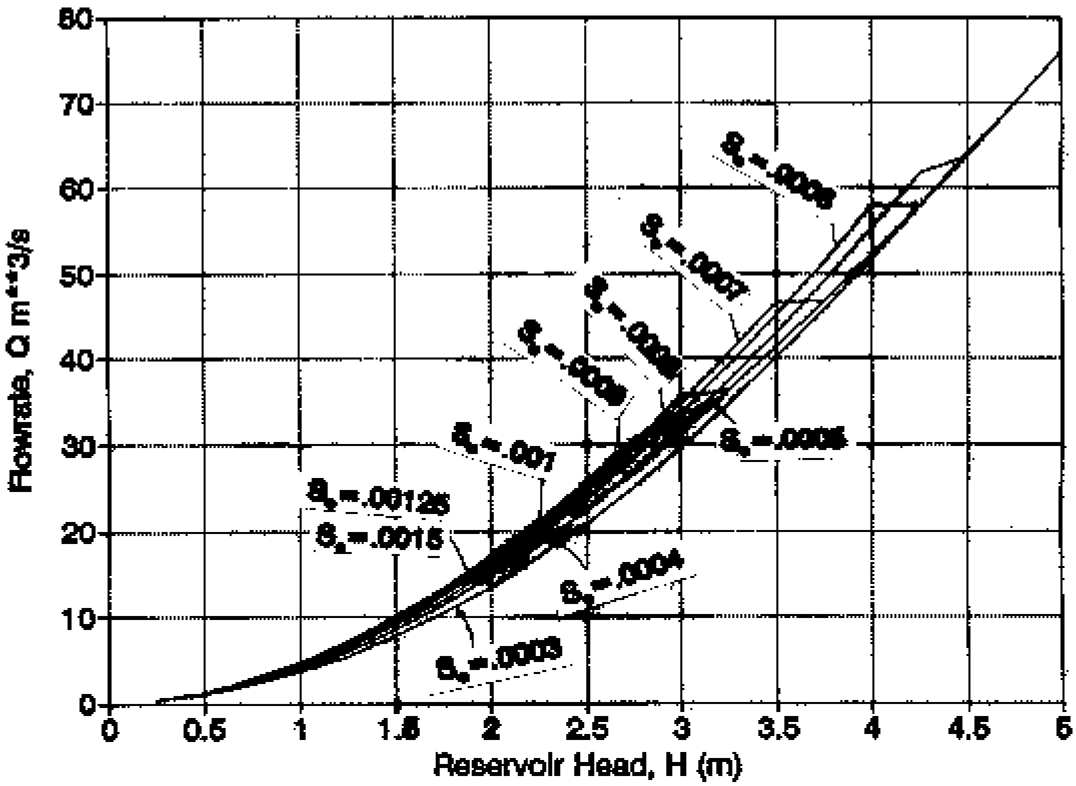
				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.364	.123	1.0	1.00	2.38
.75	.545	.222	2.2	1.00	2.14
1.00	.725	.332	3.9	1.00	2.00
1.25	.904	.447	6.0	1.00	1.92
1.50	1.082	.563	8.5	1.00	1.87
1.75	1.259	.679	11.5	1.00	1.85
2.00	1.434	.792	14.8	1.00	1.84
2.25	1.608	.903	18.5	1.00	1.84
2.50	1.780	1.010	22.5	1.00	1.84
2.75	1.950	1.112	26.8	1.00	1.85
3.00	2.119	1.211	31.4	1.00	1.87
3.25	2.285	1.306	36.3	1.00	1.89
3.50	2.448	1.396	41.4	1.00	1.91
3.75	2.609	1.483	46.8	1.00	1.93
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

So = .001500

				SQUARED	
H	Y1	Y2	Q	Fr1	Fr2
.25	.183	.044	.3	1.00	2.92
.50	.364	.123	1.0	1.00	2.38
.75	.545	.222	2.2	1.00	2.14
1.00	.725	.332	3.9	1.00	2.00
1.25	.904	.447	6.0	1.00	1.92
1.50	1.082	.563	8.5	1.00	1.87
1.75	1.259	.679	11.5	1.00	1.85
2.00	1.434	.792	14.8	1.00	1.84

2.25	1.608	.903	18.5	1.00	1.84
2.50	1.780	1.010	22.5	1.00	1.84
2.75	1.950	1.112	26.8	1.00	1.85
3.00	2.119	1.211	31.4	1.00	1.87
3.25	2.285	1.306	36.3	1.00	1.89
3.50	2.448	1.396	41.4	1.00	1.91
3.75	2.609	1.483	46.8	1.00	1.93
4.00	2.767	1.565	52.3	1.00	1.96
4.25	2.921	1.643	58.0	1.00	1.98
4.50	3.071	1.718	63.8	1.00	2.01
4.75	3.217	1.788	69.8	1.00	2.04
5.00	3.358	1.855	75.9	1.00	2.07

The following graph show the above data plotted.



The following TK-Solver model might also be used to develop the delivery diagrams for different bottom slopes as long as the flow in the downstream trapezoidal channel is uniform, i.e. critical flow in the upstream circular channel does not control. Only bottom slopes of S_o = .0002, S_o = .0003, and S_o = .0004 are given in the solution table, and for the largest of these bottom slopes the table stops for H = 4.6 m.

PRB2_72.TK

```
==================== VARIABLE SHEET ====================
St Input─────  Name───  Output───  Unit─────  Comment─────────
   9.81        g
   1           Cu
   5           D
   3           b
   1.4         m
   .013        n
L  .001        So
L  3           H
   .1          KE
```

```
   .1        KL
   .8        Delz
LG 25.527868 Q
LG 2.6826585 Y1
LG 1.8026504 Y2
             beta    1.6439249
             g2      19.62
             KE1     .05606524
             KL1     .05606524
             Cun     76.923077
             SoS     .03162278
             DS2     6.25
             A1      10.729956
             A2      9.9573194
L            Fr1     .5178297
L            Fr2     .73585924
```

```
==================== RULE SHEET =========================
S Rule---------------------------------------------------
  g2=2*g
  KE1=(1+KE)/g2
  KL1=(1+KL)/g2
  Cun=Cu/n
  SoS=sqrt(So)
  DS2=D*D/4
  beta=acos(1-2*Y1/D)
  A1=DS2*(beta-cos(beta)*sin(beta))
  A2=(b+m*Y2)*Y2
  Q=Cun*A2^1.666667/(b+2*Y2*sqrt(m*m+1))^.6666667*SoS
  H=Y1+KE1*(Q/A1)^2
  Y1+(Q/A1)^2/g2=Y2+KL1*(Q/A2)^2+Delz
  Fr1=sqrt(Q^2*(D*sin(beta))/(g*A1^3))
  Fr2=sqrt(Q^2*(b+2*m*Y2)/(g*A2^3))
```

==================== TABLE: ============================

Title:

Element	So	H	Q	Y1	Y2	Fr1	Fr2
1	.0002	.85	.02134037	.84999479	.04886374	.00401022	.20785585
2	.0002	1.1	.42218873	1.0990234	.28963981	.04794599	.26861683
3	.0002	1.35	1.1903641	1.3456179	.52934428	.09109973	.28872608
4	.0002	1.6	2.3026997	1.5896855	.76879204	.12754231	.30081731
5	.0002	1.85	3.7678966	1.8312416	1.0080405	.15882401	.30957027
6	.0002	2.1	5.6024413	2.0701564	1.2470467	.18656129	.31653802
7	.0002	2.35	7.8252233	2.3061596	1.4857503	.21190103	.32240445
8	.0002	2.6	10.45563	2.538853	1.7240881	.23561911	.32752351
9	.0002	2.85	13.51269	2.7677078	1.9619938	.25824153	.33209947
10	.0002	3.1	17.014592	2.9920475	2.1993948	.28012671	.33626
11	.0002	3.35	20.978322	3.2110163	2.4362079	.30151568	.3400899
12	.0002	3.6	25.419315	3.423535	2.6723339	.32256108	.3436482
13	.0002	3.85	30.351053	3.6282402	2.9076516	.34334366	.34697752
14	.0002	4.1	35.784555	3.8234066	3.14201	.36388403	.35010954
15	.0002	4.35	41.727721	4.0068523	3.3752204	.38416076	.35306823
16	.0002	4.6	48.184512	4.1758304	3.6070459	.40415784	.35587195
17	.0002	4.85	55.153963	4.326922	3.8371926	.42399219	.35853483
18	.0002	5.1	62.629109	4.4559529	4.0653018	.44422667	.3610677
19	.0003	.85	.02565316	.84999247	.04832009	.00482069	.25412402
20	.0003	1.1	.50238672	1.0986156	.28476742	.05709509	.32827975
21	.0003	1.35	1.4116924	1.3438136	.51965963	.10832045	.35287643
22	.0003	1.6	2.7248883	1.5854499	.75418679	.15170773	.36766712
23	.0003	1.85	4.4509076	1.8235232	.9884386	.18914684	.37836414
24	.0003	2.1	6.6076905	2.0578214	1.2223553	.22257363	.38686862
25	.0003	2.35	9.215804	2.2879256	1.4558498	.25337518	.39401944
26	.0003	2.6	12.296146	2.5132236	1.6888294	.28251667	.40025123
27	.0003	2.85	15.86886	2.7328998	1.9211954	.31069134	.40581491
28	.0003	3.1	19.952666	2.9459026	2.1528386	.33842784	.41086716
29	.0003	3.35	24.564284	3.1508891	2.383632	.36616636	.41551179
30	.0003	3.6	29.717824	3.3461456	2.6134224	.3943218	.41982082
31	.0003	3.85	35.424017	3.5294762	2.8420191	.42335249	.4238459
32	.0003	4.1	41.689187	3.6980484	3.0691806	.45385877	.42762493
33	.0003	4.35	48.513797	3.8481697	3.2945946	.48675	.431186

34	.0003	4.6	55.890276	3.9749328	3.5178478	.52354944	.43454972
35	.0003	4.85	63.799258	4.0715378	3.7383683	.56697151	.43773027
36	.0003	5.1	72.201448	4.1276259	3.9552844	.62210413	.44073504
37	.0004	.85	.02908332	.84999032	.04779176	.0054653	.2929312
38	.0004	1.1	.56400102	1.0982536	.28008352	.06413872	.37826557
39	.0004	1.35	1.5796418	1.3422284	.51036737	.12148589	.40663144
40	.0004	1.6	3.0426694	1.5817406	.74018882	.17016972	.4236911
41	.0004	1.85	4.9616685	1.816765	.96967206	.21236588	.43601852
42	.0004	2.1	7.3550722	2.0470024	1.1987437	.25026729	.44580756
43	.0004	2.35	10.244024	2.2718826	1.4272925	.28546194	.45402817
44	.0004	2.6	13.649761	2.4905731	1.6551975	.31908716	.46118347
45	.0004	2.85	17.59232	2.7019558	1.8823283	.35200698	.46756395
46	.0004	3.1	22.089639	2.9045685	2.1085377	.38494673	.47335079
47	.0004	3.35	27.156717	3.0965064	2.3336507	.4186033	.47866373
48	.0004	3.6	32.804575	3.2752706	2.5574496	.45376006	.48358535
49	.0004	3.85	39.038808	3.4375259	2.7796521	.49144419	.48817422
50	.0004	4.1	45.857295	3.5786865	2.9998761	.53319197	.49247221
51	.0004	4.35	53.246064	3.6920984	3.217571	.58156634	.49650835
52	.0004	4.6	61.170097	3.7670447	3.4318518	.64134827	.50029947

73.

In developing the delivery diagrams in the previous problem you should have computed supercritical depths at the beginning of the trapezoidal section when critical flow occurs in the circular entrance. To have smooth flow from this position on, when this condition of critical flow occurs, the channel must have a bottom slope sufficient to maintain uniform flow at this depth, or a steep slope. Compute these slopes and compare them with S_c needed to separate a mild from s steep channel if the trapezoidal channel receives water directly from the reservoir.

Wanted: Compute slopes to maintain uniform flow., and compare them with S_c

Solution: The needed slope S_o is computed from Mannings Equation, or

$S_o = \{nQ/C_u(P/A)^{2/3}/A\}^2$

To determine what the slope S_c is if the trapezoidal channel received the water directly from the reservoir it is necessary to first solve for Y_c (in the trapezoidal section), and then use this depth to compute the critical slope, or

$S_c = \{nQ/C_u(P_c/A_c)^{2/3}/A_c\}^2$

The program PRB2_66H.FOR reads in the table of data from the previous problem that provides the delivery diagrams, and computes Y_c and these slopes as well as the difference between them. The output from the program is given after its listing. The difference S_o-S_c is negative whenever the flow into the channel is not controlled by critical flow at the entrance, and these values say nothing about how steep the trapezoidal channel would need to be to be considered steep. However, whenever critical flow occurs at the entrance then this difference S_o-S_c shows that for a channel with the given circular entrance, the slope of the trapezoidal channel's bottom must be larger than would be required if the entrance were not circular.

An alternative would be to have the change in the bottom elevation across the transition form the circular to the trapezoidal sections, or change the bottom elevation so that supercritical flow does not exit at the beginning of the trapezoidal section.

Program PRB2_66H.FOR

```
      CHARACTER*30 CHAR
      REAL B/3./,FM/1.4/,FN/.013/,G/9.91/
      FM2=2.*SQRT(FM*FM+1.)
15    READ(3,'(A30)',END=99) CHAR
      WRITE(4,'(A30)') CHAR
      DO 20 I=1,20
      READ(3,110) H,Y1,Y2,Q,Fr1,Fr2
110   FORMAT(F6.2,2F8.3,F8.1,2F7.2)
      A=(B+FM*Y2)*Y2
      So=(FN*Q*((B+FM2*Y1)/A)**.66666667/A)**2
      NCT=0
      Yc=1.1*Y2
16    F=Q**2*(B+2.*FM*Yc)-(G*((B+FM*Yc)*Yc)**3)
      Yi=1.005*Yc
      F1=Q**2*(B+2.*FM*Yi)-(G*((B+FM*Yi)*Yi)**3)
      DIF=(Yi-Yc)*F/(F1-F)
      Yc=Yc-DIF
      NCT=NCT+1
      IF(NCT.LT.30 .AND. ABS(DIF).GT. 1.E-5) GO TO 16
      IF(NCT.EQ.30) WRITE(*,*)' DID NOT CONVERGE',DIF
      Ac=(B+FM*Yc)*Yc
      Sc=(FN*Q*((B+FM2*Yc)/Ac)**.66666667/Ac)**2
20    WRITE(4,112) H,Y1,Y2,Q,Fr1,Fr2,Yc,So,Sc,So-Sc
112   FORMAT(F6.2,2F7.3,F8.1,2F7.2,F7.3,2F8.6,F9.6)
```

```
      GO TO 15
99    STOP
      END
```

So = .000200

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.201	.189	.2	.85	.26	.076	.000193	.004086	-.003893
.50	.438	.428	.9	.64	.30	.202	.000241	.003077	-.002836
.75	.673	.667	1.8	.57	.32	.315	.000204	.002742	-.002538
1.00	.908	.906	3.1	.55	.32	.443	.000200	.002525	-.002325
1.25	1.140	1.144	4.7	.54	.33	.572	.000194	.002382	-.002188
1.50	1.371	1.383	6.7	.54	.35	.708	.000192	.002273	-.002081
1.75	1.600	1.621	9.0	.55	.35	.843	.000187	.002189	-.002003
2.00	1.826	1.859	11.7	.56	.35	.982	.000183	.002120	-.001937
2.25	2.050	2.098	14.8	.57	.36	1.124	.000179	.002061	-.001881
2.50	2.271	2.335	18.2	.59	.36	1.263	.000174	.002011	-.001837
2.75	2.490	2.573	21.9	.61	.36	1.400	.000168	.001968	-.001800
3.00	2.705	2.810	26.1	.63	.37	1.542	.000164	.001929	-.001764
3.25	2.917	3.047	30.6	.66	.37	1.681	.000159	.001894	-.001734
3.50	3.125	3.284	35.4	.69	.37	1.817	.000154	.001862	-.001709
3.75	3.329	3.520	40.5	.72	.39	1.952	.000148	.001834	-.001686
4.00	3.528	3.756	45.9	.75	.39	2.085	.000142	.001808	-.001666
4.25	3.722	3.992	51.6	.79	.39	2.216	.000136	.001784	-.001648
4.50	3.910	4.228	57.5	.84	.39	2.344	.000130	.001762	-.001632
4.75	4.090	4.463	63.5	.89	.39	2.466	.000123	.001742	-.001619
5.00	4.261	4.697	69.5	.96	.40	2.583	.000116	.001724	-.001608

So = .000300

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.187	.185	.2	.97	.32	.076	.000205	.004086	-.003881
.50	.416	.419	1.0	.75	.36	.216	.000316	.003022	-.002705
.75	.645	.653	2.1	.69	.37	.347	.000296	.002677	-.002382
1.00	.873	.887	3.5	.66	.39	.477	.000273	.002482	-.002209
1.25	1.098	1.120	5.4	.64	.40	.622	.000274	.002338	-.002064
1.50	1.322	1.354	7.7	.64	.41	.769	.000271	.002232	-.001961
1.75	1.543	1.587	10.3	.65	.42	.913	.000262	.002153	-.001891
2.00	1.761	1.820	13.3	.66	.42	1.058	.000254	.002087	-.001833
2.25	1.976	2.053	16.8	.68	.44	1.208	.000249	.002030	-.001781
2.50	2.188	2.286	20.6	.69	.45	1.353	.000240	.001982	-.001742
2.75	2.397	2.519	24.9	.71	.45	1.503	.000234	.001939	-.001705
3.00	2.603	2.751	29.5	.74	.45	1.648	.000226	.001902	-.001676
3.25	2.804	2.983	34.4	.77	.46	1.790	.000216	.001868	-.001652
3.50	3.002	3.215	39.8	.79	.46	1.934	.000209	.001838	-.001629
3.75	3.195	3.447	45.4	.82	.47	2.073	.000200	.001810	-.001611
4.00	3.383	3.678	51.4	.87	.47	2.212	.000191	.001785	-.001593
4.25	3.566	3.909	57.5	.91	.47	2.344	.000181	.001762	-.001580
4.50	3.742	4.140	63.9	.95	.48	2.474	.000172	.001741	-.001569
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .000400

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.399	.411	1.0	.84	.41	.216	.000335	.003022	-.002687
.75	.621	.640	2.2	.77	.44	.357	.000345	.002658	-.002313
1.00	.842	.869	3.8	.73	.45	.502	.000343	.002453	-.002110
1.25	1.061	1.098	5.9	.72	.47	.656	.000348	.002311	-.001962
1.50	1.278	1.327	8.3	.72	.48	.804	.000337	.002211	-.001875
1.75	1.493	1.555	11.2	.73	.49	.958	.000332	.002131	-.001799
2.00	1.704	1.784	14.4	.74	.49	1.107	.000318	.002067	-.001749
2.25	1.912	2.013	18.2	.75	.50	1.263	.000312	.002011	-.001699
2.50	2.117	2.241	22.3	.77	.51	1.414	.000302	.001964	-.001662
2.75	2.319	2.469	26.8	.79	.51	1.564	.000291	.001923	-.001632
3.00	2.516	2.697	31.7	.82	.52	1.713	.000279	.001886	-.001607
3.25	2.710	2.924	37.0	.85	.53	1.861	.000269	.001853	-.001584
3.50	2.900	3.152	42.7	.88	.53	2.007	.000258	.001823	-.001565
3.75	3.084	3.379	48.7	.91	.54	2.150	.000246	.001796	-.001549
4.00	3.264	3.606	54.9	.94	.54	2.288	.000234	.001771	-.001537
4.25	3.439	3.833	61.4	.98	.54	2.424	.000222	.001749	-.001527

4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .000500

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	$S_o - S_c$
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.383	.404	1.1	.91	.45	.230	.000426	.002973	-.002547
.75	.599	.628	2.3	.84	.48	.367	.000399	.002640	-.002241
1.00	.815	.853	4.0	.80	.50	.518	.000403	.002436	-.002033
1.25	1.028	1.077	6.2	.79	.52	.676	.000410	.002296	-.001886
1.50	1.240	1.302	8.8	.79	.53	.832	.000403	.002195	-.001792
1.75	1.448	1.526	11.8	.79	.54	.987	.000392	.002117	-.001725
2.00	1.653	1.750	15.2	.81	.55	1.141	.000379	.002054	-.001675
2.25	1.856	1.974	19.1	.82	.56	1.298	.000369	.002000	-.001631
2.50	2.054	2.198	23.4	.84	.57	1.452	.000356	.001953	-.001597
2.75	2.250	2.422	28.2	.86	.57	1.608	.000345	.001912	-.001567
3.00	2.441	2.646	33.3	.89	.58	1.759	.000330	.001875	-.001545
3.25	2.628	2.869	38.8	.91	.58	1.908	.000316	.001843	-.001526
3.50	2.811	3.093	44.7	.94	.59	2.056	.000302	.001813	-.001511
3.75	2.990	3.316	50.9	.97	.59	2.200	.000288	.001787	-.001498
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .000600

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	$S_o - S_c$
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.370	.397	1.1	.96	.49	.230	.000449	.002973	-.002523
.75	.580	.617	2.4	.89	.53	.377	.000459	.002623	-.002164
1.00	.790	.837	4.2	.86	.55	.534	.000472	.002419	-.001947
1.25	.999	1.058	6.4	.85	.57	.689	.000463	.002286	-.001823
1.50	1.205	1.278	9.1	.85	.57	.849	.000459	.002186	-.001727
1.75	1.408	1.498	12.2	.85	.59	1.006	.000447	.002109	-.001662
2.00	1.608	1.718	15.8	.87	.60	1.166	.000437	.002045	-.001608
2.25	1.805	1.938	19.8	.88	.61	1.324	.000423	.001991	-.001568
2.50	1.998	2.158	24.3	.90	.62	1.483	.000410	.001945	-.001535
2.75	2.188	2.378	29.2	.92	.62	1.639	.000395	.001904	-.001509
3.00	2.374	2.598	34.4	.94	.63	1.790	.000377	.001868	-.001492
3.25	2.556	2.818	40.1	.97	.64	1.942	.000361	.001836	-.001475
3.50	2.733	3.037	46.2	1.00	.64	2.092	.000346	.001807	-.001461
3.75	2.609	1.483	46.8	1.00	1.93	2.106	.012132	.001804	.010328
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .000700

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	$S_o - S_c$
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.364	.123	1.0	1.00	2.38	.216	.026824	.003022	.023803
.75	.563	.607	2.5	.94	.57	.387	.000524	.002607	-.002083
1.00	.768	.823	4.3	.91	.59	.541	.000522	.002411	-.001889
1.25	.971	1.039	6.6	.90	.61	.702	.000523	.002277	-.001754
1.50	1.172	1.255	9.3	.90	.62	.860	.000510	.002180	-.001670
1.75	1.371	1.472	12.5	.91	.63	1.021	.000499	.002103	-.001604
2.00	1.566	1.688	16.2	.92	.65	1.183	.000489	.002039	-.001550
2.25	1.758	1.904	20.3	.93	.66	1.342	.000473	.001986	-.001512
2.50	1.947	2.120	24.9	.95	.66	1.503	.000459	.001939	-.001480
2.75	2.132	2.336	29.9	.97	.67	1.660	.000442	.001899	-.001456
3.00	2.313	2.552	35.3	.99	.68	1.815	.000424	.001863	-.001439
3.25	2.285	1.306	36.3	1.00	1.89	1.842	.011564	.001857	.009707
3.50	2.448	1.396	41.4	1.00	1.91	1.975	.011822	.001829	.009993
3.75	2.609	1.483	46.8	1.00	1.93	2.106	.012132	.001804	.010328
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484

4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .000800

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.364	.123	1.0	1.00	2.38	.216	.026824	.003022	.023803
.75	.547	.598	2.5	.99	.60	.387	.000549	.002607	-.002058
1.00	.747	.810	4.4	.96	.62	.549	.000575	.002404	-.001828
1.25	.946	1.022	6.7	.95	.65	.708	.000569	.002273	-.001703
1.50	1.143	1.234	9.5	.95	.66	.870	.000564	.002174	-.001610
1.75	1.337	1.446	12.8	.95	.68	1.035	.000557	.002097	-.001540
2.00	1.528	1.659	16.5	.96	.69	1.195	.000540	.002034	-.001495
2.25	1.716	1.871	20.7	.97	.70	1.357	.000525	.001981	-.001456
2.50	1.900	2.084	25.3	.99	.71	1.516	.000505	.001936	-.001431
2.75	1.950	1.112	26.8	1.00	1.85	1.564	.011252	.001923	.009329
3.00	2.119	1.211	31.4	1.00	1.87	1.704	.011367	.001888	.009479
3.25	2.285	1.306	36.3	1.00	1.89	1.842	.011564	.001857	.009707
3.50	2.448	1.396	41.4	1.00	1.91	1.975	.011822	.001829	.009993
3.75	2.609	1.483	46.8	1.00	1.93	2.106	.012132	.001804	.010328
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .000900

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.364	.123	1.0	1.00	2.38	.216	.026824	.003022	.023803
.75	.545	.222	2.2	1.00	2.14	.357	.018870	.002658	.016212
1.00	.725	.332	3.9	1.00	2.00	.510	.015627	.002444	.013183
1.25	.923	1.005	6.8	.99	.69	.714	.000621	.002268	-.001648
1.50	1.116	1.214	9.6	.99	.70	.876	.000609	.002172	-.001562
1.75	1.305	1.423	13.0	.99	.71	1.044	.000607	.002093	-.001485
2.00	1.434	.792	14.8	1.00	1.84	1.124	.011472	.002061	.009411
2.25	1.608	.903	18.5	1.00	1.84	1.275	.011271	.002007	.009263
2.50	1.780	1.010	22.5	1.00	1.84	1.421	.011187	.001962	.009225
2.75	1.950	1.112	26.8	1.00	1.85	1.564	.011252	.001923	.009329
3.00	2.119	1.211	31.4	1.00	1.87	1.704	.011367	.001888	.009479
3.25	2.285	1.306	36.3	1.00	1.89	1.842	.011564	.001857	.009707
3.50	2.448	1.396	41.4	1.00	1.91	1.975	.011822	.001829	.009993
3.75	2.609	1.483	46.8	1.00	1.93	2.106	.012132	.001804	.010328
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .001000

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.364	.123	1.0	1.00	2.38	.216	.026824	.003022	.023803
.75	.545	.222	2.2	1.00	2.14	.357	.018870	.002658	.016212
1.00	.725	.332	3.9	1.00	2.00	.510	.015627	.002444	.013183
1.25	.904	.447	6.0	1.00	1.92	.663	.013591	.002306	.011286
1.50	1.082	.563	8.5	1.00	1.87	.815	.012407	.002205	.010203
1.75	1.259	.679	11.5	1.00	1.85	.973	.011871	.002124	.009747
2.00	1.434	.792	14.8	1.00	1.84	1.124	.011472	.002061	.009411
2.25	1.608	.903	18.5	1.00	1.84	1.275	.011271	.002007	.009263
2.50	1.780	1.010	22.5	1.00	1.84	1.421	.011187	.001962	.009225
2.75	1.950	1.112	26.8	1.00	1.85	1.564	.011252	.001923	.009329
3.00	2.119	1.211	31.4	1.00	1.87	1.704	.011367	.001888	.009479
3.25	2.285	1.306	36.3	1.00	1.89	1.842	.011564	.001857	.009707
3.50	2.448	1.396	41.4	1.00	1.91	1.975	.011822	.001829	.009993
3.75	2.609	1.483	46.8	1.00	1.93	2.106	.012132	.001804	.010328
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976

5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .001250

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.364	.123	1.0	1.00	2.38	.216	.026824	.003022	.023803
.75	.545	.222	2.2	1.00	2.14	.357	.018870	.002658	.016212
1.00	.725	.332	3.9	1.00	2.00	.510	.015627	.002444	.013183
1.25	.904	.447	6.0	1.00	1.92	.663	.013591	.002306	.011286
1.50	1.082	.563	8.5	1.00	1.87	.815	.012407	.002205	.010203
1.75	1.259	.679	11.5	1.00	1.85	.973	.011871	.002124	.009747
2.00	1.434	.792	14.8	1.00	1.84	1.124	.011472	.002061	.009411
2.25	1.608	.903	18.5	1.00	1.84	1.275	.011271	.002007	.009263
2.50	1.780	1.010	22.5	1.00	1.84	1.421	.011187	.001962	.009225
2.75	1.950	1.112	26.8	1.00	1.85	1.564	.011252	.001923	.009329
3.00	2.119	1.211	31.4	1.00	1.87	1.704	.011367	.001888	.009479
3.25	2.285	1.306	36.3	1.00	1.89	1.842	.011564	.001857	.009707
3.50	2.448	1.396	41.4	1.00	1.91	1.975	.011822	.001829	.009993
3.75	2.609	1.483	46.8	1.00	1.93	2.106	.012132	.001804	.010328
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

So = .001500

H	Y_1	Y_2	Q_3	F_{r1}	$F_{r@}$	Y_C	S_c	S_o	S_o-S_c
.25	.183	.044	.3	1.00	2.92	.099	.067705	.003771	.063934
.50	.364	.123	1.0	1.00	2.38	.216	.026824	.003022	.023803
.75	.545	.222	2.2	1.00	2.14	.357	.018870	.002658	.016212
1.00	.725	.332	3.9	1.00	2.00	.510	.015627	.002444	.013183
1.25	.904	.447	6.0	1.00	1.92	.663	.013591	.002306	.011286
1.50	1.082	.563	8.5	1.00	1.87	.815	.012407	.002205	.010203
1.75	1.259	.679	11.5	1.00	1.85	.973	.011871	.002124	.009747
2.00	1.434	.792	14.8	1.00	1.84	1.124	.011472	.002061	.009411
2.25	1.608	.903	18.5	1.00	1.84	1.275	.011271	.002007	.009263
2.50	1.780	1.010	22.5	1.00	1.84	1.421	.011187	.001962	.009225
2.75	1.950	1.112	26.8	1.00	1.85	1.564	.011252	.001923	.009329
3.00	2.119	1.211	31.4	1.00	1.87	1.704	.011367	.001888	.009479
3.25	2.285	1.306	36.3	1.00	1.89	1.842	.011564	.001857	.009707
3.50	2.448	1.396	41.4	1.00	1.91	1.975	.011822	.001829	.009993
3.75	2.609	1.483	46.8	1.00	1.93	2.106	.012132	.001804	.010328
4.00	2.767	1.565	52.3	1.00	1.96	2.232	.012470	.001781	.010689
4.25	2.921	1.643	58.0	1.00	1.98	2.354	.012858	.001760	.011098
4.50	3.071	1.718	63.8	1.00	2.01	2.472	.013225	.001741	.011484
4.75	3.217	1.788	69.8	1.00	2.04	2.588	.013699	.001723	.011976
5.00	3.358	1.855	75.9	1.00	2.07	2.700	.014170	.001707	.012463

74.

For the trapezoidal channel with a circular entrance of Problem 71 compute the change in bottom elevation Δz needed so that critical flow occurs in both the circular section and at the beginning of the trapezoidal channel when the reservoir head is H = 2.5 m.

Wanted: For the trapezoidal channel with a circular entrance, Problem 71 compute Δz so critical flow occurs in both sections.

Solution:
First critical conditions are computed in the circular section (D=5 m) for H=2.5 m, and the solution gives:

$Q_c = 22.375\ m^3/s$, $Y_c = 1.780$ m and $E_c = 2.430$ m.

Next for this flow rate of 22.375 m^3/s the critical depth is computed in the trapezoidal channel (b = 3 m, m=1.4) and $Y_c = 1.404$ m, with $E_c = 1.901$ m. If we ignore the local loss between the sections then $\Delta z = 2.430 - 1.901 = 0.529$ m (rise in bottom).

75.

Develop an iterative solution for either the subcritical, or supercritical depth corresponding to a given value of the specific energy E in a trapezoidal channel based on being able to solve the cubic equation for the three real roots if the channel is rectangular. The development of this iterative solution can be based on the follow two observations. (1) When the channel is rectangular, the root Y_2 (or the root obtained by adding 2π to θ to make the argument for the cosine) is the subcritical depth, and root Y_3 (obtained by adding 4π to θ to make the argument of the cosine) is the supercritical depth. (2) A mean flow rate per unit width q in a trapezoidal channel can be define by $q = 2Q/(T+b) = Q/(mY+b) = Q/(A/Y)$. Use this iterative approach to solve both the sub- and supercritical depth in a 10 ft wide trapezoidal channel with m = 1 if E = 5 ft, and the flow rate is Q = 400 cfs.

Wanted: Develop iterative solution as requested

Solution:

The program ROOTSTR.FOR implements the requested solution. Note that it iteratively modifies the average width b+2mY (which is AW in the program) until its value doesn't change between consecutive iterations. This average width is divided into Q to get q (qs in the program).

Program ROOTSTR.FOR

```
C Iteratively solve for subcr. or supercr. depth in a
C trapezoidal channel for a given E
      PARAMETER (PI=3.14159265)
1     WRITE(*,*)' Give: Q,b,m,E,g & 1=SUBCR. or 2=SUPERCR.'
      READ(*,*) Q,B,FM,E,g,ITYPE
      IF(Q.LT.1.E-5) STOP
      NCT=0
      IF(ITYPE.EQ.1) THEN
      PII=2.*PI
      ELSE
      PII=4.*PI
      ENDIF
      AW1=(1.+.5*FM)*B
      E3=E/3.
2     AW=AW1
      qs=Q/AW
      THETA=ACOS(((6.75*qs*qs/g-E**3)/27.)/E3**3)
      Y=E3*(1.-2.*COS((THETA+PII)/3.))
      NCT=NCT+1
      AW1=B+FM*Y
C      WRITE(*,*) NCT,AW,AW1,qs
      IF(ABS(AW1-AW).GT.1.E-4 .AND. NCT.LT.40) GO TO 2
      WRITE(6,100) Q,B,FM,E,g,Y
100   FORMAT(' Q=',F8.3,' b=',F8.3,' m=',F8.2,/,' E=',F8.3,' g=',F8.2,
     &' Depth Y=',F9.3)
      GO TO 1
      END
```

Input to solve Problem:

400 10 1 5 32.2 1

gives as the solution Y = 4.370 ft and

400 10 1 5 32.2 2

gives as the solution Y = 2.532 ft.

The above program does not have any logic that checks whether $E>E_c$. The program below first computes the critical depth and then the critical specific energy, and tells its user that the given E must be larger than the critical value of the specific energy.

Program ROOTTR1.FOR

```
C Iteratively solve for subcr. or supercr. depth in a
C trapezoidal channel for a given E.  The different between this program
C and ROOTSTR is that it computes Yc and Ec and gives a minimum allowable E.
      PARAMETER (PI=3.14159265)
1     WRITE(*,*)' Give: Q,b,m,g & 1=SUBCR. or 2=SUPERCR.'
      READ(*,*) Q,B,FM,g,ITYPE
      IF(Q.LT.1.E-5) STOP
      YS=((Q/B)**2/g)**.3333333
      Yc=CRIT(Q,B,FM,g,YS)
      Ec=Yc+(Q/((B+FM*Yc)*Yc))**2/(2.*g)
      WRITE(*,120) Yc,Ec
120   FORMAT(' Yc=',F8.3,' Ec=',F8.3,' Give E>Ec')
5     READ(*,*) E
      NCT=0
      IF(ITYPE.EQ.1) THEN
      PII=2.*PI
      ELSE
      PII=4.*PI
      ENDIF
      AW1=(1.+.5*FM)*B
      E3=E/3.
2     AW=AW1
      qs=Q/AW
      THETA=ACOS(((6.75*qs*qs/g-E**3)/27.)/E3**3)
      Y=E3*(1.-2.*COS((THETA+PII)/3.))
      NCT=NCT+1
      AW1=B+FM*Y
C      WRITE(*,*) NCT,AW,AW1,qs
      IF(ABS(AW1-AW).GT.1.E-4 .AND. NCT.LT.40) GO TO 2
      WRITE(6,100) Q,B,FM,E,g,Y
100   FORMAT(' Q=',F8.3,' b=',F8.3,' m=',F8.2,/,' E=',F8.3,' g=',F8.2,
     &' Depth Y=',F9.3)
      WRITE(6,130)
130   FORMAT(/,' Give: 0=STOP, 1=Dif. E but same size, 2=Dif. size')
      READ(*,*) NCT
      IF(NCT.EQ.1) THEN
      GO TO 5
      ELSE IF(NCT.EQ.2) THEN
      GO TO 1
      ELSE
      STOP
      ENDIF
      END
      FUNCTION CRIT(Q,B,FM,g,YS)
      M=0
1     F=Q*Q*(B+2.*FM*YS)-g*((B+FM*YS)*YS)**3
      M=M+1
      IF(MOD(M,2).NE.0) THEN
      F1=F
      YS1=YS
      YS=1.005*YS
      GO TO 1
      ENDIF
      DIF=(YS-YS1)*F1/(F-F1)
      YS=YS1-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 1
      CRIT=YS
      RETURN
      END
```

76.

Modify the iterative solution method developed to solve for the depths associated with a given specific energy E in a trapezoidal channel of the previous problem to find the subcritical depth, or the supercritical depth in a circular channel with a known diameter, and a specified value of the specific energy E.

Wanted: Modify iterative solution in previous problem to find subcritical or supercritical depth as requested for circular channel.

Solution:

Note for a circular section that an equivalent width to a rectangular channel can be defined by dividing the area by the depth or b = A/Y. Thus for a circular section the depth associated with a specified E is obtained by finding the appropriate root, as is done in the solution method for the previous problem, by iteratively adjusting this width. The program listed below obtains the solution for either the subcritical or supercritical depth using this approach. Note that it will converge rapidly when the width doesn't change much with depth as when Y=D/2, but when Y is very small, or near D, then the width changes rapidly with Y and its convergence is slow, and may even fail.

Program ROOTCIR.FOR

```
C Iteratively solve for subcr. or supercr. depth in a
C Circular channel for a given E.  The program
C computes Yc and Ec and gives a minimum allowable E.
      PARAMETER (PI=3.14159265)
1     WRITE(*,*)' Give: Q,D,g & 1=SUBCR. or 2=SUPERCR.'
      READ(*,*) Q,D,g,ITYPE
      IF(Q.LT.1.E-5) STOP
      IF(ITYPE.EQ.1) THEN
      PII=2.*PI
      AW1=.95*D
      ELSE
      PII=4.*PI
      AW1=.8*D
      ENDIF
      YS=((Q/(.5*D))**2/g)**.3333333
      Yc=CRIT(Q,D,g,YS)
      BETA=ACOS(1.-2.*Yc/D)
      Ec=Yc+(Q/(.25*D*D*(BETA-COS(BETA)*SIN(BETA))))**2/(2.*g)
5     NCT=0
      WRITE(*,120) Yc,Ec
120   FORMAT(' Yc=',F8.3,' Ec=',F8.3,' Give E>Ec')
      READ(*,*) E
      E3=E/3.
2     AW=AW1
      qs=Q/AW
      THETA=ACOS(((6.75*qs*qs/g-E**3)/27.)/E3**3)
      Y=E3*(1.-2.*COS((THETA+PII)/3.))
      NCT=NCT+1
      BETA=ACOS(1.-2.*Y/D)
      AW1=D*D*(BETA-COS(BETA)*SIN(BETA))/(4.*Y)
      WRITE(*,*) NCT,AW,AW1,qs
      IF(ABS(AW1-AW).GT.1.E-4 .AND. NCT.LT.40) GO TO 2
      WRITE(6,100) Q,D,E,g,Y
100   FORMAT(' Q=',F8.3,' D=',F8.3,' E=',F8.3,' g=',F8.2,
     &' Depth Y=',F9.3)
      WRITE(6,130)
130   FORMAT(/,' Give: 0=STOP, 1=Dif. E but same size, 2=Dif. size')
      READ(*,*) NCT
      IF(NCT.EQ.1) THEN
      GO TO 5
      ELSE IF(NCT.EQ.2) THEN
      GO TO 1
      ELSE
      STOP
      ENDIF
      END
```

```
      FUNCTION CRIT(Q,D,g,YS)
      M=0
1     BETA=ACOS(1.-2.*YS/D)
      F=Q*Q*(D*SIN(BETA))-g*(.25*D*D*(BETA-COS(BETA)*SIN(BETA)))**3
      M=M+1
      IF(MOD(M,2).NE.0) THEN
      F1=F
      YS1=YS
      YS=1.005*YS
      GO TO 1
      ENDIF
      DIF=(YS-YS1)*F1/(F-F1)
      YS=YS1-DIF
      IF(ABS(DIF).GT.1.E-5 .AND. M.LT.30) GO TO 1
      CRIT=YS
      RETURN
      END
```

As an example problem set D = 6 ft, Q = 100 cfs, and E = 5 ft. Solve for both the subcritical and supercritical depths. These are: 4.728 ft 🡐 and 1.762 ft 🡐, respectively.

77.

Water is taken from a reservoir by means of a rectangular inlet channel that is 10 ft wide. A short distance downstream therefrom the channel divides in a trapezoidal section with $b_2 = 4$ ft, $m_2 = 1.5$, $n_2 = .015$, and $S_{o2} = 0.0008$, and a pipe with a diameter $D_3 = 3$ ft, $n_3 = .013$, and $S_{o3} = 0.0014$. The bottom of the pipe is 1.8 ft above the bottom of the rectangular channel, and the trapezoidal and rectangular channel have the same bottom elevation. When the water surface elevation in the reservoir is 4.5 ft above the bottom of the channel determine the depths and flowrates in all three channel (6 unknowns). Assume the entrance loss coefficient equals 0.12.

Given: An upstream channel branches into a trapezoidal and a circular channels.
Wanted: Depth and flow rates when reservoir head H = 4.5 ft.

Solution:
There are 6 unknown variables: $Y_1, Y_2, Y_3, Q_1, Q_2, Q_3$

The 6 needed equations are:

$H = Y_1 + (1+K_e)(Q_1/A_1)^2/(2g)$ Energy at entrance (1)

$Y_1 + (Q_1/A_1)^2/(2g) = Y_2 + (Q_2/A_2)^2/(2g)$ Energy Chan. 1 to 2 (2)

$Y_1 + (Q_1/A_1)^2/(2g) = Y_3 + (Q_3/A_3)^2/(2g) + 1.8$ Energy Chan. 1 to 3 (3)

$Q_1 = Q_2 + Q_3$ Continuity at Junction (4)

$Q_2 = (1.486/n_2)A_2^{5/3}/P_2^{2/3}S_{o2}^{1/2}$ Uniform Flow Chan. 2 (5)

$Q_3 = (1.486/n_3)A_3^{5/3}/P_3^{2/3}S_{o3}^{1/2}$ Uniform Flow Chan. 3 (6)

The following solution was obtained using the Program BRANCHCH.

```
 For Channel 1
 H =   4.500
 Q = 221.466   <--
 Y =   3.955   <--
 b =  10.000
 m =    .000
 n =    .013
 For Channel 2
 S = .000800
 Q = 197.171  <--
 Y =   4.090  <--
 b =   4.000
 m =   1.500
 n =    .015
 For Channel 3
 S = .001400
 Q =  24.295  <--
 Y =   2.390  <--
 D =   3.000
 n =    .013
```

A TK-Solver model to solve this problem PRB2_63.TK is given below.

```
--------------- VARIABLE SHEET --------------
St Input---- Name--- Output--- Unit----- Comment
   4.5       H
             Y1      3.9545531
   .12       Ke
             Q1      221.46627
```

```
   10        b1
   64.4      g2
             Q2      197.17078
             Q3      24.295482
             Y2      4.0903278
   4         b2
   1.5       m2
             beta    2.206155
   3         D
             A3      6.0385759
             Y3      2.3901993
   1.8       dz
   1.486     Cu
   .015      n2
   .0014     So3
   .013      n3
   .0008     So2
```

```
 --------------- RULE SHEET --------------
S Rule-------------------------------------
* H=Y1+(1+Ke)*(Q1/(b1*Y1))^2/g2
* Q1=Q2+Q3
* Y1+(Q1/(b1*Y1))^2/g2=Y2+(Q2/((b2+m2*Y2)*Y2))^2/g2
* cos(beta)=1-2*Y3/D
* A3=.25*D*D*(beta-cos(beta)*sin(beta))
* Y1+(Q1/(b1*Y1))^2/g2=Y3+(Q3/A3)^2/g2+dz
* Q2=Cu/n2*((b2+m2*Y2)*Y2)^1.66666667/(b2+2.*Y2*sqrt(m2^2+1))^.66666667*sqrt(So
* Q3=Cu/n3*A3^1.66666667/(D*beta)^.666666667*sqrt(So3)
```

Below this problem is solved using a spreadsheet with EXCEL's Solver. To implement this solution the 6 cells in row 6 (B6,C6,D6,E6 & G6) are the 6 equations written so they equal zero. Cell G7 which is used to optimize the problem with solver contains =B6+C6+D6+E6+G6. and solver seeks a solution in which this becomes zero. Solver is told to adjust the values in cells B4,C4,D4,F4,G4 and H4 (the cells that contain the 3 flow rates Q_1, Q_2 & Q_3 and the 3 depths Y_1, Y_2 & Y_3.)

Excel Sheet PRB2_67.XLS

A	B	C	D	E	F	G	H	I	J	
Three Channel Problem (Prob. 2:67)										
H =	4.5	Ke =	0.12	b1 =	10	b2 =	4	m2 =	1.5	So2 =
D =	3	n3 =	0.013	So3 =	0.0014	dz =	1.8	g2 =	64.4	beta =
Qs =	221.4664	197.1709	24.29548	Ys =	3.954552	4.09032699	2.390199		Cu =	1.48
	A1 =	39.54552	A2 =	41.45747	A3 =	6.038575777				
	-5E-09	3.2E-14	-6.1E-09	7.86E-09	3.88E-06	-1.66139E-06				
						2.21315E-06				

The following is a MathCAD model THREECH1.MCD to solve this problem.

Variables $H := 4.5$ $Ke := .12$ $b1 := 10$ $m2 := 1.5$ $n2 := .015$ $So2 := .0008$ $D := 3$ $n3 := .013$ $So3 := .0014$
$dz := 1.8$ $g := 32.2$ $Cu := 1.486$ $Q1 := 130$ $Q2 := 110$ $Q3 := 12$ $Y1 := 3.8$ $Y2 := 3.9$ $Y3 := 2.25$ $Beta := 1.8$
Given

$$Q1 = Q2 + Q3 \qquad H = Y1 + (1 + Ke)\cdot\frac{Q1^2}{2\cdot g\cdot(b1\cdot Y1)^2} \qquad Y1 + \frac{Q1^2}{2\cdot g\cdot(b1\cdot Y1)^2} = Y2 + \frac{Q2^2}{2\cdot g\cdot((b2 + m2\cdot Y2)\cdot Y2)^2}$$

$$\cos(Beta) = 1 - 2\cdot\frac{Y3}{D} \qquad Y1 + \frac{Q1^2}{2\cdot g\cdot(b1\cdot Y1)^2} = dz + Y3 + \frac{Q3^2}{2\cdot g\cdot\left[D\cdot\frac{D}{4}\cdot(Beta - \cos(Beta)\cdot\sin(Beta))\right]^2}$$

$$Q2 = \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667} \cdot Cu}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2 \cdot m2 + 1}\right)^{.6666667}} \cdot \frac{\sqrt{So2}}{n2} \qquad Q3 = \frac{\left[D \cdot \frac{D}{4} \cdot (Beta - \cos(Beta) \cdot \sin(Beta))\right]^{1.6666667} \cdot Cu}{(D \cdot Beta)^{.6666667}} \cdot \frac{\sqrt{So3}}{n3}$$

$$Find(Q1, Q2, Q3, Y1, Y2, Y3, Beta) = \begin{bmatrix} 221.466 \\ 197.171 \\ 24.295 \\ 3.955 \\ 4.09 \\ 2.39 \\ 2.206 \end{bmatrix}$$

Using the MATLAB script "BRANCHCH.m", which is on the CD in the back cover of the book to solve this problem results in the following in the command window:

```
>> BRANCHCH
Give g = 32.2
Give number of channels 3
For Channel # 1 type is:(0=trap.,1=cir.) 0
b1 = 10
m1 = 0
H= 4.5
KL = .12
For Channel #2 give 0=trap,1=cir. 0
b2 = 4
m2 = 1.5
n2 = .015
So2 = .0008
Dz2 = 0
KL2 = .12
For Channel #3 give 0=trap,1=cir. 1
D3 = 3
n3 = .013
So3 = .0014
Dz3 = 1.8
KL3 = .12
Give number of gates = 0
Provide initial guesses
Q1 = 220
Y1 = 3.8
Q2 = 200
Y2 = 3.8
Q3 = 20
Y3 = 3.8
'    Q1        Y1        Q2        Y2        Q3        Y3
'   234.8    3.854    226.1    3.746      8.7    2.451
```

78.

Modify Program THREECH to accommodate any number of channels branching from the upstream main channel, i.e. allow the number of channels to be 4, 5 ,etc. Also make the following changes: (1) In place of the FUNCTION F make this a subroutine (a void function in C) that supplies all of the equations each time it is called, and (2) Rather than have a built in linear algebra solver when implementing the Newton method, call on a linear algebra subroutine such as SOLVEQ. Use this modified program to solve the following problem. A reservoir with a head H = 5 ft, and an entrance loss coefficient, K_e = .1, supply a main trapezoidal channel with b_1 = 10 ft and m = 1.5. A short distance downstream this channel branches into 3 identical long trapezoidal channels with b = 4, m = 1, n = .013 and S_o = .0005. Solve for the flowrates and depths in the four channels.

Wanted: Modify Program THREECH to accommodate any number of channels branching from a main channel. Then solve problem with: H=5 ft, K_e=.1, b_1=10 ft, m_1=1.5 with 3 identical long downstream channels with b= 4 ft, m=1, n=.013 and S_o=.0005.

Solution:

Program THREECH1.FOR is such a program:

```
      PARAMETER (ND=5,NT=10)
      INTEGER*2 Cir(ND),INDX(NT)
      REAL F(NT),F1(NT),D(NT,NT)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      WRITE(*,*) ' Give number of channel'
      READ(*,*) NC
      NU=2*NC
      WRITE(*,90) NC
90    FORMAT(' Give g,H,KL & for',I2,' Channels:0=Trap or 1=Cir,b,m,n,So
     & & z(or Cir D & 0 for b,m)')
      READ(*,*) g,H,FKL,(Cir(I),b(I),FM(I),FN(I),So(I),z(I),I=1,NC)
      WRITE(*,*)' Give Est. of Q and Y for',NC,' Channels'
      READ(*,*)(Q(I),Y(I),I=1,NC)
      g2=2.*g
      C=1.486
      IF(G.LT.20.) C=1.
      FKL=FKL+1.
      NCT=0
10    SUM=0.
      CALL FUN(F)
      DO 20 I=1,NU
      IF(I.GT.NC) THEN
      XX=Y(I-NC)
      Y(I-NC)=1.005*Y(I-NC)
      DX=Y(I-NC)-XX
      ELSE
      XX=Q(I)
      Q(I)=1.005*Q(I)
      DX=Q(I)-XX
      ENDIF
      CALL FUN(F1)
      DO 18 J=1,NU
18    D(J,I)=(F1(J)-F(J))/DX
      IF(I.GT.NC) THEN
      Y(I-NC)=XX
      ELSE
      Q(I)=XX
      ENDIF
20    CONTINUE
      CALL SOLVEQ(NU,1,NT,D,F,1,DD,INDX)
      DO 30 I=1,NU
      IF(I.GT.NC) THEN
      Y(I-NC)=Y(I-NC)-F(I)
      ELSE
      Q(I)=Q(I)-F(I)
      ENDIF
30    SUM=SUM+ABS(F(I))
      NCT=NCT+1
```

```
      IF(NCT.LT.30 .AND. SUM.GT.1.E-4) GO TO 10
      DO 40 I=1,NC
40    WRITE(3,100) I,Q(I)
100   FORMAT(' Q(',I1,') =',F8.2)
      DO 50 I=1,NC
50    WRITE(3,101) I,Y(I)
101   FORMAT(' Y(',I1,') =',F8.3)
      END
      FUNCTION A(K)
      PARAMETER (ND=5,NT=10)
      INTEGER*2 Cir(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      IF(Cir(K).GT.0) THEN
      COSB=1.-2.*Y(K)/b(K)
      FM(K)=ACOS(COSB)
      P=FM(K)*b(K)
      A=.25*b(K)**2*(FM(K)-COSB*SIN(FM(K)))
      ELSE
      P=b(K)+2.*Y(K)*SQRT(FM(K)**2+1.)
      A=(b(K)+FM(K)*Y(K))*Y(K)
      ENDIF
      RETURN
      END
      SUBROUTINE FUN(F)
      PARAMETER (ND=5,NT=10)
      REAL F(NT)
      INTEGER*2 Cir(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      QQ=Q(2)
      DO 2 J=3,NC
2     QQ=QQ+Q(J)
      F(1)=Q(1)-QQ
      F(2)=H-Y(1)-FKL*(Q(1)/A(1))**2/g2
      VELS=(Q(1)/A(1))**2
      DO 3 J=2,NC
      AAA=A(J)
      F(J+1)=Y(1)-Y(J)+(VELS-(Q(J)/AAA)**2)/g2-Z(J)
3     F(J+NC)=Q(J)-C*AAA*(AAA/P)**.6666667*SQRT(So(J))/FN(J)
      RETURN
      END
```

Input to Program to solve Problem:

```
4
32.2 5 .1 0 10 1.5 .013 .0005 0 0 4 1 .013 .0005 0 0 4 1 .013 .0005 0 0 4 1 .013
.0005 0 0 4 1 .013 .0005 0
```

Output:

```
Q(1) =  524.41
Q(2) =  174.80
Q(3) =  174.80
Q(4) =  174.80
Y(1) =   3.641
Y(2) =   4.566
Y(3) =   4.566
Y(4) =   4.566
```

79.

For the four channel system of the previous problem develop the delivery diagram for reservoir heads varying from H = 1 ft to H = 8 ft in increments of ΔH = .25 ft.

Wanted: Obtain delivery diagrams for previous problem

Solution: The program below is a modification of Program THREECH1.FOR that includes a DO loop to obtain a series of solutions.

Program THREECHD.FOR

```
C Has Do loop added to THREECH1.FOR so data for a delivery diagram can be obtained.
      PARAMETER (ND=5,NT=10)
      INTEGER*2 Cir(ND),INDX(NT)
      REAL F(NT),F1(NT),D(NT,NT),Fr(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),

     &Z(ND),Cir,NC
      WRITE(*,*) ' Give number of channel, Hend,Del H'
      READ(*,*) NC,H2,DH
      NU=2*NC
      WRITE(*,90) NC
90    FORMAT(' Give g,H,KL & for',I2,' Channels:0=Trap or 1=Cir,b,m,n,So
     & & z(or Cir D & 0 for b,m)')
      READ(*,*) g,H,FKL,(Cir(I),b(I),FM(I),FN(I),So(I),z(I),I=1,NC)
      WRITE(*,*)' Give Est. of Q and Y for',NC,' Channels'
      READ(*,*)(Q(I),Y(I),I=1,NC)
      g2=2.*g
      C=1.486
      IF(G.LT.20.) C=1.
      FKL=FKL+1.
      NUM=(H2-H)/DH+1.5
      DO 80 KK=1,NUM
      NCT=0
10    SUM=0.
      CALL FUN(F)
      DO 20 I=1,NU
      IF(I.GT.NC) THEN
      XX=Y(I-NC)
      Y(I-NC)=1.005*Y(I-NC)
      DX=Y(I-NC)-XX
      ELSE
      XX=Q(I)
      Q(I)=1.005*Q(I)
      DX=Q(I)-XX
      ENDIF
      CALL FUN(F1)
      DO 18 J=1,NU
18    D(J,I)=(F1(J)-F(J))/DX
      IF(I.GT.NC) THEN
      Y(I-NC)=XX
      ELSE
      Q(I)=XX
      ENDIF
20    CONTINUE
      CALL SOLVEQ(NU,1,NT,D,F,1,DD,INDX)
      DO 30 I=1,NU
      IF(I.GT.NC) THEN
      Y(I-NC)=Y(I-NC)-F(I)
      ELSE
      Q(I)=Q(I)-F(I)
      ENDIF
30    SUM=SUM+ABS(F(I))
      NCT=NCT+1
      IF(NCT.LT.30 .AND. SUM.GT.1.E-4) GO TO 10
      DO 40 I=1,NC
      IF(Cir(I).GT.0) THEN
      BETA=ACOS(1.-2.*Y(I)/b(I))
      Top=b(I)*SIN(BETA)
      ELSE
```

```
      Top=b(I)+2.*FM(I)*Y(I)
      ENDIF
40    Fr(I)=SQRT(Q(I)**2*Top/(G*A(I)**3))
      WRITE(3,100) H,(Q(I),Y(I),Fr(I),I=1,NC)
100   FORMAT(F5.2,5(F8.2,F8.3,F7.2))
80    H=H+DH
      END
      FUNCTION A(K)
      PARAMETER (ND=5,NT=10)
      INTEGER*2 Cir(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      IF(Cir(K).GT.0) THEN
      COSB=1.-2.*Y(K)/b(K)
      FM(K)=ACOS(COSB)
      P=FM(K)*b(K)
      A=.25*b(K)**2*(FM(K)-COSB*SIN(FM(K)))
      ELSE
      P=b(K)+2.*Y(K)*SQRT(FM(K)**2+1.)
      A=(b(K)+FM(K)*Y(K))*Y(K)
      ENDIF
      RETURN
      END
      SUBROUTINE FUN(F)
      PARAMETER (ND=5,NT=10)
      REAL F(NT)
      INTEGER*2 Cir(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      QQ=Q(2)
      DO 2 J=3,NC
2     QQ=QQ+Q(J)
      F(1)=Q(1)-QQ
      F(2)=H-Y(1)-FKL*(Q(1)/A(1))**2/g2
      VELS=(Q(1)/A(1))**2
      DO 3 J=2,NC
      AAA=A(J)
      F(J+1)=Y(1)-Y(J)+(VELS-(Q(J)/AAA)**2)/g2-Z(J)
3     F(J+NC)=Q(J)-C*AAA*(AAA/P)**.6666667*SQRT(So(J))/FN(J)
      RETURN
      END
```

Input:

```
4 8 .25
32.2 1 .1 0 10 1.5 .013 .0005 0 0 4 1 .013 .0005 0 0 4 1 .013 .0005 0 0 4 1 .013 .0005 0
27 .9 9 .9 9 .9 9 .9
```

Output:

H	Q1	Y1	Fr1	Q2	Y2	Fr2	Q3	Y3	Fr3	Q4	Y4	Fr4
1.00	27.30	.868	.56	9.10	.926	.40	9.10	.926	.40	9.10	.926	.40
1.25	40.04	1.074	.58	13.35	1.156	.41	13.35	1.156	.41	13.35	1.156	.41
1.50	55.02	1.276	.61	18.34	1.386	.41	18.34	1.386	.41	18.34	1.386	.41
1.75	72.26	1.475	.63	24.09	1.616	.42	24.09	1.616	.42	24.09	1.616	.42
2.00	91.80	1.670	.66	30.60	1.845	.42	30.60	1.845	.42	30.60	1.845	.42
2.25	113.69	1.860	.68	37.90	2.074	.43	37.90	2.074	.43	37.90	2.074	.43
2.50	137.97	2.044	.71	45.99	2.303	.43	45.99	2.303	.43	45.99	2.303	.43
2.75	164.68	2.223	.73	54.89	2.531	.43	54.89	2.531	.43	54.89	2.531	.43
3.00	193.86	2.393	.76	64.62	2.758	.44	64.62	2.758	.44	64.62	2.758	.44
3.25	225.52	2.553	.80	75.17	2.985	.44	75.17	2.985	.44	75.17	2.985	.44
3.50	259.64	2.699	.83	86.55	3.210	.44	86.55	3.210	.44	86.55	3.210	.44
3.75	296.10	2.820	.88	98.70	3.433	.44	98.70	3.433	.44	98.70	3.433	.44
4.00	333.80	2.847	.98	111.27	3.648	.45	111.27	3.648	.45	111.27	3.648	.45

The solution failed with H = 4.25. It is clear the reason for this is that the Froude Number for the upstream channel has become critical. Critical flow in Channel #1 will therefore control the flow rate into the system for H's larger than 4.0 ft. The program THREECHC.FOR has been written to handle this case. The critical flow equation is solved for Y_{c1} = Y(1) and Q_{c1} = Q(1). Now the energy equation between Channel #1 and #2, #3 and #4 are not available, i.e. the energy in the downstream channels will be different from that in Channel #1. In place of these the energy equations between channels #2, #3 and #4 are available. Since the critical flow and energy between the reservoir and Channel #1 can be solved first, the number of equations in the system is reduced by 2, or in this case 1 continuity equation, 2 energy equations, and 3 Mannings Equations, for a total of 6, rather than 8.

Program THREECHC.FOR

```
C Critical flow occurs in upstream channel to limit the flow to Qc1.
C  Any number of channels (5 with ND=5) may exist downstream from Channel 1
C  Contains DO KK to obtain a series of solutions.
      PARAMETER (ND=5,NT=10)
      INTEGER*2 Cir(ND),INDX(NT)
      REAL F(NT),F1(NT),D(NT,NT),Fr(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      WRITE(*,*) ' Give number of channel, Hend,Del H'
      READ(*,*) NC,H2,DH
      NU=2*NC
      NU2=NU-2
      NCP=NC+1
      WRITE(*,90) NC
90    FORMAT(' Give g,H,KL & for',I2,' Channels:0=Trap or 1=Cir,b,m,n,So
     & & z(or Cir D & 0 for b,m)')
      READ(*,*) g,H,FKL,(Cir(I),b(I),FM(I),FN(I),So(I),z(I),I=1,NC)
      WRITE(*,*)' Give Est. of Q and Y for',NC,' Channels'
      READ(*,*)(Q(I),Y(I),I=1,NC)
      g2=2.*g
      C=1.486
      IF(G.LT.20.) C=1.
      FKL=.5*(FKL+1.)
C Solves Critical depth in upstream channel
      NUM=(H2-H)/DH+1.5
      DO 80 KK=1,NUM
      Yc=Y(1)
      NCT=0
1     FF=FUNC(Yc,Top)
      DIF=.05*FF/(FUNC(Yc+.05,TT)-FF)
      Yc=Yc-DIF
      NCT=NCT+1
      IF(NCT.LT.30 .AND. ABS(DIF).GT. 1.E-5) GO TO 1
      IF(NCT.EQ.30) WRITE(*,*)' failed to converge',KK,Yc,Y(1)
      Y(1)=Yc
      Q(1)=SQRT(G*A(1)**3/Top)
      NCT=0
10    SUM=0.
      CALL FUN(F)
      II=0
      DO 20 I=2,NU
      IF(I.EQ.NCP) GO TO 20
      II=II+1
      IF(I.GT.NC) THEN
      XX=Y(I-NC)
      Y(I-NC)=1.005*Y(I-NC)
      DX=Y(I-NC)-XX
      ELSE
      XX=Q(I)
      Q(I)=1.005*Q(I)
      DX=Q(I)-XX
      ENDIF
      CALL FUN(F1)
      DO 18 J=1,NU2
18    D(J,II)=(F1(J)-F(J))/DX
      IF(I.GT.NC) THEN
      Y(I-NC)=XX
      ELSE
      Q(I)=XX
      ENDIF
20    CONTINUE
      CALL SOLVEQ(NU2,1,NT,D,F,1,DD,INDX)
      II=0
      DO 30 I=2,NU
      IF(I.EQ.NCP) GO TO 30
      II=II+1
```

```
      IF(I.GT.NC) THEN
      Y(I-NC)=Y(I-NC)-F(II)
      ELSE
      Q(I)=Q(I)-F(II)
      ENDIF
      SUM=SUM+ABS(F(II))
30    CONTINUE
      NCT=NCT+1
      WRITE(*,*) ' NCT=',NCT,' SUM=',SUM
      IF(NCT.LT.30 .AND. SUM.GT.1.E-4) GO TO 10
      IF(NCT.EQ.30) WRITE(*,*)' Newton failed',SUM
      DO 40 I=1,NC
      IF(Cir(I).GT.0) THEN
      BETA=ACOS(1.-2.*Y(I)/b(I))
      Top=b(I)*SIN(BETA)
      ELSE
      Top=b(I)+2.*FM(I)*Y(I)
      ENDIF
40    Fr(I)=SQRT(Q(I)**2*Top/(G*A(I)**3))
      WRITE(3,100) H,(Q(I),Y(I),Fr(I),I=1,NC)
100   FORMAT(F5.2,5(F8.2,F8.3,F7.2))
80    H=H+DH
      END
      FUNCTION A(K)
      PARAMETER (ND=5,NT=10)
      INTEGER*2 Cir(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      IF(Cir(K).GT.0) THEN
      COSB=1.-2.*Y(K)/b(K)
      FM(K)=ACOS(COSB)
      P=FM(K)*b(K)
      A=.25*b(K)**2*(FM(K)-COSB*SIN(FM(K)))
      ELSE
      P=b(K)+2.*Y(K)*SQRT(FM(K)**2+1.)
      A=(b(K)+FM(K)*Y(K))*Y(K)
      ENDIF
      RETURN
      END
      SUBROUTINE FUN(F)
      PARAMETER (ND=5,NT=10)
      REAL F(NT)
      INTEGER*2 Cir(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      QQ=Q(2)
      DO 2 J=3,NC
2     QQ=QQ+Q(J)
      F(1)=Q(1)-QQ
      VELS=(Q(2)/A(2))**2
      II=1
      DO 3 J=3,NC
      II=II+1
3     F(II)=Y(2)-Y(J)+(VELS-(Q(J)/A(J))**2)/g2-(Z(J)-Z(2))
      DO 4 J=2,NC
      II=II+1

      AAA=A(J)
4     F(II)=Q(J)-C*AAA*(AAA/P)**.6666667*SQRT(So(J))/FN(J)
      RETURN
      END
      FUNCTION FUNC(Yc,T)
      PARAMETER (ND=5,NT=10)
      INTEGER*2 Cir(ND)
      COMMON g2,C,H,FKL,P,b(ND),FM(ND),FN(ND),So(ND),Y(ND),Q(ND),
     &Z(ND),Cir,NC
      IF(Cir(1).GT.0) THEN
      BETA=ACOS(1.-2.*Yc/b(1))
      T=b(1)*SIN(BETA)
      ELSE
      T=b(1)+2.*FM(1)*Yc
```

```
      ENDIF
      Y(1)=Yc
      FUNC=H-Yc-A(1)*FKL/T
      RETURN
      END
```

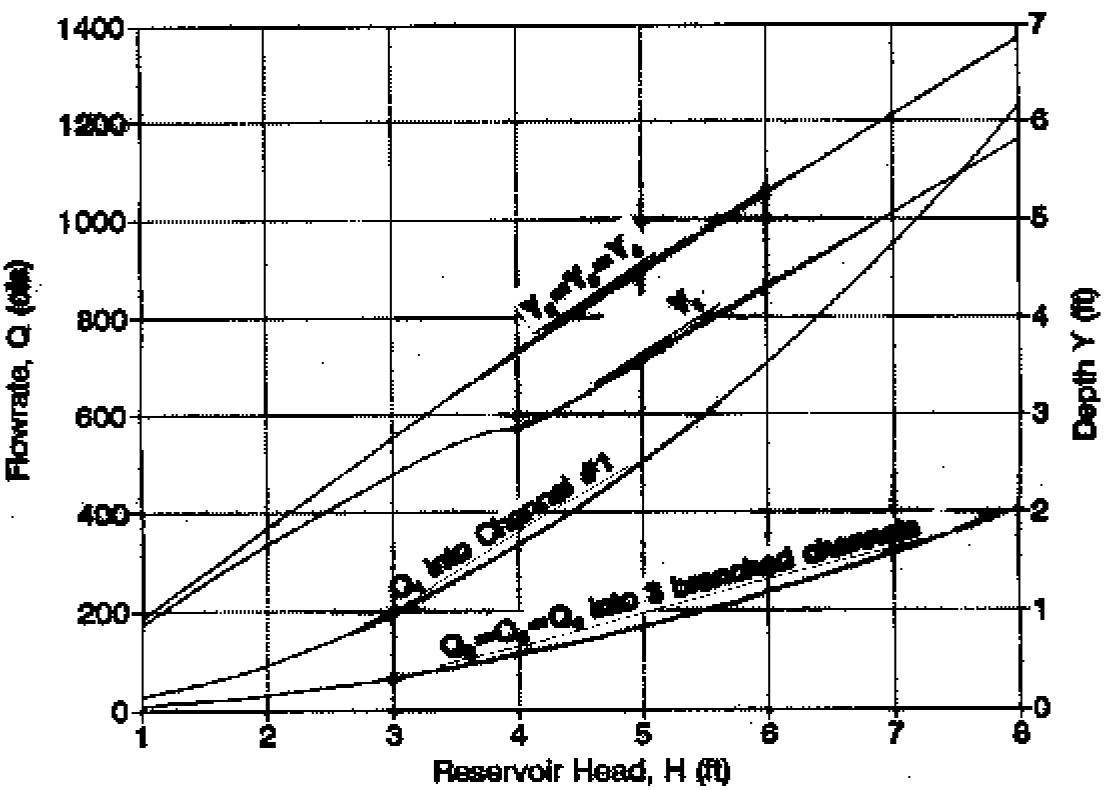
Input:

```
4 8 .25
32.2 4.25 .1 0 10 1.5 .013 .0005 0 0 4 1 .013 .0005 0 0 4 1 .013 .0005 0 0 4 1 .013 .0005 0
370 2.9 120 3.9 120 3.9 120 3.9
```

Output:

H	Q1	Y1	Fr1	Q2	Y2	Fr2	Q3	Y3	Fr3	Q4	Y4	Fr4
4.25	372.03	2.993	1.00	124.01	3.853	.45	124.01	3.853	.45	124.01	3.853	.45
4.50	412.87	3.178	1.00	137.62	4.058	.45	137.62	4.058	.45	137.62	4.058	.45
4.75	455.94	3.364	1.00	151.98	4.263	.45	151.98	4.263	.45	151.98	4.263	.45
5.00	501.27	3.551	1.00	167.09	4.466	.46	167.09	4.466	.46	167.09	4.466	.46
5.25	548.88	3.738	1.00	182.96	4.669	.46	182.96	4.669	.46	182.96	4.669	.46
5.50	598.80	3.925	1.00	199.60	4.870	.46	199.60	4.870	.46	199.60	4.870	.46
5.75	651.06	4.113	1.00	217.02	5.071	.46	217.02	5.071	.46	217.02	5.071	.46
6.00	705.69	4.301	1.00	235.23	5.271	.46	235.23	5.271	.46	235.23	5.271	.46
6.25	762.72	4.489	1.00	254.24	5.470	.46	254.24	5.470	.46	254.24	5.470	.46
6.50	822.18	4.678	1.00	274.06	5.669	.47	274.06	5.669	.47	274.06	5.669	.47
6.75	884.09	4.867	1.00	294.70	5.867	.47	294.70	5.867	.47	294.70	5.867	.47
7.00	948.48	5.057	1.00	316.16	6.064	.47	316.16	6.064	.47	316.16	6.064	.47
7.25	1015.39	5.247	1.00	338.46	6.261	.47	338.46	6.261	.47	338.46	6.261	.47
7.50	1084.83	5.437	1.00	361.61	6.457	.47	361.61	6.457	.47	361.61	6.457	.47
7.75	1156.83	5.627	1.00	385.61	6.653	.47	385.61	6.653	.47	385.61	6.653	.47
8.00	1231.43	5.817	1.00	410.48	6.848	.48	410.48	6.848	.48	410.48	6.848	.48

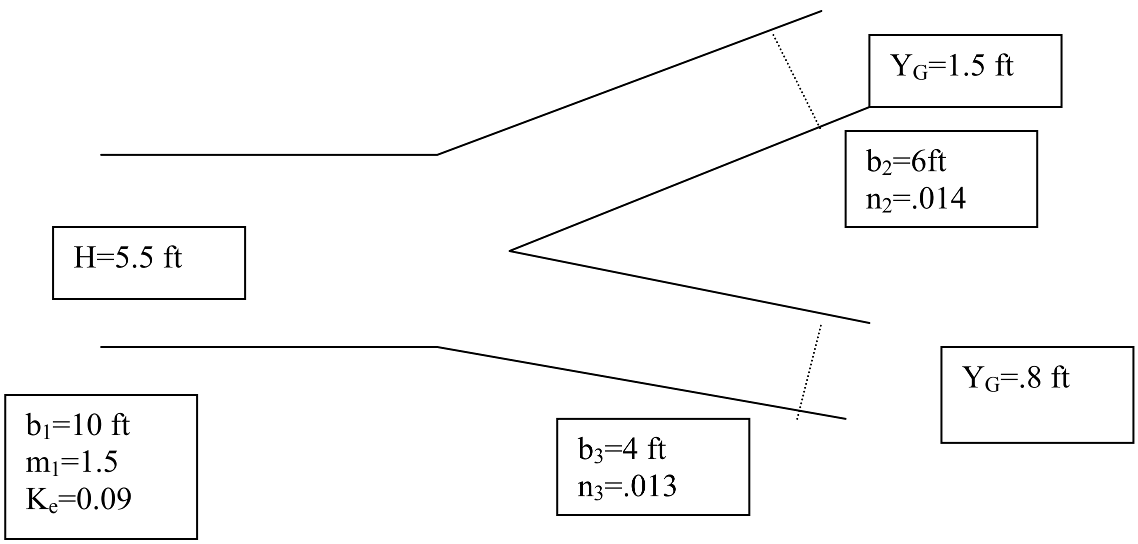
The plot of the above data is given below:



Delivery Diagram for 4 channel system of Problem 2.67b

80.

A trapezoidal channel with b_1 = 10 ft, and m_1 = 1.5 takes water from a reservoir with a water surface elevation 5.5 ft above its bottom. A short distance downstream therefrom it divides into two rectangular channels with the following properties: b_2 = 6 ft, and n_2 = .014, and b_3 = 4 ft, and n_3 = .013. A short distance downstream in the two rectangular channel there are gates to control the flowrate. The gates are set at y_{G2} = 1.5 ft and y_{G3} = 0.8 ft above the bottom of the channel, respectively. The bottom of all three channels are at the same elevation, and the contraction coefficient for the gates are both 0.6. If the entrance loss coefficient equals K_L = 0.09 Determine the depths in all three channels upstream from the gates, and the flowrates in each of the three channels.



As in Problem 77 there are 6 unknown variable (3 flow rates and 3 depths) or, $Y_1, Y_2, Y_3,\ Q_1, Q_2, Q_3$

The 6 needed equations are the same as in Problem 77 except the two Manning Equations are replaced by Energy equations across the gates, or:

$H = Y_1 + (1+K_e)(Q_1/A_1)^2/(2g)$ Energy at entrance (1)

$Y_1 + (Q_1/A_1)^2/(2g) = Y_2 + (Q_2/A_2)^2/(2g)$ Energy Chan. 1 to 2 (2)

$Y_1 + (Q_1/A_1)^2/(2g) = Y_3 + (Q_3/A_3)^2/(2g)$ Energy Chan. 1 to 3 (3)

$Q_1 = Q_2 + Q_3$ Continuity at Junction (4)

$Y_{2u}+(Q_2/A_{2u})^2/(2g)=Y_{2d}+(Q_2/A_{2d})^2/(2g)$ Energy across gate in # 2 (5)

$Y_{3u}+(Q_3/A_{3u})^2/(2g)=Y_{3d}+(Q_3/A_{3d})^2/(2g)$ Energy across gate in # 3 (6)

<u>Solution:</u> See program of next problem. The Mannings n and bottom slopes given for this problem are not used, and do not affect the solution, because both downstream channel have their flows controlled by the gates, and the channels are assume to be short enough that no GVF's develop.

<u>With Cc = 0.6</u> <u>Assuming the depth</u> of water downstr. from gates are 1.5 and 0.8 ft

Q_1 = 127.43 cfs Q_1 = 199.98 cfs

Q_2 = 92.92 cfs Q_2 = 144.35 cfs

Q_3 = 34.51 cfs Q_3 = 55.64 cfs

Y_1 = 5.472 ft Y_1 = 5.430 ft

Y_2 = 5.368 ft Y_2 = 5.156 ft

Y_3 = 5.459 ft Y_3 = 5.391 ft

A MathCAD model is given below that solves the above 6 equations; however since the areas in Channel 1 and upstream of the gates in Channels 2 and 3 depend upon the depth of flow, the MathCAD model solves 9 equations for 9 unknowns. PRB2_80.MCD

H := 5.5 b1 := 10 m1 := 1.5 Ke := .09 Cc := .6 Yg := 1.5·Cc b2 := 6 b3 := 4 Yg3 := .8·Cc g := 32.2

Q1 := 130 Q2 := 95 Q3 := 35 Y1 := 5.5 Y2 := 5.4 Y3 := 5.5 A2d := b2·Yg A3d := b3·Yg3 A1 := 60 A2u := 30 A3u := 22

Given

$$H = Y1 + (1 + Ke)\cdot\frac{\left(\frac{Q1}{A1}\right)^2}{2\cdot g} \qquad A1 = (b1 + m1\cdot Y1)\cdot Y1 \qquad A2u = b2\cdot Y2 \qquad A3u = b3\cdot Y3$$

$$Y2 + \frac{\left(\frac{Q2}{A2u}\right)^2}{2\cdot g} = Yg + \frac{\left(\frac{Q2}{A2d}\right)^2}{2\cdot g} \qquad Y3 + \frac{\left(\frac{Q3}{A3u}\right)^2}{2\cdot g} = Yg3 + \frac{\left(\frac{Q3}{A3d}\right)^2}{2\cdot g} \qquad Q1 = Q2 + Q3$$

$$Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2\cdot g} = Y2 + \frac{\left(\frac{Q2}{A2u}\right)^2}{2\cdot g} \qquad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2\cdot g} = Y3 + \frac{\left(\frac{Q3}{A3u}\right)^2}{2\cdot g}$$

$$\text{Find}(Q1, Q2, Q3, Y1, Y2, Y3, A1, A2u, A3u) = \begin{bmatrix} 127.434 \\ 92.92 \\ 34.514 \\ 5.472 \\ 5.368 \\ 5.459 \\ 99.643 \\ 32.211 \\ 21.836 \end{bmatrix}$$

81.

Write a computer program, or use an available software program capable of solving system of nonlinear simultaneous equations to develop the depths and discharges that will occur in all three channels of the previous problem with the water surface elevation in the reservoir at 5.5 ft, but with different gate setting in channel two varying from y_{G2} = 0 ft to wide open. This second rectangular channel has a bottom slope of S_{o2} = 0.0005, and extends downstream for a very long distance.

Wanted: Write a program to solve the three channel problem with gates in the downstream channels.

Solution: The program PRB2_69.FOR is a Fortran program that allows a gate to exist in the downstream channels, or to have uniform flow occur in these channels. It modifies Program THREECH.FOR by adding the integer array IGATE(WC) that is given a 1 if the gate exists and 0 if uniform flow occurs. Notice that in providing the input data that n and So are meaningless for the upstream channel, as well as, the downstream channels that contain a gate, i.e., these values are used only when uniform flow occurs in the downstream channel. Below the listing of the program, the solution to the previous problem is given using the following input:

```
32.2 5.5 .09 0 10 1.5 .013 .001 0 0 6 0 .013 .001 0 0 4 0 .013 .001 0
1 1
.9
.48
130 5.5  90 5.4 35 5.5
```

Program PRB2_69.FOR

```
C  Solves three channels, with downstream channel either uniform or containing a gate
C Program THREECH has been modified to allow gate to be in either downstream channel
      PARAMETER (NC=3,NU=6)
      INTEGER*2 Cir(NC),IGATE(NC)
      CHARACTER*49 FMT/'(3(3H Q(,I1,2H)=,F10.3),/,3(3H Y(,I1,2H)=,F10.4)
     &)'/
      COMMON g,g2,C,H,FKL,P,b(NC),FM(NC),FN(NC),So(NC),Y(NC),Q(NC),
     &Z(NC),EQ(NU),D(NU,NU),Dif(NU),Yd(NC),ARd(NC),Cir,IGATE
      FMT(2:2)=CHAR(48+NC)
      FMT(27:27)=CHAR(48+NC)
      WRITE(*,90) NC
90    FORMAT(' Give g,H,KL & for',I2,' Channels:0=Trap or 1=Cir,b,m,n,So
     & & z(or Cir D & 0 for b,m)')
      READ(*,*) g,H,FKL,(Cir(I),b(I),FM(I),FN(I),So(I),z(I),I=1,NC)
      WRITE(*,*)' For',NC-1,' downs. ch. give 0 or 1 (unif. or gate)'
      READ(*,*) (IGATE(I),I=2,NC)
      DO 1 I=2,NC
      IF(IGATE(I).EQ.0) GO TO 1
      WRITE(*,*)' Give depth downst. from gate in ch.',I
      READ(*,*) Yd(I)
      ARd(I)=(b(I)+FM(I)*Yd(I))*Yd(I)
1     CONTINUE
      WRITE(*,*)' Give Est. of Q and Y for',NC,' Channels'
      READ(*,*)(Q(I),Y(I),I=1,NC)
      g2=2.*g
      C=1.486
      IF(G.LT.20.) C=1.
      FKL=FKL+1
10    DO 20 I=1,NU
      EQ(I)=F(I)
      DO 20 J=1,NU
      IF(J.GT.NC) THEN
      Y(J-NC)=Y(J-NC)-.001
      ELSE
      Q(J)=Q(J)-.001
      ENDIF
      D(I,J)=(EQ(I)-F(I))/.001
```

```
      IF(J.GT.NC) THEN
      Y(J-NC)=Y(J-NC)+.001
      ELSE
      Q(J)=Q(J)+.001
      ENDIF
20    CONTINUE
      DO 40 KI=1,NU-1
      DO 40 I=KI+1,NU
      IF(ABS(D(I,KI)).LT.1.E-7) GO TO 40
      FAC=D(I,KI)/D(KI,KI)
      DO 30 J=KI+1,NU
30    D(I,J)=D(I,J)-FAC*D(KI,J)
      EQ(I)=EQ(I)-FAC*EQ(KI)
40    CONTINUE
      Dif(NU)=EQ(NU)/D(NU,NU)
      Y(NC)=Y(NC)-Dif(NU)
      ADIF=ABS(Dif(NU))
      DO 60 I=NU-1,1,-1
      SUM=0.
      DO 50 J=I+1,NU
50    SUM=SUM+Dif(J)*D(I,J)
      Dif(I)=(EQ(I)-SUM)/D(I,I)
      IF(I.GT.NC) THEN
      Y(I-NC)=Y(I-NC)-Dif(I)
      ELSE
      Q(I)=Q(I)-Dif(I)
      ENDIF
60    ADIF=ADIF+ABS(Dif(I))
      IF(ADIF.GT. .0001) GO TO 10
      WRITE(6,FMT) (I,Q(I),I=1,NC),(I,Y(I),I=1,NC)
      WRITE(3,FMT) (I,Q(I),I=1,NC),(I,Y(I),I=1,NC)
      END
      FUNCTION A(K)
      PARAMETER (NC=3,NU=6)
      INTEGER*2 Cir(NC),IGATE(NC)
      COMMON g,g2,C,H,FKL,P,b(NC),FM(NC),FN(NC),So(NC),Y(NC),Q(NC),
     &Z(NC),EQ(NU),D(NU,NU),Dif(NU),Yd(NC),ARd(NC),Cir,IGATE
      IF(Cir(K).GT.0) THEN
      COSB=1.-2.*Y(K)/b(K)
      FM(K)=ACOS(COSB)
      P=FM(K)*b(K)
      A=.25*b(K)**2*(FM(K)-COSB*SIN(FM(K)))
      ELSE
      P=b(K)+2.*Y(K)*SQRT(FM(K)**2+1.)
      A=(b(K)+FM(K)*Y(K))*Y(K)
      ENDIF
      RETURN
      END
      FUNCTION F(K)
      PARAMETER (NC=3,NU=6)
      INTEGER*2 Cir(NC),IGATE(NC)
      COMMON g,g2,C,H,FKL,P,b(NC),FM(NC),FN(NC),So(NC),Y(NC),Q(NC),
     &Z(NC),EQ(NU),D(NU,NU),Dif(NU),Yd(NC),ARd(NC),Cir,IGATE
      IF(K.LT.2) THEN
      QQ=Q(2)
      DO 2 J=3,NC
2     QQ=QQ+Q(J)
      F=Q(1)-QQ
      ELSE IF(K.EQ.2) THEN
      F=H-Y(1)-FKL*(Q(1)/A(1))**2/g2
      ELSE IF(K.LT.NC+2) THEN
      F=Y(1)-Y(K-1)+((Q(1)/A(1))**2-(Q(K-1)/A(K-1))**2)/g2-Z(K-1)
      ELSE
      ICH=K-NC
      AAA=A(ICH)
      IF(IGATE(ICH).GT.0) THEN
      F=Y(ICH)-Yd(ICH)+((Q(ICH)/AAA)**2-(Q(ICH)/ARd(ICH))**2)/g2
      ELSE
      F=Q(ICH)-C*AAA*(AAA/P)**.6666667*SQRT(So(ICH))/FN(ICH)
      ENDIF
      ENDIF
```

```
      RETURN
      END
```

Output for Problem 81

```
 Q(1)=   127.434 Q(2)=    92.920 Q(3)=    34.514
 Y(1)=    5.4723 Y(2)=    5.3685 Y(3)=    5.4589
```

The input to solve problem with the gate in Channel 2 wide open is:

```
32.2 5.5 .09 0 10 1.5 .013 .001 0 0 6 0 .013 .0005 0 0 4 0 .013 .001 0
0 1
.48
180 5.5 140 5.4 35 5.5
```

Output with gate 2 side open (uniform flow in this channel)

```
 Q(1)=   158.592 Q(2)=   124.082 Q(3)=    34.510
 Y(1)=    5.4568 Y(2)=    5.2560 Y(3)=    5.4576
```

To handle the case when the gate in Channel 2 is completely closed the problem consists of solving the following equations:

$H = Y_1 + (1+K_e)(Q_1/A_1)^2/(2g)$ Energy at entrance (1)

$Y_1 + (Q_1/A_1)^2/(2g) = Y_3 + (Q_3/A_3)^2/(2g)$ Energy Chan. 1 to 3 (2)

$Q_1 = Q_3$ Continuity at Junction (3)

$Y_{3u}+(Q_3/A_{3u})^2/(2g)=Y_{3d}+(Q_3/A_{3d})^2/(2g)$ Energy across gate in # 3 (4)

Notice because of Eq. 3 the subscripts to Q can be dropped, and Eq. (3) eliminated.

A TK-Solver model (PRB2_69C.TK) for this problem with the solution is given below.

```
--------------- VARIABLE SHEET -----------------
St Input---- Name--- Output--- Unit----- Comment
   5.5       H                           Model for gate 2 closed
             Y1      5.4979959
             Q       34.521437
             A1      100.3219
   64.4      g2
   .09       KL
             Y3      5.4610535
             A3      21.844214
   10        b1
   1.5       m1
             Ad3     1.92
   4         b3
   .48       Yd3

_______________ RULE SHEET _________________
S Rule________________________________________
* A1=(b1+m1*Y1)*Y1
* A3=b3*Y3
* Ad3=b3*Yd3
* H=Y1+(Q/A1)^2/g2*(1+KL)
* Y1+(Q/A1)^2/g2=Y3+(Q/A3)^2/g2
* Y3+(Q/A3)^2/g2=Yd3+(Q/Ad3)^2/g2
```

This model could be used to solve the series of problems with the gate in channel having different settings, by changing Y_{d2} = Yd(2). The above Fortran program has been modified with a GO TO 10 at the end of the program to accomplish this series of solutions. The series of solutions is started with the gate in Channel 2 set so it just begins restricting the flow, i.e., with a gate setting that produces a downstream depth that is just slightly below the alternative depth to the above solved uniform depth Y_{o2}= 5.256 ft, with a flow rate Q_2 = 124.1 cfs, or Y_{d2} = 1.2 (Alternative depth is Y_2 = 1.251 ft)

Program PRB2_69S.FOR

```
C  Solves three channels, with downstream channel either uniform or containing a gate
C Program THREECH has been modified to allow gate to be in either downstream channel
C  Has a return for new solution for series of solutions added to Program PRB2_69.FOR
      PARAMETER (NC=3,NU=6)
      INTEGER*2 Cir(NC),IGATE(NC)
      COMMON g,g2,C,H,FKL,P,b(NC),FM(NC),FN(NC),So(NC),Y(NC),Q(NC),
     &Z(NC),EQ(NU),D(NU,NU),Dif(NU),Yd(NC),ARd(NC),Cir,IGATE
      WRITE(*,90) NC
90    FORMAT(' Give g,H,KL & for',I2,' Channels:0=Trap or 1=Cir,b,m,n,So
     & & z(or Cir D & 0 for b,m)')
      READ(*,*) g,H,FKL,(Cir(I),b(I),FM(I),FN(I),So(I),z(I),I=1,NC)
      WRITE(*,*)' For',NC-1,' downs. ch. give 0 or 1 (unif. or gate)'
      READ(*,*) (IGATE(I),I=2,NC)
      DO 1 I=2,NC
      IF(IGATE(I).EQ.0) GO TO 1
      WRITE(*,*)' Give depth downst. from gate in ch.',I
      READ(*,*) Yd(I)
      ARd(I)=(b(I)+FM(I)*Yd(I))*Yd(I)
1     CONTINUE
      WRITE(*,*)' Give Est. of Q and Y for',NC,' Channels'
      READ(*,*)(Q(I),Y(I),I=1,NC)
      WRITE(*,*) ' Give increment decrease of gate 2 downstream depth'
      READ(*,*) DY2
      DY2=ABS(DY2)
      g2=2.*g
      C=1.486
      IF(G.LT.20.) C=1.
      FKL=FKL+1
10    DO 20 I=1,NU
      EQ(I)=F(I)
      DO 20 J=1,NU
      IF(J.GT.NC) THEN
      Y(J-NC)=Y(J-NC)-.001
      ELSE
      Q(J)=Q(J)-.001
      ENDIF
      D(I,J)=(EQ(I)-F(I))/.001
      IF(J.GT.NC) THEN
      Y(J-NC)=Y(J-NC)+.001
      ELSE
      Q(J)=Q(J)+.001
      ENDIF
20    CONTINUE
      DO 40 KI=1,NU-1
      DO 40 I=KI+1,NU
      IF(ABS(D(I,KI)).LT.1.E-7) GO TO 40
      FAC=D(I,KI)/D(KI,KI)
      DO 30 J=KI+1,NU
30    D(I,J)=D(I,J)-FAC*D(KI,J)
      EQ(I)=EQ(I)-FAC*EQ(KI)
40    CONTINUE
      Dif(NU)=EQ(NU)/D(NU,NU)
      Y(NC)=Y(NC)-Dif(NU)
      ADIF=ABS(Dif(NU))
      DO 60 I=NU-1,1,-1
      SUM=0.
      DO 50 J=I+1,NU
50    SUM=SUM+Dif(J)*D(I,J)
      Dif(I)=(EQ(I)-SUM)/D(I,I)
      IF(I.GT.NC) THEN
      Y(I-NC)=Y(I-NC)-Dif(I)
      ELSE
      Q(I)=Q(I)-Dif(I)
      ENDIF
60    ADIF=ADIF+ABS(Dif(I))
```

```
      IF(ADIF.GT. .0001) GO TO 10
      WRITE(6,100) Yd(2),(Q(I),Y(I),I=1,NC)
      READ(*,*)
      WRITE(3,100) Yd(2),(Q(I),Y(I),I=1,NC)
100   FORMAT(F8.3,3(F9.2,F8.3))
      Yd(2)=Yd(2)-DY2
      IF(Yd(2).GT. .01) GO TO 10
      END
      FUNCTION A(K)
      PARAMETER (NC=3,NU=6)
      INTEGER*2 Cir(NC),IGATE(NC)
      COMMON g,g2,C,H,FKL,P,b(NC),FM(NC),FN(NC),So(NC),Y(NC),Q(NC),
     &Z(NC),EQ(NU),D(NU,NU),Dif(NU),Yd(NC),ARd(NC),Cir,IGATE
      IF(Cir(K).GT.0) THEN
      COSB=1.-2.*Y(K)/b(K)
      FM(K)=ACOS(COSB)
      P=FM(K)*b(K)
      A=.25*b(K)**2*(FM(K)-COSB*SIN(FM(K)))
      ELSE
      P=b(K)+2.*Y(K)*SQRT(FM(K)**2+1.)
      A=(b(K)+FM(K)*Y(K))*Y(K)
      ENDIF
      RETURN
      END
      FUNCTION F(K)
      PARAMETER (NC=3,NU=6)
      INTEGER*2 Cir(NC),IGATE(NC)
      COMMON g,g2,C,H,FKL,P,b(NC),FM(NC),FN(NC),So(NC),Y(NC),Q(NC),
     &Z(NC),EQ(NU),D(NU,NU),Dif(NU),Yd(NC),ARd(NC),Cir,IGATE
      IF(K.LT.2) THEN
      QQ=Q(2)
      DO 2 J=3,NC
2     QQ=QQ+Q(J)
      F=Q(1)-QQ
      ELSE IF(K.EQ.2) THEN
      F=H-Y(1)-FKL*(Q(1)/A(1))**2/g2
      ELSE IF(K.LT.NC+2) THEN
      F=Y(1)-Y(K-1)+((Q(1)/A(1))**2-(Q(K-1)/A(K-1))**2)/g2-Z(K-1)
      ELSE
      ICH=K-NC
      AAA=A(ICH)
      IF(IGATE(ICH).GT.0) THEN
      F=Y(ICH)-Yd(ICH)+((Q(ICH)/AAA)**2-(Q(ICH)/ARd(ICH))**2)/g2
      ELSE
      F=Q(ICH)-C*AAA*(AAA/P)**.6666667*SQRT(So(ICH))/FN(ICH)
      ENDIF
      ENDIF
      RETURN
      END
```

Input:

```
32.2 5.5 .09 0 10 1.5 .013 .001 0 0 6 0 .013 .001 0 0 4 0 .013 .001 0
1 1
1.2
.48
157 5.46 120 5.26 34.5 5.46
.1
```

Output:

Y_{d2}	Q_1	Y_1	Q_2	Y_2	Q_3	Y_3
1.200	154.28	5.459	119.77	5.274	34.51	5.458
1.100	155.66	5.458	121.15	5.268	34.51	5.458
1.000	157.03	5.458	122.52	5.263	34.51	5.458
.900	158.39	5.457	123.88	5.257	34.51	5.458
.800	159.72	5.456	125.22	5.251	34.51	5.458
.700	161.05	5.455	126.54	5.245	34.51	5.458
.600	162.36	5.455	127.85	5.239	34.51	5.457

.500	163.66	5.454	129.15	5.234	34.51	5.457
.400	164.94	5.453	130.44	5.228	34.51	5.457
.300	166.22	5.452	131.71	5.222	34.51	5.457
.200	167.48	5.452	132.97	5.216	34.51	5.457
.100	168.73	5.451	134.22	5.210	34.51	5.457

82.

Assume the gate in channel 2 of problem 80 is wide open and that this channel has a steep bottom slope. Also its bottom is 1.5 ft above the bottom of the other two channels. What will the depths and flowrates be in all three channels now.

Given: The gate in channel 2 of Problem 80 is wide open and this channel is steep.
Wanted: Depth and Flow rates

Solution:

Now critical depth will exist at the beginning of Channel 2, or $Y_c = (q_2^2/g)^{1/3}$, so the energy equation between channel 1 and 2 consists of:

$$Y_1+(Q_1/A_1)^2/(2g) = 1.5Y_c + \Delta z = 1.5[(Q_2/b_2)^2/g]^{1/3}+1.5$$

The TK-Solver model PRB2_70.TK for this problem consists of:

```
═══════════════ VARIABLE SHEET ═══════════════
St Input──── Name─── Output─── Unit─────
   5.5       H
             Y1       5.4423073
             Q        182.5037
             A1       98.851137
   64.4      g2
   .09       KL
             Y3       5.4564251
             A3       21.825701
   10        b1
   1.5       m1
             Ad3      1.92
   4         b3
   .48       Yd3
   6         b2
             Y2       2.6634909
             Q2       147.99808
             Q3       34.505622
   32.2      g
   1.5       delz

═══════════════ RULE SHEET ═══════════════
S Rule──────────────────────────────────
* A1=(b1+m1*Y1)*Y1
* A3=b3*Y3
* Ad3=b3*Yd3
* Q=Q2+Q3
* H=Y1+(Q/A1)^2/g2*(1+KL)
* Y1+(Q/A1)^2/g2=Y3+(Q3/A3)^2/g2
* Y2=((Q2/b2)^2/g)^.3333333
* Y1+(Q/A1)^2/g2=1.5*Y2+delz
* Y3+(Q3/A3)^2/g2=Yd3+(Q3/Ad3)^2/g2
```

The MathCAD model used in Problem 80 can be modified replacing the equation across the gate by the above equation. (PRB2_82.MCD)

$H := 5.5$ $b1 := 10$ $m1 := 1.5$ $Ke := .09$ $Cc := .6$ $Yg := 1.5 \cdot Cc$ $b2 := 6$ $b3 := 4$ $Yg3 := .8 \cdot Cc$ $g := 32.2$

$Q1 := 130$ $Q2 := 95$ $Q3 := 35$ $Y1 := 5.5$ $Y2 := 5.4$ $Y3 := 5.5$ $A2d := b2 \cdot Yg$ $A3d := b3 \cdot Yg3$ $A1 := 60$ $A2u := 30$ $A3u := 22$

$Delz := 1.5$

Given

$$H = Y1 + (1+Ke) \cdot \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g} \qquad A1 = (b1 + m1 \cdot Y1) \cdot Y1 \qquad A2u = b2 \cdot Y2 \qquad A3u = b3 \cdot Y3$$

$$Y2 + \frac{\left(\frac{Q2}{A2u}\right)^2}{2 \cdot g} = 1.5 \cdot \left[\frac{\left(\frac{Q2}{b2}\right)^2}{g}\right]^{.33333333} + Delz \qquad Y3 + \frac{\left(\frac{Q3}{A3u}\right)^2}{2 \cdot g} = Yg3 + \frac{\left(\frac{Q3}{A3d}\right)^2}{2 \cdot g} \qquad Q1 = Q2 + Q3$$

$$Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g} = Y2 + \frac{\left(\frac{Q2}{A2u}\right)^2}{2 \cdot g} \qquad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g} = Y3 + \frac{\left(\frac{Q3}{A3u}\right)^2}{2 \cdot g}$$

$$\text{Find}(Q1, Q2, Q3, Y1, Y2, Y3, A1, A2u, A3u) = \begin{bmatrix} 182.504 \\ 147.998 \\ 34.506 \\ 5.442 \\ 5.137 \\ 5.456 \\ 98.851 \\ 30.824 \\ 21.826 \end{bmatrix}$$

Note that in the MathCAD model the depth solved as Y2 is the depth that would exist in Channel 2 corresponding to the specific energy created by critical condition, i.e. $1.5Y_c$. In other words last two equations in the first column could be consolidated into one equation and eliminate Y2 as an unknown. Alternatively the critical flow equation could be added to give Y_{c2} and then multiply this depth by 1.5 to get the specific energy as has been done in the TK-Solver model.

83.

Two long channels are joined by a smooth transition. The upstream channel has the following properties: Its bottom width is b_1 = 10 ft; its side slope is m_1 = 2; its Mannings roughness coefficient is n_1 = .014, and its bottom slope is, S_{o1} = 0.0002. The downstream channel has the following properties: b_2 = 8 ft, m_2 = 1.0, n_2 = .014, and S_{o2} = 0.001. The bottom rises by Δz = 0.5 feet through the transition. The design flowrate is Q = 450 cfs. Determine the depths both immediately upstream and downstream from the transition.

Given: Two long channel are connected by a transition with Δz=0.5 ft, Q = 450 cfs, b_1=10 ft, m_1=2, n_1=.014, S_{o1}=.0002, b_2=8 ft, m_2=1, n_2=.014, S_{o2}=.001

Find: Upstream and Downstream depths

Solution: Downstream controls. Therefore uniform flow will occur in downstream channel. Thus solving Mannings equation for Y_2 with Q = 450 cfs and , b_2=8 ft, m_2=1, n_2=.014, S_{o2}=.001, gives

Y_2 = 5.016 ft ← E_{o2}= 5.754 ft

Next solve energy across the transition, i.e., $E_1 = E_{o2}+\Delta z$ = 5.754+.5 = 6.254 ft

Gives Y_1 = 6.081 ft ←

The normal depth in the upstream channel is obtained by solving Mannings equation for the depth Y, which is Y_{o1} = 5.912 ft, with the corresponding specific energy E_{o1} = 6.101 ft (A=129.04 ft^2 and F_r=.314). Since $E_{o1} < 6.254 = E_1 = E_{o2}+\Delta z$, there will be a gradual increase in depth in the upstream channel from 5.912 ft to 6.081 ft immediately upstream from the transition. Had E_{o1} be greater than E_1=6.254 ft, then the specific energy in the upstream channel would have governed, and established the value for the specific energy immediately downstream from the transition. In Chapter 4 means for solving how the depth increases from 5.912 ft to 6.081 ft will be handles, i.e. the solution of gradually varied flows (GVF) will be dealt with.

84.

Write a computer program capable of solving any of the variables that may be unknown through a transition between two trapezoidal channels. The variables of the problem are: Y_1, b_1, m_1, $\in z$, Y_2, b_2, m_2 and Q.

Wanted: Write a program that is capable of solving for any of the variables associated with a transition between two trapezoidal channels.

Program ENERSOL is designed to obtain such a solution.

ENERSOL.FOR

```
      REAL X(9)
      CHARACTER*2 CV(9)
      LOGICAL*2 REP
      DATA G2/64.4/,CV/'Y1','Y2','b1','b2','m1','m2','Q ','hf','Dz'/
      WRITE(*,"(' Give accel. gravity & output unit (6=screen)')")
      READ(*,*) G2,IOUT
      G2=2.*G2
1     WRITE(*,"(' Give No. of IUNKown',/,(I3,1X,A2))") (I,CV(I),I=1,9)
      READ(*,*) IUNK
      WRITE(*,"(' Provide variable values',/,' (Including guess UNK)')")
      DO 10 I=1,9
      WRITE(*,"(I3,3X,A2,' = ',\)") I,CV(I)
10    READ(*,*) X(I)
      M=0
20    REP=.TRUE.
30    A1=(X(3)+X(5)*X(1))*X(1)
      A2=(X(4)+X(6)*X(2))*X(2)
      F=X(1)-X(2)+X(7)**2/G2*(1./A1**2-1./A2**2)-X(8)-X(9)
      IF(REP) THEN
      F1=F
      XX=X(IUNK)
      X(IUNK)=1.05*X(IUNK)
      REP=.FALSE.
      GO TO 30
      ENDIF
      X(IUNK)=XX-(X(IUNK)-XX)*F1/(F-F1)
      M=M+1
      IF(ABS(X(IUNK)-XX).GT. .00001 .AND. M.LT.20) GO TO 20
      WRITE(IOUT,"(I3,1X,A2,' =',F12.4)")(I,CV(I),X(I),I=1,9)
      IF(IOUT.NE.6) WRITE(6,"(I3,1X,A2,' =',F12.4)")(I,CV(I),X(I),I=1,9)
      WRITE(*,"(//' Give 1 to solve another problem ')")
      READ(*,*) M
      IF(M.EQ.1) GO TO 1
      END
```

ENERSOL.PAS

```
Program SolveEnergy;
Label L0,L1,L2;
Const
 SC:array[1..9] of string[2]=('Y1','Y2','b1','b2','m1','m2','Q ','hf','Dz');
Var
 i,IUNK,m,io:Integer; A1,A2,F,F1,XX,G2:Real;
 x:array[1..9] of real;  rept:boolean;
IOUT:TEXT;  fln:STRING[20];
BEGIN
fln:='ENERSOL.OUT         ';
 Writeln('Give accel. gravity & output unit(if not 0 also file, 6=ENERSOL.OUT');
 Readln(G2,io);G2:=2.*G2;
 If io>0 Then Begin
  if io<>6 then begin Writeln('Give filename ');Readln(fln) end;
  Assign(IOUT,fln);Rewrite(IOUT) End;
L0: Writeln('Give No. of unknown');
 For i:=1 to 9 do Writeln(i:3,' - ',SC[i]); Readln(IUNK);
 Writeln('Give values to knowns');Writeln('(and guess for unknown)');
 For i:=1 to 9 do begin Write(SC[i],' = ');Readln(x[i]) end;
 m:=0;
L1: rept:=true;
L2:A1:=(x[3]+x[5]*x[1])*x[1];A2:=(x[4]+x[6]*x[2])*x[2];
```

```
 F:=x[1]-x[2]+Sqr(x[7])/G2*(1.0/Sqr(A1)-1.0/Sqr(A2))-x[8]-x[9];
 if rept then Begin F1:=F; rept:=false;XX:=x[IUNK];
 x[IUNK]:=1.05*x[IUNK]; goto L2; End;
 m:=m+1; x[IUNK]:=xx-(x[IUNK]-XX)*F1/(F-F1);
 If (m<20) and (Abs(x[iunk]-XX)>0.00001) then goto L1;
 For i:=1 to 9 do begin Writeln(i:3,' ',SC[i],' =',x[i]:10:6);
 if io>0 then Writeln(IOUT,i:3,' ',SC[i],' =',x[i]:10:6) end;
Writeln;Writeln;Writeln('Give 1 to solve another problem');Readln(m);
if m=1 then goto L0; close(IOUT);
END.
```

ENERSOL.C

```
#include <math.h>
#include <stdio.h>
#include <conio.h>
char *sc[]={"Y1","Y2","b1","b2","m1","m2","Q ","hf","Dz"};
int i,iunk,m,rept,io=0;  float a1,a2,f,f1,xx,g2,x[9];
 char *fln="enersol.out    ";
 FILE *fil;
void main(void){
 cprintf("Give accel. of gravity,  and output unit (0=no file,6=ENERSOL.OUT)\r\n");
 scanf("%f %d",&g2,&io); if(io){if(io!=6){cprintf("Give file name ");scanf("%s",fln);}
 fil=fopen(fln,"w");}  g2=2.*g2;
L0:cprintf("Give No. of unknown\r\n");
 for(i=0;i<9;i++) cprintf("%3d - %s\r\n",i+1,sc[i]); scanf("%d",&iunk);
 cprintf("Give values to knowns\r\n (and guess for unknown)\r\n");
 for(i=0;i<9;i++){cprintf("%s =",sc[i]);scanf("%f",&x[i]);}
 m=0; iunk--;
L1: rept=1;
L2: a1=(x[2]+x[4]*x[0])*x[0]; a2=(x[3]+x[5]*x[1])*x[1];
    f=x[0]-x[1]+x[6]*x[6]/g2*(1./a1/a1-1./a2/a2)-x[7]-x[8];
   if(rept){f1=f;rept=0;xx=x[iunk];x[iunk]*=1.05; goto L2;}
   m++;x[iunk]=xx-(x[iunk]-xx)*f1/(f-f1);
   if((m<20) && (fabs(x[iunk]-xx)>.00001)) goto L1;
 for(i=0;i<9;i++){ cprintf("%3d %s = %10.4f\r\n",i+1,sc[i],x[i]);
 if(io) fprintf(fil,"%3d %s = %10.4f\r\n",i+1,sc[i],x[i]);}
 cprintf("Give 1 to solve another problem ");scanf("%d",&m);
 if(m==1) goto L0; if(io) close(fil);
}
```

Examples of the output from Program ENERSOL follows:

```
  1 Y1 =  5.000000
  2 Y2 =  4.500000
  3 b1 = 12.000000
  4 b2 = 10.000000
  5 m1 =  1.500000
  6 m2 =  1.300000
  7 Q  =400.000000
  8 hf =  0.000000
  9 Dz =  0.272980

  1 Y1 =  5.000000
  2 Y2 =  4.300000
  3 b1 = 15.000000
  4 b2 = 12.000000
  5 m1 =  1.500000
  6 m2 =  1.250000
  7 Q  =670.959403
  8 hf =  0.000000
  9 Dz =  0.000000

  1 Y1 =  5.000000
  2 Y2 =  4.300000
  3 b1 = 15.000000
  4 b2 = 12.000000
  5 m1 =  1.500000
  6 m2 =  1.250000
  7 Q  =400.000000
  8 hf =  0.451214
  9 Dz =  0.000000
```

MathCAD Model PRB2 84.MCD

$Y1 := 5$ $b1 := 15$ $m1 := 1.4$ $Delz := .2$ $Y2 := 4.5$ $b2 := 12$ $m2 := 1.3$ $Q := 700$ $Delh := 0$ $Cu := 1.486$ $g := 32.2$

Given

$$\frac{Q^2}{2\cdot g\cdot((b1+m1\cdot Y1)\cdot Y1)^2}+Y1=\frac{Q^2}{2\cdot g\cdot((b2+m2\cdot Y2)\cdot Y2)^2}+Delz-Delh+Y2$$

$Find(Q) = 516.778$ or solving for Delz $Find(Delz) = 0.320265$ etc.

85.

Write a program that solves energy between two circular channels

Program ENERCIR.FOR

```
      REAL X(7)
      CHARACTER*2 CV(7)
      LOGICAL*2 REP
      DATA G2/64.4/,CV/'Y1','Y2','D1','D2','Q ','hf','Dz'/
      WRITE(*,"(' Give accel. gravity & output unit (6=screen)')")
      READ(*,*) G2,IOUT
      G2=2.*G2
1     WRITE(*,"(' Give No. of IUNKown',/,(I3,1X,A2))") (I,CV(I),I=1,7)
      READ(*,*) IUNK
      WRITE(*,"(' Provide variable values',/,' (Including guess UNK)')")
      DO 10 I=1,7
      WRITE(*,"(I3,3X,A2,' = ',\)") I,CV(I)
10    READ(*,*) X(I)
      M=0
20    REP=.TRUE.
30    BET1=ACOS(1.-2.*X(1)/X(3))
      A1=.25*X(3)**2*(BET1-SIN(BET1)*COS(BET1))
      BET2=ACOS(1.-2.*X(2)/X(4))
      A2=.25*X(4)**2*(BET2-SIN(BET2)*COS(BET2))
      F=X(1)-X(2)+X(5)**2/G2*(1./A1**2-1./A2**2)-X(6)-X(7)
      IF(REP) THEN
      F1=F
      XX=X(IUNK)
      X(IUNK)=1.05*X(IUNK)
      REP=.FALSE.
      GO TO 30
      ENDIF
      X(IUNK)=XX-(X(IUNK)-XX)*F1/(F-F1)
      M=M+1
      IF(ABS(X(IUNK)-XX).GT. .00001 .AND. M.LT.20) GO TO 20
      WRITE(IOUT,"(I3,1X,A2,' =',F12.4)")(I,CV(I),X(I),I=1,7)
      IF(IOUT.NE.6) WRITE(6,"(I3,1X,A2,' =',F12.4)")(I,CV(I),X(I),I=1,7)
      WRITE(*,"(//' Give 1 to solve another problem ')")
      READ(*,*) M
      IF(M.EQ.1) GO TO 1
      END
```

As an example problem to solve assume: D_1= 2 ft, Y_1 = ? ft, D_2 = 1.8 ft, Y_2 = 1.5 ft.

MathCAD Model PRB2 85.MCD

$D1 := 2 \quad D2 := 1.8 \quad Y2 := 1.5 \quad Y1 := 1.6 \quad Ke := .1 \quad Delz := 0 \quad Q := 10 \quad g := 32.2 \quad \beta1 := 2 \quad \beta2 := 2$

Given

$$\cos(\beta1) = 1 - 2\cdot\frac{Y1}{D1} \qquad \cos(\beta2) = 1 - 2\cdot\frac{Y2}{D2}$$

$$Y1 + \frac{Q^2}{2\cdot g\cdot\left[.25\cdot D1^2\cdot(\beta1 - \cos(\beta1)\cdot\sin(\beta1))\right]^2} = Y2 + \frac{(1+Ke)\cdot Q^2}{2\cdot g\cdot\left[.25\cdot D2^2\cdot(\beta2 - \cos(\beta2)\cdot\sin(\beta2))\right]^2} + Delz$$

$$\text{Find}(\beta1, \beta2, Y1) = \begin{pmatrix} 2.246 \\ 2.301 \\ 1.625 \end{pmatrix}$$

86. & 87.

Write a computer program to solve the problem of water entering a trapezoidal channel from a reservoir at uniform flow. This program should be able to solve any of the variables in the following list in addition to determine the flow rate Q: H, b, m, K_L, S_o or n. (H is the head of water in reservoir above the channel bottom.)

87. Write a computer program to solve the problem of water entering a circular channel from a reservoir at uniform flow. This program should be able to solve any of the variables in the following list in addition to determine the flow rate Q: H, D, K_L, S_o or n.

Wanted: Write a program that solves Energy at a Reservoir and Manning's Equation simultaneously.

The program E_UN, listed below, solves this type of problem for both trapezoidal and circular channels. Thus it satisfies the request of both of these problems.

E_UN.FOR

```
C Solves Mannings (uniform flow) and Energy simultaneously for any 2 unknowns
C  See E_UN1 to solve Q & Y and method that can be used with calculator
      CHARACTER*17 FMT/'(1X,A1,'' ='',F9.3)'/
      CHARACTER*1 CX(8)/'b','m','S','n','Q','H','Y','K'/

      CHARACTER*5 CH(0:1)/'value','guess'/
      REAL F(2),F1(2),D(2,2)
      INTEGER*2 ID(8),INDX(2)
      COMMON X(8),G,FKE,Cu,ITYPE
      WRITE(*,*)' Give: g,entrance loss C. & 0=TRAP or 1=CIRLCE'
      READ(*,*) G,FKE,ITYPE
      IF(ITYPE.EQ.1) THEN
      CX(1)='D'
      CX(2)=' '
      ENDIF
      IF(G.GT.15.) THEN
      Cu=1.486
      ELSE
      Cu=1.
      ENDIF
      X(8)=FKE
      FKE=(1.+FKE)/(2.*G)
1     DO 10 I=1,7
      IF(ITYPE.EQ.1 .AND. I.EQ.2) GO TO 10
      WRITE(*,'(I2,2X,A1)') I,CX(I)
10    ID(I)=0
2     WRITE(*,*)' Give two numbers for 2 unknown variables'
      READ(*,*) I1,I2
      IF(I1.LT.1 .OR. I1.GT.7 .OR. I2.LT.1 .OR. I2.GT.7) GO TO 2
      ID(I1)=1
      ID(I2)=1
      DO 20 I=1,7
      IF(ITYPE.EQ.1 .AND. I.EQ.2) GO TO 20
      WRITE(*,100) CH(ID(I)),CX(I)
100   FORMAT(' Give ',A5,' for ',A1,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
      NCT=0
30    SUM=0.
      CALL FUN(F)
      I1=0
      DO 40 I=1,7
      IF(ID(I).EQ.0) GO TO 40
      XX=X(I)
      I1=I1+1
      X(I)=1.005*X(I)
      CALL FUN(F1)
      DO 35 J=1,2
35    D(J,I1)=(F1(J)-F(J))/(X(I)-XX)
```

```
      X(I)=XX
40    CONTINUE
      CALL SOLVEQ(2,1,2,D,F,1,DD,INDX)
      I1=0
      DO 50 I=1,7
      IF(ID(I).EQ.0) GO TO 50
      I1=I1+1
      SUM=SUM+ABS(F(I1))
      X(I)=X(I)-F(I1)
50    CONTINUE
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,' SUM=',SUM
      IF(NCT.LT.30 .AND. SUM.GT.1.E-5) GO TO 30
      WRITE(*,*)' Solution:'
      DO 60 I=1,8
      IF(ITYPE.EQ.1 .AND. I.EQ.2) GO TO 60
      IF(I.EQ.3) THEN
      FMT(16:16)='6'
      ELSEIF(I.EQ.4) THEN
      FMT(16:16)='3'
      ELSE
      FMT(16:16)='3'
      ENDIF
      WRITE(*,FMT) CX(I),X(I)
60    CONTINUE
      WRITE(*,*)' Give 1 to solve another prob. (0=STOP)'
      READ(*,*) I2
      IF(I2.EQ.1) GO TO 1
      END
      SUBROUTINE FUN(F)
      REAL F(2)
      COMMON X(8),G,FKE,Cu,ITYPE
      IF(ITYPE.EQ.1) THEN
      BETA=ACOS(1.-2.*X(7)/X(1))
      A=.25*X(1)**2*(BETA-COS(BETA)*SIN(BETA))
      P=X(1)*BETA
      ELSE
      A=(X(1)+X(2)*X(7))*X(7)
      P=X(1)+2.*X(7)*SQRT(X(2)**2+1.)
      ENDIF
      F(1)=X(4)*X(5)*P**.6666667-Cu*A**1.6666667*SQRT(X(3))
      F(2)=X(6)-X(7)-FKE*(X(5)/A)**2
      RETURN
      END
```

E_UN.C

```
// Solves Mannings (uniform flow) and Energy simultaneously for any 2 unknowns
// See E_UN1 to solve Q & Y and method that can be used with calculator
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
float x[8],g,fke,cu;  int itype;
extern void solveq(int n,float **d,float *f,int itype,float *dd,int *indx);
void fun(float *f){float beta,a,p;
 if(itype){beta=acos(1.-2.*x[6]/x[0]);a=.25*x[0]*x[0]*(beta-cos(beta)*sin(beta));p=x[0]*beta;}
  else {a=(x[0]+x[1]*x[6])*x[6];p=x[0]+2.*x[6]*sqrt(x[1]*x[1]+1.);}
 f[0]=x[3]*x[4]*pow(p,.6666667)-cu*pow(a,1.6666667)*sqrt(x[2]);
 f[1]=x[5]-x[6]-fke*pow(x[4]/a,2.);return;} //End of fun
void main(void){char *fmt=" %c =%9.3f\n",*ch[]={"value","guess"},*cx="bmSnQHYK";
 float f[2],f1[2],xx,sum,**d,*dd; int id[8],indx[2],i,j,i1,i2,nct;
 d=(float**)malloc(2*sizeof(float*));for(i=0;i<2;i++)d[i]=(float*)malloc(2*sizeof(float));
 printf("Give: g,entrance loss C. & 0=TRAP or 1=CIRLCE\n");scanf("%f %f %d",&g,&fke,&itype);
 if(itype){stpcpy(cx[0],"D");stpcpy(cx[1]," ");} if(g>15.) cu=1.486; else cu=1.;
x[7]=fke;fke=(1.+fke)/(2.*g);
L1: for(i=0;i<7;i++){if((!itype) || ((itype)&&(i != 1)))printf("%2d  %c\n",i+1,cx[i]);id[i]=0;}
 do{printf(" Give two numbers for 2 unknown variables\n"); scanf("%d %d",&i1,&i2);
  }while((i1<1)||(i1>7)||(i2<1)||(i2>7)); id[i1-1]=1;id[i2-1]=1;
 for(i=0;i<7;i++){if((!itype) || ((itype)&&(i != 1))){printf("Give %s for %c =",ch[id[i]],cx[i]);
 scanf("%f",&x[i]);}} nct=0;
do{sum=0.; fun(f); i1=-1;for(i=0;i<7;i++){if(id[i]){xx=x[i];i1++;x[i]*=1.005;fun(f1);
  for(j=0;j<2;j++) d[j][i1]=(f1[j]-f[j])/(x[i]-xx); x[i]=xx;}}
```

```
solveq(2,d,f,1,dd,indx); i1=-1;
for(i=0;i<7;i++){if(id[i]){sum+=fabs(f[++i1]);x[i]-=f[i1];}}
 printf("nct= %d SUM=%f\n",nct,sum);}while((++nct<30)&&(sum>1.e-5));
printf("Solution:\n");for(i=0;i<8;i++){if((!itype) || ((itype)&&(i != 1))){
 if(i==2)stpcpy(fmt[8],"6"); else stpcpy(fmt[8],"3");printf(fmt,cx[i],x[i]);}}
printf("Give 1 to solve another prob. (0=STOP)\n");scanf("%d",&i2); if(i2) goto L1;}
```

The following is a MathCAD model designed to solve the energy equation and the Mannings equation simultaneously at the entrance to a trapezoidal channel. Two variables are solve; however since the area A is given by a separate equation, it is also solved, making the Find statement solve for A, but 2 other variable.

PRB2_86.MCD

$H := 5 \quad Q := 100 \quad b := 6 \quad m := 1 \quad K := .05 \quad So := .0006 \quad n := .013 \quad Y := 3 \quad g := 32.2 \quad Cu := 1.486 \quad A := 40$

Given

$$A = (b + m \cdot Y) \cdot Y \qquad H = Y + (1 + K) \cdot \frac{\left(\frac{Q}{A}\right)^2}{2 \cdot g} \qquad Q = \frac{Cu}{n} \cdot \left[\frac{A}{\left(b + 2 \cdot Y \cdot \sqrt{m^2 + 1}\right)}\right]^{.6666667} \cdot A \cdot \sqrt{So}$$

$$\text{Find}(A, Y, Q) = \begin{pmatrix} 48.087 \\ 4.556 \\ 251.06 \end{pmatrix}$$

The following is a MathCAD mode designed to solve the energy at the entrance to a channel, and Mannings equations simultaneously for a circular channel

PRB2_87.MCD

$H := 5 \quad Q := 100 \quad D := 5 \quad K := .05 \quad So := .0006 \quad n := .013 \quad Y := 3 \quad g := 32.2 \quad Cu := 1.486 \quad A := 40 \quad \beta := 1.9$

Given

$$\cos(\beta) = 1 - 2 \cdot \frac{Y}{D} \qquad Q = \frac{Cu}{n} \cdot \left(\frac{A}{D \cdot \beta}\right)^{.6666667} \cdot A \cdot \sqrt{So}$$

$$A = .25 \cdot D^2 \cdot (\beta - \cos(\beta) \cdot \sin(\beta)) \qquad H = Y + (1 + K) \cdot \frac{\left(\frac{Q}{A}\right)^2}{2 \cdot g}$$

$$\text{Find}(\beta, A, Y, Q) = \begin{bmatrix} 2.736 \\ 19.365 \\ 4.797 \\ 68.366 \end{bmatrix}$$

88.

A trapezoidal channel is to be designed to carry a flowrate Q = 400 cfs. The single channel divides into two trapezoidal channels with the first having a bottom width b_2 = 6 ft, a side slope m_2 = 1.5, a Manning's n_2 = .013 and a bottom slope S_{o2} = .0005, and the second having a bottom width b_3 = 8 ft, a side slope m_3 = 1.2, a Manning's n_3 = .013 and a bottom slope S_{o3} = .0006. The bottom of the channel with a bottom width of 8 ft is 0.5 ft above the other divided channel. The upstream single channel is to have a bottom slope of S_{o1} = .0008, m_1 = 2.0 and a Manning's n_1 = .014. Determine the depths and flowrates in the downstream divided channels, and the bottom width and depth in the upstream channel so that uniform flow will occur in it when the above design flowrate occurs.

Given: trapezoidal channel, Q = 400 cfs. The upstream channel divides into two downstream channels.
Wanted: The depths and flow rates

Solution:
There are 6 unknowns: b_1, Y_1, Q_2, Y_2, Q_3, Y_3
Equations (6):

$$F_1 = n_1Q_1P_1^{2/3} - C_uA_1^{5/3}S_{o1}^{1/2} = 0$$

$$F_2 = n_2Q_2P_2^{2/3} - C_uA_2^{5/3}S_{o2}^{1/2} = 0$$

$$F_3 = n_3Q_3P_3^{2/3} - C_uA_3^{5/3}S_{o3}^{1/2} = 0$$

$$F_4 = Q_1 - Q_2 - Q_3 = 0$$

$$F_5 = Y_1+(Q_1/A_1)^2/(2g) - Y_2-(Q_2/A_2)^2/(2g) = 0$$

$$F_6 = Y_1+(Q_1/A_1)^2/(2g) - Y_3-(Q_3/A_3)^2/(2g)- \Delta z = 0$$

Solution: (Using Program BRANCHCH with Δz=0)
b_1 = 13.795 ft ←
Y_1 = 3.503 ft ←
Q_2 = 183.333 cfs ←
Y_2 = 3.680 ft ←
Q_3 = 216.67 cfs ←
Y_3 = 3.600 ft

Solution: (Using Program BRANCHCH with Δz=0.5 ft)
b_1 = 11.710 ft ←
Y_1 = 3.743 ft ←
Q_2 = 207.51 cfs ←
Y_2 = 3.914 ft ←
Q_3 = 192.43 cfs ←
Y_3 = 3.377 ft

TK-Solver Model PRB1 68.TK (Notice the solution is for Δz=0)

VARIABLE SHEET

St	Input	Name	Output	Unit	Comment
	.014	n1			
	400	Q1			
		b1	13.795412		
		Y1	3.5025865		
	2	m1			
	1.486	Cu			
		A1	72.855846		
	.0008	So1			

```
   64.4      g2
             Y2      3.6803277
             Q2      183.33316
   6         b2
   1.5       m2
             Y3      3.6001088
             Q3      216.66684
   8         b3
   1.2       m3
   .013      n2
   .0005     So2
   .013      n3
   .0006     So3

=============== RULE SHEET =================
S Rule-------------------------------------
* A1=(b1+m1*Y1)*Y1
* n1*Q1*(b1+2*Y1*sqrt(m1^2+1))^.6666667-Cu*A1^1.6666667*sqrt(So1)=0
* Y1+(Q1/A1)^2/g2=Y2+(Q2/((b2+m2*Y2)*Y2))^2/g2
* Y1+(Q1/A1)^2/g2=Y3+(Q3/((b3+m3*Y3)*Y3))^2/g2
* n2*Q2*(b2+2*Y2*sqrt(m2^2+1))^.6666667-Cu*((b2+m2*Y2)*Y2)^1.6666667*sqrt(So2)=0
* n3*Q3*(b3+2*Y3*sqrt(m3^2+1))^.6666667-Cu*((b3+m3*Y3)*Y3)^1.6666667*sqrt(So3)=0
* Q1=Q2+Q3
```

MathCAD Model PRB2_88.MCD

$Q := 400 \quad b1 := 13 \quad m1 := 2 \quad n1 := .014 \quad So1 := .0008 \quad Delz := .5 \quad b2 := 6 \quad m2 := 1.5 \quad n2 := .013 \quad g := 32.2 \quad So2 := .0005$

$b3 := 8 \quad m3 := 1.2 \quad n3 := .013 \quad So3 := .0006 \quad Cu := 1.486 \quad Y2 := 3.7 \quad Y3 := 3.6 \quad Y1 := 3.6 \quad Q2 := 180 \quad Q3 := 220$

$A1 := (b1 + m1 \cdot Y1) \cdot Y1 \quad A2 := (b2 + m2 \cdot Y2) \cdot Y2 \quad A3 := (b3 + m3 \cdot Y3) \cdot Y3$

Given $\quad Q = Q2 + Q3 \quad A2 = (b2 + m2 \cdot Y2) \cdot Y2 \quad A3 = (b3 + m3 \cdot Y3) \cdot Y3 \quad A1 = (b1 + m1 \cdot Y1) \cdot Y1$

$$n1 \cdot Q \cdot \left(b1 + 2 \cdot Y1 \cdot \sqrt{m1^2 + 1}\right)^{.666667} = Cu \cdot A1^{1.6666667} \cdot \sqrt{So1} \qquad n2 \cdot Q2 \cdot \left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.666667} = Cu \cdot A2^{1.6666667} \cdot \sqrt{So2}$$

$$n3 \cdot Q3 \cdot \left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.666667} = Cu \cdot A3^{1.6666667} \cdot \sqrt{So3}$$

$$Y1 + \frac{\left(\frac{Q}{A1}\right)^2}{2 \cdot g} = Y2 + \frac{\left(\frac{Q2}{A2}\right)^2}{2 \cdot g} \qquad Y1 + \frac{\left(\frac{Q}{A1}\right)^2}{2 \cdot g} = Y3 + \frac{\left(\frac{Q3}{A3}\right)^2}{2 \cdot g} + Delz$$

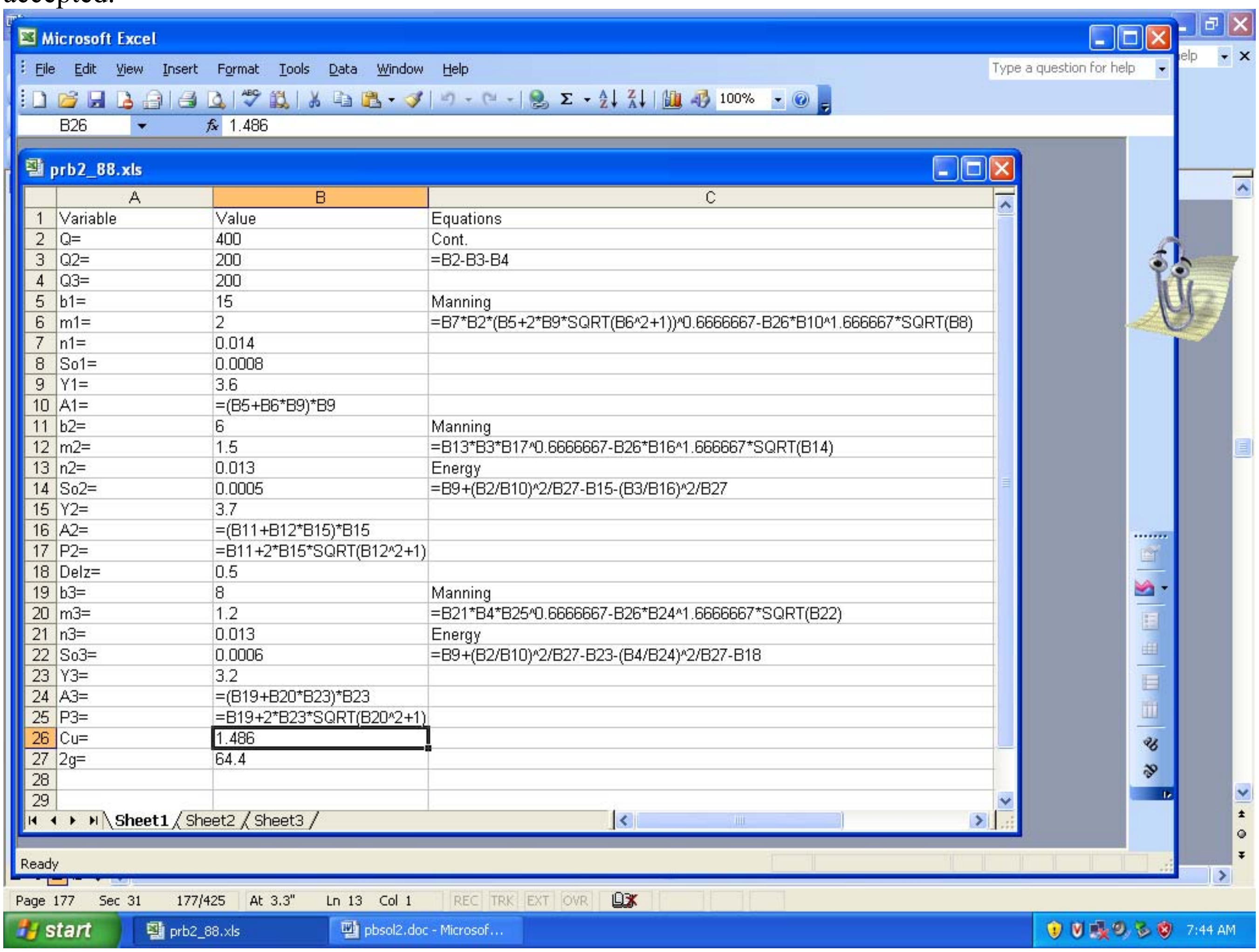
$$Find(b1, Y1, A1, Q2, Y2, A2, Q3, Y3, A3) = \begin{bmatrix} 11.71 \\ 3.743 \\ 71.845 \\ 207.571 \\ 3.914 \\ 46.468 \\ 192.429 \\ 3.377 \\ 40.702 \end{bmatrix}$$

This problem can also be solved using a spreadsheet such as EXCEL. In EXCEL an "add-in" **Solver** allows a system of equations to be solved. The spreadsheet is created as below. Column A defines the variables, whose values are given in Column B (guesses are provided for the unknown variables.) Column C contains the 6 equations, which are shown in the first image below. To have EXCEL solve the problem **Solver** is accessed from the "Tools" menu and the following set:

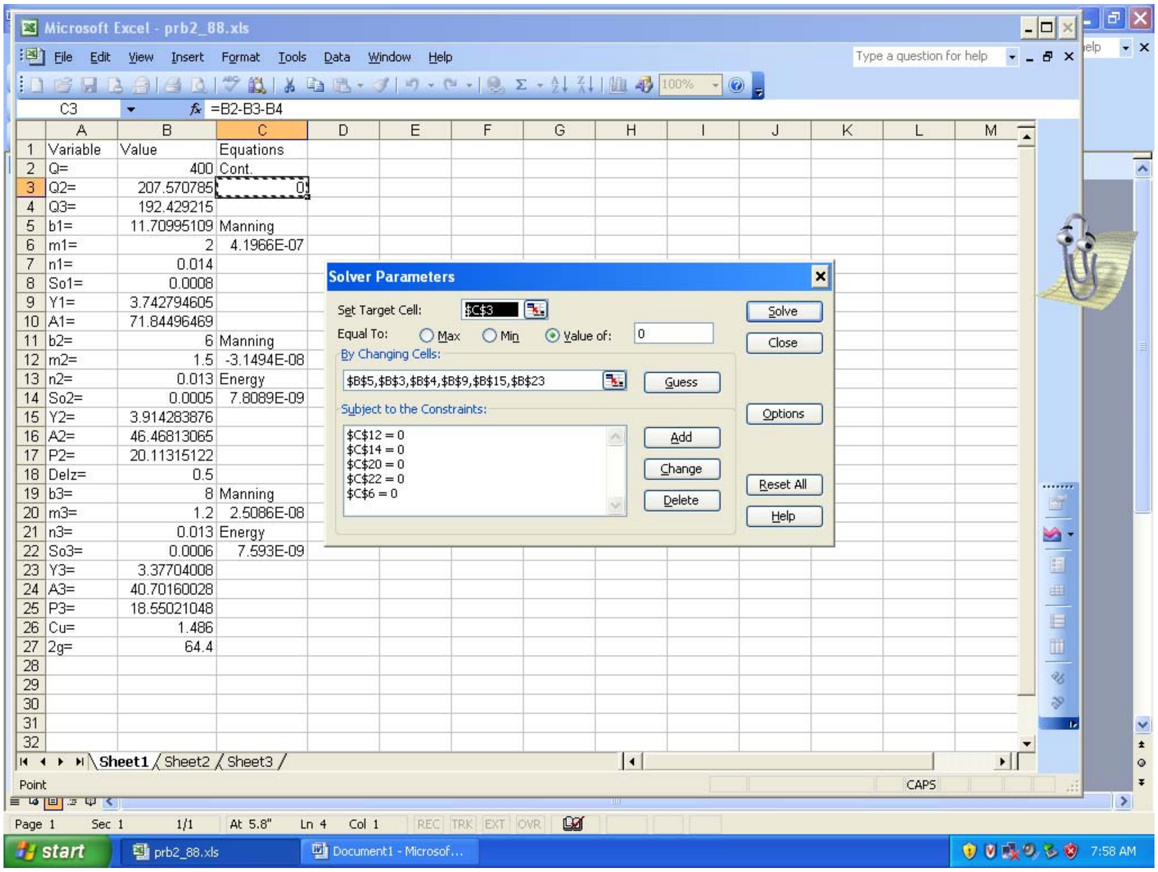
Set Target Cell **C3 = 0**

By Changing Cells
B5,B3,B4,B9,B15,B23

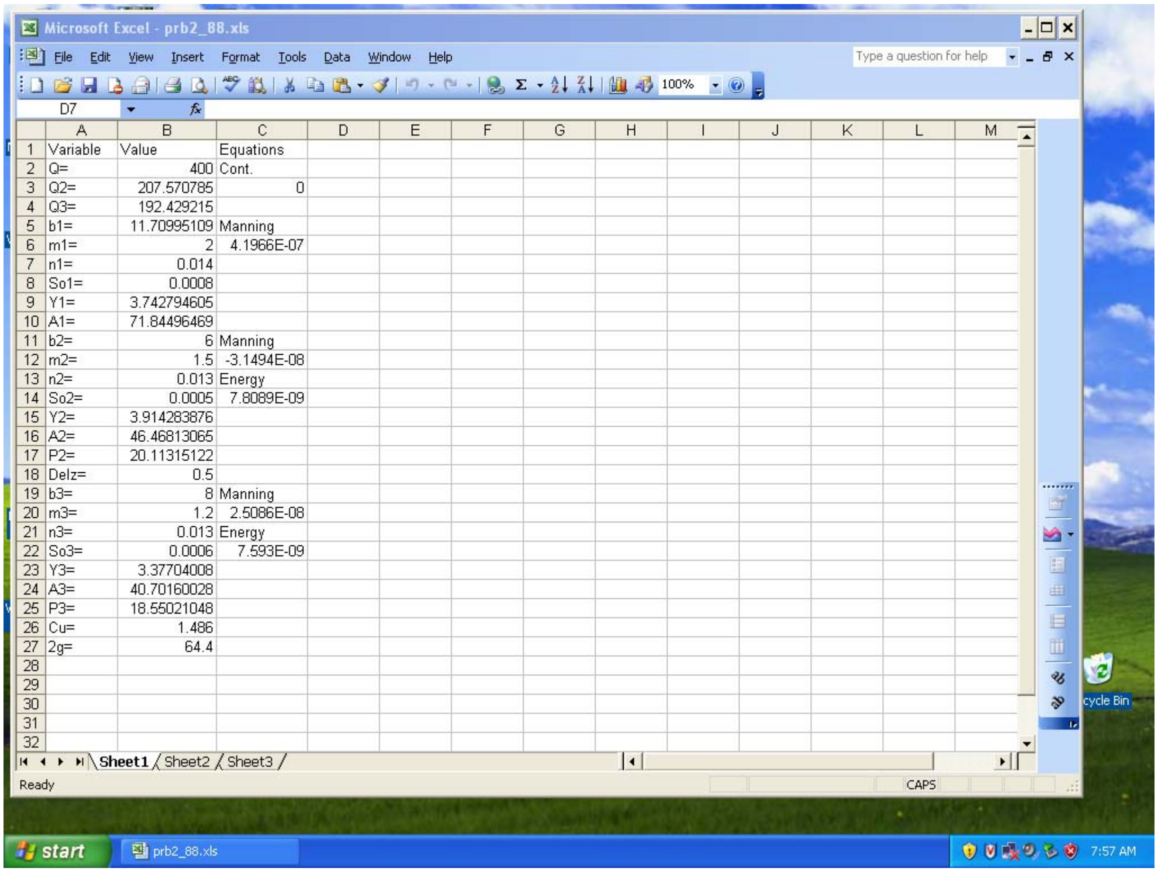
Subject to the Constraints
C6=0
C12=0
C14=0
C20=0
C22=0
There are three EXCEL worksheets shown below. The first displays the equation that were used in Column C. The second shows the solver window displayed that is activated by clicking on Tools and then Solver and filling in what is needed in this window to solve the problem. The third shows the solution in which the values of the variables are given in Column B after the solution from Solver has be accepted.



	A	B	C
1	Variable	Value	Equations
2	Q=	400	Cont.
3	Q2=	200	=B2-B3-B4
4	Q3=	200	
5	b1=	15	Manning
6	m1=	2	=B7*B2*(B5+2*B9*SQRT(B6^2+1))^0.6666667-B26*B10^1.666667*SQRT(B8)
7	n1=	0.014	
8	So1=	0.0008	
9	Y1=	3.6	
10	A1=	=(B5+B6*B9)*B9	
11	b2=	6	Manning
12	m2=	1.5	=B13*B3*B17^0.6666667-B26*B16^1.666667*SQRT(B14)
13	n2=	0.013	Energy
14	So2=	0.0005	=B9+(B2/B10)^2/B27-B15-(B3/B16)^2/B27
15	Y2=	3.7	
16	A2=	=(B11+B12*B15)*B15	
17	P2=	=B11+2*B15*SQRT(B12^2+1)	
18	Delz=	0.5	
19	b3=	8	Manning
20	m3=	1.2	=B21*B4*B25^0.6666667-B26*B24^1.6666667*SQRT(B22)
21	n3=	0.013	Energy
22	So3=	0.0006	=B9+(B2/B10)^2/B27-B23-(B4/B24)^2/B27-B18
23	Y3=	3.2	
24	A3=	=(B19+B20*B23)*B23	
25	P3=	=B19+2*B23*SQRT(B20^2+1)	
26	Cu=	1.486	
27	2g=	64.4	
28			
29			



The EXCEL sheet that show the dialog box from selecting **Solver** and how this box is filled in to solve the problem.



	A	B	C
1	Variable	Value	Equations
2	Q=	400	Cont.
3	Q2=	207.570785	0
4	Q3=	192.429215	
5	b1=	11.70995109	Manning
6	m1=	2	4.1966E-07
7	n1=	0.014	
8	So1=	0.0008	
9	Y1=	3.742794605	
10	A1=	71.84496469	
11	b2=	6	Manning
12	m2=	1.5	-3.1494E-08
13	n2=	0.013	Energy
14	So2=	0.0005	7.8089E-09
15	Y2=	3.914283876	
16	A2=	46.46813065	
17	P2=	20.11315122	
18	Delz=	0.5	
19	b3=	8	Manning
20	m3=	1.2	2.5086E-08
21	n3=	0.013	Energy
22	So3=	0.0006	7.593E-09
23	Y3=	3.37704008	
24	A3=	40.70160028	
25	P3=	18.55021048	
26	Cu=	1.486	
27	2g=	64.4	

The EXCEL sheet that gives the solution after the results from **Solver** have been accepted.

89.

Four channels branch from the upstream main channel as shown in the sketch below. The sizes, etc. of the channel are as shown on the sketch, include the proposed width of 16 feet for the upstream main channel. A gate exist a short distance downstream in channel 4, with a setting that produces a depth downstream from it of 0.6 ft. Also notice that the bottom of channel 2 is 0.3 ft above the bottoms of the other channels. Is is possible to have this upstream channel 16 feet and not restrict the flow that the other channels can carry under the conditions given? Solve for the flowrates and depths in all of the channels if the upstream trapezoidal channel as a bottom width b_1 = 20 ft, and a side slope m_1 = 2.

Given an upstream channel that branches into four downstream channel making a system of 5 channel.

When using the input to Program BRANCHCH that is given below it fails to obtain a solution. Also the TK-Solver model BRANCH5.TK, when b1 is given a value of 16 ft, or the MathCAD model BRANCH5.MCD when b1 is given a value of 16 ft fail. Using a bottom width of b1=16 ft for Channel 1, and a reservoir head H = 5 ft, and solving the energy at the entrance of Channel 1 and critical flow equations simultaneously, gives a critical flow rate of 790.4 cfs. Therefore this Q_c=790.4 cfs is the maxiumum flow rate that Channel 1 will allow into the downstream system of 4 channel. It turns out that their combined carrying capacity is larger than this Q_c as will be demonstrated below.

Input to BRANCHCH that fails to produce a solution (BRANCH5.DAT)

```
1
1
1
1
1
6 876.2 5.8 16 2 .013  .0
.0008 200 5.7 5 1 .013 0 0
.0005 50 5.85 5 0 .013 0 0
.001 40 5.8 5 0 .013 0 .3
.00075 11 5.7 6 1 .013 0 0
4 .6
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4 Q5 Y5
```

Since it appears that critical flow limits the amount that Channel 1 can carry, and that this amount is less than the carrying capacity of the four downstream channels let's increase its bottom width to b_1= 20 ft. The input to and output from BRANCHCH to solve this problem are given below.

Input for BRANCHCH with b_1=20 ft (BRANCH5A.DAT)

```
1
1
1
1
1
5 880 4.8 20 2 .013 0/
.001 220 4.56 5 1 .013 0 .3
.0007 320 4.86 6 1.5 .013 0. 0.
.0004 90 4.86 8 0 .013 0 0.
.0005 250 4.86 6 1 .013 0 0.
4 .6
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4 Q5 Y5
```

Output from BRANCHCH

```
 For Channel 1
 H =   5.000
 Q = 876.160
 Y =   4.119
 b =  20.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 231.680
 Y =   4.102
 b =   5.000
 m =   1.000
 n =    .013
```

```
For Channel 3
S = .000700
Q = 326.142
Y =   4.497
b =   6.000
  m =   1.500
  n =    .013
  For Channel 4
  S = .000400
  Q =  80.800
  Y =   4.935
  b =   8.000
  m =    .000
  n =    .013
  For Channel 5
  S = .000500
  Q = 237.538
  Y =   4.641
  b =   6.000
  m =   1.000
  n =    .013
```

This solution shows us that the downstream channel can carry a flow rate Q=876.16 cfs. Since we are assuming that Channel 1 is very short and we are using an entrance loss coefficient of K_e, the head available at the beginning of Channels 3,4 and 5 is also H=5 ft, and for Channel 2 is H = 5-.3 = 4.7 ft, and this combined carrying capacity of 876.16 cfs could be determined by specifying the upstream head for each of the 4 channel and solving for it flow rate. The values given in the above solution for the Q, or Q_2=231.680, Q_3=326.142, Q_4=80.800,Q_5=237.538 cfs would result and they add to 876.16 cfs.

Using this flow rate of 876.16 cfs, one can solve the energy and critical flow equations for Channel 1 simultaneously for its bottom width and depth and get Y_{c1}=3.596 ft, and b_1=18.431 ft. (This solution was obtained using CHANNEL.) Therefore we can conclude that in order for Channel 1 to have the carrying capacity of the downstream four channels it bottom width must be 18.431 ft or larger. With a bottom width of 18.431 ft the flow in Channel 1 will be critical, and when it bottom width larger than this value the flow in it will be subcritical. For example when it width is 20 ft as given in the above solution the Froude Number associated with its flow rate is $F_{r1} = (876.161^2[20+2(4.119)2]/[32.2\{(20+2(4.119))4.119)^3\}])^{1/2} = .50$

The TK-Solver Model given below can be used to solve the same problem that was solved above using BRANCHCH in with the bottom width of Channel 1 was specified as b_1= 20 ft.

TK-Solver model BRANCH5.TK

```
═══════════════ VARIABLE SHEET ════════════════
St Input───── Name──── Output──── Unit────── Comment
             Q1      876.16076
             Y1      4.1187534
             Q2      231.68034
             Y2      4.1021758
             Q3      326.14257
             Y3      4.4973491
             Q4      80.799842
             Y4      4.9349589
             Q5      237.53801
             Y5      4.6406832
   5         H
   20        b1
   2         m1
   64.4      g2
   5         b2
   1         m2
   .3        z12
   6         b3
   1.5       m3
   8         b4
   .6        Yd4
   .013      n2
   .001      So2
   .013      n3
   .0007     So3
   6         b5
   .013      n5
   1         m5
   .0005     So5
```

```
═══════════════ RULE SHEET ═════════════════
S Rule──────────────────────────────────────────────
* Q1-Q2-Q3-Q4-Q5=0
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-1.486*((b2+m2*Y2)*Y2)^1.666667*sqrt(So
* n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-1.486*((b3+m3*Y3)*Y3)^1.666667*sqrt(So
* n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.6666667-1.486*((b5+m5*Y5)*Y5)^1.666667*sqrt(So
* H-Y1-(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(Q3/((b3+m3*Y3)*Y3))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y5-(Q5/((b5+m5*Y5)*Y5))^2/g2=0
* Yd4+(Q4/(b4*Yd4))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
```

While we can using the flow rate of 871.161 cfs for Channel 1 and solve the energy and critical flow equations simultaneously to find the limiting width for Channel 1 as given above, this TK-Solver model could be modified by adding the critical flow equation for Channel 1 to the other equations and adding b1 to the list of guesses (unknowns). The model to solve this problem is given below.

<u>TK-Solver model BRANCH5C.TK</u>

```
═══════════════ VARIABLE SHEET ═════════════════
St Input──── Name─── Output─── Unit───── Comment─
             Q1      876.16076
             Y1      3.5960442
             Q2      231.68034
             Y2      4.1021758
             Q3      326.14257
             Y3      4.4973491
             Q4      80.799842
             Y4      4.9349589
             Q5      237.53801
             Y5      4.6406832
   5         H
             b1      18.431452
   2         m1
   64.4      g2
   5         b2
   1         m2
   .3        z12
   6         b3
   1.5       m3
   8         b4
   .6        Yd4
   .013      n2
   .001      So2
   .013      n3
   .0007     So3
   6         b5
   .013      n5
   1         m5
   .0005     So5
             g

═══════════════ RULE SHEET ═════════════════
S Rule────────────────────────────────────────
  Q1-Q2-Q3-Q4-Q5=0
  n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-1.486*((b2+m2*Y2)*Y2)^1.666667*sqrt(So
  n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-1.486*((b3+m3*Y3)*Y3)^1.666667*sqrt(So
  n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.6666667-1.486*((b5+m5*Y5)*Y5)^1.666667*sqrt(So
  H-Y1-(Q1/((b1+m1*Y1)*Y1))^2/g2=0
  Q1^2*(b1+2*m1*Y1)-.5*g2*((b1+m1*Y1)*Y1)^3=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(Q3/((b3+m3*Y3)*Y3))^2/g2=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y5-(Q5/((b5+m5*Y5)*Y5))^2/g2=0
  Yd4+(Q4/(b4*Yd4))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
```

If the bottom width of Channel 1 is 16 ft, then critical flow will occur in its entrance, and supercritical flow will occur in its upstream portion, with a hydraulic jump (as discussed in Chapter 3) which will dissipate some energy thus reducing the head H available to the four downstream channels. This problem can be solved by modifying the above TK-Solver model to specifying the flow rate in Channel 1 as 790.4 cfs (the critical value with H=5 ft, and b_1=16 ft) and solving for H rather than Q1. This solution gives a head H = 4.733 ft. Therefore if the reservoir head were 5 ft as given in the problem statement, and the bottom width of Channel 1 is 16 ft, then there would be a headloss of 5-4.733 = 0.267 ft through Channel 1, and the head available at their beginnings to cause the flow in the remaining 4 channel as given below would 4.733 ft.

```
TK-Solver model BRANCH5O.TK
================ VARIABLE SHEET ================
St Input----- Name---- Output---- Unit------ Comment
   790.4      Q1
              Q2      206.79883
              Y2      3.8693062
              Q3      291.59808
              Y3      4.2588222
              Q4      78.311432
              Y4      4.6647788
              Q5      213.69166
              Y5      4.3929954
              H       4.7331578
   16         b1
   2          m1
   64.4       g2
   5          b2
   1          m2
   .3         z12
   6          b3
   1.5        m3
   8          b4
   .6         Yd4
   .013       n2
   .001       So2
   .013       n3
   .0007      So3
   6          b5
   .013       n5
  1         m5
  .0005     So5

================ RULE SHEET ================
S Rule----------------------------------------
* Q1-Q2-Q3-Q4-Q5=0
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-1.486*((b2+m2*Y2)*Y2)^1.666667*sqrt(So
* n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-1.486*((b3+m3*Y3)*Y3)^1.666667*sqrt(So
* n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.6666667-1.486*((b5+m5*Y5)*Y5)^1.666667*sqrt(So
* H-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
* H-Y3-(Q3/((b3+m3*Y3)*Y3))^2/g2=0
* H-Y4-(Q4/(b4*Y4))^2/g2=0
* H-Y5-(Q5/((b5+m5*Y5)*Y5))^2/g2=0
* Yd4+(Q4/(b4*Yd4))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
```

Below are MathCAD models to solve the problems above. The first limits the flow rate to 790.4 cfs, the critical value for Channel 1 with a bottom width of 16 ft. The second solves the problem with the width of Channel 1 specified as 18.45 ft.

MathCAD model BRANCH50.MCD with $Q_1 = Q_c = 790.4$ cfs controlling

$H := 5$ $Q1 := 790.4$ $Y1 := 4.87$ $b1 := 16$ $m1 := 2$ $n1 := .013$ $dz := .3$ $g2 := 64.4$ $Yd4 := .6$ $Q2 := 230$ $Y2 := 4.1$
$b2 := 5$ $m2 := 1$ $n2 := .013$ $So2 := .001$ $Q3 := 325$ $Y3 := 4.5$ $b3 := 6$ $m3 := 1.5$ $n3 := .013$ $So3 := .0007$ $Q4 := 81$
$Y4 := 5.25$ $b4 := 8$ $n4 := .013$ $So4 := .0004$ $Q5 := 235$ $Y5 := 4.64$ $b5 := 6$ $m5 := 1$ $n5 := .013$ $So5 := .0005$

Given

$$Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5}$$

$$Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2} \qquad Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3}$$

$$H = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz \qquad H = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$H = Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} \qquad H = Y5 + \frac{\left[\frac{Q5}{(b5 + m5 \cdot Y5) \cdot Y5}\right]^2}{g2}$$

$$\text{Find}(H, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) = \begin{bmatrix} 4.73316 \\ 3.86931 \\ 206.79883 \\ 4.25882 \\ 291.59808 \\ 4.66478 \\ 78.31143 \\ 4.393 \\ 213.69166 \end{bmatrix}$$

MathCAD model BRANCH5.MCD with b1 specified equal to 18.45 ft.

$H := 5$ $Q1 := 876$ $Y1 := 4.87$ $b1 := 18.45$ $m1 := 2$ $n1 := .013$ $dz := .3$ $g2 := 64.4$ $Yd4 := .6$ $Q2 := 230$ $Y2 := 4.1$
$b2 := 5$ $m2 := 1$ $n2 := .013$ $So2 := .001$ $Q3 := 325$ $Y3 := 4.5$ $b3 := 6$ $m3 := 1.5$ $n3 := .013$ $So3 := .0007$
$Q4 := 81$ $Y4 := 5.25$ $b4 := 8$ $n4 := .013$ $So4 := .0004$ $Q5 := 235$ $Y5 := 4.64$ $b5 := 6$ $m5 := 1$ $n5 := .013$ $So5 := .0005$

Given

$$Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5}\cdot\frac{((b5 + m5\cdot Y5)\cdot Y5)^{1.666667}}{\left(b5 + 2\cdot Y5\cdot\sqrt{m5^2+1}\right)^{.66666667}}\cdot\sqrt{So5}$$

$$Q2 = \frac{1.486}{n2}\cdot\frac{((b2 + m2\cdot Y2)\cdot Y2)^{1.666667}}{\left(b2 + 2\cdot Y2\cdot\sqrt{m2^2+1}\right)^{.66666667}}\cdot\sqrt{So2} \qquad Q3 = \frac{1.486}{n3}\cdot\frac{((b3 + m3\cdot Y3)\cdot Y3)^{1.666667}}{\left(b3 + 2\cdot Y3\cdot\sqrt{m3^2+1}\right)^{.66666667}}\cdot\sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2\cdot Y2)\cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3\cdot Y3)\cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4\cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5\cdot Y5)\cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	3.661
1	876.161
2	4.102
3	231.68
4	4.497
5	326.143
6	4.935
7	80.8
8	4.641
9	237.538

90.

Determine the minimum width b_1 that the upstream channel of the previous channel system must have so that critical flow will not occur at its entrance, thus limiting the flowrate that the downstream channels can carry. You might use one of the models you used to solve the previous in which you attempt to successively reduct the width from 20 ft until it fails to produce a solution to get an ideal how the upstream depth decrease with decreasing b_1 until critical depth occurs. Why are the flowrates and depths of the channels downstream from the junction not effected by changing widths of the upstream channel until critical conditions occur?

Wanted: the minimum width of Channel 1 so critical flow at its entrance will not control the flow rate into the system.

Solution: This question was answered in the previous problem and is b_1 = 18.431 ft.

As stated in the solution to the previous problem, if we assume there is no loss of head in Channel 1 then the four downstream channels can have their flow rates and depths determined individually as follows:

Channel 2

$5-\Delta z = 4.7 = Y_2+(Q/A)_2^2/(2g)$ and Mannings Equation give:

Q_2 = 231.7 cfs, Y_2 = 4.102 ft

Channel 3

$5 = Y_3+(Q/A)_3^2/(2g)$ and Mannings Equation give:

Q_3 = 326.1 cfs, Y_3 = 4.497 ft

Channel 4

Energy across gate $5 = Y_{d4}+Q_4^2/(2gA_{d4}^2)$ gives:

Q_4 = 80.8 cfs, and $E = 5 = Y_4+(Q/A)_4^2/(2g)$ → Y_4 = 4.935 ft

Channel 5

$5= Y_5+(Q/A)_5^2/(2g)$ and Mannings Equation give:

Q_5 = 237.5 cfs, Y_5= 4.641 ft

So the flow rate in Channel 1 is the sum of these individual flow rate or 876.2 cfs

Now solveing $H = Y_1 + (Q/A)_1^2/(2g)$ and $Q_1^2T/(gA_1^3) = 1$

For Y_1 = 3.596 ft and b = 18.433 ft ←

Below the MathCAD model is used with b = 19 ft, b=18.5 ft, and b = 18.45 ft, respectively. Notice all answers are the same except Y_1 from these three solutions. When b = 18.4 ft is used the solution fails, as one would expect.

MathCAD model BRANCH5.MCD with b1=19 ft

H := 5 Q1 := 876 Y1 := 4.87 b1 := 19.0 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1 b2 := 5
m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007 Q4 := 81 Y4 := 5.25
b4 := 8 n4 := .013 So4 := .0004 Q5 := 235 Y5 := 4.64 b5 := 6 m5 := 1 n5 := .013 So5 := .0005

Given

$$Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5}\cdot\frac{((b5 + m5\cdot Y5)\cdot Y5)^{1.666667}}{\left(b5 + 2\cdot Y5\cdot\sqrt{m5^2 + 1}\right)^{.66666667}}\cdot\sqrt{So5}$$

$$Q2 = \frac{1.486}{n2}\cdot\frac{((b2 + m2\cdot Y2)\cdot Y2)^{1.666667}}{\left(b2 + 2\cdot Y2\cdot\sqrt{m2^2 + 1}\right)^{.66666667}}\cdot\sqrt{So2} \qquad Q3 = \frac{1.486}{n3}\cdot\frac{((b3 + m3\cdot Y3)\cdot Y3)^{1.666667}}{\left(b3 + 2\cdot Y3\cdot\sqrt{m3^2 + 1}\right)^{.66666667}}\cdot\sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2\cdot Y2)\cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3\cdot Y3)\cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4\cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5\cdot Y5)\cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	3.931
1	876.161
2	4.102
3	231.68
4	4.497
5	326.143
6	4.935
7	80.8
8	4.641
9	237.538

MathCAD model BRANCH5.MCD with b1=18.5 ft

H := 5 Q1 := 876 Y1 := 4.87 b1 := 18.5 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1 b2 := 5 m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007 Q4 := 81 Y4 := 5.25 b4 := 8 n4 := .013 So4 := .0004 Q5 := 235 Y5 := 4.64 b5 := 6 m5 := 1 n5 := .013 So5 := .0005

Given

$$Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5}\cdot\frac{((b5 + m5\cdot Y5)\cdot Y5)^{1.666667}}{\left(b5 + 2\cdot Y5\cdot\sqrt{m5^2 + 1}\right)^{.66666667}}\cdot\sqrt{So5}$$

$$Q2 = \frac{1.486}{n2}\cdot\frac{((b2 + m2\cdot Y2)\cdot Y2)^{1.666667}}{\left(b2 + 2\cdot Y2\cdot\sqrt{m2^2 + 1}\right)^{.66666667}}\cdot\sqrt{So2} \qquad Q3 = \frac{1.486}{n3}\cdot\frac{((b3 + m3\cdot Y3)\cdot Y3)^{1.666667}}{\left(b3 + 2\cdot Y3\cdot\sqrt{m3^2 + 1}\right)^{.66666667}}\cdot\sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5 \cdot Y5) \cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	3.719
1	876.161
2	4.102
3	231.68
4	4.497
5	326.143
6	4.935
7	80.8
8	4.641
9	237.538

MathCAD model BRANCH5.MCD with b1 specified equal to 18.45 ft.

H := 5 Q1 := 876 Y1 := 4.87 b1 := 18.45 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1
b2 := 5 m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007
Q4 := 81 Y4 := 5.25 b4 := 8 n4 := .013 So4 := .0004 Q5 := 235 Y5 := 4.64 b5 := 6 m5 := 1 n5 := .013 So5 := .0005
Given

$$Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5}$$

$$Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2} \qquad Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5 \cdot Y5) \cdot Y5}\right]^2}{g2}$$

$$\text{Find}(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =$$

	0
0	3.661
1	876.161
2	4.102
3	231.68
4	4.497
5	326.143
6	4.935
7	80.8
8	4.641
9	237.538

91.

Fix the upstream width of channel #1 in the system of the previous two problems to b_1 = 19 ft, and obtain a series of solutions in which the reservoir water surface H increases from 5 ft until a solution no longer exists. What caused this situation with rising reservoir heads?

Given: For the 5 channel system of the previous problem let b_1 = 19 ft, and obtain several solutions with H varying from 5 ft to larger values until a solution no longer exists.

This series of solution will first be obtained using the MathCAD model BRANCH5.MCD

H := 5.1 Q1 := 876 Y1 := 4.87 b1 := 19.0 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1
b2 := 5 m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007 Q4 := 81
Y4 := 5.25 b4 := 8 n4 := .013 So4 := .0004 Q5 := 235 Y5 := 4.64 b5 := 6 m5 := 1 n5 := .013 So5 := .0005

Given

$$Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5}\cdot\frac{((b5 + m5\cdot Y5)\cdot Y5)^{1.666667}}{\left(b5 + 2\cdot Y5\cdot\sqrt{m5^2+1}\right)^{.66666667}}\cdot\sqrt{So5}$$

$$Q2 = \frac{1.486}{n2}\cdot\frac{((b2 + m2\cdot Y2)\cdot Y2)^{1.666667}}{\left(b2 + 2\cdot Y2\cdot\sqrt{m2^2+1}\right)^{.66666667}}\cdot\sqrt{So2} \qquad Q3 = \frac{1.486}{n3}\cdot\frac{((b3 + m3\cdot Y3)\cdot Y3)^{1.666667}}{\left(b3 + 2\cdot Y3\cdot\sqrt{m3^2+1}\right)^{.66666667}}\cdot\sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2\cdot Y2)\cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3\cdot Y3)\cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4\cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5\cdot Y5)\cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	3.995
1	909.621
2	4.189
3	241.41
4	4.587
5	339.663
6	5.036
7	81.713
8	4.733
9	246.834

H := 5.2 Q1 := 876 Y1 := 4.87 b1 := 19.0 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1
b2 := 5 m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007 Q4 := 81

$Y4 := 5.25 \quad b4 := 8 \quad n4 := .013 \quad So4 := .0004 \quad Q5 := 235 \quad Y5 := 4.64 \quad b5 := 6 \quad m5 := 1 \quad n5 := .013 \quad So5 := .0005$

Given $\quad Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5}$

$$Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2} \qquad Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5 \cdot Y5) \cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	4.057
1	943.81
2	4.277
3	251.364
4	4.676
5	353.501
6	5.137
7	82.616
8	4.826
9	256.329

$H := 5.4 \quad Q1 := 876 \quad Y1 := 4.87 \quad b1 := 19.0 \quad m1 := 2 \quad n1 := .013 \quad dz := .3 \quad g2 := 64.4 \quad Yd4 := .6 \quad Q2 := 230 \quad Y2 := 4.1$

$b2 := 5 \quad m2 := 1 \quad n2 := .013 \quad So2 := .001 \quad Q3 := 325 \quad Y3 := 4.5 \quad b3 := 6 \quad m3 := 1.5 \quad n3 := .013 \quad So3 := .0007 \quad Q4 := 81$

$Y4 := 5.25 \quad b4 := 8 \quad n4 := .013 \quad So4 := .0004 \quad Q5 := 235 \quad Y5 := 4.64 \quad b5 := 6 \quad m5 := 1 \quad n5 := .013 \quad So5 := .0005$

Given $\quad Q1 - Q2 - Q3 - Q4 - Q5 = 0$

$$Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5} \quad Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2}$$

$$Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3} \qquad H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y4+\frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} \qquad Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y5+\frac{\left[\frac{Q5}{(b5+m5\cdot Y5)\cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	4.173
1	$1.014 \cdot 10^3$
2	4.451
3	271.947
4	4.855
5	382.139
6	5.339
7	84.393
8	5.012
9	275.919

H := 5.6 Q1 := 876 Y1 := 4.87 b1 := 19.0 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1
b2 := 5 m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007 Q4 := 81
Y4 := 5.25 b4 := 8 n4 := .013 So4 := .0004 Q5 := 235 Y5 := 4.64 b5 := 6 m5 := 1 n5 := .013 So5 := .0005

Given $Q1-Q2-Q3-Q4-Q5=0$

$$Q5=\frac{1.486}{n5}\cdot\frac{((b5+m5\cdot Y5)\cdot Y5)^{1.666667}}{\left(b5+2\cdot Y5\cdot\sqrt{m5^2+1}\right)^{.66666667}}\cdot\sqrt{So5} \qquad Q2=\frac{1.486}{n2}\cdot\frac{((b2+m2\cdot Y2)\cdot Y2)^{1.666667}}{\left(b2+2\cdot Y2\cdot\sqrt{m2^2+1}\right)^{.66666667}}\cdot\sqrt{So2}$$

$$Q3=\frac{1.486}{n3}\cdot\frac{((b3+m3\cdot Y3)\cdot Y3)^{1.666667}}{\left(b3+2\cdot Y3\cdot\sqrt{m3^2+1}\right)^{.66666667}}\cdot\sqrt{So3} \qquad H=Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}$$

$$Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y2+\frac{\left[\frac{Q2}{(b2+m2\cdot Y2)\cdot Y2}\right]^2}{g2}+dz \qquad Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y3+\frac{\left[\frac{Q3}{(b3+m3\cdot Y3)\cdot Y3}\right]^2}{g2}$$

$$Y4+\frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2}=Yd4+\frac{\left(\frac{Q4}{b4\cdot Yd4}\right)^2}{g2} \qquad Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y4+\frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2}$$

$$Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y5+\frac{\left[\frac{Q5}{(b5+m5\cdot Y5)\cdot Y5}\right]^2}{g2}$$

$$\mathrm{Find}(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =$$

	0
0	4.275
1	$1.088 \cdot 10^{3}$
2	4.625
3	293.444
4	5.033
5	412.075
6	5.541
7	86.133
8	5.197
9	296.317

$H := 5.8$ $Q1 := 876$ $Y1 := 4.87$ $b1 := 19.0$ $m1 := 2$ $n1 := .013$ $dz := .3$ $g2 := 64.4$ $Yd4 := .6$ $Q2 := 230$ $Y2 := 4.1$
$b2 := 5$ $m2 := 1$ $n2 := .013$ $So2 := .001$ $Q3 := 325$ $Y3 := 4.5$ $b3 := 6$ $m3 := 1.5$ $n3 := .013$ $So3 := .0007$ $Q4 := 81$
$Y4 := 5.25$ $b4 := 8$ $n4 := .013$ $So4 := .0004$ $Q5 := 235$ $Y5 := 4.64$ $b5 := 6$ $m5 := 1$ $n5 := .013$ $So5 := .0005$

Given $Q1 - Q2 - Q3 - Q4 - Q5 = 0$ $\quad Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5}$

$$Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2} \qquad Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5 \cdot Y5) \cdot Y5}\right]^2}{g2}$$

$$\mathrm{Find}(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =$$

	0
0	4.346
1	$1.165 \cdot 10^{3}$
2	4.8
3	315.865
4	5.211
5	443.329
6	5.743
7	87.839
8	5.383
9	317.535

$H := 5.9 \quad Q1 := 876 \quad Y1 := 4.87 \quad b1 := 19.0 \quad m1 := 2 \quad n1 := .013 \quad dz := .3 \quad g2 := 64.4 \quad Yd4 := .6 \quad Q2 := 230 \quad Y2 := 4.1$
$b2 := 5 \quad m2 := 1 \quad n2 := .013 \quad So2 := .001 \quad Q3 := 325 \quad Y3 := 4.5 \quad b3 := 6 \quad m3 := 1.5 \quad n3 := .013 \quad So3 := .0007 \quad Q4 := 81$
$Y4 := 5.25 \quad b4 := 8 \quad n4 := .013 \quad So4 := .0004 \quad Q5 := 235 \quad Y5 := 4.64 \quad b5 := 6 \quad m5 := 1 \quad n5 := .013 \quad So5 := .0005$

Given $\quad Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5}$

$$Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2} \qquad Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5 \cdot Y5) \cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	4.344
1	$1.204 \cdot 10^3$
2	4.887
3	327.427
4	5.301
5	459.455
6	5.844
7	88.679
8	5.476
9	328.455

$H := 5.91 \quad Q1 := 876 \quad Y1 := 4.87 \quad b1 := 19.0 \quad m1 := 2 \quad n1 := .013 \quad dz := .3 \quad g2 := 64.4 \quad Yd4 := .6 \quad Q2 := 230 \quad Y2 := 4.1$
$b2 := 5 \quad m2 := 1 \quad n2 := .013 \quad So2 := .001 \quad Q3 := 325 \quad Y3 := 4.5 \quad b3 := 6 \quad m3 := 1.5 \quad n3 := .013 \quad So3 := .0007 \quad Q4 := 81$
$Y4 := 5.25 \quad b4 := 8 \quad n4 := .013 \quad So4 := .0004 \quad Q5 := 235 \quad Y5 := 4.64 \quad b5 := 6 \quad m5 := 1 \quad n5 := .013 \quad So5 := .0005$

Given $\quad Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5}$

$$Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2} \qquad Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y5 + \frac{\left[\frac{Q5}{(b5 + m5 \cdot Y5) \cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	4.337
1	$1.208 \cdot 10^3$
2	4.896
3	328.596
4	5.309
5	461.087
6	5.854
7	88.763
8	5.485
9	329.558

H := 5.92 Q1 := 876 Y1 := 4.87 b1 := 19.0 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1
b2 := 5 m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007 Q4 := 81
Y4 := 5.25 b4 := 8 n4 := .013 So4 := .0004 Q5 := 235 Y5 := 4.64 b5 := 6 m5 := 1 n5 := .013 So5 := .0005

Given $\quad Q1 - Q2 - Q3 - Q4 - Q5 = 0 \qquad Q5 = \frac{1.486}{n5} \cdot \frac{((b5 + m5 \cdot Y5) \cdot Y5)^{1.666667}}{\left(b5 + 2 \cdot Y5 \cdot \sqrt{m5^2 + 1}\right)^{.66666667}} \cdot \sqrt{So5}$

$$Q2 = \frac{1.486}{n2} \cdot \frac{((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667}}{\left(b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}\right)^{.66666667}} \cdot \sqrt{So2} \qquad Q3 = \frac{1.486}{n3} \cdot \frac{((b3 + m3 \cdot Y3) \cdot Y3)^{1.666667}}{\left(b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}\right)^{.66666667}} \cdot \sqrt{So3}$$

$$H = Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} \qquad Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y2 + \frac{\left[\frac{Q2}{(b2 + m2 \cdot Y2) \cdot Y2}\right]^2}{g2} + dz$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1 \cdot Y1) \cdot Y1}\right]^2}{g2} = Y3 + \frac{\left[\frac{Q3}{(b3 + m3 \cdot Y3) \cdot Y3}\right]^2}{g2} \qquad Y4 + \frac{\left(\frac{Q4}{b4 \cdot Y4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{b4 \cdot Yd4}\right)^2}{g2}$$

$$Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y4+\frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} \qquad Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y5+\frac{\left[\frac{Q5}{(b5+m5\cdot Y5)\cdot Y5}\right]^2}{g2}$$

Find(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) =

	0
0	4.325
1	$1.212\cdot 10^3$
2	4.904
3	329.768
4	5.318
5	462.721
6	5.864
7	88.846
8	5.494
9	330.664

H := 5.93 Q1 := 876 Y1 := 4.87 b1 := 19.0 m1 := 2 n1 := .013 dz := .3 g2 := 64.4 Yd4 := .6 Q2 := 230 Y2 := 4.1
b2 := 5 m2 := 1 n2 := .013 So2 := .001 Q3 := 325 Y3 := 4.5 b3 := 6 m3 := 1.5 n3 := .013 So3 := .0007 Q4 := 81
Y4 := 5.25 b4 := 8 n4 := .013 So4 := .0004 Q5 := 235 Y5 := 4.64 b5 := 6 m5 := 1 n5 := .013 So5 := .0005

Given $\quad Q1-Q2-Q3-Q4-Q5=0 \qquad Q5=\frac{1.486}{n5}\cdot\frac{((b5+m5\cdot Y5)\cdot Y5)^{1.666667}}{\left(b5+2\cdot Y5\cdot\sqrt{m5^2+1}\right)^{.66666667}}\cdot\sqrt{So5}$

$$Q2=\frac{1.486}{n2}\cdot\frac{((b2+m2\cdot Y2)\cdot Y2)^{1.666667}}{\left(b2+2\cdot Y2\cdot\sqrt{m2^2+1}\right)^{.66666667}}\cdot\sqrt{So2} \qquad Q3=\frac{1.486}{n3}\cdot\frac{((b3+m3\cdot Y3)\cdot Y3)^{1.666667}}{\left(b3+2\cdot Y3\cdot\sqrt{m3^2+1}\right)^{.66666667}}\cdot\sqrt{So3}$$

$$H=Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2} \qquad Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y2+\frac{\left[\frac{Q2}{(b2+m2\cdot Y2)\cdot Y2}\right]^2}{g2}+dz$$

$$Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y3+\frac{\left[\frac{Q3}{(b3+m3\cdot Y3)\cdot Y3}\right]^2}{g2} \qquad Y4+\frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2}=Yd4+\frac{\left(\frac{Q4}{b4\cdot Yd4}\right)^2}{g2}$$

$$Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y4+\frac{\left(\frac{Q4}{b4\cdot Y4}\right)^2}{g2} \qquad Y1+\frac{\left[\frac{Q1}{(b1+m1\cdot Y1)\cdot Y1}\right]^2}{g2}=Y5+\frac{\left[\frac{Q5}{(b5+m5\cdot Y5)\cdot Y5}\right]^2}{g2}$$

$$\text{Find}(Y1, Q1, Y2, Q2, Y3, Q3, Y4, Q4, Y5, Q5) = $$

	0
0	4.293
1	$1.216 \cdot 10^{3}$
2	4.913
3	330.927
4	5.327
5	464.332
6	5.874
7	88.93
8	5.503
9	331.759

Solution fails when H = 5.94 ft.

The same problem is solved below using Program BRANCHCH.
Input for Program BRANCH5B.DAT

```
1
1
1
1
1
5.1 850 4.8 19 2 .013 0/
.001 220 4.56 5 1 .013 0 .3
.0007 320 4.86 6 1.5 .013 0. 0.
.0004 90 4.86 8 0 .013 0 0.
.0005 250 4.86 6 1 .013 0 0.
4 .6
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4 Q5 Y5
```

```
Output with H = 5.1 ft

 For Channel 1
 H =   5.100
 Q = 909.620
 Y =   3.995
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 241.410
 Y =   4.189
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 339.663
 Y =   4.587
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  81.713
 Y =   5.036
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 246.834
 Y =   4.733
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.2 ft
 For Channel 1
 H =   5.200
 Q = 943.809
 Y =   4.057
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 251.363
 Y =   4.277
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 353.501
 Y =   4.676
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  82.616
 Y =   5.137
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 256.329
 Y =   4.826
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.3 ft
 For Channel 1
 H =   5.300
 Q = 978.733
 Y =   4.116
 b =  19.000
```

```
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 261.542
 Y =   4.364
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 367.659
 Y =   4.765
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  83.509
 Y =   5.238
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 266.023
 Y =   4.919
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.4 ft
 For Channel 1
 H =   5.400
 Q =1014.397
 Y =   4.173
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 271.947
 Y =   4.451
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 382.138
 Y =   4.855
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  84.393
 Y =   5.339
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 275.918
 Y =   5.012
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.5 ft
 For Channel 1
 H =   5.500
 Q =1050.807
 Y =   4.226
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 282.580
 Y =   4.538
 b =   5.000
 m =   1.000
 n =    .013

 For Channel 3
 S = .000700
 Q = 396.943
 Y =   4.944
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  85.267
 Y =   5.440
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 286.016
 Y =   5.105
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.6 ft
 For Channel 1
 H =   5.600
 Q =1087.968
 Y =   4.275
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 293.444
 Y =   4.625
 b =   5.000
 m =   1.000

 n =    .013
 For Channel 3
 S = .000700
 Q = 412.074
 Y =   5.033
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  86.133
 Y =   5.541
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 296.317
 Y =   5.197
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.7 ft
 For Channel 1
 H =   5.700
 Q =1125.886
 Y =   4.316
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 304.538
 Y =   4.713
 b =   5.000

 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 427.535
 Y =   5.122
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  86.990
 Y =   5.642
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 306.823
 Y =   5.290
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.8 ft
 For Channel 1
 H =   5.800
 Q =1164.567
 Y =   4.346
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 315.865
 Y =   4.800

 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 443.328
 Y =   5.211
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  87.839
 Y =   5.743
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
```

```
 S = .000500
 Q = 317.535
 Y =   5.383
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.9 ft
 For Channel 1
 H =   5.900
 Q =1204.016
 Y =   4.344
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 327.427

 Y =   4.887
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 459.455
 Y =   5.301
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  88.679
 Y =   5.844
 b =   8.000
 m =    .000
 n =    .013

 For Channel 5
 S = .000500
 Q = 328.455
 Y =   5.476
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.91 ft
 For Channel 1
 H =   5.910
 Q =1208.003
 Y =   4.337
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 328.596
 Y =   4.896
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 461.086
 Y =   5.309
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  88.763
 Y =   5.854
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 329.558
 Y =   5.485
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.92 ft
 For Channel 1
 H =   5.920
 Q =1211.998
 Y =   4.326
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 329.768
 Y =   4.904
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 462.721
 Y =   5.318
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  88.846
 Y =   5.864
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 330.664
 Y =   5.494
 b =   6.000
 m =   1.000
 n =    .013

Output with H = 5.93 ft
 For Channel 1
 H =   5.930
 Q =1216.001
 Y =   4.413
 b =  19.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .001000
 Q = 330.941
 Y =   4.913
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 464.359
 Y =   5.327
 b =   6.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q =  88.930
 Y =   5.875
 b =   8.000
 m =    .000
 n =    .013
 For Channel 5
 S = .000500
 Q = 331.771
 Y =   5.504
 b =   6.000
 m =   1.000
 n =    .013
```

Solution fails for H = 5.94 ft

92. Enlarge the downstream steep rectangular channel in Illustrative Problem 9 from $b_2 = 8$ ft to $b_2 = 10$ ft, and repeat this problem including solving values of Q and Y for a number of different bottom slopes for the upstream channel.

Let $b_2 = 10$ ft in example problem 9 and obtain a series of solution for different bottom slopes.

Solution:

The three equations that govern when uniform flow is to be maintained in the upstream channel with critical flow in the downstream channel are:

1. Energy across the transition $Y_1+(Q/A)_1^2/(2g) = E_{c2}$

2. Critical flow in Channel 2 $Q^2T_2 = gA_2^3$

3. Uniform flow equation in Channel 2 $Q = (C_u/n_2)A_2^{5/3}/P_2^{2/3}S_{o2}^{1/2}$

When the downstream channel is rectangular these three equations can be combined into a single equation that allows Y_1 to be solved first and thereafter Q and $Y_2 = Y_{c2}$ because the critical specific energy is only a function of flow rate, i.e.,

$E_{c2} = 1.5[(Q/b_2)^2/g]^{1/3}$.

In other words the energy Eq. 1 and the critical flow Eq. 2 combine into

$F_1 = Y_1+(Q/A)_1^2/(2g) - 1.5[(Q/b_2)^2/g]^{1/3} = 0$

With Eq. 3 giving Q as a function of Y_1 (and the known variables for the upstream channel) we have a single equation from which to solve Y_1.

Program TRAPRECT.FOR is designed to obtain a series of solutions in which either S_{o1}, or b_2 is varied. Following are the outputs from first varying b_2 and then varying S_{o1}. Four graphs at the end of this problem solution plot both how Y_1 and Q vary with these variables. Note that as either b_2 gets smaller or S_{o1} gets larger the upstream depth gets smaller and the flow rate gets smaller. In other words it is very easy to choke the flow in very mild upstream channels if the steep downstream channel is much smaller in size.

The program PRB2_80.FOR, listed below solves the problem of flow from a mild upstream
trapezoidal channel to a steep downstream rectangular channel

```
Program PRB2_80.FOR
C Upstream mild trap. channel with transitin to steep downstr. rect. channel
C  Find Qmax so normal depth exists upstream.  (Solves Prob 2-80 & Exm Pr. 9)
      REAL SO(20)
      WRITE(*,*)' Give: g,b1,m1,n1,b2 & guess for Y1'
      READ(*,*) G,B1,FM1,FN,B2,Y1
      CEC=1.5/(G**.3333333*B2**.6666667)
      Cu=1.
      IF(G.GT.20.) Cu=1.486
      G2=2.*G
      FMS=2.*SQRT(FM1*FM1+1.)
      WRITE(*,*)' Give: N & this many So''s'
      READ(*,*) N,(SO(K),K=1,N)
      DO 10 K=1,N
      SSO=SQRT(SO(K))*Cu/FN
      M=0
1     A=(B1+FM1*Y1)*Y1
      Q=SSO*A*(A/(B1+FMS*Y1))**.6666667
      F=Y1+(Q/A)**2/G2-CEC*Q**.6666667
      M=M+1
```

```
      IF(MOD(M,2).EQ.0) GO TO 2
      FF=F
      YY=Y1
      Y1=1.005*Y1
      GO TO 1
2     DIF=FF*(Y1-YY)/(F-FF)
      Y1=YY-DIF
      IF(M.LT.30 .AND. ABS(DIF).GT. 1.E-5) GO TO 1
      IF(M.EQ.30) WRITE(*,*)' Failed to converge', DIF
      A=(B1+FM1*Y1)*Y1
      Q=SSO*A*(A/(B1+FMS*Y1))**.6666667
10    WRITE(3,100) SO(K),Q,Y1,CEC*Q**.6666667
100   FORMAT(F10.6,F9.2,2F8.3)
      END
```

Output

So	Q	Y1	E
.000080	1262.62	11.705	11.866
.000100	1025.92	10.161	10.333
.000120	860.60	9.010	9.191
.000140	738.22	8.110	8.297
.000180	568.70	6.775	6.973
.000200	507.51	6.262	6.463
.000300	317.99	4.520	4.732
.000400	220.59	3.494	3.708
.000500	162.21	2.809	3.021
.000600	123.96	2.318	2.525
.000700	97.40	1.950	2.150
.000800	78.18	1.664	1.857
.000900	63.83	1.438	1.622
.001000	52.85	1.254	1.431
.002000	12.69	.447	.553

The following program TRAPRECT.FOR solves this problem and is used to obtain data for plotting.

```
C Solves energy between upstream trap. and downs. rect. channels with critical flow in rect.
      WRITE(*,*)' Give: g,b1,m1,n1,So1 & b2'
      READ(*,*) G,B1,FM1,FN1,SOS,B2S
      WRITE(3,120) B1,FM1,FN1
120   FORMAT(' b1=',F8.3,' m1=',F7.3,' n=',F8.4,/,
     &'    So       b2        Y1        Q      E      Fr')
      WRITE(*,*)' Give 0=So varies, or 1=b2 varies, N,Delta, & guess Y1'
      READ(*,*) IV,N,DV,Y1
      G2=2.*G
      FMS=2.*SQRT(FM1*FM1+1.)
      FM2=2.*FM1
      Cu=1.
      IF(G.GT.20.) Cu=1.486
      COE=Cu/FN1
      B2=B2S
      SO1=SOS
1     SSO=COE*SQRT(SO1)
      DO 10 J=1,N
      IF(IV.EQ.0) THEN
      SO1=SOS+DV*FLOAT(J-1)
      SSO=COE*SQRT(SO1)
      ELSE
      B2=B2S+DV*FLOAT(J-1)
      ENDIF
      M=0
2     A1=(B1+FM1*Y1)*Y1
      Q=SSO*A1*(A1/(B1+FMS*Y1))**.6666667
      F=Y1+(Q/A1)**2/G2-1.5*((Q/B2)**2/G)**.33333333
      M=M+1
      IF(MOD(M,2).EQ.0) GO TO 4
      FF=F
      YY=Y1
      Y1=1.005*Y1
      GO TO 2
4     DIF=(Y1-YY)*FF/(F-FF)
      Y1=YY-DIF
      IF(M.LT.30 .AND. ABS(DIF).GT. 1.E-5) GO TO 2
```

```
      IF(J.EQ.1) YY1=Y1
      A1=(B1+FM1*Y1)*Y1
      Q=SSO*A1*(A1/(B1+FMS*Y1))**.6666667
      VEL2=(Q/A1)**2
10    WRITE(3,110) SO1,B2,Y1,Q,Y1+VEL2/G2,VEL2*(B1+FM2*Y1)/(G*A1)
110   FORMAT(F9.4,2F9.4,F9.2,2F8.2)
      WRITE(*,*)' Give 1 for new problem or 0=STOP'
      READ(*,*) II
      IF(II.EQ.0) STOP
      IF(IV.EQ.1) THEN
      WRITE(*,*)' Give new So1'
      READ(*,*) SO1
      ELSE
      WRITE(*,*) ' Give new b2'
      READ(*,*) B2
      ENDIF
      Y1=YY1
      WRITE(3,130)
130   FORMAT('                                  ')
      GO TO 1
      END
Output file TRAPRECT.OUT
 b1=  10.000 m1=  1.500 n=   .0130
    So       b2        Y1        Q      E      Fr
    .0010  10.0000   1.2455    54.15    1.45     .39
    .0010   9.7500   1.0887    42.98    1.27     .37
    .0010   9.5000    .9394    33.41    1.09     .36
    .0010   9.2500    .7993    25.38     .92     .35
    .0010   9.0000    .6698    18.82     .77     .33
    .0010   8.7500    .5527    13.61     .63     .31
    .0010   8.5000    .4490     9.60     .51     .29
    .0010   8.2500    .3594     6.61     .41     .28
    .0010   8.0000    .2839     4.45     .32     .26
    .0010   7.7500    .2217     2.95     .25     .24
    .0010   7.5000    .1715     1.92     .19     .22
    .0010   7.2500    .1315     1.23     .14     .20
    .0010   7.0000    .1001      .78     .11     .19
    .0010   6.7500    .0758      .49     .08     .17
    .0010   6.5000    .0570      .30     .06     .15

    .0009  10.0000   1.4495    66.76    1.67     .36
    .0009   9.7500   1.2819    53.98    1.48     .35
    .0009   9.5000   1.1208    42.85    1.29     .34
    .0009   9.2500    .9674    33.32    1.11     .33
    .0009   9.0000    .8234    25.33     .94     .31
    .0009   8.7500    .6904    18.79     .78     .30
    .0009   8.5000    .5700    13.60     .65     .28
    .0009   8.2500    .4634     9.60     .52     .27
    .0009   8.0000    .3711     6.62     .42     .25
    .0009   7.7500    .2933     4.46     .33     .23
    .0009   7.5000    .2290     2.95     .25     .22
    .0009   7.2500    .1769     1.92     .19     .20
    .0009   7.0000    .1355     1.23     .15     .18
    .0009   6.7500    .1029      .78     .11     .17
    .0009   6.5000    .0776      .48     .08     .15

    .0008  10.0000   1.7058    83.61    1.94     .33
    .0008   9.7500   1.5262    68.85    1.74     .33
    .0008   9.5000   1.3518    55.78    1.53     .32
    .0008   9.2500   1.1838    44.38    1.34     .31
    .0008   9.0000   1.0237    34.59    1.16     .30
    .0008   8.7500    .8730    26.37     .98     .28
    .0008   8.5000    .7335    19.63     .82     .27
    .0008   8.2500    .6068    14.25     .68     .26
    .0008   8.0000    .4942    10.08     .55     .24
    .0008   7.7500    .3964     6.96     .44     .23
    .0008   7.5000    .3135     4.70     .35     .21
    .0008   7.2500    .2447     3.11     .27     .20
    .0008   7.0000    .1889     2.02     .21     .18
    .0008   6.7500    .1444     1.29     .16     .17
```

.0008	6.5000	.1094	.81	.12	.15
.0007	10.0000	2.0343	106.69	2.29	.30
.0007	9.7500	1.8411	89.44	2.07	.30
.0007	9.5000	1.6518	73.93	1.85	.29
.0007	9.2500	1.4676	60.16	1.64	.28
.0007	9.0000	1.2895	48.09	1.44	.27
.0007	8.7500	1.1191	37.69	1.25	.26
.0007	8.5000	.9580	28.90	1.07	.25
.0007	8.2500	.8080	21.63	.90	.24
.0007	8.0000	.6709	15.79	.74	.23
.0007	7.7500	.5482	11.23	.60	.22
.0007	7.5000	.4409	7.79	.48	.21
.0007	7.2500	.3493	5.27	.38	.19
.0007	7.0000	.2729	3.49	.30	.18
.0007	6.7500	.2105	2.26	.23	.16
.0007	6.5000	.1606	1.44	.17	.15
.0006	10.0000	2.4655	139.30	2.73	.27
.0006	9.7500	2.2566	118.82	2.50	.27
.0006	9.5000	2.0504	100.17	2.27	.26
.0006	9.2500	1.8478	83.34	2.04	.26
.0006	9.0000	1.6497	68.30	1.82	.25
.0006	8.7500	1.4574	55.03	1.61	.24
.0006	8.5000	1.2721	43.50	1.40	.23
.0006	8.2500	1.0957	33.66	1.20	.23
.0006	8.0000	.9301	25.44	1.02	.22
.0006	7.7500	.7770	18.74	.85	.21
.0006	7.5000	.6385	13.45	.70	.20
.0006	7.2500	.5160	9.39	.56	.18
.0006	7.0000	.4101	6.39	.44	.17
.0006	6.7500	.3210	4.24	.35	.16
.0006	6.5000	.2476	2.74	.27	.15
.0005	10.0000	3.0499	187.33	3.33	.24
.0005	9.7500	2.8223	162.50	3.08	.23
.0005	9.5000	2.5964	139.62	2.83	.23
.0005	9.2500	2.3727	118.68	2.58	.22
.0005	9.0000	2.1519	99.65	2.34	.22
.0005	8.7500	1.9348	82.51	2.10	.21
.0005	8.5000	1.7226	67.25	1.87	.21
.0005	8.2500	1.5165	53.83	1.65	.20
.0005	8.0000	1.3182	42.22	1.43	.20
.0005	7.7500	1.1296	32.37	1.22	.19
.0005	7.5000	.9529	24.20	1.03	.18
.0005	7.2500	.7904	17.61	.85	.17
.0005	7.0000	.6440	12.45	.69	.16
.0005	6.7500	.5153	8.55	.55	.15
.0005	6.5000	.4051	5.71	.43	.14
.0004	10.0000	3.8778	262.24	4.16	.20
.0004	9.7500	3.6270	231.21	3.89	.20
.0004	9.5000	3.3769	202.32	3.62	.19
.0004	9.2500	3.1279	175.53	3.35	.19
.0004	9.0000	2.8802	150.84	3.09	.19
.0004	8.7500	2.6345	128.22	2.82	.18
.0004	8.5000	2.3914	107.66	2.56	.18
.0004	8.2500	2.1518	89.12	2.30	.18
.0004	8.0000	1.9168	72.59	2.05	.17
.0004	7.7500	1.6878	58.04	1.80	.17
.0004	7.5000	1.4667	45.44	1.57	.16
.0004	7.2500	1.2556	34.73	1.34	.16
.0004	7.0000	1.0570	25.84	1.13	.15
.0004	6.7500	.8736	18.67	.93	.14
.0004	6.5000	.7080	13.07	.75	.13
.0003	10.0000	5.1344	389.75	5.42	.16
.0003	9.7500	4.8523	349.04	5.12	.16
.0003	9.5000	4.5700	310.79	4.82	.16
.0003	9.2500	4.2877	274.95	4.52	.15
.0003	9.0000	4.0057	241.50	4.23	.15

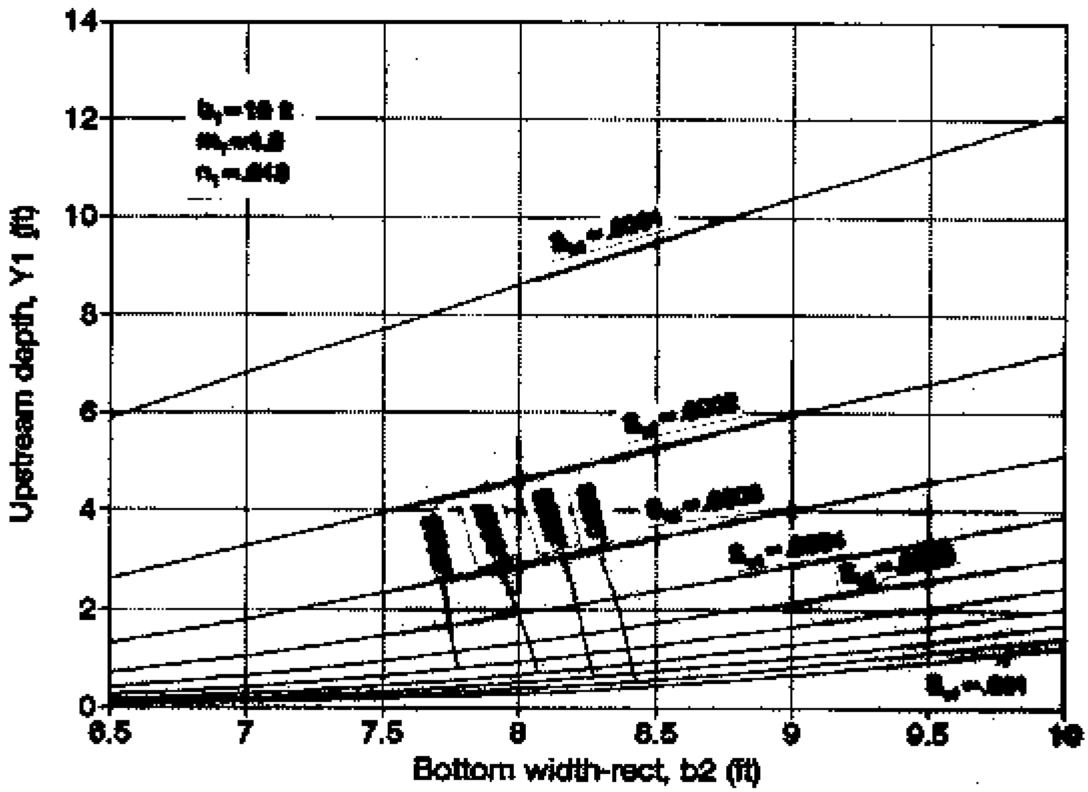
```
    .0003   8.7500   3.7240   210.41    3.93     .15
    .0003   8.5000   3.4432   181.67    3.63     .15
    .0003   8.2500   3.1635   155.23    3.34     .14
    .0003   8.0000   2.8857   131.09    3.04     .14
    .0003   7.7500   2.6104   109.21    2.75     .14
    .0003   7.5000   2.3387    89.57    2.46     .13
    .0003   7.2500   2.0718    72.15    2.18     .13
    .0003   7.0000   1.8116    56.91    1.91     .13
    .0003   6.7500   1.5603    43.82    1.64     .12
    .0003   6.5000   1.3207    32.81    1.39     .12

    .0002  10.0000   7.2881   642.51    7.56     .12
    .0002   9.7500   6.9568   584.13    7.22     .11
    .0002   9.5000   6.6247   528.80    6.87     .11
    .0002   9.2500   6.2918   476.49    6.53     .11
    .0002   9.0000   5.9581   427.16    6.18     .11
    .0002   8.7500   5.6238   380.77    5.83     .11
    .0002   8.5000   5.2888   337.28    5.49     .11
    .0002   8.2500   4.9534   296.67    5.14     .11
    .0002   8.0000   4.6176   258.89    4.79     .10
    .0002   7.7500   4.2819   223.91    4.44     .10
    .0002   7.5000   3.9464   191.68    4.09     .10
    .0002   7.2500   3.6117   162.19    3.74     .10
    .0002   7.0000   3.2783   135.39    3.40     .10
    .0002   6.7500   2.9473   111.26    3.05     .09
    .0002   6.5000   2.6198    89.75    2.71     .09

    .0001  10.0000  12.1441  1346.08   12.38     .06
    .0001   9.7500  11.7077  1242.17   11.94     .06
    .0001   9.5000  11.2697  1142.91   11.49     .06
    .0001   9.2500  10.8302  1048.25   11.04     .06
    .0001   9.0000  10.3890   958.14   10.59     .06
    .0001   8.7500   9.9463   872.52   10.14     .06
    .0001   8.5000   9.5019   791.33    9.69     .06
    .0001   8.2500   9.0558   714.53    9.23     .06
    .0001   8.0000   8.6080   642.05    8.77     .06
    .0001   7.7500   8.1585   573.84    8.31     .06
    .0001   7.5000   7.7072   509.84    7.85     .06
    .0001   7.2500   7.2542   449.99    7.39     .06
    .0001   7.0000   6.7995   394.22    6.93     .06
    .0001   6.7500   6.3431   342.48    6.46     .06
    .0001   6.5000   5.8852   294.71    6.00     .05

Output file TRAPRECT.OU1
 b1=  10.000 m1=  1.500 n=   .0130
    So       b2        Y1        Q      E       Fr
    .0001  10.0000  12.1441  1346.08   12.38     .06
    .0002  10.0000   7.2881   642.51    7.56     .12
    .0003  10.0000   5.1344   389.75    5.42     .16
    .0004  10.0000   3.8778   262.24    4.16     .20
    .0005  10.0000   3.0499   187.33    3.33     .24
    .0006  10.0000   2.4655   139.30    2.73     .27
    .0007  10.0000   2.0343   106.69    2.29     .30
    .0008  10.0000   1.7058    83.61    1.94     .33
    .0009  10.0000   1.4495    66.76    1.67     .36
    .0010  10.0000   1.2455    54.15    1.45     .39
    .0011  10.0000   1.0807    44.50    1.28     .41
    .0012  10.0000    .9456    37.00    1.13     .43
    .0013  10.0000    .8335    31.07    1.00     .45
    .0014  10.0000    .7395    26.33     .90     .47
    .0015  10.0000    .6600    22.48     .81     .49

    .0001   9.7500  11.7077  1242.17   11.94     .06
    .0002   9.7500   6.9568   584.13    7.22     .11
    .0003   9.7500   4.8523   349.04    5.12     .16
    .0004   9.7500   3.6270   231.21    3.89     .20
    .0005   9.7500   2.8223   162.50    3.08     .23
    .0006   9.7500   2.2566   118.82    2.50     .27
    .0007   9.7500   1.8411    89.44    2.07     .30
    .0008   9.7500   1.5262    68.85    1.74     .33
    .0009   9.7500   1.2819    53.98    1.48     .35
```

.0010	9.7500	1.0887	42.98	1.27	.37
.0011	9.7500	.9335	34.66	1.10	.40
.0012	9.7500	.8070	28.26	.96	.42
.0013	9.7500	.7027	23.27	.84	.43
.0014	9.7500	.6159	19.33	.74	.45
.0015	9.7500	.5428	16.17	.66	.47
.0001	9.5000	11.2697	1142.91	11.49	.06
.0002	9.5000	6.6247	528.80	6.87	.11
.0003	9.5000	4.5700	310.79	4.82	.16
.0004	9.5000	3.3769	202.32	3.62	.19
.0005	9.5000	2.5964	139.62	2.83	.23
.0006	9.5000	2.0504	100.17	2.27	.26
.0007	9.5000	1.6518	73.93	1.85	.29
.0008	9.5000	1.3518	55.78	1.53	.32
.0009	9.5000	1.1208	42.85	1.29	.34
.0010	9.5000	.9394	33.41	1.09	.36
.0011	9.5000	.7949	26.38	.93	.38
.0012	9.5000	.6782	21.06	.80	.40
.0013	9.5000	.5828	16.97	.69	.41
.0014	9.5000	.5041	13.79	.60	.43
.0015	9.5000	.4385	11.30	.53	.44
.0001	9.2500	10.8302	1048.25	11.04	.06
.0002	9.2500	6.2918	476.49	6.53	.11
.0003	9.2500	4.2877	274.95	4.52	.15
.0004	9.2500	3.1279	175.53	3.35	.19
.0005	9.2500	2.3727	118.68	2.58	.22
.0006	9.2500	1.8478	83.34	2.04	.26
.0007	9.2500	1.4676	60.16	1.64	.28
.0008	9.2500	1.1838	44.38	1.34	.31
.0009	9.2500	.9674	33.32	1.11	.33
.0010	9.2500	.7993	25.38	.92	.35
.0011	9.2500	.6667	19.59	.78	.36
.0012	9.2500	.5608	15.28	.66	.38
.0013	9.2500	.4753	12.04	.56	.39
.0014	9.2500	.4056	9.58	.48	.40
.0015	9.2500	.3483	7.68	.42	.41
.0001	9.0000	10.3890	958.14	10.59	.06
.0002	9.0000	5.9581	427.16	6.18	.11
.0003	9.0000	4.0057	241.50	4.23	.15
.0004	9.0000	2.8802	150.84	3.09	.19
.0005	9.0000	2.1519	99.65	2.34	.22
.0006	9.0000	1.6497	68.30	1.82	.25
.0007	9.0000	1.2895	48.10	1.44	.27
.0008	9.0000	1.0237	34.59	1.16	.30
.0009	9.0000	.8234	25.33	.94	.31
.0010	9.0000	.6698	18.82	.77	.33
.0011	9.0000	.5504	14.18	.64	.34
.0012	9.0000	.4563	10.81	.53	.36
.0013	9.0000	.3815	8.33	.45	.37
.0014	9.0000	.3213	6.48	.38	.37
.0015	9.0000	.2726	5.09	.32	.38
.0001	8.7500	9.9463	872.52	10.14	.06
.0002	8.7500	5.6238	380.77	5.83	.11
.0003	8.7500	3.7240	210.41	3.93	.15
.0004	8.7500	2.6345	128.22	2.82	.18
.0005	8.7500	1.9348	82.51	2.10	.21
.0006	8.7500	1.4574	55.03	1.61	.24
.0007	8.7500	1.1191	37.69	1.25	.26
.0008	8.7500	.8730	26.37	.98	.28
.0009	8.7500	.6904	18.79	.78	.30
.0010	8.7500	.5527	13.61	.63	.31
.0011	8.7500	.4473	10.00	.52	.32
.0012	8.7500	.3657	7.45	.42	.33
.0013	8.7500	.3018	5.62	.35	.34
.0014	8.7500	.2512	4.29	.29	.35
.0015	8.7500	.2108	3.32	.25	.35

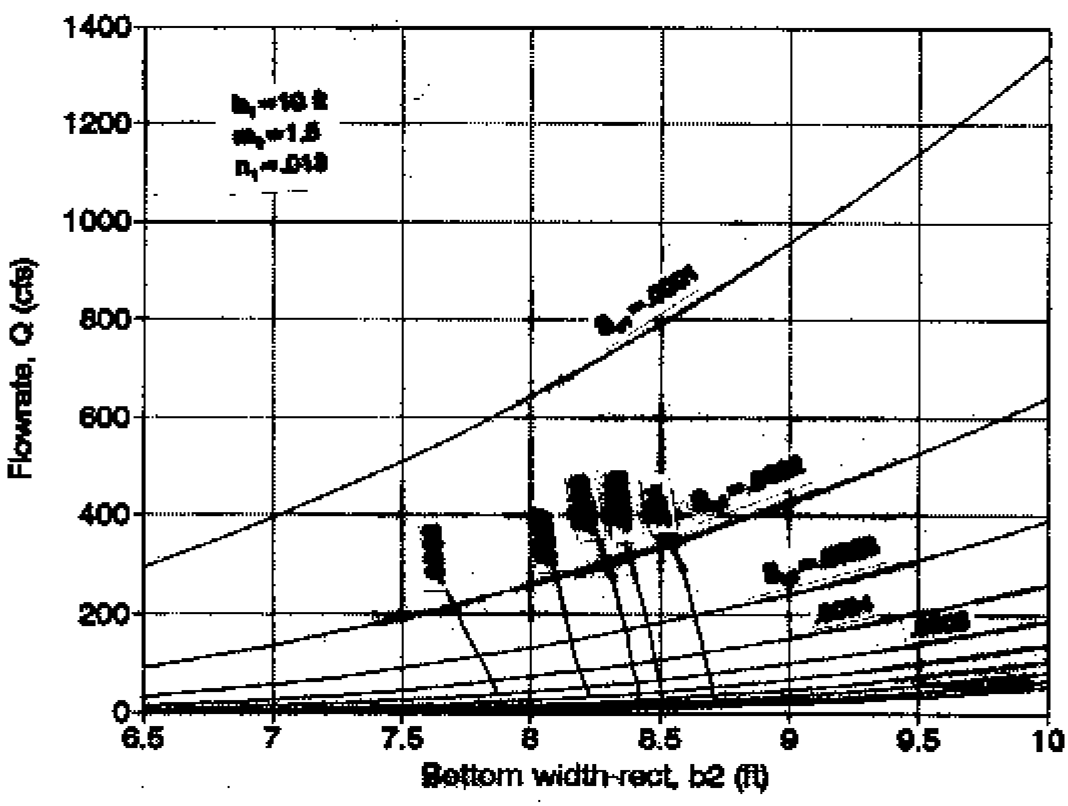
.0001	8.5000	9.5019	791.33	9.69	.06
.0002	8.5000	5.2888	337.28	5.49	.11
.0003	8.5000	3.4432	181.67	3.63	.15
.0004	8.5000	2.3914	107.66	2.56	.18
.0005	8.5000	1.7226	67.25	1.87	.21
.0006	8.5000	1.2721	43.50	1.40	.23
.0007	8.5000	.9580	28.90	1.07	.25
.0008	8.5000	.7335	19.63	.82	.27
.0009	8.5000	.5700	13.60	.65	.28
.0010	8.5000	.4490	9.60	.51	.29
.0011	8.5000	.3581	6.89	.41	.30
.0012	8.5000	.2890	5.03	.33	.31
.0013	8.5000	.2358	3.72	.27	.32
.0014	8.5000	.1943	2.80	.22	.32
.0015	8.5000	.1617	2.13	.19	.32
.0001	8.2500	9.0558	714.53	9.23	.06
.0002	8.2500	4.9534	296.67	5.14	.11
.0003	8.2500	3.1635	155.23	3.34	.14
.0004	8.2500	2.1518	89.12	2.30	.18
.0005	8.2500	1.5165	53.83	1.65	.20
.0006	8.2500	1.0957	33.66	1.20	.23
.0007	8.2500	.8080	21.63	.90	.24
.0008	8.2500	.6068	14.25	.68	.26
.0009	8.2500	.4634	9.60	.52	.27
.0010	8.2500	.3594	6.61	.41	.28
.0011	8.2500	.2829	4.64	.32	.28
.0012	8.2500	.2258	3.33	.26	.29
.0013	8.2500	.1824	2.43	.21	.29
.0014	8.2500	.1492	1.80	.17	.30
.0015	8.2500	.1233	1.35	.14	.30
.0001	8.0000	8.6080	642.05	8.77	.06
.0002	8.0000	4.6176	258.89	4.79	.10
.0003	8.0000	2.8857	131.09	3.04	.14
.0004	8.0000	1.9168	72.59	2.05	.17
.0005	8.0000	1.3182	42.22	1.43	.20
.0006	8.0000	.9301	25.44	1.02	.22
.0007	8.0000	.6709	15.79	.74	.23
.0008	8.0000	.4942	10.08	.55	.24
.0009	8.0000	.3711	6.62	.42	.25
.0010	8.0000	.2839	4.45	.32	.26
.0011	8.0000	.2210	3.07	.25	.26
.0012	8.0000	.1747	2.17	.20	.27
.0013	8.0000	.1401	1.56	.16	.27
.0014	8.0000	.1139	1.15	.13	.27
.0015	8.0000	.0937	.86	.11	.27
.0001	7.7500	8.1585	573.84	8.31	.06
.0002	7.7500	4.2819	223.91	4.44	.10
.0003	7.7500	2.6104	109.21	2.75	.14
.0004	7.7500	1.6878	58.04	1.80	.17
.0005	7.7500	1.1296	32.37	1.22	.19
.0006	7.7500	.7770	18.74	.85	.21
.0007	7.7500	.5482	11.23	.60	.22
.0008	7.7500	.3964	6.96	.44	.23
.0009	7.7500	.2933	4.46	.33	.23
.0010	7.7500	.2217	2.95	.25	.24
.0011	7.7500	.1710	2.00	.19	.24
.0012	7.7500	.1342	1.40	.15	.25
.0013	7.7500	.1070	1.00	.12	.25
.0014	7.7500	.0866	.73	.10	.25
.0015	7.7500	.0710	.54	.08	.25
.0001	7.5000	7.7072	509.84	7.85	.06
.0002	7.5000	3.9464	191.68	4.09	.10
.0003	7.5000	2.3387	89.57	2.46	.13
.0004	7.5000	1.4667	45.44	1.57	.16
.0005	7.5000	.9529	24.20	1.03	.18
.0006	7.5000	.6385	13.45	.70	.20

.0007	7.5000	.4409	7.79	.48	.21
.0008	7.5000	.3135	4.70	.35	.21
.0009	7.5000	.2290	2.95	.25	.22
.0010	7.5000	.1715	1.92	.19	.22
.0011	7.5000	.1312	1.29	.15	.22
.0012	7.5000	.1025	.89	.11	.22
.0013	7.5000	.0814	.63	.09	.23
.0014	7.5000	.0656	.46	.07	.23
.0015	7.5000	.0537	.34	.06	.23
.0001	7.2500	7.2542	449.99	7.39	.06
.0002	7.2500	3.6117	162.19	3.74	.10
.0003	7.2500	2.0718	72.15	2.18	.13
.0004	7.2500	1.2556	34.73	1.34	.16
.0005	7.2500	.7904	17.61	.85	.17
.0006	7.2500	.5160	9.39	.56	.18
.0007	7.2500	.3493	5.27	.38	.19
.0008	7.2500	.2447	3.11	.27	.20
.0009	7.2500	.1769	1.92	.19	.20
.0010	7.2500	.1315	1.23	.14	.20
.0011	7.2500	.1001	.82	.11	.20
.0012	7.2500	.0778	.56	.09	.21
.0013	7.2500	.0616	.40	.07	.21
.0014	7.2500	.0496	.29	.05	.21
.0015	7.2500	.0405	.21	.04	.21
.0001	7.0000	6.7995	394.22	6.93	.06
.0002	7.0000	3.2783	135.39	3.40	.10
.0003	7.0000	1.8116	56.91	1.91	.13
.0004	7.0000	1.0570	25.84	1.13	.15
.0005	7.0000	.6440	12.45	.69	.16
.0006	7.0000	.4101	6.39	.44	.17
.0007	7.0000	.2729	3.49	.30	.18
.0008	7.0000	.1889	2.02	.21	.18
.0009	7.0000	.1355	1.23	.15	.18
.0010	7.0000	.1001	.78	.11	.19
.0011	7.0000	.0759	.52	.08	.19
.0012	7.0000	.0589	.35	.06	.19
.0013	7.0000	.0465	.25	.05	.19
.0014	7.0000	.0374	.18	.04	.19
.0015	7.0000	.0305	.13	.03	.19

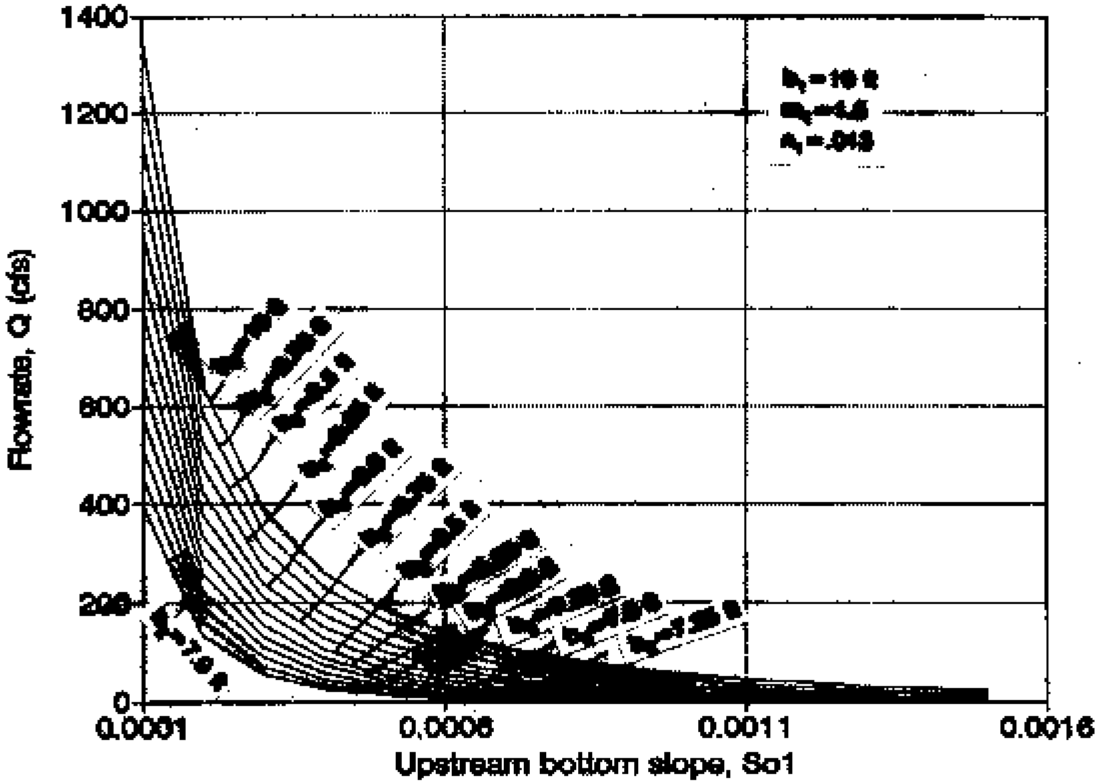
Plot of results follows.

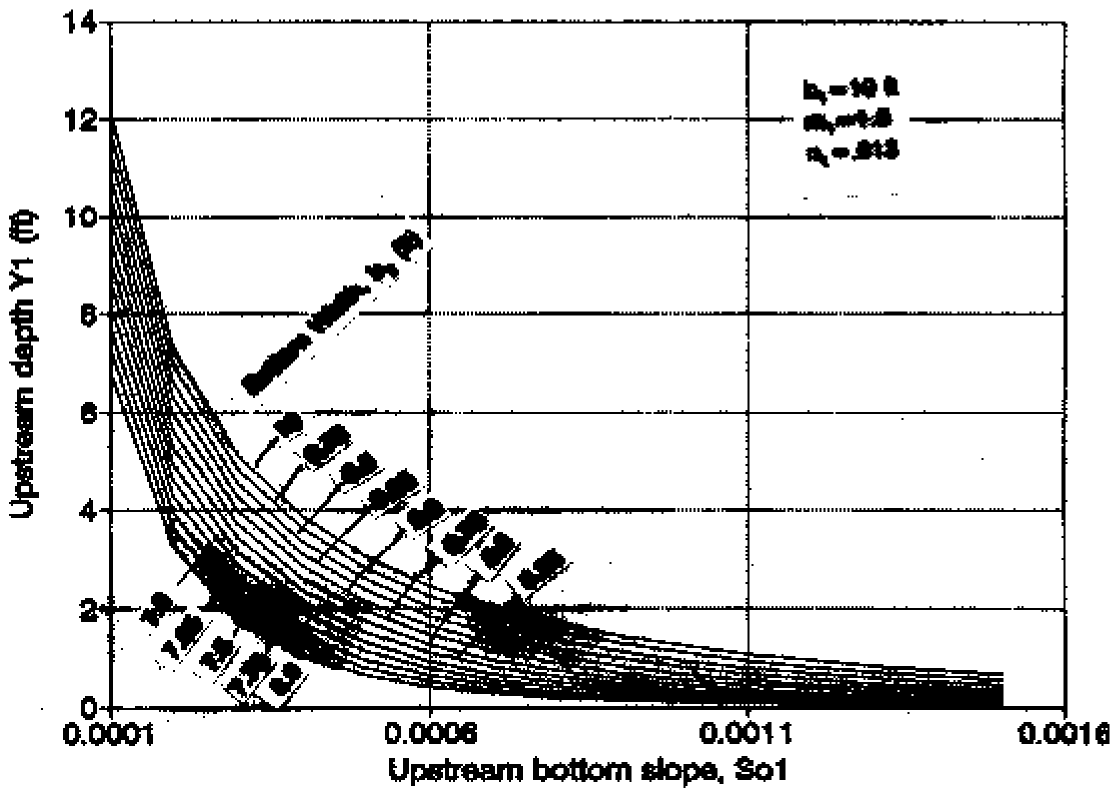


Upstream depth, Y1 (ft)
Bottom width-rect, b2 (ft)



Flowrate, Q (cfs)
Bottom width-rect, b2 (ft)


1400
1200
1000
800
600
400
200
0
Flowrate, Q (cfs)
0.0001
0.0006
0.0011
0.0016
Upstream bottom slope, So1


14
12
10
8
6
4
2
0
Upstream depth Y1 (ft)
0.0001
0.0006
0.0011
0.0016
Upstream bottom slope, So1

92.

Enlarge the downstream steep rectangular channel in Illustrative Problem 9 from $b_2 = 8$ ft to $b_2 = 10$ ft, and repeat this problem including solving values of Q and Y for a number of different bottom slopes for the upstream channel.

Given: An upstream trapezoidal channel with b_1=8 ft, m_1=1.5, n_1=.013, S_{o1}=.0005 has smooth transition to a steep downstream circular channel with D_2=18.2 ft.
Find: The maximum flow rate Q so uniform flow occurs in upstream channel. Obtain a series of solution in which S_{o1} varies.

Solution: The following three equations allow for the solution of: Q, $Y_2=Y_{c2}$, and $Y_1=Y_{o1}$.

$E_1 = Y_1+Q^2/(2gA_1^2) = Y_c+Q^2/(2gA_{2c}^2)$ Energy across transition (1)

$Q^2D_2\sin\beta - g[.25D_2^2(\beta-\sin\beta\cos\beta)] = 0$ Critical flow CH. 2 (2)

$nQP_1^{2/3} - C_uA_1^{5/3}S_{o1}^{1/2} = 0$ Uniform flow upstream (3)

The solution to these 3 equations is implemented in the TK-Solver Model PRB2_81L.TK. The "List Solve" capability of TK-Solver is utilized to obtain the series of solution given below in the "Table Sheet".

```
================ VARIABLE SHEET ================
St Input----- Name---- Output---- Unit------ Comment
              A1       66.506014
   8          b1
   1.5        m1
LG 4.5060911  Y1
LG 333.08674  Q
LG 3.6280156  Yc
L  .0005      So1
              Fr1      .50204396
              Ac       36.866133
              beta     .925648
   18.2       D
   32.2       g
   64.4       g2
   1.486      Cu
   .013       n1

=============== RULE SHEET =================
S Rule-------------------------------------------
  A1=(b1+m1*Y1)*Y1
  beta=acos(1-2*Yc/D)
  Ac=.25*D*D*(beta-cos(beta)*sin(beta))
  Y1+(Q/A1)^2/g2=Yc+(Q/Ac)^2/g2
  Q^2*D*sin(beta)=g*Ac^3
  Q=Cu/n1*A1^1.6666667/(b1+2*Y1*sqrt(m1*m1+1))^.6666667*sqrt(So1)
  Fr1=sqrt(Q^2*(b1+2*m1*Y1)/(g*A1^3))

=============== TABLE:  ====================
Title:
```

Element	So1	Y1	Q	Yc	
1	.0000504	.178225599	.355531262	.165020555	← The list solve failed to obtain the solution for these first two bottom slopes.
2	.0000503	.173198298	.338716336	.160039971	
3	.000502	3.55859802	210.491776	2.87176648	
4	.0005	4.50609107	333.086744	3.62801565	
5	.00048	5.42452448	473.68682	4.34410168	
6	.00046	6.08266295	586.410234	4.84723481	
7	.00044	6.67557737	695.904777	5.2937787	
8	.00042	7.24566918	807.882277	5.71745857	
9	.0004	7.81200288	925.286281	6.13311093	
10	.00035	9.28010186	1256.08199	7.18770884	

11	.00025	12.8377293	2190.35296	9.61112987
12	.0002	15.2038189	2889.61962	11.1092613
13	.00015	18.2858043	3854.97294	12.8858907
14	.00014	19.0283835	4092.2222	13.2777709
15	.000135	19.4200896	4217.62064	13.4780235
16	.000133	19.5808903	4269.12022	13.5588877
17	.000132	19.6622072	4295.16538	13.599478
18	.000131	19.7441472	4321.41026	13.6401717
19	.00013	19.8267193	4347.85704	13.6809672
20	.000129	19.9099332	4374.5079	13.7218628
21	.000128	19.9937983	4401.36507	13.7628567
22	.000127	20.0783246	4428.43084	13.8039471
23	.000126	20.1635222	4455.70751	13.8451319
24	.000125	20.2494015	4483.19743	13.8864092
25	.000124	20.3359732	4510.90299	13.9277768
26	.000123	20.423248	4538.82663	13.9692325
27	.000122	20.5112372	4566.97083	14.0107739
28	.000121	20.5999521	4595.33812	14.0523985
29	.00012	20.6894046	4623.93108	14.094104
30	.00011	21.6273087	4922.87686	14.5149607
31	.000105	22.1287134	5081.82623	14.727299
32	.0001	22.654539	5247.65882	14.9402276

By using PRB2_81.TK which does not use the “List Solve”, and changing So1 in small increment the solutions to the first above bottom slopes, and others, can be obtained as:

S_{o1}	Y_1(ft)	Q(cfs)	$Y_c=Y_2$(ft)
.000502	3.559	210.5	2.872
.000503	3.649	221.17	2.945
.000504	3.795	238.8	3.062
.0005041	3.820	241.9	3.082
.0005042	3.853	245.9	3.108
.0005043	3.916	255.9	3.159
.0005044	Solution fails		

The Program UNIFTRC.FOR is designed to solve problem of this type. The downstream channel can either be circular as in this problem or trapezoidal.

Program UNIFTRC.FOR

```
C  Solve for uniform flow in the upstream channel when this channel flows into
C a steep downstream channel.  The downstream channel (which has critical flow
C at its entrance) may be trapezoidal or circular.  The three equations being
C  solved are: (1) Energy across transition, (2) Critical Flow, and (3) Uniform
C flow in upstream channel.
       INTEGER*2 INDX(3)
       REAL F(3),FF(3),DD(3,3)
       COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
       EQUIVALENCE (Q,X(1)),(Y1,X(2)),(Y2,X(3))
       WRITE(*,*)' Is downstream steep channel: (1) trap, (2) circular'
       READ(*,*) ITYPE
       WRITE(*,*)' For upst. Ch. give: b,m,n,So,KL,g'
       READ(*,*) B1,FM1,FN,SO,FKL,G
       FMS=2.*SQRT(FM1*FM1+1.)
       FKL=1.+FKL
       G2=2.*G
       Cu=1.486
       IF(G.LT.20.) Cu=1.
       IF(ITYPE.EQ.1) THEN
       WRITE(*,*)' For downst. Ch. give: b,m'
       READ(*,*) B2,FM2
       ELSE
       WRITE(*,*)' For downst. Ch. give: D'
       READ(*,*) D
```

```
      D4=.25*D*D
      ENDIF
      WRITE(*,*)' Give guesses for: Q,Y1 & Yc2'
      READ(*,*) X
      SO=SQRT(SO)*Cu/FN
      NCT=0
10    CALL FUN(F)
      DO 20 I=1,3
      XX=X(I)
      X(I)=1.005*X(I)
      CALL FUN(FF)
      DO 15 J=1,3
15    DD(J,I)=(FF(J)-F(J))/(X(I)-XX)
20    X(I)=XX
      CALL SOLVEQ(3,1,3,DD,F,1,DET,INDX)
      SUM=0.
      DO 25 I=1,3
      X(I)=X(I)-F(I)
25    SUM=SUM+ABS(F(I))
      NCT=NCT+1
      WRITE(*,*) ' NCT=',NCT,SUM
      IF(SUM.GT. .001 .AND.NCT.LT.30) GO TO 10
      WRITE(*,100) X
100   FORMAT(' Solution: Q=',F9.2,' Y1=',F9.3,' Y2=Yc=',F9.3)
      END
      SUBROUTINE FUN(F)
      REAL F(3)
      COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
      A1=AREA(1)
      A2=AREA(2)
      F(1)=X(2)+(X(1)/A1)**2/G2-X(3)-FKL*(X(1)/A2)**2/G2
      F(2)=X(1)**2*T2/(G*A2**3)-1.
      F(3)=X(1)-So*A1*(A1/P1)**.6666667
      RETURN
      END
      FUNCTION AREA(K)
      COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
      IF(K.EQ.1) THEN
      P1=B1+FMS*X(2)
      AREA=(B1+FM1*X(2))*X(2)
      ELSE
      IF(ITYPE.EQ.1) THEN
      T2=B2+2.*FM2*X(3)
      AREA=(B2+FM2*X(3))*X(3)
      ELSE
      BETA=ACOS(1.-2.*X(3)/D)
      T2=D*SIN(BETA)
      AREA=D4*(BETA-COS(BETA)*SIN(BETA))
      ENDIF
      ENDIF
      RETURN
      END
```

The above program solves a single problem. The program below contains a DO loop so a series of solution in which the upstream channel bottom slope varies. This program has been used to get the series of solutions given below its listing.

<u>Program UNIFTRCS.FOR</u>

```
C This program is the same as UNIFTRC except it obtains a series of solutions
C  for the list of So's given.
C  Solve for uniform flow in the upstream channel when this channel flows into
C a steep downstream channel.  The downstream channel (which has critical flow
C at its entrance) may be trapezoidal or circular.  The three equations being
C  solved are: (1) Energy across transition, (2) Critical Flow, and (3) Uniform
C flow in upstream channel.
       INTEGER*2 INDX(3)
       REAL F(3),FF(3),DD(3,3),So1(30)
       COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
       EQUIVALENCE (Q,X(1)),(Y1,X(2)),(Y2,X(3))
       WRITE(*,*)' Is downstream steep channel: (1) trap, (2) circular'
       READ(*,*) ITYPE
       WRITE(*,*)' For upst. Ch. give: b,m,n,KL,g'
       READ(*,*) B1,FM1,FN,FKL,G
       FMS=2.*SQRT(FM1*FM1+1.)
       FKL=1.+FKL
       G2=2.*G
       Cu=1.486
       IF(G.LT.20.) Cu=1.
       IF(ITYPE.EQ.1) THEN
       WRITE(*,*)' For downst. Ch. give: b,m'
       READ(*,*) B2,FM2
       ELSE
       WRITE(*,*)' For downst. Ch. give: D'
       READ(*,*) D
       D4=.25*D*D
       ENDIF
       WRITE(*,*)' Give: N, and this many So1 values'
       READ(*,*) N,(So1(I),I=1,N)
       WRITE(*,*)' Give guesses for: Q,Y1 & Yc2'
       READ(*,*) X
       DO 50 K=1,N
       SO=SQRT(So1(K))*Cu/FN
       NCT=0
10     CALL FUN(F)
       DO 20 I=1,3
       XX=X(I)
       X(I)=1.005*X(I)
       CALL FUN(FF)
       DO 15 J=1,3
15     DD(J,I)=(FF(J)-F(J))/(X(I)-XX)
20     X(I)=XX
       CALL SOLVEQ(3,1,3,DD,F,1,DET,INDX)
       SUM=0.
       DO 25 I=1,3
       X(I)=X(I)-F(I)
25     SUM=SUM+ABS(F(I))
       NCT=NCT+1
       WRITE(*,*) ' NCT=',NCT,SUM
       IF(SUM.GT. .001 .AND.NCT.LT.30) GO TO 10
       WRITE(*,100) So1(K),X
50     WRITE(3,100) So1(K),X
100    FORMAT(F12.7,F9.2,2F9.3)
       END
       SUBROUTINE FUN(F)
       REAL F(3)
       COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
       A1=AREA(1)
       A2=AREA(2)
       F(1)=X(2)+(X(1)/A1)**2/G2-X(3)-FKL*(X(1)/A2)**2/G2
       F(2)=X(1)**2*T2/(G*A2**3)-1.
       F(3)=X(1)-So*A1*(A1/P1)**.6666667
       RETURN
       END
       FUNCTION AREA(K)
       COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
       IF(K.EQ.1) THEN
       P1=B1+FMS*X(2)
       AREA=(B1+FM1*X(2))*X(2)
       ELSE
```

```
IF(ITYPE.EQ.1) THEN
T2=B2+2.*FM2*X(3)
AREA=(B2+FM2*X(3))*X(3)
ELSE
BETA=ACOS(1.-2.*X(3)/D)
T2=D*SIN(BETA)
AREA=D4*(BETA-COS(BETA)*SIN(BETA))
ENDIF
ENDIF
RETURN
END
```

S_o	Q(cfs)	Y_1(ft)	Y_2(ft)
.0005000	333.09	4.506	3.628
.0004800	473.69	5.425	4.344
.0004600	586.41	6.083	4.847
.0004400	695.91	6.676	5.294
.0004200	807.88	7.246	5.717
.0004000	925.29	7.812	6.133
.0003500	1256.08	9.280	7.188
.0003000	1664.93	10.916	8.326
.0002500	2190.35	12.838	9.611
.0002000	2889.62	15.204	11.109
.0001500	3854.97	18.286	12.886
.0001400	4092.22	19.028	13.278
.0001350	4217.62	19.420	13.478
.0001330	4269.12	19.581	13.559
.0001320	4295.17	19.662	13.599

93. A trapezoidal channel with b_1 = 8 ft, and m_1 = 1.5 is laid on a bottom slope of S_{o1} = .0005, and has a Mannings roughness coefficient, n_1 = .013. This channel convey the water into a smooth transition to a circular channel with a diameter D_2 = 18.2 ft. The circular channel has a steep slope. Determine the maximum flowrate that can exist, and not increase the depth in the upstream channel above its normal depth, through the transition if the bottom elevation does not change between the trapezoidal and circular channels. Determine what the relationship is between the maximum flowrate possible and the slope of the upstream channel by solving the problem for several different values of S_{o1}.

Solution: As with the previous problems the solution to determine the maximum flowrate is to solve the following three equations simultaneously:
(1) Mannings Equation in the upstream channel, $Q = (Cu/n)A_t^{5/3}/P_t^{2/3}S_o^{1/2}$
(2) Critical flow in the circular channel, $Q^2T_c = gA_c^3$
(3) The energy equation across the transition, $Y_t + (Q/A_t)^2/(2g) = Y_c + (Q/A_c)^2/(2g)$
The three variables that are solved from these three equations are: Q, Y_{o1} and Y_c.
The following MathCAD model provides the solution. In this MathCAD model there are 7 equations that are solved simultaneously; in addition to the above three equations there are 4 equations for: (1) the angle β, (2) the area of the trapezoidal channel, (3) the wetted Perimeter of the trapezoidal channel and (4) the area of the circular channel.

MathCAD Model PRB2_93.MCD

$b1 := 8 \quad m1 := 1.5 \quad So1 := .0005 \quad n1 := .013 \quad D := 18.2 \quad Q := 300 \quad Yo1 := 6 \quad Yc := 5.8 \quad Cu := 1.486 \quad A1 := 50 \quad Ac := 48$

$P1 := 23 \quad \beta := 1.6 \quad g := 32.2$

Given

$$Q = \frac{Cu}{n1}\cdot\frac{A1^{1.666667}}{P1^{.6666667}}\cdot\sqrt{So1} \qquad A1 = (b1 + m1\cdot Yo1)\cdot Yo1 \qquad P1 = b1 + 2\cdot Yo1\cdot\sqrt{m1^2+1} \qquad \beta = acos\left(1 - 2\cdot\frac{Yc}{D}\right)$$

$Ac = .25 \cdot D^2 \cdot (\beta - \cos(\beta) \cdot \sin(\beta))$ $\quad Yo1 + \frac{\left(\frac{Q}{A1}\right)^2}{2 \cdot g} = Yc + \frac{\left(\frac{Q}{Ac}\right)^2}{2 \cdot g}$ $\quad Q^2 \cdot D \cdot \sin(\beta) = g \cdot Ac^3$

$$\text{Find}(Q, Yo1, Yc, A1, P1, Ac, \beta) = \begin{bmatrix} 333.074 \\ 4.506 \\ 3.628 \\ 66.504 \\ 24.247 \\ 36.865 \\ 0.926 \end{bmatrix}$$

How would you modify this model so that the wetted Perimeter in the trapezoidal channel was no longer one of the unknowns? How would you eliminate the areas of both the trapezoidal and circular channels as unknowns?

94.

Determine the maximum flowrate that can be accommodated in the channel shown below without cause the depth in the upstream channel to rise above its normal depth. The upstream channel has a bottom width $b_1 = 8$ ft, a side slope $m_1 = 2$, a Mannings roughness coefficient, $n_1 = .014$, and a bottom slope, $S_{o1} = .00025$. The downstream channel is steep, i.e. under uniform flow conditions the depth will be less than critical depth, has a bottom width $b_2 = 7$ ft, and a side slope $m_2 = 1$.. Solve the problem with the upstream slope changing and plot the maximum flowrate, and the depths upstream and at the head of the steep channel versus S_{o1}. Can you explain these trends? (Increase S_o to about .001.)

Given: An upstream trapezoidal channel is to have uniform flow with a downstream trapezoidal channel which is steep.
b_1=8 ft, m_1=2, n_1=.014; b_2=7 ft, m_2=1
Find: Q, $Y_1=Y_o$ and $Y_2=Y_c$ for different bottom slopes S_{o1}.

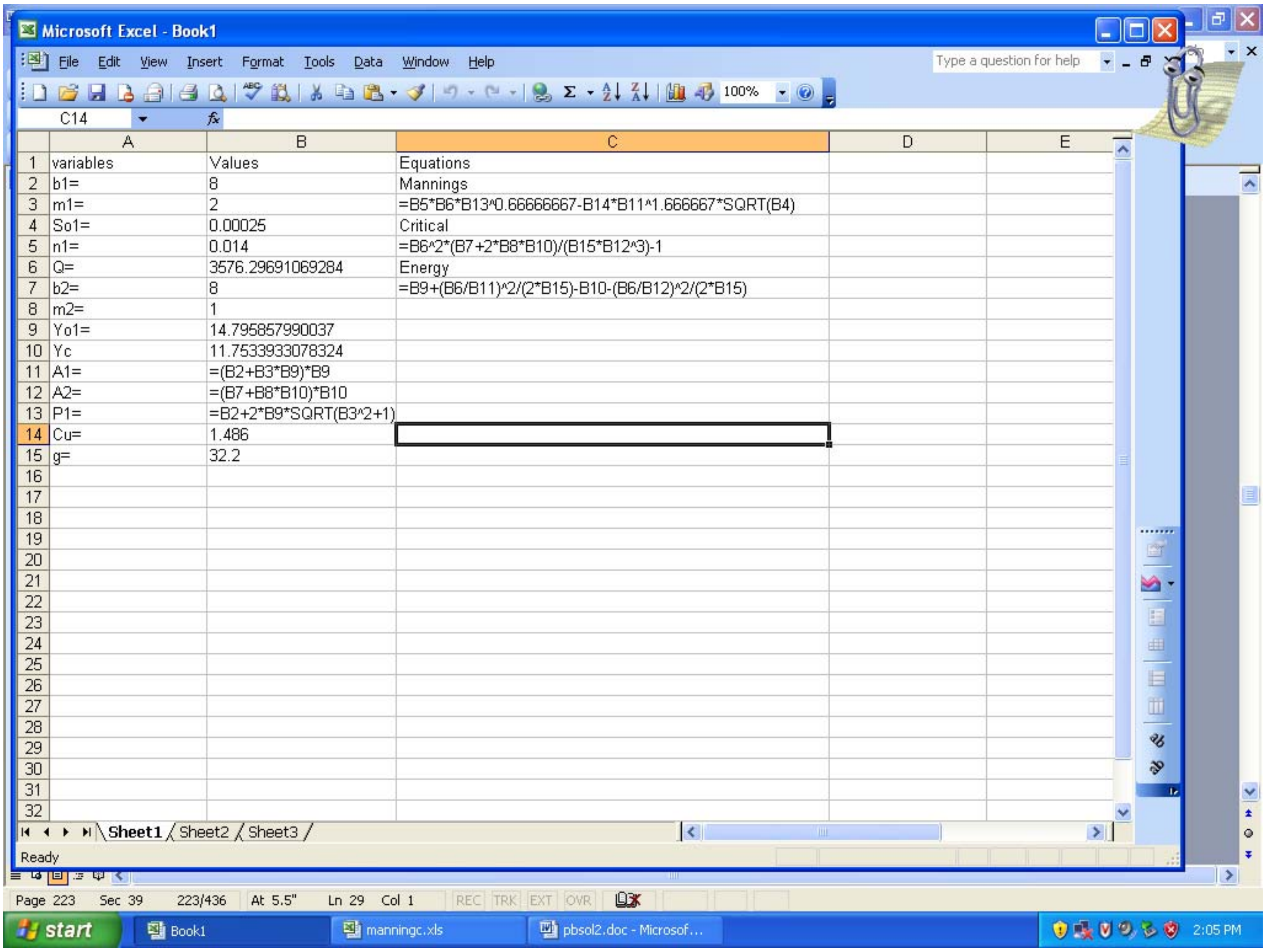
Solution: The three equations to solve are:

$$E_1 = Y_1+Q^2/(2gA_1^2) = Y_c+Q^2/(2gA_{2c}^2) \quad \text{Energy across transition} \qquad (1)$$

$$Q^2T_2 - gA_2^3 = 0 \quad \text{Critical flow CH. 2} \qquad (2)$$

$$nQP_1^{2/3} - C_uA_1^{5/3}S_{o1}^{1/2} = 0 \quad \text{Uniform flow upstream} \qquad (3)$$

The EXCEL spreadsheet solves the above three equation, with the second written as $Q^2T_2/(gA_2^3)- 1 = 0$. The **Solver** add-in is used to change the values of Cells b6 (contains Q), b9 (contains Y_{o1}) and b10 (contains Y_c the critical depth at section 2), by making the energy equation equal to 0 (Cell c7) subject to the constraints that cells C3 and C5 = 0 (the other two equation.) The display below shows the formula. The first solutions are for the bottom width of the downstream channel also equal to 8 ft, i.e. b_2 = 8 ft. After the display which shows the equations; thereafter the values in the cells are shown after the solution is obtained for S_{o1} = .00025. Q = 3,576.3 cfs, Y_{o1} = 14.796 ft and Y_c = 11.754 ft. Notice how large the depths and flow rates are. Also the solutions are provided for S_{o1} = .0003, S_{o1} = .00035, and S_{o1} = .0004 (all with b_2 = 8 ft.) Thereafter, solutions from the EXCEL spreadsheet are given in which b_2 = 7 ft.



variables	Values	Equations
b1=	8	Mannings
m1=	2	2.3135E-10
So1=	0.00025	Critical
n1=	0.014	9.9923E-07
Q=	3576.29691	Energy
b2=	8	-9.3341E-07
m2=	1	
Yo1=	14.795858	
Yc	11.7533933	
A1=	556.201691	
A2=	232.169401	
P1=	74.1690885	
Cu=	1.486	
g=	32.2	

variables	Values	Equations
b1=	8	Mannings
m1=	2	6.842E-07
So1=	0.0003	Critical
n1=	0.014	1.1722E-09
Q=	2073.78271	Energy
b2=	8	-3.3855E-09
m2=	1	
Yo1=	11.2916621	
Yc	8.92880544	
A1=	345.336561	
A2=	151.15401	
P1=	58.4978479	
Cu=	1.486	
g=	32.2	

variables	Values	Equations
b1=	8	Mannings
m1=	2	2.8917E-07
So1=	0.00035	Critical
n1=	0.014	8.197E-10
Q=	1349.70867	Energy
b2=	8	-1.9576E-09
m2=	1	
Yo1=	9.05458313	
Yc	7.13593703	
A1=	236.407617	
A2=	108.009094	
P1=	48.4933268	
Cu=	1.486	
g=	32.2	

variables	Values	Equations

variables	Values	Equations
b1=	8	Mannings
m1=	2	1.1281E-09
So1=	0.00025	Critical
n1=	0.014	-6.1747E-07
Q=	2161.01478	Energy
b2=	7	-9.8485E-07
m2=	1	
Yo1=	11.9520675	
Yc	9.4498175	
A1=	381.320374	
A2=	155.447773	
P1=	61.4512707	
Cu=	1.486	
g=	32.2	

variables	Values	Equations
b1=	8	Mannings
m1=	2	2.90929E-07
So1=	0.00035	Critical
n1=	0.014	1.94223E-09
Q=	726.115673	Energy
b2=	7	-3.59195E-09
m2=	1	
Yo1=	6.85583917	
Yc	5.35689423	
A1=	148.851775	
A2=	66.1945755	
P1=	38.6602449	
Cu=	1.486	
g=	32.2	

b1=	8	Mannings
m1=	2	7.5632E-08
So1=	0.0004	Critical
n1=	0.014	3.434E-10
Q=	947.247293	Energy
b2=	8	-6.863E-10
m2=	1	
Yo1=	7.50424795	
Yc	5.90050106	
A1=	172.661458	
A2=	82.0199213	
P1=	41.5600171	
Cu=	1.486	
g=	32.2	

variables	Values	Equations
b1=	8	Mannings
m1=	2	9.3832E-07
So1=	0.0003	Critical
n1=	0.014	3.41954E-09
Q=	1178.86288	Energy
b2=	7	-7.93861E-09
m2=	1	
Yo1=	8.82439787	
Yc	6.93138389	
A1=	226.335179	
A2=	96.5637699	
P1=	47.463907	
Cu=	1.486	
g=	32.2	

variables	Values	Equations
b1=	8	Mannings
m1=	2	2.28794E-07
So1=	0.0004	Critical
n1=	0.014	2.4288E-09
Q=	483.698844	Energy
b2=	7	-3.73557E-09
m2=	1	
Yo1=	5.50739772	
Yc	4.28562048	
A1=	104.722041	
A2=	48.3658863	
P1=	32.6298313	
Cu=	1.486	
g=	32.2	

The program UNIFTRCS listed in the solution to the previous problem has been used to solve this problem using different values for the substream bottom slope S_{o1}. The first table is for a downstream bottom width b_2 = 8 ft, and thereafter for b_2 = 7 ft.

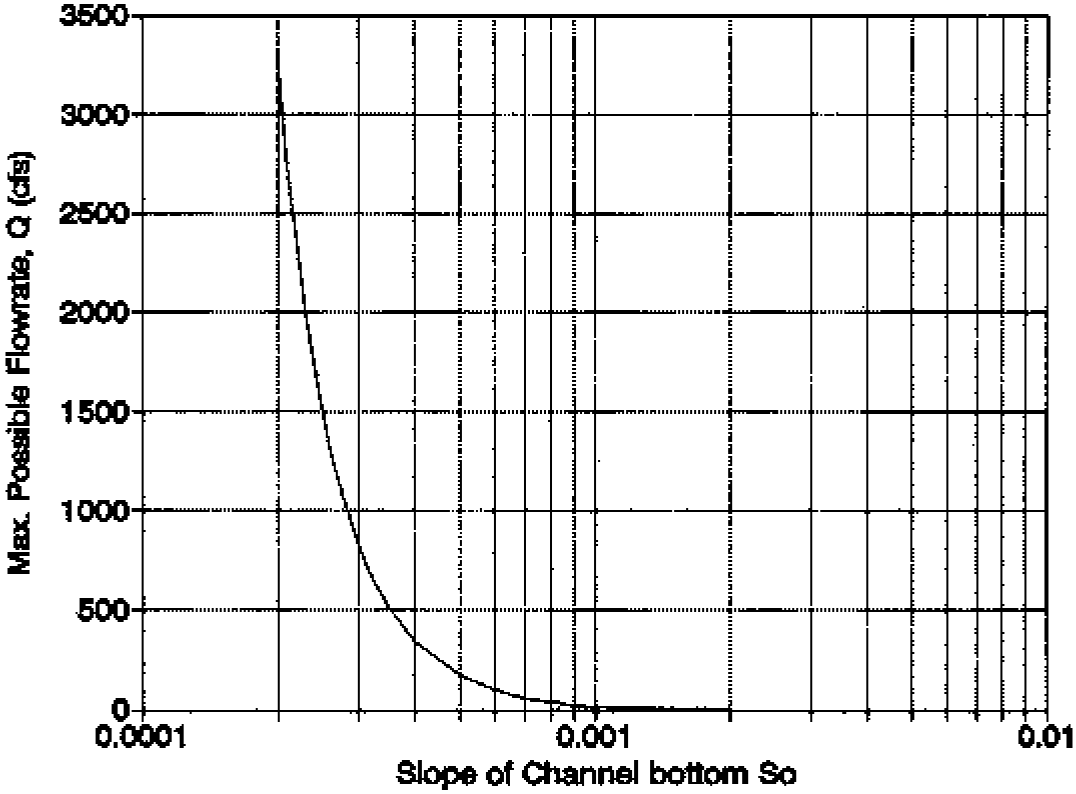
For b_2 = 8 ft

S_O	Q(cfs)	Y_1(ft)	Y_2(ft)
.0002500	3576.34	14.796	11.753
.0003000	2073.81	11.292	8.929
.0003500	1349.72	9.055	7.136
.0004000	947.26	7.504	5.901
.0004500	700.64	6.367	4.999
.0005000	538.50	5.496	4.313
.0005500	426.12	4.809	3.773
.0006000	344.97	4.253	3.339
.0006500	284.44	3.794	2.981
.0007000	238.07	3.409	2.682

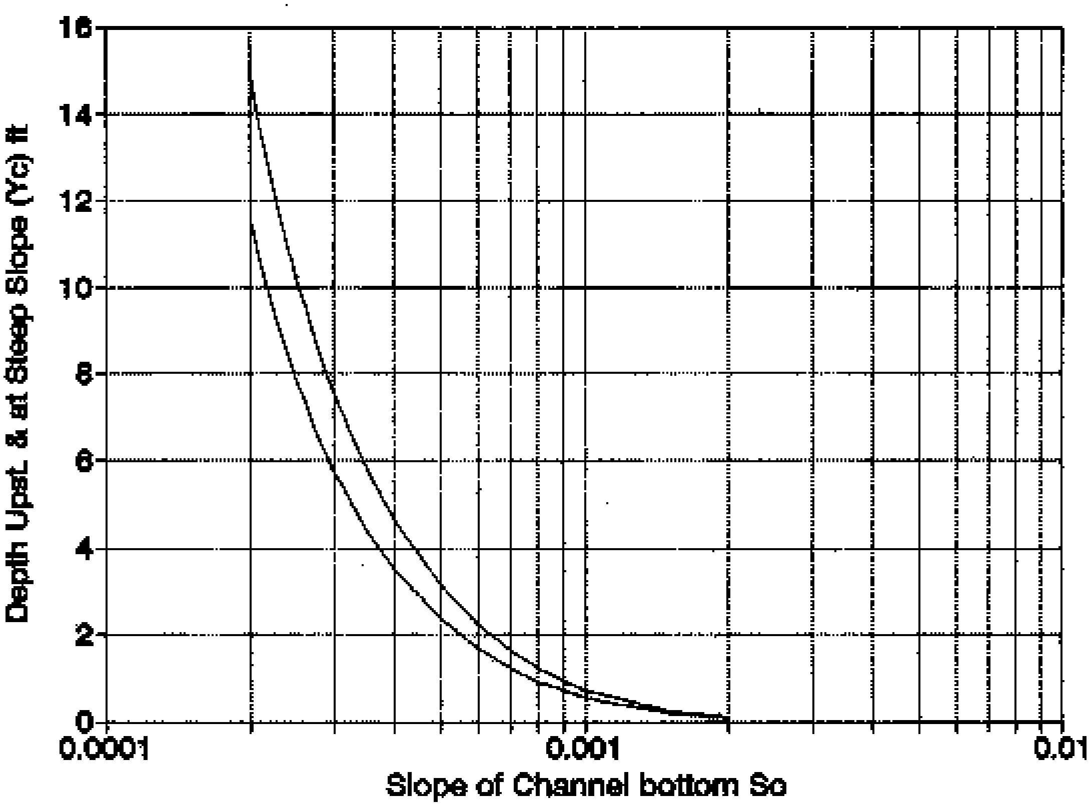
For b_2 = 7 ft (Note how dramatically the reduction of 1 ft in bottom width changes values.)

S_O	Q(cfs)	Y_1(ft)	Y_2(ft)
.0004000	339.79	4.657	3.511
.0005000	172.06	3.151	2.360
.0006000	97.16	2.237	1.669
.0007000	58.60	1.636	1.219
.0008000	36.96	1.222	.911
.0009000	24.13	.929	.693
.0009500	19.71	.814	.608
.0010000	16.20	.717	.536
.0011000	11.16	.561	.420
.0012000	7.86	.446	.334
.0013000	5.66	.358	.269
.0014000	4.15	.292	.219
.0015000	3.10	.240	.181
.0016000	2.35	.200	.151
.0017000	1.81	.168	.127

This results are plotted on the following two graphs. Note that even with the abscissa a log scale, how rapidly, the flow rate and depths decrease with increasing bottom slope of the upstream channel. In other words a smaller downstream channel can severely choke a channel’s carrying capacity if it is much smaller than the upstream channel even if it has a steep bottom slope, and this restriction in the flow becomes larger as the bottom slope of the upstream channel increases.



Max. Possible Flowrate, Q (cfs)
3500
3000
2500
2000
1500
1000
500
0
0.0001
0.001
0.01
Slope of Channel bottom So



Depth Upst. & at Steep Slope (Yc) ft
16
14
12
10
8
6
4
2
0
0.0001
0.001
0.01
Slope of Channel bottom So

95.

A transition between an upstream mild channel and a downstream steep channel is smooth with no change in the bottom position. The upstream channel has b_1 = 12 ft, m_1 = 2, n_1 = .014, and a bottom slope S_{o1} = .0002. The downstream steep channel has a bottom width b_2 = 8 ft and a side slope m_2 = 1. Determine the maximum flowrate possible in this channel if the flow in the upstream channel is to be uniform. What is the depth of this upstream uniform flow, and what is the depth at the beginning of the steep channel.

Given: An upstream trapezoidal channel contains uniform flow and goes into a steep downstream trapezoidal channel
b_1=12 ft, m_1=2, n_1=.014, S_{o1}=.0002; b_2=8 ft, m_2=1.

Find: Maximum flow rate possible and still have uniform flow upstream.

The problem could be solve with the TK-Solver model given in the solution of previous problem, or the Program UNIFTRC. The latter has been used to get:

Q = 2,382.4 cfs $\quad$ Y_{c2}=9.585 ft ←

Y_1=Y_o=12.246 ft $\quad$ E_{c2}=12.687 ft ←

The solution to this problem requires that the Mannings Equation for the upstream channel be solved simultaneously with the Critical flow equation, and the energy across the transition, or
Mannings: $Q=(C_u/n)A^{5/3}/P^{2/3}S_o^{1/2}$ for the upstream trapezoidal channel
Energy across transition: $Y_{o1} + (Q/A_{o1})^2/(2g) = Y_c + (Q/A_c)^2/(2g)$
Critical flow in downstream channel: $Q^2T_2 = gA_2^3$
The problem is also solved using the MathCAD model that is given below.
MathCAD Model PRB2 95.MCD

$b1 := 12 \quad m1 := 2 \quad n1 := .014 \quad So1 := .0002 \quad b2 := 8 \quad m2 := 1 \quad g := 32.2 \quad Y1 := 12 \quad Yc := 10 \quad Q := 2500$

$A1 := (b1 + m1\cdot Y1)\cdot Y1 \quad A2 := (b2 + m2\cdot Yc)\cdot Yc \quad P1 := 50 \quad Cu := 1.486$

Given

$$A1 = (b1 + m1\cdot Y1)\cdot Y1 \quad A2 = (b2 + m2\cdot Yc)\cdot Yc \quad P1 = b1 + 2\cdot Y1\cdot\sqrt{m1^2 + 1} \quad Q = \frac{Cu}{n1}\cdot\frac{A1^{1.666667}}{P1^{.6666667}}\cdot\sqrt{So1}$$

$$Q^2\cdot(b2 + 2\cdot m2\cdot Yc) = g\cdot A2^3 \quad Y1 + \frac{\left(\frac{Q}{A1}\right)^2}{2\cdot g} = Yc + \frac{\left(\frac{Q}{A2}\right)^2}{2\cdot g} \quad Find(Q, Y1, Yc, A1, A2, P1) = \begin{bmatrix} 2.382\cdot 10^3 \\ 12.246 \\ 9.585 \\ 446.868 \\ 168.559 \\ 66.765 \end{bmatrix}$$

The EXCEL model given in Problem 94 produces the following results.

variables	Values	Equations
b1=	12	Mannings
m1=	2	1.37323E-07
So1=	0.0002	Critical
n1=	0.014	-5.5504E-11
Q=	2382.35393	Energy
b2=	8	-2.99826E-10

m2=	1
Yo1=	12.2457779
Yc	9.58524072
A1=	446.867487
A2=	168.558765
P1=	66.7647836
Cu=	1.486
g=	32.2

variables	Values	Equations
b1=	12	Mannings
m1=	2	2.86366E-07
So1=	0.0003	Critical
n1=	0.014	4.64095E-09
Q=	367.401361	Energy
b2=	8	-6.51985E-09
m2=	1	
Yo1=	4.5683229	
Yc	3.46327041	
A1=	96.5590229	
A2=	39.7004053	
P1=	32.4301611	
Cu=	1.486	
g=	32.2	

96.

In the previous problem obtain the solutions for the same unknown for the downstream channel having a side slope of $m_2 = 0.5$, and $m_2 = 0$, respectively. The upstream mild change has the same size, slope, etc. as in the previous problem and the bottom width of the steep downstream channel is $b_2 = 8$ ft.

Wanted: Same as previous problem except m_2=0.5 and 0.

m_2=0.5

Q = 330.38 cfs, Y_1=4.797 ft, Y_2=3.479 ft, E_{c2}=3.479 ft

m_2=0.0

Q = 137.14 cfs, Y_1=3.038 ft, Y_2=2.090 ft, E_{c2}=3.134 ft

Using the above MathCAD model it is necessary to change the value of m2 to 0.5, and provide a different guess for the flow rate as shown below.

$b1 := 12 \quad m1 := 2 \quad n1 := .014 \quad So1 := .0002 \quad b2 := 8 \quad m2 := .5 \quad g := 32.2 \quad Y1 := 12 \quad Yc := 10 \quad Q := 300$

$A1 := (b1 + m1 \cdot Y1) \cdot Y1 \quad A2 := (b2 + m2 \cdot Yc) \cdot Yc \quad P1 := 50 \quad Cu := 1.486$

Given

$$A1 = (b1 + m1 \cdot Y1) \cdot Y1 \quad A2 = (b2 + m2 \cdot Yc) \cdot Yc \quad P1 = b1 + 2 \cdot Y1 \cdot \sqrt{m1^2 + 1} \quad Q = \frac{Cu}{n1} \cdot \frac{A1^{1.666667}}{P1^{.6666667}} \cdot \sqrt{So1}$$

$$Q^2 \cdot (b2 + 2 \cdot m2 \cdot Yc) = g \cdot A2^3 \quad Y1 + \frac{\left(\frac{Q}{A1}\right)^2}{2 \cdot g} = Yc + \frac{\left(\frac{Q}{A2}\right)^2}{2 \cdot g}$$

$$\text{Find}(Q, Y1, Yc, A1, A2, P1) = \begin{bmatrix} 330.38 \\ 4.797 \\ 3.479 \\ 103.597 \\ 33.887 \\ 33.454 \end{bmatrix}$$

And for m_2 = 0

$b1 := 12 \quad m1 := 2 \quad n1 := .014 \quad So1 := .0002 \quad b2 := 8 \quad m2 := .0 \quad g := 32.2 \quad Y1 := 5 \quad Yc := 4 \quad Q := 150$

$A1 := (b1 + m1 \cdot Y1) \cdot Y1 \quad A2 := (b2 + m2 \cdot Yc) \cdot Yc \quad P1 := 20 \quad Cu := 1.486$

Given

$$A1 = (b1 + m1 \cdot Y1) \cdot Y1 \quad A2 = (b2 + m2 \cdot Yc) \cdot Yc \quad P1 = b1 + 2 \cdot Y1 \cdot \sqrt{m1^2 + 1} \quad Q = \frac{Cu}{n1} \cdot \frac{A1^{1.666667}}{P1^{.6666667}} \cdot \sqrt{So1}$$

$$Q^2 \cdot (b2 + 2 \cdot m2 \cdot Yc) = g \cdot A2^3 \quad Y1 + \frac{\left(\frac{Q}{A1}\right)^2}{2 \cdot g} = Yc + \frac{\left(\frac{Q}{A2}\right)^2}{2 \cdot g}$$

$$\text{Find}(Q, Y1, Yc, A1, A2, P1) = \begin{bmatrix} 137.144 \\ 3.038 \\ 2.09 \\ 54.911 \\ 16.718 \\ 25.586 \end{bmatrix}$$

Notice that a reduction of the side slope of the downstream channel has a very large effect in reducing the flow rate (and the depths) in the channels. In brief a reduction of the size of the downstream channel reduces the flow rate a large amount because the critical flow occurs here. The assumption is that the downstream channel does have a bottom slope sufficient that critical flow, or supercritical flow can occur in it for the given flow rate.

97.

Solve Problem 95 for several width of downstream rectangular channel with bottom widths varying from $b_2 = 6$ ft to $b_2 = 12$ ft. ($m_2 = 0$) Note from these solution how significant the choking effect is as the size of the downstream steep channel is reduced in size.

Given: An upstream channel is to have uniform flow in it. It flows into a steep downstream rectangular channel through a smooth transition.
Find: The maximum flow rate for varying bottom widths for the downstream channel.

Solution: The program UNIFTRCB has been obtained from Program UNIFTRC by adding a DO loop and a few additional statements so that a series of solutions can be obtained with the downstream channel's bottom width varying.

Program UNIFTRCB.FOR

```
C  This program is the same as UNIFTRC except that it contain a DO loop to obtain
C a series of solution in which the downstream width changes
C  Solve for uniform flow in the upstream channel when this channel flows into
C a steep downstream channel.  The downstream channel (which has critical flow
C at its entrance) may be trapezoidal or circular.  The three equations being
C  solved are: (1) Energy across transition, (2) Critical Flow, and (3) Uniform
C flow in upstream channel.
       INTEGER*2 INDX(3)
       REAL F(3),FF(3),DD(3,3),B2A(20)
       COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
       EQUIVALENCE (Q,X(1)),(Y1,X(2)),(Y2,X(3))
       WRITE(*,*)' Is downstream steep channel: (1) trap, (2) circular'
       READ(*,*) ITYPE
       WRITE(*,*)' For upst. Ch. give: b,m,n,So,KL,g'
       READ(*,*) B1,FM1,FN,SO,FKL,G
       FMS=2.*SQRT(FM1*FM1+1.)
       FKL=1.+FKL
       G2=2.*G
       Cu=1.486
       IF(G.LT.20.) Cu=1.
       IF(ITYPE.EQ.1) THEN
       WRITE(*,*)' For downst. channel give side slope m'
       READ(*,*) FM2
       ENDIF
       WRITE(*,*)' Give guesses for: Q,Y1 & Yc2'
       READ(*,*) X
       WRITE(*,*) ' Give:N & this many values for b2 (or D)'
       READ(*,*) N,(B2A(I),I=1,N)
       SO=SQRT(SO)*Cu/FN
       DO 50 K=1,N
       IF(ITYPE.EQ.1) THEN
       B2=B2A(K)
       ELSE
       D=B2A(K)
       D4=.25*D*D
       ENDIF
       NCT=0
10     CALL FUN(F)
       DO 20 I=1,3
       XX=X(I)
       X(I)=1.005*X(I)
       CALL FUN(FF)
       DO 15 J=1,3
15     DD(J,I)=(FF(J)-F(J))/(X(I)-XX)
20     X(I)=XX
       CALL SOLVEQ(3,1,3,DD,F,1,DET,INDX)
       SUM=0.
       DO 25 I=1,3
       X(I)=X(I)-F(I)
25     SUM=SUM+ABS(F(I))
       NCT=NCT+1
       WRITE(*,*) ' NCT=',NCT,SUM
       IF(SUM.GT. .001 .AND.NCT.LT.30) GO TO 10
```

```
      WRITE(*,100) B2,X
50    WRITE(3,100) B2,X
100   FORMAT(F8.2,F9.2,2F9.3)
      END
      SUBROUTINE FUN(F)
      REAL F(3)
      COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
      A1=AREA(1)
      A2=AREA(2)
      F(1)=X(2)+(X(1)/A1)**2/G2-X(3)-FKL*(X(1)/A2)**2/G2
      F(2)=X(1)**2*T2/(G*A2**3)-1.
      F(3)=X(1)-So*A1*(A1/P1)**.6666667
      RETURN
      END
      FUNCTION AREA(K)
      COMMON X(3),B1,FM1,SO,B2,FM2,D,G,G2,FKL,ITYPE,P1,T2,D4,FMS
      IF(K.EQ.1) THEN
      P1=B1+FMS*X(2)
      AREA=(B1+FM1*X(2))*X(2)
      ELSE
      IF(ITYPE.EQ.1) THEN
      T2=B2+2.*FM2*X(3)
      AREA=(B2+FM2*X(3))*X(3)
      ELSE
      BETA=ACOS(1.-2.*X(3)/D)
      T2=D*SIN(BETA)
      AREA=D4*(BETA-COS(BETA)*SIN(BETA))
      ENDIF
      ENDIF
      RETURN
      END
```

Output:

b(ft)	Q(cfs)	Y_1(ft)	Y_2(ft)
6.00	22.81	1.116	.766
6.50	40.66	1.554	1.067
7.00	65.45	2.030	1.395
7.50	97.53	2.528	1.738
8.00	137.14	3.038	2.090
8.50	184.49	3.554	2.446
9.00	239.77	4.073	2.804
10.00	374.88	5.112	3.521
11.00	544.02	6.147	4.235
12.00	748.70	7.173	4.945

98.

Water is taken from a reservoir with a water surface elevation 5 ft above a 12 ft wide, trapezoidal channel with a side slope $m_1 = 1.5$. A short distance downstream from the channel entrance it divides into two trapezoidal channels with $b_2 = 8$ ft, $m_2 = 1.0$, and $b_3 = 4$ ft, and $m_3 = 2$. The second channel has a gate in it a short distance downstream from the branch that causes the depth of flow downstream from it to be at $Y_{22} = 0.8$ ft. The third channel is long, has a Mannings $n_3 = .015$, and a bottom slope $S_{o3} = .0005$. The bottom rises by $Z_{13} = 0.3$ ft between the upstream channel and the third channel. Just upstream from the junction a 12" diameter pipe takes water from near the bottom of the channel. This pipe is 2000 feet long, has an equivalent sand roughness e = .012 inches, and delivers water at it end with a head $H_e = 0$ ft. Determine the flowrates in all channels and the pipe, as well as the depths in the channels.

Given: A three channel system with an additional pipe that removes water at end of upstream channel.

The equations that govern this problem consist of: (1) Energy at the reservoir, (2) Junction continuity, (3) Energy between Channels 1 and 2, (4) Energy between Channels 1 and 3, (5) Energy across the gate in Channel 3, (6) Mannings Equation in Channel 2, (7) The frictional headloss in the pipe as defined by the Darcy-Weisbach equation. (8)The Colebrook-White equation to define the friction factor for the Darcy-Weisbach equation as a function of the flow conditions.
The TK-Solver Model PRB2_75.TK, below contains these 8 equations, so that the first 7 variables in the VARIABLE SHEET, as well as the friction factor f, can be solved.

```
=============== VARIABLE SHEET =================
St Input---- Name--- Output--- Unit---- Comment
             Q1      367.59122          Solves Prob. 75 (Chap. 2) 3 branched c
             Y1      4.6827386          With pipe out from main channel also.
             Q2      114.90699
             Y2      4.8850241
             Q3      250.57477
             Y3      4.6386959
             Q4      2.1094613          (Flow out from pipeline)
   5         H
   12        b1
   1.5       m1
   64.4      g2
   8         b2
   1         m2
   0         z12
   0         z13
   4         b3
   2         m3
   .015      n3
   .0005     So3
   .2        K1
   .2        K2
   .2        K3
   1.486     Cman
             f       .02090222
   2000      L
   1         D
   .001      e
   .00001217 v
   .8        Y22

=============== RULE SHEET ================
S Rule---------------------------------------
* Q1-Q2-Q3-Q4=0
* H-Y1-(1+K1)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(1+K2)*(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(1+K3)*(Q3/((b3+m3*Y3)*Y3))^2/g2-z13=0
* Y1=f*L/D*(Q4/(pi()/4*D*D))^2/g2
* n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-Cman*((b3+m3*Y3)*Y3)^1.666667*sqrt(So3
* Y2+(Q2/((b2+m2*Y2)*Y2))^2/g2-Y22-(Q2/((b2+m2*Y22)*Y22))^2/g2=0
* 1/sqrt(f)=1.14-2*log(e/D+7.3434728*v*D/Q4/sqrt(f))
```

The following MathCAD model PRB2_75.MCD solves the same system of 8 equations for the 8 unknown variables.

$Q1 := 370 \quad Y1 := 4.7 \quad Q2 := 110 \quad Y2 := 4.9 \quad Q3 := 250 \quad Y3 := 4.6 \quad Q4 := 2 \quad H := 5 \quad b1 := 12 \quad m1 := 1.5 \quad g2 := 64.4 \quad b2 := 8$

$m2 := 1 \quad z12 := 0 \quad z13 := 0 \quad b3 := 4 \quad m3 := 2 \quad n3 := .015 \quad So3 := .0005 \quad K1 := .2 \quad K2 := .2 \quad K3 := .2 \quad Cman := 1.486$

$f := .02 \quad L := 2000 \quad D := 1 \quad e := .001 \quad vis := .0000121 \quad Y22 := .8$

Given

$$Q1 - Q2 - Q3 - Q4 = 0 \qquad H - Y1 - (1 + K1)\cdot\frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = 0$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} - Y2 - (1 + K2)\cdot\frac{\left[\frac{Q2}{(b2 + m2\cdot Y2)\cdot Y2}\right]^2}{g2} - z12 = 0$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} - Y3 - (1 + K3)\cdot\frac{\left[\frac{Q3}{(b3 + m3\cdot Y3)\cdot Y3}\right]^2}{g2} - z13 = 0$$

$$n3\cdot Q3\cdot\left(b3 + 2\cdot Y3\cdot\sqrt{m3^2 + 1}\right)^{.66666667} = Cman\cdot((b3 + m3\cdot Y3)\cdot Y3)^{1.6666667}\cdot\sqrt{So3}$$

$$Y1 = f\cdot\frac{L}{D}\cdot\frac{\left(\frac{Q4}{\pi\cdot\frac{D^2}{4}}\right)^2}{g2}$$

$$Y2 + \frac{\left[\frac{Q2}{((b2 + m2\cdot Y2)\cdot Y2)}\right]^2}{g2} - Y22 - \frac{\left[\frac{Q2}{(b2 + m2\cdot Y22)\cdot Y22}\right]^2}{g2} = 0$$

$$\frac{1}{\sqrt{f}} = 1.14 - 2\cdot\log\left(\frac{e}{D} + 7.3434728vis\cdot\frac{D}{Q4\cdot\sqrt{f}}\right)$$

$$\mathrm{Find}(Q1, Y1, Q2, Y2, Q3, Y3, Q4, f) = \begin{bmatrix} 367.59102 \\ 4.68274 \\ 114.90699 \\ 4.88502 \\ 250.57457 \\ 4.6387 \\ 2.10946 \\ 0.0209 \end{bmatrix}$$

Notice that the flow rate in the pipe is relatively small compared with the channel flow rates. The diameter of the pipe in the above MathCAD model has been changed from 1 ft to 2 ft, and thereafter to 4 ft in the solutions given below. The flow rate in the pipe changes from 2.11 cfs to 13.04 cfs (an increase of 10.93 cfs) when the diameter is 2 ft, and to 50.36 cfs (an increase of 48.25 cfs) when the diameter is 4 ft. When the pipe diameter is increase to 2 ft, the total flow into the channel system increases from 367.09 cfs to 378.04 cfs (an increase of 10.95 cfs, which is almost identical to the increase in the pipe flow rate.)

However, when the diameter is given as 4 ft the total flow rate into channel 1 decreases to 359.06 cfs (a decrease of 8.03 cfs.) However, the depth associated with this flow rate in Channel 1 is 1.871 ft, which is supercritical. Thus the solution suggests that critical flow will control in Channel 1 when too much water is withdrawn through the pipe. However, as can be seen from the added critical flow equations at the end of this last model this is not the case since the critical flow rate in Channel 1 with a reservoir head of H = 5 ft is Qc=539.5 cfs, with Yc =3.42 ft.

Diameter of pipe changed to 2 ft

$Q1 := 370\quad Y1 := 4.7\quad Q2 := 110\quad Y2 := 4.9\quad Q3 := 250\quad Y3 := 4.6\quad Q4 := 2\quad H := 5\quad b1 := 12\quad m1 := 1.5\quad g2 := 64.4$

$b2 := 8\quad m2 := 1\quad z12 := 0\quad z13 := 0\quad b3 := 4\quad m3 := 2\quad n3 := .015\quad So3 := .0005\quad K1 := .2\quad K2 := .2\quad K3 := .2$

$Cman := 1.486\quad f := .02\quad L := 2000\quad D := 2\quad e := .001\quad vis := .0000121\quad Y22 := .8$

Given

$$Q1 - Q2 - Q3 - Q4 = 0 \qquad H - Y1 - (1 + K1)\cdot\frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = 0$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} - Y2 - (1 + K2)\cdot\frac{\left[\frac{Q2}{(b2 + m2\cdot Y2)\cdot Y2}\right]^2}{g2} - z12 = 0$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} - Y3 - (1 + K3)\cdot\frac{\left[\frac{Q3}{(b3 + m3\cdot Y3)\cdot Y3}\right]^2}{g2} - z13 = 0$$

$$n3\cdot Q3\cdot\left(b3 + 2\cdot Y3\cdot\sqrt{m3^2 + 1}\right)^{.66666667} = Cman\cdot((b3 + m3\cdot Y3)\cdot Y3)^{1.6666667}\cdot\sqrt{So3} \qquad Y1 = f\cdot\frac{L}{D}\cdot\frac{\left(\frac{Q4}{\pi\cdot\frac{D^2}{4}}\right)^2}{g2}$$

$$Y2 + \frac{\left[\frac{Q2}{((b2 + m2\cdot Y2)\cdot Y2)}\right]^2}{g2} - Y22 - \frac{\left[\frac{Q2}{(b2 + m2\cdot Y22)\cdot Y22}\right]^2}{g2} = 0 \qquad \frac{1}{\sqrt{f}} = 1.14 - 2\cdot\log\left(\frac{e}{D} + 7.3434728 vis\cdot\frac{D}{Q4\cdot\sqrt{f}}\right)$$

$$Find(Q1, Y1, Q2, Y2, Q3, Y3, Q4, f) = \begin{bmatrix} 378.03741 \\ 4.65994 \\ 114.85402 \\ 4.88115 \\ 250.14138 \\ 4.63517 \\ 13.04202 \\ 0.01741 \end{bmatrix}$$

Diameter of pipe changed to 4 ft

$Q1 := 370\quad Y1 := 4.7\quad Q2 := 110\quad Y2 := 4.9\quad Q3 := 250\quad Y3 := 4.6\quad Q4 := 2\quad H := 5\quad b1 := 12\quad m1 := 1.5\quad g2 := 64.4\quad b2 := 8$

$m2 := 1\quad z12 := 0\quad z13 := 0\quad b3 := 4\quad m3 := 2\quad n3 := .015\quad So3 := .0005\quad K1 := .2\quad K2 := .2\quad K3 := .2\quad Cman := 1.486$

$f := .02\quad L := 2000\quad D := 4\quad e := .001\quad vis := .0000121\quad Y22 := .8$

Given

$$Q1 - Q2 - Q3 - Q4 = 0 \qquad H - Y1 - (1 + K1)\cdot\frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} = 0$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} - Y2 - (1 + K2)\cdot\frac{\left[\frac{Q2}{(b2 + m2\cdot Y2)\cdot Y2}\right]^2}{g2} - z12 = 0$$

$$Y1 + \frac{\left[\frac{Q1}{(b1 + m1\cdot Y1)\cdot Y1}\right]^2}{g2} - Y3 - (1 + K3)\cdot\frac{\left[\frac{Q3}{(b3 + m3\cdot Y3)\cdot Y3}\right]^2}{g2} - z13 = 0$$

$$n3\cdot Q3\cdot\left(b3 + 2\cdot Y3\cdot\sqrt{m3^2 + 1}\right)^{.66666667} = Cman\cdot((b3 + m3\cdot Y3)\cdot Y3)^{1.6666667}\cdot\sqrt{So3} \qquad Y1 = f\cdot\frac{L}{D}\cdot\frac{\left(\frac{Q4}{\pi\cdot\frac{D^2}{4}}\right)^2}{g2}$$

$$Y2 + \frac{\left[\frac{Q2}{((b2 + m2\cdot Y2)\cdot Y2)}\right]^2}{g2} - Y22 - \frac{\left[\frac{Q2}{(b2 + m2\cdot Y22)\cdot Y22}\right]^2}{g2} = 0 \qquad \frac{1}{\sqrt{f}} = 1.14 - 2\cdot\log\left(\frac{e}{D} + 7.3434728vis\cdot\frac{D}{Q4\cdot\sqrt{f}}\right)$$

$$Find(Q1, Y1, Q2, Y2, Q3, Y3, Q4, f) = \begin{bmatrix} 359.05797 \\ 1.8714 \\ 108.17715 \\ 4.40557 \\ 200.51865 \\ 4.203 \\ 50.36217 \\ 0.01501 \end{bmatrix}$$

$Yc := 3.6$

$Qc := 370$

Given

$$H - Yc - (1 + K1)\cdot\frac{\left[\frac{Qc}{(b1 + m1\cdot Yc)\cdot Yc}\right]^2}{g2} = 0$$

$$Qc^2\cdot\frac{b1 + 2\cdot m1\cdot Yc}{.5\cdot g2\cdot((b1 + m1\cdot Yc)\cdot Yc)^3} = 1$$

$$find(Yc, Qc) = \begin{pmatrix} 3.42065 \\ 539.48851 \end{pmatrix}$$

99.

Example Problem 18 obtains a series of solution for a four channel branched system in which the height of the gate in channel 3 varies from 1.7 ft to being closed. Solve this same channel system except the upstream channel # 1 has a width of 12 ft rather than 15 ft. The gates contraction coefficient is $C_c = .6$. Obtain this series of solutions with the gates position starting at 0.1 ft.

Wanted: Solve Example Problem 18 with the gate height varying, but Channel 1 bottom width is 12 ft, rather than 15 ft as in the example problem.

Solution:

Program PRB18.FOR

```
      REAL X[ALLOCATABLE](:),F[ALLOCATABLE](:),F1[ALLOCATABLE](:),
     &D[ALLOCATABLE](:,:),DZ[ALLOCATABLE](:),KLOS[ALLOCATABLE](:)
      INTEGER*2 IT[ALLOCATABLE](:),INDX[ALLOCATABLE](:),
     &IV[ALLOCATABLE](:)
      LOGICAL*1 STEEP[ALLOCATABLE](:),KEYB
      CHARACTER*2 CU[ALLOCATABLE](:)
      CHARACTER*6 S(2)/'SQYbmn','SQYDn '/
      CHARACTER*1 CH
      CHARACTER*38  FMT/"(1X,A1,' =',F8.6,/,(1X,A1,' =',F8.3))"/
      COMMON CMAN,G,G2,YGA(5),EYG(5),IYG(5),IGATE(6),NGATE
      WRITE(*,*)' Give: (1) 1=ES, 2=SI; (2) No. channels; (3) No. gates
     &(4) IN-unit; (5) OUT-unit'
      KEYB=.FALSE.                          !  Trapezodial    Circular
      READ(*,*) II,NC,NGATE,IN,IOUT         ! -------------   ---------
      IGATE(NGATE+1)=NC+1                   !  H  = x(1)       H  = x(1)
      IF(IN.EQ.0 .OR. IN.EQ.5) KEYB=.TRUE.  !  Q1 = x(2)       Q1 = x(2)
      IF(II.EQ.2) THEN                      !  Y1 = x(3)       Y1 = x(3)
      G=9.81                                !  b1 = x(4)       D1 = x(4)
      CMAN=1.                               !  m1 = x(5)       n1 = x(5)
      ELSE                                  !  n1 = x(6)
      G=32.2                                !  So2= x(7)       So2= x(6)
      CMAN=1.486                            !  Q2 = x(8)       Q2= x(7)
      ENDIF                                 !  Y2 = x(9)       Y2 = x(8)
      G2=2.*G                               !  b2 = x(10)      D2 = x(9)
      N=2*NC                                !  m2 = x(11)      n2 = x(10)
      ALLOCATE(IT(NC),KLOS(NC),DZ(NC),F(N),F1(N),D(N,N),CU(N),STEEP(NC))
      IF(KEYB) WRITE(*,*)' For each channel give 1 = trap., or 2 = cir.'
      NV=0
      DO 10 I=1,NC
      IF(KEYB) WRITE(*,"(' Channel #',I2,' = ',\)") I
      READ(IN,*) IT(I)
10    NV=NV-IT(I)+7
      ALLOCATE(X(NV),IV(N),INDX(N))
      II=0
      DO 30 I=1,NC
      STEEP(I)=.FALSE.
      IF(KEYB) THEN
      WRITE(*,"(' Give variables for Channel',I2)") I
      DO 20 J=1,7-IT(I)
      CH=S(IT(I))(J:J)
      IF(I.EQ.1 .AND. J.EQ.1) CH='H'
      WRITE(*,"(5X,A1,' = ',\)") CH
20    READ(*,*) X(J+II)
      WRITE(*,"(' Loss Coef =',\)")
      READ(*,*) KLOS(I)
      IF(I.GT.1) THEN
      WRITE(*,"(' Change in bottom position =',\)")
      READ(*,*) DZ(I)
      ENDIF
      ELSE
      IF(I.EQ.1) THEN
      READ(IN,*)(X(J),J=1,7-IT(I)),KLOS(I)
      ELSE
      READ(IN,*)(X(J+II),J=1,7-IT(I)),KLOS(I),DZ(I)
      IF(X(II+1).GT. .008) STEEP(I)=.TRUE.
      ENDIF
```

```
      ENDIF
30    II=II+7-IT(I)
      IF(NGATE.GT.0) THEN
      IF(KEYB) WRITE(*,*)' Give pairs of values: channel no. and depth d
     &owns. from gate(s)'
      READ(IN,*) (IGATE(I),YGA(I),I=1,NGATE)
      ENDIF
      IF(KEYB) WRITE(*,*)' Give symbols for',N,' Unknowns, i.e. Y1 Q1 b2
     & etc.'
      READ(IN,120) (CU(I),I=1,N)
120   FORMAT(26(A2,1X))
      IU=0
      DO 40 I=1,N
      II=ICHAR(CU(I)(2:2))-48
      DO 35 J=1,7-IT(II)
      IF(S(IT(II))(J:J).NE.CU(I)(1:1)) GO TO 35
      IU=IU+1
      IPOS=0
      DO 36 K=1,II-1
36    IPOS=IPOS+7-IT(K)
      IV(IU)=IPOS+J
      GO TO 40
35    CONTINUE
      WRITE(*,*)' Do not have ',CU(I),' as a variable'
        STOP
  40    CONTINUE
        YGG=YGA(1)
        DO 88 KK=1,17
        YGA(1)=YGG+.06*FLOAT(KK-1)
        NCT=0
  42    CALL FUNCT(NC,N,NV,IT,STEEP,X,F,DZ,KLOS)
        DO 50 J=1,N
        XX=X(IV(J))
        X(IV(J))=1.005*X(IV(J))
        CALL FUNCT(NC,N,NV,IT,STEEP,X,F1,DZ,KLOS)
        DO 45 I=1,N
  45    D(I,J)=(F1(I)-F(I))/(X(IV(J))-XX)
  50    X(IV(J))=XX
        CALL SOLVEQ(N,1,N,D,F,1,DD,INDX)
        NCT=NCT+1
        SUM=0.
        DO 60 I=1,N
        X(IV(I))=X(IV(I))-F(I)
  60    SUM=SUM+ABS(F(I))
        WRITE(*,*)' Iteration=',NCT,' SUM=',SUM
        IF(NCT.LT.20 .AND. SUM.GT. .0005) GO TO 42
        DO 65 I=1,NGATE
        NCT=2
        II=IYG(I)
  63    FF=EYG(I)-X(II)-(X(II-1)/((X(II+1)+X(II+2)*X(II))*X(II)))**2/G2
        IF(MOD(NCT,2) .NE. 0) GO TO 64
        NCT=NCT+1
        F11=FF
        XX=X(II)
        X(II)=1.005*X(II)
        GO TO 63
  64    NCT=NCT+1
        DIF=(X(II)-XX)*F11/(FF-F11)
        X(II)=XX-DIF
        WRITE(*,*)' NCT=',NCT,'  DIF=',DIF
        IF(NCT.LT. 40 .AND. ABS(DIF).GT. .0001) GO TO 63
        IF(NCT.GE.40) WRITE(*,*)' Failed to converge with gates',DIF
  65    CONTINUE
  88    WRITE(IOUT,89) YGA(1)/.6,(X(6*(I-1)+2),X(6*(I-1)+3),I=1,NC)
  89    FORMAT(F5.2,5(F8.1,F8.3))
  C      II=0
  C      S(IT(1))(1:1)='H'
  C      FMT(16:16)='3'
  C      DO 70 I=1,NC
  C      WRITE(IOUT,"(' For Channel',I2)") I
  C      WRITE(IOUT,FMT)(S(IT(I))(J:J),X(J+II),J=1,7-IT(I))
```

```
C      IF(I.EQ.1) S(IT(1))(1:1)='S'
C      FMT(16:16)='6'
C70    II=II-IT(I)+7
      END
      SUBROUTINE FUNCT(NC,N,NV,IT,STEEP,X,F,DZ,KLOS)
      REAL X(NV),F(N),DZ(N),KLOS(N)
      INTEGER IT(NC)
      LOGICAL*1 STEEP(NC)
      COMMON CMAN,G,G2,YGA(5),EYG(5),IYG(5),IGATE(6),NGATE
      II=0
      IG=1
      DO 20 I=1,NC
      IF(IT(I).EQ.1) THEN
      A=(X(II+4)+X(II+5)*X(II+3))*X(II+3)
      IF(STEEP(I)) THEN
      T=X(II+4)+2.*X(II+5)*X(II+3)
      ELSE
      P=X(II+4)+2.*SQRT(X(II+5)**2+1.)*X(II+3)
      ENDIF
      ELSE
      ARG=1.-2.*X(II+3)/X(II+4)
      BETA=ACOS(ARG)
      A=.25*X(II+4)**2*(BETA-SIN(BETA)*ARG)
      IF(STEEP(I)) THEN
      T=X(II+4)*SIN(BETA)
      ELSE
      P=BETA*X(II+4)
      ENDIF
      ENDIF
      IF(I.EQ.1) THEN
      E1=X(3)+(X(2)/A)**2/G2
      F(1)=X(1)-E1-KLOS(1)*(X(2)/A)**2/G2
      F(2)=X(2)
      JJ=7-IT(1)
      DO 10 J=2,NC
      F(2)=F(2)-X(JJ+2)
10    JJ=JJ+7-IT(J)
      ELSE
      IF(I.EQ.IGATE(IG)) THEN
      F(2*I-1)=E1-YGA(IG)-(1.+KLOS(I))*(X(II+2)/((X(II+4)+X(II+5)*YGA(IG
     &))*YGA(IG)))**2/G2-DZ(I)
      EYG(IG)=E1
      IYG(IG)=II+3
      IG=IG+1
      ELSE
      F(2*I-1)=E1-X(II+3)-(1.+KLOS(I))*(X(II+2)/A)**2/G2-DZ(I)
      ENDIF
      IF(STEEP(I)) THEN
      F(2*I)=T*X(II+2)**2-G*A**3
      ELSE
      F(2*I)=X(II+7-IT(I))*X(II+2)-CMAN*A*(A/P)**.66666667*SQRT(X(II+1))
      ENDIF
      ENDIF
20    II=II+7-IT(I)
      RETURN
      END
```

Program PRB18.C

```
#include <stdio.h>
#include <stdlib.h>
#include <math.h>
float cman,g,g2,yga[5],eyg[5]; int ngate,iyg[5],igate[6];
extern void solveq(int n,float **a,float *b,int itype,float *dd,int *indx);
void funct(int nc,int *it,int *steep,float *x,float *f,float *dz,float *klos){
 int ii,ig,j,jj,i;  float a,t,p,arg,beta,e1;
 ii=0; ig=0;
 for(i=0;i<nc;i++){if(it[i]){arg=1.-2.*x[ii+2]/x[ii+3]; beta=acos(arg);
  a=.25*x[ii+3]*x[ii+3]*(beta-sin(beta)*arg);
  if(steep[i])t=x[ii+3]*sin(beta); else p=beta*x[ii+3];} else {
  a=(x[ii+3]+x[ii+4]*x[ii+2])*x[ii+2];
  if(steep[i]) t=x[ii+3]+2.*x[ii+4]*x[ii+2]; else p=x[ii+3]+2.*sqrt(x[ii+4]*x[ii+4]+1.)*x[ii+2];}
```

```
  if(i){if((i+1)==igate[ig]){f[2*i]=e1-yga[ig]-(1.+klos[i])*pow(x[ii+1]/((x[ii+3]+x[ii+4]*yga[ig])*
  yga[ig]),2)/g2-dz[i];
    eyg[ig]=e1;iyg[ig++]=ii+3;}
   else f[2*i]=e1-x[ii+2]-(1.+klos[i])*pow(x[ii+1]/a,2.)/g2-dz[i];
    if(steep[i]) f[2*i+1]=t*x[ii+1]*x[ii+1]-g*pow(a,3.); else
     f[2*i+1]=x[ii+5-it[i]]*x[ii+1]-cman*a*pow(a/p,.6666667)*sqrt(x[ii]); } else
   {e1=x[2]+pow(x[1]/a,2.)/g2; f[0]=x[0]-e1-klos[0]*pow(x[1]/a,2.)/g2; f[1]=x[1];
    jj=6-it[0]; for(j=1;j<nc;j++){f[1]-=x[jj+1];jj+=6-it[j];} }ii+=6-it[i]; }
  } // end funct
  void main(void){ int
  *it,*indx,*iv,*steep,keyb=0,screen=0,ii,nc,n,nv=0,in,iout,i,j,iu,ipos,k,nct=0,kk;
   float *x,*f,*f1,**d,*dz,*klos,xx,sum,ff,f11,dif,*dd,ygg;
   char ch2[2],*cu1,*cu2,*s[]={"SQYbmn","SQYDn "},ch;
   FILE *fili,*filo; char filnam[20];
   printf("Give:(1) 1=ES,2=SI;(2) No. of channels;(3) No. gates,(4) IN-unit,(5) OUT-unit\n");
   scanf("%d %d %d %d %d",&ii,&nc,&ngate,&in,&iout);  igate[ngate]=nc+1;
   if((in==0) || (in==5)) keyb=1; else {printf("Give input file name\n");scanf("%s",filnam);
   if((fili=fopen(filnam,"r"))==NULL){printf("\nFile %s cannot be opened\n",filnam);exit(0);}}
   if((iout==0) || (iout==6)) screen=1; else {printf("Give output file name\n");scanf("%s",filnam);
   if((filo=fopen(filnam,"w"))==NULL){printf("\nFile %s cannot be opened\n",filnam);exit(0);}}
   if(ii==2){g=9.81;cman=1.;} else {g=32.2;cman=1.486;} g2=2.*g;n=2*nc;
   it=(int *)calloc(nc,sizeof(int)); steep=(int *)calloc(nc,sizeof(int));
   klos=(float *)calloc(nc,sizeof(float)); dz=(float *)calloc(nc,sizeof(float));
   f=(float *)calloc(n,sizeof(float)); f1=(float *)calloc(n,sizeof(float));
   cu1=(char *)calloc(n,sizeof(char)); cu2=(char *)calloc(n,sizeof(char));
   d=(float **)malloc(n*sizeof(float*));for(i=0;i<n;i++)d[i]=(float*)malloc(n*sizeof(float));
   if(keyb) printf("For each channel give: 1=Trap., or 2=cir.\n");
   for(i=0;i<nc;i++){if(keyb){printf("Channel # %2d =",i+1);scanf("%d",&it[i]);} else
    fscanf(fili,"%d",&it[i]); nv+=7-it[i]--;}
   x=(float *)calloc(nv,sizeof(float));iv=(int *)calloc(n,sizeof(int));indx=(int
  *)calloc(n,sizeof(int));
   for(i=0,ii=0;i<nc;i++){steep[i]=0;if(keyb){printf("Give variables for Channel %2d\n",i+1);
    for(j=0;j<(6-it[i]);j++){ch=s[it[i]][j]; if((i==0) && (j==0)) ch='H'; printf("     %c
  =",ch);scanf("%f",&x[j+ii]);}
    printf("Loss Coef =");scanf("%f",&klos[i]);
     if(i){printf("Change in bottom position =");scanf("%f",&dz[i]);}}  else {
   if(i==0){for(j=0;j<(6-it[i]);j++) fscanf(fili,"%f",&x[j]);fscanf(fili,"%f",&klos[i]);} else
   {for(j=0;j<(6-it[i]);j++) fscanf(fili,"%f",&x[j+ii]);fscanf(fili,"%f %f",&klos[i],&dz[i]);
    if(x[ii]>.008)steep[i]=1;}}
   ii+=6-it[i];} // end for i
   if(ngate>0) {if(keyb)printf("Give pairs of values: channel no and depth downs. from gate(s)\n");
    for(i=0;i<ngate;i++){if(keyb) scanf("%d %f",&igate[i],&yga[i]); else fscanf(fili,"%d
  %f",&igate[i],&yga[i]);}}
 if(keyb) printf("Give symbols for %d Unknowns, i.e. Y1 Q1 b2 etc.\n",n);
 for(i=0;i<n;i++){if(keyb) scanf("%s",ch2); else fscanf(fili,"%s",ch2);cu1[i]=ch2[0];cu2[i]=ch2[1];}
 for(iu=0,i=0;i<n;i++){
L30:ii=cu2[i]-49; for(j=0;j<(6-it[ii]);j++){if(s[it[ii]][j]==cu1[i]){ipos=0;
 for(k=0;k<ii;k++) ipos+=6-it[k]; iv[iu]=ipos+j; goto L40;} }
 printf("Do not have %c%c as a variable\n",cu1[i],cu2[i]);
 printf("Give correct variable (z1=stop) ");scanf("%s",ch2);if(ch2[0]=='z')exit(0);
  cu1[i]=ch2[0];cu2[i]=ch2[1]; goto L30; L40:iu++; } // end for i
ygg=yga[0];for(kk=0;kk<18;kk++){yga[0]=ygg+.06*(float)kk;
do{funct(nc,it,steep,x,f,dz,klos);
 for(j=0;j<n;j++){ xx=x[iv[j]];x[iv[j]]*=1.005; funct(nc,it,steep,x,f1,dz,klos);
 for(i=0;i<n;i++) d[i][j]=(f1[i]-f[i])/(x[iv[j]]-xx); x[iv[j]]=xx;}
 solveq(n,d,f,1,dd,indx); nct++; sum=0.;
 for(i=0;i<n;i++){x[iv[i]]-=f[i]; sum+=fabs(f[i]);} printf("Iteration=%d sum=%f\n",nct,sum);
}while ((nct<20) && (sum>.0005));
 for(i=0,nct=1;i<ngate;i++){ii=iyg[i]-1;
L63:ff=eyg[i]-x[ii]-pow(x[ii-1]/((x[ii+1]+x[ii+2]*x[ii])*x[ii]),2.)/g2;nct++;
 if((nct%2)==0){f11=ff;xx=x[ii];x[ii]*=1.005; goto L63;}
 dif=(x[ii]-xx)*f11/(ff-f11); x[ii]=xx-dif; printf("NCT= %d DIF=%f\n",nct,dif);
 if((nct<40) && (fabs(dif)>.0001)) goto L63;if(nct==40)printf("Not converge with
gates\n");}s[it[0]][0]='H';
 fprintf(filo,"%6.2f",yga[0]/.6);
for(i=0,ii=0;i<nc;i++) fprintf(filo,"%8.1f %8.3f",x[6*i+1],x[6*i+2]);fprintf(filo,"\n"); }
 if(keyb==0) fclose(fili); if(screen==0) fclose(filo);
}
```

Input Data to Program PRB18 (file: PRB18A.DAT)

```
1
1
1
1
5. 392. 4.66 12. 2. .013 .2
.0008 215. 4.70 8. 1. .013 .2 0.
.0008   8. 4.94 5. 1. .013 .2 0.
.0008 169. 4.68 5. 1. .013 .2 0.
3 .06
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4
```

Solution:

Height gate	Channel # 1		Channel # 2		Channel # 3		Channel #4	
(ft)	Q (cfs)	Y (ft)	Q (cfs)	Y (ft)	Q (cfs)	Y (ft)	Q (cfs)	Y (ft)
.10	541.5	4.307	311.0	4.197	4.9	4.884	225.7	4.285
.20	545.8	4.286	310.6	4.194	9.8	4.880	225.4	4.282
.30	550.0	4.265	310.2	4.191	14.8	4.876	225.0	4.279
.40	554.2	4.242	309.7	4.187	19.8	4.871	224.7	4.275
.50	558.5	4.218	309.2	4.184	24.9	4.865	224.3	4.272
.60	562.7	4.192	308.7	4.180	30.0	4.859	223.9	4.268
.70	566.9	4.165	308.2	4.176	35.1	4.852	223.5	4.264
.80	571.0	4.135	307.6	4.172	40.3	4.845	223.1	4.260
.90	575.1	4.104	307.0	4.168	45.5	4.836	222.6	4.255
1.00	579.2	4.069	306.4	4.163	50.7	4.827	222.1	4.250
1.10	583.1	4.032	305.6	4.157	55.9	4.817	221.5	4.245
1.20	586.9	3.990	304.8	4.151	61.2	4.806	220.9	4.239
1.30	590.5	3.943	303.9	4.145	66.4	4.793	220.2	4.232
1.40	593.9	3.889	302.9	4.137	71.6	4.778	219.4	4.224
1.50	596.9	3.824	301.6	4.127	76.9	4.761	218.4	4.214
1.60	599.1	3.737	299.9	4.115	82.0	4.741	217.2	4.202
1.70	598.8	3.579	296.9	4.092	87.0	4.707	214.8	4.179

100.

A reservoir with a water surface elevation 4.8 ft above the bottom of a trapezoidal channel with b_1 =20 ft, m_1 = 1, n_1 = .014, supplies water to a branched channel system as shown, in which this upstream channel divides into three channels; channels 2 and 3 consisting of pipes with D_2 = 5 ft, n_2 = .013, S_{o2} = .0006 and D_3 = 6 ft, n_2 = .013, S_{o3} = .0005. Channel 4 is trapezoidal with b_4 = 8 ft, m_4 = 1, n_4 = .015. This channel contains a gate that produces a depth of 1.7 ft downstream from it. The entrance loss coefficient is K_e = .15, and the loss coefficients to the 3 branched channels are all 0.1. The bottoms of the pipes for channels 2 and 3 are $\Delta z_{12} = \Delta z_{13}$ = 0.2 ft above the bottom of channel 1. Solve for the flowrates and depths in these four channels.

Wanted: Solve the 4 channel system in which Channels 2 and 3 are pipes.

Solution: There is the continuity equation between the four channels available. Also since it is assumed that uniform flow will occur in Channels 2 and 3, there are two additional Mannings Equations for these Channel. The energy equation can be written across the gate in Channel 4 and also at the entrance to Channel 1, making a total of 7 equations. The unknowns are: 4 flow rates Q_1, Q_2, Q_3 and Q_4 and 4 depths Y_1, Y_2, Y_3 and Y_4. The available equations are:

$F_1 = Q_1 - Q_2 - Q_3 - Q_4 = 0$

$F_2 = n_2Q_2P_2^{.666667} - C_uA_2^{1.666667}S_{o2}^{1/2} = 0$

$F_3 = n_3Q_3P_3^{.666667} - C_uA_3^{1.666667}S_{o3}^{1/2} = 0$

$F_4 = Y_4 + (Q_4/A_4)^2/(2g) - Y_{d4} - (1+K_L)(Q_4/A_{d4})^2/(2g) = 0$

$F_5 = Y_1 + (Q_1/A_1)^2/(2g) - Y_2 - (1+K_L)(Q_2/A_2)^2/(2g) = 0$

$F_6 = Y_1 + (Q_1/A_1)^2/(2g) - Y_3 - (1+K_L)(Q_3/A_3)^2/(2g) = 0$

$F_7 = Y_1 + (Q_1/A_1)^2/(2g) - Y_4 - (1+K_L)(Q_4/A_4)^2/(2g) = 0$

$F_8 = H - Y_1 - (1+K_e)(Q_1/A_1)^2/(2g) = 0$

This problem can be solved using the Program BRANCHCH

Input (file: BRANCHCH.DA1)

```
Input BRANCHCH.DA1
1
2
2
1
4.8 381 4.6 20 1 .014 .15
.0006 67 4.6 5 .013 .1 .2
.0005 84 4.58 6 .013 .1 .2
.0012 230 4.65 8 1 .015 .1 0
4 1.7
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4

Output from Program BRANCHCH
 For Channel 1
 H =   4.800
 Q = 370.473
 Y =   4.610
 b =  20.000
 m =   1.000
 n =    .014
 For Channel 2
 S = .000600
 Q =  66.732
 Y =   4.343
 D =   5.000
 n =    .013
 For Channel 3
 S = .000500
 Q =  82.482
 Y =   4.332
 D =   6.000
 n =    .013
 For Channel 4
```

```
 S = .001200
 Q = 221.259
 Y =   4.541
 b =   8.000
 m =   1.000
 n =    .015
```

The following MathCAD model solves the Problem.
<u>PRB2 100.MCD</u>

$H := 4.8 \quad Ke := .15 \quad b1 := 20 \quad m1 := 1 \quad n1 := .014 \quad \Delta z12 := .2 \quad \Delta z13 := .2 \quad KL := .1 \quad Cu := 1.486 \quad D2 := 5 \quad n2 := .013 \quad So2 := .0006$

$D3 := 6 \quad n3 := .013 \quad So3 := .0005 \quad b4 := 8 \quad m4 := 1 \quad n4 := .015 \quad So4 := .0012 \quad Yd4 := 1.7 \quad Y1 := 4.5 \quad Y2 := 4.3 \quad Y3 := 4.3 \quad Y4 := 4.5$

$Q1 := 350 \quad Q2 := 70 \quad Q3 := 80 \quad Q4 := 200 \quad A1 := (b1 + m1 \cdot Y1) \cdot Y1 \quad \beta 2 := \operatorname{acos}\left(1 - 2 \cdot \frac{Y2}{D2}\right) \quad A2 := \frac{D2^2}{4} \cdot (\beta 2 - \cos(\beta 2) \cdot \sin(\beta 2))$

$\beta 3 := \operatorname{acos}\left(1 - 2 \cdot \frac{Y3}{D3}\right) \quad A3 := \frac{D3^2}{4} \cdot (\beta 3 - \cos(\beta 3) \cdot \sin(\beta 3)) \quad Ad4 := (b4 + m4 \cdot Yd4) \cdot Yd4 \quad A4 := (b4 + m4 \cdot Y4) \cdot Y4$

$P2 := D2 \cdot \beta 2 \quad P3 := D3 \cdot \beta 3 \quad g := 32.2 \quad g2 := 2 \cdot g \quad Ke := .15 \quad KL := .1 \quad \Delta z12 := .2 \quad \Delta z13 := .2$

Given $\quad A1 = (b1 + m1 \cdot Y1) \cdot Y1 \quad \cos(\beta 2) = 1 - 2 \cdot \frac{Y2}{D2} \quad \cos(\beta 3) = 1 - 2 \cdot \frac{Y3}{D3} \quad P3 = D3 \cdot \beta 3 \quad P2 = D2 \cdot \beta 2$

$A2 = \frac{D2^2}{4} \cdot (\beta 2 - \cos(\beta 2) \cdot \sin(\beta 2)) \quad A3 = \frac{D3^2}{4} \cdot (\beta 3 - \cos(\beta 3) \cdot \sin(\beta 3)) \quad A4 = (b4 + m4 \cdot Y4) \cdot Y4 \quad Q1 - Q2 - Q3 - Q4 = 0$

$n2 \cdot Q2 \cdot P2^{.6666667} - Cu \cdot A2^{1.666667} \cdot \sqrt{So2} = 0 \quad n3 \cdot Q3 \cdot P3^{.6666667} - Cu \cdot A3^{1.666667} \cdot \sqrt{So3} = 0$

$H = Y1 + (1 + Ke) \cdot \frac{\left(\frac{Q1}{A1}\right)^2}{g2} \quad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y2 + \frac{\left(\frac{Q2}{A2}\right)^2}{g2} \cdot (1 + KL) + \Delta z12 \quad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y3 + \frac{\left(\frac{Q3}{A3}\right)^2}{g2} \cdot (1 + KL) + \Delta z13$

$Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y4 + \frac{\left(\frac{Q4}{A4}\right)^2}{g2} \cdot (1 + KL) \quad Y4 + \frac{\left(\frac{Q4}{A4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{Ad4}\right)^2}{g2}$

Find(Q1, Q2, Q3, Q4, Y1, Y2, Y3, Y4, A1, A2, A3, A4, P2, P3, β2, β3) =

	0
0	380.167
1	66.718
2	82.447
3	231.002
4	4.598
5	4.342
6	4.33
7	4.483
8	113.11
9	18.107
10	21.85
11	55.954
12	11.995
13	12.182
14	2.399

The following is a TK-Solver model to solve this same problem. The use of TK-Solver is easier than MathCAD in that: (1) the variables are automatically placed in the variable sheet when they are first introduced into an equation, and (2) it is not necessary to provide guesses for variables that can be solve explicitly. Therefore, no guesses are needed for the areas, angles, or wetted perimeters. Guesses are only needed for the four depths and the four flow rates. (The reason for the small differences in the solution is that the bottom slope of Channel 3 has been used as S_{o3}=.0006, as shown on the figure rather than S_{o3}=.0005 as stated in the problem statement.)

TK-Solver Model PB2 100.TK

```
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit----- Comment---------
             Y1      4.5904136
             Y2      4.3407539
             Y3      4.2820613
             Y4      4.4814392
             Q1      386.717
             Q2      66.707483
             Q3      89.046772
             Q4      230.96274
             A1      112.88017
   20        b1
   1         m1
             Beta2   2.3983845
   5         D2
             Beta3   2.0123601
   6         D3
             A2      18.103785
             A3      21.588516
             P2      11.991923
             P3      12.074161
   .013      n2
   1.486     Cu
   .0006     So2
   .013      n3
   .0006     So3
   4.8       H
   .15       Ke
   64.4      g2
             A4      55.934811
   8         b4
   1         m4
   .1        KL
   .2        Dz
   1.7       Yd4

==================== RULE SHEET ========================
S Rule---------------------------------------------------
  A1=(b1+m1*Y1)*Y1
  Beta2=acos(1-2*Y2/D2)
  Beta3=acos(1-2*Y3/D3)
  A2=.25*D2^2*(Beta2-cos(Beta2)*sin(Beta2))
  A3=.25*D3^2*(Beta3-cos(Beta3)*sin(Beta3))
  P2=Beta2*D2
  P3=Beta3*D3
  A4=(b4+m4*Y4)*Y4
  Q1=Q2+Q3+Q4
  n2*Q2*P2^.6666667=Cu*A2^1.666667*sqrt(So2)
  n3*Q3*P3^.6666667=Cu*A3^1.666667*sqrt(So3)
  H=Y1+(1+Ke)*(Q1/A1)^2/g2
  Y1+(Q1/A1)^2/g2=Y2+(1+KL)*(Q2/A2)^2/g2+Dz
  Y1+(Q1/A1)^2/g2=Y3+(1+KL)*(Q3/A3)^2/g2+Dz
  Y1+(Q1/A1)^2/g2=Y4+(1+KL)*(Q4/A4)^2/g2
  Y4+(Q4/A4)^2/g2=Yd4+(Q4/((b4+m4*Yd4)*Yd4))^2/g2
```

101.

Example Problem 19 gave the answers that were obtained by solving the problem using TK-Solver, but did not actually provide the sheets of this solution. Using TK-Solver, MathCAD, or other software capable of solving a system of equations verify the given answers. Obtain these answers: (1) by using the 7 equations given in this example problem (You may add areas and wetted perimeters to the list of unknown variables if you want) but specify the value of Q_1 = 531.4, and (2) Also include Q_1 as an unknown and include the additional equations needed.

Wanted: Solve the four branched channel system.

```
Model PRB19.TK
=============== VARIABLE SHEET ================
St Input──── Name─── Output─── Unit───── Comment
             H       4.0357674
   531.41    Q1
             Q2      219.61028
             Q3      155.89986
             Q4      155.89986
             Y2      3.4673228
             Y3      3.5404031
             Y4      3.5404031
             A2      39.76091
             A3      30.23647
             A4      30.23647
             P2      17.80707
             P3      15.013772
             P4      15.013772
   8         b2
   1         m2
   5         b3
   1         m3
   5         b4
   1         m4
   .013      n
   .0008     So
   .2        KL
   32.2      g

=============== RULE SHEET ===============
S Rule────────────────────────────────────
* A2=(b2+m2*Y2)*Y2
* P2=b2+2.*Y2*sqrt(m2*m2+1)
* A3=(b3+m3*Y3)*Y3
* P3=b3+2.*Y3*sqrt(m3*m3+1)
* A4=(b4+m4*Y4)*Y4
* P4=b4+2.*Y4*sqrt(m4*m4+1)
* Q1=Q2+Q4+Q4
* Q2=1.486/n*A2*(A2/P2)^.66666667*sqrt(So)
* Q3=1.486/n*A3*(A3/P3)^.66666667*sqrt(So)
* Q4=1.486/n*A4*(A4/P4)^.66666667*sqrt(So)
* H=Y2+(1+KL)*Q2^2/(2*g*A2^2)
* H=Y3+(1+KL)*Q3^2/(2*g*A3^2)
* H=Y4+(1+KL)*Q4^2/(2*g*A4^2)
```

Critical flow and Energy at reservoir added to solve Q_1 (Model PRB19A.TK)

```
=============== VARIABLE SHEET =========
St Input──── Name─── Output─── Unit─────
             H       4.0470267
             Yc      3.5092455
             Q1      534.18314
             Q2      220.71631
             Q3      156.73342
             Q4      156.73342
             Y2      3.4769934
             Y3      3.5502844
```

```
             Y4      3.5502844
             A1      59.722064
             A2      39.905431
             A3      30.355941
             A4      30.355941
             P2      17.834423
             P3      15.041721
             P4      15.041721
   8         b2
   1         m2
   5         b3
   1         m3
   5         b4
   1         m4
   .013      n
   .0008     So
   .2        KL
   32.2      g
   10        b1
   2         m1

═══════════════ RULE SHEET ═══════════════
S Rule──────────────────────────────────────────
* A2=(b2+m2*Y2)*Y2
* P2=b2+2.*Y2*sqrt(m2*m2+1)
* A3=(b3+m3*Y3)*Y3
* P3=b3+2.*Y3*sqrt(m3*m3+1)
* A4=(b4+m4*Y4)*Y4
* P4=b4+2.*Y4*sqrt(m4*m4+1)
* Q1=Q2+Q4+Q4
* Q2=1.486/n*A2*(A2/P2)^.66666667*sqrt(So)
* Q3=1.486/n*A3*(A3/P3)^.66666667*sqrt(So)
* Q4=1.486/n*A4*(A4/P4)^.66666667*sqrt(So)
* H=Y2+(1+KL)*Q2^2/(2*g*A2^2)
* H=Y3+(1+KL)*Q3^2/(2*g*A3^2)
* H=Y4+(1+KL)*Q4^2/(2*g*A4^2)
* A1=(b1+m1*Yc)*Yc
* Yc+(1+KL)*.5*A1/(b1+2.*Yc*m1)=5
* Q1^2*(b1+2.*Yc*m1)=g*A1^3
```

The MathCAD model PRB19.MCD is given below to solve the problem when Q_1=531.41 cfs

Variables

$Q1 := 531.41 \quad b1 := 10 \quad m1 := 2 \quad n := .013 \quad KL := .2 \quad b2 := 8 \quad m2 := 1 \quad So := .0008 \quad b3 := 5 \quad m3 := 1 \quad b4 := 5 \quad m4 := 1$

Guesses

$Q2 := 217 \quad Q3 := 157 \quad Q4 := 157 \quad Y1 := 3.88 \quad Y2 := 4 \quad Y3 := 4 \quad Y4 := 4 \quad H := 4.7 \quad A1 := 80 \quad P1 := 20 \quad A2 := 50 \quad P2 := 16$

$A3 := 30 \quad P3 := 12 \quad A4 := 30 \quad P4 := 12$

Given

$Q1 = Q2 + Q3 + Q4 \qquad A2 = (b2 + m2 \cdot Y2) \cdot Y2 \qquad H = Y2 + \left(\frac{1+KL}{64.4}\right) \cdot \left(\frac{Q2}{A2}\right)^2 \qquad P2 = b2 + 2 \cdot Y2 \cdot \sqrt{m2 \cdot m2 + 1}$

$Q2 = \frac{1.486}{n} \cdot A2 \cdot \left(\frac{A2}{P2}\right)^{.66666667} \cdot \sqrt{So} \qquad A3 = (b3 + m3 \cdot Y3) \cdot Y3 \qquad H = Y3 + \left(\frac{1+KL}{64.4}\right) \cdot \left(\frac{Q3}{A3}\right)^2 \qquad P3 = b3 + 2 \cdot Y3 \cdot \sqrt{m3 \cdot m3 + 1}$

$Q3 = \frac{1.486}{n} \cdot A3 \cdot \left(\frac{A3}{P3}\right)^{.66666667} \cdot \sqrt{So} \qquad A4 = (b4 + m4 \cdot Y4) \cdot Y4 \qquad H = Y4 + \left(\frac{1+KL}{64.4}\right) \cdot \left(\frac{Q4}{A4}\right)^2 \qquad P4 = b4 + 2 \cdot Y4 \cdot \sqrt{m4 \cdot m4 + 1}$

$Q4 = \frac{1.486}{n} \cdot A4 \cdot \left(\frac{A4}{P4}\right)^{.66666667} \cdot \sqrt{So}$

$\text{Find}(H, Q2, Y2, Q3, Y3, Q4, Y4, A2, P2, A3, P3, A4, P4) =$

	0
0	4.036
1	219.61
2	3.467
3	155.9
4	3.54
5	155.9
6	3.54
7	39.761
8	17.807
9	30.236
10	15.014
11	30.236
12	15.014

In this MathCAD model when Critical Flow and Energy Equations are added to solve for Q_1 and Y_c the following results: (PRB19A.MCD)

Variables

$b1 := 10 \quad m1 := 2 \quad n := .013 \quad KL := .2 \quad b2 := 8 \quad m2 := 1 \quad So := .0008 \quad b3 := 5 \quad m3 := 1 \quad b4 := 5 \quad m4 := 1$

Guesses

$Q2 := 217 \quad Q3 := 157 \quad Q4 := 157 \quad Y1 := 3.88 \quad Y2 := 4 \quad Y3 := 4 \quad Y4 := 4 \quad H := 4.7 \quad A1 := 80 \quad P1 := 20 \quad A2 := 50 \quad P2 := 16$

$A3 := 30 \quad P3 := 12 \quad A4 := 30 \quad P4 := 12 \quad Yc := 3 \quad A1 := 80 \quad Q1 := 531.41$

Given

$$Q1 = Q2 + Q3 + Q4 \qquad A2 = (b2 + m2 \cdot Y2) \cdot Y2 \qquad H = Y2 + \left(\frac{1+KL}{64.4}\right) \cdot \left(\frac{Q2}{A2}\right)^2 \qquad P2 = b2 + 2 \cdot Y2 \cdot \sqrt{m2 \cdot m2 + 1}$$

$$Q2 = \frac{1.486}{n} \cdot A2 \cdot \left(\frac{A2}{P2}\right)^{.66666667} \cdot \sqrt{So} \qquad A3 = (b3 + m3 \cdot Y3) \cdot Y3 \qquad H = Y3 + \left(\frac{1+KL}{64.4}\right) \cdot \left(\frac{Q3}{A3}\right)^2 \qquad P3 = b3 + 2 \cdot Y3 \cdot \sqrt{m3 \cdot m3 + 1}$$

$$Q3 = \frac{1.486}{n} \cdot A3 \cdot \left(\frac{A3}{P3}\right)^{.66666667} \cdot \sqrt{So} \qquad A4 = (b4 + m4 \cdot Y4) \cdot Y4 \qquad H = Y4 + \left(\frac{1+KL}{64.4}\right) \cdot \left(\frac{Q4}{A4}\right)^2 \qquad P4 = b4 + 2 \cdot Y4 \cdot \sqrt{m4 \cdot m4 + 1}$$

$$Q4 = \frac{1.486}{n} \cdot A4 \cdot \left(\frac{A4}{P4}\right)^{.66666667} \cdot \sqrt{So} \qquad A1 = (b1 + m1 \cdot Yc) \cdot Yc \qquad Yc + (1+KL) \cdot \frac{.5 \cdot A1}{b1 + 2 \cdot Yc \cdot m1} = 5$$

$$Q1^2 \cdot (b1 + 2. \cdot Yc \cdot m1) = 32.2 \cdot A1^3$$

Find(H, Q1, Q2, Y2, Q3, Y3, Q4, Y4, A1, Yc, A2, P2, A3, P3, A4, P4) =

	0
0	4.047
1	534.183
2	220.716
3	3.477
4	156.733
5	3.55
6	156.733
7	3.55
8	59.722
9	3.509
10	39.905
11	17.834
12	30.356
13	15.042
14	30.356

102.

Water enters a rectangular channel with a 20 ft bottom width from a reservoir whose water surface is 9 ft above the channel bottom. The channel has a Mannings roughness coefficient n = .014. (a) If the slope of the channel bottom is S_o = .055 what is the flowrate into the channel? (b) If the slope of the channel bottom is S_o = .0005 what is the flowrate into the channel? (c) A vertical gate is place a short distance downstream from the channel entrance with its tip 2 ft above the channel bottom. The gates contraction coefficient is C_c = .6. Now what is the flowrate?

Given: rectangular channel with b = 20 ft, and H = 9 ft.
Wanted: Q if (a) S_o=.055 (Steep), (b) S_o=.0005, n=.014, and (c) has gate with tip 2 ft above bottom and contraction coefficient equal .6

(a) Critical flow will occur at entrance to $Y_c=(2/3)E_c = (2/3)9 = 6$ ft

$q = (gY_c)^{1/3} = 83.40$ cfs/ft, $Q = qb = 1,668$ cfs ←

(b) Solve Energy and Uniform flow equations simultaneously

$9 = Y_o + (Q/A)^2/(2g)$

$Q = (1.486/n)A(A/P)^{2/3}\sqrt{S_o}$

Solution give: $Y_o = 8.341$ ft, $Q = 1083.8$ cfs ←

(c) Energy across gate

$9 = Y + \{Q/(bY)\}^2/(2g) = 1.2 + Q^2/(64.4x(24)^2) = 537.9$ cfs ←

103.

A trapezoidal channel with b_1 = 5 m, m_1 = 1.2, and S_{o1} = .0008 changes to a rectangular channel with a bottom width of b_2 = 4.5 m and a bottom slope S_{o2} = .015. Both channel have n = .014 and the bottom elevation remains constant through the transition. For a flowrate Q = 50 m^3/s what will the depths be immediately upstream and at the end of the transition? What is the change in water surface through the transition? As a (b) part to this problem, what will the depths be if the bottom drops by 1 m through the transition, i.e. Δz = - 1.

Given: A transition between a trapezoidal and rectangular channel, with Q = 50 m^3/s
Wanted: (a) Depths upstream and downstream of transition and water surface change.
(b) if bottom drops 1 m what are depths, etc.

Solution:
Uniform Flow Conditions and upstream and downstream channels

Y_{o1} = 2.391 m

E_{o1} = 2.751 m

F_{ro1} = .641

Y_{o2} = 2.391 m

E_{o2} = 4.607 m

F_{ro2} = 2.14

q_2 = 50/4.5 = 11.11 m^2/s, Y_c=2.326 m

E_c =3.489 >E_{o1}

So upstream depth is above normal depth.

$E_1 = E_c$ = 3.489 → Y_1 = 3.349 m ←

and water surface drops 3.35-2.33=1.02 m ←

(b) E_1 = 3.489 - 1 = 2.489 Won't solve

So critical flow in trapezoidal section controls

$Q^2T/(gA^3) = 1$ → Y_{c1} = 1.855 m ← E_{c1} = 2.564 m

Depth at beginning of transition will be 1.855 m so

E_2 = 2.564 + 1 = 3.564 m → Y_2 = 0.839 m ← below normal depth

104.

Give the equations that need to be solved to determine the flowrate in each of the 4 channels shown in the sketch below. The upstream main channel is short and is supplied by a reservoir with a water surface elevation of H = 5 ft. The bottom elevation of channel 2 is 0.3 ft above the bottom of the main channel and the other channel's bottoms are at the same elevation. Channel 4 contains a sluice gate with a contraction coefficient, $C_c = 0.6$, and its gate's position above the channel bottom is $Y_G = 1.0$ ft. Ignore all minor losses

Wanted: Solve a four channel system in which an upstream channel branches into 3 downstream channels.

Solution:

Unknown Variables: $Q_1, Y_1, Q_2, Y_2, Q_3, Y_3, Q_4, Y_4$ (8)

Equations: (8 needed)

$$F_1 = Q_1 - Q_2 - Q_3 - Q_4 = 0 \quad (1)$$

$$F_2 = H - Y_1 - (1+K_e)(Q/A)_1^2/(2g) = 0 \quad (2)$$

$$F_3 = Y_1 + (Q/A)_1^2/(2g) - Y_2 - (1+K_L)(Q/A)_2^2/(2g) - \Delta z_{12} = 0 \quad (3)$$

$$F_4 = Y_1 + (Q/A)_1^2/(2g) - Y_3 - (1+K_L)(Q/A)_3^2/(2g) = 0 \quad (4)$$

$$F_5 = Y_1 + (Q/A)_1^2/(2g) - Y_{u4} - (1+K_L)(Q/A)_{u4}^2/(2g) = 0 \quad (5)$$

$$F_6 = Y_{u4} + (Q/A)_{u4}^2/(2g) - Y_{d4} - (1+K_L)(Q/A)_{d4}^2/(2g) = 0 \quad (6)$$

$$F_7 = n_2 Q_2 P_2^{2/3} - C_u A_2^{5/3} S_{o2}^{1/2} = 0 \quad (7)$$

$$F_8 = n_3 Q_3 P_3^{2/3} - C_u A_3^{5/3} S_{o3}^{1/2} = 0 \quad (8)$$

Below the solution is obtained by using the Program BRANCHCH and also a TK-Solver Model.

Input for Program BRANCHCH (File: PRB1_76.DAT)

```
1
1
1
1
5 500 4.9 15 2 .013 0/
.001 180 4.56 6 1 .013 0 .3
.0007 220 4.86 8 1.5 .013 0. 0.
.0004 100 4.86 10 0 .0013 0 0.
4 .6
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4
```

Output:

```
 For Channel 1
 H =   5.000
 Q = 748.354
 Y =   3.901
 b =  15.000
 m =   2.000
```

```
 n =    .013
 For Channel 2
 S = .001000
 Q = 261.105
 Y =   4.070
 b =   6.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000700
 Q = 386.249
 Y =   4.460
 b =   8.000
 m =   1.500
 n =    .013
 For Channel 4
 S = .000400
 Q = 101.000
 Y =   4.935
 b =  10.000
 m =    .000
 n =    .001
```

```
TK-Solver Model PRB1_76.TK
================ VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment---------------------------------
              Q1      748.35433           Solves Prob. 104 (Chap. 2) 4 branched c
              Y1      3.900655            with gate in 4th channel
              Q2      261.10509
              Y2      4.0695981
              Q3      386.24944
              Y3      4.4604935
              Q4      100.9998
              Y4      4.9349589
   5          H
   15         b1
   2          m1
   64.4       g2
   6          b2
   1          m2
   .3         z12
   8          b3
   1.5        m3
   10         b4
   .6         Yd4
   .013       n2
   .001       So2
   .013       n3
   .0007      So3

==================== RULE SHEET ========================
S Rule---------------------------------------------------------
* Q1-Q2-Q3-Q4=0
* H-Y1-(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(Q3/((b3+m3*Y3)*Y3))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
* Yd4+(Q4/(b4*Yd4))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-1.486*((b2+m2*Y2)*Y2)^1.666667*sqrt(So
* n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-1.486*((b3+m3*Y3)*Y3)^1.666667*sqrt(So
```

The Problem is also solved using the following MathCAD model.

PRB2_104.MCD

$H := 5$ $b1 := 15$ $m1 := 2$ $\Delta z12 := .3$ $Cu := 1.486$ $b2 := 6$ $m2 := 1$ $n2 := .013$ $So2 := .001$ $b3 := 8$ $m3 := 1.5$ $n3 := .013$

$So3 := .0007$ $b4 := 10$ $Yd4 := 0.6$ $Ad4 := b4 \cdot Yd4$ $g2 := 64.4$

$Y1 := 4.$ $Y2 := 4.5$ $Y3 := 4.5$ $Y4 := 4.8$ $Q1 := 750$ $Q2 := 250$ $Q3 := 400$ $Q4 := 100$ $A1 := (b1 + m1 \cdot Y1) \cdot Y1$

$A2 := (b2 + m2 \cdot Y2) \cdot Y2 \quad A3 := (b3 + m3 \cdot Y3) \cdot Y3 \quad A4 := b4 \cdot Y4 \quad P2 := b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1} \quad P3 := b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}$

Given $\quad A1 = (b1 + m1 \cdot Y1) \cdot Y1 \quad A2 = (b2 + m2 \cdot Y2) \cdot Y2 \quad A3 = (b3 + m3 \cdot Y3) \cdot Y3 \quad A4 = b4 \cdot Y4$

$P2 = b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1} \quad P3 = b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1} \quad Q1 - Q2 - Q3 - Q4 = 0 \quad n2 \cdot Q2 \cdot P2^{.6666667} - Cu \cdot A2^{1.666667} \cdot \sqrt{So2} = 0$

$$n3 \cdot Q3 \cdot P3^{.6666667} - Cu \cdot A3^{1.666667} \cdot \sqrt{So3} = 0 \quad H = Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} \quad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y2 + \frac{\left(\frac{Q2}{A2}\right)^2}{g2} + \Delta z12$$

$$Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y3 + \frac{\left(\frac{Q3}{A3}\right)^2}{g2} \quad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y4 + \frac{\left(\frac{Q4}{A4}\right)^2}{g2} \quad Y4 + \frac{\left(\frac{Q4}{A4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{Ad4}\right)^2}{g2}$$

$\text{Find}(Q1, Y1, Q2, Y2, Q3, Y3, Q4, Y4, A1, A2, P2, A3, P3, A4) =$

	0
0	748.354
1	3.901
2	261.105
3	4.07
4	386.249
5	4.46
6	101
7	4.935
8	88.94
9	40.979
10	17.511
11	65.528
12	24.083
13	49.35

From the above solutions it is rather interesting that the depth in Channel 1 is considerable less than in the three downstream channels, i.e. the velocity head in Channel 1 is considerably larger than in the other channels. In other word the capacity of the upstream channel is less than the combined capacities of the downstream channels. Below the solution (using the MathCAD model) is given with the width of the upstream channel increased from 15 ft to 20 ft. Notice that the flow rates have not changed, but the depth in Channel 1 has increased from 3.901 ft to 4.485 ft, more in line with the depths in the downstream channels. Also the depths in the downstream channels have not changed. Why is this so? It doen't take much thinking to realize that since the entrance loss coefficient K_e=0 and the loss coefficients K_L=0 between Channel 1 and the downstream channels is zero, and the assumption is that Channel 1 is very short so no loss occurs in it, and therefore all channel have the same specific energy, i.e. H the reservoir water surface elevation. Thus this problem could be solved as separate problems. The flow rate and depth in Channel 2 can be obtained by solving Mannings equation simultaneously with it specifice energy equal to H. The same type of solution will provide the flow rate and depth in Channel 3. For Channel 4 the flow rate can be determined by equating the specific energy downstream from the gate to H and solve for the flow rate, and thereafter solve for the depth upstream from the gate. Using Program E_UN1 (given in the text of this chapter), or Program Channel, to solve the energy and Mannings equations simultaneously in a piecemeal fashon gives the following:

Channel # 2 (H=5-0.3=4.7 ft) Channel # 3 (H=5 ft)

Q_2=261.105 cfs, Y_2=4.070 ft Q_3=386.249 cfs, Y_3=4.460 ft

Energy across the gate in Channel # 4 gives: $Y_{d4} + q^2/(2gY_{d4}^2) = 5$, or $0.6 + q^2/(64.4 \times 0.6^2)$, q=10.0999 cfs/ft, or Q_4 = 101.000 cfs and Y_4 = 4.935 ft. Adding these flow rates gives Q_1=261.105+386.249+101.000 = 748.354 cfs, and the upstream depth can now be determined from solving the energy equation $5 = Y_1 + (Q/A)_1^2/(64.4)$ or Y_1 =3.901 ft.

MathCAD solution with the width of upstream channel increased to 20 ft.

$H := 5$ $b1 := 20$ $m1 := 2$ $\Delta z12 := .3$ $Cu := 1.486$ $b2 := 6$ $m2 := 1$ $n2 := .013$ $So2 := .001$ $b3 := 8$ $m3 := 1.5$ $n3 := .013$ $So3 := .0007$ $b4 := 10$ $Yd4 := 0.6$ $Ad4 := b4 \cdot Yd4$ $g2 := 64.4$ $Y1 := 4.$ $Y2 := 4.5$ $Y3 := 4.5$ $Y4 := 4.8$ $Q1 := 750$ $Q2 := 250$ $Q3 := 400$ $Q4 := 100$ $A1 := (b1 + m1 \cdot Y1) \cdot Y1$ $A2 := (b2 + m2 \cdot Y2) \cdot Y2$ $A3 := (b3 + m3 \cdot Y3) \cdot Y3$ $A4 := b4 \cdot Y4$

$P2 := b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}$ $P3 := b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}$

Given $A1 = (b1 + m1 \cdot Y1) \cdot Y1$ $A2 = (b2 + m2 \cdot Y2) \cdot Y2$ $A3 = (b3 + m3 \cdot Y3) \cdot Y3$ $A4 = b4 \cdot Y4$

$P2 = b2 + 2 \cdot Y2 \cdot \sqrt{m2^2 + 1}$ $P3 = b3 + 2 \cdot Y3 \cdot \sqrt{m3^2 + 1}$ $Q1 - Q2 - Q3 - Q4 = 0$ $n2 \cdot Q2 \cdot P2^{.6666667} - Cu \cdot A2^{1.666667} \cdot \sqrt{So2} = 0$

$n3 \cdot Q3 \cdot P3^{.6666667} - Cu \cdot A3^{1.666667} \cdot \sqrt{So3} = 0$ $H = Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2}$ $Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y2 + \frac{\left(\frac{Q2}{A2}\right)^2}{g2} + \Delta z12$

$Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y3 + \frac{\left(\frac{Q3}{A3}\right)^2}{g2}$ $Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{g2} = Y4 + \frac{\left(\frac{Q4}{A4}\right)^2}{g2}$ $Y4 + \frac{\left(\frac{Q4}{A4}\right)^2}{g2} = Yd4 + \frac{\left(\frac{Q4}{Ad4}\right)^2}{g2}$

Find(Q1, Y1, Q2, Y2, Q3, Y3, Q4, Y4, A1, A2, P2, A3, P3, A4) =

	0
0	748.354
1	4.485
2	261.105
3	4.07
4	386.249
5	4.46
6	101
7	4.935
8	129.924
9	40.979
10	17.511
11	65.528
12	24.083
13	49.35

105.

Write a computer program, or use a software package such as MathCAD, or TK-SOLVER to obtain the solution to Example Problem 19.

The following is a TK-Solver model designed to solve Example Problem 19. Note in this model that the critical flow rate is specified for Channel 1, and the head H is solved as one of the unknowns. This H is the head at the entrance to the 3 downstream channels and will generally be less than the specific energy E_o in Channel 1. Note the energy equations for Channels 2, 3 and 4 now do include the energy at the reservoir, and this head is H. An alternative would be to eliminate H and equate E_2=E_3 and E_2=E_3. (See the TK-Solver model that solves the next problem with S_o=.0003 and .0002 for when critical flow in CHannel 1 does not restrict the flow.) This model is named EP2_19.TK

```
==================== VARIABLE SHEET ============
St Input──── Name─── Output─── Unit───── Comment
   531.4     Q1
             Q2      219.6063
             Q3      155.89685
             Q4      155.89685
             Y2      3.4672858
             Y3      3.5403655
             Y4      3.5403655
             H       4.0357255
   64.4      g2
   10        b1
   2         m1
   8         b2
   1         m2
   5         b3
   1         m3
   5         b4
   1         m4
   0         z12
   0         z13
   0         z14
   .013      n2
   .0008     So2
   .013      n3
   .0008     So3
   .013      n4
   .0008     So4
   .2        K1
   .2        K2
   .2        K3
   .2        K4
   1.486     Cman

==================== RULE SHEET =========================
S Rule──────────────────────────────────────────────────
* Q1-Q2-Q3-Q4=0
* H-Y2-(1+K2)*(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
* H-Y3-(1+K3)*(Q3/((b3+m3*Y3)*Y3))^2/g2-z13=0
* H-Y4-(1+K4)*(Q4/((b4+m4*Y4)*Y4))^2/g2-z14=0
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-Cman*((b2+m2*Y2)*Y2)^1.666667*sqrt(So2
* n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-Cman*((b3+m3*Y3)*Y3)^1.666667*sqrt(So3
* n4*Q4*(b4+2*Y4*sqrt(m4*m4+1))^.6666667-Cman*((b4+m4*Y4)*Y4)^1.666667*sqrt(So4
```

106.

Solve the same problem as given as Example Problem 19 with the exception that the bottom slopes of the 3 channels downstream from the branch are: $S_{o2} = S_{o3} = S_{o4} = .0005$.

Wanted: Decreasing the bottom slope of the downstream channels from .0008 to .0005 is not enough to prevent critical flow from occurring in Channel 1. Therefore the solution methodoly is the same as used in Example Problem 19 and the previous problem. The difference is that the depths in the three downstream channelS are larger, with H also larger, but less than 5 ft.

In solving this problem no depth has been computed for Channel 1, since it is not needed as part solution to the system of equations, since the energy equations have been written with H replacing $E_1=Y_1+(Q/A)_1^2/(2g)$. The depth Y_1 could be solve by a simultaneous solution of the energy equation from the reservoir with H=5 ft (and K_e=.2), and the critical flow equation to give Y_{c1}=3.509 ft with an associated energy of E_1=4.742 ft, which is less than the computed H=4.350 ft, i.e. the difference represents energy loss through a mild hydraulic jump as the flow in Channel 1 attempts to becomes supercritical and then must again become subcritical as it branches into the subcritical flows. into the 3 downstream channels.

```
TK-Solver Model EP2 106.TK
==================== VARIABLE SHEET ============
St Input----- Name---- Output---- Unit------ Comment
   531.4      Q1
              Q2       219.50138
              Q3       155.94931
              Q4       155.94931
              Y2       3.9452814
              Y3       3.9997797
              Y4       3.9997797
              H        4.3495045
   64.4       g2
   8          b2
   1          m2
   5          b3
   1          m3
   5          b4
   1          m4
   0          z12
   0          z13
   0          z14
   .013       n2
   .0005      So2
   .013       n3
   .0005      So3
   .013       n4
   .0005      So4
   .2         K1
   .2         K2
   .2         K3
   .2         K4
   1.486      Cman

==================== RULE SHEET ======================
S Rule-----------------------------------------------
  Q1-Q2-Q3-Q4=0
  H-Y2-(1+K2)*(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
  H-Y3-(1+K3)*(Q3/((b3+m3*Y3)*Y3))^2/g2-z13=0
  H-Y4-(1+K4)*(Q4/((b4+m4*Y4)*Y4))^2/g2-z14=0
  n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-Cman*((b2+m2*Y2)*Y2)^1.666667*sqrt(So2
  n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-Cman*((b3+m3*Y3)*Y3)^1.666667*sqrt(So3
  n4*Q4*(b4+2*Y4*sqrt(m4*m4+1))^.6666667-Cman*((b4+m4*Y4)*Y4)^1.666667*sqrt(So4
```

An alternative model that does include Channel 1 might be as given below, in which the local loss coefficient for Channel 1 is added to the list of knowns. This approach to the problem uses the energy equation at the reservoir, and then equations the energy in the three downstream channels to the energy in Channel 1. Thus this loss coefficient, that is solved as part of the solution, represents not only the head loss due to entering the channel from the reservoir but also that due to the flow attempting to go supercritical. The solution shows that K_{L1}= .2143. Should we attempt to use the model below to solve for H rather than K1 a solution would not be possible because of the incompatibility of the specifications. (Note in Chapter 4 we will account for losses due to fluid friction in the channel, i.e. they will be given lengths, and the gradually varied flow in the channels will be part of the solution process. In other word later we will more adequately handle what actually occurs.)

```
TK-Solver Model PB2 106A.TK
==================== VARIABLE SHEET ======================
St Input---- Name--- Output--- Unit---- Comment---------
   531.41    Q1                          Solve for local loss in Channel 1
             Y1      3.8715379
             Q2      218.72942
               Y2      4.5229443
               Q3      156.34029
               Y3      4.5613785
               Q4      156.34029
               Y4      4.5613785
    5          H
    10         b1
    2          m1
    64.4       g2
    8          b2
    1          m2
    0          z12
    0          z13
    0          z14
    5          b3
    1          m3
    .013       n2
    .0003      So2
    .013       n3
    .0003      So3
    5          b4
    .013       n4
    1          m4
    .0003      So4
               K1      .21433425
    .2         K2
    .2         K3
    .2         K4
    1.486      Cman

==================== RULE SHEET ========================
S Rule-------------------------------------------------
  Q1-Q2-Q3-Q4=0
  H-Y1-(1+K1)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(1+K2)*(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(1+K3)*(Q3/((b3+m3*Y3)*Y3))^2/g2-z13=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-(1+K4)*(Q4/((b4+m4*Y4)*Y4))^2/g2-z14=0
  n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-Cman*((b2+m2*Y2)*Y2)^1.666667*sqrt(So2
  n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-Cman*((b3+m3*Y3)*Y3)^1.666667*sqrt(So3
  n4*Q4*(b4+2*Y4*sqrt(m4*m4+1))^.6666667-Cman*((b4+m4*Y4)*Y4)^1.666667*sqrt(So4
```

The solution with the bottom slopes of the three downstream channels further deceased to .0004 is given below. Note that with this bottom slope that critical flow still governs since the computed H=4.533 ft is still less than 5.0 ft.

```
==================== VARIABLE SHEET ===================
St Input---- Name--- Output--- Unit---- Comment-------
   531.4     Q1
             Q2      219.23752
```

```
             Q3      156.08124
             Q4      156.08124
             Y2      4.1898387
             Y3      4.2367837
             Y4      4.2367837
             H       4.5331867
   64.4      g2
   10        b1
   2         m1
   8         b2
   1         m2
   5         b3
   1         m3
   5         b4
   1         m4
   0         z12
   0         z13
   0         z14
   .013      n2
   .0004     So2
   .013      n3
   .0004     So3
   .013      n4
   .0004     So4
   .2        K1
   .2        K2
   .2        K3
   .2        K4
   1.486     Cman
```

Using the MATLAB script "BRANCHCH.m" to solve this problem might give the following: (Note that the downstream channels have the reverse numbering, i.e. 2 is 4 and 4 is 2.

```
>> BRANCHCH
Give g = 32.2
Give number of channels 4
For Channel # 1 type is:(0=trap.,1=cir.) 0
b1 = 16
m1 = 1.5
H= 5
KL = .2
For Channel #2 give 0=trap,1=cir. 1
D2 = 8
n2 = .013
So2 = .0008
Dz2 = .2
KL2 = 0
For Channel #3 give 0=trap,1=cir. 0
b3 = 8
m3 = 1
n3 = .013
So3 = .001
Dz3 = 0
KL3 = 0
For Channel #4 give 0=trap,1=cir. 0
b4 = 8
m4 = 0
n4 = .013
So4 = .001
Dz4 = 0
KL4 = 0
Give number of gates = 1
Numbers must be consecutive
Channel No. containing gate # 1 = 4
This gates contraction coef. = 1
Its heigth above channel bottom = 1
Provide initial guesses
Q1 = 530
Y1 = 4.8
Q2 = 200
Y2 = 4.8
Q3 = 155
```

```
Y3 = 4.8
Q4 = 140
Y4 = 4.8
'    Q1       Y1        Q2       Y2       Q3       Y3       Q4       Y4
'   547.0    4.454     44.2    4.481    375.8    3.868    126.9    4.735
```

107.

Solve the same problem as given as Example Problem 19 with the exception that the bottom slopes of the 3 channels downstream from the branch are: $S_{o2} = S_{o3} = S_{o4} = .0003$. What are the flowrates and depths in the four channels when $S_{o2} = S_{o3} = S_{o4} = .0002$.

Solution: We will assume that having bottom slopes of .0003 for the downstream channels will now not result in critical flow in Channel 1. Therefore the energy equation at the reservoir is added, and the energy equations between the three downstream channel will be equated to the specific energy in Channel 1. This modified TK-Solver model is given below (File: PRB2_79.TK). Note that the solution when the bottom slopes are .0003 actually still produce critical flow in Channel 1. This can be seen since 534.158 cfs is very slightly larger than the critical flow rate of 531.4 cfs. In other words the bottom slope of the 3 downstream channels must be less than .0003 for critical conditions in Channel 1 not to restrict the flow rate, i.e., with this bottom slope, and flatter downstream channels, the control is transferred from upstream to downstream. Below the solution that assumed that the downstream channels control, a solution is obtained in which the flow rate in Channel 1 is specified equal to 531.4 cfs.

TK-Solver Model PRB2_79.TK

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment--------------------------------
              Q1       534.15817             Solves Prob. 107 (Chap. 2) 4 branched
              Y1       3.881989
              Q2       219.81859
              Y2       4.5350472
              Q3       157.16979
              Y3       4.5735751
              Q4       157.16979
              Y4       4.5735751
   5          H
   10         b1
   2          m1
   64.4       g2
   8          b2
   1          m2
   0          z12
   0          z13
   0          z14
   5          b3
   1          m3
   .013       n2
   .0003      So2
   .013       n3
   .0003      So3
   5          b4
   .013       n4
   1          m4
   .0003      So4
   .2         K1
   .2         K2
   .2         K3
   .2         K4
   1.486      Cman

==================== RULE SHEET ====================
S Rule--------------------------------------------------
  Q1-Q2-Q3-Q4=0
  H-Y1-(1+K1)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(1+K2)*(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(1+K3)*(Q3/((b3+m3*Y3)*Y3))^2/g2-z13=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-(1+K4)*(Q4/((b4+m4*Y4)*Y4))^2/g2-z14=0
  n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-Cman*((b2+m2*Y2)*Y2)^1.666667*sqrt(So2
  n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-Cman*((b3+m3*Y3)*Y3)^1.666667*sqrt(So3
  n4*Q4*(b4+2*Y4*sqrt(m4*m4+1))^.6666667-Cman*((b4+m4*Y4)*Y4)^1.666667*sqrt(So4
```

Solution assuming critical flow in Channel 1 controls:

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment-----------
```

```
   531.41    Q1                              Solves Prob. 79 (Chap. 2) 4 branched c
             Y1      3.8715379
             Q2      218.72942
             Y2      4.5229443
             Q3      156.34029
             Y3      4.5613785
             Q4      156.34029
             Y4      4.5613785
             H       4.9866794
   10        b1
   2         m1
   64.4      g2
   8         b2
   1         m2
   0         z12
   0         z13
   0         z14
   5         b3
   1         m3
   .013      n2
   .0003     So2
   .013      n3
   .0003     So3
   5         b4
   .013      n4
   1         m4
   .0003     So4
   .2        K1
   .2        K2
   .2        K3
   .2        K4
   1.486     Cman
```

Notice that this correct solution gives H = 4.987 ft, i.e. .013 ft of head must be lost at the entrance of Channel 1 if the reservoir head is H = 5.0 ft.

```
Bottom Slopes of the 3 downstream Channels decreased to .0002.(File: PRB2 79B.TK)
==================== VARIABLE SHEET =====================
St Input───── Name──── Output─── Unit────── Comment──────────────────────────────
             Q1      467.51645               Solves Prob. 107 (Chap. 2) 4 branched
             Y1      4.4079332               Except slope of downstream channels is
             Q2      192.4842                now specified  as .0002
             Y2      4.7085126
             Q3      137.51612
             Y3      4.7355376
             Q4      137.51612
             Y4      4.7355376
   5         H
   10        b1
   2         m1
   64.4      g2
   8         b2
   1         m2
   0         z12
   0         z13
   0         z14
   5         b3
   1         m3
   .013      n2
   .0002     So2
   .013      n3
   .0002     So3
   5         b4
   .013      n4
   1         m4
   .0002     So4
   .2        K1
   .2        K2
   .2        K3
```

```
.2        K4
1.486     Cman
```

Now the flow rate in Channel 1 is 467.52 cfs, which is clearly less than 531.4 cfs, so downstream control does occur.

108.

The program BRANCHCH.FOR will solve problems in which the critical depth at the entrance of the channel from the reservoir restricts the flowrate. However, to obtain such a solution it is necessary that you previously determine what this restrictive flowrate is. Modify the program so that if it fail to converge, or if you specify that critical flow governs that it will solve the problem without the necessity of giving what this restricting flowrate is.

The Program BRANCHC2.FOR given below is a modification of Program BRANCHCH.FOR that allows its user to indicate whether critical flow in the upstream channel controls. If so it computes this controlling critical flow rate and depth and solves the problem accordingly. Read the comments at the top of the program listing and note that if critical flow is specified to govern that the number of unknowns is reduced by 1, or N=2*NC-1. However, since Q_{c1} and Y_{c1} are computed before solving the system of equations the number of variables solves is actually 1 more than 2 time the number of channels.

Program BRANCHC2.FOR

```
C This program is a modification of BRANCH that allow the user to specify that
C critical flow occurs in Channel 1, by giving 1 as item (6) of the first line
C of input.  If so given, then the critical flow rate Qc1 and critical depth
C  Yc1 are solve by using the Energy Eq. from res. and critical flow eq. These
C  are then eliminated from the unknown, and the Number of unknowns becomes
C N=2*NC-1.  The program then computes H as the head that is available at the
C downstream branched channel, i.e. the difference between the given head H and
C that computed equal the loss in Channel 1 as its flow attempts to become
C supercritical.  Notice when the list of unknowns is given, that S1 is given
C to specify H since for the rest of the channel the bottom slope is the first
C variable.  Only 1 unknown is given for Channel 1, whereas 2 unknowns are given
C for the downstream channels.
      REAL X[ALLOCATABLE](:),F[ALLOCATABLE](:),F1[ALLOCATABLE](:),
     &D[ALLOCATABLE](:,:),DZ[ALLOCATABLE](:),KLOS[ALLOCATABLE](:)
      INTEGER*2 IT[ALLOCATABLE](:),INDX[ALLOCATABLE](:),
     &IV[ALLOCATABLE](:)
      LOGICAL*1 STEEP[ALLOCATABLE](:),KEYB,CONTR
      CHARACTER*2 CU[ALLOCATABLE](:)
      CHARACTER*6 S(2)/'SQYbmn','SQYDn '/
      CHARACTER*1 CH
      CHARACTER*38  FMT/"(1X,A1,' =',F8.6,/,(1X,A1,' =',F8.3))"/
      COMMON CMAN,G,G2,YGA(5),EYG(5),IYG(5),IGATE(6),NGATE
      WRITE(*,*)' Give: (1) 1=ES, 2=SI; (2) No. channels; (3) No. gates
     &(4) IN-unit; (5) OUT-unit; (6) 0 or 1 if Crit.'
      CONTR=.FALSE.
      KEYB=.FALSE.
      READ(*,*) II,NC,NGATE,IN,IOUT,N       !  Trapezodial    Circular
      IF(N.EQ.1) CONTR=.TRUE.               ! -------------   ---------
      IGATE(NGATE+1)=NC+1                   !  H  = x(1)       H  = x(1)
      IF(IN.EQ.0 .OR. IN.EQ.5) KEYB=.TRUE.  !  Q1 = x(2)       Q1 = x(2)
      IF(II.EQ.2) THEN                      !  Y1 = x(3)       Y1 = x(3)
      G=9.81                                !  b1 = x(4)       D1 = x(4)
      CMAN=1.                               !  m1 = x(5)       n1 = x(5)
      ELSE                                  !  n1 = x(6)
      G=32.2                                !  So2= x(7)       So2= x(6)
      CMAN=1.486                            !  Q2 = x(8)       Q2= x(7)
      ENDIF                                 !  Y2 = x(9)       Y2 = x(8)
      G2=2.*G                               !  b2 = x(10)      D2 = x(9)
      N=2*NC                                !  m2 = x(11)      n2 = x(10)
      IF(CONTR) N=N-1
      ALLOCATE(IT(NC),KLOS(NC),DZ(NC),F(N+1),F1(N+1),D(N,N),CU(N),STEEP(
     &NC))
      IF(KEYB) WRITE(*,*)' For each channel give 1 = trap., or 2 = cir.'
5     NV=0
      DO 10 I=1,NC
      IF(KEYB) WRITE(*,"(' Channel #',I2,' = ',\)") I
      READ(IN,*) IT(I)
10    NV=NV-IT(I)+7
```

```
      ALLOCATE(X(NV),IV(N),INDX(N))
      II=0
      DO 30 I=1,NC
      STEEP(I)=.FALSE.
      IF(KEYB) THEN
      WRITE(*,"(' Give variables for Channel',I2)") I
      DO 20 J=1,7-IT(I)
      CH=S(IT(I))(J:J)
      IF(I.EQ.1 .AND. J.EQ.1) CH='H'
      WRITE(*,"(5X,A1,' = ',\)") CH
20    READ(*,*) X(J+II)
      WRITE(*,"(' Loss Coef =',\)")
      READ(*,*) KLOS(I)
      IF(I.GT.1) THEN
      WRITE(*,"(' Change in bottom position =',\)")
      READ(*,*) DZ(I)
      ENDIF
      ELSE
      IF(I.EQ.1) THEN
      READ(IN,*)(X(J),J=1,7-IT(I)),KLOS(I)
      ELSE
      READ(IN,*)(X(J+II),J=1,7-IT(I)),KLOS(I),DZ(I)
      IF(X(II+1).GT. .008) STEEP(I)=.TRUE.
      ENDIF
      ENDIF
30    II=II+7-IT(I)
      IF(NGATE.GT.0) THEN
      IF(KEYB) WRITE(*,*)' Give pairs of values: channel no. and depth d
     &owns. from gate(s)'
      READ(IN,*) (IGATE(I),YGA(I),I=1,NGATE)
      ENDIF
      IF(KEYB) WRITE(*,*)' Give symbols for',N,' Unknowns, i.e. Q1 Y1 b2
     & etc.'
      READ(IN,120) (CU(I),I=1,N)
120   FORMAT(26(A2,1X))
      IU=0
      IPOS=0
      II1=1
      DO 40 I=1,N
      II=ICHAR(CU(I)(2:2))-48
      DO 35 J=1,7-IT(II)
      IF(S(IT(II))(J:J).NE.CU(I)(1:1)) GO TO 35
      IF(II.NE.II1) THEN
      IPOS=IPOS+7-IT(II1)
      II1=II
      ENDIF
      IU=IU+1
      IV(IU)=IPOS+J
      GO TO 40
35    CONTINUE
40    CONTINUE
      IF(CONTR) GO TO 80
41    NCT=0
42    CALL FUNCT(NC,N,NV,IT,STEEP,X,F,DZ,KLOS,CONTR)
      DO 50 J=1,N
      XX=X(IV(J))
      X(IV(J))=1.005*X(IV(J))
      CALL FUNCT(NC,N,NV,IT,STEEP,X,F1,DZ,KLOS,CONTR)
      DO 45 I=1,N
45    D(I,J)=(F1(I)-F(I))/(X(IV(J))-XX)
50    X(IV(J))=XX
      CALL SOLVEQ(N,1,N,D,F,1,DD,INDX)
      NCT=NCT+1
      SUM=0.
      DO 60 I=1,N
      X(IV(I))=X(IV(I))-F(I)
60    SUM=SUM+ABS(F(I))
      WRITE(*,*)' Iteration=',NCT,' SUM=',SUM
      IF(NCT.LT.20 .AND. SUM.GT. .0005) GO TO 42
      IF(NCT.GE.20) THEN
      WRITE(*,*)' Failed to converge',SUM
```

```
      IF(.NOT.CONTR) THEN
      WRITE(*,*)' Critical depth at entrance may control.  Give 1 if I s
     &hould try sol., otherwise 0'
      READ(*,*) II
      IF(II.EQ.1) GO TO 80
      ENDIF
      ENDIF
      DO 65 I=1,NGATE
      NCT=2
      II=IYG(I)
63    FF=EYG(I)-X(II)-(X(II-1)/((X(II+1)+X(II+2)*X(II))*X(II)))**2/G2
      IF(MOD(NCT,2) .NE. 0) GO TO 64
      NCT=NCT+1
      F11=FF
      XX=X(II)
      X(II)=1.005*X(II)
      GO TO 63
64    NCT=NCT+1
      DIF=(X(II)-XX)*F11/(FF-F11)
      X(II)=XX-DIF
      WRITE(*,*)' NCT=',NCT,'  DIF=',DIF
      IF(NCT.LT. 40 .AND. ABS(DIF).GT. .0001) GO TO 63
      WRITE(*,*)' Failed to converge with gates',DIF
65    CONTINUE
      II=0
      S(IT(1))(1:1)='H'
      FMT(16:16)='3'
      IG=1
      DO 70 I=1,NC
      WRITE(IOUT,"(' For Channel',I2)") I
      IF(I.EQ.IGATE(IG) .AND. ABS(DZ(I)).GT.1.E-3) THEN
      WRITE(*,130) I
130   FORMAT(' Is Dz at gate for channel',I3,' give 0; if at beg. give
     & 1')
      READ(*,*) NCT
      IF(NCT.EQ.1) X(II+3)=X(II+3)-DZ(I)
      IG=IG+1
      ENDIF
      WRITE(IOUT,FMT)(S(IT(I))(J:J),X(J+II),J=1,7-IT(I))
      IF(I.EQ.1) S(IT(1))(1:1)='S'
      FMT(16:16)='6'
70    II=II-IT(I)+7
      STOP
80    NCT=0
      IV(1)=1
90    AA=(X(4)+X(3)*X(5))*X(3)
      TOPW=X(4)+2.*X(3)*X(5)
      FF=X(3)+.5*(1.+KLOS(1))*AA/TOPW-X(1)
      F11=1.+(1.+KLOS(1))*(.5-AA/TOPW**2*X(5))
      DIF=FF/F11
      X(3)=X(3)-DIF
      NCT=NCT+1
      WRITE(*,99)NCT,DIF,FF,F11,(X(I),I=1,4)
99    FORMAT(I3,6F11.4,F10.4)
      IF(NCT.LT.20 .AND. ABS(DIF).GT. .00001) GO TO 90
      AA=(X(4)+X(3)*X(5))*X(3)
      TOPW=X(4)+2.*X(3)*X(5)
      X(2)=SQRT(G*AA**3/TOPW)
      EC=X(3)+(X(2)/AA)**2/G2
      X(1)=.9*EC
      WRITE(3,301) EC,AA,TOPW,X(2),X(3)
      WRITE(*,301) EC,AA,TOPW,X(2),X(3)
301   FORMAT(5F12.5)
      READ(*,*)
      IF(CONTR) GO TO 41
      CONTR=.TRUE.
      IV(1)=1
      GO TO 41
      END
      SUBROUTINE FUNCT(NC,N,NV,IT,STEEP,X,F,DZ,KLOS,CONTR)
      REAL X(NV),F(N+1),DZ(NC),KLOS(NC)
```

```
      INTEGER IT(NC)
      LOGICAL*1 STEEP(NC),CONTR
      COMMON CMAN,G,G2,YGA(5),EYG(5),IYG(5),IGATE(6),NGATE
      II=0
      IG=1
      DO 20 I=1,NC
      IF(IT(I).EQ.1) THEN
      A=(X(II+4)+X(II+5)*X(II+3))*X(II+3)
      IF(STEEP(I)) THEN
      T=X(II+4)+2.*X(II+5)*X(II+3)
      ELSE
      P=X(II+4)+2.*SQRT(X(II+5)**2+1.)*X(II+3)
      ENDIF
      ELSE
      ARG=1.-2.*X(II+3)/X(II+4)
      BETA=ACOS(ARG)
      A=.25*X(II+4)**2*(BETA-SIN(BETA)*ARG)
      IF(STEEP(I)) THEN
      T=X(II+4)*SIN(BETA)
      ELSE
      P=BETA*X(II+4)
      ENDIF
      ENDIF
      IF(I.EQ.1) THEN
      IF(CONTR) THEN
      E1=X(1)
      ELSE
      E1=X(3)+(X(2)/A)**2/G2
      F(1)=X(1)-E1-KLOS(1)*(X(2)/A)**2/G2
      ENDIF
      F(2)=X(2)
      JJ=7-IT(1)
Continuity equation
      DO 10 J=2,NC
      F(2)=F(2)-X(JJ+2)
10    JJ=JJ+7-IT(J)
      ELSE
      IF(I.EQ.IGATE(IG)) THEN
      F(2*I-1)=E1-YGA(IG)-(1.+KLOS(I))*(X(II+2)/((X(II+4)+X(II+5)*YGA(IG
     &))*YGA(IG)))**2/G2-DZ(I)
      EYG(IG)=E1
      IYG(IG)=II+3
      IG=IG+1
      ELSE
      F(2*I-1)=E1-X(II+3)-(1.+KLOS(I))*(X(II+2)/A)**2/G2-DZ(I)
      ENDIF
      IF(STEEP(I)) THEN
      F(2*I)=T*X(II+2)**2-G*A**3
      ELSE
      F(2*I)=X(II+7-IT(I))*X(II+2)-CMAN*A*(A/P)**.66666667*SQRT(X(II+1))
      ENDIF
      ENDIF
20    II=II+7-IT(I)
      IF(CONTR) THEN
      DO 30 I=1,N
30    F(I)=F(I+1)
      ENDIF
      RETURN
      END
```

To solve the original problem in which the 3 downstream channels have bottom slopes of .0008 the input would be:

From Keyboard: 1 4 0 2 3 1

File: BRANCHCR.DAT

```
1
1
1
1
5.0 531.41 4. 10 2 .013 .2
.0008 217.3 4. 8 1 .013 .2 0.
.0008 157.05  4. 5 1 .013 .2 0.
.0008 157.05  4. 5 1 .013 .2 0.
```

```
S1 Q2 Y2 Q3 Y3 Q4 Y4
```

Output:

```
     4.75154     59.72206    24.03698   534.18320     3.50925
 For Channel 1
 H =   4.047
 Q = 534.183
 Y =   3.509
 b =  10.000
 m =   2.000
 n =    .013
 For Channel 2
 S = .000800
 Q = 220.716
 Y =   3.477
 b =   8.000
 m =   1.000
 n =    .013
 For Channel 3
 S = .000800
 Q = 156.733
 Y =   3.550
 b =   5.000
 m =   1.000
 n =    .013
 For Channel 4
 S = .000800
 Q = 156.733
 Y =   3.550
 b =   5.000
 m =   1.000
 n =    .013
```

Since this critical flow rate is Q_{c1}=534.183, which is slightly different than that used previously, the TK-Solver solution using this value is given below. (Model: EP2_19.TK with a slightly different value given for Q1)

```
==================== VARIABLE SHEET =====================
St Input───── Name──── Output──── Unit────── Comment─────────
   534.183    Q1
              Q2      220.71626
              Q3      156.73337
              Q4      156.73337
              Y2      3.4769908
              Y3      3.5502819
              Y4      3.5502819
              H       4.0470249
   64.4       g2
   8          b2
   1          m2
   5          b3
   1          m3
   5          b4
   1          m4
   0          z12
   0          z13
   0          z14
   .013       n2
   .0008      So2
   .013       n3
   .0008      So3
   .013       n4
   .0008      So4
   .2         K1
   .2         K2
   .2         K3
   .2         K4
   1.486      Cman

==================== RULE SHEET =========================
S Rule───────────────────────────────────────────────────
```

```
Q1-Q2-Q3-Q4=0
H-Y2-(1+K2)*(Q2/((b2+m2*Y2)*Y2))^2/g2-z12=0
H-Y3-(1+K3)*(Q3/((b3+m3*Y3)*Y3))^2/g2-z13=0
H-Y4-(1+K4)*(Q4/((b4+m4*Y4)*Y4))^2/g2-z14=0
n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-Cman*((b2+m2*Y2)*Y2)^1.666667*sqrt(So2)
n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-Cman*((b3+m3*Y3)*Y3)^1.666667*sqrt(So3)
n4*Q4*(b4+2*Y4*sqrt(m4*m4+1))^.6666667-Cman*((b4+m4*Y4)*Y4)^1.666667*sqrt(So4)
```

109.

At the location of sluice gates in a trapezoidal channel with b = 5 m and m = 1.5, n=.013 and S_o = .0005, the bottom of the channel rises by 0.3 m and the cross-section changes to a rectangle that contains 3 gates each 1 meter wide as shown in the sketch below. The two outside gates are set 0.5 m above the channel bottom and the center gate is set 0.7 m above the channel bottom. All three gates have contraction coefficients equal to C_c = 0.6, and a pier 0.5 m wide exists between the gates to support them. If the flowrate in the upstream channel is $Q_1 = 10\ m^3/s$, write the system of equations, and then solve them, that give the upstream depth, Y_1, and the flowrates past each of the gates.

Given: Flow in channel upstream from three gates is Q_1=10 m^3/s, with depth downstream from two gates Y_{2o}-.6(.5) = .3 m, and from one gate Y_{2m}=.6(.7)=.42 m
Find: Depth upstream from gates and flow rates past each gate.

Solution:

Since the settings of gates 1 and 3 are the same their flow rates will be identical, so let

q_o= flow rate from out side gates 1 and 2, and q_i= flow rate from middle gate. Three equation are needed to solve: Y_1, q_i, and q_o, and are:

$$F_1 = 2q_o + q_i - Q_1 = 0$$

$$F_2 = Y_1 + (Q/A)_1^2/(2g) - Y_o + (q/Y)_o^2/(2g) = 0$$

$$F_3 = Y_1 + (Q/A)_1^2/(2g) - Y_i + (q/Y)_i^2/(2g) = 0$$

Solution as given by TK-Solver Model below (PB2_108.TK) is:

q_o = 2.956 m^3/s <== q_i = 4.088 m^3/s <== and Y_1 = 5.247 m <==

TK-Solver Model PB2 108.TK

```
==================== VARIABLE SHEET ============
St Input----- Name---- Output---- Unit------ Comment
              qo       2.9560253
              qi       4.0879495
              Y1       5.2473963
   10         Q1
   5          b1
   1.5        m1
   19.62      g2
   .3         Yo
   .42        Yi

==================== RULE SHEET ========================
S Rule----------------------------------------------------------
  Q1=2*qo+qi
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2=Yo+(qo/Yo)^2/g2
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2=Yi+(qi/Yi)^2/g2
```

110.

Write a computer program that is capable of completely solving the critical flow equation for a trapezoidal channel and that uses the dimensionless form of this equation in the computations. By completely solving the equation is meant that any of the variables Y_c, E_c, Q_c, b or m may be unknown with the other variables known. Also accomplish this with a software package such as TK-Solver or MathCAD.

Wanted: A program and model that completely solve the dimensionless critical flow equation in a trapezoidal channel.

Solution:
Critical Flow Equation for trapezoidal section

$$Q^2(b+2mY)-g(bY+mY^2)^3 = 0$$

$$Q^2b(1+2mY/b)-(gb^6/m^3)(mY/b+m^2Y^2/b^2)^3 = 0$$

Let $Y'=mY/b$

$[m^3Q^2/(gb^5)][1+2Y'] - [Y'+Y'^2] = 0$ let $Q'= m^3Q^2/(gb^5)$ and

$$F(Q',Y') = Q'[1+2Y'] - [Y'+Y'^2] = 0$$

$$\frac{\partial F}{\partial \xi} = \frac{\partial F}{\partial Q'}\frac{\partial Q'}{\partial \xi} + \frac{\partial F}{\partial Y'}\frac{\partial Y'}{\partial \xi}$$

Variables: Y,b,m,Q

in which $\frac{\partial Q'}{\partial Y'} = 0$, $\frac{\partial Q'}{\partial b} = -\frac{5m^3Q^2}{gb^6}$, $\frac{\partial Q'}{\partial m} = \frac{3m^3Q^2}{gb^5}$

$$\frac{\partial Q'}{\partial Q} = \frac{2m^3Q}{gb^5},\quad \frac{\partial Y'}{\partial Y} = \frac{m}{b},\quad \frac{\partial Y'}{\partial b} = -\frac{mY}{b^2},\quad \frac{\partial Y'}{\partial m} = \frac{Y}{b},\quad \frac{\partial Y'}{\partial Q} = 0$$

Below is a Fortran Program that implements the requested complete solution. Note it uses numerical differentiation. Also a TK-Solver Model is listed below.

Listing of Fortran Program, CRITDIML.FOR, to solve Critical Flow

```
      CHARACTER*1 VAR(4)/'Y','b','m','Q'/
      DOUBLE PRECISION X(4),Y,b,m,Q,G,F,F1,DIF,XX,Yp,Qp,g
      EQUIVALENCE (Y,X(1)),(b,X(2)),(m,X(3)),(Q,X(4))
      WRITE(*,*)' Give: Y,b,m,Q & g (includes guess for unkown)'
      READ(*,*) X,g
      WRITE(*,100) (I,VAR(I),I=1,4)
100   FORMAT(' Give No. for unknown'/,(I4,1X,A1))
      READ(*,*) IU
      MM=0
10    Yp=m*Y/b
      Qp=m**3*Q*Q/(g*b**5)
      F=Qp*(1.+2.*Yp)-(Yp+Yp**2)**3
      MM=MM+1
      IF(MOD(MM,2).NE.0) THEN
      F1=F
      XX=X(IU)
      X(IU)=1.005*X(IU)
      GO TO 10
      ENDIF
      DIF=(X(IU)-XX)*F1/(F-F1)
      X(IU)=XX-DIF
```

```
      IF(ABS(DIF).GT. 1.D-7 .AND. MM.LT.20) GO TO 10
      WRITE(*,110) (VAR(I),X(I),I=1,4)
110   FORMAT(1X,A1,' = ',F10.3)
      END
```

TK-Solver CRITDIML.TK

```
==================== VARIABLE SHEET ====================
St Input---- Name--- Output--- Unit----- Comment--------
             Qp      .04968944
   1         m
   400       Q
   32.2      g
   10        b
             Yp      .3275732
             Y       3.275732

==================== RULE SHEET ========================
S Rule--------------------------------------------------
* Qp=m^3*Q^2/(g*b^5)
* Yp=m*Y/b
* Qp*(1+2*Yp)=(Yp+Yp^2)^3
```

111.

Write a computer program that is capable of completely solving the critical flow equation for a circular channel and that uses the dimensionless form of this equation in the computations. By completely solving the equation is meant that nay of the variables Y_c, E_c, Q_c or D may be unknown with the other variables known. Also accomplish this with a software package such as TK-Solver or MathCAD.

Wanted: A program and model that completely solve the dimensionless critical flow equation in a trapezoidal channel.

Solution:

Critical Flow Equation for trapezoidal section

$$Q^2 D\sin\beta - g[(D^2/4)(\beta-\sin\beta\cos\beta)]^3 = 0$$

Let $Y' = Y/D$, $\cos\beta = 1-2Y/D = 1-2Y'$

$$Q^2\sin\beta - (gD^5/64)(\beta-\sin\beta\cos\beta)^3 = 0$$

$$(Q^2/(gD^5))\sin\beta = Q'\sin\beta = [(\beta-\sin\beta\cos\beta)/4]^3$$

with $Q' = (Q^2/(gD^5)$

$$F(Q',\beta) = Q'\sin\beta - [(\beta-\sin\beta\cos\beta)/4]^3 = 0$$ in which $\beta=\cos^{-1}(1-2Y')$

The Fortran Program and TK-Solver Model below implement solution.

Fortran Program to Solve Critical Flow Eq. in a Circular Channel (PRB2_83.FOR)

```
      CHARACTER*1 VAR(3)/'Y','D','Q'/
      DOUBLE PRECISION X(3),Y,D,Q,b,g,F,F1,DIF,XX,Yp,Qp
      EQUIVALENCE (Y,X(1)),(D,X(2)),(Q,X(3))
      WRITE(*,*)' Give: Y,D,Q & g (includes guess for unkown)'
      READ(*,*) X,g
      WRITE(*,100) (I,VAR(I),I=1,3)
100   FORMAT(' Give No. for unknown'/,(I4,1X,A1))
      READ(*,*) IU
      MM=0
10    Yp=Y/D
      Qp=Q*Q/(g*D**5)
      b=ACOS(1.-2.*Yp)
      F=Qp*SIN(b)-((b-COS(b)*SIN(b))/4.)**3
      MM=MM+1
      IF(MOD(MM,2).NE.0) THEN
      F1=F
      XX=X(IU)
      X(IU)=1.005*X(IU)
      GO TO 10
      ENDIF
      DIF=(X(IU)-XX)*F1/(F-F1)
      X(IU)=XX-DIF
      IF(ABS(DIF).GT. 1.D-7 .AND. MM.LT.20) GO TO 10
      WRITE(*,110) (VAR(I),X(I),I=1,3)
110   FORMAT(1X,A1,' = ',F10.3)
```

```
      END
```

TK-Solver Model PRB2 83.TK to solve the dimensionless critical flow in a circular channel

```
==================== VARIABLE SHEET ====================
St Input───── Name─── Output─── Unit───── Comment──────────
              Qp      .00998454
   50         Q
   32.2       g
              b       1.1994594
              Yp      .31856923
              Y       1.9114154
   6          D
==================== RULE SHEET ========================
S Rule──────────────────────────────────────────────────
* Qp=Q^2/(g*D^5)
* Yp=Y/D
* cos(b)=1-2*Yp
* Qp=((b-cos(b)*sin(b))/4.)^3
```

112.

Write a computer program that provides a table of values relating the depth upstream from a sluice gate to the depth downstream therefrom. Write this program so it only needs to evaluate explicit equations. Execute the program for a flowrate Q = 40 m^3/s in a 4 m wide channel with the downstream depth varying from Y_2 = 0.2 m to Y_2 = 1.8 m in increments of 0.05 m and plot the results. Note if this were done for several flowrates, q, the graph could be used to solve the specific energy across the gate. Verify the results from your program with CHANNEL. Also use a spreadsheet, TK-Solver, MathCAD or similar software to solve the problem, and plot the results.

Obtain data relating the depth upstream from a sluice gate to the depth downstream.

TK-Solver Model, PRB2 100.TK, that solves energy across gate in rectangular channel

```
==================== VARIABLE SHEET ============
St Input───── Name─── Output─── Unit───── Comment
   10.19368   qsg
   10         q
   9.81       g
L             qsy
L  .2         Y2
L             Y1

==================== RULE SHEET ========================
S Rule──────────────────────────────────────────────────
* qsg=q^2/g
* qsy=qsg/(2.*Y2^2)
* Y1=.5*(qsy+sqrt(qsy*(qsy+4.*Y2)))

==================== TABLE: SluiceG ====================
Title:
Element Y2────────── Y1──────────
1       .2         127.620686
2       .25        81.7986776
3       .3         56.9299825
4       .35        41.9539611
5       .4         32.2503493
6       .45        25.6118101
7       .5         20.8756643
8       .55        17.3821886
9       .6         14.7344122
10      .65        12.6818355
11      .7         11.0600477
12      .75        9.75751571
13      .8         8.69641861
```

14	.85	7.82112727
15	.9	7.09103137
16	.95	6.47593457
17	1	5.95301751
18	1.05	5.50478639
19	1.1	5.11765737
20	1.15	4.78096084
21	1.2	4.48622878
22	1.25	4.22667682
23	1.3	3.99682242
24	1.35	3.7921997
25	1.4	3.60914414
26	1.45	3.44462817
27	1.5	3.2961344
28	1.55	3.1615571
29	1.6	3.0391249
30	1.65	2.92733974
31	1.7	2.82492829
32	1.75	2.73080303
33	1.8	2.64403092

113.

An upstream channel receives water from a reservoir with a head of H = 5 ft, and shortly downstream therefrom divides into three branch channel, with the following sizes: Channel (2) is rectangular with a bottom width of 8 ft, and it contains a gate a short distance downstream from its beginning that cause the depth of flow downstream therefrom to be Y_{d2} = 1.0 ft; Channel (3) is trapezoidal with b_3 = 8 ft, a side slope m_3 = 1, and a bottom slope of S_{o3} = 0.001; Channel (4) is circular with a diameter D_4 = 8 ft,and has a bottom slope S_{o4} = 0.0008. The bottom drops by Δz_{12} = 0.2 ft between channels (1) and (2) and also by Δz_{14} = -0.2 ft between channels (1) and (4). All Mannings roughness coefficients are n = .013, and the entrance loss to the reservoir is K_e = 0.2. If the upstream main channel has a side slope m_1 = 1.5, and a bottom width b_1 = 16 ft, what are the flow rates in the four channels and the depths? If the width of the upstream channel is reduced to b_1 = 14 ft what are the flow rates and the depths? What is the critical flow rate, i.e. the maximum flow rate, for channel (1)? What will occur if the width of channel (1) is reduced to 14 ft?

Input to have BRANCHCH solve problem (file: PRB2 101.IN)

```
1
1
1
2
5 650 4 16 1.5 .013 .2
.001 130 4.8 8 0 .013 0 -.2
.001 350 4.2 8 1 .013 0 0
.0008 170 4.2 8 .013 0 -.2
2 1
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4
```

Output from Program BRANCHCH

```
 For Channel 1
 H =   5.000
 Q = 630.681
 Y =   4.103
 b =  16.000
 m =   1.500
 n =    .013
 For Channel 2
 S = .001000
 Q = 129.207
 Y =   4.664
 b =   8.000
 m =    .000
 n =    .013
 For Channel 3
 S = .001000
 Q = 339.567
 Y =   4.143
 b =   8.000
 m =   1.000
 n =    .013
 For Channel 4
 S = .000800
 Q = 161.907
 Y =   4.594
 D =   8.000
 n =    .013
```

TK-Solver Model PRB2 100.TK to solve problem

```
==================== VARIABLE SHEET ====================
St Input──── Name─── Output─── Unit───── Comment─────────
             Q1      630.68125
             Y1      4.1030195
             Q2      129.20741
```

```
              Y2      4.8804482
              Q3      339.56716
              Y3      4.1431346
              Q4      161.90668
              Y4      4.594263
    5         H
    16        b1
    1.5       m1
    64.4      g2
    8         b2
    8         b3
    1         m3
    .013      n3
    .001      So3
    .013      n4
    .0008     So4
    .2        Ke
              E1      4.8505032
    -.2       Dz12
    1         Yd2
              beta    1.7199141
    8         D4
    -.2       Dz14
              A4      29.869298

  ==================== RULE SHEET ========================
  S Rule---------------------------------------------------
  * Q1-Q2-Q3-Q4=0
  * n3*Q3*(b3+2*Y3*sqrt(m3*m3+1))^.6666667-1.486*((b3+m3*Y3)*Y3)^1.666667*sqrt(So
  * beta=acos(1-2*Y4/D4)
  * A4=.25*D4^2*(beta-cos(beta)*sin(beta))
* n4*Q4*(D4*beta)^.6666667-1.486*A4^1.666667*sqrt(So4)=0
* H-Y1-(1+Ke)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1-Y2-(Q2/(b2*Y2))^2/g2-Dz12=0
* E1-Y3-(Q3/((b3+m3*Y3)*Y3))^2/g2=0
* E1-Y4-(Q4/A4)^2/g2-Dz14=0
* Yd2+(Q2/(b2*Yd2))^2/g2-Y2-(Q2/(b2*Y2))^2/g2=0
```

Program BRANCHCH, as it it written, allows one Δz for changes in bottom elevation for each channel. If a channel contains a grate there could be a Δz at the junction of this channel and the upstream channel as well as a Δz at the gate. When Program BRANCHCH notes that Δz is given for a channel that contains a gate it prompt the user with a question whether this is at the gate or the beginning of the channel. Modify program BRANCHCH so that channels with gates as the downstream boundary condition have two Δz's read in, one at the channel's beginning, and one at the channel's gate. Use this program to solve the previous problem in which there is a rise of Δz=.2 ft at the gate in Channel 2.

Program BRANCHCZ.FOR contains the additional array DZGA(5). Values for Δz at the gates are read as a 3rd itiem of data for gates. After the program's listing, the input to solve the problem with a rise of Δz=.2 at the gate is given, and the solution produced by BRANCHCZ.

Program BRANCHCZ.FOR

```
C This version of BRANCHCH has two Del z for channel with gates, one at
C beg. of channel and one at gate.
      REAL X[ALLOCATABLE](:),F[ALLOCATABLE](:),F1[ALLOCATABLE](:),
     &D[ALLOCATABLE](:,:),DZ[ALLOCATABLE](:),KLOS[ALLOCATABLE](:)
      INTEGER*2 IT[ALLOCATABLE](:),INDX[ALLOCATABLE](:),
     &IV[ALLOCATABLE](:)
      LOGICAL*1 STEEP[ALLOCATABLE](:),KEYB
      CHARACTER*2 CU[ALLOCATABLE](:)
      CHARACTER*6 S(2)/'SQYbmn','SQYDn '/
      CHARACTER*1 CH
      CHARACTER*38  FMT/"(1X,A1,' =',F8.6,/,(1X,A1,' =',F8.3))"/
      COMMON CMAN,G,G2,YGA(5),DZGA(5),EYG(5),IYG(5),IGATE(6),NGATE
      WRITE(*,*)' Give: (1) 1=ES, 2=SI; (2) No. channels; (3) No. gates
     &(4) IN-unit; (5) OUT-unit'
      KEYB=.FALSE.                          !  Trapezodial    Circular
      READ(*,*) II,NC,NGATE,IN,IOUT         ! -------------   ---------
      IGATE(NGATE+1)=NC+1                   !  H  = x(1)       H  = x(1)
      IF(IN.EQ.0 .OR. IN.EQ.5) KEYB=.TRUE.  !  Q1 = x(2)       Q1 = x(2)
      IF(II.EQ.2) THEN                      !  Y1 = x(3)       Y1 = x(3)
      G=9.81                                !  b1 = x(4)       D1 = x(4)
```

```
      CMAN=1.                              !  m1 = x(5)       n1 = x(5)
      ELSE                                 !  n1 = x(6)
      G=32.2                               !  So2= x(7)       So2= x(6)
      CMAN=1.486                           !  Q2 = x(8)       Q2= x(7)
      ENDIF                                !  Y2 = x(9)       Y2 = x(8)
      G2=2.*G                              !  b2 = x(10)      D2 = x(9)
      N=2*NC                               !  m2 = x(11)      n2 = x(10)
      ALLOCATE(IT(NC),KLOS(NC),DZ(NC),F(N),F1(N),D(N,N),CU(N),STEEP(NC))
      IF(KEYB) WRITE(*,*)' For each channel give 1 = trap., or 2 = cir.'
      NV=0
      DO 10 I=1,NC
      IF(KEYB) WRITE(*,"(' Channel #',I2,' = ',\)") I
      READ(IN,*) IT(I)
10    NV=NV-IT(I)+7
      ALLOCATE(X(NV),IV(N),INDX(N))
      II=0
      DO 30 I=1,NC
      STEEP(I)=.FALSE.
      IF(KEYB) THEN
      WRITE(*,"(' Give variables for Channel',I2)") I
      DO 20 J=1,7-IT(I)
      CH=S(IT(I))(J:J)
      IF(I.EQ.1 .AND. J.EQ.1) CH='H'
      WRITE(*,"(5X,A1,' = ',\)") CH
20    READ(*,*) X(J+II)
      WRITE(*,"(' Loss Coef =',\)")
      READ(*,*) KLOS(I)
      IF(I.GT.1) THEN
      WRITE(*,"(' Change in bottom position =',\)")
      READ(*,*) DZ(I)
      ENDIF
      ELSE
      IF(I.EQ.1) THEN
      READ(IN,*)(X(J),J=1,7-IT(I)),KLOS(I)
      ELSE
      READ(IN,*)(X(J+II),J=1,7-IT(I)),KLOS(I),DZ(I)
      IF(X(II+1).GT. .008) STEEP(I)=.TRUE.
      ENDIF
      ENDIF
30    II=II+7-IT(I)
      IF(NGATE.GT.0) THEN
      IF(KEYB) WRITE(*,*)' Give 3 values: channel no., depth downs. from
     & gate(s), & Del z at gate'
      READ(IN,*) (IGATE(I),YGA(I),DZGA(I),I=1,NGATE)
      ENDIF
      IF(KEYB) WRITE(*,*)' Give symbols for',N,' Unknowns, i.e. Y1 Q1 b2
     & etc.'
      READ(IN,120) (CU(I),I=1,N)
120   FORMAT(26(A2,1X))
      IU=0
      DO 40 I=1,N
      II=ICHAR(CU(I)(2:2))-48
      DO 35 J=1,7-IT(II)
      IF(S(IT(II))(J:J).NE.CU(I)(1:1)) GO TO 35
      IU=IU+1
      IPOS=0
      DO 36 K=1,II-1
36    IPOS=IPOS+7-IT(K)
      IV(IU)=IPOS+J
      GO TO 40
35    CONTINUE
      WRITE(*,*)' Do not have ',CU(I),' as a variable'
      STOP
40    CONTINUE
      WRITE(*,*) ' IU=',IU
      WRITE(*,*) ' IV=',(IV(I),I=1,IU)
      NCT=0
42    CALL FUNCT(NC,N,NV,IT,STEEP,X,F,DZ,KLOS)
      DO 50 J=1,N
      XX=X(IV(J))
      X(IV(J))=1.005*X(IV(J))
```

```
      CALL FUNCT(NC,N,NV,IT,STEEP,X,F1,DZ,KLOS)
      DO 45 I=1,N
45    D(I,J)=(F1(I)-F(I))/(X(IV(J))-XX)
50    X(IV(J))=XX
      CALL SOLVEQ(N,1,N,D,F,1,DD,INDX)
      NCT=NCT+1
      SUM=0.
      DO 60 I=1,N
      X(IV(I))=X(IV(I))-F(I)
60    SUM=SUM+ABS(F(I))
      WRITE(*,*)' Iteration=',NCT,' SUM=',SUM
      IF(NCT.LT.20 .AND. SUM.GT. .0005) GO TO 42
      DO 65 I=1,NGATE
      NCT=2
      II=IYG(I)
63    FF=EYG(I)-X(II)-(X(II-1)/((X(II+1)+X(II+2)*X(II))*X(II)))**2/G2
      IF(MOD(NCT,2) .NE. 0) GO TO 64
      NCT=NCT+1
      F11=FF
      XX=X(II)
      X(II)=1.005*X(II)
      GO TO 63
64    NCT=NCT+1
      DIF=(X(II)-XX)*F11/(FF-F11)
      X(II)=XX-DIF
      WRITE(*,*)' NCT=',NCT,'  DIF=',DIF
      IF(NCT.LT. 40 .AND. ABS(DIF).GT. .0001) GO TO 63
      IF(NCT.GE.40) WRITE(*,*)' Failed to converge with gates',DIF
65    CONTINUE
      II=0
      S(IT(1))(1:1)='H'
      FMT(16:16)='3'
      DO 70 I=1,NC
      WRITE(IOUT,"(' For Channel',I2)") I
      WRITE(IOUT,FMT)(S(IT(I))(J:J),X(J+II),J=1,7-IT(I))
      IF(I.EQ.1) S(IT(1))(1:1)='S'
      FMT(16:16)='6'
70    II=II-IT(I)+7
      END
      SUBROUTINE FUNCT(NC,N,NV,IT,STEEP,X,F,DZ,KLOS)
      REAL X(NV),F(N),DZ(N),KLOS(N)
      INTEGER IT(NC)
      LOGICAL*1 STEEP(NC)
      COMMON CMAN,G,G2,YGA(5),DZGA(5),EYG(5),IYG(5),IGATE(6),NGATE
      II=0
      IG=1
      DO 20 I=1,NC
      IF(IT(I).EQ.1) THEN
      A=(X(II+4)+X(II+5)*X(II+3))*X(II+3)
      IF(STEEP(I)) THEN
```

```
      T=X(II+4)+2.*X(II+5)*X(II+3)
      ELSE
      P=X(II+4)+2.*SQRT(X(II+5)**2+1.)*X(II+3)
      ENDIF
      ELSE
      ARG=1.-2.*X(II+3)/X(II+4)
      BETA=ACOS(ARG)
      A=.25*X(II+4)**2*(BETA-SIN(BETA)*ARG)
      IF(STEEP(I)) THEN
      T=X(II+4)*SIN(BETA)
      ELSE
      P=BETA*X(II+4)
      ENDIF
      ENDIF
      IF(I.EQ.1) THEN
      E1=X(3)+(X(2)/A)**2/G2
      F(1)=X(1)-E1-KLOS(1)*(X(2)/A)**2/G2
      F(2)=X(2)
      JJ=7-IT(1)
      DO 10 J=2,NC
      F(2)=F(2)-X(JJ+2)
10    JJ=JJ+7-IT(J)
      ELSE
      IF(I.EQ.IGATE(IG)) THEN
      F(2*I-1)=E1-YGA(IG)-(1.+KLOS(I))*(X(II+2)/((X(II+4)+X(II+5)*YGA(IG
     &))*YGA(IG)))**2/G2-DZGA(IG)-DZ(I)
      EYG(IG)=E1
      IYG(IG)=II+3
      IG=IG+1
      ELSE
      F(2*I-1)=E1-X(II+3)-(1.+KLOS(I))*(X(II+2)/A)**2/G2-DZ(I)
      ENDIF
      IF(STEEP(I)) THEN
      F(2*I)=T*X(II+2)**2-G*A**3
      ELSE
      F(2*I)=X(II+7-IT(I))*X(II+2)-CMAN*A*(A/P)**.66666667*SQRT(X(II+1))
      ENDIF
      ENDIF
20    II=II+7-IT(I)
      RETURN
      END
```

Input: file: PRB2_101.INA

```
1
1
1
2
5 650 4 16 1.5 .013 .2
.001 130 4.8 8 0 .013 0 -.2
.001 350 4.2 8 1 .013 0 0
.0008 170 4.2 8 .013 0 -.2
2 1 .2
Q1 Y1 Q2 Y2 Q3 Y3 Q4 Y4
```

Output:

```
 For Channel 1
 H =   5.000
 Q = 628.005
 Y =   4.120
 b =  16.000
 m =   1.500
 n =    .013
 For Channel 2
 S = .001000
 Q = 126.023
 Y =   4.677
 b =   8.000
 m =    .000
 n =    .013
 For Channel 3
 S = .001000
 Q = 339.930
```

```
Y =   4.146
b =   8.000
m =   1.000
n =    .013
For Channel 4
S = .000800
Q = 162.053
Y =   4.597
D =   8.000
n =    .013
```

114.

The plan view of the inlet portion of a channel system is shown in the sketch below. It consist of two parallel channel (1) and (2) that receive water from a reservoir with a water surface elevation H = 6 ft above the bottom of these channel. Channel (1) has a bottom width b_1 = 8 ft, and a side slope m_1 = 1.5, and channel (2) has b_2 = 10 ft, and m_2 = 1.2. A short distance downstream these two channels join to form channel (3), but shortly thereafter branch into 3 separate channels denoted by (4), (5) and (6) on the sketch. A gate controls the flow in channel (4) and is set so it produces a depth Y_{d4} = 1.2 ft immediately downstream from the gate. Channel (4) is rectangular with b_4 = 8 ft. Channel (5) is a pipe with a diameter D_5 = 8 ft, and its bottom is 1 ft below the other channels. Channel (5) has a bottom slope S_{o5} = 0.00095, and a Mannings n_5 = .012. Channel (6) has b_6 = 10 ft, m_6 = 1.5, S_{o6} =0.0006, and n_6 = .013. The minor loss coefficients are K_e = .05 (entrance from reservoirs to both channels (1) & (2)), K_{13} = K_{23} =.08, K_{34} = K_{36} = .1, and K_{35} = .15, in which the double subscripts denote from which channel to what channel the flow occurs.

Do the following: (a) Determine the variables that are unknown and list them, (b) Write out the system of equations that will provide the solution to these unknowns, (c) Solve this system of equations.

If instead of dropping 1 ft to the bottom of channel (5) its bottom were 1 ft above the other channels what would the flow rates and depths in the six channels be?

```
TK-Solver Model TEST1 94.TK

==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
              Q1       438.49519
              Q2       446.7412
              Q3       885.23639
              Q4       167.6231
              Q5       129.93272
              Q6       587.68057
   6          H
              Y1       5.6353254
   .05        Ke
   8          b1
   1.5        m1
   64.4       g2
              Y2       5.6353254
   10         b2
   1.2        m2
              Y3       5.6083855
   .08        K13
   25         b3
   1.5        m3
   .08        K23
              Yu4      5.7262161
   .1         K34
   8          b4
   0          m4
   1.2        Yd4
              Y5       4.6218934
   .15        K35
              Y6       5.304833
   .1         K36
   10         b6
   1.5        m6
   .012       n5
   8          D5
   .00095     So5
   .013       n6
   .0006      So6

==================== RULE SHEET =========================
```

```
S Rule────────────────────────────────────────────────────
* Q1+Q2-Q3=0
* Q3-Q4-Q5-Q6=0
* H-Y1-(1+Ke)*(Q1/A(b1,m1,Y1))^2/g2=0
* H-Y2-(1+Ke)*(Q2/A(b2,m2,Y2))^2/g2=0
* Y1+(Q1/A(b1,m1,Y1))^2/g2-Y3-(1+K13)*(Q3/A(b3,m3,Y3))^2/g2=0
* Y2+(Q2/A(b2,m2,Y2))^2/g2-Y3-(1+K23)*(Q3/A(b3,m3,Y3))^2/g2=0
* Y3+(Q3/A(b3,m3,Y3))^2/g2-Yu4-(1+K34)*(Q4/A(b4,m4,Yu4))^2/g2=0
* Yu4+(Q4/A(b4,m4,Yu4))^2/g2-Yd4-(Q4/A(b4,m4,Yd4))^2/g2=0
* Y3+(Q3/A(b3,m3,Y3))^2/g2-Y5-(1+K35)*(Q5/Ac(D5,Y5))^2/g2+1=0
* Y3+(Q3/A(b3,m3,Y3))^2/g2-Y6-(1+K36)*(Q6/A(b6,m6,Y6))^2/g2=0
* n5*Q5*(D5*acos(1-2*Y5/D5))^.66666667-1.486*Ac(D5,Y5)^1.666667*sqrt(So5)=0
* n6*Q6*(b6+2*Y6*sqrt(m6*m6+1))^.6666667-1.486*A(b6,m6,Y6)^1.6666667*sqrt(So6)=

════════════════════ RULE FUNCTION: A ═══════════════════
Comment:
Parameter Variables:
Argument Variables:   b,m,Y
Result Variables:     A
S Rule─────────────────────────────────────────────────────
  A=(b+m*Y)*Y

════════════════════ PROCEDURE FUNCTION: Ac ═════════════
Comment:
Parameter Variables:
Input Variables:      D,Y
Output Variables:     Ac
S Statement────────────────────────────────────────────────
  cosb=1-2*Y/D
  beta=acos(cosb)
  Ar=D*D/4*(beta-cosb*sin(beta))
```

115.

The sketch below shows two channels that take water from separate reservoirs, and combine this flow into a single channel a short distance downstream. Using the sizes of trapezoidal channels shown on the sketch below solve for the flow rates and depth in the three channels. (The loss coefficient at both entrances, as well as between the channels is K = .05.)

The following is a TK-Solver model for this problem. Note it solves the following 6 equations (their details are provided in the Rule Sheet) for Q_1, Y_1, Q_2, Y_2, Q_3, & Y_3: (1) Continuity, (2) Energy at reservoir 1, (3) Energy at reservoir 2, (4) Energy between Channels 1 and 3, (5) Energy between Channels 2 and 3, and (6) Uniform flow (Manning's equation) in Channel 3.

TK-Solver Model PRB2_86.TK

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
              Q1       16.15576
              Q2       14.929998
              Q3       31.085758
              Y1       1.8840705
              Y2       1.8840705
              Y3       1.7760892
   2          H1
   .05        KE1
   3          b1
   1.5        m1
   19.62      G2
   2          H2
   .05        KE2
   3.5        b2
   1          m2
   .05        K12
   6          b3
   1.5        m3
   .014       n3
   1          C
   .0006      SO3
```

116.

The same channel system as in the previous problem, except that the depth of the water surface in reservoir 2 is H_2 = 2.04 m, or .04 m above that in reservoir 1. How much has this effected the flow. Why do you think the flow from the reservoir into channel 2 approaches critical conditions so rapidly as H_2 increases? What is the limiting value of H_2 for critical conditions to occur. What would happen if this watersurface rose to 2.1 m?

The reservoir H_2 of the previous problem is raised .04 m to H_2=2.04 m.

Now a solution using the TK-Solver model of the previous problem fails. Critical flow at the entrance of Channel 2 limits it flow rate. There are two approaches: (1) Add the critical flow equation to the previous model, and eliminate the energy equation between Channels 2 and 3. This TK-Solver model is shown below. (PRB2_86B.TK)

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment-----------
              Q1       7.3819524
              Q2       23.832587
              Q3       31.214539
              Y1       1.9791001
              Y2       1.4509103
              Y3       1.7800949
   2          H1
   .05        KE1
   3          b1
   1.5        m1
   19.62      G2
   2.04       H2
   .05        KE2
   3.5        b2
   1          m2
   .05        K12
   6          b3
   1.5        m3
   .014       n3
   1          C
   .0006      SO3
              Fr2      1

==================== RULE SHEET =========================
S Rule------------------------------------------------------
  Q1+Q2-Q3=0
  H1-Y1-(1+KE1)*abs(Q1)*Q1/((b1+m1*Y1)*Y1)^2/G2=0
  H2-Y2-(1+KE2)*(Q2/((b2+m2*Y2)*Y2))^2/G2=0
  Y1+abs(Q1)*Q1/((b1+m1*Y1)*Y1)^2/G2-Y3-(1+K12)*(Q3/((b3+m3*Y3)*Y3))^2/G2=0
  Q2^2*(b2+2*m2*Y2)/(.5*G2*((b2+m2*Y2)*Y2)^3)=1
  n3*Q3*(b3+2*Y3*SQRT(m3*m3+1))^.66666667-C*((b3+m3*Y3)*Y3)^1.6666667*SQRT(SO3)
  Fr2=SQRT(Q2^2*(b2+2.*m2*Y2)/(.5*G2)/((b2+m2*Y2)*Y2)^3)
```

(2) Solve the critical flow and energy at reservoir equations for Channel 2 separately for Q_c=Q_2 and Y_c=Y_2 and then specify Q_2 as a known value. The equations now reduce to (1) continuity, (2) the energy at Reservoir 1, (3) the energy between Channels 1 and 3 and (4) Mannings equation for Channel 3. The TK-Solver model for this approach is given below. (Note Y_2 is given as a known but not used in any equation.)

PRB2_86D.TK

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment----------
              Q1       7.3819524
   23.832587  Q2
              Q3       31.214539
              Y1       1.9791001
   1.4509103  Y2
              Y3       1.7800949
   2          H1
   .05        KE1
   3          b1
   1.5        m1
   19.62      G2
   .05        K12
   6          b3
```

```
   1.5       m3
   .014      n3
   1         C
   .0006     SO3

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Q1+Q2-Q3=0
* H1-Y1-(1+KE1)*(Q1/((b1+m1*Y1)*Y1))^2/G2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/G2-Y3-(1+K12)*(Q3/((b3+m3*Y3)*Y3))^2/G2=0
* n3*Q3*(b3+2*Y3*SQRT(m3*m3+1))^.66666667-C*((b3+m3*Y3)*Y3)^1.6666667*SQRT(SO3)
```

The following TK-Solver model has the Froude Number for Channel 2 added, and the head of Reservoir 2 is slowly increased from 2 m until the Froude Number approaches unity. Note that with H_2=2.0268 m, that F_{r2}=.99954374. Thus this is the limit to the amount (.0268 m) that the reservoir that supplies reservoir 2 can be raised above the head that supplies Channel 1 without having critical flow limit the flow that Channel 2 will supply. It is clear that raising H_2 to 2.1 m will produce the same results as above, i.e., Q_2=Q_c= 23.8 m^3/s. The additional head from reservoir will be dissipated in a hydraulic jump, or at the junction between Channels 2 and 3 so that the head at the beginning of Channel 3 2ill be governed by the flow coming from Channel 1.

PRB2 86A.TK

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment----------
              Q1       7.655488
              Q2       23.556836
              Q3       31.212324
              Y1       1.9774668
              Y2       1.4414668
              Y3       1.780026
   2          H1
   .05        KE1
   3          b1
   1.5        m1
   19.62      G2
   2.0268     H2
   .05        KE2
   3.5        b2
   1          m2
   .05        K12
   6          b3
   1.5        m3
   .014       n3
   1          C
   .0006      SO3
              Fr2      .99954374

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Q1+Q2-Q3=0
* H1-Y1-(1+KE1)*abs(Q1)*Q1/((b1+m1*Y1)*Y1)^2/G2=0
* H2-Y2-(1+KE2)*(Q2/((b2+m2*Y2)*Y2))^2/G2=0
* Y1+abs(Q1)*Q1/((b1+m1*Y1)*Y1)^2/G2-Y3-(1+K12)*(Q3/((b3+m3*Y3)*Y3))^2/G2=0
* Y2+(Q2/((b2+m2*Y2)*Y2))^2/G2-Y3-(1+K12)*(Q3/((b3+m3*Y3)*Y3))^2/G2=0
* n3*Q3*(b3+2*Y3*SQRT(m3*m3+1))^.66666667-C*((b3+m3*Y3)*Y3)^1.6666667*SQRT(SO3)
* Fr2=SQRT(Q2^2*(b2+2.*m2*Y2)/(.5*G2)/((b2+m2*Y2)*Y2)^3)
```

117.

The same channel system as in Problem 115, except that a gate is placed in Channel 3 that causes a depth of Y_{3d} downstream from it equal to 0.7 m, solve for the depths and flow rates in all channels.

Place a gate in Channel 3 of the previous problem. Now Mannings equation is replaced by an energy equation across the gate.

```
TK-Solver Model PRB2_88.TK
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit----- Comment---------
              Q1       12.928675
              Q2       11.908018
              Q3       24.836693
              Y1       1.9310007
              Y2       1.9310007
              Y3       1.8759641
   2          H1
   .05        KE1
   3          b1
   1.5        m1
   19.62      G2
   2          H2
   .05        KE2
   3.5        b2
   1          m2
   .05        K12
   6          b3
   1.5        m3
   1          C
   .7         Y3d

==================== RULE SHEET ========================
S Rule---------------------------------------------------
* Q1+Q2-Q3=0
* H1-Y1-(1+KE1)*(Q1/((b1+m1*Y1)*Y1))^2/G2=0
* H2-Y2-(1+KE2)*(Q2/((b2+m2*Y2)*Y2))^2/G2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/G2-Y3-(1+K12)*(Q3/((b3+m3*Y3)*Y3))^2/G2=0
* Y2+(Q2/((b2+m2*Y2)*Y2))^2/G2-Y3-(1+K12)*(Q3/((b3+m3*Y3)*Y3))^2/G2=0
* Y3+(Q3/((b3+m3*Y3)*Y3))^2/G2=Y3d+(Q3/((b3+m3*Y3d)*Y3d))^2/G2
```

118.

A system of 6 channels are involved in branches as shown in the sketch below. Assume all minor loss coefficients between an upstream channel and its branch downstream, as well as the entrance loss coefficient equal .05, i.e. $K_e = K_{ij} = .05$. Channel (3), which contains a gate that produces a depth of $Y_{d3} = 0.7$ m downstream therefrom, is rectangular with a bottom width of $b_3 = 3$ m. The sizes and properties of the other channels, which are trapezoidal, are: $b_1 = 5$ m, $m_1 = 1.5$, $b_2 = 2$ m, $m_2 = 1.$, $n_2 = .013$, $S_{o2} = .0008$, $b_4 = 2$ m, $m_4 = 1.$, $b_5 = 1.5$ m, $m_5 = 1.$, $n_5 = .013$, $S_{o5} = .0005$, and $b_6 = 1$ m, $m_6 = 1.$, $n_6 = .013$, $S_{o6} = .001$. The bottoms of all channels are at the same elevation at the junctions. Write out the system of equations whose solution will provide the depths and flow rates in all 6 channels, and obtain a solution to these equations.

One might be inclined from a cursory examination of the problem to assume that the equations needed to solve for the 12 unknowns (6 Q's and 6 Y's) would consist of: 2 continuity equations, 7 energy equations and 3 Manning equations, as used in the TK-Solver (PRB2_106.TK) shown below. However, it fails to obtain a solution with b1=5 m and b4=2 m. In fact both of these bottom widths need to be increased to about b1=7.46 m and b4=3.175 m, as given by the solution below. Note for this solution that both the Froude Number in Cahnnels 1 and 4 are very close to unity. Thus critical flow controls in Channel 4 and in Channel 1, when b1=5 m and b4=2 m.

```
TK-Solver Model PRB2_106.TK
==================== VARIABLE SHEET =====================
St Input───── Name──── Output──── Unit────── Comment──────────
              Q1       66.081802
              Y1       1.7537076
              Q2       21.421332
              Y2       2.1846023
              Q3       12.355917
              Y3       2.3011939
              Q4       32.304552
              Y4       1.8369915
              Q5       15.870125
              Y5       2.2953066
              Q6       16.434427
              Y6       2.1764288
   2.5        H
   7.46       b1
   1.5        m1
   19.62      g2
   2          b2
   .013       n2
   1          m2
   .0008      So2
   3          b3
   3.175      b4
   1          m4
   1.5        b5
   1          m5
   .013       n5
   .0005      So5
   1          b6
   1          m6
   .013       n6
   .001       So6
   .05        Ke
              E1       2.4644623
              E4       2.4644623
   0          Dz12
   0          Dz13
   0          Dz14
   0          Dz45
   0          Dz46
   .7         Yd3
              Fr1      .98112592
              Fr4      .95575685
```

```
==================== RULE SHEET ========================
S Rule---------------------------------------------------
  Q1-Q2-Q3-Q4=0
  Q4-Q5-Q6=0
  H-Y1-(1+Ke)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
  E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
  E1-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-Dz12=0
  E1-Y3-(Q3/(b3*Y3))^2/g2-Dz13=0
  E4=Y4+(Q4/((b4+m4*Y4)*Y4))^2/g2
  E1-E4-Dz14=0
  E4-Y5-(Q5/((b5+m5*Y5)*Y5))^2/g2-Dz45=0
  E4-Y6-(Q6/((b6+m6*Y6)*Y6))^2/g2-Dz46=0
  n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-((b2+m2*Y2)*Y2)^1.666667*sqrt(So2)=0
  n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.6666667-((b5+m5*Y5)*Y5)^1.666667*sqrt(So5)=0
  n6*Q6*(b6+2*Y6*sqrt(m6*m6+1))^.6666667-((b6+m6*Y6)*Y6)^1.666667*sqrt(So6)=0
  Yd3+(Q3/(b3*Yd3))^2/g2-Y3-(Q3/(b3*Y3))^2/g2=0
  Fr1=Q1^2*(b1+2*m1+Y1)/(9.81*((b1+m1*Y1)*Y1)^3)
  Fr4=Q4^2*(b4+2*m4+Y4)/(9.81*((b4+m4*Y4)*Y4)^3)
```

The 12 equations that need to be solved are: (1) Energy at the reservoir, (2) Critical flow in Channel 1, (3) Energy between Channels 2 and 3, (4) Energy betwen Channels 3 and 4, (5) Energy between Channels 5 and 6, (6) Energy across the gate in Channel 3, (7) Mannings equation in Channel 2, (8) Mannings equation in Channel 5, (9) Mannings equation in Channel 6, (10) Continuity equation at 1st junction, (11) Continuity equation at 2nd junction, and (12) Critical flow Equation in Channel 4. This last critical flow equaiton needs to use the energy in Channels 2, 3 and 4. Notice it is now not possible to write energy between Channel 1 and the three downstream channels, or energy between Channel 4 and its two downstream channels, since the energy in the downstream channels will be less with critical flow controlling.

Since Equations (1) and (2) are not linked to the other 10 equations, these might be solved first for Q1 = Qc1 and Y1 =Yc1 to give Q1=49.95 m3/s and Y1=2.462 m. Thereafter, the reamining 10 equations can be solved as shown in the TK-Solver model PB2_106B.TK below. Note since the energy E4 is needed as the head to solve the critical flow equation in Channel 4, that it is not possible to solve Q4=Qc4 independently of the other equations, as can be done for critical conditons in Channel 1 werhe H is given.

TK-Solver Model PB2_106B.TK

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment-----------
              Q2       17.772377
              Y2       1.9916665
              Q3       11.567161
              Y3       2.0694485
              Q4       20.610462
              Y4       1.6719548
              Q5       10.220556
              Y5       1.8617543
              Q6       10.389906
              Y6       1.7678885
   9.81       g
   19.62      g2
   49.95      Q1
   2          b2
   1          m2
   .013       n2
   .0008      So2
   3          b3
   2          b4
   1          m4
   1.5        b5
```

```
   1          m5
   .013       n5
   .0005      So5
   1          b6
   1          m6
   .013       n6
   .001       So6
              E2      2.246379
              E4      2.246379
              E5      1.9976712
              E6      1.9976712
   .7         Yd3

==================== RULE SHEET ==========================
S Rule---------------------------------------------------
* Q4^2*(b4+2.*m4*Y4)/(g*((b4+m4*Y4)*Y4)^3)=1
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.666667-((b2+m2*Y2)*Y2)^1.666667*sqrt(So2)=0
* n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.666667-((b5+m5*Y5)*Y5)^1.666667*sqrt(So5)=0
* n6*Q6*(b6+2*Y6*sqrt(m6*m6+1))^.666667-((b6+m6*Y6)*Y6)^1.666667*sqrt(So6)=0
* E4=Y4+(Q4/((b4+m4*Y4)*Y4))^2/g2
* E5=Y5+(Q5/((b5+m5*Y5)*Y5))^2/g2
* E6=Y6+(Q6/((b6+m6*Y6)*Y6))^2/g2
* E5=E6
* Y3+(Q3/(b3*Y3))^2/g2=Yd3+(Q3/(b3*Yd3))^2/g2
* Q4=Q5+Q6
* Q1=Q2+Q3+Q4
* E2=Y2+(Q2/((b2+m2*Y2)*Y2))^2/g2
* E2=Y3+(Q3/(b3*Y3))^2/g2
* E4=E2
```

Notice from this solution that the critical flow controls at the end of Channel 4 and
reduces the energy from E2 =E3=E4 = 2.246 m to E5=E6=1.998 m, or 0.247 m. Also
E1c=Y1c+(Qc1/A1)2/(2g)
= 2.462 m, and so the critical conditions at the beginning of Channel 1 dissipate 2.462-2.246 =
0.216 m of head.

Part (b) If b1 is increased to 8 m, the gate is raised to produce a depth Yd3=.95 m, and b4 is
increased to 3.2 m, then what are the depths and flow rates?

The equations now are the 12 fist equations that do not assume that critical flow controls
in either Channel 1 or 4. The solution follows:

TK-Solver Model PB2_106G.TK

```
==================== VARIABLE SHEET ======================
St Input----- Name---- Output---- Unit------ Comment----------
              Q1      69.488823
              Y1      1.8469796
              Q2      21.499709
              Y2      2.1885295
              Q3      15.558205
              Y3      2.1806225
              Q4      32.430909
              Y4      1.8878556
              Q5      15.930967
              Y5      2.2994272
              Q6      16.499942
              Y6      2.1803058
   2.5        H
   8          b1
   1.5        m1
   19.62      g2
   2          b2
   .013       n2
   1          m2
   .0008      So2
   3          b3
   3.2        b4
```

```
   1         m4
   1.5       b5
   1         m5
   .013      n5
   .0005     So5
   1         b6
   1         m6
   .013      n6
   .001      So6
   .05       Ke
             E1      2.4689038
             E4      2.4689038
   0         Dz12
   0         Dz13
   0         Dz14
   0         Dz45
   0         Dz46
   .95       Yd3
             Fr1     .80328887
             Fr4     .85753811

==================== RULE SHEET ========================
S Rule----------------------------------------------------------
* Q1-Q2-Q3-Q4=0
* Q4-Q5-Q6=0
* H-Y1-(1+Ke)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-Dz12=0
* E1-Y3-(Q3/(b3*Y3))^2/g2-Dz13=0
* E4=Y4+(Q4/((b4+m4*Y4)*Y4))^2/g2
* E1-E4-Dz14=0
* E4-Y5-(Q5/((b5+m5*Y5)*Y5))^2/g2-Dz45=0
* E4-Y6-(Q6/((b6+m6*Y6)*Y6))^2/g2-Dz46=0
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-((b2+m2*Y2)*Y2)^1.666667*sqrt(So2)=0
* n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.6666667-((b5+m5*Y5)*Y5)^1.666667*sqrt(So5)=0
* n6*Q6*(b6+2*Y6*sqrt(m6*m6+1))^.6666667-((b6+m6*Y6)*Y6)^1.666667*sqrt(So6)=0
* Yd3+(Q3/(b3*Yd3))^2/g2-Y3-(Q3/(b3*Y3))^2/g2=0
* Fr1=Q1^2*(b1+2*m1+Y1)/(9.81*((b1+m1*Y1)*Y1)^3)
* Fr4=Q4^2*(b4+2*m4+Y4)/(9.81*((b4+m4*Y4)*Y4)^3)
```

Part (c) If the gate in Channel 3 is completely closed and Channel 4 has a bottom width b4=3.5 m, what is the minimum bottom width that Channel 1 can have without critical flow occurring here?

The two equations involving Channel 3 have been removed from the Rule Sheet, and Q3, Y3, b3 and Yd3 removed from the Variable Sheet to give the model shown below. The input value for b1 has been successively decreased until the solution fails. A value for b1=5.57 m is the minimum, and this produces a Froude Number Fr1=0.956.

```
TK-Solver Model PB2_106S.TK
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment---------
             Q1      53.844329
             Y1      1.8076938
             Q2      21.466677
             Y2      2.1868754
             Q4      32.377652
             Y4      2.0594306
             Q5      15.905324
             Y5      2.2976916
             Q6      16.472329
             Y6      2.1786729
   2.5       H
   5.57      b1
   1.5       m1
   19.62     g2
   2         b2
   .013      n2
```

```
   1          m2
   .0008      So2
   3.5        b4
   1          m4
   1.5        b5
   1          m5
   .013       n5
   .0005      So5
   1          b6
   1          m6
   .013       n6
   .001       So6
   .05        Ke
              E1      2.467033
              E4      2.467033
   0          Dz12
   0          Dz13
   0          Dz14
   0          Dz45
   0          Dz46
              Fr1     .95609685
              Fr4     .73365042

==================== RULE SHEET ======================
S Rule--------------------------------------------------
* Q1-Q2-Q3-Q4=0
* Q4-Q5-Q6=0
* H-Y1-(1+Ke)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-Dz12=0
* E1-Y3-(Q3/(b3*Y3))^2/g2-Dz13=0
* E4=Y4+(Q4/((b4+m4*Y4)*Y4))^2/g2
* E1-E4-Dz14=0
* E4-Y5-(Q5/((b5+m5*Y5)*Y5))^2/g2-Dz45=0
* E4-Y6-(Q6/((b6+m6*Y6)*Y6))^2/g2-Dz46=0
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-((b2+m2*Y2)*Y2)^1.666667*sqrt(So2)=0
* n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.6666667-((b5+m5*Y5)*Y5)^1.666667*sqrt(So5)=0
* n6*Q6*(b6+2*Y6*sqrt(m6*m6+1))^.6666667-((b6+m6*Y6)*Y6)^1.666667*sqrt(So6)=0
* Yd3+(Q3/(b3*Yd3))^2/g2-Y3-(Q3/(b3*Y3))^2/g2=0
* Fr1=Q1^2*(b1+2*m1+Y1)/(9.81*((b1+m1*Y1)*Y1)^3)
* Fr4=Q4^2*(b4+2*m4+Y4)/(9.81*((b4+m4*Y4)*Y4)^3)
```

Part (d) What is the solution if the gate is closed and b1=5.0 m and b4=2 m?
From the previous experience one would expect critical flow to occur in Channel 4, and
with Channel 3 removed because the gate is closed it is likely that critical flow will not occur
in Channel 1. The TK-SOlver model below solves the 10 equation for the 10 unknowns.

TK-Solver Model PB2_106T.TK

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment----------
              Q1      46.774635
              Y1      2.1005899
              Q2      21.713646
              Y2      2.1992074
              Q4      25.060989
              Y4      1.8548408
              Q5      12.375638
              Y5      2.0407298
              Q6      12.68535
              Y6      1.9366893
   2.5        H
   5          b1
   1.5        m1
   19.62      g2
   2          b2
   .013       n2
   1          m2
   .0008      So2
   2          b4
```

```
   1         m4
   1.5       b5
   1         m5
   .013      n5
   .0005     So5
   1         b6
   1         m6
   .013      n6
   .001      So6
   .05       Ke
             E1      2.4809805
             E4      2.4809805
   0         Dz12
   0         Dz13
   0         Dz14

   0         Dz45
   0         Dz46
             Fr1     .66993123
   9.81      g
             E5      2.190243
             E6      2.190243

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Q1-Q2-Q4=0
* Q4-Q5-Q6=0
* H-Y1-(1+Ke)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2-Dz12=0
* Q4^2*(b4+2*m4*Y4)=(g*((b4+m4*Y4)*Y4)^3)
* E4=Y4+(Q4/((b4+m4*Y4)*Y4))^2/g2
* E1-E4-Dz14=0
* E5=Y5+(Q5/((b5+m5*Y5)*Y5))^2/g2
* E6=Y6+(Q6/((b6+m6*Y6)*Y6))^2/g2
* E5=E6
* n2*Q2*(b2+2*Y2*sqrt(m2*m2+1))^.6666667-((b2+m2*Y2)*Y2)^1.666667*sqrt(So2)=0
* n5*Q5*(b5+2*Y5*sqrt(m5*m5+1))^.6666667-((b5+m5*Y5)*Y5)^1.666667*sqrt(So5)=0
* n6*Q6*(b6+2*Y6*sqrt(m6*m6+1))^.6666667-((b6+m6*Y6)*Y6)^1.666667*sqrt(So6)=0
* Fr1=SQRT(Q1^2*(b1+2*m1+Y1)/(9.81*((b1+m1*Y1)*Y1)^3))
```

119.

Solve for the flowrates and depth in the various channels shown in the sketch below if the head of water from both of the supply reservoirs equals 5 ft, i.e. H_1 = 5' and H_2 = 5'. The flow in channel 6 is controlled by a vertical gate, so that the depth downstream from it is Y_{d6} = 0.6 ft, and Channels 3 and 4 are very long both with bottom slopes of $S_{o3} = S_{o4}$ = .0008, and Mannings roughness coefficients $n_3 = n_4$ = .014 so that uniform flow exist in these channels. The entrance loss coefficients are K_e = .05, and the loss coefficients from each upstream to downstream channel is K_j = .04. There is no loss across the gate.

In this problem both supply reservoirs have a water surface elevation of H_1=H_2= 5 ft. The TK-Solver model PRB2_91.TK given below solves this problem. Notice that the total flow rate into the system is the sum of Q1 and Q4 = 341.7+127.2 = 468.9 cfs.

Model PRB2 91.TK

```
==================== VARIABLE SHEET ======================
St Input----- Name---- Output---- Unit------ Comment---------
              Q1       341.67016
              Q2       148.17408
              Q3       320.74052
              Q4       127.24444
              Q5       97.803607
              Q6       50.370474
              Y1       4.7043837
              Y2       4.8342033
              Y3       4.4420372
              Y4       4.7043837
              Y5       4.7004188
              Y6       4.9121652
   5          H1
   5          H2
   .05        KE
   .04        Kj
   64.4       g2
   10         b1
   1.5        m1
   10         b2
   8          b3
   1          m3
   .014       n3
   .0008      So3
   4          b4
   .5         m4
   5          b5
   .014       n5
   .0008      So5
   5          b6
   .6         Yd6

==================== RULE SHEET ==========================
S Rule-----------------------------------------------------
* Q1+Q4-Q3-Q2=0
* Q2-Q5-Q6=0
* H1-Y1-(1+KE)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* H2-Y4-(1+KE)*(Q4/((b4+m4*Y4)*Y4))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(1+Kj)*(Q2/(b2*Y2))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-(Q4/((b4+m4*Y4)*Y4))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(1+Kj)*(Q3/((b3+m3*Y3)*Y3))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Y5-(1+Kj)*(Q5/(b5*Y5))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Y6-(1+Kj)*(Q6/(b6*Y6))^2/g2=0
* Y6+(Q6/(b6*Y6))^2/g2-Yd6-(Q6/(b6*Yd6))^2/g2=0
* n3*Q3-1.486*((b3+m3*Y3)*Y3)^1.6666667/(b3+sqrt(m3^2+1)*2*Y3)^.6666667*sqrt(So
* n5*Q5-1.486*(b5*Y5)^1.6666667/(b5+2*Y5)^.6666667*sqrt(So5)=0
```

120.

In the previous problem solve for the flowrates and depths if the water surface elevation of the reservoir that supplies channel 4 increases a small amount to H_2 = 5.05 ft. Solve for the flowrates and depths if this reservoir water surface fall a small amount to 4.98 ft. How do you explain why the depth in channel 4 decreases as the reservoir water surface elevation rises, and vice versus? What is the minimum H_2 possible to have the flow in channel 4 not reverse itself and flow into reservoir 2? What is the maximum head H_2 of reservoir 2 to have the condition in the other channels as specified in the previous problem? What is the minimum H_2 possible to have the other conditions as specified in the previous problem with the flow reversing itself and flowing into reservoir 2 from channel 4?

The channel system of the previous problem is solved with H_2= 5.05 ft, i.e. raised from 5 ft.
(Use same TK-SOlver Model with H2 altered.

```
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit----- Comment---------
             Q1      273.71437
             Q2      148.34778
             Q3      321.41639
             Q4      196.04981
             Q5      97.944745
             Q6      50.403039
             Y1      4.8232303
             Y2      4.8398621
             Y3      4.4470821
             Y4      3.8232303
             Y5      4.7059265
             Y6      4.917895
   5         H1
   5.05      H2
   .05       KE
   .04       Kj
   64.4      g2
   10        b1
   1.5       m1
   10        b2
   8         b3
   1         m3
   .014      n3
   .0008     So3
   4         b4
   .5        m4
   5         b5
   .014      n5
   .0008     So5
   5         b6
   .6        Yd6
```

Note that raising H_2 by .05 ft has caused the depth in Channel 4 to drop from 4.704 to 3.823 or .881 ft. This decrease in depth is caused by the increasing flow rate in Channel 4 with an accompanying increase in the velocity head which causes the depth to decrease. In other words the increase in velocity head is greater than the rise in the reservoir water surface elevation, H_2.

Reservoir 2 has its ws elevation lowered slightly to H_2=4.98 ft.
The same TK-SOlver model as used to solve Problem 119 can be used with the variable H2 changed to 4.98 ft as shown in the Variable Sheet below.

```
==================== VARIABLE SHEET =====================
```

St	Input	Name	Output	Unit	Comment
		Q1	409.70703		
		Q2	147.91338		
		Q3	319.72753		
		Q4	57.933873		
		Q5	97.591822		
		Q6	50.321558		
		Y1	4.5260106		
		Y2	4.8257101		
		Y3	4.4344654		
		Y4	4.9260106		
		Y5	4.6921528		
		Y6	4.9035651		
	5	H1			
	4.98	H2			
	.05	KE			
	.04	Kj			
	64.4	g2			
	10	b1			
	1.5	m1			
	10	b2			
	8	b3			
	1	m3			
	.014	n3			
	.0008	So3			
	4	b4			
	.5	m4			
	5	b5			
	.014	n5			
	.0008	So5			
	5	b6			
	.6	Yd6			

Now notice that decreasing H_2 to 4.98 ft has caused the depth in Channel 4 to increase from Y_4=4.704 to 4.926 ft, or .222 ft. This is caused by the flow rate being decreased in Channel 4, and again the change in velocity head is greater than the change in the reservoir's water surface elevation.

<u>Minimum elevation of Reservoir 2 so flow does not reverse itself and flow into this reservoir.</u>

To solve this problem the flow rate Q4 is set to zero, and the unknown is H2. The answer is that should the water surface elevation in Reservoir 2 fall below 4.968 ft, then water will start to flow into this reservoir from Reservoir 1.

VARIABLE SHEET

St	Input	Name	Output	Unit	Comment
		Q1	465.93144		
		Q2	147.56295		
		Q3	318.3685		
	0	Q4			
		Q5	97.307217		
		Q6	50.255729		
		Y1	4.2862369		
		Y2	4.8142933		
		Y3	4.4242874		
		Y4	4.9660113		
		Y5	4.6810422		
		Y6	4.8920041		
	5	H1			
		H2	4.9660113		
	.05	KE			
	.04	Kj			

```
   64.4      g2
   10        b1
   1.5       m1
   10        b2
   8         b3
   1         m3
   .014      n3
   .0008     So3
   4         b4
   .5        m4
   5         b5
   .014      n5
   .0008     So5
   5         b6
   .6        Yd6
```

Maximum H_2 possible

Since a rise in H_2 causes the depth in Channel 4 to fall, the limiting water surface elevation of Reservoir 2 will occur when critical flow occurs in Channel 4. Therefore the critical flow equation is added to the other 12 equations an H_2 is added as an additional unknown, making 13 equations and 13 unknowns as shown below. The limiting water surface elevation for reservoir 2 (that supplies channel 4) is 5.0182 ft.

TK-Solver Model prb2_91B.TK

```
═══════════════════ VARIABLE SHEET ════════════════════
St Input───── Name──── Output──── Unit────── Comment──────────
              Q1       513.7354
              Q2       146.56079
              Q3       314.49862
              Q4       -52.67598
              Q5       96.493806
              Q6       50.066986
              Y1       3.6005087
              Y2       4.7816424
              Y3       4.3951793
              Y4       4.9747256
              Y5       4.6492715
              Y6       4.8589376
   5          H1
              H2       5.018162
   .05        KE
   .04        Kj
   64.4       g2
   10         b1
   1.5        m1
   10         b2
   8          b3
   1          m3
   .014       n3
   .0008      So3
   4          b4
   .5         m4
   5          b5
   .014       n5
   .0008      So5
   5          b6
   .6         Yd6

═════════════════ RULE SHEET ═════════════════════
```

```
S Rule─────────────────────────────────────────────────────
* Q1+Q4-Q3-Q2=0
* Q2-Q5-Q6=0
* H1-Y1-(1+KE)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* H2-Y4-(1+KE)*(Q4/((b4+m4*Y4)*Y4))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(1+Kj)*(Q2/(b2*Y2))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-abs(Q4)*Q4/(((b4+m4*Y4)*Y4))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(1+Kj)*(Q3/((b3+m3*Y3)*Y3))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Y5-(1+Kj)*(Q5/(b5*Y5))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Y6-(1+Kj)*(Q6/(b6*Y6))^2/g2=0
* Y6+(Q6/(b6*Y6))^2/g2-Yd6-(Q6/(b6*Yd6))^2/g2=0
* n3*Q3-1.486*((b3+m3*Y3)*Y3)^1.6666667/(b3+sqrt(m3^2+1)*2*Y3)^.6666667*sqrt(So3)=0
* n5*Q5-1.486*(b5*Y5)^1.6666667/(b5+2*Y5)^.6666667*sqrt(So5)=0
* Q1^2*(b1+2*m1*Y1)-32.2*((b1+m1*Y1)*Y1)^3=0
```

Below the TK-Solver model PRB2_91D.TK is utilized to "list solve" the variables over the range of water surface elevation for reservoir 2 (that feeds Channel 4) from 4.966 to 5.018 ft. (The tables are done in three parts because of the limiting width that TK-Solver has when printing a table that is too wide.)

```
═══════════════════ VARIABLE SHEET ═══════════════════
St Input───── Name──── Output──── Unit────── Comment──────────
LG 272.10239 Q1
LG 148.35117 Q2
LG 321.42956 Q3
LG 197.67834 Q4
LG 97.947492 Q5
LG 50.403673 Q6
LG 4.825544  Y1
LG 4.8399723 Y2
LG 4.4471803 Y3
LG 3.6136517 Y4
LG 4.7060338 Y5
LG 4.9180066 Y6
   5         H1
L  4.966     H2
   .05       KE
   .04       Kj
   64.4      g2
   10        b1
   1.5       m1
   10        b2
   8         b3
   1         m3
   .014      n3
   .0008     So3
   4         b4
   .5        m4
   5         b5
   .014      n5
   .0008     So5
   5         b6
   .6        Yd6
 ═══════════════════ RULE SHEET ═══════════════════════
S Rule─────────────────────────────────────────────────────
  Q1+Q4-Q3-Q2=0
  Q2-Q5-Q6=0
  H1-Y1-(1+KE)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
  H2-Y4-(1+KE)*(Q4/((b4+m4*Y4)*Y4))^2/g2=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(1+Kj)*(Q2/(b2*Y2))^2/g2=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-(Q4/((b4+m4*Y4)*Y4))^2/g2=0
  Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(1+Kj)*(Q3/((b3+m3*Y3)*Y3))^2/g2=0
  Y2+(Q2/(b2*Y2))^2/g2-Y5-(1+Kj)*(Q5/(b5*Y5))^2/g2=0
  Y2+(Q2/(b2*Y2))^2/g2-Y6-(1+Kj)*(Q6/(b6*Y6))^2/g2=0
  Y6+(Q6/(b6*Y6))^2/g2-Yd6-(Q6/(b6*Yd6))^2/g2=0
  n3*Q3-1.486*((b3+m3*Y3)*Y3)^1.6666667/(b3+sqrt(m3^2+1)*2*Y3)^.6666667*sqrt(So3)=0
  n5*Q5-1.486*(b5*Y5)^1.6666667/(b5+2*Y5)^.6666667*sqrt(So5)=0
 ═════════════════════ TABLE:  ═══════════════════════════
```

Title:

Element	H2	Y1	Y2	Y3	Y4	Y5
1	4.966	4.28599997	4.81428203	4.42427729	4.96599997	4.68103117
2	4.96972571	4.36078908	4.8178431	4.42745199	4.9662748	4.68449666
3	4.97345143	4.4278552	4.82103644	4.43029886	4.95882663	4.68760437
4	4.97717714	4.48672067	4.82383931	4.43279763	4.94317781	4.69033212
5	4.98090286	4.53766185	4.82626486	4.43496002	4.91960471	4.69269272
6	4.98462857	4.58139806	4.82834735	4.43681658	4.88882663	4.69471947
7	4.98835429	4.61881956	4.83012916	4.43840508	4.85173385	4.69645361
8	4.99208	4.65081785	4.83165275	4.43976337	4.80921785	4.69793646
9	4.99580571	4.67820342	4.83295671	4.44092586	4.76208913	4.69920556
10	4.99953143	4.70167753	4.83407442	4.44192231	4.71104896	4.7002934
11	5.00325714	4.72183153	4.83503405	4.44277782	4.65668867	4.70122739
12	5.00698286	4.73915771	4.83585903	4.4435133	4.59950057	4.70203033
13	5.01070857	4.75406366	4.83656877	4.44414605	4.53989223	4.70272112
14	5.01443429	4.76688639	4.83717932	4.44469036	4.47820068	4.70331536
15	5.01816	4.77790487	4.83770396	4.44515808	4.41470487	4.703826

TABLE:

Title:

Element	H2	Q1	Q2	Q3	Q4	Q5
1	4.966	465.972934	147.5626	318.367156	-.04317778	97.3069361
2	4.96972571	451.650027	147.671905	318.79073	14.812608	97.3956984
3	4.97345143	436.510326	147.769923	319.170817	30.4304145	97.4753026
4	4.97717714	421.164977	147.855956	319.504625	46.1956039	97.5451792
5	4.98090286	406.104096	147.930409	319.793645	61.6199571	97.6056538
6	4.98462857	391.66611	147.994331	320.041896	76.3701166	97.6575785
7	4.98835429	378.055985	148.049024	320.254385	90.2474246	97.7020087
8	4.99208	365.380023	148.095791	320.436139	103.151907	97.7400018
9	4.99580571	353.678685	148.135817	320.591735	115.048867	97.7725194
10	4.99953143	342.951401	148.170126	320.725139	125.943864	97.8003935
11	5.00325714	333.173491	148.199582	320.839698	135.865789	97.8243259
12	5.00698286	324.307047	148.224906	320.9382	144.856059	97.8449008
13	5.01070857	316.307702	148.246692	321.022956	152.961946	97.8626021
14	5.01443429	309.128731	148.265433	321.095876	160.232578	97.8778298
15	5.01816	302.723475	148.281537	321.158542	166.716604	97.8909151

TABLE:

Title:

Element	H2	Y6	Q6
1	4.966	4.89199272	50.2556638
2	4.96972571	4.89559882	50.2762062
3	4.97345143	4.8988325	50.2946203
4	4.97717714	4.90167073	50.3107771
5	4.98090286	4.90412686	50.3247547
6	4.98462857	4.90623558	50.3367523
7	4.98835429	4.90803982	50.3470154
8	4.99208	4.90958258	50.3557895
9	4.99580571	4.91090293	50.3632975
10	4.99953143	4.91203469	50.3697323
11	5.00325714	4.91300638	50.3752563
12	5.00698286	4.91384172	50.3800047
13	5.01070857	4.91456037	50.3840895
14	5.01443429	4.91517859	50.3876032
15	5.01816	4.91570981	50.3906222

121.

The same channel system as in Problem 119 except that a gate is placed in Channel 3 that causes a depth of Y_{3d} downstream from it equal to 0.7 m. Solve for the depths and flowrates in all channels.

A gate is placed in Channel 3 with Y_{3d}=0.7 ft

```
TK-Solver Model PRB2 91C.TK
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit----- Comment---------
              Q1       182.10061
              Q2       148.49847
              Q3       101.2836
              Q4       67.681455
              Q5       98.067197
              Q6       50.431272
              Y1       4.9263274
              Y2       4.8447711
              Y3       4.9563179
              Y4       4.9263274
              Y5       4.7107046
              Y6       4.9228655
   5          H1
   5          H2
   .05        KE
   .04        Kj
   64.4       g2
   10         b1
   1.5        m1
   10         b2
   8          b3
   1          m3
   .014       n3
   .0008      So3
   4          b4
   .5         m4
   5          b5
   .014       n5
   .0008      So5
   5          b6
   .6         Yd6
   .7         Yd3

==================== RULE SHEET ========================
S Rule-----------------------------------------------------
* Q1+Q4-Q3-Q2=0
* Q2-Q5-Q6=0
* H1-Y1-(1+KE)*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* H2-Y4-(1+KE)*(Q4/((b4+m4*Y4)*Y4))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y2-(1+Kj)*(Q2/(b2*Y2))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y4-abs(Q4)*Q4/(((b4+m4*Y4)*Y4))^2/g2=0
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2-Y3-(1+Kj)*(Q3/((b3+m3*Y3)*Y3))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Y5-(1+Kj)*(Q5/(b5*Y5))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Y6-(1+Kj)*(Q6/(b6*Y6))^2/g2=0
* Y6+(Q6/(b6*Y6))^2/g2-Yd6-(Q6/(b6*Yd6))^2/g2=0
* Y3+(Q3/((b3+m3*Y3)*Y3))^2/g2-Yd3-(Q3/((b3+m3*Yd3)*Yd3))^2/g2=0
* n5*Q5-1.486*(b5*Y5)^1.6666667/(b5+2*Y5)^.6666667*sqrt(So5)=0
```

MathCAD Model PRB2_120.MCD

Variables $b1 := 5 \quad m1 := 1.3 \quad H := 3 \quad Ke := .15 \quad n2 := .013 \quad So2 := .00085 \quad z13 := 1.2 \quad D := 1.5$
$m2 := .5773502 \quad Q1 := 33 \quad Q2 := 25 \quad Q3 := 8 \quad b2 := 2.8 \quad Y2 := 2.4 \quad Y1 := 2.9 \quad Y3 := 1.2 \quad beta := 2$
$g := 9.81 \quad dz := 1.2 \quad A1 := 12 \quad A2 := 4 \quad A3 := 1.4$

Given

$A1 = (b1 + m1 \cdot Y1) \cdot Y1 \qquad A2 = (b2 + m2 \cdot Y2) \cdot Y2 \qquad A3 = .25 \cdot D^2 \cdot (beta - \cos(beta) \cdot \sin(beta))$

$\cos(beta) = 1 - 2 \cdot \frac{Y3}{D} \qquad Q1 = Q2 + Q3 \qquad H = Y1 + (1 + Ke) \cdot \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g}$

$Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g} = Y2 + \frac{\left(\frac{Q2}{A2}\right)^2}{2 \cdot g} \qquad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g} = Y3 + \frac{\left(\frac{Q3}{A3}\right)^2}{2 \cdot g} + dz \qquad Q3^2 \cdot (D \cdot \sin(beta)) = g \cdot A3^3$

$Y2 = .8660254 b2 \qquad n2 \cdot Q2 \cdot (b2 + 2 \cdot m2 \cdot Y2)^{.6666667} = ((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667} \cdot \sqrt{So2}$

find(Q1, Q2, Q3, Y1, Y2, Y3, b2, beta , A1 , A2 , A3) =

	0
0	38.798
1	33.676
2	5.123
3	2.858
4	2.415
5	1.178
6	2.789
7	2.177
8	24.906
9	10.105
10	1.488

122.

Water is taken from a reservoir whose water surface is 6 feet above a large channel bottom that branches into several channels as shown in the plan view of the system sketched below. The properties of the channels are given in the table below. The distances between branches are small enough that losses can be ignored, except that the entrance loss from the reservoir which has a loss coefficient $K_e = 0.2$. A gate exist downstream in channel 5 that produces a depth of $Y_d5 = 0.8$ ft downstream from it. An 8-inch diameter pipe also takes water from channel 1 near where it branches and delivers the water at 40 psi at its end where the elevation is 300 ft. The pipe is 2000 ft long and has an equivalent sand roughness for use in the Darcy-Weisbach, Colebrook-White equations of e = .002 inches. The elevation of the bottoms of all the channels where they branch is 400 ft.

There are 14 unknown variable to solve for. They are:
$Q_1, Q_2, Q_3, Q_4, Q_5, Q_6, Q_p, Y_1, Y_2, Y_3, Y_4, Y_5, Y_6$, & f
in which Q_p is the flow rate in the pipe.

The 14 equations need are:
2 Continuity equations at the two junctions
  The headloss equation for the pipe which consist of: $400+Y_1-392.3077 - f(L/D)(Q/A)_p^2/(2g)=0$
  The Colebrook-White equation to solve for the friction factor f
  The energy equation at the reservoir
3 Energy equations between Channel 1 and Channels 2, 3 and 6
2 Energy equations between Channel 3 and Channels 4 and 5
1 Energy equation across the gate in Channel 5
3 Mannings equations for channel 2, 4 and 6.
The details in writing these equations can be seen in the TK-Solver model below.

```
TK-Solver Model CHANNP6.TK to solve 6 channel system with a pipe at the junction of the
channels
==================== VARIABLE SHEET ======================
St Input----- Name---- Output---- Unit------ Comment---------
              Q1       802.24849
              Q2       393.9381
              Q3       90.202259
              Q6       316.70105
              Qp       1.4070951
              Q4       46.585502
              Q5       43.616756
              Y1       5.5715682
              f        .01752276
   6          H
   .666667    D
   64.4       g2
              E1       5.9285947
   20         b1
   1.8        m1
              Y2       5.2300478
   6          b2
   1          m2
              Y3       5.8713288
   8          b3
              Y4       5.8179758
   3          b4
              Y5       5.8320943
   3          b5
   .8         Yd5
              Y6       5.4478584
   5          b6
```

```
   1          m6
   .014       n2
   .001       So2
   .014       n4
   .0005      So4
   .014       n6
   .0007      So6


==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Q1-Q2-Q3-Q6-Qp=0
* Q3-Q4-Q5=0
* 7.6923+Y1-382.3144*f*Qp^2=0
* 1/sqrt(f)-1.14+2*log(.00025+6.902864e-5/(Qp*sqrt(f)))=0
* H-Y1-1.2*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2=0
* E1-Y3-(Q3/(b3*Y3))^2/g2=0
* Y3+(Q3/(b3*Y3))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
* Y3+(Q3/(b3*Y3))^2/g2-Y5-(Q5/(b5*Y5))^2/g2=0
* Y5-Yd5+Q5^2/g2*(1/(b5*Y5)^2-1/(b5*Yd5)^2)=0
* E1-Y6-(Q6/((b6+m6*Y6)*Y6))^2/g2=0
* n2*Q2*(b2+2*Y2*sqrt(m2^2+1))^.6666667-1.486*sqrt(So2)*((b2+m2*Y2)*Y2)^1.66666
* n4*Q4*(b4+2*Y4)^.6666667-1.486*sqrt(So4)*(b4*Y4)^1.666667=0
* n6*Q6*(b6+2*Y6*sqrt(m6^2+1))^.6666667-1.486*sqrt(So6)*((b6+m6*Y6)*Y6)^1.66666
```

123.

In the previous 6 channel pipe system, it is desired that channel 4 delivers 100 cfs. What size should it be. In addition the pipe size is to be increased to 2 ft diameter (same e, etc.) and the gate in channel 5 is raised so the depth downstream from it is 1.1 ft. Now what are the flowrates and depths in the channels (and the width need for channel 4.)

From the previous problem Channel 4 has Q_4=100 cfs, pipe dia = 2 ft, Y_{d5}=1.1 ft, Find b_4

TK-Solver Model CHANNP6A.TK

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment---------
              Q1       866.37669
              Q2       391.88142
              Q3       158.09751
              Q6       314.99604
              Qp       1.4017269
   100        Q4
              Q5       58.097506
              Y1       5.4770277
              f        .01753138
   6          H
   2          D
   64.4       g2
              E1       5.9128379
   20         b1
   1.8        m1
              Y2       5.2161568
   6          b2
   1          m2
              Y3       5.7280059
   8          b3
              Y4       5.7277798
              b4       5.0572755
              Y5       5.7358294
   3          b5
   1.1        Yd5
              Y6       5.4334048
   5          b6
   1          m6
   .014       n2
   .001       So2
   .014       n4
   .0005      So4
   .014       n6
   .0007      So6


==================== RULE SHEET ====================
S Rule------------------------------------------------------
  Q1-Q2-Q3-Q6-Qp=0
  Q3-Q4-Q5=0
  7.6923+Y1-382.3144*f*Qp^2=0
  1/sqrt(f)-1.14+2*log(.00025+6.902864e-5/(Qp*sqrt(f)))=0
  H-Y1-1.2*(Q1/((b1+m1*Y1)*Y1))^2/g2=0
  E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
  E1-Y2-(Q2/((b2+m2*Y2)*Y2))^2/g2=0
  E1-Y3-(Q3/(b3*Y3))^2/g2=0
  Y3+(Q3/(b3*Y3))^2/g2-Y4-(Q4/(b4*Y4))^2/g2=0
  Y3+(Q3/(b3*Y3))^2/g2-Y5-(Q5/(b5*Y5))^2/g2=0
  Y5-Yd5+Q5^2/g2*(1/(b5*Y5)^2-1/(b5*Yd5)^2)=0
  E1-Y6-(Q6/((b6+m6*Y6)*Y6))^2/g2=0
  n2*Q2*(b2+2*Y2*sqrt(m2^2+1))^.6666667-1.486*sqrt(So2)*((b2+m2*Y2)*Y2)^1.66666
  n4*Q4*(b4+2*Y4)^.6666667-1.486*sqrt(So4)*(b4*Y4)^1.666667=0
  n6*Q6*(b6+2*Y6*sqrt(m6^2+1))^.6666667-1.486*sqrt(So6)*((b6+m6*Y6)*Y6)^1.66666
```

124.

A trapezoidal channel with b_1 = 10 ft and m_1 = 1 receives water from a reservoirs whose water surface is H = 5 ft above the channel bottom. A short distance downstream from the entrance the channel branches into channel 2, 3 and 4. Channels 2 and 4 have gates a short distance downstream that cause downstream depths of Y_{d2} = 1.5 ft and Y_{d4} = 1.2 ft, respectively. These channels are rectangular with b_2 = 4 ft, and b_4 = 3 ft, respectively. Channel 3 consists of a pipe with a diameter D_3 = 8 ft, a Mannings n_3 = .014, and a bottom slope S_{o3} = .0008. There is a diversion at the junction of the channel of Q_{out} = 50 cfs. At the branch all channel have the same elevation. Write out the equations whose solution will provide the flowrates and depths in all channels.

There are 8 unknown variables (4 flow rates and 4 depths):
$Q_1,Q_2,Q_3,Q_4,Y_1,Y_2,Y_3$ and Y_4

Equations needed to solve these 8 unknowns are:

$F_1=Q_1-Q_2-Q_3-Q_4- 50 = 0$ Continuity at junction

$F_2=H -Y_1-(Q/A)_1^2/(2g) = 0$ Energy at Reservoir

$F_3=Y_1+(Q/A)_1^2/(2g) - Y_2+(Q/A)_2^2/(2g) = 0$ Energy between 1 & 2

$F_4=Y_1+(Q/A)_1^2/(2g) - Y_3+(Q/A)_3^2/(2g) = 0$ Energy between 1 & 3

$F_5=Y_1+(Q/A)_1^2/(2g) - Y_4+(Q/A)_4^2/(2g) = 0$ Energy between 1 & 4

$F_6=Y_2+(Q/A)_2^2/(2g) - Y_{d2}+(Q/A)_{d2}^2/(2g) = 0$ Energy across gate in 2

$F_7=Y_4+(Q/A)_4^2/(2g) - Y_{d4}+(Q/A)_{d4}^2/(2g) = 0$ Energy across gate in 4

$F_8=n_3Q_3P_3^{2/3}-C_uA_3^{5/3}S_{o3}^{1/2} = 0$ Uniform flow in Channel 3

The following TK-Solver model implements these equations in the Rule Sheet and gives a solution in the Variable Sheet.

```
EXMPRB4_TK
==================== VARIABLE SHEET ======================
St Input───── Name──── Output─── Unit────── Comment──────────
              Q1       347.33885
              Q2       90.079964
              Q3       150.94218
              Q4       56.316704
              E1       5
              Y1       4.57984
   10         b1
   1          m1
   64.4       g2
              Y2       4.6331406
   4          b2
              Y3       4.6058926
              A3       29.961282
   3          b4
              Y4       4.7583212
```

```
            beta    1.7228548
   8        D3
   1.5      Yd2
   1.2      Yd4
   .014     n3
   1.486    Cu
   .0008    So3

==================== RULE SHEET =========================
S Rule-----------------------------------------------------------------------------
* Q1-Q2-Q3-Q4-50=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1=5
* E1-Y2-(Q2/(b2*Y2))^2/g2=0
* beta=acos(1-2*Y3/D3)
* A3=.25*D3^2*(beta-cos(beta)*sin(beta))
* E1-Y3-(Q3/A3)^2/g2=0
* E1-Y4-(Q4/(b4*Y4))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Yd2-(Q2/(Yd2*b2))^2/g2=0
* Y4+(Q4/(b4*Y4))^2/g2-Yd4-(Q4/(Yd4*b4))^2/g2=0
* n3*Q3*(D3*beta)^.6666667-Cu*A3^1.66666667*sqrt(So3)=0
```

125.

For the branched channel system of the previous problem make up a table that gives the discharges and depths in the 4 channels as a function of the gate setting Y_{G2} in channel 2. The contraction coefficient for this gate is C_c = .6, and the channel downstream from the gate is rectangular with b_2 = 4 ft, n_2 = .014, and S_{o2} = .0009.

Wanted: A table of solution to the channel system in the previous problem in which the position of the gate in Channel 2 varies.

When the gate is completely opened the solution is obtained from the TK-Solver Model PRB2_97.TK. Notice in this model that rather than the equation that equates energy across the gate in Channel 2 that the uniform flow equation (Mannings equation) is used by adding it to the Rules Sheet and commenting out the energy equation across the gate.

TK-Solver Model PRB2_97.TK

```
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit---- Comment---------
             Q1      333.27246
             Q2      76.013578
             Q3      150.94218
             Q4      56.316704
             E1      5
             Y1      4.6225012
   10        b1
   1         m1
   64.4      g2
             Y2      4.751635
   4         b2
             Y3      4.6058926
             A3      29.961282
   3         b4
             Y4      4.7583212
             beta    1.7228548
   8         D3
   1.5       Yd2
   1.2       Yd4
   .014      n3
   1.486     Cu
   .0008     So3
   .014      n2
   .0009     So2

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Q1-Q2-Q3-Q4-50=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1=5
* E1-Y2-(Q2/(b2*Y2))^2/g2=0
* beta=acos(1-2*Y3/D3)
* A3=.25*D3^2*(beta-cos(beta)*sin(beta))
* E1-Y3-(Q3/A3)^2/g2=0
* E1-Y4-(Q4/(b4*Y4))^2/g2=0
* n2*Q2*(b2+2*Y2)^.6666667-Cu*(b2*Y2)^1.6666667*sqrt(So2)=0
C Y2+(Q2/(b2*Y2))^2/g2-Yd2-(Q2/(Yd2*b2))^2/g2=0
* Y4+(Q4/(b4*Y4))^2/g2-Yd4-(Q4/(Yd4*b4))^2/g2=0
* n3*Q3*(D3*beta)^.6666667-Cu*A3^1.66666667*sqrt(So3)=0
```

The TK-Solver Model PRB2_97A.TK, given below is used to solve the problem for gate setting from 2.0 ft to .0833 ft. (Actually the depth downstream of the gate Yd2 were specified that are given in the last column of the table below. Notice that Yd2 = Cc time the gate height) One might modify this model to use the "list" feature to obtain the table of solutions.

TK-Solver Model PRB2_97A.TK

```
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit---- Comment---------
             Q1      260.82977
             Q2      3.5708822
```

```
            Q3      150.94218
            Q4      56.316704
            E1      5
            Y1      4.7894512
   10       b1
   1        m1
   64.4     g2
            Y2      4.9995049
   4        b2
            Y3      4.6058926
            A3      29.961282
   3        b4
            Y4      4.7583212
            beta    1.7228548
   8        D3
   .05      Yd2
   1.2      Yd4
   .014     n3
```

The list solve capability of TK-Solver is utilized below to solve the problem in which Yd2 changes from .05 (Y_G = .0833) to Yd2 = 1.5 (Y_G = 2.0). The table is printed in two parts because TK-Solver doesnot print a wide table.

TK Solver Model   PRB2 97D.TK

```
==================== VARIABLE SHEET ====================
St Input──── Name─── Output─── Unit───── Comment────────
LG 260.82977 Q1
LG 3.5708822 Q2
LG 150.94218 Q3
LG 56.316704 Q4
             E1      5
LG 4.7894512 Y1
   10        b1
   1         m1
   64.4      g2
LG 4.9995049 Y2
   4         b2
LG 4.6058926 Y3
             A3      29.961282
   3         b4
LG 4.7583212 Y4
LG 1.7228548 beta
   8         D3
L  .05       Yd2
   1.2       Yd4
   .014      n3
   1.486     Cu
   .0008     So3

==================== RULE SHEET ========================
S Rule──────────────────────────────────────────────────
  Q1-Q2-Q3-Q4-50=0
  E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
  E1=5
  E1-Y2-(Q2/(b2*Y2))^2/g2=0
  beta=acos(1-2*Y3/D3)
  A3=.25*D3^2*(beta-cos(beta)*sin(beta))
  E1-Y3-(Q3/A3)^2/g2=0
  E1-Y4-(Q4/(b4*Y4))^2/g2=0
C n2*Q2*(b2+2*Y2)^.6666667-Cu*(b2*Y2)^1.6666667*sqrt(So2)=0
  Y2+(Q2/(b2*Y2))^2/g2-Yd2-(Q2/(Yd2*b2))^2/g2=0
  Y4+(Q4/(b4*Y4))^2/g2-Yd4-(Q4/(Yd4*b4))^2/g2=0
  n3*Q3*(D3*beta)^.6666667-Cu*A3^1.66666667*sqrt(So3)=0

==================== TABLE:  ===========================
Title:
```

Element	Yd2	Q1	Y1	Q2	Y2	Q3
1	.05	260.829767	4.78945117	3.57088224	4.9995049	150.942181
2	.1	264.364489	4.78292348	7.10560342	4.99803846	150.942181
3	.15	267.862771	4.77632453	10.6038861	4.99562736	150.942181

4	.2	271.324332	4.7696561	14.065447	4.99229628	150.942181
5	.25	274.748882	4.76292001	17.4899971	4.98806812	150.942181
6	.3	278.136126	4.75611819	20.8772412	4.98296411	150.942181
7	.35	281.485763	4.7492526	24.2268776	4.97700396	150.942181
8	.4	284.797484	4.74232531	27.5385984	4.97020598	150.942181
9	.45	288.070974	4.73533845	30.8120885	4.96258721	150.942181
10	.5	291.305912	4.72829425	34.0470263	4.95416346	150.942181
11	.55	294.501968	4.72119502	37.2430826	4.94494945	150.942181
12	.6	297.658806	4.7140432	40.3999208	4.93495887	150.942181
13	.65	300.776082	4.70684129	43.5171966	4.92420443	150.942181
14	.7	303.853443	4.69959193	46.5945576	4.91269796	150.942181
15	.75	306.890528	4.69229787	49.6316431	4.90045041	150.942181
16	.8	309.886969	4.68496198	52.6280838	4.88747197	150.942181
17	.85	312.842386	4.67758728	55.5835011	4.87377205	150.942181
18	.9	315.756393	4.6701769	58.4975076	4.85935936	150.942181
19	.95	318.628591	4.66273413	61.3697059	4.84424192	150.942181
20	1	321.458574	4.65526242	64.1996885	4.82842712	150.942181
21	1.05	324.245923	4.64776538	66.9870376	4.81192175	150.942181
22	1.1	326.99021	4.64024679	69.7313244	4.79473197	150.942181
23	1.15	329.690994	4.6327106	72.4321089	4.77686343	150.942181
24	1.2	332.347824	4.62516098	75.0889393	4.75832119	150.942181
25	1.25	334.960237	4.61760226	77.7013513	4.73910981	150.942181
26	1.3	337.527753	4.61003902	80.2688682	4.71923335	150.942181
27	1.35	340.049885	4.60247603	82.7909995	4.69869536	150.942181
28	1.4	342.526126	4.59491831	85.2672411	4.67749891	150.942181
29	1.45	344.955959	4.58737112	87.6970741	4.65564663	150.942181
30	1.5	347.33885	4.57983997	90.0799645	4.63314065	150.942181

═══════════════════ TABLE: ═══════════════════════════

Title:

Element	Yd2	Y3	Q4	Y4	beta
1	.05	4.60589261	56.3167044	4.75832119	1.72285478
2	.1	4.60589261	56.3167044	4.75832119	1.72285478
3	.15	4.60589261	56.3167044	4.75832119	1.72285478
4	.2	4.60589261	56.3167044	4.75832119	1.72285478
5	.25	4.60589261	56.3167044	4.75832119	1.72285478
6	.3	4.60589261	56.3167044	4.75832119	1.72285478
7	.35	4.60589261	56.3167044	4.75832119	1.72285478
8	.4	4.60589261	56.3167044	4.75832119	1.72285478
9	.45	4.60589261	56.3167044	4.75832119	1.72285478
10	.5	4.60589261	56.3167044	4.75832119	1.72285478
11	.55	4.60589261	56.3167044	4.75832119	1.72285478
12	.6	4.60589261	56.3167044	4.75832119	1.72285478
13	.65	4.60589261	56.3167044	4.75832119	1.72285478
14	.7	4.60589261	56.3167044	4.75832119	1.72285478
15	.75	4.60589261	56.3167044	4.75832119	1.72285478
16	.8	4.60589261	56.3167044	4.75832119	1.72285478
17	.85	4.60589261	56.3167044	4.75832119	1.72285478
18	.9	4.60589261	56.3167044	4.75832119	1.72285478
19	.95	4.60589261	56.3167044	4.75832119	1.72285478
20	1	4.60589261	56.3167044	4.75832119	1.72285478
21	1.05	4.60589261	56.3167044	4.75832119	1.72285478
22	1.1	4.60589261	56.3167044	4.75832119	1.72285478
23	1.15	4.60589261	56.3167044	4.75832119	1.72285478
24	1.2	4.60589261	56.3167044	4.75832119	1.72285478
25	1.25	4.60589261	56.3167044	4.75832119	1.72285478
26	1.3	4.60589261	56.3167044	4.75832119	1.72285478
27	1.35	4.60589261	56.3167044	4.75832119	1.72285478
28	1.4	4.60589261	56.3167044	4.75832119	1.72285478
29	1.45	4.60589261	56.3167044	4.75832119	1.72285478
30	1.5	4.60589261	56.3167044	4.75832119	1.72285478

126.

For the branched channel system of Problem 124 make up a table that gives the discharges and depths in the 4 channels as a function of the gate setting Y_{G4} in channel 4. The contraction coefficient for this gate is C_c = .6, and the channel downstream from the gate is rectangular with b_4 = 3 ft, n_4 = .014, and S_{o4} = .0012.

Wanted: A series of solution to the channel system of the previous problems in which the position of the gate in Channel 4 varies.

Solution: First for the gate completely open the energy equation across this gate is changed to the uniform flow equation in Channel 4. The TK-Solver model to solve this problem is given below

```
TK-Solver Model PRB2 98.TK
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit---- Comment---------
             Q1      348.20058
             Q2      90.079964
             Q3      150.94218
             Q4      57.178434
             E1      5
             Y1      4.5770841
   10        b1
   1         m1
   64.4      g2
             Y2      4.6331406
   4         b2
             Y3      4.6058926
             A3      29.961282
   3         b4
             Y4      4.7499943
             beta    1.7228548
   8         D3
   1.5       Yd2
   1.2       Yd4
   .014      n3
   1.486     Cu
   .0008     So3
   .0012     So4
   .014      n4

==================== RULE SHEET =========================
S Rule--------------------------------------------------
* Q1-Q2-Q3-Q4-50=0
* E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
* E1=5
* E1-Y2-(Q2/(b2*Y2))^2/g2=0
* beta=acos(1-2*Y3/D3)
* A3=.25*D3^2*(beta-cos(beta)*sin(beta))
* E1-Y3-(Q3/A3)^2/g2=0
* E1-Y4-(Q4/(b4*Y4))^2/g2=0
* Y2+(Q2/(b2*Y2))^2/g2-Yd2-(Q2/(Yd2*b2))^2/g2=0
C Y4+(Q4/(b4*Y4))^2/g2-Yd4-(Q4/(Yd4*b4))^2/g2=0
* n4*Q4*(b4+2*Y4)^.66666667-Cu*(b4*Y4)^1.6666667*sqrt(So4)=0
* n3*Q3*(D3*beta)^.6666667-Cu*A3^1.66666667*sqrt(So3)=0
```

For gate setting from 2.0 ft to .0833 ft the TK-Solver Model PRB2_98A.TK has been used. In this model the "list" feature has been used to obtain the table of solutions below. In this table the results of the previous solution for the gate wide open, and the solution that follow for the gate completely closed have been added.

```
TK-Solver Model PRB2 98A.TK
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit---- Comment---------
```

```
LG 347.33885 Q1
LG 90.079964 Q2
LG 150.94218 Q3
LG 56.316704 Q4
LG 4.57984   Y1
LG 4.6331406 Y2
LG 4.6058926 Y3
LG 4.7583212 Y4
L  1.2       Yd4
             E1      5
   10        b1
   1         m1
   64.4      g2
   4         b2
             A3      29.961282
   3         b4
             beta    1.7228548
   8         D3
   1.5       Yd2
   .014      n3
   1.486     Cu
   .0008     So3

==================== RULE SHEET ========================
S Rule--------------------------------------------------
  Q1-Q2-Q3-Q4-50=0
  E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
  E1=5
  E1-Y2-(Q2/(b2*Y2))^2/g2=0
  beta=acos(1-2*Y3/D3)
  A3=.25*D3^2*(beta-cos(beta)*sin(beta))
  E1-Y3-(Q3/A3)^2/g2=0
  E1-Y4-(Q4/(b4*Y4))^2/g2=0
  Y2+(Q2/(b2*Y2))^2/g2-Yd2-(Q2/(Yd2*b2))^2/g2=0
  Y4+(Q4/(b4*Y4))^2/g2-Yd4-(Q4/(Yd4*b4))^2/g2=0
  n3*Q3*(D3*beta)^.6666667-Cu*A3^1.66666667*sqrt(So3)=0

==================== TABLE: Prb126 =====================
Title:
```

Y_G	Q1	Y1	Q2	Y2	Q3	Y3	Q4	Y4	Yd4
Open	348.20	4.577	90.08	4.633	150.94	4.606	57.18	4.750	
2.0	347.34	4.58	90.08	4.633	150.94	4.606	56.317	4.758	1.2
1.667	339.17	4.605	90.08	4.633	150.94	4.606	48.15	4.828	1
1.333	330.49	4.63	90.08	4.633	150.94	4.606	39.471	4.887	.8
1.000	321.32	4.656	90.08	4.633	150.94	4.606	30.3	4.935	.6
0.667	311.68	4.681	90.08	4.633	150.94	4.606	20.654	4.97	.4
0.333	301.57	4.705	90.08	4.633	150.94	4.606	10.549	4.992	.2
0.167	296.35	4.717	90.08	4.633	150.94	4.606	5.3292	4.998	.1
0.8333	293.7	4.723	90.08	4.633	150.94	4.606	2.6782	5	.05
Closed	291.02	4.729	90.08	4.633	150.94	4.606	0.0	5.000	.0

For the case in which the gate in Channel 4 is completely closed the flow rate and depth for Channel 4 are removed as well as the equation across the gate. This model is given below

TK-Solver PRB2_98B.TK

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment-----------
             Q1      291.02215
             Q2      90.079964
             Q3      150.94218
             E1      5
             Y1      4.7289178
   10        b1
   1         m1
   64.4      g2
             Y2      4.6331406
   4         b2
             Y3      4.6058926
             A3      29.961282
   3         b4
             beta    1.7228548
```

```
   8         D3
   1.5       Yd2
   .014      n3
   1.486     Cu
   .0008     So3
   .0012     So4
   .014      n4

==================== RULE SHEET ========================
S Rule-----------------------------------------------------
  Q1-Q2-Q3-50=0
  E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
  E1=5
  E1-Y2-(Q2/(b2*Y2))^2/g2=0
  beta=acos(1-2*Y3/D3)
  A3=.25*D3^2*(beta-cos(beta)*sin(beta))
  E1-Y3-(Q3/A3)^2/g2=0
* E1-Y4-(Q4/(b4*Y4))^2/g2=0
  Y2+(Q2/(b2*Y2))^2/g2-Yd2-(Q2/(Yd2*b2))^2/g2=0
  n3*Q3*(D3*beta)^.6666667-Cu*A3^1.66666667*sqrt(So3)=0
```

127.

Repeat problem 124 with all 4 channel with the same sizes and gate settings, etc. but the bottom of channel 3 (the pipe) at its beginning is 1 ft below the bottom of the other channels at the junction position, i.e. Δz_{13} = - 1.0 ft.

Solve the 4 Channel system of Problem 124 except that the pipe Channel 3 drops 1 ft.

The TK-Solver Model EXMPRB4.TK has been modified below to include Dz13, the drop in the bottom position of Channel 3.

TK-Solver Model PB2 127.TK

```
===================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
              Q1       395.03793
              Q2       90.079964
              Q3       198.64126
              Q4       56.316704
              E1       5
              Y1       4.3943574
   10         b1
   1          m1
   64.4       g2
              Y2       4.6331406
   4          b2
              Y3       5.5592079
              A3       37.282896
   3          b4
              Y4       4.7583212
              beta     1.9712129
   8          D3
   1.5        Yd2
   1.2        Yd4
   .014       n3
   1.486      Cu
   .0008      So3
   -1         Dz13

===================== RULE SHEET =========================
S Rule-----------------------------------------------------
  Q1-Q2-Q3-Q4-50=0
  E1=Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2
  E1=5
  E1-Y2-(Q2/(b2*Y2))^2/g2=0
  beta=acos(1-2*Y3/D3)
  A3=.25*D3^2*(beta-cos(beta)*sin(beta))
  E1-Y3-(Q3/A3)^2/g2-Dz13=0
  E1-Y4-(Q4/(b4*Y4))^2/g2=0
  Y2+(Q2/(b2*Y2))^2/g2-Yd2-(Q2/(Yd2*b2))^2/g2=0
  Y4+(Q4/(b4*Y4))^2/g2-Yd4-(Q4/(Yd4*b4))^2/g2=0
  n3*Q3*(D3*beta)^.6666667-Cu*A3^1.66666667*sqrt(So3)=0
```

128.

In the branched channel system of the previous problems, what is the maximum flowrate that can be obtained through channel 3 if its diameter were to be increased? What are the flowrates in the other channels when this condition occurs and the gates are both completely open? What is the minimum diameter that the pipe can have for this maximum flowrate to occur? Solve the flowrates and depths for both $\Delta z_{13} = 0$ and $\Delta z_{13} = -1.0$ ft.

Wanted: The maximum Q for Channel 3 of previous problems? Also diameter? Do for $\Delta z = 0$ and -1 ft.

Solution:

Critical flow in Channel 1 will limit the flow rate of the system of channels. Solve Energy and Critical Flow Equations for Channel 1 simultaneously, with H = 5 ft. Give Q_{c1}=464.91 cfs, and Y_{c1}=3.583 ft.

Next for Channel 2 and 4 solve the Energy and Mannings Equations simultaneously with the head equal to 5 ft to give:

Q_2=75.8c cfs and Y_{o2}=4.752 ft

and

Q_4=57.05 cfs and Y_{o4}=4.750 ft

Thus the flow rate in Channel 3 is Q_3=469.91-75.83-57.05 = 332.03 cfs <==

for Δz=0

For Channel 3 (the circular channel) solve the Energy and Uniform Flow Equations simultaneously for Y and the diameter D to get

Y_3=4.472 ft and D_3= 23.099 ft <==

for Δz=-1

For Channel 3 (the circular channel) solve the Energy and Uniform Flow Equations simultaneously for Y and the diameter D. Now the total specific energy equals 6 ft, rather than 5 ft.

Y_3=5.418 ft and D_3= 13.706 ft <==

129.

Modify the model used to obtain the dimensionless graph of Example Problem 23 so that it can solve the specific energy between section 1 and 2 (with the filled bottom) of a pipe for other variables without the flow being critical as section 2.

Wanted: Add the capability to model for Example Problem 23 so the downstream Froude Number does not need to be unity.

All that is needed is to add F_{r2} and its square to the dimensionless energy and continuity equations as shown in the TK-Solver model PRB2_117.TK below, i.e. PAPER6.TK has had the Rule Sheet modified. The downstream Froude Number has been given a value of F_{r2}=.9 and Yb and Yr solved for varying upstream Froude Numbers. The list solver failed for values of F_{r2} larger than 0.891.

```
===================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit----- Comment---------
   .6         Y1
   1.7721542  B1
   .49202836  A1
   .50217434  Yd1
L  .01        Fr2
LG .54897512  Yr
LG .44221115  Yb
LG 1.6689039  Br
LG .44159578  Ar
LG 1.4549598  Bb
LG .33503915  Ab
L             r6
L             Fr
L             r7
   .9         Fr2s
              Fr22

===================== RULE SHEET ==========================
S Rule------------------------------------------------------
* A1*sqrt(Yd1*Fr2)=Fr2s*(Ar-Ab)*sqrt((Ar-Ab)/sin(Br))
* Y1+Fr2*Yd1/2=Yr+.5*Fr22*(Ar-Ab)/sin(Br)
* Br=acos(1-2*Yr)
* Ar=.25*(Br-cos(Br)*sin(Br))
* Bb=acos(1-2*Yb)
* Ab=.25*(Bb-cos(Bb)*sin(Bb))
* r6=Yb/Y1
* Fr=sqrt(Fr2)
* r7=Yr/Y1
* Fr22=Fr2s^2

===================== TABLE:  ============================
Title:
Element Fr--------- r6--------- r7---------
1       .1          .735734191 .926576003
2       .16673332   .632671451 .902713387
3       .213541565 .570042435 .890722486
4       .251793566 .522994484 .883321998
5       .284956137 .484652391 .878492021
6       .314642654 .45199529  .87534309
7       .34176015  .42339511  .873394814
8       .366878727 .397862497 .872354622
9       .390384426 .374746162 .872029469
10      .412553027 .353592019 .872284425
11      .433589668 .334069451 .873020988
12      .453651849 .315929429 .874164751
13      .472863617 .298979187 .875657948
14      .49132474  .283066114 .8774547
```

15	.509116882	.268067094	.879517875
16	.526307895	.2538812	.881816929
17	.542954878	.240424536	.884326385
18	.55910643	.22762652	.887024735
19	.574804315	.21542713	.889893623
20	.59008474	.203774834	.892917242
21	.604979338	.192625003	.896081857
22	.61951594	.181938681	.899375445
23	.633719181	.171681609	.902787414
24	.647610994	.16182345	.906308367
25	.66121101	.152337154	.909929926
26	.674536878	.143198442	.913644579
27	.687604538	.134385371	.917445558
28	.70042844	.125877962	.921326741
29	.713021739	.117657885	.925282565
30	.725396443	.109708166	.929307958
31	.737563557	.102012925	.933398275
32	.749533188	.094557111	.937549254
33	.761314652	.087326247	.941756966
34	.772916554	.080306127	.946017782
35	.784346862	.073482486	.950328339
36	.795612971	.06684056	.954685514
37	.806721761	.060364509	.959086398
38	.817679644	.054036578	.963528276
39	.828492607	.047835798	.968008606
40	.839166253	.041735857	.972525007
41	.849705831	.035701302	.97707524
42	.860116271	.029680105	.981657198
43	.870402206	.023587064	.986268892
44	.880567999	.017258717	.990908445
45	.890617763	.010277865	.995574078

130.

Develop a model that is capable of solving problems involving the specific energy between section 1 and 2 in a pipe such as in Example Problem 23 in which the bottom of the pipe is filled to a depth Δz, but do this so that the actual, rather than the dimensionless, variables are solved.

Wanted: The area filled in the bottom of the circular channel is computed by the same formula, etc. that provide the area for the flow to a depth Y in a circular channel. Thus the area will by the area computed for a depth Y_r (also measured from the bottom) minus the area computed for a depth Y_b, the depth of fill in the channel bottom.

Energy equation

$$Y_1 + Q^2/(2gA_1^2) = Y_r + Q^2/[2g(A_r-A_b)^2]$$

$F_{r1}^2 = Q^2T_1/(gA_1^3)$, $F_{r2}^2 = Q^2T_r/[g(A_r-A_b)^3]$ or $Q^2 = gA_1^3F_{r1}^2/T_1$ and $Q^2 = g(A_r-A_b)^3F_{r2}^2/T_r$

Thus the Energy equation becomes the following after replacing the Q's:

$Y_1 + Y_{d1}F_{r1}^2/2 = Y_r + Y_{d2}F_{r2}^2/2$ in which $Y_d = A/T$ the hydraulic radius, and $F_{r2}^2 = 1$ and $Y_{d2} = (A_r-A_b)/T_r$, so the energy equation becomes:

$Y_1 + Y_{d1}F_{r1}^2/2 = Y_r + Y_{d2}/2$ for critical flow at the hump

$\beta_b = \cos^{-1}(1-2Y_b/D)$, $A_b = (D^2/4)[\beta_b - \cos\beta_b\sin\beta_b]$

$\beta_r = \cos^{-1}(1-2Y_r/D)$, $A_r = (D^2/4)[\beta_r - \cos\beta_r\sin\beta_r]$

$Y_1 + .125DF_{r1}^2[\beta_1/\sin\beta_1 - \cos\beta_1] = .125D[(\beta_r-\beta_b)/\sin\beta_r - \cos\beta_r + \cos\beta_b]$ <==

Continuity equation $Q_1 = Q_2$

$$g^{1/2}A_1^{1.5}F_{r1}/T_1^{1/2} = g^{1/2}(A_r-A_b)^{1.5}F_{r1}/T_r^{1/2}$$

$$A_1Y_{d1}^{.5}F_{r1} = (A_r-A_b)^{1.5}/T_r^{.5}F_{r2} = (A_r-A_b)Y_{d2}^{.5}F_{r2}$$

$Y_{d1} = A_1/T_1 = .25D^2(\beta_1 - \cos\beta_1\sin\beta_1)/(D\sin\beta_1) = .125D(\beta_1/\sin\beta_1 - \cos\beta_1)$

$Y_{d2} = (A_r-A_b)/T_r = .125D[(\beta_r-\beta_b)/\sin\beta_r - \cos\beta_r + \cos\beta_b]$

when $F_{r2} = 1$ then the continuity equation becomes:

$A_1[Y_{d1}F_{r1}]^{1/2} = (A_r-A_b)[(A_r-A_b)/(D\sin\beta_r)]$ <==

(The following TK-Solver model PRB2_118.TK uses variables with dimensions, rather than the dimensionless variables used in Example Problem 23. Note that about all the changes needed over the model used in the example problem is to add the variable D, and then change the equations (rules) to include this diameter where needed. D has been assigned 1, so the results in the table are the same as obtain from model PAPER6.TK

```
=========== VARIABLE SHEET ===========
St Input---- Name--- Output--- Unit
   .6        Y1
   1.7721542 B1
   .49202836 A1
   .50217434 Yd1
L  .01       Fr2
LG .54897512 Yr
LG .44221115 Yb
LG 1.6689039 Br
LG .44159578 Ar
LG 1.4549598 Bb
LG .33503915 Ab
L            r6
L            Fr
L            r7
   1         D

============= RULE SHEET =============
S Rule-------------------------------
  A1*sqrt(Yd1*Fr2)=(Ar-Ab)*sqrt((Ar-Ab)/(D*sin(Br)))
  Y1+Fr2*Yd1/2=Yr+.5*(Ar-Ab)/(D*sin(Br))
  Br=acos(1-2*Yr/D)
  Ar=.25*D*D*(Br-cos(Br)*sin(Br))
  Bb=acos(1-2*Yb/D)
  Ab=.25*D*D*(Bb-cos(Bb)*sin(Bb))
r6=Yb/Y1
Fr=sqrt(Fr2)
r7=Yr/Y1
```

============ TABLE: =============

Title:

Element	Fr	r6	r7
1	.1	.737018591	.914958538
2	.16673332	.634520787	.886406031
3	.213541565	.572274906	.871492998
4	.251793566	.52554281	.861855741
5	.284956137	.487480849	.855174567
6	.314642654	.45508205	.850428074
7	.34176015	.426725883	.84706462
8	.366878727	.401427647	.844748235
9	.390384426	.378539187	.843257215
10	.412553027	.357608681	.842436586
11	.433589668	.338307236	.842173194
12	.453651849	.320387205	.842381553
13	.472863617	.303656974	.842995278
14	.49132474	.287964936	.843961636
15	.509116882	.273188878	.845237934
16	.526307895	.25922871	.846789047
17	.542954878	.246001341	.848585666
18	.55910643	.233436979	.850603042
19	.574804315	.221476397	.852820049
20	.59008474	.210068877	.855218482
21	.604979338	.199170647	.857782524
22	.61951594	.188743657	.860498326
23	.633719181	.178754626	.863353678
24	.647610994	.169174282	.86633775
25	.66121101	.159976755	.869440882
26	.674536878	.151139075	.87265441
27	.687604538	.142640777	.875970531
28	.70042844	.134463563	.87938218
29	.713021739	.126591029	.882882942
30	.725396443	.119008433	.886466963
31	.737563557	.111702508	.890128888
32	.749533188	.104661298	.893863797
33	.761314652	.097874023	.897667161
34	.772916554	.091330968	.901534796
35	.784346862	.085023384	.905462824
36	.795612971	.078943416	.909447649
37	.806721761	.073084033	.913485919
38	.817679644	.067438983	.917574509
39	.828492607	.062002747	.921710497
40	.839166253	.05677052	.925891146
41	.849705831	.051738194	.930113884
42	.860116271	.046902357	.934376296
43	.870402206	.042260309	.938676101
44	.880567999	.037810097	.943011152
45	.890617763	.033550569	.947379415
46	.900555384	.029481461	.951778966
47	.910384534	.025603532	.956207979
48	.920108689	.021918749	.96066472
49	.929731144	.018430578	.96514754
50	.939255024	.015144418	.969654868
51	.948683298	.012068289	.974185207
52	.95131488	.011243487	.975461716
53	.953939201	.010436485	.976739897
54	.956556323	.009647652	.978019718
55	.959166305	.008877403	.979301151
56	.961769203	.00812621	.980584165
57	.964365076	.007394604	.981868732
58	.96695398	.006683192	.983154821
59	.969535971	.005992672	.984442405
60	.972111105	.005323851	.985731455
61	.974679434	.004677674	.987021943

Problem 2:118 (Solved by writing program)

The Fortran Program PRB2_118.FOR given below is designed to generate tables of values for Yr, Yb, Q, Yr/Y1 and Yb/Y1.

Program PRB2 118.FOR

```
      REAL Jac(2,2),F(2),FF(2),Y11(8)
      COMMON C1,C2,D,DS,X(2)
      WRITE(*,*)' Give: g,D,N & this many Y1''s'
      READ(*,*) G,D,N,(Y11(I),I=1,N)
      WRITE(*,*)' Give: (Fr1)s,(Fr1)e,Del(Fr1) & Guesses for
Yr & Yb'
      READ(*,*) Fr1s,Fr1e,DFr,Yr,Yb
      No=(Fr1e-Fr1s)/DFr+1.9
      DS=.25*D*D
      DO 10 K=1,N
      X(1)=Yr
      X(2)=Yb
      Y1=Y11(K)
      COSB1=1.-2.*Y1/D
      BETA1=ACOS(COSB1)
      A1=DS*(BETA1-SIN(BETA1)*COSB1)
      Yd1=A1/(D*SIN(BETA1))
      C22=A1*SQRT(Yd1)
      WRITE(3,100) Y1
100   FORMAT(' For Y1 =',F8.2)
      DO 10 I=1,No
      Fr1=Fr1s+DFr*FLOAT(I-1)
      C1=Y1+.5*Yd1*Fr1**2
      C2=C22*Fr1
      M=0
1     CALL FUN(F)
      DO 5 J=1,2
      XX=X(J)
      X(J)=1.005*X(J)
      CALL FUN(FF)
      Jac(1,J)=(FF(1)-F(1))/(X(J)-XX)
      Jac(2,J)=(FF(2)-F(2))/(X(J)-XX)
5     X(J)=XX
      FAC=Jac(1,2)/Jac(1,1)
      Jac(2,2)=Jac(2,2)-FAC*Jac(1,2)
      F(2)=F(2)-FAC*F(1)
      Z2=F(2)/Jac(2,2)
      X(2)=ABS(X(2)-Z2)
      Z1=(F(1)-Z2*Jac(1,2))/Jac(1,1)
      X(1)=X(1)-Z1
      M=M+1
      IF(M.LT.15 .AND. ABS(Z1)+ABS(Z2).GT. 1.E-5) GO TO
1
      IF(M.EQ.15) WRITE(*,*)' Did not
converge',I,K,Z1,Z2
      IF(I.EQ.1) THEN
      Yr=X(1)
      Yb=X(2)
      ENDIF
10    WRITE(3,110)
Fr1,X,SQRT(G*(A1*Fr1)**2*Yd1),X(1)/Y1,X(2)/Y1
110   FORMAT(F7.2,5F9.4)
      END
      SUBROUTINE FUN(F)
      REAL F(2)
      COMMON C1,C2,D,DS,X(2)
      COSr=1.-2.*X(1)/D
      BETAr=ACOS(COSr)
      COSb=1.-2.*X(2)/D
      BETAb=ACOS(COSb)
      SINr=SIN(BETAr)
      Ar=DS*(BETAr-SINr*COSr)
      Ab=DS*(BETAb-SIN(BETAb)*COSb)
      F(1)=C1-X(1)-.5*(Ar-Ab)/(D*SINr)
      F(2)=C2-(Ar-Ab)*SQRT(ABS(Ar-Ab)/(D*SINr))
      RETURN
      END
```

Input:

```
32.2 1 1 .6
.1 .9 .02 .9 .7
```

Output:

For Y1 = .60

.10	.5490	.4423	.1979	.9150	.7371
.12	.5432	.4225	.2374	.9053	.7041
.14	.5380	.4039	.2770	.8967	.6732
.16	.5333	.3864	.3166	.8889	.6440
.18	.5291	.3697	.3561	.8818	.6162
.20	.5253	.3538	.3957	.8754	.5897
.22	.5218	.3385	.4353	.8697	.5642
.24	.5188	.3238	.4749	.8646	.5396
.26	.5160	.3096	.5144	.8601	.5159
.28	.5136	.2958	.5540	.8561	.4930
.30	.5116	.2825	.5936	.8526	.4709
.32	.5098	.2696	.6331	.8497	.4494
.34	.5084	.2571	.6727	.8473	.4285
.36	.5072	.2450	.7123	.8453	.4083
.38	.5063	.2331	.7518	.8438	.3886
.40	.5057	.2216	.7914	.8428	.3694
.42	.5054	.2104	.8310	.8423	.3507
.44	.5053	.1995	.8706	.8422	.3325
.46	.5055	.1889	.9101	.8425	.3148
.48	.5060	.1785	.9497	.8433	.2975
.50	.5067	.1684	.9893	.8445	.2807
.52	.5077	.1586	1.0288	.8462	.2643
.54	.5089	.1490	1.0684	.8482	.2483
.56	.5104	.1396	1.1080	.8507	.2327
.58	.5122	.1305	1.1476	.8536	.2176
.60	.5141	.1217	1.1871	.8569	.2028
.62	.5164	.1130	1.2267	.8606	.1884
.64	.5188	.1046	1.2663	.8647	.1744
.66	.5215	.0965	1.3058	.8692	.1608
.68	.5244	.0885	1.3454	.8740	.1476
.70	.5276	.0808	1.3850	.8793	.1347
.72	.5309	.0734	1.4246	.8849	.1223
.74	.5345	.0662	1.4641	.8909	.1103
.76	.5383	.0592	1.5037	.8972	.0986
.78	.5424	.0524	1.5433	.9040	.0874
.80	.5466	.0460	1.5828	.9110	.0766
.82	.5511	.0398	1.6224	.9185	.0663
.84	.5557	.0338	1.6620	.9262	.0564
.86	.5606	.0282	1.7015	.9343	.0470
.88	.5657	.0228	1.7411	.9428	.0381
.90	.5709	.0178	1.7807	.9515	.0297

By adding the dimensionless flowrate $Q' = \{Q^2/(gA^5)\}^{1/2}$ to the write statement, this program can be used to generate data to replace (or plot) the graphs given with Example Problem 23. This modified program PRB2_188.FOR is listed below. Executing this program twice, with the following two sets of input, the output listed below has been obtained. The last column in these output tables gives the value for Q' as function of the first column which gives the upstream Froude Number F_{r1}. This data has been plotted on the attached graph.

```
32.2 1 4 .6 .7 .8 .9
.1 .9 .02 .9 .7
```

and

```
32.2 1 6 .6 .5 .4 .3 .2 .1
32.2 1 .02 .9 .7
```

```
      REAL Jac(2,2),F(2),FF(2),Y11(8)
      COMMON C1,C2,D,DS,X(2)
      WRITE(*,*)' Give: g,D,N & this many Y1''s'
      READ(*,*) G,D,N,(Y11(I),I=1,N)
      WRITE(*,*)' Give: (Fr1)s,(Fr1)e,Del(Fr1) & Guesses
for',
     &' Yr & Yb'
      READ(*,*) Fr1s,Fr1e,DFr,Yr,Yb
      No=(Fr1e-Fr1s)/DFr+1.9
```

```
      DS=.25*D*D
      GD5=SQRT(G*D**5)
      DO 10 K=1,N
      X(1)=Yr
      X(2)=Yb
      Y1=Y11(K)
      COSB1=1.-2.*Y1/D
      BETA1=ACOS(COSB1)
      A1=DS*(BETA1-SIN(BETA1)*COSB1)
      Yd1=A1/(D*SIN(BETA1))
      C22=A1*SQRT(Yd1)
      WRITE(3,100) Y1
100   FORMAT(' For Y1 =',F8.2)
      DO 10 I=1,No
      Fr1=Fr1s+DFr*FLOAT(I-1)
      C1=Y1+.5*Yd1*Fr1**2
      C2=C22*Fr1
      M=0
1     CALL FUN(F)
      DO 5 J=1,2
      XX=X(J)
      X(J)=1.005*X(J)
      CALL FUN(FF)
      Jac(1,J)=(FF(1)-F(1))/(X(J)-XX)
      Jac(2,J)=(FF(2)-F(2))/(X(J)-XX)
5     X(J)=XX
      FAC=Jac(1,2)/Jac(1,1)
      Jac(2,2)=Jac(2,2)-FAC*Jac(1,2)
      F(2)=F(2)-FAC*F(1)
      Z2=F(2)/Jac(2,2)
      X(2)=ABS(X(2)-Z2)
      Z1=(F(1)-Z2*Jac(1,2))/Jac(1,1)
      X(1)=X(1)-Z1
      M=M+1
      IF(M.LT.15 .AND. ABS(Z1)+ABS(Z2).GT. 1.E-5) GO TO 1
      IF(M.EQ.15) WRITE(*,*)' Did not converge',I,K,Z1,Z2
      IF(I.EQ.1) THEN
      Yr=X(1)
      Yb=X(2)
      ENDIF
      Q=SQRT(G*(A1*Fr1)**2*Yd1)
10    WRITE(3,110) Fr1,X,Q,X(1)/Y1,X(2)/Y1,Q/GD5
110   FORMAT(F7.2,6F9.4)
      END
```

```
      SUBROUTINE FUN(F)
      REAL F(2)
      COMMON C1,C2,D,DS,X(2)
      COSr=1.-2.*X(1)/D
      BETAr=ACOS(COSr)
      COSb=1.-2.*X(2)/D
      BETAb=ACOS(COSb)
      SINr=SIN(BETAr)
      Ar=DS*(BETAr-SINr*COSr)
      Ab=DS*(BETAb-SIN(BETAb)*COSb)
      F(1)=C1-X(1)-.5*(Ar-Ab)/(D*SINr)
      F(2)=C2-(Ar-Ab)*SQRT(ABS(Ar-Ab)/(D*SINr))
      RETURN
   END
```

The output tables are:

For Y1 = .60

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.5490	.4423	.1979	.9150	.7371	.0349
.12	.5432	.4225	.2374	.9053	.7041	.0418
.14	.5380	.4039	.2770	.8967	.6732	.0488
.16	.5333	.3864	.3166	.8889	.6440	.0558
.18	.5291	.3697	.3561	.8818	.6162	.0628
.20	.5253	.3538	.3957	.8754	.5897	.0697
.22	.5218	.3385	.4353	.8697	.5642	.0767
.24	.5188	.3238	.4749	.8646	.5396	.0837
.26	.5160	.3096	.5144	.8601	.5159	.0907
.28	.5136	.2958	.5540	.8561	.4930	.0976
.30	.5116	.2825	.5936	.8526	.4709	.1046
.32	.5098	.2696	.6331	.8497	.4494	.1116
.64	.5188	.1046	1.2663	.8647	.1744	.2232
.66	.5215	.0965	1.3058	.8692	.1608	.2301
.68	.5244	.0885	1.3454	.8740	.1476	.2371
.70	.5276	.0808	1.3850	.8793	.1347	.2441
.72	.5309	.0734	1.4246	.8849	.1223	.2510
.74	.5345	.0662	1.4641	.8909	.1103	.2580
.76	.5383	.0592	1.5037	.8972	.0986	.2650
.78	.5424	.0524	1.5433	.9040	.0874	.2720
.80	.5466	.0460	1.5828	.9110	.0766	.2789
.82	.5511	.0398	1.6224	.9185	.0663	.2859
.84	.5557	.0338	1.6620	.9262	.0564	.2929
.86	.5606	.0282	1.7015	.9343	.0470	.2999
.88	.5657	.0228	1.7411	.9428	.0381	.3068
.90	.5709	.0178	1.7807	.9515	.0297	.3138

For Y1 = .70

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.6364	.5061	.2667	.9091	.7230	.0470
.12	.6294	.4825	.3201	.8991	.6893	.0564
.14	.6231	.4605	.3734	.8901	.6578	.0658
.16	.6174	.4397	.4268	.8820	.6281	.0752
.18	.6123	.4200	.4801	.8748	.6000	.0846
.20	.6078	.4012	.5335	.8683	.5731	.0940
.22	.6037	.3832	.5868	.8625	.5474	.1034
.24	.6001	.3659	.6401	.8573	.5228	.1128
.26	.5969	.3493	.6935	.8528	.4990	.1222
.28	.5942	.3333	.7468	.8488	.4761	.1316
.30	.5918	.3178	.8002	.8455	.4540	.1410

.34	.5084	.2571	.6727	.8473	.4285	.1185
.36	.5072	.2450	.7123	.8453	.4083	.1255
.38	.5063	.2331	.7518	.8438	.3886	.1325
.40	.5057	.2216	.7914	.8428	.3694	.1395
.42	.5054	.2104	.8310	.8423	.3507	.1464
.44	.5053	.1995	.8706	.8422	.3325	.1534
.46	.5055	.1889	.9101	.8425	.3148	.1604
.48	.5060	.1785	.9497	.8433	.2975	.1674
.50	.5067	.1684	.9893	.8445	.2807	.1743
.52	.5077	.1586	1.0288	.8462	.2643	.1813
.54	.5089	.1490	1.0684	.8482	.2483	.1883
.56	.5104	.1396	1.1080	.8507	.2327	.1953
.58	.5122	.1305	1.1476	.8536	.2176	.2022
.60	.5141	.1217	1.1871	.8569	.2028	.2092
.62	.5164	.1130	1.2267	.8606	.1884	.2162
.32	.5898	.3028	.8535	.8426	.4326	.1504
.34	.5882	.2883	.9069	.8403	.4119	.1598
.36	.5870	.2742	.9602	.8385	.3918	.1692
.38	.5861	.2606	1.0136	.8373	.3723	.1786
.40	.5855	.2473	1.0669	.8365	.3533	.1880
.42	.5853	.2344	1.1203	.8362	.3349	.1974
.44	.5854	.2219	1.1736	.8363	.3170	.2068
.46	.5859	.2097	1.2270	.8369	.2996	.2162
.48	.5866	.1979	1.2803	.8380	.2827	.2256
.50	.5877	.1864	1.3336	.8395	.2662	.2350
.52	.5890	.1752	1.3870	.8415	.2502	.2444
.54	.5907	.1643	1.4403	.8439	.2347	.2538
.56	.5927	.1537	1.4937	.8467	.2195	.2632
.58	.5950	.1434	1.5470	.8499	.2048	.2726
.60	.5975	.1334	1.6004	.8536	.1905	.2820
.62	.6003	.1237	1.6537	.8576	.1767	.2914
.64	.6034	.1142	1.7071	.8621	.1632	.3008
.66	.6068	.1051	1.7604	.8669	.1502	.3102
.68	.6105	.0963	1.8138	.8721	.1375	.3196
.70	.6144	.0877	1.8671	.8777	.1253	.3290
.72	.6185	.0794	1.9204	.8836	.1135	.3384
.74	.6229	.0714	1.9738	.8899	.1020	.3478
.76	.6276	.0637	2.0271	.8966	.0911	.3572
.78	.6325	.0564	2.0805	.9035	.0805	.3666
.80	.6376	.0493	2.1338	.9109	.0704	.3760
.82	.6430	.0425	2.1872	.9185	.0607	.3854
.84	.6485	.0361	2.2405	.9265	.0515	.3948
.86	.6543	.0300	2.2939	.9347	.0428	.4042

.88	.6603	.0242	2.3472	.9433	.0346	.4136
.90	.6665	.0188	2.4006	.9521	.0269	.4230

For Y1 = .80

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.7202	.5622	.3507	.9003	.7028	.0618
.12	.7118	.5345	.4209	.8897	.6681	.0742
.14	.7043	.5087	.4910	.8804	.6359	.0865
.16	.6976	.4845	.5612	.8720	.6056	.0989
.18	.6917	.4616	.6313	.8646	.5770	.1113
.20	.6864	.4398	.7014	.8580	.5498	.1236
.22	.6818	.4191	.7716	.8522	.5238	.1360
.24	.6777	.3992	.8417	.8471	.4990	.1483
.26	.6742	.3801	.9119	.8427	.4752	.1607
.28	.6711	.3618	.9820	.8389	.4523	.1731
.30	.6686	.3441	1.0522	.8358	.4302	.1854
.32	.6666	.3271	1.1223	.8332	.4088	.1978
.34	.6650	.3106	1.1925	.8313	.3883	.2101
.36	.6639	.2947	1.2626	.8298	.3683	.2225
.38	.6632	.2793	1.3327	.8290	.3491	.2349
.40	.6629	.2643	1.4029	.8286	.3304	.2472
.42	.6630	.2498	1.4730	.8288	.3123	.2596
.44	.6636	.2358	1.5432	.8294	.2948	.2719
.46	.6645	.2222	1.6133	.8306	.2778	.2843
.48	.6658	.2090	1.6835	.8322	.2613	.2967
.50	.6675	.1963	1.7536	.8343	.2453	.3090
.52	.6695	.1839	1.8237	.8369	.2299	.3214
.54	.6719	.1719	1.8939	.8399	.2149	.3338
.56	.6746	.1603	1.9640	.8433	.2003	.3461
.58	.6777	.1490	2.0342	.8471	.1863	.3585
.60	.6811	.1381	2.1043	.8514	.1726	.3708
.62	.6848	.1276	2.1745	.8560	.1595	.3832
.64	.6888	.1174	2.2446	.8610	.1468	.3956
.66	.6931	.1076	2.3148	.8664	.1345	.4079
.68	.6978	.0981	2.3849	.8722	.1227	.4203
.70	.7026	.0890	2.4550	.8783	.1113	.4326
.72	.7078	.0803	2.5252	.8847	.1004	.4450
.74	.7132	.0719	2.5953	.8915	.0899	.4574
.76	.7188	.0639	2.6655	.8985	.0798	.4697
.78	.7247	.0562	2.7356	.9059	.0702	.4821
.80	.7308	.0489	2.8058	.9135	.0611	.4945
.82	.7371	.0419	2.8759	.9213	.0524	.5068
.84	.7435	.0354	2.9461	.9294	.0443	.5192
.86	.7502	.0292	3.0162	.9377	.0366	.5315
.88	.7570	.0235	3.0863	.9462	.0294	.5439
.90	.7639	.0182	3.1565	.9549	.0227	.5563

For Y1 = .90

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.7964	.6017	.4706	.8848	.6686	.0829
.12	.7863	.5690	.5647	.8737	.6323	.0995
.14	.7776	.5388	.6589	.8640	.5987	.1161
.16	.7700	.5106	.7530	.8555	.5673	.1327
.18	.7633	.4840	.8471	.8481	.5378	.1493
.20	.7575	.4588	.9412	.8417	.5098	.1659
.22	.7526	.4349	1.0354	.8362	.4833	.1825
.24	.7484	.4121	1.1295	.8316	.4579	.1990
.26	.7449	.3903	1.2236	.8277	.4337	.2156
.28	.7421	.3695	1.3177	.8246	.4105	.2322
.30	.7400	.3494	1.4119	.8222	.3883	.2488
.32	.7385	.3302	1.5060	.8205	.3669	.2654
.34	.7376	.3117	1.6001	.8195	.3463	.2820
.36	.7372	.2939	1.6942	.8191	.3265	.2986
.38	.7374	.2767	1.7884	.8194	.3075	.3152
.40	.7382	.2602	1.8825	.8202	.2891	.3317
.42	.7395	.2443	1.9766	.8216	.2714	.3483
.44	.7412	.2290	2.0707	.8236	.2544	.3649
.46	.7434	.2142	2.1648	.8261	.2380	.3815
.48	.7461	.2000	2.2590	.8290	.2222	.3981
.50	.7493	.1863	2.3531	.8325	.2070	.4147
.52	.7528	.1731	2.4472	.8365	.1924	.4313
.54	.7568	.1605	2.5413	.8409	.1783	.4479
.56	.7611	.1483	2.6355	.8457	.1648	.4644
.58	.7658	.1367	2.7296	.8509	.1519	.4810
.60	.7708	.1255	2.8237	.8565	.1395	.4976
.62	.7762	.1148	2.9178	.8624	.1276	.5142
.64	.7818	.1046	3.0120	.8687	.1162	.5308
.66	.7877	.0949	3.1061	.8752	.1054	.5474
.68	.7938	.0856	3.2002	.8820	.0951	.5640
.70	.8002	.0768	3.2943	.8891	.0854	.5805
.54	.4261	.1294	.7541	.8522	.2588	.1329
.56	.4272	.1214	.7820	.8544	.2429	.1378
.58	.4286	.1137	.8099	.8571	.2273	.1427
.60	.4301	.1061	.8379	.8602	.2122	.1477
.62	.4318	.0987	.8658	.8636	.1974	.1526
.64	.4337	.0915	.8937	.8675	.1829	.1575
.66	.4359	.0844	.9216	.8717	.1689	.1624
.68	.4382	.0776	.9496	.8764	.1552	.1673
.70	.4407	.0710	.9775	.8814	.1419	.1723
.72	.4434	.0645	1.0054	.8868	.1290	.1772
.74	.4463	.0582	1.0334	.8925	.1165	.1821
.76	.4493	.0522	1.0613	.8987	.1043	.1870
.78	.4526	.0463	1.0892	.9052	.0926	.1919
.80	.4560	.0406	1.1171	.9120	.0813	.1969
.82	.4596	.0352	1.1451	.9193	.0704	.2018

.72	.8067	.0685	3.3885	.8963	.0761	.5971
.74	.8133	.0606	3.4826	.9037	.0674	.6137
.76	.8201	.0532	3.5767	.9113	.0591	.6303
.78	.8270	.0462	3.6708	.9189	.0514	.6469
.80	.8339	.0397	3.7650	.9266	.0441	.6635
.82	.8409	.0336	3.8591	.9343	.0374	.6801
.84	.8479	.0280	3.9532	.9421	.0311	.6967
.86	.8548	.0228	4.0473	.9498	.0254	.7132
.88	.8617	.0181	4.1414	.9574	.0201	.7298
.90	.8684	.0138	4.2356	.9649	.0153	.7464

For Y1 = .60

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.5490	.4423	.1979	.9150	.7371	.0349
.12	.5432	.4225	.2374	.9053	.7041	.0418
.14	.5380	.4039	.2770	.8967	.6732	.0488
.16	.5333	.3864	.3166	.8889	.6440	.0558
.18	.5291	.3697	.3561	.8818	.6162	.0628
.20	.5253	.3538	.3957	.8754	.5897	.0697
.22	.5218	.3385	.4353	.8697	.5642	.0767
.24	.5188	.3238	.4749	.8646	.5396	.0837
.26	.5160	.3096	.5144	.8601	.5159	.0907
.28	.5136	.2958	.5540	.8561	.4930	.0976
.30	.5116	.2825	.5936	.8526	.4709	.1046
.32	.5098	.2696	.6331	.8497	.4494	.1116
.34	.5084	.2571	.6727	.8473	.4285	.1185
.36	.5072	.2450	.7123	.8453	.4083	.1255
.38	.5063	.2331	.7518	.8438	.3886	.1325
.40	.5057	.2216	.7914	.8428	.3694	.1395
.42	.5054	.2104	.8310	.8423	.3507	.1464
.44	.5053	.1995	.8706	.8422	.3325	.1534
.46	.5055	.1889	.9101	.8425	.3148	.1604
.48	.5060	.1785	.9497	.8433	.2975	.1674
.50	.5067	.1684	.9893	.8445	.2807	.1743
.52	.5077	.1586	1.0288	.8462	.2643	.1813
.54	.5089	.1490	1.0684	.8482	.2483	.1883
.56	.5104	.1396	1.1080	.8507	.2327	.1953
.58	.5122	.1305	1.1476	.8536	.2176	.2022
.60	.5141	.1217	1.1871	.8569	.2028	.2092
.62	.5164	.1130	1.2267	.8606	.1884	.2162
.64	.5188	.1046	1.2663	.8647	.1744	.2232
.66	.5215	.0965	1.3058	.8692	.1608	.2301
.68	.5244	.0885	1.3454	.8740	.1476	.2371
.70	.5276	.0808	1.3850	.8793	.1347	.2441
.72	.5309	.0734	1.4246	.8849	.1223	.2510
.74	.5345	.0662	1.4641	.8909	.1103	.2580
.76	.5383	.0592	1.5037	.8972	.0986	.2650
.78	.5424	.0524	1.5433	.9040	.0874	.2720
.80	.5466	.0460	1.5828	.9110	.0766	.2789
.82	.5511	.0398	1.6224	.9185	.0663	.2859
.84	.5557	.0338	1.6620	.9262	.0564	.2929
.86	.5606	.0282	1.7015	.9343	.0470	.2999
.88	.5657	.0228	1.7411	.9428	.0381	.3068
.90	.5709	.0178	1.7807	.9515	.0297	.3138

For Y1 = .50

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.4596	.3738	.1396	.9191	.7475	.0246
.12	.4549	.3576	.1676	.9099	.7153	.0295
.14	.4507	.3425	.1955	.9015	.6849	.0345
.16	.4469	.3281	.2234	.8939	.6561	.0394
.18	.4435	.3143	.2514	.8870	.6287	.0443
.20	.4404	.3012	.2793	.8808	.6024	.0492
.22	.4376	.2885	.3072	.8751	.5770	.0541
.24	.4351	.2763	.3351	.8701	.5526	.0591
.26	.4328	.2645	.3631	.8656	.5290	.0640
.28	.4308	.2531	.3910	.8616	.5061	.0689
.30	.4291	.2420	.4189	.8582	.4839	.0738
.32	.4276	.2312	.4469	.8552	.4624	.0787
.34	.4263	.2207	.4748	.8527	.4415	.0837
.36	.4253	.2105	.5027	.8507	.4211	.0886
.38	.4245	.2006	.5306	.8491	.4012	.0935
.40	.4240	.1909	.5586	.8480	.3818	.0984
.42	.4236	.1815	.5865	.8473	.3630	.1034
.44	.4235	.1723	.6144	.8471	.3445	.1083
.46	.4236	.1633	.6424	.8472	.3266	.1132
.48	.4239	.1545	.6703	.8478	.3090	.1181
.50	.4244	.1459	.6982	.8489	.2919	.1230
.52	.4252	.1376	.7261	.8503	.2752	.1280
.84	.4634	.0300	1.1730	.9269	.0600	.2067
.86	.4674	.0250	1.2009	.9348	.0501	.2116
.88	.4715	.0203	1.2289	.9431	.0406	.2166
.90	.4759	.0159	1.2568	.9517	.0318	.2215

For Y1 = .40

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.3689	.3023	.0911	.9223	.7557	.0161
.12	.3653	.2896	.1093	.9133	.7240	.0193
.14	.3621	.2777	.1275	.9051	.6942	.0225
.16	.3591	.2663	.1457	.8977	.6658	.0257
.18	.3564	.2554	.1640	.8910	.6386	.0289
.20	.3540	.2450	.1822	.8849	.6125	.0321
.22	.3517	.2350	.2004	.8794	.5874	.0353
.24	.3498	.2252	.2186	.8744	.5631	.0385

.26	.3480	.2158	.2368	.8700	.5396	.0417
.28	.3464	.2067	.2551	.8660	.5167	.0449
.30	.3450	.1978	.2733	.8626	.4946	.0482
.32	.3438	.1892	.2915	.8596	.4730	.0514
.34	.3428	.1808	.3097	.8571	.4520	.0546
.36	.3420	.1726	.3279	.8550	.4315	.0578
.38	.3413	.1646	.3462	.8534	.4115	.0610
.40	.3409	.1568	.3644	.8522	.3920	.0642
.42	.3406	.1492	.3826	.8514	.3730	.0674
.44	.3404	.1417	.4008	.8511	.3544	.0706
.46	.3405	.1345	.4190	.8512	.3362	.0738
.48	.3407	.1274	.4372	.8517	.3184	.0771
.50	.3410	.1204	.4555	.8526	.3010	.0803
.52	.3416	.1136	.4737	.8539	.2840	.0835
.54	.3422	.1070	.4919	.8556	.2674	.0867
.56	.3431	.1005	.5101	.8577	.2512	.0899
.58	.3441	.0941	.5283	.8602	.2353	.0931
.60	.3452	.0879	.5466	.8631	.2198	.0963
.62	.3466	.0819	.5648	.8664	.2047	.0995
.64	.3480	.0760	.5830	.8701	.1899	.1027
.66	.3497	.0702	.6012	.8742	.1755	.1059
.68	.3514	.0646	.6194	.8786	.1614	.1092
.70	.3534	.0591	.6376	.8835	.1477	.1124
.72	.3555	.0538	.6559	.8887	.1344	.1156
.74	.3577	.0486	.6741	.8943	.1215	.1188
.76	.3601	.0436	.6923	.9002	.1089	.1220
.78	.3626	.0387	.7105	.9066	.0968	.1252
.80	.3653	.0340	.7287	.9133	.0851	.1284
.82	.3681	.0295	.7470	.9203	.0738	.1316
.84	.3711	.0252	.7652	.9278	.0629	.1348
.86	.3742	.0210	.7834	.9356	.0526	.1381
.88	.3775	.0171	.8016	.9437	.0427	.1413
.90	.3809	.0134	.8198	.9522	.0334	.1445

For Y1 = .30

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.2774	.2287	.0523	.9248	.7625	.0092
.12	.2748	.2193	.0627	.9160	.7311	.0111
.14	.2724	.2105	.0732	.9080	.7017	.0129
.16	.2702	.2021	.0837	.9008	.6737	.0147
.18	.2683	.1940	.0941	.8942	.6468	.0166
.20	.2665	.1863	.1046	.8882	.6209	.0184
.22	.2648	.1788	.1150	.8828	.5960	.0203
.24	.2634	.1715	.1255	.8779	.5718	.0221
.26	.2620	.1645	.1360	.8735	.5484	.0240
.28	.2609	.1577	.1464	.8696	.5256	.0258
.30	.2598	.1510	.1569	.8661	.5034	.0276
.32	.2590	.1446	.1673	.8632	.4819	.0295
.34	.2582	.1382	.1778	.8606	.4608	.0313
.36	.2576	.1321	.1882	.8586	.4403	.0332
.38	.2571	.1261	.1987	.8569	.4202	.0350
.40	.2567	.1202	.2092	.8557	.4006	.0369
.42	.2565	.1144	.2196	.8549	.3814	.0387
.44	.2563	.1088	.2301	.8545	.3627	.0405
.46	.2563	.1033	.2405	.8545	.3443	.0424
.48	.2565	.0979	.2510	.8549	.3263	.0442
.50	.2567	.0926	.2614	.8557	.3087	.0461
.52	.2571	.0875	.2719	.8569	.2915	.0479
.54	.2576	.0824	.2824	.8585	.2747	.0498
.56	.2582	.0775	.2928	.8605	.2582	.0516
.58	.2589	.0726	.3033	.8629	.2421	.0534
.60	.2597	.0679	.3137	.8657	.2263	.0553
.62	.2607	.0633	.3242	.8689	.2109	.0571
.64	.2617	.0587	.3346	.8724	.1958	.0590
.66	.2629	.0543	.3451	.8764	.1811	.0608
.68	.2642	.0500	.3556	.8807	.1667	.0627
.70	.2656	.0458	.3660	.8854	.1527	.0645
.72	.2671	.0417	.3765	.8905	.1390	.0663
.74	.2688	.0377	.3869	.8959	.1257	.0682
.76	.2705	.0339	.3974	.9017	.1128	.0700
.78	.2724	.0301	.4079	.9079	.1003	.0719
.80	.2744	.0265	.4183	.9145	.0883	.0737
.82	.2764	.0230	.4288	.9214	.0766	.0756
.84	.2786	.0196	.4392	.9287	.0654	.0774
.86	.2809	.0164	.4497	.9364	.0547	.0792
.88	.2833	.0133	.4601	.9444	.0445	.0811
.90	.2858	.0104	.4706	.9528	.0348	.0829

For Y1 = .20

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.1855	.1540	.0237	.9275	.7701	.0042
.12	.1836	.1474	.0285	.9182	.7370	.0050
.14	.1821	.1416	.0332	.9104	.7080	.0059
.16	.1807	.1360	.0380	.9033	.6802	.0067
.18	.1794	.1307	.0427	.8968	.6536	.0075
.20	.1782	.1256	.0474	.8909	.6280	.0084
.22	.1771	.1206	.0522	.8856	.6032	.0092
.24	.1761	.1158	.0569	.8807	.5792	.0100
.26	.1753	.1112	.0617	.8764	.5558	.0109
.28	.1745	.1066	.0664	.8725	.5331	.0117
.30	.1738	.1022	.0712	.8691	.5110	.0125
.32	.1732	.0979	.0759	.8662	.4895	.0134
.34	.1727	.0937	.0807	.8636	.4684	.0142
.36	.1723	.0896	.0854	.8615	.4478	.0151
.38	.1720	.0855	.0902	.8599	.4277	.0159
.40	.1717	.0816	.0949	.8586	.4080	.0167
.42	.1716	.0777	.0996	.8578	.3887	.0176
.44	.1715	.0740	.1044	.8573	.3699	.0184
.46	.1715	.0703	.1091	.8573	.3513	.0192
.48	.1715	.0666	.1139	.8576	.3332	.0201
.50	.1717	.0631	.1186	.8584	.3154	.0209
.52	.1719	.0596	.1234	.8595	.2980	.0217
.54	.1722	.0562	.1281	.8611	.2810	.0226
.56	.1726	.0529	.1329	.8630	.2643	.0234
.58	.1731	.0496	.1376	.8653	.2479	.0242
.60	.1736	.0464	.1423	.8680	.2319	.0251
.62	.1742	.0433	.1471	.8711	.2163	.0259
.64	.1749	.0402	.1518	.8745	.2009	.0268
.66	.1757	.0372	.1566	.8784	.1859	.0276
.68	.1765	.0343	.1613	.8826	.1713	.0284
.70	.1774	.0314	.1661	.8872	.1570	.0293
.72	.1784	.0286	.1708	.8921	.1430	.0301
.74	.1795	.0259	.1756	.8975	.1295	.0309
.76	.1806	.0233	.1803	.9032	.1163	.0318
.78	.1818	.0207	.1850	.9092	.1034	.0326
.80	.1831	.0182	.1898	.9157	.0910	.0334
.82	.1845	.0158	.1945	.9225	.0791	.0343
.84	.1859	.0135	.1993	.9297	.0675	.0351
.86	.1874	.0113	.2040	.9372	.0565	.0360
.88	.1890	.0092	.2088	.9451	.0460	.0368
.90	.1907	.0072	.2135	.9534	.0360	.0376

For Y1 = .10

F_{r1}	Y_r	Y_b	Q	Y_r/Y_1	Y_b/Y_1	Q'
.10	.0949	.0838	.0061	.9491	.8377	.0011
.12	.0920	.0743	.0073	.9202	.7426	.0013
.14	.0912	.0713	.0085	.9123	.7134	.0015
.16	.0905	.0686	.0097	.9054	.6859	.0017
.18	.0899	.0660	.0109	.8990	.6595	.0019
.20	.0893	.0634	.0121	.8932	.6341	.0021
.22	.0888	.0609	.0133	.8879	.6095	.0023
.24	.0883	.0586	.0145	.8831	.5856	.0026
.26	.0879	.0562	.0157	.8788	.5623	.0028
.28	.0875	.0540	.0170	.8750	.5398	.0030
.30	.0872	.0518	.0182	.8716	.5177	.0032
.32	.0869	.0496	.0194	.8687	.4961	.0034
.34	.0866	.0475	.0206	.8662	.4751	.0036
.36	.0864	.0454	.0218	.8641	.4545	.0038
.38	.0862	.0434	.0230	.8624	.4343	.0041
.40	.0861	.0415	.0242	.8611	.4146	.0043
.42	.0860	.0395	.0254	.8603	.3952	.0045
.44	.0860	.0376	.0266	.8598	.3762	.0047
.46	.0860	.0358	.0278	.8597	.3576	.0049
.48	.0860	.0339	.0291	.8600	.3392	.0051
.50	.0861	.0321	.0303	.8607	.3213	.0053
.52	.0862	.0304	.0315	.8618	.3038	.0055
.54	.0863	.0287	.0327	.8633	.2865	.0058
.56	.0865	.0270	.0339	.8651	.2696	.0060
.58	.0867	.0253	.0351	.8674	.2531	.0062
.60	.0870	.0237	.0363	.8700	.2369	.0064
.62	.0873	.0221	.0375	.8730	.2210	.0066
.64	.0876	.0205	.0387	.8764	.2054	.0068
.66	.0880	.0190	.0400	.8801	.1902	.0070
.68	.0884	.0175	.0412	.8842	.1753	.0073
.70	.0889	.0161	.0424	.8887	.1608	.0075
.72	.0894	.0147	.0436	.8936	.1466	.0077
.74	.0899	.0133	.0448	.8988	.1327	.0079
.76	.0904	.0119	.0460	.9044	.1192	.0081
.78	.0910	.0106	.0472	.9104	.1061	.0083
.80	.0917	.0093	.0484	.9167	.0935	.0085
.82	.0923	.0081	.0496	.9235	.0812	.0087
.84	.0931	.0069	.0509	.9305	.0694	.0090
.86	.0938	.0058	.0521	.9380	.0581	.0092
.88	.0946	.0047	.0533	.9457	.0473	.0094
.90	.0954	.0037	.0545	.9539	.0371	.0096

An alternative is to use the TK-Solver model with the dimensionless flowrate Q' added as an additional list variable. This model is given below along with the output obtained for Y_1' = .6, .7, .8 & .9. (In using TK-Solver it is necessary that the guesses for the list variable Y_b' (Yb) be given quite accurately, especially for the larger values of F^{r1}, and it was necessary to adjust the guess to get convergence.
Model P2_119*.TK (also see PRB2_118.TK)

```
=========== VARIABLE SHEET ===========
St Input---- Name--- Output--- Unit---
   .6         Y1
   1.7721543  B1
   .49202834  A1
   .50217435  Yd1
L  .01        Fr2
LG .54897512  Yr
LG .44221115  Yb
LG 1.6689039  Br
LG .44159578  Ar
LG 1.4549598  Bb
LG .33503915  Ab
L             r6
L             Fr
L             r7
L             Qp
              Qpc
```

```
============= RULE SHEET =============
S Rule--------------------------------
  A1*sqrt(Yd1*Fr2)=(Ar-Ab)*sqrt((Ar-Ab)/sin(Br))
  Y1+Fr2*Yd1/2=Yr+.5*(Ar-Ab)/sin(Br)
  Br=acos(1-2*Yr)
  Ar=.25*(Br-cos(Br)*sin(Br))
  Bb=acos(1-2*Yb)
  Ab=.25*(Bb-cos(Bb)*sin(Bb))
  r6=Yb/Y1
  Fr=sqrt(Fr2)
  r7=Yr/Y1
  Qpc=sqrt(A1^3/sin(B1))
  Qp=Fr*Qpc
```

TABLE:

Title:

Element	Fr	r6	r7	Qp
1	.1	.737018591	.914958538	.034867226
2	.16673332	.634520787	.886406031	.058135283
3	.213541565	.572274906	.871492998	.074456019
4	.251793566	.52554281	.861855741	.087793431
5	.284956137	.487480849	.855174567	.099356299
6	.314642654	.45508205	.850428074	.109707164
7	.34176015	.426725883	.84706462	.119162282
8	.366878727	.401427647	.844748235	.127920433
9	.390384426	.378539187	.843257215	.136116218
10	.412553027	.357608681	.842436586	.143845794
11	.433589668	.338307236	.842173194	.151180687
12	.453651849	.320387205	.842381553	.158175813
13	.472863617	.303656974	.842995278	.164874424
14	.49132474	.287964936	.843961636	.171311305
15	.509116882	.273188878	.845237934	.177514932
16	.526307895	.25922871	.846789047	.183508961
17	.542954878	.246001341	.848585666	.189313302
18	.55910643	.233436979	.850603042	.1949449
19	.574804315	.221476397	.852820049	.200418317
20	.59008474	.210068877	.855218482	.205746177
21	.604979338	.199170647	.857782524	.21093951
22	.61951594	.188743657	.860498326	.21600802
23	.633719181	.178754626	.863353678	.220960296
24	.647610994	.169174282	.86633775	.225803986
25	.66121101	.159976755	.869440882	.230545934
26	.674536878	.151139075	.87265441	.235192295
27	.687604538	.142640777	.875970531	.239748625
28	.70042844	.134463563	.87938218	.244219964
29	.713021739	.126591029	.882882942	.248610898
30	.725396443	.119008433	.886466963	.252925614
31	.737563557	.111702508	.890128888	.257167949
32	.749533188	.104661298	.893863797	.261341427
33	.761314652	.097874023	.897667161	.265449297
34	.772916554	.091330968	.901534796	.269494558
35	.784346862	.085023384	.905462824	.273479989
36	.795612971	.078943416	.909447649	.277408169
37	.806721761	.073084033	.913485919	.281281496
38	.817679644	.067438983	.917574509	.285102206
39	.828492607	.062002747	.921710497	.288872386
40	.839166253	.05677052	.925891146	.29259399
41	.849705831	.051738194	.930113884	.296268849
42	.860116271	.046902357	.934376296	.29989868
43	.870402206	.042260309	.938676101	.3034851
44	.880567999	.037810097	.943011152	.30702963
45	.890617763	.033550569	.947379415	.310533704
46	.900555384	.029481461	.951778966	.313998677
47	.910384534	.025603532	.956207979	.317425829
48	.920108689	.021918749	.96066472	.320816372
49	.929731144	.018430578	.96514754	.324171455
50	.939255024	.015144418	.969654868	.327492168
51	.948683298	.012068289	.974185207	.330779545
52	.95131488	.011243487	.975461716	.331697105
53	.953939201	.010436485	.976739897	.332612133
54	.956556323	.009647652	.978019718	.333524651
55	.959166305	.008877403	.979301151	.334434679
56	.961769203	.00812621	.980584165	.335342237
57	.964365076	.007394604	.981868732	.336247346
58	.96695398	.006683192	.983154821	.337150025
59	.969535971	.005992672	.984442405	.338050294
60	.972111105	.005323851	.985731455	.338948171
61	.974679434	.004677674	.987021943	.339843677

```
=========== VARIABLE SHEET ===========
St Input---- Name--- Output--- Unit----
   .7         Y1
   1.9823132  B1
   .5872298   A1
   .64072025  Yd1
L  .01        Fr2
LG .54897512  Yr
LG .44221115  Yb
LG 1.6689039  Br
LG .44159578  Ar
LG 1.4549598  Bb
LG .33503915  Ab
L             r6
L             Fr
L             r7
L             Qp
              Qpc
```

TABLE:

Title:

Element	Fr	r6	r7	Qp
1	.1	.722989946	.909115662	.047004811
2	.16673332	.618480042	.879491251	.078372682
3	.213541565	.555601056	.864260648	.100374809
4	.251793566	.508653001	.854557823	.11835509
5	.284956137	.470572394	.847936984	.133943094
6	.314642654	.438267586	.843324857	.147897185
7	.34176015	.410077063	.840143256	.160643713
8	.366878727	.384993423	.838040744	.172450652
9	.390384426	.362354758	.836785792	.183499462
10	.412553027	.341700346	.836216767	.193919771
11	.433589668	.322695243	.836215782	.203808004
12	.453651849	.305087524	.836693851	.213238194
13	.472863617	.288682448	.837581919	.222268649
14	.49132474	.273326057	.838825156	.230946265
15	.509116882	.258894328	.840379189	.239309428
16	.526307895	.24528575	.842207516	.247390031
17	.542954878	.232416092	.844279681	.255214914
18	.55910643	.220214627	.846569962	.262806921
19	.574804315	.208621354	.849056397	.270185682
20	.59008474	.197584899	.851720057	.277368217
21	.604979338	.187060925	.854544483	.284369395
22	.61951594	.177010889	.857515253	.291202297
23	.633719181	.167401073	.860619646	.297878503
24	.647610994	.15820181	.863846364	.304408324
25	.66121101	.149386864	.86718532	.310800986
26	.674536878	.140932931	.870627454	.317064785
27	.687604538	.132819227	.874164594	.323207213

28	.70042844	.125027149	.877789332	.329235065
29	.713021739	.117539996	.881494927	.335154521
30	.725396443	.110342738	.885275216	.340971227
31	.737563557	.103421818	.889124549	.346690356
32	.749533188	.096764992	.893037724	.352316658
33	.761314652	.090361187	.897009939	.357854514
34	.772916554	.084200387	.901036745	.363307966
35	.784346862	.078273536	.905114011	.36868076
36	.795612971	.072572456	.909237888	.373976373
37	.806721761	.067089783	.913404782	.379198039
38	.817679644	.061818907	.917611328	.384348771
39	.828492607	.056753936	.921854367	.389431384
40	.839166253	.051889662	.92613093	.394448511
41	.849705831	.047221544	.930438217	.39940262
42	.860116271	.042745702	.934773583	.404296028
43	.870402206	.038458925	.939134524	.409130912
44	.880567999	.034358694	.943518668	.413909323
45	.890617763	.030443231	.947923759	.418633196
46	.900555384	.026711569	.952347651	.423304356
47	.910384534	.023163664	.956788298	.42792453
48	.920108689	.019800562	.961243746	.43249535
49	.929731144	.016624648	.965712125	.437018367
50	.939255024	.013640025	.970191648	.441495049
51	.948683298	.010853125	.974680595	.445926791
52	.95131488	.010107089	.975943004	.447163761
53	.953939201	.009377674	.97720599	.448397319
54	.956556323	.00866519	.978469519	.449627492
55	.959166305	.007969993	.979733554	.450854309
56	.961769203	.007292482	.980998063	.452077796
57	.964365076	.006633113	.982263011	.453297981
58	.96695398	.005992411	.983528363	.454514891
59	.969535971	.005370977	.984794086	.455728551
60	.972111105	.004769511	.986060148	.456938988
61	.974679434	.004188835	.987326516	.458146226

═══════════ VARIABLE SHEET ═══════════

St	Input	Name	Output	Unit
	.8	Y1		
	2.2142974	B1		
	.67357436	A1		
	.8419679	Yd1		
L	.01	Fr2		
LG	.54897512	Yr		
LG	.44221115	Yb		
LG	1.6689039	Br		
LG	.44159578	Ar		
LG	1.4549598	Bb		
LG	.33503915	Ab		
L		r6		
L		Fr		

L		r7
L		Qp
		Qpc

═══════════════ TABLE: ═══════════════

Title:

Element	Fr	r6	r7	Qp
1	.1	.702795997	.900278451	.061806383
2	.16673332	.595783393	.869435227	.103051834
3	.213541565	.532090038	.854004636	.131982317
4	.251793566	.484837704	.844429572	.155624496
5	.284956137	.446698512	.838095941	.176121081
6	.314642654	.414479159	.833862223	.194469244
7	.34176015	.386468695	.831115555	.211229587
8	.366878727	.361632109	.829484346	.22675447
9	.390384426	.339290571	.828724234	.241282493
10	.412553027	.318972265	.828664851	.254984103
11	.433589668	.30033459	.829182045	.26798609
12	.453651849	.283120109	.830182149	.280385799
13	.472863617	.267129977	.831592482	.292259897
14	.49132474	.25220709	.833355329	.30367005
15	.509116882	.23822495	.835423958	.31466673
16	.526307895	.225080052	.837759894	.325291872
17	.542954878	.212686525	.840331005	.335580771
18	.55910643	.20097227	.843110116	.345563461
19	.574804315	.18987611	.846073985	.355265755
20	.59008474	.179345654	.849202542	.364710034
21	.604979338	.169335661	.8524783	.373915846
22	.61951594	.159806782	.855885899	.382900394
23	.633719181	.150724559	.859411752	.391678903
24	.647610994	.142058644	.863043759	.400264931
25	.66121101	.133782166	.866771079	.408670609
26	.674536878	.125871213	.870583945	.416906846
27	.687604538	.11830442	.874473512	.424983493
28	.70042844	.111062621	.878431733	.432909484
29	.713021739	.104128563	.882451252	.440692946
30	.725396443	.09748667	.886525318	.448341303
31	.737563557	.091122839	.890647711	.455861356
32	.749533188	.085024278	.894812682	.463259352
33	.761314652	.079179359	.899014895	.470541049
34	.772916554	.073577502	.903249385	.477711765
35	.784346862	.06820907	.907511517	.484776425
36	.795612971	.063065286	.911796955	.491739599
37	.806721761	.058138164	.916101627	.49860554
38	.817679644	.053420446	.920421704	.505378211
39	.828492607	.048905557	.924753577	.512061313
40	.839166253	.044587569	.929093834	.518658307
41	.849705831	.040461176	.933439249	.52517244
42	.860116271	.036521677	.937786761	.531606756
43	.870402206	.032764979	.942133461	.53796412
44	.880567999	.029187604	.946476587	.544247229
45	.890617763	.02578672	.950813505	.550458625
46	.900555384	.022560196	.955141704	.556600709
47	.910384534	.019506682	.959458788	.562675751
48	.920108689	.016625737	.963762469	.568685899
49	.929731144	.013918028	.968050558	.574633191
50	.939255024	.011385624	.972320963	.580519556
51	.948683298	.009032488	.97657168	.586346831
52	.95131488	.008404572	.977761896	.587973317
53	.953939201	.007791499	.978950366	.589595315
54	.956556323	.007193496	.980137048	.591212864
55	.959166305	.00661082	.981321903	.592825999
56	.961769203	.006043767	.98250489	.594434756
57	.964365076	.005492679	.98368597	.596039171
58	.96695398	.004957951	.984865106	.597639279
59	.969535971	.004440043	.986042257	.599235115
60	.972111105	.003939492	.987217386	.600826711
61	.974679434	.003456935	.988390456	.602414103

═══════════════ VARIABLE SHEET ═══════════════

St	Input	Name	Output	Unit
	.9	Y1		
	2.4980915	B1		
	.74452289	A1		
	1.2408715	Yd1		
L	.01	Fr2		
LG	.54897512	Yr		
LG	.44221115	Yb		
LG	1.6689039	Br		
LG	.44159578	Ar		
LG	1.4549598	Bb		
LG	.33503915	Ab		
L		r6		
L		Fr		

L r7
L Qp
Qpc

═══════ TABLE: ═══════

Title:

Element	Fr	r6	r7	Qp
1	.1	.668571849	.884842201	.082935686
2	.16673332	.55716393	.852911877	.138281423
3	.213541565	.491684451	.837885471	.177102163
4	.251793566	.443517226	.829183315	.208826723
5	.284956137	.404920061	.823944872	.236330328
6	.314642654	.372531455	.820927835	.260951045
7	.34176015	.34455447	.819467233	.283441126
8	.366878727	.319903779	.819160473	.30427339
9	.390384426	.297868316	.81974289	.323768003
10	.412553027	.277954215	.821029846	.342153685
11	.433589668	.259803014	.822886596	.359600567
12	.453651849	.24314539	.825211258	.376239275
13	.472863617	.227773288	.827924555	.392172686
14	.49132474	.213522261	.830963315	.407483546
15	.509116882	.200259805	.834276183	.422239581
16	.526307895	.187877391	.837820691	.436497065
17	.542954878	.176284861	.841561198	.450303355
18	.55910643	.165406389	.845467404	.463698756
19	.574804315	.155177505	.849513258	.476717904
20	.59008474	.145542857	.853676139	.489390829
21	.604979338	.136454516	.857936228	.501743767
22	.61951594	.127870647	.862276025	.513799797
23	.633719181	.119754479	.866679974	.525579353
24	.647610994	.112073481	.87113416	.537100624
25	.66121101	.104798704	.875626068	.54837989
26	.674536878	.097904246	.880144392	.55943179
27	.687604538	.091366816	.884678876	.570269543
28	.70042844	.085165376	.889220185	.580905135
29	.713021739	.079280842	.893759799	.591349474
30	.725396443	.073695834	.898289923	.60161252
31	.737563557	.068394473	.902803416	.611703399
32	.749533188	.0633622	.907293729	.621630495
33	.761314652	.058585632	.91175485	.631401533
34	.772916554	.054052433	.916181262	.64102365
35	.784346862	.049751211	.920567908	.650503454
36	.795612971	.045671423	.924910152	.659847079
37	.806721761	.041803304	.929203761	.66906023
38	.817679644	.038137796	.933444871	.678148225
39	.828492607	.034666501	.937629972	.687116031
40	.839166253	.031381631	.941755889	.695968292
41	.849705831	.028275981	.945819763	.704709364
42	.860116271	.025342896	.949819036	.713343334
43	.870402206	.022576261	.95375144	.721874044
44	.880567999	.019970492	.957614979	.730305114
45	.890617763	.017520545	.961407923	.738639955
46	.900555384	.015221937	.96512879	.74688179
47	.910384534	.013070785	.968776338	.755033663
48	.920108689	.011063879	.972349555	.763098457
49	.929731144	.00919879	.975847645	.771078906
50	.939255024	.007474046	.97927002	.778977601
51	.948683298	.005889424	.982616285	.786797005
52	.95131488	.005469666	.983542523	.788979525
53	.953939201	.005061134	.984462736	.791156025
54	.956556323	.004663908	.985376923	.793326553
55	.959166305	.004278093	.986285084	.795491159
56	.961769203	.003903816	.987187221	.797649891
57	.964365076	.003541231	.988083335	.799802796
58	.96695398	.003190527	.98897343	.801949921
59	.969535971	.002851932	.98985751	.804091313
60	.972111105	.002525723	.990735579	.806227018
61	.974679434	.002212238	.991607644	.80835708

131.

Make a graph similar to that given in Example Problem 23 except relate the dimensionless flowrate to the Froude Number.

<u>Wanted:</u> Since the dimensionless flowrate is only a function of the upstream conditions $Q' = F_{r1}(A_1'/T_1')^{1/2}$, i.e. obtained from solving Q from the upstream Froude Number definition, it is possible to let a spreadsheet generate the data to plot Q' versus F_{r1}. This has been done in obtaining the graph given below. An alternative is to use the data obtain in the previous problem from either the program PRB2_118.FOR, or the TK-Solver model PRB2_119.TK.

Below the data to graph $Q' = Q/(gD^5)^{1/2}$ the dimensionless flowrate Q' has been added to the WRITE statement in the Fortran Program PRB2_118.FOR (given in the previous problem). Executing this program twice, with the following two sets of input, the output listed below has been obtained. The last column in these output tables gives the value for Q' as function of the first column which gives the upstream Froude Number F_{r1}. This data has been plotted on the attached graph.

```
32.2 1 4 .6 .7 .8 .9
.1 .9 .02 .9 .7

and
32.2 1 6 .6 .5 .4 .3 .2 .1
32.2 1 .02 .9 .7
```

```
 For Y1 =     .60
   .10    .5490    .4423    .1979    .9150    .7371    .0349
   .12    .5432    .4225    .2374    .9053    .7041    .0418
   .14    .5380    .4039    .2770    .8967    .6732    .0488
   .16    .5333    .3864    .3166    .8889    .6440    .0558
   .18    .5291    .3697    .3561    .8818    .6162    .0628
   .20    .5253    .3538    .3957    .8754    .5897    .0697
   .22    .5218    .3385    .4353    .8697    .5642    .0767
   .24    .5188    .3238    .4749    .8646    .5396    .0837
   .26    .5160    .3096    .5144    .8601    .5159    .0907
   .28    .5136    .2958    .5540    .8561    .4930    .0976
   .30    .5116    .2825    .5936    .8526    .4709    .1046
   .32    .5098    .2696    .6331    .8497    .4494    .1116
   .34    .5084    .2571    .6727    .8473    .4285    .1185
   .36    .5072    .2450    .7123    .8453    .4083    .1255
   .38    .5063    .2331    .7518    .8438    .3886    .1325
   .40    .5057    .2216    .7914    .8428    .3694    .1395
   .42    .5054    .2104    .8310    .8423    .3507    .1464
   .44    .5053    .1995    .8706    .8422    .3325    .1534
   .46    .5055    .1889    .9101    .8425    .3148    .1604
   .48    .5060    .1785    .9497    .8433    .2975    .1674
   .50    .5067    .1684    .9893    .8445    .2807    .1743
   .52    .5077    .1586   1.0288    .8462    .2643    .1813
   .54    .5089    .1490   1.0684    .8482    .2483    .1883
   .56    .5104    .1396   1.1080    .8507    .2327    .1953
   .58    .5122    .1305   1.1476    .8536    .2176    .2022
   .60    .5141    .1217   1.1871    .8569    .2028    .2092
   .62    .5164    .1130   1.2267    .8606    .1884    .2162
   .64    .5188    .1046   1.2663    .8647    .1744    .2232
   .66    .5215    .0965   1.3058    .8692    .1608    .2301
   .68    .5244    .0885   1.3454    .8740    .1476    .2371
   .70    .5276    .0808   1.3850    .8793    .1347    .2441
   .72    .5309    .0734   1.4246    .8849    .1223    .2510
   .74    .5345    .0662   1.4641    .8909    .1103    .2580
   .76    .5383    .0592   1.5037    .8972    .0986    .2650
   .78    .5424    .0524   1.5433    .9040    .0874    .2720
   .80    .5466    .0460   1.5828    .9110    .0766    .2789
   .82    .5511    .0398   1.6224    .9185    .0663    .2859
   .84    .5557    .0338   1.6620    .9262    .0564    .2929
   .86    .5606    .0282   1.7015    .9343    .0470    .2999
   .88    .5657    .0228   1.7411    .9428    .0381    .3068
   .90    .5709    .0178   1.7807    .9515    .0297    .3138
 For Y1 =     .70
   .10    .6364    .5061    .2667    .9091    .7230    .0470
   .12    .6294    .4825    .3201    .8991    .6893    .0564
   .14    .6231    .4605    .3734    .8901    .6578    .0658
```

.16	.6174	.4397	.4268	.8820	.6281	.0752
.18	.6123	.4200	.4801	.8748	.6000	.0846
.20	.6078	.4012	.5335	.8683	.5731	.0940
.22	.6037	.3832	.5868	.8625	.5474	.1034
.24	.6001	.3659	.6401	.8573	.5228	.1128
.26	.5969	.3493	.6935	.8528	.4990	.1222
.28	.5942	.3333	.7468	.8488	.4761	.1316
.30	.5918	.3178	.8002	.8455	.4540	.1410
.32	.5898	.3028	.8535	.8426	.4326	.1504
.34	.5882	.2883	.9069	.8403	.4119	.1598
.36	.5870	.2742	.9602	.8385	.3918	.1692
.38	.5861	.2606	1.0136	.8373	.3723	.1786
.40	.5855	.2473	1.0669	.8365	.3533	.1880
.42	.5853	.2344	1.1203	.8362	.3349	.1974
.44	.5854	.2219	1.1736	.8363	.3170	.2068
.46	.5859	.2097	1.2270	.8369	.2996	.2162
.48	.5866	.1979	1.2803	.8380	.2827	.2256
.50	.5877	.1864	1.3336	.8395	.2662	.2350
.52	.5890	.1752	1.3870	.8415	.2502	.2444
.54	.5907	.1643	1.4403	.8439	.2347	.2538
.56	.5927	.1537	1.4937	.8467	.2195	.2632
.58	.5950	.1434	1.5470	.8499	.2048	.2726
.60	.5975	.1334	1.6004	.8536	.1905	.2820
.62	.6003	.1237	1.6537	.8576	.1767	.2914
.64	.6034	.1142	1.7071	.8621	.1632	.3008
.66	.6068	.1051	1.7604	.8669	.1502	.3102
.68	.6105	.0963	1.8138	.8721	.1375	.3196
.70	.6144	.0877	1.8671	.8777	.1253	.3290
.72	.6185	.0794	1.9204	.8836	.1135	.3384
.74	.6229	.0714	1.9738	.8899	.1020	.3478
.76	.6276	.0637	2.0271	.8966	.0911	.3572
.78	.6325	.0564	2.0805	.9035	.0805	.3666
.80	.6376	.0493	2.1338	.9109	.0704	.3760
.82	.6430	.0425	2.1872	.9185	.0607	.3854
.84	.6485	.0361	2.2405	.9265	.0515	.3948
.86	.6543	.0300	2.2939	.9347	.0428	.4042
.88	.6603	.0242	2.3472	.9433	.0346	.4136
.90	.6665	.0188	2.4006	.9521	.0269	.4230
For Y1 =	.80					
.10	.7202	.5622	.3507	.9003	.7028	.0618
.12	.7118	.5345	.4209	.8897	.6681	.0742
.14	.7043	.5087	.4910	.8804	.6359	.0865
.16	.6976	.4845	.5612	.8720	.6056	.0989
.18	.6917	.4616	.6313	.8646	.5770	.1113
.20	.6864	.4398	.7014	.8580	.5498	.1236
.22	.6818	.4191	.7716	.8522	.5238	.1360
.24	.6777	.3992	.8417	.8471	.4990	.1483
.26	.6742	.3801	.9119	.8427	.4752	.1607
.28	.6711	.3618	.9820	.8389	.4523	.1731
.30	.6686	.3441	1.0522	.8358	.4302	.1854
.32	.6666	.3271	1.1223	.8332	.4088	.1978
.34	.6650	.3106	1.1925	.8313	.3883	.2101
.36	.6639	.2947	1.2626	.8298	.3683	.2225
.38	.6632	.2793	1.3327	.8290	.3491	.2349
.40	.6629	.2643	1.4029	.8286	.3304	.2472
.42	.6630	.2498	1.4730	.8288	.3123	.2596
.44	.6636	.2358	1.5432	.8294	.2948	.2719
.46	.6645	.2222	1.6133	.8306	.2778	.2843
.48	.6658	.2090	1.6835	.8322	.2613	.2967
.50	.6675	.1963	1.7536	.8343	.2453	.3090
.52	.6695	.1839	1.8237	.8369	.2299	.3214
.54	.6719	.1719	1.8939	.8399	.2149	.3338
.56	.6746	.1603	1.9640	.8433	.2003	.3461
.58	.6777	.1490	2.0342	.8471	.1863	.3585
.60	.6811	.1381	2.1043	.8514	.1726	.3708
.62	.6848	.1276	2.1745	.8560	.1595	.3832
.64	.6888	.1174	2.2446	.8610	.1468	.3956
.66	.6931	.1076	2.3148	.8664	.1345	.4079
.68	.6978	.0981	2.3849	.8722	.1227	.4203
.70	.7026	.0890	2.4550	.8783	.1113	.4326
.72	.7078	.0803	2.5252	.8847	.1004	.4450
.74	.7132	.0719	2.5953	.8915	.0899	.4574
.76	.7188	.0639	2.6655	.8985	.0798	.4697
.78	.7247	.0562	2.7356	.9059	.0702	.4821
.80	.7308	.0489	2.8058	.9135	.0611	.4945
.82	.7371	.0419	2.8759	.9213	.0524	.5068
.84	.7435	.0354	2.9461	.9294	.0443	.5192
.86	.7502	.0292	3.0162	.9377	.0366	.5315
.88	.7570	.0235	3.0863	.9462	.0294	.5439
.90	.7639	.0182	3.1565	.9549	.0227	.5563

For Y1 = .90

.10	.7964	.6017	.4706	.8848	.6686	.0829
.12	.7863	.5690	.5647	.8737	.6323	.0995
.14	.7776	.5388	.6589	.8640	.5987	.1161
.16	.7700	.5106	.7530	.8555	.5673	.1327
.18	.7633	.4840	.8471	.8481	.5378	.1493
.20	.7575	.4588	.9412	.8417	.5098	.1659
.22	.7526	.4349	1.0354	.8362	.4833	.1825
.24	.7484	.4121	1.1295	.8316	.4579	.1990
.26	.7449	.3903	1.2236	.8277	.4337	.2156
.28	.7421	.3695	1.3177	.8246	.4105	.2322
.30	.7400	.3494	1.4119	.8222	.3883	.2488
.32	.7385	.3302	1.5060	.8205	.3669	.2654
.34	.7376	.3117	1.6001	.8195	.3463	.2820
.36	.7372	.2939	1.6942	.8191	.3265	.2986
.38	.7374	.2767	1.7884	.8194	.3075	.3152
.40	.7382	.2602	1.8825	.8202	.2891	.3317
.42	.7395	.2443	1.9766	.8216	.2714	.3483
.44	.7412	.2290	2.0707	.8236	.2544	.3649
.46	.7434	.2142	2.1648	.8261	.2380	.3815
.48	.7461	.2000	2.2590	.8290	.2222	.3981
.50	.7493	.1863	2.3531	.8325	.2070	.4147
.52	.7528	.1731	2.4472	.8365	.1924	.4313
.54	.7568	.1605	2.5413	.8409	.1783	.4479
.56	.7611	.1483	2.6355	.8457	.1648	.4644
.58	.7658	.1367	2.7296	.8509	.1519	.4810
.60	.7708	.1255	2.8237	.8565	.1395	.4976
.62	.7762	.1148	2.9178	.8624	.1276	.5142
.64	.7818	.1046	3.0120	.8687	.1162	.5308
.66	.7877	.0949	3.1061	.8752	.1054	.5474
.68	.7938	.0856	3.2002	.8820	.0951	.5640
.70	.8002	.0768	3.2943	.8891	.0854	.5805
.72	.8067	.0685	3.3885	.8963	.0761	.5971
.74	.8133	.0606	3.4826	.9037	.0674	.6137
.76	.8201	.0532	3.5767	.9113	.0591	.6303
.78	.8270	.0462	3.6708	.9189	.0514	.6469
.80	.8339	.0397	3.7650	.9266	.0441	.6635
.82	.8409	.0336	3.8591	.9343	.0374	.6801
.84	.8479	.0280	3.9532	.9421	.0311	.6967
.86	.8548	.0228	4.0473	.9498	.0254	.7132
.88	.8617	.0181	4.1414	.9574	.0201	.7298
.90	.8684	.0138	4.2356	.9649	.0153	.7464

For Y1 = .60

.10	.5490	.4423	.1979	.9150	.7371	.0349
.12	.5432	.4225	.2374	.9053	.7041	.0418
.14	.5380	.4039	.2770	.8967	.6732	.0488
.16	.5333	.3864	.3166	.8889	.6440	.0558
.18	.5291	.3697	.3561	.8818	.6162	.0628
.20	.5253	.3538	.3957	.8754	.5897	.0697
.22	.5218	.3385	.4353	.8697	.5642	.0767
.24	.5188	.3238	.4749	.8646	.5396	.0837
.26	.5160	.3096	.5144	.8601	.5159	.0907
.28	.5136	.2958	.5540	.8561	.4930	.0976
.30	.5116	.2825	.5936	.8526	.4709	.1046
.32	.5098	.2696	.6331	.8497	.4494	.1116
.34	.5084	.2571	.6727	.8473	.4285	.1185
.36	.5072	.2450	.7123	.8453	.4083	.1255
.38	.5063	.2331	.7518	.8438	.3886	.1325
.40	.5057	.2216	.7914	.8428	.3694	.1395
.42	.5054	.2104	.8310	.8423	.3507	.1464
.44	.5053	.1995	.8706	.8422	.3325	.1534
.46	.5055	.1889	.9101	.8425	.3148	.1604
.48	.5060	.1785	.9497	.8433	.2975	.1674
.50	.5067	.1684	.9893	.8445	.2807	.1743
.52	.5077	.1586	1.0288	.8462	.2643	.1813
.54	.5089	.1490	1.0684	.8482	.2483	.1883
.56	.5104	.1396	1.1080	.8507	.2327	.1953
.58	.5122	.1305	1.1476	.8536	.2176	.2022
.60	.5141	.1217	1.1871	.8569	.2028	.2092
.62	.5164	.1130	1.2267	.8606	.1884	.2162
.64	.5188	.1046	1.2663	.8647	.1744	.2232
.66	.5215	.0965	1.3058	.8692	.1608	.2301
.68	.5244	.0885	1.3454	.8740	.1476	.2371
.70	.5276	.0808	1.3850	.8793	.1347	.2441
.72	.5309	.0734	1.4246	.8849	.1223	.2510
.74	.5345	.0662	1.4641	.8909	.1103	.2580
.76	.5383	.0592	1.5037	.8972	.0986	.2650
.78	.5424	.0524	1.5433	.9040	.0874	.2720
.80	.5466	.0460	1.5828	.9110	.0766	.2789
.82	.5511	.0398	1.6224	.9185	.0663	.2859
.84	.5557	.0338	1.6620	.9262	.0564	.2929
.86	.5606	.0282	1.7015	.9343	.0470	.2999
.88	.5657	.0228	1.7411	.9428	.0381	.3068
.90	.5709	.0178	1.7807	.9515	.0297	.3138

For Y1 =	.50					
.10	.4596	.3738	.1396	.9191	.7475	.0246
.12	.4549	.3576	.1676	.9099	.7153	.0295
.14	.4507	.3425	.1955	.9015	.6849	.0345
.16	.4469	.3281	.2234	.8939	.6561	.0394
.18	.4435	.3143	.2514	.8870	.6287	.0443
.20	.4404	.3012	.2793	.8808	.6024	.0492
.22	.4376	.2885	.3072	.8751	.5770	.0541
.24	.4351	.2763	.3351	.8701	.5526	.0591
.26	.4328	.2645	.3631	.8656	.5290	.0640
.28	.4308	.2531	.3910	.8616	.5061	.0689
.30	.4291	.2420	.4189	.8582	.4839	.0738
.32	.4276	.2312	.4469	.8552	.4624	.0787
.34	.4263	.2207	.4748	.8527	.4415	.0837
.36	.4253	.2105	.5027	.8507	.4211	.0886
.38	.4245	.2006	.5306	.8491	.4012	.0935
.40	.4240	.1909	.5586	.8480	.3818	.0984
.42	.4236	.1815	.5865	.8473	.3630	.1034
.44	.4235	.1723	.6144	.8471	.3445	.1083
.46	.4236	.1633	.6424	.8472	.3266	.1132
.48	.4239	.1545	.6703	.8478	.3090	.1181
.50	.4244	.1459	.6982	.8489	.2919	.1230
.52	.4252	.1376	.7261	.8503	.2752	.1280
.54	.4261	.1294	.7541	.8522	.2588	.1329
.56	.4272	.1214	.7820	.8544	.2429	.1378
.58	.4286	.1137	.8099	.8571	.2273	.1427
.60	.4301	.1061	.8379	.8602	.2122	.1477
.62	.4318	.0987	.8658	.8636	.1974	.1526
.64	.4337	.0915	.8937	.8675	.1829	.1575
.66	.4359	.0844	.9216	.8717	.1689	.1624
.68	.4382	.0776	.9496	.8764	.1552	.1673
.70	.4407	.0710	.9775	.8814	.1419	.1723
.72	.4434	.0645	1.0054	.8868	.1290	.1772
.74	.4463	.0582	1.0334	.8925	.1165	.1821
.76	.4493	.0522	1.0613	.8987	.1043	.1870
.78	.4526	.0463	1.0892	.9052	.0926	.1919
.80	.4560	.0406	1.1171	.9120	.0813	.1969
.82	.4596	.0352	1.1451	.9193	.0704	.2018
.84	.4634	.0300	1.1730	.9269	.0600	.2067
.86	.4674	.0250	1.2009	.9348	.0501	.2116
.88	.4715	.0203	1.2289	.9431	.0406	.2166
.90	.4759	.0159	1.2568	.9517	.0318	.2215
For Y1 =	.40					
.10	.3689	.3023	.0911	.9223	.7557	.0161
.12	.3653	.2896	.1093	.9133	.7240	.0193
.14	.3621	.2777	.1275	.9051	.6942	.0225
.16	.3591	.2663	.1457	.8977	.6658	.0257
.18	.3564	.2554	.1640	.8910	.6386	.0289
.20	.3540	.2450	.1822	.8849	.6125	.0321
.22	.3517	.2350	.2004	.8794	.5874	.0353
.24	.3498	.2252	.2186	.8744	.5631	.0385
.26	.3480	.2158	.2368	.8700	.5396	.0417
.28	.3464	.2067	.2551	.8660	.5167	.0449
.30	.3450	.1978	.2733	.8626	.4946	.0482
.32	.3438	.1892	.2915	.8596	.4730	.0514
.34	.3428	.1808	.3097	.8571	.4520	.0546
.36	.3420	.1726	.3279	.8550	.4315	.0578
.38	.3413	.1646	.3462	.8534	.4115	.0610
.40	.3409	.1568	.3644	.8522	.3920	.0642
.42	.3406	.1492	.3826	.8514	.3730	.0674
.44	.3404	.1417	.4008	.8511	.3544	.0706
.46	.3405	.1345	.4190	.8512	.3362	.0738
.48	.3407	.1274	.4372	.8517	.3184	.0771
.50	.3410	.1204	.4555	.8526	.3010	.0803
.52	.3416	.1136	.4737	.8539	.2840	.0835
.54	.3422	.1070	.4919	.8556	.2674	.0867
.56	.3431	.1005	.5101	.8577	.2512	.0899
.58	.3441	.0941	.5283	.8602	.2353	.0931
.60	.3452	.0879	.5466	.8631	.2198	.0963
.62	.3466	.0819	.5648	.8664	.2047	.0995
.64	.3480	.0760	.5830	.8701	.1899	.1027
.66	.3497	.0702	.6012	.8742	.1755	.1059
.68	.3514	.0646	.6194	.8786	.1614	.1092
.70	.3534	.0591	.6376	.8835	.1477	.1124
.72	.3555	.0538	.6559	.8887	.1344	.1156
.74	.3577	.0486	.6741	.8943	.1215	.1188
.76	.3601	.0436	.6923	.9002	.1089	.1220
.78	.3626	.0387	.7105	.9066	.0968	.1252
.80	.3653	.0340	.7287	.9133	.0851	.1284
.82	.3681	.0295	.7470	.9203	.0738	.1316
.84	.3711	.0252	.7652	.9278	.0629	.1348
.86	.3742	.0210	.7834	.9356	.0526	.1381
.88	.3775	.0171	.8016	.9437	.0427	.1413
.90	.3809	.0134	.8198	.9522	.0334	.1445
For Y1 =	.30					

.10	.2774	.2287	.0523	.9248	.7625	.0092
.12	.2748	.2193	.0627	.9160	.7311	.0111
.14	.2724	.2105	.0732	.9080	.7017	.0129
.16	.2702	.2021	.0837	.9008	.6737	.0147
.18	.2683	.1940	.0941	.8942	.6468	.0166
.20	.2665	.1863	.1046	.8882	.6209	.0184
.22	.2648	.1788	.1150	.8828	.5960	.0203
.24	.2634	.1715	.1255	.8779	.5718	.0221
.26	.2620	.1645	.1360	.8735	.5484	.0240
.28	.2609	.1577	.1464	.8696	.5256	.0258
.30	.2598	.1510	.1569	.8661	.5034	.0276
.32	.2590	.1446	.1673	.8632	.4819	.0295
.34	.2582	.1382	.1778	.8606	.4608	.0313
.36	.2576	.1321	.1882	.8586	.4403	.0332
.38	.2571	.1261	.1987	.8569	.4202	.0350
.40	.2567	.1202	.2092	.8557	.4006	.0369
.42	.2565	.1144	.2196	.8549	.3814	.0387
.44	.2563	.1088	.2301	.8545	.3627	.0405
.46	.2563	.1033	.2405	.8545	.3443	.0424
.48	.2565	.0979	.2510	.8549	.3263	.0442
.50	.2567	.0926	.2614	.8557	.3087	.0461
.52	.2571	.0875	.2719	.8569	.2915	.0479
.54	.2576	.0824	.2824	.8585	.2747	.0498
.56	.2582	.0775	.2928	.8605	.2582	.0516
.58	.2589	.0726	.3033	.8629	.2421	.0534
.60	.2597	.0679	.3137	.8657	.2263	.0553
.62	.2607	.0633	.3242	.8689	.2109	.0571
.64	.2617	.0587	.3346	.8724	.1958	.0590
.66	.2629	.0543	.3451	.8764	.1811	.0608
.68	.2642	.0500	.3556	.8807	.1667	.0627
.70	.2656	.0458	.3660	.8854	.1527	.0645
.72	.2671	.0417	.3765	.8905	.1390	.0663
.74	.2688	.0377	.3869	.8959	.1257	.0682
.76	.2705	.0339	.3974	.9017	.1128	.0700
.78	.2724	.0301	.4079	.9079	.1003	.0719
.80	.2744	.0265	.4183	.9145	.0883	.0737
.82	.2764	.0230	.4288	.9214	.0766	.0756
.84	.2786	.0196	.4392	.9287	.0654	.0774
.86	.2809	.0164	.4497	.9364	.0547	.0792
.88	.2833	.0133	.4601	.9444	.0445	.0811
.90	.2858	.0104	.4706	.9528	.0348	.0829
For Y1 =	.20					
.10	.1855	.1540	.0237	.9275	.7701	.0042
.12	.1836	.1474	.0285	.9182	.7370	.0050
.14	.1821	.1416	.0332	.9104	.7080	.0059
.16	.1807	.1360	.0380	.9033	.6802	.0067
.18	.1794	.1307	.0427	.8968	.6536	.0075
.20	.1782	.1256	.0474	.8909	.6280	.0084
.22	.1771	.1206	.0522	.8856	.6032	.0092
.24	.1761	.1158	.0569	.8807	.5792	.0100
.26	.1753	.1112	.0617	.8764	.5558	.0109
.28	.1745	.1066	.0664	.8725	.5331	.0117
.30	.1738	.1022	.0712	.8691	.5110	.0125
.32	.1732	.0979	.0759	.8662	.4895	.0134
.34	.1727	.0937	.0807	.8636	.4684	.0142
.36	.1723	.0896	.0854	.8615	.4478	.0151
.38	.1720	.0855	.0902	.8599	.4277	.0159
.40	.1717	.0816	.0949	.8586	.4080	.0167
.42	.1716	.0777	.0996	.8578	.3887	.0176
.44	.1715	.0740	.1044	.8573	.3699	.0184
.46	.1715	.0703	.1091	.8573	.3513	.0192
.48	.1715	.0666	.1139	.8576	.3332	.0201
.50	.1717	.0631	.1186	.8584	.3154	.0209
.52	.1719	.0596	.1234	.8595	.2980	.0217
.54	.1722	.0562	.1281	.8611	.2810	.0226
.56	.1726	.0529	.1329	.8630	.2643	.0234
.58	.1731	.0496	.1376	.8653	.2479	.0242
.60	.1736	.0464	.1423	.8680	.2319	.0251
.62	.1742	.0433	.1471	.8711	.2163	.0259
.64	.1749	.0402	.1518	.8745	.2009	.0268
.66	.1757	.0372	.1566	.8784	.1859	.0276
.68	.1765	.0343	.1613	.8826	.1713	.0284
.70	.1774	.0314	.1661	.8872	.1570	.0293
.72	.1784	.0286	.1708	.8921	.1430	.0301
.74	.1795	.0259	.1756	.8975	.1295	.0309
.76	.1806	.0233	.1803	.9032	.1163	.0318
.78	.1818	.0207	.1850	.9092	.1034	.0326
.80	.1831	.0182	.1898	.9157	.0910	.0334
.82	.1845	.0158	.1945	.9225	.0791	.0343
.84	.1859	.0135	.1993	.9297	.0675	.0351
.86	.1874	.0113	.2040	.9372	.0565	.0360
.88	.1890	.0092	.2088	.9451	.0460	.0368
.90	.1907	.0072	.2135	.9534	.0360	.0376
For Y1 =	.10					
.10	.0949	.0838	.0061	.9491	.8377	.0011

.12	.0920	.0743	.0073	.9202	.7426	.0013
.14	.0912	.0713	.0085	.9123	.7134	.0015
.16	.0905	.0686	.0097	.9054	.6859	.0017
.18	.0899	.0660	.0109	.8990	.6595	.0019
.20	.0893	.0634	.0121	.8932	.6341	.0021
.22	.0888	.0609	.0133	.8879	.6095	.0023
.24	.0883	.0586	.0145	.8831	.5856	.0026
.26	.0879	.0562	.0157	.8788	.5623	.0028
.28	.0875	.0540	.0170	.8750	.5398	.0030
.30	.0872	.0518	.0182	.8716	.5177	.0032
.32	.0869	.0496	.0194	.8687	.4961	.0034
.34	.0866	.0475	.0206	.8662	.4751	.0036
.36	.0864	.0454	.0218	.8641	.4545	.0038
.38	.0862	.0434	.0230	.8624	.4343	.0041
.40	.0861	.0415	.0242	.8611	.4146	.0043
.42	.0860	.0395	.0254	.8603	.3952	.0045
.44	.0860	.0376	.0266	.8598	.3762	.0047
.46	.0860	.0358	.0278	.8597	.3576	.0049
.48	.0860	.0339	.0291	.8600	.3392	.0051
.50	.0861	.0321	.0303	.8607	.3213	.0053
.52	.0862	.0304	.0315	.8618	.3038	.0055
.54	.0863	.0287	.0327	.8633	.2865	.0058
.56	.0865	.0270	.0339	.8651	.2696	.0060
.58	.0867	.0253	.0351	.8674	.2531	.0062
.60	.0870	.0237	.0363	.8700	.2369	.0064
.62	.0873	.0221	.0375	.8730	.2210	.0066
.64	.0876	.0205	.0387	.8764	.2054	.0068
.66	.0880	.0190	.0400	.8801	.1902	.0070
.68	.0884	.0175	.0412	.8842	.1753	.0073
.70	.0889	.0161	.0424	.8887	.1608	.0075
.72	.0894	.0147	.0436	.8936	.1466	.0077
.74	.0899	.0133	.0448	.8988	.1327	.0079
.76	.0904	.0119	.0460	.9044	.1192	.0081
.78	.0910	.0106	.0472	.9104	.1061	.0083
.80	.0917	.0093	.0484	.9167	.0935	.0085
.82	.0923	.0081	.0496	.9235	.0812	.0087
.84	.0931	.0069	.0509	.9305	.0694	.0090
.86	.0938	.0058	.0521	.9380	.0581	.0092
.88	.0946	.0047	.0533	.9457	.0473	.0094
.90	.0954	.0037	.0545	.9539	.0371	.0096

An alternative is to use the TK-Solver model with the dimensionless flowrate Q' added as an additional list variable. This model is given below along with the output obtained for Y_1' = .6, .7, .8 & .9. (In using TK-Solver it is necessary that the guesses for the list variable Y_b' (Yb) be given quite accurately, especially for the larger values of F^{r1}, and it was necessary to adjust the guess to get convergence.
Model P2_119*.TK (also see PRB2_118.TK)

```
============ VARIABLE SHEET ============
St Input───── Name──── Output──── Unit───
   .6         Y1
   1.7721543  B1
   .49202834  A1
   .50217435  Yd1
L  .01        Fr2
LG .54897512  Yr
LG .44221115  Yb
LG 1.6689039  Br
LG .44159578  Ar
LG 1.4549598  Bb
LG .33503915  Ab
L             r6
L             Fr
L             r7
L             Qp
              Qpc

=============== RULE SHEET ===============
S Rule─────────────────────────────────
  A1*sqrt(Yd1*Fr2)=(Ar-Ab)*sqrt((Ar-Ab)/sin(Br))
  Y1+Fr2*Yd1/2=Yr+.5*(Ar-Ab)/sin(Br)
  Br=acos(1-2*Yr)
  Ar=.25*(Br-cos(Br)*sin(Br))
  Bb=acos(1-2*Yb)
  Ab=.25*(Bb-cos(Bb)*sin(Bb))
  r6=Yb/Y1
  Fr=sqrt(Fr2)
  r7=Yr/Y1
  Qpc=sqrt(A1^3/sin(B1))
  Qp=Fr*Qpc
```

═══════════ TABLE: ══════════════

Title:

Element	Fr	r6	r7	Qp
1	.1	.737018591	.914958538	.034867226
2	.16673332	.634520787	.886406031	.058135283
3	.213541565	.572274906	.871492998	.074456019
4	.251793566	.52554281	.861855741	.087793431
5	.284956137	.487480849	.855174567	.099356299
6	.314642654	.45508205	.850428074	.109707164
7	.34176015	.426725883	.84706462	.119162282
8	.366878727	.401427647	.844748235	.127920433
9	.390384426	.378539187	.843257215	.136116218
10	.412553027	.357608681	.842436586	.143845794
11	.433589668	.338307236	.842173194	.151180687
12	.453651849	.320387205	.842381553	.158175813
13	.472863617	.303656974	.842995278	.164874424
14	.49132474	.287964936	.843961636	.171311305
15	.509116882	.273188878	.845237934	.177514932
16	.526307895	.25922871	.846789047	.183508961
17	.542954878	.246001341	.848585666	.189313302
18	.55910643	.233436979	.850603042	.1949449
19	.574804315	.221476397	.852820049	.200418317
20	.59008474	.210068877	.855218482	.205746177
21	.604979338	.199170647	.857782524	.21093951
22	.61951594	.188743657	.860498326	.21600802
23	.633719181	.178754626	.863353678	.220960296
24	.647610994	.169174282	.86633775	.225803986
25	.66121101	.159976755	.869440882	.230545934
26	.674536878	.151139075	.87265441	.235192295
27	.687604538	.142640777	.875970531	.239748625
28	.70042844	.134463563	.87938218	.244219964
29	.713021739	.126591029	.882882942	.248610898
30	.725396443	.119008433	.886466963	.252925614
31	.737563557	.111702508	.890128888	.257167949
32	.749533188	.104661298	.893863797	.261341427
33	.761314652	.097874023	.897667161	.265449297
34	.772916554	.091330968	.901534796	.269494558
35	.784346862	.085023384	.905462824	.273479989
36	.795612971	.078943416	.909447649	.277408169
37	.806721761	.073084033	.913485919	.281281496
38	.817679644	.067438983	.917574509	.285102206
39	.828492607	.062002747	.921710497	.288872386
40	.839166253	.05677052	.925891146	.29259399
41	.849705831	.051738194	.930113884	.296268849
42	.860116271	.046902357	.934376296	.29989868
43	.870402206	.042260309	.938676101	.3034851
44	.880567999	.037810097	.943011152	.30702963
45	.890617763	.033550569	.947379415	.310533704
46	.900555384	.029481461	.951778966	.313998677
47	.910384534	.025603532	.956207979	.317425829
48	.920108689	.021918749	.96066472	.320816372
49	.929731144	.018430578	.96514754	.324171455
50	.939255024	.015144418	.969654868	.327492168
51	.948683298	.012068289	.974185207	.330779545
52	.95131488	.011243487	.975461716	.331697105
53	.953939201	.010436485	.976739897	.332612133
54	.956556323	.009647652	.978019718	.333524651
55	.959166305	.008877403	.979301151	.334434679
56	.961769203	.00812621	.980584165	.335342237
57	.964365076	.007394604	.981868732	.336247346
58	.96695398	.006683192	.983154821	.337150025
59	.969535971	.005992672	.984442405	.338050294
60	.972111105	.005323851	.985731455	.338948171
61	.974679434	.004677674	.987021943	.339843677

═══════════ VARIABLE SHEET ══════════

St	Input	Name	Output	Unit
	.7	Y1		
	1.9823132	B1		
	.5872298	A1		
	.64072025	Yd1		
L	.01	Fr2		
LG	.54897512	Yr		
LG	.44221115	Yb		
LG	1.6689039	Br		
LG	.44159578	Ar		
LG	1.4549598	Bb		
LG	.33503915	Ab		
L		r6		
L		Fr		

```
L           r7
L           Qp
            Qpc
```

═══════════════ TABLE: ═════════════════

Title:

Element	Fr	r6	r7	Qp
1	.1	.722989946	.909115662	.047004811
2	.16673332	.618480042	.879491251	.078372682
3	.213541565	.555601056	.864260648	.100374809
4	.251793566	.508653001	.854557823	.11835509
5	.284956137	.470572394	.847936984	.133943094
6	.314642654	.438267586	.843324857	.147897185
7	.34176015	.410077063	.840143256	.160643713
8	.366878727	.384993423	.838040744	.172450652
9	.390384426	.362354758	.836785792	.183499462
10	.412553027	.341700346	.836216767	.193919771
11	.433589668	.322695243	.836215782	.203808004
12	.453651849	.305087524	.836693851	.213238194
13	.472863617	.288682448	.837581919	.222268649
14	.49132474	.273326057	.838825156	.230946265
15	.509116882	.258894328	.840379189	.239309428
16	.526307895	.24528575	.842207516	.247390031
17	.542954878	.232416092	.844279681	.255214914
18	.55910643	.220214627	.846569962	.262806921
19	.574804315	.208621354	.849056397	.270185682
20	.59008474	.197584899	.851720057	.277368217
21	.604979338	.187060925	.854544483	.284369395
22	.61951594	.177010889	.857515253	.291202297
23	.633719181	.167401073	.860619646	.297878503
24	.647610994	.15820181	.863846364	.304408324
25	.66121101	.149386864	.86718532	.310800986
26	.674536878	.140932931	.870627454	.317064785
27	.687604538	.132819227	.874164594	.323207213
28	.70042844	.125027149	.877789332	.329235065
29	.713021739	.117539996	.881494927	.335154521
30	.725396443	.110342738	.885275216	.340971227
31	.737563557	.103421818	.889124549	.346690356
32	.749533188	.096764992	.893037724	.352316658
33	.761314652	.090361187	.897009939	.357854514
34	.772916554	.084200387	.901036745	.363307966
35	.784346862	.078273536	.905114011	.36868076
36	.795612971	.072572456	.909237888	.373976373
37	.806721761	.067089783	.913404782	.379198039
38	.817679644	.061818907	.917611328	.384348771
39	.828492607	.056753936	.921854367	.389431384
40	.839166253	.051889662	.92613093	.394448511
41	.849705831	.047221544	.930438217	.39940262
42	.860116271	.042745702	.934773583	.404296028
43	.870402206	.038458925	.939134524	.409130912
44	.880567999	.034358694	.943518668	.413909323
45	.890617763	.030443231	.947923759	.418633196
46	.900555384	.026711569	.952347651	.423304356
47	.910384534	.023163664	.956788298	.42792453
48	.920108689	.019800562	.961243746	.43249535
49	.929731144	.016624648	.965712125	.437018367
50	.939255024	.013640025	.970191648	.441495049
51	.948683298	.010853125	.974680595	.445926791
52	.95131488	.010107089	.975943004	.447163761
53	.953939201	.009377674	.97720599	.448397319
54	.956556323	.00866519	.978469519	.449627492
55	.959166305	.007969993	.979733554	.450854309
56	.961769203	.007292482	.980998063	.452077796
57	.964365076	.006633113	.982263011	.453297981
58	.96695398	.005992411	.983528363	.454514891
59	.969535971	.005370977	.984794086	.455728551
60	.972111105	.004769511	.986060148	.456938988
61	.974679434	.004188835	.987326516	.458146226

══════════════ VARIABLE SHEET ══════════════

St	Input	Name	Output	Unit
	.8	Y1		
	2.2142974	B1		
	.67357436	A1		
	.8419679	Yd1		
L	.01	Fr2		
LG	.54897512	Yr		
LG	.44221115	Yb		
LG	1.6689039	Br		
LG	.44159578	Ar		

```
LG 1.4549598 Bb
LG .33503915 Ab
L            r6
L            Fr
L            r7
L            Qp
             Qpc
```

════════════════ TABLE: ════════════════

Title:

Element	Fr	r6	r7	Qp
1	.1	.702795997	.900278451	.061806383
2	.16673332	.595783393	.869435227	.103051834
3	.213541565	.532090038	.854004636	.131982317
4	.251793566	.484837704	.844429572	.155624496
5	.284956137	.446698512	.838095941	.176121081
6	.314642654	.414479159	.833862223	.194469244
7	.34176015	.386468695	.831115555	.211229587
8	.366878727	.361632109	.829484346	.22675447
9	.390384426	.339290571	.828724234	.241282493
10	.412553027	.318972265	.828664851	.254984103
11	.433589668	.30033459	.829182045	.26798609
12	.453651849	.283120109	.830182149	.280385799
13	.472863617	.267129977	.831592482	.292259897
14	.49132474	.25220709	.833355329	.30367005
15	.509116882	.23822495	.835423958	.31466673
16	.526307895	.225080052	.837759894	.325291872
17	.542954878	.212686525	.840331005	.335580771
18	.55910643	.20097227	.843110116	.345563461
19	.574804315	.18987611	.846073985	.355265755
20	.59008474	.179345654	.849202542	.364710034
21	.604979338	.169335661	.8524783	.373915846
22	.61951594	.159806782	.855885899	.382900394
23	.633719181	.150724559	.859411752	.391678903
24	.647610994	.142058644	.863043759	.400264931
25	.66121101	.133782166	.866771079	.408670609
26	.674536878	.125871213	.870583945	.416906846
27	.687604538	.11830442	.874473512	.424983493
28	.70042844	.111062621	.878431733	.432909484
29	.713021739	.104128563	.882451252	.440692946
30	.725396443	.09748667	.886525318	.448341303
31	.737563557	.091122839	.890647711	.455861356
32	.749533188	.085024278	.894812682	.463259352
33	.761314652	.079179359	.899014895	.470541049
34	.772916554	.073577502	.903249385	.477711765
35	.784346862	.06820907	.907511517	.484776425
36	.795612971	.063065286	.911796955	.491739599
37	.806721761	.058138164	.916101627	.49860554
38	.817679644	.053420446	.920421704	.505378211
39	.828492607	.048905557	.924753577	.512061313
40	.839166253	.044587569	.929093834	.518658307
41	.849705831	.040461176	.933439249	.52517244
42	.860116271	.036521677	.937786761	.531606756
43	.870402206	.032764979	.942133461	.53796412
44	.880567999	.029187604	.946476587	.544247229
45	.890617763	.02578672	.950813505	.550458625
46	.900555384	.022560196	.955141704	.556600709
47	.910384534	.019506682	.959458788	.562675751
48	.920108689	.016625737	.963762469	.568685899
49	.929731144	.013918028	.968050558	.574633191
50	.939255024	.011385624	.972320963	.580519556
51	.948683298	.009032488	.97657168	.586346831
52	.95131488	.008404572	.977761896	.587973317
53	.953939201	.007791499	.978950366	.589595315
54	.956556323	.007193496	.980137048	.591212864
55	.959166305	.00661082	.981321903	.592825999
56	.961769203	.006043767	.98250489	.594434756
57	.964365076	.005492679	.98368597	.596039171
58	.96695398	.004957951	.984865106	.597639279
59	.969535971	.004440043	.986042257	.599235115
60	.972111105	.003939492	.987217386	.600826711
61	.974679434	.003456935	.988390456	.602414103

═══════════════ VARIABLE SHEET ═══════════════

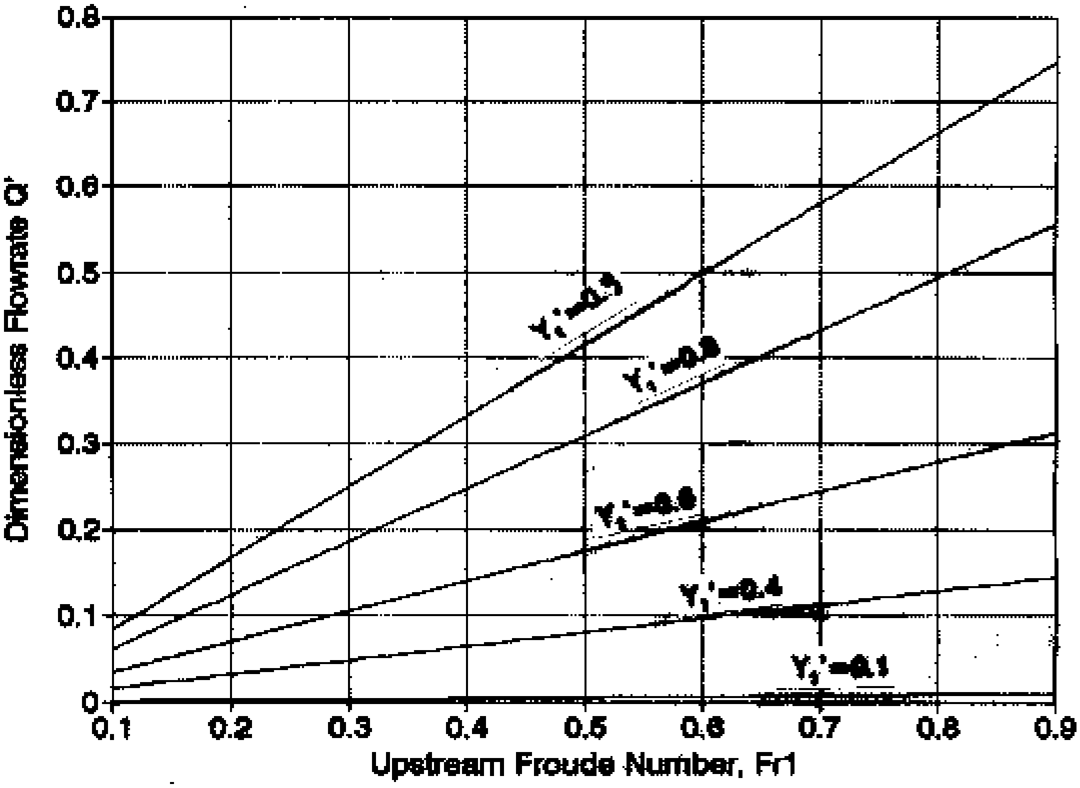
St	Input	Name	Output	Unit
	.9	Y1		
	2.4980915	B1		
	.74452289	A1		
	1.2408715	Yd1		
L	.01	Fr2		

```
LG .54897512 Yr
LG .44221115 Yb
LG 1.6689039 Br
LG .44159578 Ar
LG 1.4549598 Bb
LG .33503915 Ab
L            r6
L            Fr
L            r7
L            Qp
             Qpc
```

═════════════════ TABLE: ═════════════════════

Title:

Element	Fr	r6	r7	Qp
1	.1	.668571849	.884842201	.082935686
2	.16673332	.55716393	.852911877	.138281423
3	.213541565	.491684451	.837885471	.177102163
4	.251793566	.443517226	.829183315	.208826723
5	.284956137	.404920061	.823944872	.236330328
6	.314642654	.372531455	.820927835	.260951045
7	.34176015	.34455447	.819467233	.283441126
8	.366878727	.319903779	.819160473	.30427339
9	.390384426	.297868316	.81974289	.323768003
10	.412553027	.277954215	.821029846	.342153685
11	.433589668	.259803014	.822886596	.359600567
12	.453651849	.24314539	.825211258	.376239275
13	.472863617	.227773288	.827924555	.392172686
14	.49132474	.213522261	.830963315	.407483546
15	.509116882	.200259805	.834276183	.422239581
16	.526307895	.187877391	.837820691	.436497065
17	.542954878	.176284861	.841561198	.450303355
18	.55910643	.165406389	.845467404	.463698756
19	.574804315	.155177505	.849513258	.476717904
20	.59008474	.145542857	.853676139	.489390829
21	.604979338	.136454516	.857936228	.501743767
22	.61951594	.127870647	.862276025	.513799797
23	.633719181	.119754479	.866679974	.525579353
24	.647610994	.112073481	.87113416	.537100624
25	.66121101	.104798704	.875626068	.54837989
26	.674536878	.097904246	.880144392	.55943179
27	.687604538	.091366816	.884678876	.570269543
28	.70042844	.085165376	.889220185	.580905135
29	.713021739	.079280842	.893759799	.591349474
30	.725396443	.073695834	.898289923	.60161252
31	.737563557	.068394473	.902803416	.611703399
32	.749533188	.0633622	.907293729	.621630495
33	.761314652	.058585632	.91175485	.631401533
34	.772916554	.054052433	.916181262	.64102365
35	.784346862	.049751211	.920567908	.650503454
36	.795612971	.045671423	.924910152	.659847079
37	.806721761	.041803304	.929203761	.66906023
38	.817679644	.038137796	.933444871	.678148225
39	.828492607	.034666501	.937629972	.687116031
40	.839166253	.031381631	.941755889	.695968292
41	.849705831	.028275981	.945819763	.704709364
42	.860116271	.025342896	.949819036	.713343334
43	.870402206	.022576261	.95375144	.721874044
44	.880567999	.019970492	.957614979	.730305114
45	.890617763	.017520545	.961407923	.738639955
46	.900555384	.015221937	.96512879	.74688179
47	.910384534	.013070785	.968776338	.755033663
48	.920108689	.011063879	.972349555	.763098457
49	.929731144	.00919879	.975847645	.771078906
50	.939255024	.007474046	.97927002	.778977601
51	.948683298	.005889424	.982616285	.786797005
52	.95131488	.005469666	.983542523	.788979525
53	.953939201	.005061134	.984462736	.791156025
54	.956556323	.004663908	.985376923	.793326553
55	.959166305	.004278093	.986285084	.795491159
56	.961769203	.003903816	.987187221	.797649891
57	.964365076	.003541231	.988083335	.799802796
58	.96695398	.003190527	.98897343	.801949921
59	.969535971	.002851932	.98985751	.804091313
60	.972111105	.002525723	.990735579	.806227018
61	.974679434	.002212238	.991607644	.80835708


Dimensionless Flowrate Q'
Upstream Froude Number, Fr1
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
Y1'=0.4
Y1'=0.1

131a.

The upstream Froude Number equals F_{r1} = 0.3 for the flow at a depth of 3 ft in a 5 ft diameter pipe upstream of a section whose bottom is filled with a flat hump. What rise must this hump have so critical depth occurs at this restricted section? What is the critical depth over the hump? What is the flowrate? If Mannings n = .013, what upstream bottom slope should the channel have so uniform flow occurs here? Solve the problem using the dimensionless values in the graphs, etc. from Example Problem 23, and verify the results by using the equations that apply that use the dimensions of the problem.

Solution using dimensionless variables:

$Y_b = Y_b'D = (Y_b'/Y_1)Y_1D = .4709(.6)(5) = 1.4126$ ft

$Y_r = Y_r'D = (Y_r'/Y_1)Y_1D = .8526(.6)(5) = 2.5579$ ft

$Y_c = Y_r - Y_b = 1.145$ ft.

$Q = (gD^5)^{1/2}Q' = (32.2x5^5)^{.5}(.1046) = 33.18$ cfs

Solve Mannings equation for S_o = .000360.

Program PRB2_118 for the previous problem solve the equations using the dimensioned variables. Using this program with the following input gives the solution table below. Note the values are as given above associated with F_{r1} = .3.

32.2 5 1 3
.1 .9 .1 4.5 3.5

```
 For Y1 =    3.00
    .10   2.7449   2.2111  11.0604    .9150    .7370    .0349
    .20   2.6263   1.7690  22.1208    .8754    .5897    .0697
    .30   2.5579   1.4126  33.1812    .8526    .4709    .1046
    .40   2.5285   1.1082  44.2416    .8428    .3694    .1395
    .50   2.5336    .8422  55.3019    .8445    .2807    .1743
    .60   2.5707    .6084  66.3623    .8569    .2028    .2092
    .70   2.6378    .4042  77.4227    .8793    .1347    .2441
    .80   2.7331    .2298  88.4831    .9110    .0766    .2789
    .90   2.8546    .0891  99.5435    .9515    .0297    .3138
```

132.

A complex branching channel system takes water from a reservoir with a water surface elevation of H = 2 meter above the channel bottom as shown. The sizes of the channels are shown on the sketch below. All channels are rectangular except number 3 which is trapezoidal. Channel 2 contains two gates each 0.6 meter wide that both have contraction coefficients $C_c = .6$. The first gate is set so its tip is .2 m above the channel bottom and the other gate is set .15 m above the channel bottom. Channel 3 and 4 are long with the Mannings roughness coefficients, and bottom slope as shown on the sketch. Identify what unknowns you would solve for and then give the equations that need to be solved. Then solve these equations

Wanted: the equations and solution to the given 4 channel system in which Channel 2 contains two gates.

Solution:

The unknown variables are: four flow rates in the 4 channels, the flow rates past the two gates in Channel 2, Q_{2a} and Q_{2b}, and four depths in the 4 channels, for a total of 10. These are: $Q_1,Q_2,Q_3,Q_4,Q_{2a},Q_{2b},Y_1,Y_2,Y_3,Y_4$.
The equations consist of 2 continuity equations; one at the junction between the channels, and on across the gate, 6 energy equations and 2 Mannings equations. There equations are given in the TK-Solver "Variable Sheet" given below.

```
TK-SOlver Model PRB2 110.TK
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
              Q1       13.147476
              Q2       1.4006664
              Q3       9.070333
              Q4       2.6764765
              Q2A      .70033318
              Q2B      .70033318
              Y1       .6558483
              Y2       1.9292766
              Y3       1.8214206
              Y4       1.8905935
   .2         Y2A
   .2         Y2B
   4          B1
   19.62      g2
   2          H
   .05        KE
   2          B2
   .6         B2A
   .6         B2B
   1.5        B3
   1          m3
   1.5        B4
   .014       n3
   1          Cu
   .0005      So3
   .014       n4
   .0004      So4

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Q1-Q2-Q3-Q4=0
* Q2-Q2A-Q2B=0
* H-Y1-(1+KE)*(Q1/(B1*Y1))^2/g2=0
* Y1+(Q1/(B1*Y1))^2/g2-Y2-(Q2/(B2*Y2))^2/g2=0
* Y2+(Q2/(B2*Y2))^2/g2-Y2A-(Q2A/(B2A*Y2A))^2/g2=0
* Y2+(Q2/(B2*Y2))^2/g2-Y2B-(Q2B/(B2B*Y2B))^2/g2=0
* Y1+(Q1/(B1*Y1))^2/g2-Y3-(Q3/((B3+m3*Y3)*Y3))^2/g2=0
```

```
* Y1+(Q1/(B1*Y1))^2/g2-Y4-(Q4/(B4*Y4))^2/g2=0
* n3*Q3*(B3+2*Y3*sqrt(m3^2+1))^.66666667-Cu*((B3+m3*Y3)*Y3)^1.666667*sqrt(So3)=
* n4*Q4*(B4+2*Y4)^.66666667-Cu*(B4*Y4)^1.666667*sqrt(So4)=0
```

133.

The same channel exists as in the previous problem except now channel 4 contains a pier over a length of it that divides the flow into two parts Q_5 and Q_6. Set up and solve the equations that now govern the problem. (Note: Under the assumptions that we do not need to solve the gradually varied flow in the channel upstream and near the junctions, and that the pier's length is short so that the energy at it upstream and downstream ends are the same we end up with one less equation than unknown unless some condition is established across the pier. Since under our assumptions one would expect the flow in the two channel through the pier length to be close to the ratio of the channel widths, this might be used as the condition.)

Given: Previous problem with a pier in Channel 4.

Solution:

Notice in the following TK-SOlver model that one additional continuity, and two additional energy equations have been added to the model of the previous problem. Since we have added 4 additional variables: Q_5,Q_6,Y_5 & Y_6 one additional equation is still needed, and as suggested in the problem statement this equation is:

$Q_5b_6=Q_6b_5$.

```
TK-Solver Model PRB2_111.TK
==================== VARIABLE SHEET =====================
St Input───── Name──── Output──── Unit────── Comment─────────
              Q1       13.147476
              Q2       1.4006664
              Q3       9.070333
              Q4       2.6764765
              Q2A      .70033318
              Q2B      .70033318
              Y1       .6558483
              Y2       1.9292766
              Y3       1.8214206
              Y4       1.8905935
              Q5       1.0705906
              Q6       1.6058859
              Y5       1.8265564
              Y6       1.8265564
   .2         Y2A
   .2         Y2B
   4          B1
   19.62      g2
   2          H
   .05        KE
   2          B2
   .6         B2A
   .6         B2B
   1.5        B3
   1          m3
   1.5        B4
   .014       n3
   1          Cu
   .0005      So3
   .014       n4
   .0004      So4
   .4         B5
   .6         B6

==================== RULE SHEET =========================
S Rule───────────────────────────────────────────────────
* Q1-Q2-Q3-Q4=0
```

```
* Q2-Q2A-Q2B=0
* Q4=Q5+Q6
* H-Y1-(1+KE)*(Q1/(B1*Y1))^2/g2=0
* Y1+(Q1/(B1*Y1))^2/g2-Y2-(Q2/(B2*Y2))^2/g2=0
* Y2+(Q2/(B2*Y2))^2/g2-Y2A-(Q2A/(B2A*Y2A))^2/g2=0
* Y2+(Q2/(B2*Y2))^2/g2-Y2B-(Q2B/(B2B*Y2B))^2/g2=0
* Y1+(Q1/(B1*Y1))^2/g2-Y3-(Q3/((B3+m3*Y3)*Y3))^2/g2=0
* Y1+(Q1/(B1*Y1))^2/g2-Y4-(Q4/(B4*Y4))^2/g2=0
* n3*Q3*(B3+2*Y3*sqrt(m3^2+1))^.66666667-Cu*((B3+m3*Y3)*Y3)^1.666667*sqrt(So3)=
* n4*Q4*(B4+2*Y4)^.66666667-Cu*(B4*Y4)^1.666667*sqrt(So4)=0
* Y4+(Q4/(B4*Y4))^2/g2-Y5-(Q5/(B5*Y5))^2/g2=0
* Y4+(Q4/(B4*Y4))^2/g2-Y6-(Q6/(B6*Y6))^2/g2=0
* Q5*B6=Q6*B5
```

134.

Parshall and Cutthroat flumes that are widely used to measure flowrates in open channels and ditch make use of the energy principle, and if free flow conditions exist critical depth occurs close to the throat. A Cutthroat flume always has its throat width, W, 2 feet less than its upstream and downstream width, and its bottom is flat. The upstream stilling well is 2.11 ft upstream from the throat whereas the upstream convengence sides are 3.16 ft long. (The length of the flume downstream from the throat is 3 ft, and flume beginning is 2 ft upstream from the throat, so the diverging portion is 1 on 6 and the upstream convergine portion of the flume converges 1 on 3.) The laboratory calibration eqution for Cutthroat flumes is Q (cfs) = $Ch_a^{1.56}$ (h_a in ft) in which C = $3.50W^{1.025}$ (with the width W in ft). Use this free flow equation for a Cutthroat flume to locate the position where critical flow will occur in a 6 foot wide Cutthroat flume for a range of flowrates. Use one dimensional hydraulics in this determination, and assume there is no energy loss between the upstream stilling well position where depth h_a is measured and the critical flow section.

The Fortran program given below generates the table of values requested.

Input: 6 32.2 51 5 2.5

Output:

Q(cfs)	h_a(ft)	E_a(ft)	q(ft^2/s)	Width	x_c(ft)
5.00	.387	.435	.887	5.634	-1.097
7.50	.502	.567	1.317	5.693	-.921
10.00	.604	.683	1.744	5.734	-.797
12.50	.697	.790	2.168	5.767	-.700
15.00	.783	.889	2.589	5.793	-.622
17.50	.865	.983	3.010	5.815	-.556
20.00	.942	1.072	3.428	5.834	-.499
22.50	1.016	1.157	3.846	5.851	-.448
25.00	1.087	1.239	4.262	5.866	-.403
27.50	1.155	1.319	4.678	5.879	-.363
30.00	1.221	1.396	5.092	5.891	-.326
32.50	1.286	1.470	5.506	5.903	-.292
35.00	1.348	1.543	5.919	5.913	-.260
37.50	1.409	1.614	6.331	5.923	-.231
40.00	1.469	1.683	6.743	5.932	-.204
42.50	1.527	1.751	7.154	5.941	-.178
45.00	1.584	1.817	7.565	5.949	-.154
47.50	1.640	1.882	7.975	5.956	-.131
50.00	1.694	1.946	8.384	5.964	-.109
52.50	1.748	2.009	8.793	5.970	-.089
55.00	1.801	2.070	9.202	5.977	-.069
57.50	1.853	2.131	9.610	5.983	-.051
60.00	1.905	2.191	10.018	5.989	-.033
62.50	1.955	2.250	10.426	5.995	-.016
65.00	2.005	2.308	10.833	6.000	.000
67.50	2.054	2.366	11.240	6.006	.008
70.00	2.102	2.422	11.646	6.011	.016
72.50	2.150	2.478	12.052	6.016	.023
75.00	2.197	2.534	12.458	6.020	.030
77.50	2.244	2.588	12.863	6.025	.037
80.00	2.290	2.643	13.269	6.029	.044
82.50	2.336	2.696	13.674	6.034	.050
85.00	2.381	2.749	14.078	6.038	.056
87.50	2.426	2.801	14.483	6.042	.063

90.00	2.470	2.853	14.887	6.046	.068
92.50	2.514	2.905	15.291	6.049	.074
95.00	2.557	2.956	15.694	6.053	.080
97.50	2.600	3.006	16.098	6.057	.085
100.00	2.642	3.056	16.501	6.060	.090
102.50	2.685	3.106	16.904	6.064	.095
105.00	2.726	3.155	17.307	6.067	.100
107.50	2.768	3.203	17.709	6.070	.105
110.00	2.809	3.252	18.112	6.073	.110
112.50	2.850	3.300	18.514	6.076	.115
115.00	2.890	3.347	18.916	6.080	.119
117.50	2.930	3.395	19.318	6.082	.124
120.00	2.970	3.441	19.719	6.085	.128
122.50	3.010	3.488	20.121	6.088	.132
125.00	3.049	3.534	20.522	6.091	.137
127.50	3.088	3.580	20.923	6.094	.141
130.00	3.126	3.626	21.324	6.096	.145

Note: A positive x is upstream from throat (neck) of flume, and negative x is downstream therefrom.

135.

Repeat the previous problem except for a 2 foot wide Cutthroat flume.

The Fortran program given below generates the table of values requested.

Q(cfs)	h_a(ft)	E_a(ft)	q(ft^2/s)	Width	x_c(ft)
2.00	.443	.471	1.000	2.000	.000
4.00	.691	.738	1.957	2.044	.066
6.00	.896	.959	2.899	2.070	.105
8.00	1.077	1.154	3.831	2.088	.132
10.00	1.243	1.333	4.756	2.103	.154
12.00	1.397	1.500	5.676	2.114	.171
14.00	1.542	1.657	6.591	2.124	.186
16.00	1.680	1.807	7.502	2.133	.199
18.00	1.812	1.950	8.409	2.141	.211
20.00	1.938	2.087	9.314	2.147	.221
22.00	2.061	2.220	10.216	2.154	.230
24.00	2.179	2.348	11.115	2.159	.239
26.00	2.293	2.473	12.012	2.164	.247
28.00	2.405	2.594	12.908	2.169	.254
30.00	2.514	2.713	13.801	2.174	.261
32.00	2.620	2.828	14.693	2.178	.267
34.00	2.724	2.941	15.583	2.182	.273
36.00	2.825	3.052	16.471	2.186	.278
38.00	2.925	3.161	17.358	2.189	.284
40.00	3.023	3.268	18.244	2.193	.289
42.00	3.119	3.372	19.128	2.196	.294
44.00	3.213	3.475	20.011	2.199	.298
46.00	3.306	3.577	20.893	2.202	.303
48.00	3.398	3.676	21.774	2.204	.307
50.00	3.488	3.775	22.654	2.207	.311
52.00	3.576	3.872	23.533	2.210	.315
54.00	3.664	3.968	24.410	2.212	.318
56.00	3.750	4.062	25.287	2.215	.322
58.00	3.836	4.155	26.163	2.217	.325
60.00	3.920	4.247	27.038	2.219	.329
62.00	4.003	4.338	27.912	2.221	.332
64.00	4.086	4.429	28.786	2.223	.335
66.00	4.167	4.518	29.658	2.225	.338
68.00	4.247	4.606	30.530	2.227	.341
70.00	4.327	4.693	31.401	2.229	.344
72.00	4.406	4.779	32.272	2.231	.347
74.00	4.484	4.865	33.141	2.233	.349
76.00	4.561	4.949	34.010	2.235	.352
78.00	4.638	5.033	34.879	2.236	.354
80.00	4.714	5.116	35.747	2.238	.357
82.00	4.789	5.199	36.614	2.240	.359
84.00	4.864	5.280	37.480	2.241	.362
86.00	4.938	5.361	38.346	2.243	.364
88.00	5.011	5.442	39.211	2.244	.366
90.00	5.084	5.522	40.076	2.246	.369
92.00	5.156	5.601	40.940	2.247	.371
94.00	5.227	5.679	41.804	2.249	.373
96.00	5.298	5.757	42.667	2.250	.375
98.00	5.369	5.834	43.530	2.251	.377
100.00	5.439	5.911	44.392	2.253	.379
102.00	5.508	5.987	45.253	2.254	.381

Note: A positive x is upstream from throat (neck) of flume, and negative x is downstream therefrom.

136.

A 6 foot wide Cutthroat flume is installed in a trapezoidal channel with a bottom width of 12 ft, and a side slope of m = 2. The slope of the downstream channel bottom is S_o = .00287. When the flume was first install the downstream channel consisted of a large sand grain materials that had a Mannings roughness coefficient n = .018. However because this material was transported downstream the downstream channel was lined to a depth of 6 inches around it entire cross-section with riprap rock with a Manning's n = .035. Assuming the downstream channel is very long so it will flow at uniform depth, and that the local loss from the downstream stilling well into the wider trapezoidal channel equals the difference in velocity heads between these two section, or that the normal depth in the downstream channel equals the depth in stilling well h_b determine the effect of the riprap in changing the flow from free flow to submerged flow through the Cutthroat flume. The transition submergence ratio, $S_t = h_b/h_a$ equals 0.88 for a 6 foot wide Cutthroat flume.

The following Fortran Program has been written to obtain the table of values requested for the problem.

```
Program KENNEC.FOR
      REAL YS(6),ES(6),FN(6)/.018,.02,.0225,.025,.03,.035/
      DATA SO,FM/.00287,2./
      READ(*,*) B,W
      C=3.5*W**1.025
      FMS=2.*SQRT(FM*FM+1.)
      SSO=1.486*SQRT(SO)
      Y=.2
      DO 50 I=500,30000,500
      Q=FLOAT(I)/448.83
      DO 30 J=1,6
      M=0
5     P=B+FMS*Y
      A=(B+FM*Y)*Y
      F=FN(J)*Q*P**.66666667-SSO*A**1.66666667
      M=M+1
      IF(MOD(M,2).NE.0) THEN
      F1=F
      YY=Y
      Y=1.01*Y
      GO TO 5
      ENDIF
      DIF=(Y-YY)*F1/(F-F1)
      Y=YY-DIF
      IF(M.LT.40 .AND. ABS(DIF).GT. 1.E-6) GO TO 5
      IF(M.GE.40) WRITE(*,*) I,DIF,Y
      YS(J)=Y
30    ES(J)=Y+Q*Q/(64.4*((B+FM*Y)*Y)**2)
      HA=(Q/C)**.641025641
      EHA=HA+Q*Q/(64.6*(7.3354*HA)**2)
      YC=((Q/6.)**2/32.2)**.333333333
50    WRITE(3,100)I,Q,HA,EHA,YC,(YS(J),ES(J),YS(J)/HA,J=1,6)
100   FORMAT(I6,F6.2,3F6.3,6(1X,3F6.3))
      END
```

Effects of Riprap on **Free-** versus **Submerged-Conditions** through Cutthroat Flume

This section is a discussion about the effects of placing riprap in the channel downstream from a Cutthroat Flume. These analyses are contained in the three following tables, Table 1, 2 and 3. The placement of the riprap has two effects on the hydraulics of the channel downstream from the Cutthroat flume: (1) the gravel lining causes a larger resistance roughness coefficient, and (2) reduces the bottom width of the channel by the thickness of the lining when accounting for the 2:1 side slope. One half of this bottom width reduction occurs from both sides that the bottom width reduction is approximately equal to the thickness of the lining. Thus 6 inches of lining if place uniformly over the cross-section will reduce the bottom width to 11.5 ft.

To determine the effect of placing a 6 inch thickness of gravel lining a computer program was written to solve for the following for a range of flowrates starting with 1000 gpm to 30,000 gpm in increments of 500 gpm (the first columns in Tables 3, 4 and 5:

1. The depth in the upstream stilling well h_a of the 6 ft Cutthroat flume from the calibration equation given in the laboratory report calibration these flumes, UWRL WG 31-4. (Column 3 in Table 1, 2 and 3.)

2. From this computed depth, and the flowrate in column 2, compute the specific energy by adding the velocity head to the depth h_a. (Column 4)

3. Compute the critical depth at the throat (neck) of the flume $Y_c = (q^2/g)^{1/3} = \{(Q/6)/g\}^{1/3}$.

(Cutthroat flumes produce critical flow at their throats, or slightly upstream therefrom when free flow conditions exist.)

4. For 6 values of Mannings n, computed the following three items for the downstream channel: (a) the normal depth obtained by solving Mannings equation of Y_o.

(b) the specific energy associated with this normal depth.

(c) the ratio of the downstream normal depth to the upstream stilling well depth h_a. This latter ratio is believed to be close to the submergence ratio S, or the depth in the downstream stilling well h_b divided by the depth h_a, or $S = h_b/h_a$. That the downstream channel depth is approximately equal to the downstream stilling well h_b is justified on the basis that the difference in velocity head at section b (where stilling well h_b exists) and the downstream channel will be dissipated, thus making the depths equal. Laboratory tests on the 6 ft wide Cutthroat flume showed that the transition from submerged to free flow conditions occurs with the submergence ratio $S = h_b/h_a = .88$; free flow conditions occurring for smaller ratio and submerged conditions occurring when S is greater than .88. The positions where this critical submergence ratio occurs in Tables 1, 2 and 3 are bolded. By noting where the bolded line occurs the following can be noted:

1. If the downstream bottom width of the channel were 12 ft (Table 1), only when Manning's n = .035 does a submerged condition occur at the flume. When n = .035 submerged conditions will occur for discharges less than 6,000 gpm, and free flow conditions for discharges larger than about 6,000 gpm. For a Mannings roughness coefficient of n = .030 and smaller free flow conditions will exist for discharges of 1,000 gpm (and even smaller) and all larger discharges.

2. If the downstream bottom width of the channel is 11 ft (Table 3), probably a larger reduction in width than caused by the riprap and lining, when Manning's n = .030 flow through the flume will change from submerged to free flow for a discharge of about 3,000 gpm, and when Manning's n = .035 that the transition from submerged to free flow conditions occurs for a discharge of 14,500 gpm.

3. If the downstream bottom width of the channel is 11.5 ft, that which would result from a uniform 6 inch depth of lining across the entire cross-section, and if

Mannings n = 0.030, then free flow conditions will occur at the flume for discharges larger than 1,500 gpm, and submerged conditions for discharges less than this amount. If Mannings n = 0.035 then submerged condition will occur at the Cutthroat flume for discharges less than 9,500 gpm. (See Table 2 for these conclusions.)

These analyses assume that the bottom of the Cutthroat flume is at the same elevation as the bottom of the downstream channel at its beginning. In other words they are based on the assumption that the flume's bottom is raised so that its bottom elevation is not below that of the downstream channel as is now the case. The Mannings n caused by the riprap is estimated to be in the range of 0.030 to 0.035. If one wished to be conservative then the larger value should be used. A gradually varied flow profile very likely actually exists in the channel downstream from the Cutthroat flume in which the depth will decrease in it toward the end. This GVF profile will occur if the water level into which the channel discharges is not equal to the normal depth in the channel. However, that assumption of uniform flow in the downstream channel has been used since information is not available regarding depths into which the channel discharges under various flow conditions. The assumption of uniform flow is likely on the conservative side, since an M_2 GVF profile is more likely than and M_1 GVF profile.

The conclusion is that placing the 6 inch lining and riprap in the channel downstream from the Cutthroat flume has resulted in submerged conditions occurring for smaller discharges, whereas if the downstream channel where 12 ft wide with material such as the original small size material with a Mannings roughness n = .018 to .020 then free flow conditions would exist for discharges less than 1,000 gpm.

Table 1.
Bottom width b = 11 ft, Side Sope m = 2.0, Bottom Slope S_o = .00287

Discharge gpm	cfs	h_a ft	E_a ft	Y_c ft	n = .018 Y, ft	E, ft	S	n = .020 Y, ft	E, ft	S	n = .0225 Y, ft	E, ft	S	n = .025 Y, ft	E, ft	S	n = .030 Y, ft	E, ft	S	n = .035 Y, ft	E, ft	S
1000	2.23	.231	.258	.162	.157	.181	.679	.167	.188	.723	.179	.198	.776	.191	.207	.827	.213	.226	.922	.233	.244	1.010
1500	3.34	.299	.335	.213	.200	.233	.667	.213	.242	.711	.228	.253	.762	.243	.265	.812	.271	.288	.905	.297	.311	.992
2000	4.46	.360	.404	.258	.237	.279	.659	.252	.289	.702	.271	.302	.753	.288	.316	.801	.321	.343	.893	.352	.370	.979
2500	5.57	.415	.467	.299	.271	.320	.652	.288	.332	.695	.309	.347	.745	.329	.362	.793	.367	.393	.884	.402	.423	.969
3000	6.68	.466	.526	.338	.302	.358	.647	.321	.371	.689	.345	.387	.739	.367	.404	.786	**.409**	**.438**	**.876**	.448	.472	.960
3500	7.80	.515	.581	.374	.331	.394	.642	.352	.408	.684	.378	.426	.733	.402	.444	.781	.448	.481	.870	.491	.518	.953
4000	8.91	.561	.634	.409	.358	.428	.638	.381	.443	.680	.409	.462	.729	.435	.481	.776	.485	.521	.864	.531	.561	.946
4500	10.03	.605	.684	.443	.384	.460	.635	.409	.476	.676	.438	.496	.725	.466	.517	.771	.520	.559	.859	.569	.602	.941
5000	11.14	.647	.732	.475	.409	.491	.632	.435	.507	.672	.466	.529	.721	.496	.551	.767	.553	.596	.854	.605	.641	.935
5500	12.25	.688	.779	.506	.432	.521	.629	.460	.538	.669	.494	.560	.717	.525	.583	.763	.585	.631	.850	.640	.678	.931
6000	13.37	.727	.825	.536	.455	.550	.626	.485	.567	.666	.520	.590	.714	.553	.615	.760	.616	.664	.846	.674	.714	.926
6500	14.48	.766	.869	.566	.477	.577	.624	.508	.595	.664	.545	.620	.711	.580	.645	.757	.645	.697	.843	.706	.749	.922
7000	15.60	.803	.912	.594	.499	.604	.621	.531	.623	.661	.569	.648	.709	.605	.675	.754	.674	.728	.839	.737	.782	.918
7500	16.71	.839	.953	.622	.520	.630	.619	.553	.650	.659	.593	.676	.706	.630	.703	.751	.702	.759	.836	.768	.815	.915
8000	17.82	.875	.994	.650	.540	.656	.617	.574	.676	.657	.616	.703	.704	.655	.731	.749	.729	.789	.833	.797	.846	.911
8500	18.94	.909	1.034	.676	.559	.681	.615	.595	.701	.654	.638	.729	.701	.679	.758	.746	.755	.817	.830	.826	.877	.908
9000	20.05	.943	1.073	.703	.579	.705	.613	.616	.726	.652	.660	.754	.699	.702	.784	.744	.781	.846	.827	.854	.907	.905
9500	21.17	.977	1.112	.728	.597	.728	.612	.635	.750	.651	.681	.779	.697	.724	.810	.742	.806	.873	.825	.881	.936	.902
10000	22.28	1.009	1.149	.754	.616	.752	.610	.655	.773	.649	.702	.803	.695	.746	.835	.739	.830	.900	.822	.908	.965	.899
10500	23.39	1.041	1.187	.779	.633	.774	.608	.674	.797	.647	.722	.827	.693	.768	.860	.737	.854	.926	.820	.934	.993	.897
11000	24.51	1.073	1.223	.803	.651	.796	.607	.692	.819	.645	.742	.851	.691	.789	.884	.735	.877	.952	.818	.959	1.020	.894
11500	25.62	1.104	1.259	.827	.668	.818	.605	.711	.842	.644	.761	.874	.690	.810	.907	.733	.900	.977	.816	.984	1.047	.892
12000	26.74	1.134	1.294	.851	.685	.840	.604	.729	.863	.642	.781	.896	.688	.830	.931	.732	.923	1.002	.813	1.009	1.073	.889
12500	27.85	1.164	1.329	.875	.702	.861	.603	.746	.885	.641	.799	.918	.686	.850	.953	.730	.945	1.026	.811	1.033	1.099	.887
13000	28.96	1.194	1.363	.898	.718	.881	.601	.764	.906	.639	.818	.940	.685	.870	.976	.728	.967	1.050	.809	1.057	1.124	.885
13500	30.08	1.223	1.397	.921	.734	.902	.600	.781	.927	.638	.836	.961	.683	.889	.998	.727	.988	1.073	.808	1.080	1.149	.883
14000	31.19	1.252	1.431	.943	.750	.922	.599	.797	.947	.637	.854	.982	.682	.908	1.019	.725	1.009	1.096	.806	1.103	1.174	.881
14500	32.31	1.281	1.464	.966	.765	.942	.598	.814	.967	.635	.872	1.003	.681	.926	1.041	.723	1.030	1.119	.804	**1.125**	**1.198**	**.879**
15000	33.42	1.309	1.496	.988	.781	.961	.596	.830	.987	.634	.889	1.023	.679	.945	1.062	.722	1.050	1.142	.802	1.147	1.222	.877
15500	34.53	1.337	1.529	1.010	.796	.980	.595	.846	1.007	.633	.906	1.043	.678	.963	1.083	.720	1.070	1.164	.800	1.169	1.245	.875
16000	35.65	1.364	1.561	1.031	.810	.999	.594	.862	1.026	.632	.923	1.063	.676	.981	1.103	.719	1.090	1.185	.799	1.191	1.268	.873
16500	36.76	1.391	1.592	1.052	.825	1.018	.593	.877	1.045	.631	.939	1.083	.675	.998	1.123	.718	1.109	1.207	.797	1.212	1.291	.871
17000	37.88	1.418	1.623	1.074	.840	1.036	.592	.893	1.064	.629	.956	1.102	.674	1.016	1.143	.716	1.128	1.228	.796	1.233	1.313	.869
17500	38.99	1.445	1.654	1.095	.854	1.054	.591	.908	1.082	.628	.972	1.121	.673	1.033	1.163	.715	1.147	1.249	.794	1.253	1.336	.867
18000	40.10	1.471	1.685	1.115	.868	1.072	.590	.923	1.101	.627	.988	1.140	.672	1.050	1.182	.714	1.166	1.269	.793	1.273	1.357	.866
18500	41.22	1.497	1.715	1.136	.882	1.090	.589	.938	1.119	.626	1.004	1.158	.670	1.067	1.201	.712	1.184	1.290	.791	1.293	1.379	.864
19000	42.33	1.523	1.745	1.156	.896	1.108	.588	.952	1.136	.625	1.019	1.177	.669	1.083	1.220	.711	1.203	1.310	.790	1.313	1.400	.862
19500	43.45	1.549	1.775	1.176	.909	1.125	.587	.967	1.154	.624	1.035	1.195	.668	1.099	1.239	.710	1.221	1.330	.788	1.333	1.421	.861
20000	44.56	1.574	1.804	1.196	.923	1.142	.586	.981	1.172	.623	1.050	1.213	.667	1.116	1.257	.709	1.238	1.349	.787	1.352	1.442	.859
20500	45.67	1.599	1.834	1.216	.936	1.159	.585	.995	1.189	.622	1.065	1.231	.666	1.131	1.275	.708	1.256	1.368	.786	1.371	1.463	.858
21000	46.79	1.624	1.863	1.236	.949	1.176	.585	1.009	1.206	.621	1.080	1.248	.665	1.147	1.293	.706	1.273	1.388	.784	1.390	1.483	.856
21500	47.90	1.649	1.891	1.256	.962	1.193	.584	1.023	1.223	.620	1.095	1.266	.664	1.163	1.311	.705	1.291	1.407	.783	1.409	1.503	.855
22000	49.02	1.673	1.920	1.275	.975	1.209	.583	1.036	1.240	.619	1.109	1.283	.663	1.178	1.329	.704	1.308	1.425	.782	1.427	1.523	.853
22500	50.13	1.697	1.948	1.294	.988	1.225	.582	1.050	1.256	.619	1.124	1.300	.662	1.194	1.346	.703	1.324	1.444	.780	1.446	1.542	.852
23000	51.24	1.721	1.976	1.313	1.001	1.242	.581	1.063	1.273	.618	1.138	1.317	.661	1.209	1.364	.702	1.341	1.462	.779	1.464	1.562	.850
23500	52.36	1.745	2.004	1.332	1.013	1.258	.580	1.077	1.289	.617	1.152	1.333	.660	1.224	1.381	.701	1.358	1.480	.778	1.482	1.581	.849
24000	53.47	1.769	2.032	1.351	1.025	1.273	.580	1.090	1.305	.616	1.166	1.350	.659	1.238	1.398	.700	1.374	1.498	.777	1.499	1.600	.848
24500	54.59	1.793	2.059	1.370	1.038	1.289	.579	1.103	1.321	.615	1.180	1.366	.658	1.253	1.415	.699	1.390	1.516	.776	1.517	1.619	.846
25000	55.70	1.816	2.087	1.388	1.050	1.305	.578	1.116	1.337	.614	1.194	1.382	.657	1.268	1.431	.698	1.406	1.534	.774	1.534	1.638	.845
25500	56.81	1.839	2.114	1.407	1.062	1.320	.577	1.128	1.352	.614	1.207	1.398	.656	1.282	1.448	.697	1.422	1.551	.773	1.551	1.656	.844
26000	57.93	1.862	2.141	1.425	1.074	1.335	.577	1.141	1.368	.613	1.221	1.414	.656	1.296	1.464	.696	1.438	1.569	.772	1.569	1.674	.842
26500	59.04	1.885	2.167	1.443	1.086	1.350	.576	1.154	1.383	.612	1.234	1.430	.655	1.310	1.480	.695	1.453	1.586	.771	1.585	1.693	.841
27000	60.16	1.908	2.194	1.462	1.097	1.365	.575	1.166	1.399	.611	1.247	1.446	.654	1.324	1.496	.694	1.469	1.603	.770	1.602	1.711	.840
27500	61.27	1.930	2.220	1.479	1.109	1.380	.575	1.178	1.414	.610	1.260	1.461	.653	1.338	1.512	.693	1.484	1.620	.769	1.619	1.729	.839
28000	62.38	1.953	2.246	1.497	1.121	1.395	.574	1.191	1.429	.610	1.273	1.476	.652	1.352	1.528	.692	1.499	1.637	.768	1.635	1.746	.837
28500	63.50	1.975	2.272	1.515	1.132	1.410	.573	1.203	1.444	.609	1.286	1.492	.651	1.366	1.544	.692	1.514	1.653	.767	1.652	1.764	.836
29000	64.61	1.997	2.298	1.533	1.143	1.424	.573	1.215	1.458	.608	1.299	1.507	.651	1.379	1.559	.691	1.529	1.670	.766	1.668	1.781	.835
29500	65.73	2.019	2.324	1.550	1.155	1.439	.572	1.227	1.473	.607	1.312	1.522	.650	1.393	1.575	.690	1.544	1.686	.765	1.684	1.798	.834
30000	66.84	2.041	2.350	1.568	1.166	1.453	.571	1.238	1.487	.607	1.324	1.537	.649	1.406	1.590	.689	1.559	1.702	.764	1.700	1.815	.833

Table 2.

Bottom width b = 11.5 ft, Side Sope m = 2.0, Bottom Slope S_o = .00287

Discharge		h_a	E_a	Y_c	n = .018			n = .020			n = .0225			n = .025			n = .030			n = .035		
gpm	cfs	ft	ft	ft	Y, ft	E, ft	S	Y, ft	E, ft	S	Y, ft	E, ft	S	Y, ft	E, ft	S	Y, ft	E, ft	S	Y, ft	E, ft	S
1000	2.23	.231	.258	.162	.153	.176	.662	.163	.183	.705	.174	.192	.756	.186	.202	.805	.207	.220	.898	.227	.237	.984
1500	3.34	.299	.335	.213	.194	.227	.650	.207	.236	.692	.222	.247	.743	.237	.258	.791	**.264**	**.281**	**.882**	.289	.303	.966
2000	4.46	.360	.404	.258	.231	.271	.642	.246	.281	.683	.264	.294	.733	.281	.308	.781	.313	.334	.870	.343	.361	.954
2500	5.57	.415	.467	.299	.264	.312	.635	.281	.323	.677	.301	.337	.726	.321	.352	.773	.357	.383	.861	.392	.413	.944
3000	6.68	.466	.526	.338	.294	.349	.630	.313	.361	.671	.336	.377	.720	.357	.394	.766	.398	.427	.854	.436	.460	.936
3500	7.80	.515	.581	.374	.322	.384	.626	.343	.397	.666	.368	.415	.715	.392	.433	.761	.436	.469	.848	.478	.505	.929
4000	8.91	.561	.634	.409	.349	.417	.622	.371	.431	.662	.398	.450	.710	.424	.469	.756	.472	.508	.842	.517	.547	.922
4500	10.03	.605	.684	.443	.374	.448	.619	.398	.463	.658	.427	.483	.706	.455	.504	.752	.506	.545	.837	.555	.587	.917
5000	11.14	.647	.732	.475	.398	.479	.615	.424	.494	.655	.455	.515	.702	.484	.537	.748	.539	.581	.833	.590	.625	.912
5500	12.25	.688	.779	.506	.421	.508	.613	.449	.524	.652	.481	.546	.699	.512	.569	.744	.570	.615	.829	.624	.661	.907
6000	13.37	.727	.825	.536	.444	.536	.610	.472	.553	.649	.506	.576	.696	.539	.599	.741	.600	.648	.825	.657	.696	.903
6500	14.48	.766	.869	.566	.465	.563	.608	.495	.580	.647	.531	.604	.693	.565	.629	.738	.629	.680	.822	.689	.730	.899
7000	15.60	.803	.912	.594	.486	.589	.605	.517	.607	.644	.555	.632	.691	.590	.658	.735	.657	.710	.818	.719	.763	.896
7500	16.71	.839	.953	.622	.506	.614	.603	.539	.633	.642	.578	.659	.688	.615	.686	.732	.684	.740	.815	.749	.795	.892
8000	17.82	.875	.994	.650	.526	.639	.601	.560	.659	.640	.600	.685	.686	.639	.713	.730	.711	.769	.812	.778	.826	.889
8500	18.94	.909	1.034	.676	.545	.663	.600	.580	.683	.638	.622	.711	.684	.662	.739	.728	.736	.797	.810	.806	.856	.886
9000	20.05	.943	1.073	.703	.564	.687	.598	.600	.708	.636	.643	.736	.682	.684	.765	.725	.761	.825	.807	.833	.885	.883
9500	21.17	.977	1.112	.728	.582	.710	.596	.620	.731	.634	.664	.760	.680	.706	.790	.723	.786	.852	.805	**.860**	**.914**	**.880**
10000	22.28	1.009	1.149	.754	.600	.733	.595	.639	.754	.633	.684	.784	.678	.728	.815	.721	.810	.878	.802	.886	.942	.878
10500	23.39	1.041	1.187	.779	.618	.755	.593	.657	.777	.631	.704	.807	.676	.749	.839	.719	.833	.904	.800	.911	.969	.875
11000	24.51	1.073	1.223	.803	.635	.777	.592	.675	.799	.629	.724	.830	.674	.770	.862	.717	.856	.929	.798	.936	.996	.873
11500	25.62	1.104	1.259	.827	.652	.798	.590	.693	.821	.628	.743	.852	.673	.790	.885	.716	.879	.954	.796	.961	1.022	.870
12000	26.74	1.134	1.294	.851	.668	.819	.589	.711	.842	.626	.761	.874	.671	.810	.908	.714	.901	.978	.794	.985	1.048	.868
12500	27.85	1.164	1.329	.875	.684	.840	.588	.728	.863	.625	.780	.896	.670	.829	.930	.712	.922	1.002	.792	1.008	1.073	.866
13000	28.96	1.194	1.363	.898	.700	.860	.586	.745	.884	.624	.798	.917	.668	.848	.952	.711	.943	1.025	.790	1.032	1.098	.864
13500	30.08	1.223	1.397	.921	.716	.880	.585	.761	.904	.622	.816	.938	.667	.867	.974	.709	.964	1.048	.788	1.054	1.123	.862
14000	31.19	1.252	1.431	.943	.731	.899	.584	.778	.924	.621	.833	.959	.665	.886	.995	.707	.985	1.071	.786	1.077	1.147	.860
14500	32.31	1.281	1.464	.966	.746	.919	.583	.794	.944	.620	.850	.979	.664	.904	1.016	.706	1.005	1.093	.785	1.099	1.170	.858
15000	33.42	1.309	1.496	.988	.761	.938	.582	.810	.963	.619	.867	.999	.663	.922	1.037	.705	1.025	1.115	.783	1.120	1.194	.856
15500	34.53	1.337	1.529	.010	.776	.957	.581	.825	.983	.618	.884	1.019	.661	.940	1.057	.703	1.045	1.137	.782	1.142	1.217	.854
16000	35.65	1.364	1.561	.031	.791	.975	.580	.841	1.001	.616	.901	1.038	.660	.957	1.077	.702	1.064	1.158	.780	1.163	1.239	.852
16500	36.76	1.391	1.592	.052	.805	.993	.579	.856	1.020	.615	.917	1.057	.659	.975	1.097	.701	1.083	1.179	.778	1.184	1.261	.851
17000	37.88	1.418	1.623	.074	.819	1.011	.578	.871	1.039	.614	.933	1.076	.658	.992	1.116	.699	1.102	1.200	.777	1.204	1.283	.849
17500	38.99	1.445	1.654	.095	.833	1.029	.577	.886	1.057	.613	.949	1.095	.657	1.008	1.135	.698	1.120	1.220	.775	1.224	1.305	.847
18000	40.10	1.471	1.685	.115	.847	1.047	.576	.901	1.075	.612	.964	1.113	.656	1.025	1.154	.697	1.139	1.240	.774	1.244	1.327	.846
18500	41.22	1.497	1.715	.136	.861	1.064	.575	.915	1.092	.611	.980	1.131	.654	1.041	1.173	.696	1.157	1.260	.773	1.264	1.348	.844
19000	42.33	1.523	1.745	.156	.874	1.082	.574	.929	1.110	.610	.995	1.149	.653	1.058	1.192	.694	1.175	1.280	.771	1.283	1.369	.843
19500	43.45	1.549	1.775	.176	.887	1.099	.573	.943	1.127	.609	1.010	1.167	.652	1.074	1.210	.693	1.192	1.299	.770	1.302	1.389	.841
20000	44.56	1.574	1.804	.196	.901	1.115	.572	.957	1.144	.608	1.025	1.185	.651	1.089	1.228	.692	1.210	1.318	.769	1.321	1.410	.840
20500	45.67	1.599	1.834	.216	.914	1.132	.571	.971	1.161	.607	1.040	1.202	.650	1.105	1.246	.691	1.227	1.338	.767	1.340	1.430	.838
21000	46.79	1.624	1.863	.236	.926	1.149	.571	.985	1.178	.606	1.054	1.219	.649	1.120	1.264	.690	1.244	1.356	.766	1.359	1.450	.837
21500	47.90	1.649	1.891	.256	.939	1.165	.570	.998	1.195	.606												

Table 3
Bottom width b = 12 ft, Side Sope m = 2.0, Bottom Slope S_o = .00287

Discharge gpm	cfs	h_a ft	E_a ft	Y_c ft	n = .018 Y, ft	E, ft	S	n = .020 Y, ft	E, ft	S	n = .0225 Y, ft	E, ft	S	n = .025 Y, ft	E, ft	S	n = .030 Y, ft	E, ft	S	n = .035 Y, ft	E, ft	S
1000	2.23	.231	.258	.162	.149	.172	.645	.158	.179	.687	.170	.188	.737	.181	.196	.785	.202	.214	.875	.221	.232	.960
1500	3.34	.299	.335	.213	.190	.221	.634	.202	.230	.675	.217	.241	.724	.231	.252	.771	.257	.274	.860	.282	.296	.943
2000	4.46	.360	.404	.258	.225	.264	.626	.240	.274	.667	.257	.287	.715	.274	.300	.761	.305	.326	.849	.335	.352	.930
2500	5.57	.415	.467	.299	.257	.304	.620	.274	.315	.660	.294	.329	.708	.313	.344	.754	.349	.373	.840	.382	.402	.921
3000	6.68	.466	.526	.338	.287	.340	.615	.305	.352	.654	.327	.368	.702	.349	.384	.747	.389	.417	.833	.426	.449	.913
3500	7.80	.515	.581	.374	.314	.374	.610	.335	.387	.650	.359	.404	.697	.382	.422	.742	.426	.457	.827	.467	.492	.906
4000	8.91	.561	.634	.409	.340	.407	.607	.362	.420	.646	.389	.439	.693	.414	.457	.737	.461	.496	.822	.505	.533	.900
4500	10.03	.605	.684	.443	.365	.437	.603	.389	.452	.642	.417	.471	.689	.444	.491	.733	.494	.532	.817	.541	.572	.895
5000	11.14	.647	.732	.475	.389	.467	.600	.414	.482	.639	.444	.503	.685	.472	.524	.730	.526	.567	.813	.576	.610	.890
5500	12.25	.688	.779	.506	.411	.495	.598	.438	.511	.636	.469	.533	.682	.500	.555	.726	.556	.600	.809	.609	.645	.886
6000	13.37	.727	.825	.536	.433	.522	.595	.461	.539	.634	.494	.562	.679	.526	.585	.723	.586	.632	.805	**.641**	**.680**	**.882**
6500	14.48	.766	.869	.566	.454	.549	.593	.483	.566	.631	.518	.589	.677	.551	.614	.720	.614	.663	.802	.672	.713	.878
7000	15.60	.803	.912	.594	.474	.574	.591	.505	.592	.629	.541	.617	.674	.576	.642	.717	.641	.693	.799	.702	.745	.875
7500	16.71	.839	.953	.622	.494	.599	.589	.526	.618	.627	.564	.643	.672	.600	.669	.715	.668	.723	.796	.731	.776	.871
8000	17.82	.875	.994	.650	.513	.624	.587	.546	.643	.625	.586	.669	.670	.623	.696	.712	.694	.751	.793	.759	.806	.868
8500	18.94	.909	1.034	.676	.532	.647	.585	.566	.667	.623	.607	.694	.667	.646	.721	.710	.719	.779	.791	.787	.836	.865
9000	20.05	.943	1.073	.703	.550	.670	.583	.586	.691	.621	.628	.718	.666	.668	.747	.708	.743	.806	.788	.814	.864	.863
9500	21.17	.977	1.112	.728	.568	.693	.582	.605	.714	.619	.648	.742	.664	.690	.771	.706	.767	.832	.786	.840	.892	.860
10000	22.28	1.009	1.149	.754	.586	.715	.580	.623	.736	.618	.668	.765	.662	.711	.795	.704	.791	.858	.783	.865	.920	.857
10500	23.39	1.041	1.187	.779	.603	.737	.579	.641	.758	.616	.687	.788	.660	.731	.819	.702	.814	.883	.781	.890	.947	.855
11000	24.51	1.073	1.223	.803	.620	.758	.577	.659	.780	.614	.706	.810	.658	.751	.842	.700	.836	.907	.779	.915	.973	.853
11500	25.62	1.104	1.259	.827	.636	.779	.576	.677	.802	.613	.725	.832	.657	.771	.865	.699	.858	.932	.777	.939	.999	.850
12000	26.74	1.134	1.294	.851	.652	.800	.575	.694	.822	.612	.743	.854	.655	.791	.887	.697	.880	.955	.775	.962	1.024	.848
12500	27.85	1.164	1.329	.875	.668	.820	.574	.711	.843	.610	.761	.875	.654	.810	.909	.695	.901	.979	.774	.985	1.049	.846
13000	28.96	1.194	1.363	.898	.684	.840	.572	.727	.863	.609	.779	.896	.652	.829	.930	.694	.922	1.002	.772	1.008	1.073	.844
13500	30.08	1.223	1.397	.921	.699	.859	.571	.743	.883	.608	.797	.916	.651	.847	.951	.692	.942	1.024	.770	1.030	1.097	.842
14000	31.19	1.252	1.431	.943	.714	.878	.570	.759	.903	.606	.814	.937	.650	.865	.972	.691	.962	1.046	.768	1.052	1.121	.840
14500	32.31	1.281	1.464	.966	.729	.897	.569	.775	.922	.605	.830	.956	.648	.883	.993	.690	.982	1.068	.767	1.074	1.144	.838
15000	33.42	1.309	1.496	.988	.743	.916	.568	.791	.941	.604	.847	.976	.647	.901	1.013	.688	1.002	1.090	.765	1.095	1.167	.837
15500	34.53	1.337	1.529	.010	.758	.934	.567	.806	.960	.603	.863	.995	.646	.918	1.033	.687	1.021	1.111	.764	1.116	1.189	.835
16000	35.65	1.364	1.561	.031	.772	.953	.566	.821	.978	.602	.880	1.014	.645	.935	1.053	.686	1.040	1.132	.762	1.137	1.212	.833
16500	36.76	1.391	1.592	.052	.786	.970	.565	.836	.997	.601	.896	1.033	.644	.952	1.072	.684	1.058	1.152	.761	1.157	1.233	.832
17000	37.88	1.418	1.623	.074	.800	.988	.564	.851	1.015	.600	.911	1.052	.643	.969	1.091	.683	1.077	1.173	.759	1.177	1.255	.830
17500	38.99	1.445	1.654	.095	.814	1.006	.563	.865	1.033	.599	.927	1.070	.641	.985	1.110	.682	1.095	1.193	.758	1.197	1.276	.828
18000	40.10	1.471	1.685	.115	.827	1.023	.562	.880	1.050	.598	.942	1.088	.640	1.002	1.128	.681	1.113	1.213	.757	1.216	1.297	.827
18500	41.22	1.497	1.715	.136	.840	1.040	.561	.894	1.067	.597	.957	1.106	.639	1.018	1.147	.680	1.131	1.232	.755	1.236	1.318	.825
19000	42.33	1.523	1.745	.156	.854	1.057	.561	.908	1.085	.596	.972	1.124	.638	1.033	1.165	.679	1.148	1.251	.754	1.255	1.339	.824
19500	43.45	1.549	1.775	.176	.867	1.074	.560	.922	1.102	.595	.987	1.141	.637	1.049	1.183	.677	1.166	1.271	.753	1.274	1.359	.822
20000	44.56	1.574	1.804	.196	.880	1.090	.559	.935	1.118	.594	1.002	1.158	.636	1.065	1.201	.676	1.183	1.289	.751	1.292	1.379	.821
20500	45.67	1.599	1.834	.216	.892	1.106	.558	.949	1.135	.593	1.016	1.175	.635	1.080	1.218	.675	1.200	1.308	.750	1.311	1.399	.820
21000	46.79	1.624	1.863	.236	.905	1.123	.557	.962	1.152	.593	1.030	1.192	.634	1.095	1.236	.674	1.216	1.327	.749	1.329	1.418	.818
21500	47.90	1.649	1.891	.256	.917	1.139	.557	.975	1.168	.592	1.044	1.209	.634	1.110	1.253	.673	1.233	1.345	.748	1.347	1.438	.817
22000	49.02	1.673	1.920	.275	.930	1.154	.556	.989	1.184	.591	1.058	1.225	.633	1.125	1.270	.672	1.249	1.363	.747	1.365	1.457	.816
22500	50.13	1.697	1.948	.294	.942	1.170	.555	1.002	1.200	.590	1.072	1.242	.632	1.139	1.287	.671	1.265	1.381	.746	1.382	1.476	.814
23000	51.24	1.721	1.976	.313	.954	1.186	.554	1.014	1.216	.589	1.086	1.258	.631	1.154	1.304	.670	1.282	1.399	.744	1.400	1.495	.813
23500	52.36	1.745	2.004	.332	.966	1.201	.554	1.027	1.231	.588	1.100	1.274	.630	1.168	1.320	.669	1.297	1.416	.743	1.417	1.513	.812
24000	53.47	1.769	2.032	.351	.978	1.216	.553	1.040	1.247	.588	1.113	1.290	.629	1.183	1.336	.669	1.313	1.434	.742	1.434	1.532	.811
24500	54.59	1.793	2.059	.370	.990	1.231	.552	1.052	1.262	.587	1.126	1.306	.628	1.197	1.353	.668	1.329	1.451	.741	1.451	1.550	.809
25000	55.70	1.816	2.087	.388	1.002	1.246	.552	1.065	1.277	.586	1.139	1.321	.628	1.211	1.369	.667	1.344	1.468	.740	1.468	1.568	.808
25500	56.81	1.839	2.114	.407	1.013	1.261	.551	1.077	1.293	.586	1.153	1.337	.627	1.225	1.385	.666	1.359	1.485	.739	1.484	1.586	.807
26000	57.93	1.862	2.141	.425	1.025	1.276	.550	1.089	1.308	.585	1.166	1.352	.626	1.238	1.400	.665	1.375	1.501	.738	1.501	1.604	.806
26500	59.04	1.885	2.167	.443	1.036	1.291	.550	1.101	1.322	.584	1.178	1.368	.625	1.252	1.416	.664	1.390	1.518	.737	1.517	1.621	.805
27000	60.16	1.908	2.194	.462	1.047	1.305	.549	1.113	1.337	.583	1.191	1.383	.624	1.265	1.432	.663	1.405	1.534	.736	1.533	1.639	.804
27500	61.27	1.930	2.220	.479	1.058	1.320	.548	1.125	1.352	.583	1.204	1.398	.624	1.279	1.447	.663	1.419	1.551	.735	1.549	1.656	.803
28000	62.38	1.953	2.246	.497	1.069	1.334	.548	1.137	1.366	.582	1.216	1.412	.623	1.292	1.462	.662	1.434	1.567	.734	1.565	1.673	.802
28500	63.50	1.975	2.272	.515	1.080	1.348	.547	1.148	1.381	.581	1.229	1.427	.622	1.305	1.477	.661	1.448	1.583	.733	1.581	1.690	.800
29000	64.61	1.997	2.298	.533	1.091	1.362	.546	1.160	1.395	.581	1.241	1.442	.621	1.318	1.492	.660	1.463	1.599	.732	1.597	1.707	.799
29500	65.73	2.019	2.324	.550	1.102	1.376	.546	1.171	1.409	.580	1.253	1.456	.621	1.331	1.507	.659	1.477	1.615	.732	1.612	1.723	.798
30000	66.84	2.041	2.350	.568	1.113	1.390	.545	1.183	1.423	.579	1.265	1.471	.620	1.344	1.522	.659	1.491	1.630	.731	1.627	1.740	.797

137.

Example Problem 23 examined how the ratio of step height in the bottom of a circular channel divided by the upstream depth that produced critical flow varied as a function of the Froude Number. Derive the dimensionless equations, whose solution provided this relationship.

Wanted{ Derive dimensionless relationships used in Example Problem 23

Let Y_b be depth from channel bottom to "fill in bottom of channel."

Y_r be depth from channel bottom to water surface in circular channel

$Y_2 = Y_r - Y_b$ be depth from top of fill to water surface

$A_r=D^2/4(\beta_r - \cos \beta_r \sin \beta_r)$ in which $\beta_r=\cos^{-1}(1-2Y_r/D)$

$A_b=D^2/4(\beta_b - \cos \beta_b \sin \beta_b)$ in which $\beta_b=\cos^{-1}(1-2Y_b/D)$

$A = A_r - A_b$

$E = Y+Q^2/(2gA^2)$ but $F_r^2=Q^2T/(gA^3)$

$E = Y + F_r^2A/(2T)$

Divide by D with $Y_d=A/T$ hydraulic radius $Y_d' = Y_d/D$, $T'=T/D, A'=A/D^2$

$E'=Y' + F_r^2Y_d'/2$

$Y_{d2}' = (A_r-A_b)/(DT) = A/(DT)$, $Y_d' = (A_r'-A_b')/T'$

Energy Equation

$Y_1'+F_{r1}^2Y_{d1}'/2 = Y_r'+F_{r2}^2Y_{d2}'/2$

$Y_1'+F_{r1}^2Y_{d1}'/2 = Y_r'+F_{r2}^2(A_r'-A_b')/(2T')$ ← with $F_{r2}^2=1$ at hump if critical here.

Continuity

$A_1V_1 = A_2V_2$ divide by D^2

$A_1'V_1' = A_2'V_2'$ $F_r^2=Q^2T/(gA^3) = V^2T/(gA)$

$V=F_r(gA/T)^{1/2} = F_rg^{1/2}Y_d^{1/2}$

$A_2' = A_r' - A_b'$ & $V' = F_r g^{1/2} (Y_d')^{1/2}$

$A_1' F_{r1} (Y_{d1})^{1/2} = F_{r2} (A_r' - A_b')^{3/2} / (T_2')^{1/2}$

$T = D\sin\beta$ $T' = \sin\beta$

Two equations to solve for Y_b' and Y_r' with Y_1 and F_{r1} upstream specified

Actually A_r' depends upon β_r and A_b' depends upon β_b

$A_r' = (\beta_r - \cos\beta_r \sin\beta_r)/4$ and $A_b' = (\beta_b - \cos\beta_b \sin\beta_b)/4$

and $\beta_r = \cos^{-1}(1 - 2Y_r')$ and $\beta_v = \cos^{-1}(1 - 2Y_b')$

so there are actually 6 unknowns Y_b', Y_r', A_r', A_b', β_r and β_b

138.

A circular channel with a diameter D = 3 m, with a Mannings n = .014 is laid on a slope of S_o = .0008. If the flowrate being carried by this channel is Q = 10 m^3/s, what is the maximum height of hump that can fill the bottom of the channel (but the diameter at this section is still the same, i.e., D_2 = 3 m) if the upstream depth is not to be effected. How does this height compare with a reduction in radius that produces critical flow?

Given: D=3 m, n=.014, S_o=.0008, Q=10 m^3/s

Solution:
Solving Mannings Equation gives:
Y_{o1}=2.121 m, E_{o1}=2.300 m, A_{o1}=5.344 m^2, T_{o1}=2.371 m, and F_{ro1}=0.427

$Y_1' = 2.121/3 = 0.707$

From Fig. of Example Problem 2.23

$Y_b/Y_1 = 0.33 \quad Y_b=0.33(2.121) = 0.70$ ←

$\beta_b=\cos^{-1}(1-1.4/3) = 1.00826$ rad

$A_b=(9/4)(1.00826-\cos\beta_b\sin\beta_b) = 1.2535\ m^2$

$F_{rr}^2 = 1 = [Q^2(3)\sin\beta_r]/[9.81(9/4)(\beta_r-\cos\beta_r\sin\beta_r)-1.2535]^3$

Solving for β_r = 1.756989 rad $\cos\beta_r=1-2Y_r/D$

$Y_r = (D/2)(1-\cos\beta_r) = 1.5(1-\cos(1.756989)) = 1.778$ m ←

Alternatively Solve the following 6 equations for 6 unknown variables

$Y_1+(Q/A_1)^2/(2g) = Y_r+Q^2/[(A_r-A_b)^2(2g)] = 2.300$ (1)

$F_{r2}^2=100(3\sin\beta_r)/[9.81(A_r-A_b)^3]=30.58104\sin\beta_r/(A_r-A_b)^3=1$ (2)

$\cos\beta_r=1-2Y_r/3$ (3)

$\cos\beta_b=1-2Y_b/3$ (4)

$A_r=2.25(\beta_r-\cos\beta_r\sin\beta_r)$ (5)

$A_b=2.25(\beta_b-\cos\beta_b\sin\beta_b)$ (6)

More precise answers:

Y_r=1.773 m, A_r=4.3485 m^2, A_b=1.2389 m^2, β_r=1.75376 rad, β_b=1.0037 rad,

Y_b=0.6942 m ←

MathCAD Model PB2 138.MCD

Variables $E1 := 2.3 \quad Q := 10 \quad Yr := 2 \quad Ar := 4 \quad Ab := 1 \quad Betar := 1.8 \quad Betab := 1 \quad Yb := .7$

Given

$$\frac{30.58104 \sin(Betar)}{(Ar - Ab)^3} = 1 \qquad Yr + \frac{Q^2}{19.62 \cdot (Ar - Ab)^2} = E1 \qquad \cos(Betar) = 1 - 2 \cdot \frac{Yr}{3} \qquad \cos(Betab) = 1 - 2 \cdot \frac{Yb}{3}$$

$$Ar = 2.25 \cdot (Betar - \cos(Betar) \cdot \sin(Betar)) \qquad Ab = 2.25 \cdot (Betab - \cos(Betab) \cdot \sin(Betab))$$

$$\text{Find}(Yr, Ar, Ab, Betar, Betab, Yb) = \begin{bmatrix} 1.77292 \\ 4.34852 \\ 1.23886 \\ 1.75376 \\ 1.0037 \\ 0.69422 \end{bmatrix}$$

139.

A flowrate Q = 400 cfs is taking place in a circular channel with a diameter D = 15 ft. The bottom slope of the channel is S_o = .0005, and its Mannings roughness is n = .013. Concrete has been placed in the bottom of this channel to a depth of 1.5 ft at a section, so that its bottom is flat here. The filled in portion of the channel is 1,200 ft long and then it discharges into a reservoir whose water surface is 7.0 ft above the channel bottom. Determine the depths immediately upstream and downstream from where the filled in bottom occurs, and the GVF profiles upstream from this position and to the downstream reservoir.

Given: Q=400 cfs, D=15 ft, S_o=.0005, n=.013, Downstream length of L=1200 ft filled to a depth of Y_b=1.5 ft, and discharge into a reservoir with ws=7.0 ft
Wanted: Depth throughout channel system.

Upstream Uniform Conditions:
Y_{o1}=6.287 ft, E_{o1}=6.791 ft, and F_{ro1}=.461

Fill in channel bottom
$\&_b=\cos^{-1}(1-3/15) = .6435$ rad
$A_b=D^2/4(\&_b-\cos\&_b\sin\&_b) = 9.196937$ ft^2, $T_b=\sin\&_b=9.000$ ft, $P_b=D\&_b=9.652517$ ft

At End of Channel (Reservoir)
Y_3=7.0 ft, &=1.50408 rad, A=71.666 ft^2, V=Q/A=5.58145 fps, E=7.484 ft

Program GVFCIRBO.FOR is designed to solve GVF problem in Circular channel with fill. (See Chapter 4 for methods for obtaining GVF solutions.)

Program GVFCIRBO.FOR

```
      REAL Y(1),DY(1),XP(1),YP(1,1),WK1(1,13)
      EXTERNAL DYX
      COMMON NGOOD,NBAD,KMAX,KOUNT,DXSAVE,D,FN,SO,Q2,FNQ,AF,DP
1     WRITE(6,* )'GIVE IOUT,TOL,DELX,YB,Q,FN,SO,D,XBEG,XEND,YF'
      READ(5,*) IOUT,TOL,DELX,YB,Q,FN,SO,D,XBEG,XEND,YF
      IF(IOUT.LT.0) STOP
      BETAF=ACOS(1.-2.*YF/D)
      TF=D*SIN(BETAF)
      PF=D*BETAF
      DP=PF-TF
      AF=.25*D**2*(BETAF-COS(BETAF)*SIN(BETAF))
      H1=.01
      HMIN=1.E-5
      Y(1)=YB
      FNQ=FN*Q/1.486
      Q2=Q*Q/32.2
      X=XBEG
      D2=D/2.
      D6=D*D/6.
      COSB=1.-2.*Y(1)/D
      SINB=SQRT(1.-COSB*COSB)
      BETA=ATAN2(SINB,COSB)
      A=.25*D*D*(BETA-COSB*SINB)-AF
      WRITE(IOUT,100) X,Y,BETA,A
2     XZ=X+DELX
      CALL ODESOL(Y,DY,1,X,XZ,TOL,H1,HMIN,1,XP,YP,WK1,DYX)
      X=XZ
      COSB=1.-2.*Y(1)/D
      SINB=SQRT(1.-COSB*COSB)
      BETA=ATAN2(SINB,COSB)
      A=.25*D*D*(BETA-COSB*SINB)-AF
      WRITE(IOUT,100) X,Y,BETA,A
```

```
100    FORMAT(6X,3F10.3,F10.2)
       IF(DELX .LT. 0.) GO TO 8
       IF(X .LT. XEND) GO TO 2
       GO TO 1
8      IF(X .GT. XEND) GO TO 2
       END
       SUBROUTINE DYX(X,Y,DY)
       REAL Y(1),DY(1)
       COMMON NGOOD,NBAD,KMAX,KOUNT,DXSAVE,D,FN,SO,Q2,FNQ,AF,DP
       COSB=1.-2.*Y(1)/D
       SINB=SQRT(1.-COSB*COSB)
       BETA=ATAN2(SINB,COSB)
       A=.25*D*D*(BETA-COSB*SINB)-AF
       T=D*SINB
       P=D*BETA-DP
       SF=(FNQ*(P/A)**.66666667/A)**2
       A3=A**3
       FR2=Q2*T/A3
40     DY(1)=(SO-SF)/(1.-FR2)
       RETURN
       END
```

Input: 3 1.e-6 -200 7 400 .013 .0005 15 1200 0 1.5

Output: Solution to GVF in downstream Channel with bottom filled to a depth of 1.5 ft

Position x(ft)	Depth Y(ft)	Angle β(rad)	Area A(ft**2)
1200.000	7.000	1.504	71.67
1000.000	6.998	1.504	71.63
800.000	6.996	1.504	71.60
600.000	6.994	1.503	71.57
400.000	6.992	1.503	71.55
200.000	6.990	1.503	71.52
.000	6.989	1.503	71.50

From this solution depth at the beginning of the filled portion of the channel is Y_2=6.989 ft.
V_2=400/71.50 = 5.594 fps, E_2=7.47498 ft, Now find depth at the downstream end of the channel that is not filled. E_1=E_2=7.475 --> Y_1 = 7.110 ft <==

Solution for Normal Depth in filled channel (TK-Solver Model PB2_139.TK)

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment---------
              Beta     1.4998669
              Y        6.9684753
   15         D
              P        21.845487
              A        71.194165
   400        Q
   .013       n
   .0005      So

==================== RULE SHEET ========================
S Rule--------------------------------------------------
  Beta=acos(1-2*Y/D)
  P=D*Beta-.652517
  A=D*D/4*(Beta-cos(Beta)*sin(Beta))-9.196937
  Q=1.486/n*(A/P)^.6666667*A*sqrt(So)
```

Notice this normal depth of 6.968 ft is just a little less than what the GVF-solution give for Y_2=6.989, or 0.021 ft less.

GVF- M_1 profile upstream from filled section. Obtained from Program GVFCIR.FOR

Program GVFCIR.FOR

```
       REAL Y(1),DY(1),XP(1),YP(1,1),WK1(1,13)
       EXTERNAL DYX
       COMMON NGOOD,NBAD,KMAX,KOUNT,DXSAVE,D,FN,SO,Q2,FNQ
1      WRITE(6,* )'GIVE IOUT,TOL,DELX,YB,Q,FN,SO,D,XBEG,XEND,g'
```

```
      READ(5,*) IOUT,TOL,DELX,YB,Q,FN,SO,D,XBEG,XEND,G
      IF(IOUT.LT.0) STOP
      H1=.01
      HMIN=1.E-5
      Y(1)=YB
      IF(G.GT. 25.) THEN
      Cu=1.486
      ELSE
      Cu=1.
      ENDIF
      G2=2.*G
      FNQ=FN*Q/Cu
      Q2=Q*Q/G
      X=XBEG
      D2=D/2.
      D6=D*D/6.
      COSB=1.-2.*Y(1)/D
      SINB=SQRT(1.-COSB*COSB)
      BETA=ATAN2(SINB,COSB)
      A=.25*D*D*(BETA-COSB*SINB)
      FMOM=D2*(D6*SINB**3-A*COSB)+Q2/A
      E=Y(1)+(Q/A)**2/G2
      WRITE(IOUT,100) X,Y,BETA,A,E,FMOM
2     XZ=X+DELX
      CALL ODESOL(Y,DY,1,X,XZ,TOL,H1,HMIN,1,XP,YP,WK1,DYX)
      X=XZ
      COSB=1.-2.*Y(1)/D
      SINB=SQRT(1.-COSB*COSB)
      BETA=ATAN2(SINB,COSB)
      A=.25*D*D*(BETA-COSB*SINB)
      FMOM=D2*(D6*SINB**3-A*COSB)+Q2/A
      E=Y(1)+(Q/A)**2/G2
      WRITE(IOUT,100) X,Y,BETA,A,E,FMOM
100   FORMAT(6X,3F10.3,3F10.2)
      IF(DELX .LT. 0.) GO TO 8
      IF(X .LT. XEND) GO TO 2
      GO TO 1
8     IF(X .GT. XEND) GO TO 2
      END
      SUBROUTINE DYX(X,Y,DY)
      REAL Y(1),DY(1)
      COMMON NGOOD,NBAD,KMAX,KOUNT,DXSAVE,D,FN,SO,Q2,FNQ
      COSB=1.-2.*Y(1)/D
      SINB=SQRT(1.-COSB*COSB)
      BETA=ATAN2(SINB,COSB)
      A=.25*D*D*(BETA-COSB*SINB)
      T=D*SINB
      P=D*BETA
      SF=(FNQ*(P/A)**.66666667/A)**2
      A3=A**3
      FR2=Q2*T/A3
40    DY(1)=(SO-SF)/(1.-FR2)
      RETURN
      END
```

<u>Input:</u> 3 1.e-6 -200 7.11 400 .013 .0005 12 0 -2000 32.2

<u>Output:</u>

Position x(ft)	Depth Y(ft)	Angle &(rad)	Area (ft**2)	Energy (ft)	Momentum (ft**3)
.000	7.110	1.519	82.51	7.47	308.15
-200.000	7.070	1.513	81.91	7.44	305.30
-400.000	7.031	1.508	81.33	7.41	302.58
-600.000	6.994	1.503	80.77	7.37	299.97
-800.000	6.958	1.498	80.23	7.34	297.49
-1000.000	6.923	1.494	79.71	7.31	295.11
-1200.000	6.890	1.489	79.21	7.29	292.86
-1400.000	6.858	1.485	78.74	7.26	290.71
-1600.000	6.827	1.481	78.28	7.23	288.66

-1800.000	6.798	1.477	77.84	7.21	286.72
-2000.000	6.769	1.473	77.41	7.18	284.88

140.

Write a program, or a computer model, that will generate tables of values to plot the graphs in Fig. 13, i.e. make tables for different side slopes m_1 that solve Eq. 41 (or Eq. 40). It will be instructive for you to use the Laguerre method described in the next chapter and extract all roots from this 5th degree polynomial, at least for smaller values of m_1.

Wanted: A program to generate data to plot Figure 13.

Solution:

Program TRARECT1.FOR generates the requested data. It uses Laguerre's Method to solve the roots in $E_1 = E_{o2}$ for trapezoidal to rectangular channels.

Program TRARECT1.FOR

```
      PARAMETER (ND=5,EPS=1.E-5)
      REAL M1(8)/.25,.5,.75,1.,1.25,1.5,1.75,2.0/,YP(6),
     &BP1(6)/.8,1.,1.25,1.5,1.75,2.0/
      COMPLEX C(ND+1),ROOTS(ND),AD(ND+1),Z1,Z2,Z3
      EPS1=2.*EPS*EPS
      DO 90 M=1,8
      FM3=.5*M1(M)**3
      F15=1.5*M1(M)
      F3=3.*M1(M)
      WRITE(3,100) M1(M),BP1
100   FORMAT(' For upstream side slope of m =',F7.2,/1X,52('-'),/
     &'    b''',6F8.2,/1X,52('-'))
      DO 90 I=1,91
      BP=.5+.05*FLOAT(I-1)
      CCC=FM3/BP**3
      C(6)=CMPLX(1.,0.)
      C(5)=CMPLX(2.-F15/BP,0.)
      C(4)=CMPLX(1.-F3/BP,0.)
      C(3)=CMPLX(-F15/BP,0.)
      C(2)=CMPLX(0.,0.)
      DO 85 K=1,6
      C(1)=CMPLX(CCC/BP1(K)**2,0.)
      DO 20 J=1,ND+1
20    AD(J)=C(J)
      DO 30 J=ND,1,-1
      Z1=CMPLX(0.,0.)
      CALL LAGU(AD,J,Z1,EPS)
      IF(ABS(AIMAG(Z1)).LE.EPS1*ABS(REAL(Z1))) Z1=CMPLX(REAL(Z1),0.)
      ROOTS(J)=Z1
      Z2=AD(J+1)
      DO 30 JJ=J,1,-1
      Z3=AD(JJ)
      AD(JJ)=Z2
30    Z2=Z1*Z2+Z3
      DO 50 J=2,ND

      Z1=ROOTS(J)
      DO 40 I1=J-1,1,-1
      IF(REAL(ROOTS(I1)).LE.REAL(Z1)) GO TO 50
40    ROOTS(I1+1)=ROOTS(I1)
      I1=0
50    ROOTS(I1+1)=Z1
      IF(ABS(AIMAG(ROOTS(5))).LT. 0.00004) THEN
      YP(K)=REAL(ROOTS(5))
      ELSE
      YP(K)=0.
      ENDIF
85    CONTINUE
90    WRITE(3,70) BP,YP
70    FORMAT(F5.2,6F8.4)
      END
```

```
      SUBROUTINE LAGU(C,ND,Z1,EPS)
      COMPLEX C(ND+1),Z1,DX,ZO,Z2,Z3,Z4,Z5,DZ,SS,Z6,Z7,Z8,ZERO,XX,FF
      ZERO=CMPLX(0.,0.)
      DO 20 ITER=1,50
      Z2=C(ND+1)
      Z3=ZERO
      Z4=ZERO
      DO 10 J=ND,1,-1
      Z4=Z1*Z4+Z3
      Z3=Z1*Z3+Z2
10    Z2=Z1*Z2+C(J)
      IF(CABS(Z2).LE.1.E-8) THEN
      DX=ZERO
      ELSE IF(CABS(Z3).LE.1.E-8.AND.CABS(Z4).LE.1.E-8) THEN

      DX=CMPLX(CABS(Z2/C(ND+1))**(1./FLOAT(ND)),0.)
      ELSE
      Z5=Z3/Z2
      Z8=Z5*Z5
      DZ=Z8-2.*Z4/Z2
      XX=(ND-1)*(ND*DZ-Z8)
      YY=ABS(REAL(XX))
      ZZ=ABS(AIMAG(XX))
      IF(YY.LT.1.E-12 .AND. ZZ .LT.1.E-12) THEN
      SS=ZERO
      ELSE IF (YY.GE.ZZ) THEN
      FF=(1./YY)*XX
      SS=SQRT(YY)*CSQRT(FF)
      ELSE
      FF=(1./ZZ)*XX
      SS=SQRT(ZZ)*CSQRT(FF)
      ENDIF
      Z6=Z5+SS
      Z7=Z5-SS
      IF(CABS(Z6).LT.CABS(Z7)) Z6=Z7
      DX=FLOAT(ND)/Z6
      ENDIF
      ZO=Z1-DX
      IF(Z1.EQ.ZO) RETURN
      Z1=ZO
      IF(CABS(DX).LE.EPS*CABS(Z1)) RETURN
20    CONTINUE
      WRITE(6,*)' FAILED TO CONVERGE',DX
      RETURN
      END
```

Output From Above Program:

For upstream side slope of m = .25

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	.6730	.7071	.7244	.7328	.7376	.7406
.55	.6035	.6390	.6564	.6648	.6695	.6725
.60	.5455	.5824	.5999	.6082	.6129	.6159
.65	.4963	.5347	.5521	.5604	.5650	.5680
.70	.4541	.4939	.5113	.5195	.5241	.5269
.75	.4173	.4586	.4760	.4841	.4886	.4914
.80	.3848	.4279	.4452	.4531	.4575	.4603
.85	.3557	.4008	.4180	.4258	.4302	.4329
.90	.3292	.3769	.3939	.4016	.4059	.4086
.95	.3045	.3555	.3724	.3800	.3842	.3868
1.00	.2798	.3363	.3531	.3606	.3647	.3673
1.05	.0000	.3190	.3357	.3430	.3471	.3496
1.10	.0000	.3034	.3198	.3271	.3310	.3335
1.15	.0000	.2891	.3054	.3125	.3164	.3188
1.20	.0000	.2761	.2922	.2992	.3030	.3054
1.25	.0000	.2641	.2801	.2870	.2907	.2930
1.30	.0000	.2531	.2689	.2757	.2794	.2816
1.35	.0000	.2430	.2586	.2652	.2689	.2711
1.40	.0000	.2336	.2490	.2556	.2591	.2613
1.45	.0000	.2248	.2401	.2466	.2501	.2522
1.50	.0000	.2167	.2318	.2382	.2416	.2437
1.55	.0000	.2091	.2241	.2303	.2337	.2358
1.60	.0000	.2020	.2168	.2230	.2263	.2283
1.65	.0000	.1954	.2100	.2161	.2193	.2213

1.70	.0000	.1892	.2037	.2096	.2128	.2147
1.75	.0000	.1833	.1976	.2035	.2066	.2085
1.80	.0000	.1778	.1920	.1977	.2008	.2027
1.85	.0000	.1726	.1866	.1923	.1953	.1972
1.90	.0000	.1676	.1815	.1871	.1901	.1919
1.95	.0000	.1630	.1767	.1822	.1852	.1870
2.00	.0000	.1586	.1722	.1776	.1805	.1822
2.05	.0000	.1544	.1678	.1732	.1760	.1778
2.10	.0000	.1504	.1637	.1690	.1718	.1735
2.15	.0000	.1466	.1598	.1650	.1677	.1694
2.20	.0000	.1430	.1560	.1611	.1639	.1655
2.25	.0000	.1395	.1524	.1575	.1602	.1618
2.30	.0000	.1363	.1490	.1540	.1567	.1583
2.35	.0000	.1331	.1458	.1507	.1533	.1549
2.40	.0000	.1301	.1426	.1475	.1501	.1516
2.45	.0000	.1272	.1396	.1444	.1470	.1485
2.50	.0000	.1245	.1368	.1415	.1440	.1455
2.55	.0000	.1218	.1340	.1387	.1411	.1426
2.60	.0000	.1193	.1313	.1359	.1384	.1399
2.65	.0000	.1169	.1288	.1333	.1357	.1372
2.70	.0000	.1145	.1263	.1308	.1332	.1346
2.75	.0000	.1123	.1240	.1284	.1308	.1322
2.80	.0000	.1101	.1217	.1261	.1284	.1298
2.85	.0000	.1080	.1195	.1238	.1261	.1275
2.90	.0000	.1060	.1174	.1217	.1239	.1253
2.95	.0000	.1041	.1154	.1196	.1218	.1231
3.00	.0000	.1022	.1134	.1175	.1197	.1211
3.05	.0000	.1004	.1115	.1156	.1178	.1191
3.10	.0000	.0986	.1096	.1137	.1158	.1171
3.15	.0000	.0969	.1078	.1119	.1140	.1153
3.20	.0000	.0953	.1061	.1101	.1122	.1135
3.25	.0000	.0937	.1044	.1084	.1104	.1117
3.30	.0000	.0922	.1028	.1067	.1088	.1100
3.35	.0000	.0907	.1012	.1051	.1071	.1083
3.40	.0000	.0893	.0997	.1035	.1055	.1067
3.45	.0000	.0879	.0982	.1020	.1040	.1052
3.50	.0000	.0865	.0968	.1005	.1025	.1037
3.55	.0000	.0852	.0954	.0991	.1010	.1022
3.60	.0000	.0839	.0940	.0977	.0996	.1008
3.65	.0000	.0827	.0927	.0963	.0982	.0994
3.70	.0000	.0815	.0914	.0950	.0969	.0980
3.75	.0000	.0803	.0902	.0937	.0956	.0967
3.80	.0000	.0792	.0890	.0925	.0943	.0954
3.85	.0000	.0781	.0878	.0913	.0931	.0942
3.90	.0000	.0770	.0866	.0901	.0919	.0930
3.95	.0000	.0760	.0855	.0889	.0907	.0918
4.00	.0000	.0749	.0844	.0878	.0896	.0906
4.05	.0000	.0739	.0834	.0867	.0884	.0895
4.10	.0000	.0730	.0823	.0856	.0874	.0884
4.15	.0000	.0720	.0813	.0846	.0863	.0873
4.20	.0000	.0711	.0803	.0836	.0853	.0863
4.25	.0000	.0702	.0794	.0826	.0843	.0853
4.30	.0000	.0693	.0784	.0816	.0833	.0843
4.35	.0000	.0685	.0775	.0807	.0823	.0833
4.40	.0000	.0676	.0766	.0797	.0814	.0823
4.45	.0000	.0668	.0757	.0788	.0804	.0814
4.50	.0000	.0660	.0749	.0779	.0795	.0805
4.55	.0000	.0653	.0740	.0771	.0787	.0796
4.60	.0000	.0645	.0732	.0762	.0778	.0787
4.65	.0000	.0637	.0724	.0754	.0770	.0779
4.70	.0000	.0630	.0716	.0746	.0761	.0771
4.75	.0000	.0623	.0708	.0738	.0753	.0762
4.80	.0000	.0616	.0701	.0730	.0745	.0754
4.85	.0000	.0609	.0694	.0722	.0738	.0747
4.90	.0000	.0603	.0686	.0715	.0730	.0739
4.95	.0000	.0596	.0679	.0708	.0723	.0731
5.00	.0000	.0590	.0672	.0701	.0715	.0724

For upstream side slope of m = .50

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	1.4362	1.4614	1.4760	1.4836	1.4881	1.4909
.55	1.2976	1.3240	1.3391	1.3469	1.3515	1.3544
.60	1.1820	1.2095	1.2251	1.2330	1.2377	1.2406
.65	1.0842	1.1127	1.1286	1.1367	1.1414	1.1444
.70	1.0003	1.0297	1.0460	1.0542	1.0589	1.0619
.75	.9276	.9579	.9744	.9827	.9874	.9905
.80	.8639	.8951	.9118	.9201	.9249	.9280
.85	.8078	.8397	.8566	.8650	.8698	.8728
.90	.7579	.7905	.8076	.8160	.8208	.8238
.95	.7132	.7466	.7638	.7722	.7770	.7800

1.00	.6730	.7071	.7244	.7328	.7376	.7406
1.05	.6366	.6714	.6887	.6971	.7019	.7049
1.10	.6035	.6390	.6564	.6648	.6695	.6725
1.15	.5732	.6095	.6269	.6352	.6400	.6430
1.20	.5455	.5824	.5999	.6082	.6129	.6159
1.25	.5199	.5576	.5750	.5833	.5880	.5909
1.30	.4963	.5347	.5521	.5604	.5650	.5680
1.35	.4744	.5135	.5310	.5392	.5438	.5467
1.40	.4541	.4939	.5113	.5195	.5241	.5269
1.45	.4351	.4756	.4930	.5011	.5057	.5085
1.50	.4173	.4586	.4760	.4841	.4886	.4914
1.55	.4005	.4427	.4601	.4681	.4726	.4753
1.60	.3848	.4279	.4452	.4531	.4575	.4603
1.65	.3699	.4139	.4312	.4391	.4435	.4462
1.70	.3557	.4008	.4180	.4258	.4302	.4329
1.75	.3422	.3885	.4056	.4134	.4177	.4204
1.80	.3292	.3769	.3939	.4016	.4059	.4086
1.85	.3167	.3659	.3829	.3905	.3948	.3974
1.90	.3045	.3555	.3724	.3800	.3842	.3868
1.95	.2924	.3457	.3625	.3700	.3742	.3768
2.00	.2798	.3363	.3531	.3606	.3647	.3673
2.05	.2648	.3275	.3442	.3516	.3557	.3582
2.10	.0000	.3190	.3357	.3430	.3471	.3496
2.15	.0000	.3110	.3276	.3348	.3389	.3413
2.20	.0000	.3034	.3198	.3271	.3310	.3335
2.25	.0000	.2961	.3125	.3196	.3236	.3260
2.30	.0000	.2891	.3054	.3125	.3164	.3188
2.35	.0000	.2824	.2987	.3057	.3096	.3120
2.40	.0000	.2761	.2922	.2992	.3030	.3054
2.45	.0000	.2700	.2860	.2930	.2968	.2991
2.50	.0000	.2641	.2801	.2870	.2907	.2930
2.55	.0000	.2585	.2744	.2812	.2849	.2872
2.60	.0000	.2531	.2689	.2757	.2794	.2816
2.65	.0000	.2479	.2637	.2704	.2740	.2763
2.70	.0000	.2430	.2586	.2652	.2689	.2711
2.75	.0000	.2382	.2537	.2603	.2639	.2661
2.80	.0000	.2336	.2490	.2556	.2591	.2613
2.85	.0000	.2291	.2445	.2510	.2545	.2567
2.90	.0000	.2248	.2401	.2466	.2501	.2522
2.95	.0000	.2207	.2359	.2423	.2458	.2479
3.00	.0000	.2167	.2318	.2382	.2416	.2437
3.05	.0000	.2128	.2279	.2342	.2376	.2397
3.10	.0000	.2091	.2241	.2303	.2337	.2358
3.15	.0000	.2055	.2204	.2266	.2299	.2320
3.20	.0000	.2020	.2168	.2230	.2263	.2283
3.25	.0000	.1987	.2134	.2195	.2228	.2248
3.30	.0000	.1954	.2100	.2161	.2193	.2213
3.35	.0000	.1922	.2068	.2128	.2160	.2180
3.40	.0000	.1892	.2037	.2096	.2128	.2147
3.45	.0000	.1862	.2006	.2065	.2097	.2116
3.50	.0000	.1833	.1976	.2035	.2066	.2085
3.55	.0000	.1805	.1948	.2006	.2037	.2056
3.60	.0000	.1778	.1920	.1977	.2008	.2027
3.65	.0000	.1751	.1892	.1950	.1980	.1999
3.70	.0000	.1726	.1866	.1923	.1953	.1972
3.75	.0000	.1701	.1840	.1897	.1927	.1945
3.80	.0000	.1676	.1815	.1871	.1901	.1919
3.85	.0000	.1653	.1791	.1846	.1876	.1894
3.90	.0000	.1630	.1767	.1822	.1852	.1870
3.95	.0000	.1607	.1744	.1799	.1828	.1846
4.00	.0000	.1586	.1722	.1776	.1805	.1822
4.05	.0000	.1564	.1700	.1753	.1782	.1800
4.10	.0000	.1544	.1678	.1732	.1760	.1778
4.15	.0000	.1524	.1657	.1710	.1739	.1756
4.20	.0000	.1504	.1637	.1690	.1718	.1735
4.25	.0000	.1485	.1617	.1669	.1697	.1714
4.30	.0000	.1466	.1598	.1650	.1677	.1694
4.35	.0000	.1448	.1579	.1630	.1658	.1674
4.40	.0000	.1430	.1560	.1611	.1639	.1655
4.45	.0000	.1412	.1542	.1593	.1620	.1636
4.50	.0000	.1395	.1524	.1575	.1602	.1618
4.55	.0000	.1379	.1507	.1557	.1584	.1600
4.60	.0000	.1363	.1490	.1540	.1567	.1583
4.65	.0000	.1347	.1474	.1523	.1550	.1565
4.70	.0000	.1331	.1458	.1507	.1533	.1549
4.75	.0000	.1316	.1442	.1491	.1517	.1532
4.80	.0000	.1301	.1426	.1475	.1501	.1516
4.85	.0000	.1287	.1411	.1459	.1485	.1500
4.90	.0000	.1272	.1396	.1444	.1470	.1485
4.95	.0000	.1258	.1382	.1429	.1455	.1470
5.00	.0000	.1245	.1368	.1415	.1440	.1455

For upstream side slope of m = .75

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	2.1965	2.2168	2.2292	2.2357	2.2395	2.2420
.55	1.9894	2.0109	2.0238	2.0306	2.0346	2.0372
.60	1.8167	1.8392	1.8526	1.8597	1.8638	1.8665
.65	1.6705	1.6939	1.7078	1.7150	1.7193	1.7220
.70	1.5450	1.5693	1.5836	1.5911	1.5954	1.5982
.75	1.4362	1.4614	1.4760	1.4836	1.4881	1.4909
.80	1.3409	1.3669	1.3819	1.3896	1.3942	1.3971
.85	1.2568	1.2835	1.2989	1.3067	1.3113	1.3142
.90	1.1820	1.2095	1.2251	1.2330	1.2377	1.2406
.95	1.1151	1.1432	1.1591	1.1671	1.1718	1.1748
1.00	1.0548	1.0836	1.0997	1.1078	1.1125	1.1155
1.05	1.0003	1.0297	1.0460	1.0542	1.0589	1.0619
1.10	.9507	.9807	.9972	1.0054	1.0102	1.0132
1.15	.9055	.9360	.9526	.9609	.9657	.9687
1.20	.8639	.8951	.9118	.9201	.9249	.9280
1.25	.8258	.8574	.8743	.8826	.8874	.8905
1.30	.7905	.8227	.8396	.8480	.8528	.8559
1.35	.7579	.7905	.8076	.8160	.8208	.8238
1.40	.7275	.7607	.7778	.7863	.7911	.7941
1.45	.6993	.7330	.7502	.7586	.7634	.7664
1.50	.6730	.7071	.7244	.7328	.7376	.7406
1.55	.6483	.6829	.7002	.7086	.7134	.7164
1.60	.6252	.6603	.6776	.6860	.6908	.6938
1.65	.6035	.6390	.6564	.6648	.6695	.6725
1.70	.5830	.6190	.6364	.6448	.6496	.6525
1.75	.5637	.6002	.6176	.6260	.6307	.6337
1.80	.5455	.5824	.5999	.6082	.6129	.6159
1.85	.5282	.5656	.5831	.5914	.5961	.5990
1.90	.5119	.5497	.5672	.5755	.5802	.5831
1.95	.4963	.5347	.5521	.5604	.5650	.5680
2.00	.4816	.5204	.5378	.5461	.5507	.5536
2.05	.4675	.5068	.5242	.5324	.5370	.5399
2.10	.4541	.4939	.5113	.5195	.5241	.5269
2.15	.4413	.4816	.4990	.5071	.5117	.5145
2.20	.4290	.4698	.4872	.4953	.4999	.5027
2.25	.4173	.4586	.4760	.4841	.4886	.4914
2.30	.4060	.4479	.4653	.4733	.4778	.4806
2.35	.3952	.4377	.4550	.4630	.4674	.4702
2.40	.3848	.4279	.4452	.4531	.4575	.4603
2.45	.3747	.4185	.4357	.4436	.4481	.4508
2.50	.3651	.4095	.4267	.4346	.4390	.4417
2.55	.3557	.4008	.4180	.4258	.4302	.4329
2.60	.3466	.3925	.4097	.4175	.4218	.4245
2.65	.3378	.3845	.4016	.4094	.4137	.4164
2.70	.3292	.3769	.3939	.4016	.4059	.4086
2.75	.3209	.3695	.3865	.3942	.3984	.4011
2.80	.3126	.3624	.3793	.3870	.3912	.3938
2.85	.3045	.3555	.3724	.3800	.3842	.3868
2.90	.2964	.3489	.3657	.3733	.3775	.3801
2.95	.2883	.3425	.3593	.3668	.3710	.3736
3.00	.2798	.3363	.3531	.3606	.3647	.3673
3.05	.2705	.3304	.3471	.3545	.3586	.3612
3.10	.0000	.3246	.3413	.3487	.3527	.3553
3.15	.0000	.3190	.3357	.3430	.3471	.3496
3.20	.0000	.3136	.3302	.3375	.3416	.3440
3.25	.0000	.3084	.3249	.3322	.3362	.3387
3.30	.0000	.3034	.3198	.3271	.3310	.3335
3.35	.0000	.2985	.3149	.3221	.3260	.3285
3.40	.0000	.2937	.3101	.3172	.3211	.3236
3.45	.0000	.2891	.3054	.3125	.3164	.3188
3.50	.0000	.2846	.3009	.3080	.3118	.3142
3.55	.0000	.2803	.2965	.3035	.3074	.3097
3.60	.0000	.2761	.2922	.2992	.3030	.3054
3.65	.0000	.2720	.2881	.2950	.2988	.3012
3.70	.0000	.2680	.2840	.2909	.2947	.2970
3.75	.0000	.2641	.2801	.2870	.2907	.2930
3.80	.0000	.2603	.2763	.2831	.2868	.2891
3.85	.0000	.2567	.2725	.2793	.2831	.2853
3.90	.0000	.2531	.2689	.2757	.2794	.2816
3.95	.0000	.2496	.2654	.2721	.2758	.2780
4.00	.0000	.2463	.2619	.2686	.2723	.2745
4.05	.0000	.2430	.2586	.2652	.2689	.2711
4.10	.0000	.2397	.2553	.2619	.2655	.2677
4.15	.0000	.2366	.2521	.2587	.2623	.2645
4.20	.0000	.2336	.2490	.2556	.2591	.2613
4.25	.0000	.2306	.2460	.2525	.2560	.2582
4.30	.0000	.2277	.2430	.2495	.2530	.2552

4.35	.0000	.2248	.2401	.2466	.2501	.2522
4.40	.0000	.2221	.2373	.2437	.2472	.2493
4.45	.0000	.2193	.2345	.2409	.2444	.2465
4.50	.0000	.2167	.2318	.2382	.2416	.2437
4.55	.0000	.2141	.2292	.2355	.2389	.2410
4.60	.0000	.2116	.2266	.2329	.2363	.2383
4.65	.0000	.2091	.2241	.2303	.2337	.2358
4.70	.0000	.2067	.2216	.2278	.2312	.2332
4.75	.0000	.2043	.2192	.2254	.2287	.2307
4.80	.0000	.2020	.2168	.2230	.2263	.2283
4.85	.0000	.1998	.2145	.2206	.2239	.2259
4.90	.0000	.1976	.2123	.2183	.2216	.2236
4.95	.0000	.1954	.2100	.2161	.2193	.2213
5.00	.0000	.1933	.2079	.2139	.2171	.2191

For upstream side slope of m = 1.00

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	2.9542	2.9713	2.9818	2.9875	2.9908	2.9930
.55	2.6789	2.6970	2.7082	2.7141	2.7177	2.7199
.60	2.4493	2.4684	2.4801	2.4863	2.4900	2.4924
.65	2.2549	2.2749	2.2871	2.2935	2.2973	2.2998
.70	2.0881	2.1089	2.1216	2.1282	2.1322	2.1347
.75	1.9434	1.9651	1.9781	1.9850	1.9891	1.9917
.80	1.8167	1.8392	1.8526	1.8597	1.8638	1.8665
.85	1.7049	1.7281	1.7419	1.7491	1.7533	1.7560
.90	1.6054	1.6293	1.6434	1.6507	1.6551	1.6578
.95	1.5164	1.5409	1.5553	1.5628	1.5672	1.5700
1.00	1.4362	1.4614	1.4760	1.4836	1.4881	1.4909
1.05	1.3636	1.3894	1.4043	1.4120	1.4165	1.4194
1.10	1.2976	1.3240	1.3391	1.3469	1.3515	1.3544
1.15	1.2373	1.2642	1.2796	1.2875	1.2921	1.2950
1.20	1.1820	1.2095	1.2251	1.2330	1.2377	1.2406
1.25	1.1312	1.1591	1.1749	1.1829	1.1876	1.1906
1.30	1.0842	1.1127	1.1286	1.1367	1.1414	1.1444
1.35	1.0407	1.0696	1.0858	1.0939	1.0986	1.1016
1.40	1.0003	1.0297	1.0460	1.0542	1.0589	1.0619
1.45	.9627	.9926	1.0089	1.0172	1.0219	1.0250
1.50	.9276	.9579	.9744	.9827	.9874	.9905
1.55	.8947	.9255	.9421	.9504	.9552	.9582
1.60	.8639	.8951	.9118	.9201	.9249	.9280
1.65	.8350	.8665	.8834	.8917	.8965	.8996
1.70	.8078	.8397	.8566	.8650	.8698	.8728
1.75	.7821	.8144	.8314	.8398	.8446	.8476
1.80	.7579	.7905	.8076	.8160	.8208	.8238
1.85	.7349	.7680	.7851	.7935	.7983	.8013
1.90	.7132	.7466	.7638	.7722	.7770	.7800
1.95	.6925	.7263	.7436	.7520	.7568	.7598
2.00	.6730	.7071	.7244	.7328	.7376	.7406
2.05	.6543	.6888	.7061	.7145	.7193	.7223
2.10	.6366	.6714	.6887	.6971	.7019	.7049
2.15	.6196	.6548	.6722	.6806	.6854	.6884
2.20	.6035	.6390	.6564	.6648	.6695	.6725
2.25	.5880	.6239	.6413	.6497	.6544	.6574
2.30	.5732	.6095	.6269	.6352	.6400	.6430
2.35	.5591	.5956	.6131	.6214	.6262	.6291
2.40	.5455	.5824	.5999	.6082	.6129	.6159
2.45	.5325	.5697	.5872	.5955	.6002	.6032
2.50	.5199	.5576	.5750	.5833	.5880	.5909
2.55	.5079	.5459	.5634	.5716	.5763	.5792
2.60	.4963	.5347	.5521	.5604	.5650	.5680
2.65	.4852	.5239	.5413	.5496	.5542	.5571
2.70	.4744	.5135	.5310	.5392	.5438	.5467
2.75	.4641	.5035	.5209	.5291	.5337	.5366
2.80	.4541	.4939	.5113	.5195	.5241	.5269
2.85	.4444	.4846	.5020	.5101	.5147	.5176
2.90	.4351	.4756	.4930	.5011	.5057	.5085
2.95	.4260	.4670	.4844	.4924	.4970	.4998
3.00	.4173	.4586	.4760	.4841	.4886	.4914
3.05	.4088	.4505	.4679	.4759	.4804	.4832
3.10	.4005	.4427	.4601	.4681	.4726	.4753
3.15	.3925	.4352	.4525	.4605	.4649	.4677
3.20	.3848	.4279	.4452	.4531	.4575	.4603
3.25	.3772	.4208	.4380	.4460	.4504	.4531
3.30	.3699	.4139	.4312	.4391	.4435	.4462
3.35	.3627	.4073	.4245	.4323	.4367	.4395
3.40	.3557	.4008	.4180	.4258	.4302	.4329
3.45	.3489	.3946	.4117	.4195	.4239	.4266
3.50	.3422	.3885	.4056	.4134	.4177	.4204
3.55	.3356	.3826	.3997	.4074	.4117	.4144

3.60	.3292	.3769	.3939	.4016	.4059	.4086
3.65	.3229	.3713	.3883	.3960	.4003	.4029
3.70	.3167	.3659	.3829	.3905	.3948	.3974
3.75	.3106	.3606	.3776	.3852	.3894	.3920
3.80	.3045	.3555	.3724	.3800	.3842	.3868
3.85	.2984	.3505	.3674	.3750	.3791	.3817
3.90	.2924	.3457	.3625	.3700	.3742	.3768
3.95	.2862	.3409	.3577	.3652	.3694	.3720
4.00	.2798	.3363	.3531	.3606	.3647	.3673
4.05	.2730	.3318	.3486	.3560	.3601	.3627
4.10	.2648	.3275	.3442	.3516	.3557	.3582
4.15	.0000	.3232	.3399	.3472	.3513	.3538
4.20	.0000	.3190	.3357	.3430	.3471	.3496
4.25	.0000	.3150	.3316	.3389	.3429	.3454
4.30	.0000	.3110	.3276	.3348	.3389	.3413
4.35	.0000	.3071	.3236	.3309	.3349	.3374
4.40	.0000	.3034	.3198	.3271	.3310	.3335
4.45	.0000	.2997	.3161	.3233	.3273	.3297
4.50	.0000	.2961	.3125	.3196	.3236	.3260
4.55	.0000	.2925	.3089	.3160	.3200	.3224
4.60	.0000	.2891	.3054	.3125	.3164	.3188
4.65	.0000	.2857	.3020	.3091	.3130	.3154
4.70	.0000	.2824	.2987	.3057	.3096	.3120
4.75	.0000	.2792	.2954	.3024	.3063	.3086
4.80	.0000	.2761	.2922	.2992	.3030	.3054
4.85	.0000	.2730	.2891	.2960	.2999	.3022
4.90	.0000	.2700	.2860	.2930	.2968	.2991
4.95	.0000	.2670	.2830	.2899	.2937	.2960
5.00	.0000	.2641	.2801	.2870	.2907	.2930

For upstream side slope of m = 1.25

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	3.7100	3.7248	3.7340	3.7389	3.7419	3.7438
.55	3.3667	3.3824	3.3922	3.3974	3.4005	3.4026
.60	3.0803	3.0969	3.1072	3.1127	3.1160	3.1181
.65	2.8378	2.8553	2.8661	2.8718	2.8753	2.8775
.70	2.6297	2.6481	2.6593	2.6653	2.6689	2.6712
.75	2.4493	2.4684	2.4801	2.4863	2.4900	2.4924
.80	2.2914	2.3112	2.3233	2.3297	2.3335	2.3359
.85	2.1519	2.1724	2.1849	2.1914	2.1953	2.1978
.90	2.0278	2.0490	2.0618	2.0686	2.0725	2.0751
.95	1.9167	1.9386	1.9517	1.9586	1.9627	1.9653
1.00	1.8167	1.8392	1.8526	1.8597	1.8638	1.8665
1.05	1.7262	1.7492	1.7630	1.7701	1.7743	1.7771
1.10	1.6439	1.6675	1.6814	1.6887	1.6930	1.6958
1.15	1.5686	1.5928	1.6070	1.6144	1.6188	1.6216
1.20	1.4997	1.5243	1.5388	1.5463	1.5507	1.5535
1.25	1.4362	1.4614	1.4760	1.4836	1.4881	1.4909
1.30	1.3776	1.4032	1.4181	1.4258	1.4303	1.4332
1.35	1.3233	1.3494	1.3645	1.3722	1.3768	1.3797
1.40	1.2728	1.2994	1.3147	1.3225	1.3271	1.3300
1.45	1.2259	1.2529	1.2683	1.2762	1.2808	1.2838
1.50	1.1820	1.2095	1.2251	1.2330	1.2377	1.2406
1.55	1.1410	1.1689	1.1846	1.1926	1.1973	1.2003
1.60	1.1026	1.1308	1.1467	1.1548	1.1595	1.1624
1.65	1.0664	1.0951	1.1111	1.1192	1.1239	1.1269
1.70	1.0324	1.0614	1.0776	1.0857	1.0905	1.0935
1.75	1.0003	1.0297	1.0460	1.0542	1.0589	1.0619
1.80	.9700	.9998	1.0161	1.0244	1.0291	1.0322
1.85	.9414	.9715	.9879	.9962	1.0010	1.0040
1.90	.9142	.9447	.9612	.9695	.9743	.9773
1.95	.8884	.9192	.9359	.9442	.9490	.9520
2.00	.8639	.8951	.9118	.9201	.9249	.9280
2.05	.8407	.8721	.8889	.8972	.9021	.9051
2.10	.8185	.8502	.8671	.8755	.8803	.8833
2.15	.7973	.8294	.8463	.8547	.8595	.8626
2.20	.7771	.8095	.8265	.8349	.8397	.8428
2.25	.7579	.7905	.8076	.8160	.8208	.8238
2.30	.7394	.7724	.7895	.7979	.8027	.8057
2.35	.7217	.7550	.7722	.7806	.7854	.7884
2.40	.7048	.7384	.7556	.7640	.7688	.7718
2.45	.6885	.7224	.7396	.7480	.7529	.7559
2.50	.6730	.7071	.7244	.7328	.7376	.7406
2.55	.6580	.6924	.7097	.7181	.7229	.7259
2.60	.6436	.6783	.6956	.7040	.7088	.7118
2.65	.6297	.6647	.6820	.6904	.6952	.6982
2.70	.6163	.6516	.6690	.6774	.6821	.6851
2.75	.6035	.6390	.6564	.6648	.6695	.6725
2.80	.5910	.6269	.6443	.6526	.6574	.6604
2.85	.5791	.6152	.6326	.6409	.6457	.6487

2.90	.5675	.6039	.6213	.6296	.6344	.6374
2.95	.5563	.5930	.6104	.6187	.6235	.6264
3.00	.5455	.5824	.5999	.6082	.6129	.6159
3.05	.5350	.5722	.5897	.5980	.6027	.6057
3.10	.5249	.5624	.5798	.5881	.5928	.5958
3.15	.5151	.5528	.5703	.5786	.5833	.5862
3.20	.5056	.5436	.5611	.5693	.5740	.5769
3.25	.4963	.5347	.5521	.5604	.5650	.5680
3.30	.4874	.5260	.5435	.5517	.5563	.5592
3.35	.4787	.5176	.5351	.5433	.5479	.5508
3.40	.4703	.5095	.5269	.5351	.5397	.5426
3.45	.4621	.5015	.5190	.5272	.5318	.5346
3.50	.4541	.4939	.5113	.5195	.5241	.5269
3.55	.4463	.4864	.5038	.5120	.5166	.5194
3.60	.4388	.4792	.4966	.5047	.5093	.5121
3.65	.4314	.4721	.4895	.4976	.5022	.5050
3.70	.4242	.4653	.4827	.4907	.4953	.4981
3.75	.4173	.4586	.4760	.4841	.4886	.4914
3.80	.4104	.4521	.4695	.4775	.4820	.4848
3.85	.4038	.4458	.4632	.4712	.4757	.4785
3.90	.3973	.4397	.4570	.4650	.4695	.4723
3.95	.3910	.4337	.4510	.4590	.4634	.4662
4.00	.3848	.4279	.4452	.4531	.4575	.4603
4.05	.3787	.4222	.4395	.4474	.4518	.4546
4.10	.3728	.4166	.4339	.4418	.4462	.4489
4.15	.3670	.4112	.4285	.4363	.4407	.4435
4.20	.3613	.4060	.4232	.4310	.4354	.4381
4.25	.3557	.4008	.4180	.4258	.4302	.4329
4.30	.3502	.3958	.4130	.4208	.4251	.4278
4.35	.3448	.3909	.4080	.4158	.4202	.4228
4.40	.3396	.3861	.4032	.4110	.4153	.4180
4.45	.3344	.3814	.3985	.4063	.4106	.4132
4.50	.3292	.3769	.3939	.4016	.4059	.4086
4.55	.3242	.3724	.3894	.3971	.4014	.4040
4.60	.3192	.3680	.3850	.3927	.3970	.3996
4.65	.3143	.3638	.3807	.3884	.3926	.3952
4.70	.3094	.3596	.3765	.3841	.3884	.3910
4.75	.3045	.3555	.3724	.3800	.3842	.3868
4.80	.2997	.3515	.3684	.3760	.3802	.3827
4.85	.2948	.3476	.3644	.3720	.3762	.3788
4.90	.2899	.3437	.3606	.3681	.3723	.3748
4.95	.2849	.3400	.3568	.3643	.3684	.3710
5.00	.2798	.3363	.3531	.3606	.3647	.3673

For upstream side slope of m = 1.50

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	4.4646	4.4776	4.4857	4.4901	4.4928	4.4945
.55	4.0531	4.0670	4.0757	4.0804	4.0832	4.0850
.60	3.7100	3.7248	3.7340	3.7389	3.7419	3.7438
.65	3.4195	3.4351	3.4448	3.4500	3.4531	3.4551
.70	3.1703	3.1866	3.1968	3.2022	3.2054	3.2075
.75	2.9542	2.9713	2.9818	2.9875	2.9908	2.9930
.80	2.7650	2.7828	2.7937	2.7996	2.8030	2.8053
.85	2.5979	2.6164	2.6277	2.6337	2.6373	2.6396
.90	2.4493	2.4684	2.4801	2.4863	2.4900	2.4924
.95	2.3163	2.3360	2.3480	2.3544	2.3582	2.3606
1.00	2.1965	2.2168	2.2292	2.2357	2.2395	2.2420
1.05	2.0881	2.1089	2.1216	2.1282	2.1322	2.1347
1.10	1.9894	2.0109	2.0238	2.0306	2.0346	2.0372
1.15	1.8994	1.9213	1.9345	1.9414	1.9455	1.9481
1.20	1.8167	1.8392	1.8526	1.8597	1.8638	1.8665
1.25	1.7407	1.7636	1.7773	1.7844	1.7887	1.7914
1.30	1.6705	1.6939	1.7078	1.7150	1.7193	1.7220
1.35	1.6054	1.6293	1.6434	1.6507	1.6551	1.6578
1.40	1.5450	1.5693	1.5836	1.5911	1.5954	1.5982
1.45	1.4887	1.5135	1.5280	1.5355	1.5399	1.5427
1.50	1.4362	1.4614	1.4760	1.4836	1.4881	1.4909
1.55	1.3870	1.4126	1.4275	1.4351	1.4396	1.4425
1.60	1.3409	1.3669	1.3819	1.3896	1.3942	1.3971
1.65	1.2976	1.3240	1.3391	1.3469	1.3515	1.3544
1.70	1.2568	1.2835	1.2989	1.3067	1.3113	1.3142
1.75	1.2184	1.2455	1.2609	1.2688	1.2734	1.2764
1.80	1.1820	1.2095	1.2251	1.2330	1.2377	1.2406
1.85	1.1477	1.1755	1.1912	1.1992	1.2039	1.2068
1.90	1.1151	1.1432	1.1591	1.1671	1.1718	1.1748
1.95	1.0842	1.1127	1.1286	1.1367	1.1414	1.1444
2.00	1.0548	1.0836	1.0997	1.1078	1.1125	1.1155
2.05	1.0269	1.0560	1.0722	1.0803	1.0851	1.0881
2.10	1.0003	1.0297	1.0460	1.0542	1.0589	1.0619

2.15	.9750	1.0046	1.0210	1.0292	1.0340	1.0370
2.20	.9507	.9807	.9972	1.0054	1.0102	1.0132
2.25	.9276	.9579	.9744	.9827	.9874	.9905
2.30	.9055	.9360	.9526	.9609	.9657	.9687
2.35	.8843	.9151	.9318	.9401	.9449	.9479
2.40	.8639	.8951	.9118	.9201	.9249	.9280
2.45	.8445	.8759	.8926	.9010	.9058	.9088
2.50	.8258	.8574	.8743	.8826	.8874	.8905
2.55	.8078	.8397	.8566	.8650	.8698	.8728
2.60	.7905	.8227	.8396	.8480	.8528	.8559
2.65	.7739	.8063	.8233	.8317	.8365	.8396
2.70	.7579	.7905	.8076	.8160	.8208	.8238
2.75	.7424	.7754	.7924	.8008	.8057	.8087
2.80	.7275	.7607	.7778	.7863	.7911	.7941
2.85	.7132	.7466	.7638	.7722	.7770	.7800
2.90	.6993	.7330	.7502	.7586	.7634	.7664
2.95	.6859	.7198	.7370	.7455	.7503	.7533
3.00	.6730	.7071	.7244	.7328	.7376	.7406
3.05	.6604	.6948	.7121	.7205	.7253	.7283
3.10	.6483	.6829	.7002	.7086	.7134	.7164
3.15	.6366	.6714	.6887	.6971	.7019	.7049
3.20	.6252	.6603	.6776	.6860	.6908	.6938
3.25	.6141	.6495	.6668	.6752	.6800	.6830
3.30	.6035	.6390	.6564	.6648	.6695	.6725
3.35	.5931	.6289	.6463	.6546	.6594	.6624
3.40	.5830	.6190	.6364	.6448	.6496	.6525
3.45	.5732	.6095	.6269	.6352	.6400	.6430
3.50	.5637	.6002	.6176	.6260	.6307	.6337
3.55	.5545	.5912	.6086	.6170	.6217	.6246
3.60	.5455	.5824	.5999	.6082	.6129	.6159
3.65	.5367	.5739	.5914	.5997	.6044	.6073
3.70	.5282	.5656	.5831	.5914	.5961	.5990
3.75	.5199	.5576	.5750	.5833	.5880	.5909
3.80	.5119	.5497	.5672	.5755	.5802	.5831
3.85	.5040	.5421	.5596	.5678	.5725	.5754
3.90	.4963	.5347	.5521	.5604	.5650	.5680
3.95	.4889	.5274	.5449	.5531	.5578	.5607
4.00	.4816	.5204	.5378	.5461	.5507	.5536
4.05	.4744	.5135	.5310	.5392	.5438	.5467
4.10	.4675	.5068	.5242	.5324	.5370	.5399
4.15	.4607	.5003	.5177	.5259	.5305	.5333
4.20	.4541	.4939	.5113	.5195	.5241	.5269
4.25	.4476	.4876	.5051	.5132	.5178	.5206
4.30	.4413	.4816	.4990	.5071	.5117	.5145
4.35	.4351	.4756	.4930	.5011	.5057	.5085
4.40	.4290	.4698	.4872	.4953	.4999	.5027
4.45	.4231	.4642	.4815	.4896	.4941	.4970
4.50	.4173	.4586	.4760	.4841	.4886	.4914
4.55	.4116	.4532	.4706	.4786	.4831	.4859
4.60	.4060	.4479	.4653	.4733	.4778	.4806
4.65	.4005	.4427	.4601	.4681	.4726	.4753
4.70	.3952	.4377	.4550	.4630	.4674	.4702
4.75	.3899	.4327	.4500	.4580	.4624	.4652
4.80	.3848	.4279	.4452	.4531	.4575	.4603
4.85	.3797	.4231	.4404	.4483	.4528	.4555
4.90	.3747	.4185	.4357	.4436	.4481	.4508
4.95	.3699	.4139	.4312	.4391	.4435	.4462
5.00	.3651	.4095	.4267	.4346	.4390	.4417

For upstream side slope of m = 1.75

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	5.2182	5.2298	5.2371	5.2411	5.2435	5.2450
.55	4.7387	4.7512	4.7590	4.7632	4.7658	4.7674
.60	4.3389	4.3521	4.3605	4.3649	4.3676	4.3694
.65	4.0004	4.0144	4.0232	4.0279	4.0307	4.0325
.70	3.7100	3.7248	3.7340	3.7389	3.7419	3.7438
.75	3.4582	3.4737	3.4833	3.4885	3.4916	3.4936
.80	3.2378	3.2539	3.2640	3.2693	3.2725	3.2746
.85	3.0432	3.0600	3.0704	3.0759	3.0792	3.0813
.90	2.8701	2.8875	2.8982	2.9040	2.9074	2.9096
.95	2.7152	2.7331	2.7442	2.7501	2.7536	2.7559
1.00	2.5756	2.5942	2.6056	2.6116	2.6152	2.6175
1.05	2.4493	2.4684	2.4801	2.4863	2.4900	2.4924
1.10	2.3344	2.3541	2.3660	2.3724	2.3761	2.3786
1.15	2.2295	2.2496	2.2619	2.2684	2.2722	2.2747
1.20	2.1333	2.1539	2.1664	2.1730	2.1769	2.1794
1.25	2.0447	2.0658	2.0786	2.0853	2.0892	2.0918
1.30	1.9629	1.9845	1.9975	2.0043	2.0083	2.0109
1.35	1.8871	1.9091	1.9224	1.9293	1.9334	1.9360
1.40	1.8167	1.8392	1.8526	1.8597	1.8638	1.8665

1.45	1.7512	1.7741	1.7877	1.7948	1.7990	1.8017
1.50	1.6900	1.7133	1.7271	1.7343	1.7386	1.7413
1.55	1.6327	1.6564	1.6704	1.6777	1.6820	1.6848
1.60	1.5790	1.6031	1.6173	1.6246	1.6290	1.6318
1.65	1.5285	1.5530	1.5673	1.5748	1.5792	1.5820
1.70	1.4810	1.5058	1.5203	1.5279	1.5323	1.5351
1.75	1.4362	1.4614	1.4760	1.4836	1.4881	1.4909
1.80	1.3938	1.4194	1.4342	1.4419	1.4463	1.4492
1.85	1.3538	1.3797	1.3946	1.4023	1.4069	1.4098
1.90	1.3158	1.3420	1.3571	1.3649	1.3695	1.3724
1.95	1.2798	1.3063	1.3216	1.3294	1.3340	1.3369
2.00	1.2456	1.2724	1.2878	1.2957	1.3003	1.3032
2.05	1.2131	1.2402	1.2557	1.2636	1.2682	1.2712
2.10	1.1820	1.2095	1.2251	1.2330	1.2377	1.2406
2.15	1.1525	1.1802	1.1959	1.2039	1.2086	1.2115
2.20	1.1242	1.1523	1.1681	1.1761	1.1808	1.1838
2.25	1.0973	1.1256	1.1415	1.1495	1.1542	1.1572
2.30	1.0714	1.1000	1.1160	1.1241	1.1289	1.1318
2.35	1.0467	1.0756	1.0917	1.0998	1.1045	1.1075
2.40	1.0230	1.0522	1.0683	1.0765	1.0813	1.0843
2.45	1.0003	1.0297	1.0460	1.0542	1.0589	1.0619
2.50	.9785	1.0082	1.0245	1.0327	1.0375	1.0405
2.55	.9575	.9875	1.0039	1.0121	1.0169	1.0199
2.60	.9374	.9675	.9840	.9923	.9971	1.0001
2.65	.9180	.9484	.9649	.9732	.9780	.9810
2.70	.8993	.9300	.9466	.9549	.9597	.9627
2.75	.8813	.9122	.9289	.9372	.9420	.9450
2.80	.8639	.8951	.9118	.9201	.9249	.9280
2.85	.8472	.8786	.8953	.9037	.9085	.9115
2.90	.8310	.8626	.8794	.8878	.8926	.8956
2.95	.8154	.8472	.8641	.8724	.8773	.8803
3.00	.8003	.8323	.8492	.8576	.8624	.8655
3.05	.7857	.8179	.8349	.8433	.8481	.8511
3.10	.7715	.8040	.8210	.8294	.8342	.8373
3.15	.7579	.7905	.8076	.8160	.8208	.8238
3.20	.7446	.7775	.7946	.8030	.8078	.8108
3.25	.7317	.7649	.7820	.7904	.7952	.7982
3.30	.7193	.7526	.7697	.7781	.7830	.7860
3.35	.7072	.7407	.7579	.7663	.7711	.7741
3.40	.6954	.7292	.7464	.7548	.7596	.7626
3.45	.6840	.7180	.7352	.7436	.7484	.7515
3.50	.6730	.7071	.7244	.7328	.7376	.7406
3.55	.6622	.6965	.7138	.7222	.7270	.7300
3.60	.6517	.6863	.7036	.7120	.7168	.7198
3.65	.6415	.6763	.6936	.7020	.7068	.7098
3.70	.6316	.6666	.6839	.6923	.6971	.7001
3.75	.6220	.6572	.6745	.6829	.6877	.6907
3.80	.6126	.6480	.6653	.6737	.6785	.6815
3.85	.6035	.6390	.6564	.6648	.6695	.6725
3.90	.5945	.6303	.6477	.6561	.6608	.6638
3.95	.5858	.6218	.6392	.6476	.6523	.6553
4.00	.5774	.6135	.6309	.6393	.6441	.6470
4.05	.5691	.6055	.6229	.6312	.6360	.6390
4.10	.5610	.5976	.6150	.6234	.6281	.6311
4.15	.5532	.5899	.6074	.6157	.6204	.6234
4.20	.5455	.5824	.5999	.6082	.6129	.6159
4.25	.5380	.5751	.5926	.6009	.6056	.6085
4.30	.5306	.5680	.5854	.5937	.5984	.6014
4.35	.5235	.5610	.5785	.5868	.5915	.5944
4.40	.5165	.5542	.5716	.5799	.5846	.5876
4.45	.5096	.5475	.5650	.5733	.5779	.5809
4.50	.5029	.5410	.5585	.5668	.5714	.5743
4.55	.4963	.5347	.5521	.5604	.5650	.5680
4.60	.4899	.5285	.5459	.5542	.5588	.5617
4.65	.4836	.5224	.5398	.5481	.5527	.5556
4.70	.4775	.5164	.5339	.5421	.5467	.5496
4.75	.4714	.5106	.5281	.5363	.5409	.5438
4.80	.4655	.5049	.5224	.5305	.5352	.5380
4.85	.4598	.4993	.5168	.5249	.5295	.5324
4.90	.4541	.4939	.5113	.5195	.5241	.5269
4.95	.4485	.4885	.5059	.5141	.5187	.5215
5.00	.4431	.4833	.5007	.5088	.5134	.5163

For upstream side slope of m = 2.00

b'	.80	1.00	1.25	1.50	1.75	2.00
.50	5.9711	5.9817	5.9883	5.9919	5.9941	5.9955
.55	5.4236	5.4349	5.4420	5.4459	5.4482	5.4497
.60	4.9671	4.9791	4.9867	4.9908	4.9932	4.9948
.65	4.5806	4.5933	4.6014	4.6057	4.6083	4.6099
.70	4.2491	4.2625	4.2710	4.2755	4.2782	4.2800

.75	3.9617	3.9758	3.9846	3.9894	3.9922	3.9940
.80	3.7100	3.7248	3.7340	3.7389	3.7419	3.7438
.85	3.4879	3.5032	3.5128	3.5180	3.5210	3.5230
.90	3.2903	3.3063	3.3162	3.3215	3.3247	3.3267
.95	3.1135	3.1300	3.1402	3.1457	3.1490	3.1511
1.00	2.9542	2.9713	2.9818	2.9875	2.9908	2.9930
1.05	2.8100	2.8276	2.8385	2.8443	2.8477	2.8500
1.10	2.6789	2.6970	2.7082	2.7141	2.7177	2.7199
1.15	2.5592	2.5778	2.5892	2.5953	2.5989	2.6012
1.20	2.4493	2.4684	2.4801	2.4863	2.4900	2.4924
1.25	2.3482	2.3678	2.3797	2.3861	2.3898	2.3922
1.30	2.2549	2.2749	2.2871	2.2935	2.2973	2.2998
1.35	2.1684	2.1889	2.2013	2.2078	2.2117	2.2142
1.40	2.0881	2.1089	2.1216	2.1282	2.1322	2.1347
1.45	2.0133	2.0345	2.0474	2.0541	2.0582	2.0607
1.50	1.9434	1.9651	1.9781	1.9850	1.9891	1.9917
1.55	1.8780	1.9001	1.9134	1.9203	1.9244	1.9270
1.60	1.8167	1.8392	1.8526	1.8597	1.8638	1.8665
1.65	1.7591	1.7819	1.7956	1.8027	1.8069	1.8096
1.70	1.7049	1.7281	1.7419	1.7491	1.7533	1.7560
1.75	1.6537	1.6773	1.6912	1.6985	1.7028	1.7055
1.80	1.6054	1.6293	1.6434	1.6507	1.6551	1.6578
1.85	1.5597	1.5839	1.5982	1.6056	1.6099	1.6127
1.90	1.5164	1.5409	1.5553	1.5628	1.5672	1.5700
1.95	1.4752	1.5001	1.5147	1.5222	1.5266	1.5295
2.00	1.4362	1.4614	1.4760	1.4836	1.4881	1.4909
2.05	1.3990	1.4245	1.4393	1.4470	1.4514	1.4543
2.10	1.3636	1.3894	1.4043	1.4120	1.4165	1.4194
2.15	1.3298	1.3559	1.3710	1.3787	1.3833	1.3862
2.20	1.2976	1.3240	1.3391	1.3469	1.3515	1.3544
2.25	1.2668	1.2934	1.3087	1.3166	1.3211	1.3241
2.30	1.2373	1.2642	1.2796	1.2875	1.2921	1.2950
2.35	1.2091	1.2363	1.2518	1.2597	1.2643	1.2673
2.40	1.1820	1.2095	1.2251	1.2330	1.2377	1.2406
2.45	1.1561	1.1838	1.1995	1.2075	1.2121	1.2151
2.50	1.1312	1.1591	1.1749	1.1829	1.1876	1.1906
2.55	1.1072	1.1354	1.1513	1.1594	1.1641	1.1670
2.60	1.0842	1.1127	1.1286	1.1367	1.1414	1.1444
2.65	1.0621	1.0907	1.1068	1.1149	1.1196	1.1226
2.70	1.0407	1.0696	1.0858	1.0939	1.0986	1.1016
2.75	1.0201	1.0493	1.0655	1.0737	1.0784	1.0814
2.80	1.0003	1.0297	1.0460	1.0542	1.0589	1.0619
2.85	.9812	1.0108	1.0271	1.0353	1.0401	1.0431
2.90	.9627	.9926	1.0089	1.0172	1.0219	1.0250
2.95	.9449	.9749	.9914	.9996	1.0044	1.0074
3.00	.9276	.9579	.9744	.9827	.9874	.9905
3.05	.9109	.9414	.9580	.9662	.9710	.9741
3.10	.8947	.9255	.9421	.9504	.9552	.9582
3.15	.8791	.9100	.9267	.9350	.9398	.9428
3.20	.8639	.8951	.9118	.9201	.9249	.9280
3.25	.8493	.8806	.8974	.9057	.9105	.9135
3.30	.8350	.8665	.8834	.8917	.8965	.8996
3.35	.8212	.8529	.8698	.8781	.8830	.8860
3.40	.8078	.8397	.8566	.8650	.8698	.8728
3.45	.7948	.8269	.8438	.8522	.8570	.8600
3.50	.7821	.8144	.8314	.8398	.8446	.8476
3.55	.7698	.8023	.8193	.8277	.8325	.8356
3.60	.7579	.7905	.8076	.8160	.8208	.8238
3.65	.7462	.7791	.7962	.8046	.8094	.8124
3.70	.7349	.7680	.7851	.7935	.7983	.8013
3.75	.7239	.7571	.7743	.7827	.7875	.7905
3.80	.7132	.7466	.7638	.7722	.7770	.7800
3.85	.7027	.7363	.7535	.7619	.7667	.7698
3.90	.6925	.7263	.7436	.7520	.7568	.7598
3.95	.6826	.7166	.7338	.7422	.7471	.7501
4.00	.6730	.7071	.7244	.7328	.7376	.7406
4.05	.6635	.6978	.7151	.7235	.7283	.7314
4.10	.6543	.6888	.7061	.7145	.7193	.7223
4.15	.6453	.6800	.6973	.7057	.7105	.7135
4.20	.6366	.6714	.6887	.6971	.7019	.7049
4.25	.6280	.6630	.6804	.6888	.6935	.6966
4.30	.6196	.6548	.6722	.6806	.6854	.6884
4.35	.6114	.6468	.6642	.6726	.6774	.6804
4.40	.6035	.6390	.6564	.6648	.6695	.6725
4.45	.5956	.6314	.6488	.6571	.6619	.6649
4.50	.5880	.6239	.6413	.6497	.6544	.6574
4.55	.5805	.6166	.6340	.6424	.6471	.6501
4.60	.5732	.6095	.6269	.6352	.6400	.6430
4.65	.5661	.6025	.6199	.6283	.6330	.6360
4.70	.5591	.5956	.6131	.6214	.6262	.6291

4.75	.5522	.5890	.6064	.6147	.6195	.6224
4.80	.5455	.5824	.5999	.6082	.6129	.6159
4.85	.5389	.5760	.5935	.6018	.6065	.6094
4.90	.5325	.5697	.5872	.5955	.6002	.6032
4.95	.5261	.5636	.5811	.5894	.5941	.5970
5.00	.5199	.5576	.5750	.5833	.5880	.5909

141.

An upstream trapezoidal channel has a bottom width $b_1 = 3$ m, and a side slope $m_1 = 0.75$, and transitions to a rectangular channel with $b_2 = 3.5$ m. What flowrate will result in critical depths in both the trapezoidal and rectangular channels? What are these depths and their corresponding specific energies? (Solve the problem using dimensionless variables.)

Given: b_1=3 m, m_1=0.75 and b_2=3.5 m.

Solution:

The solution requires that Eq. 40 and F_{r1}=1 be solved simultaneously for Y_1' and b'. $b_1'=b_1/b_2$=3./3.5=.8571429. The following TK-Solver Model TRARECT.TK obtains this soution and gives Y_1' = .1702 and b'=4.601. Thus $Y_{c2}= b_1/b'$ = 3/4.6013 = 0.652 m and $q_2 = (gY_c^3)^{1/2} = (9.81x.652^2)^{.5}$ = 1.649 m^2/s. Thus $Q=b_2q_2$ = 3.5(1.649) = 5.771 m^3/s. The upstream depth $Y_1=b_1Y_1'/m_1$ = 3(.1702).75 = 0.681 m.

TK-Solver Model TRARECT.TK

```
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit----- Comment---------
             Y1p     .25707709
   .25       m
             bp      1.0308088
   .8        b1p

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* (m/bp)^3/b1p^2*(1+2*Y1p)/(Y1p+Y1p^2)^3=1
* Y1p*bp+.5*m^3/(b1p*bp*(Y1p+Y1p^2))^2=1.5*m
```

If this model is solved using dimensioned varibles then E_c,Y_1 and Q are solved from $E_c = 1.5[(Q/b_2)^2/g]^{1.3}$, $E_1 = E_c$ and F_{r1}= 1. The following TK-Solver Model PRB2_129.TK obtains this solution.

```
==================== VARIABLE SHEET =====================
St Input---- Name--- Output--- Unit----- Comment----------
             Ec      .97798628
             Q       5.771182
             Y1      .68080666
   3.5       b2
   9.81      g
   3         b1
   .75       m1

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Ec=1.5*((Q/b2)^2/g)^.3333333
* Y1+(Q/(b1*Y1+m1*Y1^2))^2/(2*g)=Ec
* Q^2*(b1+2*m1*Y1)/(g*(b1*Y1+m1*Y1^2)^3)=1
```

142.

An upstream trapezoidal channel with $m_1 = 1.5$ and $b_1 = 5$ m smoothly changes to a rectangular channel with $b_2 = 5$ m wide, and critical flow occurs here. If the flowrate is $Q = 12$ m^3/s, what are the depths upstream and downstream from the transition? What is the upstream Froude Number? If n = .014, what bottom slope of the upstream channel will result in uniform flow? (Solve this problem using both dimensionless and dimensioned variables.)

Given: $Q=12$ m^3/s, $b_1=5$ m, $m_1=1.5$, $b_2=5$ m, n=.014

Solution:

First compute $Y_c=[(Q/b_2)^2/g]^{1/3} = .97168$ m, so $b'=b_1/Y_c = 5.14571$ and $b_1' = 1.25$. Next solve Eq. 40 for $Y_1' = .41414$ (See TK-Solver Model TRARECS.TK below). Therefore $Y_1 = 5(.41415)/1.5 = 1.3905$ m, and $F_{r1} = .380$. Solving Mannings Equation gives $S_{o1} = .000305$.

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
              Y1p      .3533298
   1          m
   4.0800914  bp
   1.5        b1p
              Fr1      .31959611

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Fr1=sqrt((m/bp)^3/b1p^2*(1+2*Y1p)/(Y1p+Y1p^2)^3)
* Y1p*bp+.5*m^3/(b1p*bp*(Y1p+Y1p^2))^2=1.5*m
```

Using dimensioned variables MOdel PRB2 130.TK

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
              Ec       1.4575242
              Y1       1.3804932
              So1      .00030501
   12         Q
   4          b2
   9.81       g
   5          b1
   1.5        m1
   1          Cu
   .014       n

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Ec=1.5*((Q/b2)^2/g)^.3333333
* Y1+(Q/(b1*Y1+m1*Y1^2))^2/(2*g)=Ec
* Q=Cu/n*(b1*Y1+m1*Y1^2)^1.666667*So1^.5/(b1+2*(m1^2+1)^.5*Y1)^.666667
```

143.

An upstream trapezoidal channel has $m_1 = 1.5$, $n = .013$ and $S_{o1} = .00025$, and a bottom width ratio to a downstream rectangular channel of $b_1' = b_1/b_2 = 1.5$. For a flowrate Q = 500 cfs, solve for b_1 and b_2 so critical depth occurs in the rectangular channel, and uniform flow occurs upstream.

Given: Trapezoidal channel with m_1=1.5, n=.013, S_{o1}=.00025 with b_1'=1.5,Q=500 cfs.
Find: b_1 and b_2 so critical depth occurs in rectangular channel and uniform flow upstream.

Solution:
The TK-Solver model TRARECS1.TK can be used to solve this problem just as it was used to solve Example Problem 28. This soltion is given below.

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment----------
              Y1p      .13076804
   1.5        m
              bp       16.207468
   1.5        b1p
              Y1       3.271092
              b1       37.521692
   500        Q
   1.486      Cu
   .00025     So1
   .013       n
   32.2       g

==================== RULE SHEET ========================
S Rule-----------------------------------------------------
* Y1p=m*Y1/b1
* Q=Cu*So1^.5*((b1+m*Y1)*Y1)^1.666667/(n*(b1+2.*(m*m+1)^.5*Y1)^.6666667)
* bp=b1/(((b1p*Q/b1)^2/g)^.3333333)
* Y1p*bp+.5*m^3/(b1p*bp*(Y1p+Y1p^2))^2=1.5*m
```

The downstream width $b_2=b_1/1.5$ = 37.533/1.5 = 25.01 ft. The critical depth $Y_c=(q_2^2/g)^{1/3}$ = 2.315 ft. This depth can also be obtaine drom $Y_c=b_1/b'$ = 37.522/16.207 = 2.315 ft.

If this problem is solved using dimensioned variables, then Y_1, b_1 and Y_c can be solved from the following 3 equations. (Note that Equations 2 and 3 could be combined and Y_c eliminated.)

$$Q = (C_u/n)A_1^{5/3}(S_{o1})^{1/2}/P_1^{2/3} \qquad (1)$$

$$Y_c = [(b_1'Q/b_1)^2/g]^{1/3} \qquad (2)$$

$$Y_1 = (Q/A_1)^2/(2g) = 1.5Y_c \qquad (3)$$

The TK-Solver Model PRB2_131.TK that implements the solution to these three equations produces the same answers as above.

```
==================== VARIABLE SHEET =====================
```

```
St Input───── Name──── Output──── Unit────── Comment──────────
              b1       37.521221
              Y1       3.2711205
              Yc       2.315106
   500        Q
   1.486      Cu
   .013       n
              A1       138.78678
   1.5        m
   .00025     So1
   1.5        b1p
   32.2       g

════════════════════ RULE SHEET ═════════════════════════
S Rule──────────────────────────────────────────────────
* A1=(b1+m*Y1)*Y1
* Q=Cu/n*A1^1.6666667/(b1+2*(m*m+1)^.5*Y1)^.666667*So1^.5
* Yc=((b1p*Q/b1)^2/g)^.3333333
* Y1+(Q/A1)^2/(2*g)=1.5*Yc
```

Notice the large width b_1=37.521 ft required. This width decreases and the depths Y_1 and Y_c decrease as $b_1' = b_1/b_2$ decreases as shown in the table below in which soutions were obtained for different values of b_1'.

b_1'	b_1(ft)	Y_1(ft)	Y_c(ft)
1.5	37.521	3.271	2.315
1.4	25.745	4.026	2.842
1.3	19.271	4.663	3.281
1.25	17.018	4.943	3.973
1.2	15.166	5.202	3.649
1.15	12.609	5.443	3.813
1.1	12.274	5.668	3.966
1.05	11.110	5.880	4.109
1.00	10.081	6.080	4.243
0.95	9.161	6.270	4.371
0.90	8.330	6.450	4.492

Solution to Problem 2.143 obtained using Program SOLT RC2.FOR
(the two variable identified as unknowns are: b' and Y_1' (3 & 4))
Output from SOLT RC2

```
 Solution: b'  =  16.2072 Y1' =    .1308
 m1  =    1.500
 b1' =    1.500
 b'  =   16.207
 Y1' =     .131
 Q   =  500.000
 So1 =  .000250
 n   =     .013
 Fr1 =     .371
```

To solve for b_1 and b_2 note the following

From Critical Eq. $gY_c^3=(Q/b_2)^2$ or

$b_2^2=Q^2/(gY_c^3)$ but $Y_c=b_1/b$ & $b_2=b_1/b_1'$ so

$(b_1/b_1)^2=(Q^2b'^3)/(gb_1^3)$ or

$b_1^5=Q^2b'^3b_1^2/g$ <==

$b_1=(500^2 14.480^3 1.5^2)^{.2}$ = 37.521 ft, b_2=21.014 ft

Flow rate reduces to Q = 400 cfs

```
 Solution: b'  =  14.4798 Y1' =    .1467
 m1  =    1.500
 b1' =    1.500
 b'  =   14.480
 Y1' =     .147
 Q   =  400.000
 So1 =  .000250
 n   =     .013
 Fr1 =     .367
```

$b_1^5=Q^2b'^3b_1^2/g$

$b_1=(400^2 14.480^3 1.5^2/32.2)^{.2}$=32.074 ft, b_2=21.383 ft

Flow rate reduces to Q = 300 cfs

```
 Solution: b'  =  12.6443 Y1' =    .1684
 m1  =    1.500
```

b1' = 1.500 $b_1=(Q^2b'^3b_1^2/g)^{.2}$

b' = 12.644

Y1' = .168 $b_1=(300^2 12.644^3 1.5^2/32.2)^{.2}$=<u>26.354 ft,</u>

b_2=<u>17.569 ft</u>

Q = 300.000

So1 = .000250 $Y_1=b_1Y_1'/m_1$=26.35(.168)/1.5 = <u>2.951 ft</u>

n = .013

Fr1 = .361

Flow rate reduces to Q = 200 cfs

Solution: b' = 10.6230 Y1' = .2011

m1 = 1.500

b1' = 1.500 $b_1=(200^2 10.623^3 1.5^2/32.2)^{.2}$=<u>20.185 ft,</u>

b_2=<u>13.457 ft</u>

b' = 10.623

Y1' = .201

Q = 200.000

So1 = .000250

n = .013

Fr1 = .353

<u>Program SOLT RC2.FOR</u>

```
      CHARACTER CH(8)*3/'m1 ','b1''','b'' ','Y1''','Q  ','So1','n  ',
     &'g  '/,CH1*5,FMT*15/'(A4,'' = '',F8.3)'/
      COMMON X(8),Cu
      REAL F(2),FF(2),D(2,2)
      INTEGER*2 INDX(2)
      EQUIVALENCE (X(1),FM),(X(2),B1P),(X(3),BP),(X(4),Y1P),(X(5),Q),
     &(X(6),So1),(X(7),FN),(X(8),G)
      WRITE(*,*)' Give output unit'
      READ(*,*) IOUT
1     WRITE(*,*)' Give 2 numbers for 2 unknowns'
      WRITE(*,100) (I,CH(I),I=1,6)
100   FORMAT(I3,' - ',A3)
      READ(*,*) IU1,IU2
      IF(IU1.LT.0 .OR. IU1.GT.6 .OR. IU2.LT.0 .OR. IU2.GT.6) STOP
      WRITE(*,*)' Provide values for:'
      DO 10 I=1,8
      CH1='     '
      IF(I.EQ.IU1 .OR. I.EQ.IU2) CH1='Guess'
      WRITE(*,105) CH1,CH(I)
105   FORMAT(A6,1X,A3,' = ',\)
10    READ(*,*) X(I)
      Cu=1.486
      IF(G.LT.25.) Cu=1.
      NCT=0
15    CALL FUN(F)
      II=0
      DO 20 I=1,6
      IF(I.NE.IU1 .AND. I.NE.IU2) GO TO 20
      XX=X(I)
      X(I)=1.005*X(I)
      II=II+1
      CALL FUN(FF)
      DO 25 J=1,2
25    D(J,II)=(FF(J)-F(J))/(X(I)-XX)
      X(I)=XX
20    CONTINUE
      CALL SOLVEQ(2,1,2,D,F,1,DD,INDX)
      X(IU1)=X(IU1)-F(1)
      X(IU2)=X(IU2)-F(2)
      NCT=NCT+1
      WRITE(*,*) ' NCT=',NCT,F,X(IU1),X(IU2)
      IF(NCT.LT.30 .AND. (ABS(F(1))+ABS(F(2))).GT.5.E-5) GO TO 15
      IF(NCT.EQ.30) WRITE(*,*)' Did not converge', F
      WRITE(*,110) CH(IU1),X(IU1),CH(IU2),X(IU2)
110   FORMAT(' Solution:',2(A4,' ='F9.4))
      IF(IOUT.NE.6 .AND. IOUT.NE.0) WRITE(IOUT,110) CH(IU1),X(IU1),
     &CH(IU2),X(IU2)
      Fr1=SQRT(FM**3*(1.+2.*Y1P)/(B1P**2*(BP*(Y1P+Y1P**2))**3))
      WRITE(*,*)' Variables:'
```

```
      DO 30 I=1,7
      FMT(14:14)='3'
      IF(I.EQ.6) FMT(14:14)='6'
      IF(IOUT.NE.6 .AND. IOUT.NE.0) WRITE(IOUT,FMT) CH(I),X(I)
30    WRITE(*,FMT) CH(I),X(I)
      FMT(14:14)='3'
      IF(IOUT.NE.6 .AND. IOUT.NE.0) WRITE(IOUT,FMT) 'Fr1',Fr1
      WRITE(*,FMT) 'Fr1',Fr1
      WRITE(*,*)' To solve another problem give 1 (0=STOP)'
      READ(*,*) IU1
      IF(IU1.GT.0) GO TO 1
      END
      SUBROUTINE FUN(F)
      REAL F(2)
      COMMON X(8),Cu
      EQUIVALENCE (X(1),FM),(X(2),B1P),(X(3),BP),(X(4),Y1P),(X(5),Q),
     &(X(6),So1),(X(7),FN),(X(8),G)
      AP=Y1P+Y1P**2
      F(1)=Y1P*BP+.5*FM**3/(BP*B1P*AP)**2-1.5*FM
C      F(1)=(Y1P*BP-1.5*FM)*(BP*B1P*AP)**2+.5*FM**3
      QP=FN/(Cu*SQRT(So1)*Q**.06666667*(BP**3*B1P**2/G)**.5333333)
      F(2)=QP-AP**1.666667/(FM*(FM+2.*SQRT(FM*FM+1.)*Y1P)**.6666667)
      RETURN
      END
```

The two equations that this program solves are: (1) Equation 40 (the dimensionless equation that equates the specific energy in an upstream trapezoidal channel to the critical specific energy in a downstream rectangular channel, and (2) Mannings equation written in the same dimensionless variables, as far as possible.
If the parameter Q' is defined as:

$$Q'=nQ/\{C_uS_{o1}^{1/2}[Q^2b'^3b_1{}^2/g]^{8/15}\} = n/\{C_uS_{o1}^{1/2}Q^{1/15}[b'^3b_1'^2]^{8/15}\}$$

then Mannings equation becomes:

$$Q'=[Y_1'+Y_1'^2]^{5/3}/\{m_1[m_1+2(m_1{}^2+1)^{1/2}Y_1']^{2/3}\}$$

Notice the large widths needed to satisfy the specifications. As the reduction in width becomes smaller the widths required reduce. The table below shows the solutions. In obtaining these solutions it is important to provide relatively "good guesses."

b_1'	b'	Y_1'	b_1(ft)	b_2(ft)	Y_1(ft)
1.5	16.207	.1307	37.521	25.01	3.271
1.4	9.058	.235	25.745	18.392	4.026
1.3	5.873	.363	19.271	14.824	4.663
1.2	4.156	.515	15.167	12.639	5.202
1.1	3.095	.693	12.27	11.15	5.67

144.

Write a program, or a computer model, that will generate a table of values to plot Figure 14, i.e. make a table that solves Eq. 44 (or Eq. 43). It will be instructive for you to use the Laguerre Method described in the next chapter and extract the three roots from this 3rd degree polynomial.

Solution: The easiest way to generate such a table is to find the constant $c = b'Y_1'$ corresponding to each value of $b_1' = b_1/b_2$, and then obtain Y_1' corresponding to any value of b' by dividing b' into c, i.e. $Y_1'=c/b'$.
A 2nd degree polynomial as an approximation for c can be obtained by using the first, middle, and last pairs of b_1' and c from the table that gives this data. This polynomial is,

$$c = -.2784(b_1')^2 + 1.1920b_1' + .2224$$

The following program RECRECT3.FOR generated a table of data to plot Figures 14 by solving Eq. 43 for $c = b'Y_1'$, i.e.
$F = c + .5/(b_1'c)^2 - 1 = 0$

by the Newton Method using the above 2nd degree polynomial to provide the guess for c.

Program RECRECT3.FOR

```
      REAL YP(12),C(12),
     &BP1(12)/1.05,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,2.0,2.25,2.5/
      DO 10 I=1,12
      CC=(1.1920-.2784*BP1(I))*BP1(I)+.2224
      NCT=0
      BP2=.5/BP1(I)**2
1     F=CC+BP2/CC**2-1.5
      NCT=NCT+1
      IF(MOD(NCT,2).EQ.0) GO TO 2
      FF=F
      C1=CC
      CC=1.005*CC
      GO TO 1
2     DIF=(CC-C1)*FF/(F-FF)
      CC=C1-DIF
      IF(NCT.LT.30 .AND. ABS(DIF).GT.1.E-6) GO TO 1
      IF(NCT.EQ.30) WRITE(*,*)' Did not converge',I,DIF,CC
10    C(I)=CC
      WRITE(3,100) BP1
100   FORMAT(' For upstream rectangular channel'/1X,88('-'),/
     &'  b''=  ',40X,'b1''=b1/b2',/' b1/Yc',F5.2,11F7.2,/1X,88('-'))
      DO 20 I=1,141
      BP=.5+.05*FLOAT(I-1)
      DO 15 J=1,12
15    YP(J)=C(J)/BP
20    WRITE(3,110) BP,(YP(K),K=1,12)
110   FORMAT(F5.2,12F7.4)
      END
```

Output:

```
 For upstream rectangular channel
 ----------------------------------------------------------------------------------------
  b'=                                        b1'=b1/b2
 b1/Yc 1.05   1.10   1.20   1.30   1.40   1.50   1.60   1.70   1.80   2.00   2.25   2.50
 ----------------------------------------------------------------------------------------
```

.50	2.3340	2.4487	2.5840	2.6673	2.7252	2.7680	2.8008	2.8268	2.8478	2.8794	2.9065	2.9252
.55	2.1218	2.2261	2.3491	2.4248	2.4775	2.5163	2.5462	2.5698	2.5889	2.6176	2.6422	2.6593
.60	1.9450	2.0406	2.1533	2.2228	2.2710	2.3066	2.3340	2.3557	2.3731	2.3995	2.4221	2.4377
.65	1.7954	1.8836	1.9877	2.0518	2.0963	2.1292	2.1545	2.1745	2.1906	2.2149	2.2357	2.2502
.70	1.6671	1.7490	1.8457	1.9052	1.9466	1.9771	2.0006	2.0191	2.0341	2.0567	2.0760	2.0894
.75	1.5560	1.6324	1.7226	1.7782	1.8168	1.8453	1.8672	1.8845	1.8985	1.9196	1.9376	1.9501
.80	1.4587	1.5304	1.6150	1.6671	1.7033	1.7300	1.7505	1.7667	1.7799	1.7996	1.8165	1.8283
.85	1.3729	1.4404	1.5200	1.5690	1.6031	1.6282	1.6475	1.6628	1.6752	1.6938	1.7097	1.7207
.90	1.2967	1.3604	1.4355	1.4818	1.5140	1.5378	1.5560	1.5704	1.5821	1.5997	1.6147	1.6251
.95	1.2284	1.2888	1.3600	1.4039	1.4343	1.4568	1.4741	1.4878	1.4988	1.5155	1.5297	1.5396
1.00	1.1670	1.2243	1.2920	1.3337	1.3626	1.3840	1.4004	1.4134	1.4239	1.4397	1.4532	1.4626
1.05	1.1114	1.1660	1.2305	1.2702	1.2977	1.3181	1.3337	1.3461	1.3561	1.3711	1.3840	1.3930
1.10	1.0609	1.1130	1.1745	1.2124	1.2387	1.2582	1.2731	1.2849	1.2944	1.3088	1.3211	1.3296
1.15	1.0148	1.0646	1.1235	1.1597	1.1849	1.2035	1.2177	1.2290	1.2382	1.2519	1.2637	1.2718
1.20	.9725	1.0203	1.0767	1.1114	1.1355	1.1533	1.1670	1.1778	1.1866	1.1997	1.2110	1.2188
1.25	.9336	.9795	1.0336	1.0669	1.0901	1.1072	1.1203	1.1307	1.1391	1.1518	1.1626	1.1701
1.30	.8977	.9418	.9938	1.0259	1.0482	1.0646	1.0772	1.0872	1.0953	1.1075	1.1179	1.1251
1.35	.8644	.9069	.9570	.9879	1.0093	1.0252	1.0373	1.0470	1.0547	1.0664	1.0765	1.0834
1.40	.8336	.8745	.9228	.9526	.9733	.9886	1.0003	1.0096	1.0171	1.0284	1.0380	1.0447
1.45	.8048	.8444	.8910	.9198	.9397	.9545	.9658	.9748	.9820	.9929	1.0022	1.0087
1.50	.7780	.8162	.8613	.8891	.9084	.9227	.9336	.9423	.9493	.9598	.9688	.9751
1.55	.7529	.7899	.8335	.8604	.8791	.8929	.9035	.9119	.9186	.9288	.9376	.9436
1.60	.7294	.7652	.8075	.8335	.8516	.8650	.8753	.8834	.8899	.8998	.9083	.9141
1.65	.7073	.7420	.7830	.8083	.8258	.8388	.8487	.8566	.8630	.8725	.8807	.8864
1.70	.6865	.7202	.7600	.7845	.8015	.8141	.8238	.8314	.8376	.8469	.8548	.8604
1.75	.6669	.6996	.7383	.7621	.7786	.7908	.8002	.8077	.8136	.8227	.8304	.8358
1.80	.6483	.6802	.7178	.7409	.7570	.7689	.7780	.7852	.7910	.7998	.8074	.8126
1.85	.6308	.6618	.6984	.7209	.7365	.7481	.7570	.7640	.7697	.7782	.7855	.7906
1.90	.6142	.6444	.6800	.7019	.7172	.7284	.7371	.7439	.7494	.7577	.7649	.7698
1.95	.5985	.6279	.6626	.6839	.6988	.7097	.7182	.7248	.7302	.7383	.7452	.7501
2.00	.5835	.6122	.6460	.6668	.6813	.6920	.7002	.7067	.7119	.7198	.7266	.7313
2.05	.5693	.5972	.6302	.6506	.6647	.6751	.6831	.6895	.6946	.7023	.7089	.7135
2.10	.5557	.5830	.6152	.6351	.6489	.6590	.6669	.6730	.6780	.6856	.6920	.6965
2.15	.5428	.5695	.6009	.6203	.6338	.6437	.6514	.6574	.6623	.6696	.6759	.6803
2.20	.5305	.5565	.5873	.6062	.6194	.6291	.6365	.6425	.6472	.6544	.6606	.6648
2.25	.5187	.5441	.5742	.5927	.6056	.6151	.6224	.6282	.6328	.6399	.6459	.6500
2.30	.5074	.5323	.5617	.5799	.5924	.6017	.6089	.6145	.6191	.6260	.6318	.6359
2.35	.4966	.5210	.5498	.5675	.5798	.5889	.5959	.6014	.6059	.6126	.6184	.6224
2.40	.4862	.5101	.5383	.5557	.5678	.5767	.5835	.5889	.5933	.5999	.6055	.6094
2.45	.4763	.4997	.5273	.5444	.5562	.5649	.5716	.5769	.5812	.5876	.5932	.5970
2.50	.4668	.4897	.5168	.5335	.5450	.5536	.5602	.5654	.5696	.5759	.5813	.5850
2.55	.4576	.4801	.5067	.5230	.5344	.5427	.5492	.5543	.5584	.5646	.5699	.5736
2.60	.4488	.4709	.4969	.5129	.5241	.5323	.5386	.5436	.5476	.5537	.5589	.5625
2.65	.4404	.4620	.4875	.5033	.5142	.5223	.5285	.5334	.5373	.5433	.5484	.5519
2.70	.4322	.4535	.4785	.4939	.5047	.5126	.5187	.5235	.5274	.5332	.5382	.5417
2.75	.4244	.4452	.4698	.4850	.4955	.5033	.5092	.5140	.5178	.5235	.5284	.5319
2.80	.4168	.4373	.4614	.4763	.4866	.4943	.5001	.5048	.5085	.5142	.5190	.5224
2.85	.4095	.4296	.4533	.4680	.4781	.4856	.4914	.4959	.4996	.5052	.5099	.5132
2.90	.4024	.4222	.4455	.4599	.4699	.4772	.4829	.4874	.4910	.4964	.5011	.5043
2.95	.3956	.4150	.4380	.4521	.4619	.4691	.4747	.4791	.4827	.4880	.4926	.4958
3.00	.3890	.4081	.4307	.4446	.4542	.4613	.4668	.4711	.4746	.4799	.4844	.4875
3.05	.3826	.4014	.4236	.4373	.4468	.4538	.4592	.4634	.4668	.4720	.4765	.4795
3.10	.3764	.3949	.4168	.4302	.4395	.4464	.4517	.4559	.4593	.4644	.4688	.4718
3.15	.3705	.3887	.4102	.4234	.4326	.4394	.4446	.4487	.4520	.4570	.4613	.4643
3.20	.3647	.3826	.4037	.4168	.4258	.4325	.4376	.4417	.4450	.4499	.4541	.4571
3.25	.3591	.3767	.3975	.4104	.4193	.4258	.4309	.4349	.4381	.4430	.4471	.4500
3.30	.3536	.3710	.3915	.4041	.4129	.4194	.4244	.4283	.4315	.4363	.4404	.4432
3.35	.3484	.3655	.3857	.3981	.4067	.4131	.4180	.4219	.4250	.4298	.4338	.4366
3.40	.3432	.3601	.3800	.3923	.4008	.4071	.4119	.4157	.4188	.4234	.4274	.4302
3.45	.3383	.3549	.3745	.3866	.3950	.4012	.4059	.4097	.4127	.4173	.4212	.4239
3.50	.3334	.3498	.3691	.3810	.3893	.3954	.4001	.4038	.4068	.4113	.4152	.4179
3.55	.3287	.3449	.3639	.3757	.3838	.3899	.3945	.3981	.4011	.4055	.4094	.4120
3.60	.3242	.3401	.3589	.3705	.3785	.3844	.3890	.3926	.3955	.3999	.4037	.4063

3.65	.3197	.3354	.3540	.3654	.3733	.3792	.3837	.3872	.3901	.3944	.3981	.4007
3.70	.3154	.3309	.3492	.3604	.3683	.3740	.3785	.3820	.3848	.3891	.3928	.3953
3.75	.3112	.3265	.3445	.3556	.3634	.3691	.3734	.3769	.3797	.3839	.3875	.3900
3.80	.3071	.3222	.3400	.3510	.3586	.3642	.3685	.3719	.3747	.3789	.3824	.3849
3.85	.3031	.3180	.3356	.3464	.3539	.3595	.3637	.3671	.3698	.3739	.3775	.3799
3.90	.2992	.3139	.3313	.3420	.3494	.3549	.3591	.3624	.3651	.3692	.3726	.3750
3.95	.2954	.3100	.3271	.3376	.3450	.3504	.3545	.3578	.3605	.3645	.3679	.3703
4.00	.2917	.3061	.3230	.3334	.3407	.3460	.3501	.3533	.3560	.3599	.3633	.3657
4.05	.2881	.3023	.3190	.3293	.3364	.3417	.3458	.3490	.3516	.3555	.3588	.3611
4.10	.2846	.2986	.3151	.3253	.3323	.3376	.3416	.3447	.3473	.3511	.3544	.3567
4.15	.2812	.2950	.3113	.3214	.3283	.3335	.3374	.3406	.3431	.3469	.3502	.3524
4.20	.2779	.2915	.3076	.3175	.3244	.3295	.3334	.3365	.3390	.3428	.3460	.3482
4.25	.2746	.2881	.3040	.3138	.3206	.3256	.3295	.3326	.3350	.3388	.3419	.3441
4.30	.2714	.2847	.3005	.3102	.3169	.3219	.3257	.3287	.3311	.3348	.3380	.3401
4.35	.2683	.2815	.2970	.3066	.3132	.3182	.3219	.3249	.3273	.3310	.3341	.3362
4.40	.2652	.2783	.2936	.3031	.3097	.3145	.3183	.3212	.3236	.3272	.3303	.3324
4.45	.2622	.2751	.2903	.2997	.3062	.3110	.3147	.3176	.3200	.3235	.3266	.3287
4.50	.2593	.2721	.2871	.2964	.3028	.3076	.3112	.3141	.3164	.3199	.3229	.3250
4.55	.2565	.2691	.2840	.2931	.2995	.3042	.3078	.3106	.3129	.3164	.3194	.3215
4.60	.2537	.2662	.2809	.2899	.2962	.3009	.3044	.3073	.3095	.3130	.3159	.3180
4.65	.2510	.2633	.2778	.2868	.2930	.2976	.3012	.3040	.3062	.3096	.3125	.3145
4.70	.2483	.2605	.2749	.2838	.2899	.2945	.2980	.3007	.3030	.3063	.3092	.3112
4.75	.2457	.2578	.2720	.2808	.2869	.2914	.2948	.2976	.2998	.3031	.3059	.3079
4.80	.2431	.2551	.2692	.2778	.2839	.2883	.2918	.2945	.2966	.2999	.3028	.3047
4.85	.2406	.2524	.2664	.2750	.2809	.2854	.2887	.2914	.2936	.2968	.2996	.3016
4.90	.2382	.2499	.2637	.2722	.2781	.2824	.2858	.2884	.2906	.2938	.2966	.2985
4.95	.2358	.2473	.2610	.2694	.2753	.2796	.2829	.2855	.2877	.2908	.2936	.2955
5.00	.2334	.2449	.2584	.2667	.2725	.2768	.2801	.2827	.2848	.2879	.2906	.2925
5.05	.2311	.2424	.2558	.2641	.2698	.2741	.2773	.2799	.2820	.2851	.2878	.2896
5.10	.2288	.2401	.2533	.2615	.2672	.2714	.2746	.2771	.2792	.2823	.2849	.2868
5.15	.2266	.2377	.2509	.2590	.2646	.2687	.2719	.2744	.2765	.2796	.2822	.2840
5.20	.2244	.2354	.2485	.2565	.2620	.2662	.2693	.2718	.2738	.2769	.2795	.2813
5.25	.2223	.2332	.2461	.2540	.2595	.2636	.2667	.2692	.2712	.2742	.2768	.2786
5.30	.2202	.2310	.2438	.2516	.2571	.2611	.2642	.2667	.2687	.2716	.2742	.2760
5.35	.2181	.2288	.2415	.2493	.2547	.2587	.2618	.2642	.2661	.2691	.2716	.2734
5.40	.2161	.2267	.2393	.2470	.2523	.2563	.2593	.2617	.2637	.2666	.2691	.2709
5.45	.2141	.2246	.2371	.2447	.2500	.2539	.2570	.2593	.2613	.2642	.2666	.2684
5.50	.2122	.2226	.2349	.2425	.2477	.2516	.2546	.2570	.2589	.2618	.2642	.2659
5.55	.2103	.2206	.2328	.2403	.2455	.2494	.2523	.2547	.2566	.2594	.2618	.2635
5.60	.2084	.2186	.2307	.2382	.2433	.2471	.2501	.2524	.2543	.2571	.2595	.2612
5.65	.2065	.2167	.2287	.2360	.2412	.2450	.2479	.2502	.2520	.2548	.2572	.2589
5.70	.2047	.2148	.2267	.2340	.2391	.2428	.2457	.2480	.2498	.2526	.2550	.2566
5.75	.2030	.2129	.2247	.2319	.2370	.2407	.2435	.2458	.2476	.2504	.2527	.2544
5.80	.2012	.2111	.2228	.2299	.2349	.2386	.2414	.2437	.2455	.2482	.2506	.2522
5.85	.1995	.2093	.2209	.2280	.2329	.2366	.2394	.2416	.2434	.2461	.2484	.2500
5.90	.1978	.2075	.2190	.2260	.2309	.2346	.2374	.2396	.2413	.2440	.2463	.2479
5.95	.1961	.2058	.2171	.2241	.2290	.2326	.2354	.2375	.2393	.2420	.2442	.2458
6.00	.1945	.2041	.2153	.2223	.2271	.2307	.2334	.2356	.2373	.2399	.2422	.2438
6.05	.1929	.2024	.2136	.2204	.2252	.2288	.2315	.2336	.2354	.2380	.2402	.2418
6.10	.1913	.2007	.2118	.2186	.2234	.2269	.2296	.2317	.2334	.2360	.2382	.2398
6.15	.1898	.1991	.2101	.2169	.2216	.2250	.2277	.2298	.2315	.2341	.2363	.2378
6.20	.1882	.1975	.2084	.2151	.2198	.2232	.2259	.2280	.2297	.2322	.2344	.2359
6.25	.1867	.1959	.2067	.2134	.2180	.2214	.2241	.2261	.2278	.2304	.2325	.2340
6.30	.1852	.1943	.2051	.2117	.2163	.2197	.2223	.2243	.2260	.2285	.2307	.2322
6.35	.1838	.1928	.2035	.2100	.2146	.2179	.2205	.2226	.2242	.2267	.2289	.2303
6.40	.1823	.1913	.2019	.2084	.2129	.2162	.2188	.2208	.2225	.2250	.2271	.2285
6.45	.1809	.1898	.2003	.2068	.2113	.2146	.2171	.2191	.2208	.2232	.2253	.2268
6.50	.1795	.1884	.1988	.2052	.2096	.2129	.2154	.2174	.2191	.2215	.2236	.2250
6.55	.1782	.1869	.1972	.2036	.2080	.2113	.2138	.2158	.2174	.2198	.2219	.2233
6.60	.1768	.1855	.1958	.2021	.2065	.2097	.2122	.2142	.2157	.2181	.2202	.2216
6.65	.1755	.1841	.1943	.2006	.2049	.2081	.2106	.2125	.2141	.2165	.2185	.2199
6.70	.1742	.1827	.1928	.1991	.2034	.2066	.2090	.2110	.2125	.2149	.2169	.2183
6.75	.1729	.1814	.1914	.1976	.2019	.2050	.2075	.2094	.2109	.2133	.2153	.2167

6.80	.1716	.1800	.1900	.1961	.2004	.2035	.2059	.2079	.2094	.2117	.2137	.2151
6.85	.1704	.1787	.1886	.1947	.1989	.2020	.2044	.2063	.2079	.2102	.2122	.2135
6.90	.1691	.1774	.1872	.1933	.1975	.2006	.2030	.2048	.2064	.2087	.2106	.2120
6.95	.1679	.1762	.1859	.1919	.1961	.1991	.2015	.2034	.2049	.2072	.2091	.2104
7.00	.1667	.1749	.1846	.1905	.1947	.1977	.2001	.2019	.2034	.2057	.2076	.2089
7.05	.1655	.1737	.1833	.1892	.1933	.1963	.1986	.2005	.2020	.2042	.2061	.2075
7.10	.1644	.1724	.1820	.1878	.1919	.1949	.1972	.1991	.2005	.2028	.2047	.2060
7.15	.1632	.1712	.1807	.1865	.1906	.1936	.1959	.1977	.1991	.2014	.2032	.2046
7.20	.1621	.1700	.1794	.1852	.1893	.1922	.1945	.1963	.1978	.2000	.2018	.2031
7.25	.1610	.1689	.1782	.1840	.1879	.1909	.1932	.1950	.1964	.1986	.2004	.2017
7.30	.1599	.1677	.1770	.1827	.1867	.1896	.1918	.1936	.1951	.1972	.1991	.2004
7.35	.1588	.1666	.1758	.1815	.1854	.1883	.1905	.1923	.1937	.1959	.1977	.1990
7.40	.1577	.1655	.1746	.1802	.1841	.1870	.1892	.1910	.1924	.1946	.1964	.1976
7.45	.1566	.1643	.1734	.1790	.1829	.1858	.1880	.1897	.1911	.1932	.1951	.1963
7.50	.1556	.1632	.1723	.1778	.1817	.1845	.1867	.1885	.1899	.1920	.1938	.1950

It is informative to use the Laguerre method (See Chapter 3) to extract the roots from Eq. 44.

Program RECRECT1.FOR does this and then prints out the 3 roots.

Program RECRECT1.FOR

```
      PARAMETER (ND=3,EPS=1.E-5)
      REAL P(6),BP1(6)/1.125,1.25,1.5,1.75,2.0,2.25/
      COMPLEX C(ND+1),ROOTS(ND),AD(ND+1),Z1,Z2,Z3
      EPS1=2.*EPS*EPS
      DO 2 I=1,6
2     P(I)=.5/BP1(I)**2
      DO 90 I=1,91
      BP=.5+.05*FLOAT(I-1)
      C(4)=CMPLX(1.,0.)
      C(3)=CMPLX(-1.5/BP,0.)
      C(2)=CMPLX(0.,0.)
      DO 85 K=1,6
      C(1)=CMPLX(P(K)/BP**3,0.)
      DO 20 J=1,ND+1
20    AD(J)=C(J)
      DO 30 J=ND,1,-1
      Z1=CMPLX(0.,0.)
      CALL LAGU(AD,J,Z1,EPS)
      IF(ABS(AIMAG(Z1)).LE.EPS1*ABS(REAL(Z1))) Z1=CMPLX(REAL(Z1),0.)
      ROOTS(J)=Z1
      Z2=AD(J+1)
      DO 30 JJ=J,1,-1
      Z3=AD(JJ)
      AD(JJ)=Z2
30    Z2=Z1*Z2+Z3
      DO 50 J=2,ND
      Z1=ROOTS(J)
      DO 40 I1=J-1,1,-1
      IF(REAL(ROOTS(I1)).LE.REAL(Z1)) GO TO 50
40    ROOTS(I1+1)=ROOTS(I1)
      I1=0
50    ROOTS(I1+1)=Z1
85    WRITE(3,70) BP,BP1(K),ROOTS
90    CONTINUE
70    FORMAT(2F5.2,6(F8.4,F8.4,1X))
      END
```

Output:

b'	b_1'	root # 1		root # 2		root # 3	
.50	1.125	-.9002	.0000	1.4098	.0000	2.4904	.0000
.50	1.25	-.8188	.0000	1.1889	.0000	2.6298	.0000
.50	1.50	-.6938	.0000	.9258	.0000	2.7680	.0000
.50	1.75	-.6022	.0000	.7643	.0000	2.8378	.0000
.50	2.00	-.5321	.0000	.6527	.0000	2.8794	.0000

.50	2.25	-.4767	.0000	.5703	.0000	2.9065	.0000
.55	1.125	-.8184	.0000	1.2816	.0000	2.2640	.0000
.55	1.25	-.7443	.0000	1.0808	.0000	2.3908	.0000
.55	1.50	-.6307	.0000	.8416	.0000	2.5163	.0000
.55	1.75	-.5474	.0000	.6949	.0000	2.5798	.0000
.55	2.00	-.4837	.0000	.5934	.0000	2.6176	.0000
.55	2.25	-.4334	.0000	.5184	.0000	2.6422	.0000
.60	1.125	-.7502	.0000	1.1748	.0000	2.0754	.0000
.60	1.25	-.6823	.0000	.9908	.0000	2.1915	.0000
.60	1.50	-.5781	.0000	.7715	.0000	2.3066	.0000
.60	1.75	-.5018	.0000	.6370	.0000	2.3648	.0000
.60	2.00	-.4434	.0000	.5439	.0000	2.3995	.0000
.60	2.25	-.3973	.0000	.4752	.0000	2.4221	.0000
.65	1.125	-.6925	.0000	1.0844	.0000	1.9157	.0000
.65	1.25	-.6298	.0000	.9145	.0000	2.0230	.0000
.65	1.50	-.5337	.0000	.7121	.0000	2.1292	.0000
.65	1.75	-.4632	.0000	.5880	.0000	2.1829	.0000
.65	2.00	-.4093	.0000	.5021	.0000	2.2149	.0000
.65	2.25	-.3667	.0000	.4387	.0000	2.2357	.0000
.70	1.125	-.6430	.0000	1.0070	.0000	1.7789	.0000
.70	1.25	-.5848	.0000	.8492	.0000	1.8785	.0000
.70	1.50	-.4955	.0000	.6613	.0000	1.9771	.0000
.70	1.75	-.4301	.0000	.5460	.0000	2.0270	.0000
.70	2.00	-.3801	.0000	.4662	.0000	2.0567	.0000
.70	2.25	-.3405	.0000	.4073	.0000	2.0760	.0000
.75	1.125	-.6001	.0000	.9398	.0000	1.6603	.0000
.75	1.25	-.5458	.0000	.7926	.0000	1.7532	.0000
.75	1.50	-.4625	.0000	.6172	.0000	1.8453	.0000
.75	1.75	-.4014	.0000	.5096	.0000	1.8919	.0000
.75	2.00	-.3547	.0000	.4351	.0000	1.9196	.0000
.75	2.25	-.3178	.0000	.3802	.0000	1.9376	.0000
.80	1.125	-.5626	.0000	.8811	.0000	1.5565	.0000
.80	1.25	-.5117	.0000	.7431	.0000	1.6437	.0000
.80	1.50	-.4336	.0000	.5786	.0000	1.7300	.0000
.80	1.75	-.3763	.0000	.4777	.0000	1.7736	.0000
.80	2.00	-.3326	.0000	.4079	.0000	1.7996	.0000
.80	2.25	-.2979	.0000	.3564	.0000	1.8165	.0000
.85	1.125	-.5295	.0000	.8293	.0000	1.4650	.0000
.85	1.25	-.4816	.0000	.6994	.0000	1.5470	.0000
.85	1.50	-.4081	.0000	.5446	.0000	1.6282	.0000
.85	1.75	-.3542	.0000	.4496	.0000	1.6693	.0000
.85	2.00	-.3130	.0000	.3839	.0000	1.6938	.0000
.85	2.25	-.2804	.0000	.3354	.0000	1.7097	.0000
.90	1.125	-.5001	.0000	.7832	.0000	1.3836	.0000
.90	1.25	-.4549	.0000	.6605	.0000	1.4610	.0000
.90	1.50	-.3854	.0000	.5143	.0000	1.5378	.0000
.90	1.75	-.3345	.0000	.4246	.0000	1.5766	.0000
.90	2.00	-.2956	.0000	.3626	.0000	1.5997	.0000
.90	2.25	-.2648	.0000	.3168	.0000	1.6147	.0000
.95	1.125	-.4738	.0000	.7420	.0000	1.3108	.0000
.95	1.25	-.4309	.0000	.6257	.0000	1.3841	.0000
.95	1.50	-.3651	.0000	.4873	.0000	1.4568	.0000
.95	1.75	-.3169	.0000	.4023	.0000	1.4936	.0000
.95	2.00	-.2800	.0000	.3435	.0000	1.5155	.0000
.95	2.25	-.2509	.0000	.3001	.0000	1.5297	.0000
1.00	1.125	-.4501	.0000	.7049	.0000	1.2452	.0000
1.00	1.25	-.4094	.0000	.5945	.0000	1.3149	.0000
1.00	1.50	-.3469	.0000	.4629	.0000	1.3840	.0000
1.00	1.75	-.3011	.0000	.3822	.0000	1.4189	.0000
1.00	2.00	-.2660	.0000	.3264	.0000	1.4397	.0000
1.00	2.25	-.2384	.0000	.2851	.0000	1.4532	.0000
1.05	1.125	-.4287	.0000	.6713	.0000	1.1859	.0000
1.05	1.25	-.3899	.0000	.5661	.0000	1.2523	.0000

1.05	1.50	-.3304	.0000	.4409	.0000	1.3181	.0000
1.05	1.75	-.2867	.0000	.3640	.0000	1.3513	.0000
1.05	2.00	-.2534	.0000	.3108	.0000	1.3711	.0000
1.05	2.25	-.2270	.0000	.2715	.0000	1.3840	.0000
1.10	1.125	-.4092	.0000	.6408	.0000	1.1320	.0000
1.10	1.25	-.3722	.0000	.5404	.0000	1.1954	.0000
1.10	1.50	-.3153	.0000	.4208	.0000	1.2582	.0000
1.10	1.75	-.2737	.0000	.3474	.0000	1.2899	.0000
1.10	2.00	-.2419	.0000	.2967	.0000	1.3088	.0000
1.10	2.25	-.2167	.0000	.2592	.0000	1.3211	.0000
1.15	1.125	-.3914	.0000	.6129	.0000	1.0828	.0000
1.15	1.25	-.3560	.0000	.5169	.0000	1.1434	.0000
1.15	1.50	-.3016	.0000	.4025	.0000	1.2035	.0000
1.15	1.75	-.2618	.0000	.3323	.0000	1.2338	.0000
1.15	2.00	-.2313	.0000	.2838	.0000	1.2519	.0000
1.15	2.25	-.2073	.0000	.2479	.0000	1.2637	.0000
1.20	1.125	-.3751	.0000	.5874	.0000	1.0377	.0000
1.20	1.25	-.3412	.0000	.4954	.0000	1.0958	.0000
1.20	1.50	-.2891	.0000	.3857	.0000	1.1533	.0000
1.20	1.75	-.2509	.0000	.3185	.0000	1.1824	.0000
1.20	2.00	-.2217	.0000	.2720	.0000	1.1997	.0000
1.20	2.25	-.1986	.0000	.2376	.0000	1.2110	.0000
1.25	1.125	-.3601	.0000	.5639	.0000	.9962	.0000
1.25	1.25	-.3275	.0000	.4756	.0000	1.0519	.0000
1.25	1.50	-.2775	.0000	.3703	.0000	1.1072	.0000
1.25	1.75	-.2409	.0000	.3057	.0000	1.1351	.0000
1.25	2.00	-.2128	.0000	.2611	.0000	1.1518	.0000
1.25	2.25	-.1907	.0000	.2281	.0000	1.1626	.0000
1.30	1.125	-.3462	.0000	.5422	.0000	.9579	.0000
1.30	1.25	-.3149	.0000	.4573	.0000	1.0115	.0000
1.30	1.50	-.2668	.0000	.3561	.0000	1.0646	.0000
1.30	1.75	-.2316	.0000	.2940	.0000	1.0915	.0000
1.30	2.00	-.2046	.0000	.2510	.0000	1.1075	.0000
1.30	2.25	-.1834	.0000	.2193	.0000	1.1179	.0000
1.35	1.125	-.3334	.0000	.5221	.0000	.9224	.0000
1.35	1.25	-.3032	.0000	.4403	.0000	.9740	.0000
1.35	1.50	-.2569	.0000	.3429	.0000	1.0252	.0000
1.35	1.75	-.2230	.0000	.2831	.0000	1.0510	.0000
1.35	2.00	-.1971	.0000	.2417	.0000	1.0664	.0000
1.35	2.25	-.1766	.0000	.2112	.0000	1.0765	.0000
1.40	1.125	-.3215	.0000	.5035	.0000	.8894	.0000
1.40	1.25	-.2924	.0000	.4246	.0000	.9392	.0000
1.40	1.50	-.2478	.0000	.3306	.0000	.9886	.0000
1.40	1.75	-.2151	.0000	.2730	.0000	1.0135	.0000
1.40	2.00	-.1900	.0000	.2331	.0000	1.0284	.0000
1.40	2.25	-.1703	.0000	.2037	.0000	1.0380	.0000
1.45	1.125	-.3104	.0000	.4861	.0000	.8588	.0000
1.45	1.25	-.2823	.0000	.4100	.0000	.9068	.0000
1.45	1.50	-.2392	.0000	.3192	.0000	.9545	.0000
1.45	1.75	-.2076	.0000	.2636	.0000	.9786	.0000
1.45	2.00	-.1835	.0000	.2251	.0000	.9929	.0000
1.45	2.25	-.1644	.0000	.1966	.0000	1.0022	.0000
1.50	1.125	-.3001	.0000	.4699	.0000	.8301	.0000
1.50	1.25	-.2729	.0000	.3963	.0000	.8766	.0000
1.50	1.50	-.2313	.0000	.3086	.0000	.9227	.0000
1.50	1.75	-.2007	.0000	.2548	.0000	.9459	.0000
1.50	2.00	-.1774	.0000	.2176	.0000	.9598	.0000
1.50	2.25	-.1589	.0000	.1901	.0000	.9688	.0000
1.55	1.125	-.2904	.0000	.4548	.0000	.8034	.0000
1.55	1.25	-.2641	.0000	.3835	.0000	.8483	.0000
1.55	1.50	-.2238	.0000	.2986	.0000	.8929	.0000
1.55	1.75	-.1942	.0000	.2466	.0000	.9154	.0000
1.55	2.00	-.1716	.0000	.2105	.0000	.9288	.0000

1.55	2.25	-.1538	.0000	.1840	.0000	.9376	.0000
1.60	1.125	-.2813	.0000	.4406	.0000	.7783	.0000
1.60	1.25	-.2559	.0000	.3715	.0000	.8218	.0000
1.60	1.50	-.2168	.0000	.2893	.0000	.8650	.0000
1.60	1.75	-.1882	.0000	.2389	.0000	.8868	.0000
1.60	2.00	-.1663	.0000	.2040	.0000	.8998	.0000
1.60	2.25	-.1490	.0000	.1782	.0000	.9083	.0000
1.65	1.125	-.2728	.0000	.4272	.0000	.7547	.0000
1.65	1.25	-.2481	.0000	.3603	.0000	.7969	.0000
1.65	1.50	-.2102	.0000	.2805	.0000	.8388	.0000
1.65	1.75	-.1825	.0000	.2316	.0000	.8599	.0000
1.65	2.00	-.1612	.0000	.1978	.0000	.8725	.0000
1.65	2.25	-.1445	.0000	.1728	.0000	.8807	.0000
1.70	1.125	-.2648	.0000	.4146	.0000	.7325	.0000
1.70	1.25	-.2408	.0000	.3497	.0000	.7735	.0000
1.70	1.50	-.2040	.0000	.2723	.0000	.8141	.0000
1.70	1.75	-.1771	.0000	.2248	.0000	.8347	.0000
1.70	2.00	-.1565	.0000	.1920	.0000	.8469	.0000
1.70	2.25	-.1402	.0000	.1677	.0000	.8548	.0000
1.75	1.125	-.2572	.0000	.4028	.0000	.7116	.0000
1.75	1.25	-.2339	.0000	.3397	.0000	.7514	.0000
1.75	1.50	-.1982	.0000	.2645	.0000	.7908	.0000
1.75	1.75	-.1720	.0000	.2184	.0000	.8108	.0000
1.75	2.00	-.1520	.0000	.1865	.0000	.8227	.0000
1.75	2.25	-.1362	.0000	.1629	.0000	.8304	.0000
1.80	1.125	-.2501	.0000	.3916	.0000	.6918	.0000
1.80	1.25	-.2274	.0000	.3303	.0000	.7305	.0000
1.80	1.50	-.1927	.0000	.2572	.0000	.7689	.0000
1.80	1.75	-.1673	.0000	.2123	.0000	.7883	.0000
1.80	2.00	-.1478	.0000	.1813	.0000	.7998	.0000
1.80	2.25	-.1324	.0000	.1584	.0000	.8074	.0000
1.85	1.125	-.2433	.0000	.3810	.0000	.6731	.0000
1.85	1.25	-.2213	.0000	.3213	.0000	.7108	.0000
1.85	1.50	-.1875	.0000	.2502	.0000	.7481	.0000
1.85	1.75	-.1627	.0000	.2066	.0000	.7670	.0000
1.85	2.00	-.1438	.0000	.1764	.0000	.7782	.0000
1.85	2.25	-.1288	.0000	.1541	.0000	.7855	.0000
1.90	1.125	-.2369	.0000	.3710	.0000	.6554	.0000
1.90	1.25	-.2155	.0000	.3129	.0000	.6921	.0000
1.90	1.50	-.1826	.0000	.2436	.0000	.7284	.0000
1.90	1.75	-.1585	.0000	.2011	.0000	.7468	.0000
1.90	2.00	-.1400	.0000	.1718	.0000	.7577	.0000
1.90	2.25	-.1255	.0000	.1501	.0000	.7649	.0000
1.95	1.125	-.2308	.0000	.3615	.0000	.6386	.0000
1.95	1.25	-.2099	.0000	.3048	.0000	.6743	.0000
1.95	1.50	-.1779	.0000	.2374	.0000	.7097	.0000
1.95	1.75	-.1544	.0000	.1960	.0000	.7276	.0000
1.95	2.00	-.1364	.0000	.1674	.0000	.7383	.0000
1.95	2.25	-.1222	.0000	.1462	.0000	.7452	.0000
2.00	1.125	-.2250	.0000	.3524	.0000	.6226	.0000
2.00	1.25	-.2047	.0000	.2972	.0000	.6575	.0000
2.00	1.50	-.1734	.0000	.2314	.0000	.6920	.0000
2.00	1.75	-.1505	.0000	.1911	.0000	.7095	.0000
2.00	2.00	-.1330	.0000	.1632	.0000	.7198	.0000
2.00	2.25	-.1192	.0000	.1426	.0000	.7266	.0000
2.05	1.125	-.2196	.0000	.3438	.0000	.6074	.0000
2.05	1.25	-.1997	.0000	.2900	.0000	.6414	.0000
2.05	1.50	-.1692	.0000	.2258	.0000	.6751	.0000
2.05	1.75	-.1469	.0000	.1864	.0000	.6921	.0000
2.05	2.00	-.1298	.0000	.1592	.0000	.7023	.0000
2.05	2.25	-.1163	.0000	.1391	.0000	.7089	.0000
2.10	1.125	-.2143	.0000	.3357	.0000	.5930	.0000
2.10	1.25	-.1949	.0000	.2831	.0000	.6262	.0000

2.10	1.50	-.1652	.0000	.2204	.0000	.6590	.0000
2.10	1.75	-.1434	.0000	.1820	.0000	.6757	.0000
2.10	2.00	-.1267	.0000	.1554	.0000	.6856	.0000
2.10	2.25	-.1135	.0000	.1358	.0000	.6920	.0000
2.15	1.125	-.2093	.0000	.3279	.0000	.5792	.0000
2.15	1.25	-.1904	.0000	.2765	.0000	.6116	.0000
2.15	1.50	-.1613	.0000	.2153	.0000	.6437	.0000
2.15	1.75	-.1400	.0000	.1778	.0000	.6600	.0000
2.15	2.00	-.1237	.0000	.1518	.0000	.6696	.0000
2.15	2.25	-.1109	.0000	.1326	.0000	.6759	.0000
2.20	1.125	-.2046	.0000	.3204	.0000	.5660	.0000
2.20	1.25	-.1861	.0000	.2702	.0000	.5977	.0000
2.20	1.50	-.1577	.0000	.2104	.0000	.6291	.0000
2.20	1.75	-.1369	.0000	.1737	.0000	.6450	.0000
2.20	2.00	-.1209	.0000	.1483	.0000	.6544	.0000
2.20	2.25	-.1083	.0000	.1296	.0000	.6606	.0000
2.25	1.125	-.2000	.0000	.3133	.0000	.5534	.0000
2.25	1.25	-.1819	.0000	.2642	.0000	.5844	.0000
2.25	1.50	-.1542	.0000	.2057	.0000	.6151	.0000
2.25	1.75	-.1338	.0000	.1699	.0000	.6306	.0000
2.25	2.00	-.1182	.0000	.1450	.0000	.6399	.0000
2.25	2.25	-.1059	.0000	.1267	.0000	.6459	.0000
2.30	1.125	-.1957	.0000	.3065	.0000	.5414	.0000
2.30	1.25	-.1780	.0000	.2585	.0000	.5717	.0000
2.30	1.50	-.1508	.0000	.2013	.0000	.6017	.0000
2.30	1.75	-.1309	.0000	.1662	.0000	.6169	.0000
2.30	2.00	-.1157	.0000	.1419	.0000	.6260	.0000
2.30	2.25	-.1036	.0000	.1240	.0000	.6318	.0000
2.35	1.125	-.1915	.0000	.3000	.0000	.5299	.0000
2.35	1.25	-.1742	.0000	.2530	.0000	.5595	.0000
2.35	1.50	-.1476	.0000	.1970	.0000	.5889	.0000
2.35	1.75	-.1281	.0000	.1626	.0000	.6038	.0000
2.35	2.00	-.1132	.0000	.1389	.0000	.6126	.0000
2.35	2.25	-.1014	.0000	.1213	.0000	.6184	.0000
2.40	1.125	-.1875	.0000	.2937	.0000	.5188	.0000
2.40	1.25	-.1706	.0000	.2477	.0000	.5479	.0000
2.40	1.50	-.1445	.0000	.1929	.0000	.5767	.0000
2.40	1.75	-.1254	.0000	.1592	.0000	.5912	.0000
2.40	2.00	-.1109	.0000	.1360	.0000	.5999	.0000
2.40	2.25	-.0993	.0000	.1188	.0000	.6055	.0000
2.45	1.125	-.1837	.0000	.2877	.0000	.5083	.0000
2.45	1.25	-.1671	.0000	.2426	.0000	.5367	.0000
2.45	1.50	-.1416	.0000	.1889	.0000	.5649	.0000
2.45	1.75	-.1229	.0000	.1560	.0000	.5791	.0000
2.45	2.00	-.1086	.0000	.1332	.0000	.5876	.0000
2.45	2.25	-.0973	.0000	.1164	.0000	.5932	.0000
2.50	1.125	-.1800	.0000	.2820	.0000	.4981	.0000
2.50	1.25	-.1638	.0000	.2378	.0000	.5260	.0000
2.50	1.50	-.1388	.0000	.1852	.0000	.5536	.0000
2.50	1.75	-.1204	.0000	.1529	.0000	.5676	.0000
2.50	2.00	-.1064	.0000	.1305	.0000	.5759	.0000
2.50	2.25	-.0953	.0000	.1141	.0000	.5813	.0000
2.55	1.125	-.1765	.0000	.2764	.0000	.4883	.0000
2.55	1.25	-.1605	.0000	.2331	.0000	.5157	.0000
2.55	1.50	-.1360	.0000	.1815	.0000	.5427	.0000
2.55	1.75	-.1181	.0000	.1499	.0000	.5564	.0000
2.55	2.00	-.1043	.0000	.1280	.0000	.5646	.0000
2.55	2.25	-.0935	.0000	.1118	.0000	.5699	.0000
2.60	1.125	-.1731	.0000	.2711	.0000	.4789	.0000
2.60	1.25	-.1575	.0000	.2286	.0000	.5057	.0000
2.60	1.50	-.1334	.0000	.1780	.0000	.5323	.0000
2.60	1.75	-.1158	.0000	.1470	.0000	.5457	.0000
2.60	2.00	-.1023	.0000	.1255	.0000	.5537	.0000

2.60	2.25	-.0917	.0000	.1097	.0000	.5589	.0000
2.65	1.125	-.1698	.0000	.2660	.0000	.4699	.0000
2.65	1.25	-.1545	.0000	.2243	.0000	.4962	.0000
2.65	1.50	-.1309	.0000	.1747	.0000	.5223	.0000
2.65	1.75	-.1136	.0000	.1442	.0000	.5354	.0000
2.65	2.00	-.1004	.0000	.1232	.0000	.5433	.0000
2.65	2.25	-.0899	.0000	.1076	.0000	.5484	.0000
2.70	1.125	-.1667	.0000	.2611	.0000	.4612	.0000
2.70	1.25	-.1516	.0000	.2202	.0000	.4870	.0000
2.70	1.50	-.1285	.0000	.1714	.0000	.5126	.0000
2.70	1.75	-.1115	.0000	.1415	.0000	.5255	.0000
2.70	2.00	-.0985	.0000	.1209	.0000	.5332	.0000
2.70	2.25	-.0883	.0000	.1056	.0000	.5382	.0000
2.75	1.125	-.1637	.0000	.2563	.0000	.4528	.0000
2.75	1.25	-.1489	.0000	.2162	.0000	.4782	.0000
2.75	1.50	-.1261	.0000	.1683	.0000	.5033	.0000
2.75	1.75	-.1095	.0000	.1390	.0000	.5160	.0000
2.75	2.00	-.0967	.0000	.1187	.0000	.5235	.0000
2.75	2.25	-.0867	.0000	.1037	.0000	.5284	.0000
2.80	1.125	-.1607	.0000	.2517	.0000	.4447	.0000
2.80	1.25	-.1462	.0000	.2123	.0000	.4696	.0000
2.80	1.50	-.1239	.0000	.1653	.0000	.4943	.0000
2.80	1.75	-.1075	.0000	.1365	.0000	.5068	.0000
2.80	2.00	-.0950	.0000	.1166	.0000	.5142	.0000
2.80	2.25	-.0851	.0000	.1018	.0000	.5190	.0000
2.85	1.125	-.1579	.0000	.2473	.0000	.4369	.0000
2.85	1.25	-.1436	.0000	.2086	.0000	.4614	.0000
2.85	1.50	-.1217	.0000	.1624	.0000	.4856	.0000
2.85	1.75	-.1056	.0000	.1341	.0000	.4979	.0000
2.85	2.00	-.0933	.0000	.1145	.0000	.5052	.0000
2.85	2.25	-.0836	.0000	.1000	.0000	.5099	.0000
2.90	1.125	-.1552	.0000	.2431	.0000	.4294	.0000
2.90	1.25	-.1412	.0000	.2050	.0000	.4534	.0000
2.90	1.50	-.1196	.0000	.1596	.0000	.4772	.0000
2.90	1.75	-.1038	.0000	.1318	.0000	.4893	.0000
2.90	2.00	-.0917	.0000	.1125	.0000	.4964	.0000
2.90	2.25	-.0822	.0000	.0983	.0000	.5011	.0000
2.95	1.125	-.1526	.0000	.2389	.0000	.4221	.0000
2.95	1.25	-.1388	.0000	.2015	.0000	.4457	.0000
2.95	1.50	-.1176	.0000	.1569	.0000	.4691	.0000
2.95	1.75	-.1021	.0000	.1296	.0000	.4810	.0000
2.95	2.00	-.0902	.0000	.1106	.0000	.4880	.0000
2.95	2.25	-.0808	.0000	.0967	.0000	.4926	.0000
3.00	1.125	-.1500	.0000	.2350	.0000	.4151	.0000
3.00	1.25	-.1365	.0000	.1982	.0000	.4383	.0000
3.00	1.50	-.1156	.0000	.1543	.0000	.4613	.0000
3.00	1.75	-.1004	.0000	.1274	.0000	.4730	.0000
3.00	2.00	-.0887	.0000	.1088	.0000	.4799	.0000
3.00	2.25	-.0795	.0000	.0950	.0000	.4844	.0000
3.05	1.125	-.1476	.0000	.2311	.0000	.4083	.0000
3.05	1.25	-.1342	.0000	.1949	.0000	.4311	.0000
3.05	1.50	-.1137	.0000	.1518	.0000	.4538	.0000
3.05	1.75	-.0987	.0000	.1253	.0000	.4652	.0000
3.05	2.00	-.0872	.0000	.1070	.0000	.4720	.0000
3.05	2.25	-.0782	.0000	.0935	.0000	.4765	.0000
3.10	1.125	-.1452	.0000	.2274	.0000	.4017	.0000
3.10	1.25	-.1321	.0000	.1918	.0000	.4242	.0000
3.10	1.50	-.1119	.0000	.1493	.0000	.4464	.0000
3.10	1.75	-.0971	.0000	.1233	.0000	.4577	.0000
3.10	2.00	-.0858	.0000	.1053	.0000	.4644	.0000
3.10	2.25	-.0769	.0000	.0920	.0000	.4688	.0000
3.15	1.125	-.1429	.0000	.2238	.0000	.3953	.0000
3.15	1.25	-.1300	.0000	.1887	.0000	.4174	.0000

3.15	1.50	-.1101	.0000	.1470	.0000	.4394	.0000
3.15	1.75	-.0956	.0000	.1213	.0000	.4504	.0000
3.15	2.00	-.0845	.0000	.1036	.0000	.4570	.0000
3.15	2.25	-.0757	.0000	.0905	.0000	.4613	.0000
3.20	1.125	-.1407	.0000	.2203	.0000	.3891	.0000
3.20	1.25	-.1279	.0000	.1858	.0000	.4109	.0000
3.20	1.50	-.1084	.0000	.1447	.0000	.4325	.0000
3.20	1.75	-.0941	.0000	.1194	.0000	.4434	.0000
3.20	2.00	-.0831	.0000	.1020	.0000	.4499	.0000
3.20	2.25	-.0745	.0000	.0891	.0000	.4541	.0000
3.25	1.125	-.1385	.0000	.2169	.0000	.3831	.0000
3.25	1.25	-.1260	.0000	.1829	.0000	.4046	.0000
3.25	1.50	-.1067	.0000	.1424	.0000	.4258	.0000
3.25	1.75	-.0926	.0000	.1176	.0000	.4366	.0000
3.25	2.00	-.0819	.0000	.1004	.0000	.4430	.0000
3.25	2.25	-.0733	.0000	.0877	.0000	.4471	.0000
3.30	1.125	-.1364	.0000	.2136	.0000	.3773	.0000
3.30	1.25	-.1241	.0000	.1801	.0000	.3985	.0000
3.30	1.50	-.1051	.0000	.1403	.0000	.4194	.0000
3.30	1.75	-.0912	.0000	.1158	.0000	.4300	.0000
3.30	2.00	-.0806	.0000	.0989	.0000	.4363	.0000
3.30	2.25	-.0722	.0000	.0864	.0000	.4404	.0000
3.35	1.125	-.1344	.0000	.2104	.0000	.3717	.0000
3.35	1.25	-.1222	.0000	.1774	.0000	.3925	.0000
3.35	1.50	-.1035	.0000	.1382	.0000	.4131	.0000
3.35	1.75	-.0899	.0000	.1141	.0000	.4236	.0000
3.35	2.00	-.0794	.0000	.0974	.0000	.4298	.0000
3.35	2.25	-.0712	.0000	.0851	.0000	.4338	.0000
3.40	1.125	-.1324	.0000	.2073	.0000	.3662	.0000
3.40	1.25	-.1204	.0000	.1748	.0000	.3867	.0000
3.40	1.50	-.1020	.0000	.1361	.0000	.4071	.0000
3.40	1.75	-.0886	.0000	.1124	.0000	.4173	.0000
3.40	2.00	-.0782	.0000	.0960	.0000	.4234	.0000
3.40	2.25	-.0701	.0000	.0839	.0000	.4274	.0000
3.45	1.125	-.1305	.0000	.2043	.0000	.3609	.0000
3.45	1.25	-.1187	.0000	.1723	.0000	.3811	.0000
3.45	1.50	-.1005	.0000	.1342	.0000	.4012	.0000
3.45	1.75	-.0873	.0000	.1108	.0000	.4113	.0000
3.45	2.00	-.0771	.0000	.0946	.0000	.4173	.0000
3.45	2.25	-.0691	.0000	.0826	.0000	.4212	.0000
3.50	1.125	-.1286	.0000	.2014	.0000	.3558	.0000
3.50	1.25	-.1170	.0000	.1698	.0000	.3757	.0000
3.50	1.50	-.0991	.0000	.1323	.0000	.3954	.0000
3.50	1.75	-.0860	.0000	.1092	.0000	.4054	.0000
3.50	2.00	-.0760	.0000	.0932	.0000	.4113	.0000
3.50	2.25	-.0681	.0000	.0815	.0000	.4152	.0000
3.55	1.125	-.1268	.0000	.1986	.0000	.3508	.0000
3.55	1.25	-.1153	.0000	.1675	.0000	.3704	.0000
3.55	1.50	-.0977	.0000	.1304	.0000	.3899	.0000
3.55	1.75	-.0848	.0000	.1077	.0000	.3997	.0000
3.55	2.00	-.0749	.0000	.0919	.0000	.4055	.0000
3.55	2.25	-.0671	.0000	.0803	.0000	.4094	.0000
3.60	1.125	-.1250	.0000	.1958	.0000	.3459	.0000
3.60	1.25	-.1137	.0000	.1651	.0000	.3653	.0000
3.60	1.50	-.0964	.0000	.1286	.0000	.3844	.0000
3.60	1.75	-.0836	.0000	.1062	.0000	.3941	.0000
3.60	2.00	-.0739	.0000	.0907	.0000	.3999	.0000
3.60	2.25	-.0662	.0000	.0792	.0000	.4037	.0000
3.65	1.125	-.1233	.0000	.1931	.0000	.3412	.0000
3.65	1.25	-.1122	.0000	.1629	.0000	.3603	.0000
3.65	1.50	-.0950	.0000	.1268	.0000	.3792	.0000
3.65	1.75	-.0825	.0000	.1047	.0000	.3887	.0000
3.65	2.00	-.0729	.0000	.0894	.0000	.3944	.0000

3.65	2.25	-.0653	.0000	.0781	.0000	.3981	.0000
3.70	1.125	-.1216	.0000	.1905	.0000	.3365	.0000
3.70	1.25	-.1106	.0000	.1607	.0000	.3554	.0000
3.70	1.50	-.0938	.0000	.1251	.0000	.3740	.0000
3.70	1.75	-.0814	.0000	.1033	.0000	.3835	.0000
3.70	2.00	-.0719	.0000	.0882	.0000	.3891	.0000
3.70	2.25	-.0644	.0000	.0771	.0000	.3928	.0000
3.75	1.125	-.1200	.0000	.1880	.0000	.3321	.0000
3.75	1.25	-.1092	.0000	.1585	.0000	.3506	.0000
3.75	1.50	-.0925	.0000	.1234	.0000	.3691	.0000
3.75	1.75	-.0803	.0000	.1019	.0000	.3784	.0000
3.75	2.00	-.0709	.0000	.0870	.0000	.3839	.0000
3.75	2.25	-.0636	.0000	.0760	.0000	.3875	.0000
3.80	1.125	-.1184	.0000	.1855	.0000	.3277	.0000
3.80	1.25	-.1077	.0000	.1564	.0000	.3460	.0000
3.80	1.50	-.0913	.0000	.1218	.0000	.3642	.0000
3.80	1.75	-.0792	.0000	.1006	.0000	.3734	.0000
3.80	2.00	-.0700	.0000	.0859	.0000	.3789	.0000
3.80	2.25	-.0627	.0000	.0750	.0000	.3824	.0000
3.85	1.125	-.1169	.0000	.1831	.0000	.3234	.0000
3.85	1.25	-.1063	.0000	.1544	.0000	.3415	.0000
3.85	1.50	-.0901	.0000	.1202	.0000	.3595	.0000
3.85	1.75	-.0782	.0000	.0993	.0000	.3685	.0000
3.85	2.00	-.0691	.0000	.0848	.0000	.3739	.0000
3.85	2.25	-.0619	.0000	.0741	.0000	.3775	.0000
3.90	1.125	-.1154	.0000	.1807	.0000	.3193	.0000
3.90	1.25	-.1050	.0000	.1524	.0000	.3372	.0000
3.90	1.50	-.0889	.0000	.1187	.0000	.3549	.0000
3.90	1.75	-.0772	.0000	.0980	.0000	.3638	.0000
3.90	2.00	-.0682	.0000	.0837	.0000	.3692	.0000
3.90	2.25	-.0611	.0000	.0731	.0000	.3726	.0000
3.95	1.125	-.1139	.0000	.1785	.0000	.3152	.0000
3.95	1.25	-.1036	.0000	.1505	.0000	.3329	.0000
3.95	1.50	-.0878	.0000	.1172	.0000	.3504	.0000
3.95	1.75	-.0762	.0000	.0968	.0000	.3592	.0000
3.95	2.00	-.0674	.0000	.0826	.0000	.3645	.0000
3.95	2.25	-.0603	.0000	.0722	.0000	.3679	.0000
4.00	1.125	-.1125	.0000	.1762	.0000	.3113	.0000
4.00	1.25	-.1023	.0000	.1486	.0000	.3287	.0000
4.00	1.50	-.0867	.0000	.1157	.0000	.3460	.0000
4.00	1.75	-.0753	.0000	.0955	.0000	.3547	.0000
4.00	2.00	-.0665	.0000	.0816	.0000	.3599	.0000
4.00	2.25	-.0596	.0000	.0713	.0000	.3633	.0000
4.05	1.125	-.1111	.0000	.1740	.0000	.3075	.0000
4.05	1.25	-.1011	.0000	.1468	.0000	.3247	.0000
4.05	1.50	-.0856	.0000	.1143	.0000	.3417	.0000
4.05	1.75	-.0743	.0000	.0944	.0000	.3503	.0000
4.05	2.00	-.0657	.0000	.0806	.0000	.3555	.0000
4.05	2.25	-.0589	.0000	.0704	.0000	.3588	.0000
4.10	1.125	-.1098	.0000	.1719	.0000	.3037	.0000
4.10	1.25	-.0998	.0000	.1450	.0000	.3207	.0000
4.10	1.50	-.0846	.0000	.1129	.0000	.3376	.0000
4.10	1.75	-.0734	.0000	.0932	.0000	.3461	.0000
4.10	2.00	-.0649	.0000	.0796	.0000	.3511	.0000
4.10	2.25	-.0581	.0000	.0695	.0000	.3544	.0000
4.15	1.125	-.1085	.0000	.1699	.0000	.3001	.0000
4.15	1.25	-.0986	.0000	.1432	.0000	.3168	.0000
4.15	1.50	-.0836	.0000	.1115	.0000	.3335	.0000
4.15	1.75	-.0725	.0000	.0921	.0000	.3419	.0000
4.15	2.00	-.0641	.0000	.0786	.0000	.3469	.0000
4.15	2.25	-.0574	.0000	.0687	.0000	.3502	.0000
4.20	1.125	-.1072	.0000	.1678	.0000	.2965	.0000
4.20	1.25	-.0975	.0000	.1415	.0000	.3131	.0000

4.20	1.50	-.0826	.0000	.1102	.0000	.3295	.0000
4.20	1.75	-.0717	.0000	.0910	.0000	.3378	.0000
4.20	2.00	-.0633	.0000	.0777	.0000	.3428	.0000
4.20	2.25	-.0568	.0000	.0679	.0000	.3460	.0000
4.25	1.125	-.1059	.0000	.1659	.0000	.2930	.0000
4.25	1.25	-.0963	.0000	.1399	.0000	.3094	.0000
4.25	1.50	-.0816	.0000	.1089	.0000	.3256	.0000
4.25	1.75	-.0708	.0000	.0899	.0000	.3339	.0000
4.25	2.00	-.0626	.0000	.0768	.0000	.3388	.0000
4.25	2.25	-.0561	.0000	.0671	.0000	.3419	.0000
4.30	1.125	-.1047	.0000	.1639	.0000	.2896	.0000
4.30	1.25	-.0952	.0000	.1382	.0000	.3058	.0000
4.30	1.50	-.0807	.0000	.1076	.0000	.3219	.0000
4.30	1.75	-.0700	.0000	.0889	.0000	.3300	.0000
4.30	2.00	-.0619	.0000	.0759	.0000	.3348	.0000
4.30	2.25	-.0554	.0000	.0663	.0000	.3380	.0000
4.35	1.125	-.1035	.0000	.1620	.0000	.2863	.0000
4.35	1.25	-.0941	.0000	.1367	.0000	.3023	.0000
4.35	1.50	-.0797	.0000	.1064	.0000	.3182	.0000
4.35	1.75	-.0692	.0000	.0879	.0000	.3262	.0000
4.35	2.00	-.0612	.0000	.0750	.0000	.3310	.0000
4.35	2.25	-.0548	.0000	.0655	.0000	.3341	.0000
4.40	1.125	-.1023	.0000	.1602	.0000	.2830	.0000
4.40	1.25	-.0930	.0000	.1351	.0000	.2988	.0000
4.40	1.50	-.0788	.0000	.1052	.0000	.3145	.0000
4.40	1.75	-.0684	.0000	.0869	.0000	.3225	.0000
4.40	2.00	-.0605	.0000	.0742	.0000	.3272	.0000
4.40	2.25	-.0542	.0000	.0648	.0000	.3303	.0000
4.45	1.125	-.1011	.0000	.1584	.0000	.2798	.0000
4.45	1.25	-.0920	.0000	.1336	.0000	.2955	.0000
4.45	1.50	-.0779	.0000	.1040	.0000	.3110	.0000
4.45	1.75	-.0677	.0000	.0859	.0000	.3189	.0000
4.45	2.00	-.0598	.0000	.0733	.0000	.3235	.0000
4.45	2.25	-.0536	.0000	.0641	.0000	.3266	.0000
4.50	1.125	-.1000	.0000	.1566	.0000	.2767	.0000
4.50	1.25	-.0910	.0000	.1321	.0000	.2922	.0000
4.50	1.50	-.0771	.0000	.1029	.0000	.3076	.0000
4.50	1.75	-.0669	.0000	.0849	.0000	.3153	.0000
4.50	2.00	-.0591	.0000	.0725	.0000	.3199	.0000
4.50	2.25	-.0530	.0000	.0634	.0000	.3229	.0000
4.55	1.125	-.0989	.0000	.1549	.0000	.2737	.0000
4.55	1.25	-.0900	.0000	.1306	.0000	.2890	.0000
4.55	1.50	-.0762	.0000	.1017	.0000	.3042	.0000
4.55	1.75	-.0662	.0000	.0840	.0000	.3118	.0000
4.55	2.00	-.0585	.0000	.0717	.0000	.3164	.0000
4.55	2.25	-.0524	.0000	.0627	.0000	.3194	.0000
4.60	1.125	-.0978	.0000	.1532	.0000	.2707	.0000
4.60	1.25	-.0890	.0000	.1292	.0000	.2859	.0000
4.60	1.50	-.0754	.0000	.1006	.0000	.3009	.0000
4.60	1.75	-.0655	.0000	.0831	.0000	.3085	.0000
4.60	2.00	-.0578	.0000	.0709	.0000	.3130	.0000
4.60	2.25	-.0518	.0000	.0620	.0000	.3159	.0000
4.65	1.125	-.0968	.0000	.1516	.0000	.2678	.0000
4.65	1.25	-.0880	.0000	.1278	.0000	.2828	.0000
4.65	1.50	-.0746	.0000	.0995	.0000	.2976	.0000
4.65	1.75	-.0647	.0000	.0822	.0000	.3051	.0000
4.65	2.00	-.0572	.0000	.0702	.0000	.3096	.0000
4.65	2.25	-.0513	.0000	.0613	.0000	.3125	.0000
4.70	1.125	-.0958	.0000	.1500	.0000	.2649	.0000
4.70	1.25	-.0871	.0000	.1265	.0000	.2798	.0000
4.70	1.50	-.0738	.0000	.0985	.0000	.2945	.0000
4.70	1.75	-.0641	.0000	.0813	.0000	.3019	.0000
4.70	2.00	-.0566	.0000	.0694	.0000	.3063	.0000

4.70	2.25	-.0507	.0000	.0607	.0000	.3092	.0000
4.75	1.125	-.0948	.0000	.1484	.0000	.2621	.0000
4.75	1.25	-.0862	.0000	.1251	.0000	.2768	.0000
4.75	1.50	-.0730	.0000	.0975	.0000	.2914	.0000
4.75	1.75	-.0634	.0000	.0805	.0000	.2987	.0000
4.75	2.00	-.0560	.0000	.0687	.0000	.3031	.0000
4.75	2.25	-.0502	.0000	.0600	.0000	.3059	.0000
4.80	1.125	-.0938	.0000	.1469	.0000	.2594	.0000
4.80	1.25	-.0853	.0000	.1238	.0000	.2739	.0000
4.80	1.50	-.0723	.0000	.0964	.0000	.2883	.0000
4.80	1.75	-.0627	.0000	.0796	.0000	.2956	.0000
4.80	2.00	-.0554	.0000	.0680	.0000	.2999	.0000
4.80	2.25	-.0497	.0000	.0594	.0000	.3028	.0000
4.85	1.125	-.0928	.0000	.1453	.0000	.2567	.0000
4.85	1.25	-.0844	.0000	.1226	.0000	.2711	.0000
4.85	1.50	-.0715	.0000	.0954	.0000	.2854	.0000
4.85	1.75	-.0621	.0000	.0788	.0000	.2926	.0000
4.85	2.00	-.0549	.0000	.0673	.0000	.2968	.0000
4.85	2.25	-.0491	.0000	.0588	.0000	.2996	.0000
4.90	1.125	-.0919	.0000	.1439	.0000	.2541	.0000
4.90	1.25	-.0835	.0000	.1213	.0000	.2684	.0000
4.90	1.50	-.0708	.0000	.0945	.0000	.2824	.0000
4.90	1.75	-.0614	.0000	.0780	.0000	.2896	.0000
4.90	2.00	-.0543	.0000	.0666	.0000	.2938	.0000
4.90	2.25	-.0486	.0000	.0582	.0000	.2966	.0000
4.95	1.125	-.0909	.0000	.1424	.0000	.2516	.0000
4.95	1.25	-.0827	.0000	.1201	.0000	.2656	.0000
4.95	1.50	-.0701	.0000	.0935	.0000	.2796	.0000
4.95	1.75	-.0608	.0000	.0772	.0000	.2866	.0000
4.95	2.00	-.0537	.0000	.0659	.0000	.2908	.0000
4.95	2.25	-.0482	.0000	.0576	.0000	.2936	.0000
5.00	1.125	-.0900	.0000	.1410	.0000	.2490	.0000
5.00	1.25	-.0819	.0000	.1189	.0000	.2630	.0000
5.00	1.50	-.0694	.0000	.0926	.0000	.2768	.0000
5.00	1.75	-.0602	.0000	.0764	.0000	.2838	.0000
5.00	2.00	-.0532	.0000	.0653	.0000	.2879	.0000
5.00	2.25	-.0477	.0000	.0570	.0000	.2906	.0000

The following program uses the Laguerre Method to extract all 3 roots from Eq. 44 and prints out the larger positive real root along with the corresponding value for the Froude Number F_{r1}. Note that F_{r1} does not change with b'.

Program RECRECT.FOR

```
      PARAMETER (ND=3,EPS=1.E-5)
      REAL YP(6),Fr1(6),P(6),PF(6),
     &BP1(6)/1.125,1.25,1.5,1.75,2.0,2.25/
      COMPLEX C(ND+1),ROOTS(ND),AD(ND+1),Z1,Z2,Z3
      EPS1=2.*EPS*EPS
      DO 2 I=1,6
      PF(I)=1./BP1(I)**2
2     P(I)=.5*PF(I)
      WRITE(3,100) BP1
100   FORMAT(' For upstream rectangular channel'/1X,82('-'),/
     &'  b''=  ',30X,'b1''=b1/b2',/' b1/Yc',F9.3,5F13.2,/1X,82('-'))
      DO 90 I=1,91
      BP=.5+.05*FLOAT(I-1)
      C(4)=CMPLX(1.,0.)
      C(3)=CMPLX(-1.5/BP,0.)
      C(2)=CMPLX(0.,0.)
      DO 85 K=1,6
      C(1)=CMPLX(P(K)/BP**3,0.)
      DO 20 J=1,ND+1
20    AD(J)=C(J)
      DO 30 J=ND,1,-1
```

```
      Z1=CMPLX(0.,0.)
      CALL LAGU(AD,J,Z1,EPS)
      IF(ABS(AIMAG(Z1)).LE.EPS1*ABS(REAL(Z1))) Z1=CMPLX(REAL(Z1),0.)
      ROOTS(J)=Z1
      Z2=AD(J+1)
      DO 30 JJ=J,1,-1
      Z3=AD(JJ)
      AD(JJ)=Z2
30    Z2=Z1*Z2+Z3
      DO 50 J=2,ND
      Z1=ROOTS(J)
      DO 40 I1=J-1,1,-1
      IF(REAL(ROOTS(I1)).LE.REAL(Z1)) GO TO 50
40    ROOTS(I1+1)=ROOTS(I1)
      I1=0
50    ROOTS(I1+1)=Z1
      IF(ABS(AIMAG(ROOTS(3))).LT. 0.00004) THEN
      YP(K)=REAL(ROOTS(3))
      Fr1(K)=SQRT(PF(K)/((BP*YP(K))**3))
      ELSE
      YP(K)=0.
      Fr1(K)=0.
      ENDIF
85    CONTINUE
90    WRITE(3,70) BP,(YP(K),Fr1(K),K=1,6)
70    FORMAT(F5.2,6(F8.4,F5.3))
      END
```

Output:

For upstream rectangular channel

b'= b1/Yc	b1'=b1/b2 1.125		1.25		1.50		1.75		2.00		2.25	
.50	2.4904	.640	2.6298	.531	2.7680	.409	2.8378	.338	2.8794	.289	2.9065	.254
.55	2.2640	.640	2.3908	.531	2.5163	.409	2.5798	.338	2.6176	.289	2.6422	.254
.60	2.0754	.640	2.1915	.531	2.3066	.409	2.3648	.338	2.3995	.289	2.4221	.254
.65	1.9157	.640	2.0230	.531	2.1292	.409	2.1829	.338	2.2149	.289	2.2357	.254
.70	1.7789	.640	1.8785	.531	1.9771	.409	2.0270	.338	2.0567	.289	2.0760	.254
.75	1.6603	.640	1.7532	.531	1.8453	.409	1.8919	.338	1.9196	.289	1.9376	.254
.80	1.5565	.640	1.6437	.531	1.7300	.409	1.7736	.338	1.7996	.289	1.8165	.254
.85	1.4650	.640	1.5470	.531	1.6282	.409	1.6693	.338	1.6938	.289	1.7097	.254
.90	1.3836	.640	1.4610	.531	1.5378	.409	1.5766	.338	1.5997	.289	1.6147	.254
.95	1.3108	.640	1.3841	.531	1.4568	.409	1.4936	.338	1.5155	.289	1.5297	.254
1.00	1.2452	.640	1.3149	.531	1.3840	.409	1.4189	.338	1.4397	.289	1.4532	.254
1.05	1.1859	.640	1.2523	.531	1.3181	.409	1.3513	.338	1.3711	.289	1.3840	.254
1.10	1.1320	.640	1.1954	.531	1.2582	.409	1.2899	.338	1.3088	.289	1.3211	.254
1.15	1.0828	.640	1.1434	.531	1.2035	.409	1.2338	.338	1.2519	.289	1.2637	.254
1.20	1.0377	.640	1.0958	.531	1.1533	.409	1.1824	.338	1.1997	.289	1.2110	.254
1.25	.9962	.640	1.0519	.531	1.1072	.409	1.1351	.338	1.1518	.289	1.1626	.254
1.30	.9579	.640	1.0115	.531	1.0646	.409	1.0915	.338	1.1075	.289	1.1179	.254
1.35	.9224	.640	.9740	.531	1.0252	.409	1.0510	.338	1.0664	.289	1.0765	.254
1.40	.8894	.640	.9392	.531	.9886	.409	1.0135	.338	1.0284	.289	1.0380	.254
1.45	.8588	.640	.9068	.531	.9545	.409	.9786	.338	.9929	.289	1.0022	.254
1.50	.8301	.640	.8766	.531	.9227	.409	.9459	.338	.9598	.289	.9688	.254
1.55	.8034	.640	.8483	.531	.8929	.409	.9154	.338	.9288	.289	.9376	.254
1.60	.7783	.640	.8218	.531	.8650	.409	.8868	.338	.8998	.289	.9083	.254
1.65	.7547	.640	.7969	.531	.8388	.409	.8599	.338	.8725	.289	.8807	.254
1.70	.7325	.640	.7735	.531	.8141	.409	.8347	.338	.8469	.289	.8548	.254
1.75	.7116	.640	.7514	.531	.7908	.409	.8108	.338	.8227	.289	.8304	.254
1.80	.6918	.640	.7305	.531	.7689	.409	.7883	.338	.7998	.289	.8074	.254
1.85	.6731	.640	.7108	.531	.7481	.409	.7670	.338	.7782	.289	.7855	.254
1.90	.6554	.640	.6921	.531	.7284	.409	.7468	.338	.7577	.289	.7649	.254
1.95	.6386	.640	.6743	.531	.7097	.409	.7276	.338	.7383	.289	.7452	.254

2.00	.6226	.640	.6575	.531	.6920	.409	.7095	.338	.7198	.289	.7266	.254
2.05	.6074	.640	.6414	.531	.6751	.409	.6921	.338	.7023	.289	.7089	.254
2.10	.5930	.640	.6262	.531	.6590	.409	.6757	.338	.6856	.289	.6920	.254
2.15	.5792	.640	.6116	.531	.6437	.409	.6600	.338	.6696	.289	.6759	.254
2.20	.5660	.640	.5977	.531	.6291	.409	.6450	.338	.6544	.289	.6606	.254
2.25	.5534	.640	.5844	.531	.6151	.409	.6306	.338	.6399	.289	.6459	.254
2.30	.5414	.640	.5717	.531	.6017	.409	.6169	.338	.6260	.289	.6318	.254
2.35	.5299	.640	.5595	.531	.5889	.409	.6038	.338	.6126	.289	.6184	.254
2.40	.5188	.640	.5479	.531	.5767	.409	.5912	.338	.5999	.289	.6055	.254
2.45	.5083	.640	.5367	.531	.5649	.409	.5791	.338	.5876	.289	.5932	.254
2.50	.4981	.640	.5260	.531	.5536	.409	.5676	.338	.5759	.289	.5813	.254
2.55	.4883	.640	.5157	.531	.5427	.409	.5564	.338	.5646	.289	.5699	.254
2.60	.4789	.640	.5057	.531	.5323	.409	.5457	.338	.5537	.289	.5589	.254
2.65	.4699	.640	.4962	.531	.5223	.409	.5354	.338	.5433	.289	.5484	.254
2.70	.4612	.640	.4870	.531	.5126	.409	.5255	.338	.5332	.289	.5382	.254
2.75	.4528	.640	.4782	.531	.5033	.409	.5160	.338	.5235	.289	.5284	.254
2.80	.4447	.640	.4696	.531	.4943	.409	.5068	.338	.5142	.289	.5190	.254
2.85	.4369	.640	.4614	.531	.4856	.409	.4979	.338	.5052	.289	.5099	.254
2.90	.4294	.640	.4534	.531	.4772	.409	.4893	.338	.4964	.289	.5011	.254
2.95	.4221	.640	.4457	.531	.4691	.409	.4810	.338	.4880	.289	.4926	.254
3.00	.4151	.640	.4383	.531	.4613	.409	.4730	.338	.4799	.289	.4844	.254
3.05	.4083	.640	.4311	.531	.4538	.409	.4652	.338	.4720	.289	.4765	.254
3.10	.4017	.640	.4242	.531	.4464	.409	.4577	.338	.4644	.289	.4688	.254
3.15	.3953	.640	.4174	.531	.4394	.409	.4504	.338	.4570	.289	.4613	.254
3.20	.3891	.640	.4109	.531	.4325	.409	.4434	.338	.4499	.289	.4541	.254
3.25	.3831	.640	.4046	.531	.4258	.409	.4366	.338	.4430	.289	.4471	.254
3.30	.3773	.640	.3985	.531	.4194	.409	.4300	.338	.4363	.289	.4404	.254
3.35	.3717	.640	.3925	.531	.4131	.409	.4236	.338	.4298	.289	.4338	.254
3.40	.3662	.640	.3867	.531	.4071	.409	.4173	.338	.4234	.289	.4274	.254
3.45	.3609	.640	.3811	.531	.4012	.409	.4113	.338	.4173	.289	.4212	.254
3.50	.3558	.640	.3757	.531	.3954	.409	.4054	.338	.4113	.289	.4152	.254
3.55	.3508	.640	.3704	.531	.3899	.409	.3997	.338	.4055	.289	.4094	.254
3.60	.3459	.640	.3653	.531	.3844	.409	.3941	.338	.3999	.289	.4037	.254
3.65	.3412	.640	.3603	.531	.3792	.409	.3887	.338	.3944	.289	.3981	.254
3.70	.3365	.640	.3554	.531	.3740	.409	.3835	.338	.3891	.289	.3928	.254
3.75	.3321	.640	.3506	.531	.3691	.409	.3784	.338	.3839	.289	.3875	.254
3.80	.3277	.640	.3460	.531	.3642	.409	.3734	.338	.3789	.289	.3824	.254
3.85	.3234	.640	.3415	.531	.3595	.409	.3685	.338	.3739	.289	.3775	.254
3.90	.3193	.640	.3372	.531	.3549	.409	.3638	.338	.3692	.289	.3726	.254
3.95	.3152	.640	.3329	.531	.3504	.409	.3592	.338	.3645	.289	.3679	.254
4.00	.3113	.640	.3287	.531	.3460	.409	.3547	.338	.3599	.289	.3633	.254
4.05	.3075	.640	.3247	.531	.3417	.409	.3503	.338	.3555	.289	.3588	.254
4.10	.3037	.640	.3207	.531	.3376	.409	.3461	.338	.3511	.289	.3544	.254
4.15	.3001	.640	.3168	.531	.3335	.409	.3419	.338	.3469	.289	.3502	.254
4.20	.2965	.640	.3131	.531	.3295	.409	.3378	.338	.3428	.289	.3460	.254
4.25	.2930	.640	.3094	.531	.3256	.409	.3339	.338	.3388	.289	.3419	.254
4.30	.2896	.640	.3058	.531	.3219	.409	.3300	.338	.3348	.289	.3380	.254
4.35	.2863	.640	.3023	.531	.3182	.409	.3262	.338	.3310	.289	.3341	.254
4.40	.2830	.640	.2988	.531	.3145	.409	.3225	.338	.3272	.289	.3303	.254
4.45	.2798	.640	.2955	.531	.3110	.409	.3189	.338	.3235	.289	.3266	.254
4.50	.2767	.640	.2922	.531	.3076	.409	.3153	.338	.3199	.289	.3229	.254
4.55	.2737	.640	.2890	.531	.3042	.409	.3118	.338	.3164	.289	.3194	.254
4.60	.2707	.640	.2859	.531	.3009	.409	.3085	.338	.3130	.289	.3159	.254
4.65	.2678	.640	.2828	.531	.2976	.409	.3051	.338	.3096	.289	.3125	.254
4.70	.2649	.640	.2798	.531	.2945	.409	.3019	.338	.3063	.289	.3092	.254
4.75	.2621	.640	.2768	.531	.2914	.409	.2987	.338	.3031	.289	.3059	.254
4.80	.2594	.640	.2739	.531	.2883	.409	.2956	.338	.2999	.289	.3028	.254
4.85	.2567	.640	.2711	.531	.2854	.409	.2926	.338	.2968	.289	.2996	.254
4.90	.2541	.640	.2684	.531	.2824	.409	.2896	.338	.2938	.289	.2966	.254
4.95	.2516	.640	.2656	.531	.2796	.409	.2866	.338	.2908	.289	.2936	.254
5.00	.2490	.640	.2630	.531	.2768	.409	.2838	.338	.2879	.289	.2906	.254

An alternative means for obtaining the first table of values to use to plot Figures 14 is to us the Laguerre Method to extract the roots. The best way to do this is to first extract the values of c (the larger positive root from the equation $c^3 - 1.5c^2 + .5/b_1' = 0$ and then use $Y_1' = c/b'$. The following program does not do this but rather extracts the roots for Y_1' from Eq. 44 and prints the larger positive real root corresponding to each b' and in columns for b_1'. Its output table is identical the table obtain from program RECRECT3.

<u>Program RECRECT2.FOR</u>

```
      PARAMETER (ND=3,EPS=1.E-5)
      REAL YP(12),P(12),
     &BP1(12)/1.05,1.1,1.2,1.3,1.4,1.5,1.6,1.7,1.8,2.0,2.25,2.5/
      COMPLEX C(ND+1),ROOTS(ND),AD(ND+1),Z1,Z2,Z3
      EPS1=2.*EPS*EPS
      DO 2 I=1,12
2     P(I)=.5/BP1(I)**2
      WRITE(3,100) BP1
100   FORMAT(' For upstream rectangular channel'/1X,88('-'),/
     &'  b''=  ',40X,'b1''=b1/b2',/' b1/Yc',F5.2,11F7.2,/1X,88('-'))
      DO 90 I=1,141
      BP=.5+.05*FLOAT(I-1)
      C(4)=CMPLX(1.,0.)
      C(3)=CMPLX(-1.5/BP,0.)
      C(2)=CMPLX(0.,0.)
      DO 85 K=1,12
      C(1)=CMPLX(P(K)/BP**3,0.)
      DO 20 J=1,ND+1
20    AD(J)=C(J)
      DO 30 J=ND,1,-1
      Z1=CMPLX(0.,0.)
      CALL LAGU(AD,J,Z1,EPS)
      IF(ABS(AIMAG(Z1)).LE.EPS1*ABS(REAL(Z1))) Z1=CMPLX(REAL(Z1),0.)
      ROOTS(J)=Z1
      Z2=AD(J+1)
      DO 30 JJ=J,1,-1
      Z3=AD(JJ)
      AD(JJ)=Z2
30    Z2=Z1*Z2+Z3
      DO 50 J=2,ND
      Z1=ROOTS(J)
      DO 40 I1=J-1,1,-1
      IF(REAL(ROOTS(I1)).LE.REAL(Z1)) GO TO 50
40    ROOTS(I1+1)=ROOTS(I1)
      I1=0
50    ROOTS(I1+1)=Z1
      IF(ABS(AIMAG(ROOTS(3))).LT. 0.00004) THEN
      YP(K)=REAL(ROOTS(3))
      ELSE
      YP(K)=0.
      ENDIF
85    CONTINUE
90    WRITE(3,70) BP,(YP(K),K=1,12)
70    FORMAT(F5.2,12F7.4)
      END
```

145.

A flowrate $Q = 11.5\ m^3/s$ occurs in a rectangular channel that reduces its width by 2/3 through a transition. The upstream channel width is $b_1 = 5$ m. Determine the upstream depth and Froude Number. If n = .014 what bottom slope of upstream channel will result in uniform flow?

This problem can be solved with model RECRECT.TK that solves Equation 43. To use this model we compute $b_2=5(2/3) = 3.33333$ m and then $Y_c=[(Q/b_2)^2/g]^{1/3} = 1.066571$ m, so $b' =b_1/Y_c = 5/1.066571 = 4.6892$.

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment-----------
               Y1p      .29522296
   4.68792     bp
   1.5         b1p
               Fr1      .40946168

==================== RULE SHEET =========================
S Rule---------------------------------------------------------
* bp*Y1p+.5/(bp*b1p*Y1p)^2=1.5
* Fr1=(3/(bp*Y1p)-2)^.5
```

Thus $Y_1=Y_1' = .295223(5) = 1.476$ m, with $F_{r1}=.409$. Solving Mannings Equation gives $S_{o1}=.000526$.

146.

What width b_2 of a steep downstream rectangular channel will result in uniform flow upstream if the upstream rectangular channel's width is b_1 = 12 ft, n = .015 and S_{o1} = .0005 for a flowrate Q = 350 cfs.

Since $b'=b_1/Y_{c2} = 12/[(Qb_1'/b_1)^2]^{1/3} = .4029195/b_1'^{2.3}$ Manning's Equation can be solve with Eq. 43 to bive b_1' and Y_1. The TK-Solver model PRB2_134.TK does this solution.

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit----- Comment---------
              bp       2.7335207
              b1p      1.7894904
              Y1p      .52052962
              Y1       6.2463555
   350        Q
   .0005      So
   .015       n

==================== RULE SHEET ==========================
S Rule----------------------------------------------------
* bp=4.029106/b1p^.666667
* bp*Y1p+.5/(bp*b1p*Y1p)^2=1.5
* Y1=12*Y1p
* Q=1.486*(12*Y1)^1.666667*So^.5/(n*(12+2*Y1)^.666667)
```

Thus the upstream dpeth Y_1=6.246 ft, and b_2=12/1.7895 = 6.706 ft.

147.

Determine the value of c in the equation $Y_1' = c/b'$ that defines the upstream dimensionless depth $Y_1' = Y_1/b_1$ as a function of the width ratio $b' = b_1/Y_{c2}$ for a transition from an upstream rectangular channel with $b_1 = 6$ ft and a downstream rectangular channel with $b_2 = 4.8$ ft. What is the upstream Froude Number?

For this problem $b_1' = b_1/b_2 = 6/4.8 = 1.25$. The value of c in $Y_1' = c/b'$ can be obtained by solving Eq. 43 for y_1' with $b'=1$. This solution gives $y_1' = c = 1.314925$. $F_{r1} = .5306$ from Eq. 45.

An alternative would be to use the table that was made for Problem 144, and multiply the b′ by Y_1' for the column for a width ratio $b_1' = 1.25$. If a column for 1.25 was not made, as is the case in Problem 144, then interpolation must be done between the columns that bracket this value. In the solution to Problem 44, columns for $b_1' = 1.2$ and 1.3 were used. Thus if the first line is used (corresponding to $b'=.5$)

$$c=(.5)\{(2.5840+2.6673)/2\} = 1.3128$$

pr of the line corresponding to $b'=1/1$ is used

$$c=(1.2920+1.3333)/2 = 1.3128$$

of if the line corresponding to $b'=2$ isused

$$c=(2)\{(.6460+.6668)/2\} = 1.3128$$

Now the relation for the dimensionless depth as a function of b′ for $b_1'=1.25$ is defined by the simple equation

$$Y_1' = 1.3128/b' = 1.3128 Y_{c2} b_1$$

148.

What ratio of upstream to downstream widths b_1/b_2 of rectangular channels should be used if the upstream width b_1 should equal twice the critical depth in the downstream channel, and the depth Y_1 is to be 10 percent larger than the critical depth. For a flowrate Q = 20 m^3/s and the upstream width b_1 = 5 m, what are these depths? What is the upstream Froude Number?

Wanted: What ratio of upstream to downstream widths b_1/b_2 of rectangular channels should be used if the upstream width b_1 should equal twice the critical depth in the downstream channel, and the depth Y_1 is to be 10 percent larger than the critical depth. For a flowrate Q = 20 m^3/s and the upstream width b_1 = 5 m, what are these depths? What is the upstream Froude Number?

Solution:

If the upstream channel has a width twice the critical depth in the downstream channel, then $b' = b_1/Y_c = 2$ and if $Y_1' = Y_1/b_1 = 0.56$. Equation 43 needs to be solved for b_1' with $c = b'Y_1' = 2(.56) = 1.1$ or solve

$C+.5(b_1'c)^2 = 1.5$ or solving for $b_1'^2 = 1/\{2c^2(1.5-c)\} = 1/\{(2x1.21(.4)\} = 1.03306$, or $b_1' = 1.0164$. Therefore, $b_2 = 5/1.0164 = 4.919$ m. $Y_c = \{(20/4.919)^2/9.81\}^{1/3} = 1.190$ m and $Y_1 = 1.1(1.190) = 1.309$ m. From Eq. 45 $\{3/c-2\}^{.5} = \{3/1.1-2\}^{.5} = .853$.

149.

Write a computer program, or develop a computer model, that solves Eq. 43 for any of the variables: b_1', b' or Y_1' given the other two and use this model to solve Problems 145 and 148.

Wanted: Write a computer program, or develop a computer model, that solves Eq. 43 for any of the variables: b_1', b' or Y_1' given the other two and use this model to solve Problems 133 and 136.

Solution:

A model that can be used is the TK-Solver model RECRECT.TK given in the solution to Problem 144. To solve problems in which different variables are the unknown, simply provide the other variables, and supply a guess for this variable. The program SOLR_RCR.FOR listed below is designed to solve for b_1', b' or Y_1' with the other two values given. When b_1' is unknown, then it is solved explicitly from $b_1' = \{1[2c^2(1.5-c)]\}^{1/2}$ in which $c=b'Y_1'$ (the constant associated with this b_1'. If b' or Y_1' is unknown, c is first obtained using the Newton Method with a guess supplied by $c=-.2784b_1'^2 + 1.192b_1' + .2224$.

Program SOLR_RCR.FOR

```
      REAL X(3)
      CHARACTER*3 V(3)/'b1''','b'' ','Y1'''/
20    WRITE(*,100)(I,V(I),I=1,3)
100   FORMAT(' Identify unknown by its number(0=STOP',/,(I4,' - ',A3))
      READ(*,*) IU
      IF(IU.LT.1 .OR. IU.GT.3) STOP
      WRITE(*,*)' Give values for'
      DO 10 I=1,3
      IF(I.EQ.IU) GO TO 10
      WRITE(*,110) V(I)
110   FORMAT(1X,A3,' = ',\)
      READ(*,*) X(I)
10    CONTINUE
      IF(IU.EQ.1) THEN
      C=X(2)*X(3)
      X(1)=SQRT(.5/(C**2*(1.5-C)))
      ELSE
      NCT=0
      C=(1.1920-.2784*X(1))*X(1)+.2224
      BP2=.5/X(1)**2
1     F=C+BP2/C**2-1.5
      NCT=NCT+1
      IF(MOD(NCT,2).EQ.0) GO TO 2
      FF=F
      C1=C
      C=1.005*C
      GO TO 1
2     DIF=(C-C1)*FF/(F-FF)
      C=C1-DIF
      IF(NCT.LT.30 .AND. ABS(DIF).GT.1.E-6) GO TO 1
      IF(IU.EQ.2) THEN
      X(2)=C/X(3)
      ELSE
      X(3)=C/X(2)
      ENDIF
      ENDIF
      Fr1=SQRT(3./C-2.)
      WRITE(*,120) V(IU),X(IU),(V(I),X(I),I=1,3),Fr1
120   FORMAT('  Solution for',A4,' =',F10.4,/' Variable are:',
```

```
&3(/,A4,' -',F9.3),/' Fr1 =',F9.3)
 GO TO 20
 END
```

Input to solve this problem: 1 followed by 2.55 with Solution: b_1'=1.0164 & F_{r1}=.853

Input to solve Problem 145: 3 followed by 1.5 4.6892 with Solution: Y_1'=.2951 & F_{r1}=.409

150.

The eight graphs in Figure 13 use linear graph paper to display the relationship of the dimensionless variables, Y_1', b_1' and b' for different values of m_1. Plot these relationships on eight log-log graphs and obtain an approximate equation of the form $Y_1' = a(b')^b$ for each b_1' and m_1. From these equations can you suggest one equation that may be used to provide a "guess" to start the Newton solution to Equation 40.

The eight graphs in Figure 13 use linear graph paper to display the relationship of the dimensionless variables, Y_1', b_1' and b' for different values of m_1. Plot these relationships on eight log-log graphs and obtain an approximate equation of the form $Y_1' = a(b')^b$ for each b_1' and m_1. From these equations can you suggest one equation that may be used to provide a "guess" to start the Newton solution to Equation 40.

Solution:

To evaluate a and b in the equation $Y_1' = a(b')^b$ take the log of the equation to get $\text{Log}(Y_1') = \text{Log}(a) + b\text{Log}(b')$. For the table below the two points selected to pass the equation through were b'=.7 and b'=3 (except for m_1=.25, .5 and .75) so

$$b = \frac{\text{Log}Y_1'|_3 - \text{Log}Y_1'|_{.7}}{\text{Log}(3) - \text{Log}(.7)} \qquad a = \frac{Y_1'|_{.7}}{.7^b}$$

Program PRB2_138.FOR reads in the Table from Problem 140 and computes a and b for the eight m_1's corresponding to 6 values of b_1' used in that table.

Table giving coefficients in model $Y_1' = a(b')^b$ as an approximation to the solution of Eq. 40 (or 41).

	b_1'											
	.8		1.		1.25		1.5		1.75		2.0	
m_1	a	b	a	b	a	b	a	b	a	b	a	b
.25	.2798	-1.3577	.3363	-1.0775	.3531	-1.0379	.3606	-1.0236	.3647	-1.0166	.3673	-1.0117
.50	.6519	-1.2005	.7042	-1.0653	.7233	-1.0344	.7322	-1.0220	.7372	-1.0153	.7403	-1.0113
.75	1.0164	-1.1741	1.0758	-1.0585	1.0962	-1.0312	1.1058	-1.0200	1.1112	-1.0141	1.1146	-1.0104
1.0	1.4072	-1.1065	1.4510	-1.0484	1.4709	-1.0270	1.4805	-1.0175	1.4859	-1.0124	1.4893	-1.0093
1.25	1.7885	-1.0808	1.8270	-1.0406	1.8462	-1.0232	1.8555	-1.0153	1.8609	-1.0109	1.8644	-1.0082
1.5	2.1684	-1.0650	2.2033	-1.0345	2.2218	-1.0201	2.2309	-1.0134	2.2361	-1.0096	2.2395	-1.0072
1.75	2.5475	-1.0540	2.5798	-1.0297	2.5974	-1.0176	2.6063	-1.0118	2.6114	-1.0085	2.6147	-1.0064
2.0	2.9262	-1.0457	2.9564	-1.0258	2.9733	-1.0155	2.9818	-1.0104	2.9867	-1.0075	2.9900	-1.0056

Representative Equation for b

Notice that the values of b in the above table increase with b_1'. A straight line equation that approximate these b's is,

$$b = .04b_1' - 1.09$$ ←

Straight line approximations for a might be as follows:

$b_1' = .8$	$a=1.512m_1 - .098$
$b_1' = 1.$	$a=1.497m_1 - .038$
$b_1' = 1.25$	$a=1.497m_1 - .021$
$b_1' = 1.5$	$a=1.498m_1 - .014$
$b_1' = 1.75$	$a=1.498m_1 - .010$
$b_1' = 2.0$	$a=1.499m_1 - .007$

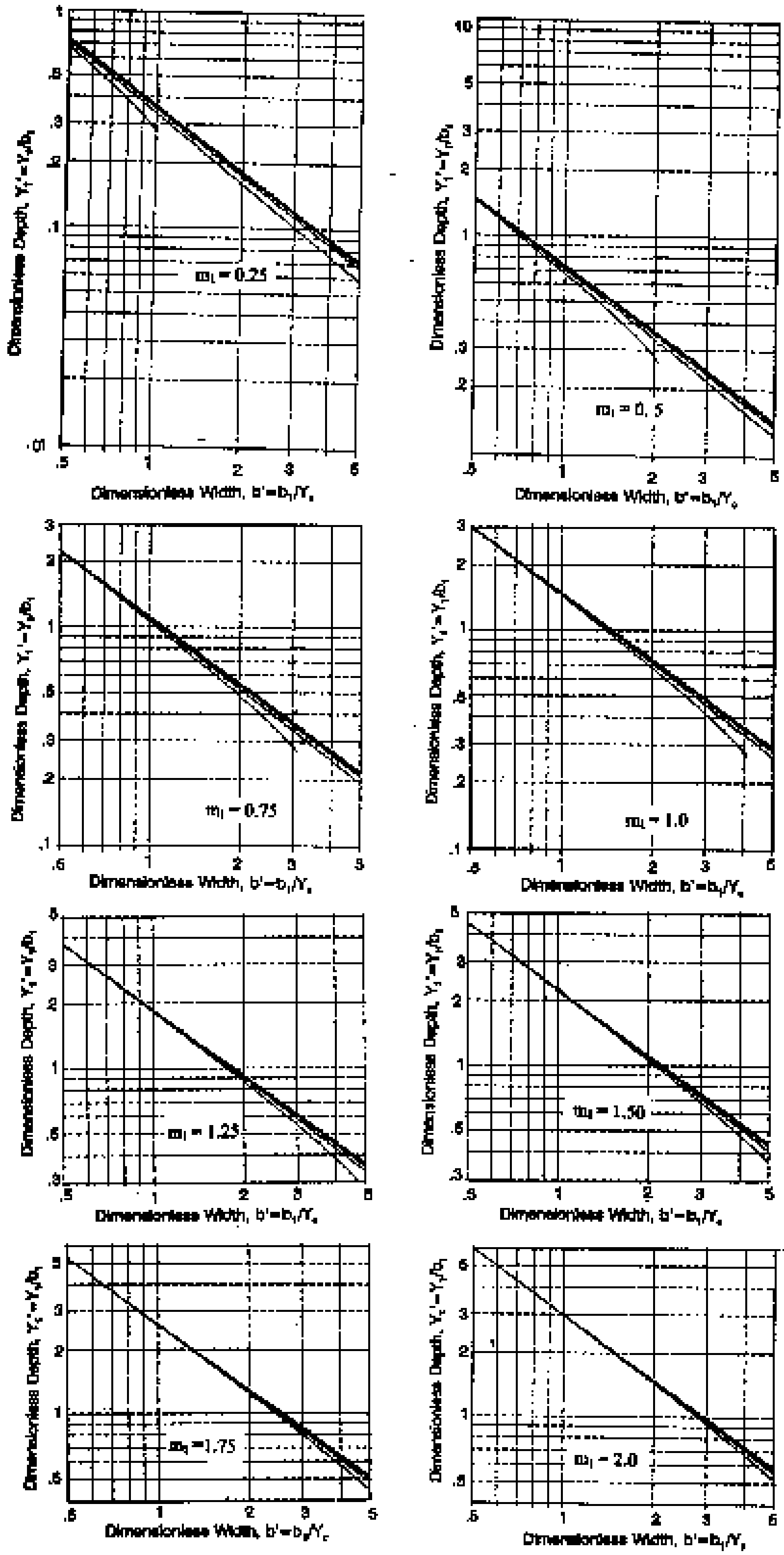
For the multiplier of m_1 a representative value is 1.498 – the constant and for variation with b_1' use $.031b_1'-.069$ so

$$a = 1.498m_1 + .031b_1' - .069$$

and "Overall Equation" is:

$$Y_1' = (1.498m_1 + .031b_1' - .069)b'^{(.04b1'-1.09)}$$ ←

$$b' = (Y_1'/a)^{1/b}$$ ←



Dimensionless Depth, Y1'=Y1/b1
Dimensionless Width, b'=b1/Yc
m1 = 0.25
m1 = 0.5
m1 = 0.75
m1 = 1.0
m1 = 1.25
m1 = 1.50
m1 = 1.75
m1 = 2.0

151.

Write a computer program, or develop a computer model, that solves Eq. 40 (or Eq. 41) for any of the variables: b_1', b' or Y_1', and use this program (model) to solve Problem 142.

Wanted: Write a computer program, or develop a computer model, that solves Eq. 40 (or Eq. 41) for any of the variables: b_1', b' or Y_1', and use this program (model) to solve Problem 130.

Problem 2:139

The program SOLT_RCR.FOR is designed to solve any of the variables m_1, b_1', b' or Y_1' from Eq. 40. It uses the approximate equation developed in the previous problem to provide the guess for the Newton Method, i.e. in

$Y_1' = a(b')^b$

$a = 1.498m_1 + .031\ b_1' - .069$ and $b = .04b_1' - 1.09$

When Y_1' is given then b' is approximated by $b' = \{Y_1'/a\}^{1/b}$

Eq. 40 provides the following explicit solution for b_1'

$(b_1')^2 = .5m_1^3/\{b'^2(Y_1'+Y_1'^2)^2(1.5m_1-Y_1'b')\}$

or

$b_1' = /.5m_1^{1.5}/\{b'(Y_1'+Y_1'^2)(1.5m_1-Y_1'b')^{.5}\}$

In solving for m_1 the following rewritten Eq. 40 is used.

$F = .5m_1^3 + [b^1'b'(Y_1'+Y_1'^2)]^2(Y_1'b'-1.5m_1) = 0$

```
Program SOLT RCR.FOR
Completely solves Eq. 40 using Newton's Method
      CHARACTER*3 CH(4)/'m1 ','b1''','b'' ','Y1'''/
      REAL X(4)
      EQUIVALENCE (X(1),FM),(X(2),B1P),(X(3),BP),(X(4),Y1P)
5     WRITE(*,*)' Give number of unknown (0=STOP)'
      DO 10 I=1,4
10    WRITE(*,100) I,CH(I)
100   FORMAT(I4,' -',A4)
      READ(*,*) IU
      IF(IU.LT.1 .OR. IU.GT.4) STOP
      WRITE(*,*)' Give values to other variables'
      DO 20 I=1,4
      IF(I.EQ.IU) GO TO 20
      WRITE(*,105) CH(I)
105   FORMAT(A4,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
      IF(IU.EQ.2) THEN
      B1P=.70710678*FM**1.5/((BP*(Y1P+Y1P**2))*SQRT(1.5*FM-Y1P*BP))
      GO TO 50
      ENDIF
```

```
      IF(IU.EQ.1) THEN
      A=Y1P/BP**(.04*B1P-1.09)
      FM=(A+.069-.031*B1P)/1.498
      ELSE
      A=1.498*FM+.031*B1P-.069
      B=.04*B1P-1.09
      IF(IU.EQ.4) THEN
      Y1P=A*BP**B
      ELSE
      BP=(Y1P/A)**(1./B)
      ENDIF
      FM3=.5*FM**3/B1P**2
      FM15=1.5*FM
      ENDIF
      NCT=0                                                         To solve Problem 130
1      IF(IU.EQ.1) THEN                                                  give 4 to select
unknown
      F=.5*FM**3+(B1P*BP*(Y1P+Y1P**2))**2*(Y1P*BP-1.5*FM)              then give to the
variable prompts:
      ELSE                                                          m1  = 1.5
      F=Y1P*BP+FM3/((BP*(Y1P+Y1P**2))**2)-FM15                      b1' = 1.25
      ENDIF                                                         b'  = 5.14571
      NCT=NCT+1
      IF(MOD(NCT,2).EQ.0) GO TO 2                                    Solution:
      FF=F                                                           Y1' = .4141
      XX=X(IU)                                                       Fr1 = .380
      X(IU)=1.005*X(IU)
      GO TO 1
2      DIF=(X(IU)-XX)*FF/(F-FF)                                      To solve for m1 when
b1' = b' = 1.2 and
      X(IU)=XX-DIF                                                 Y1' = 0.5     Select 1
as unknown and
      IF(NCT.LT.30 .AND. ABS(DIF).GT. 1.E-6) GO TO 1                provide known values.
The solution is:
      IF(NCT.EQ.30) WRITE(*,*)' Did not converge'                     m1 = .4214  and Fr1 =
.378
50    WRITE(*,110) CH(IU),X(IU)
110   FORMAT(' Solution for',A4,' =',F10.4)
      Fr1=SQRT(FM**3*(1.+2.*Y1P)/(B1P**2*(BP*(Y1P+Y1P**2))**3))
      WRITE(*,120) (CH(I),X(I),I=1,4),Fr1
120   FORMAT(' Variables for problem',4(/,A4,' =',F8.3),/' Fr1 =',F8.3,/
     &/' To Solve another problem give 1 (0=STOP')
      READ(*,*) IU
      IF(IU.GT.0) GO TO 5
      END
```

Problem 2:151Continued)

An alternative to the above approach that uses a reasonable estimate of the solution and then the Newton Method is to use Laguerre's Method described in Chapter 3, and Appendix B, that by an iterative method extract a root from a nth degree polynomial, as was used Problem 128. Expressing Eq. 40 as a polynomial gives:

<u>In terms of Y_1'</u> Eq. 41

<u>In terms of b'</u>

$$Y_1'(b')^3 - 1.5m_1(b')^2 + .5m_1^3/[b_1'(Y_1'+Y_1'^2)]^2 = 0$$

<u>In terms of m_1</u>

$$.5m_1^3 - 1.5[b_1'b'(Y_1'+Y_1'^2)]^2m_1 + Y_1'b'[(Y_1'+Y_1'^2)]^2 = 0$$

In solving the polynomial for Y_1' and b' the largest real root is the desired one; however when solving the polynomial for m_1 it is the second from the largest real root that is the desired one. Program SOLT_RCL.FOR used the Laguerre Method to completely solve Eq. 40 for: m_1, b_1', b' or Y_1' given the other variables, i.e. it represents an alternative approach to that used in program SOLT_RCR.FOR.

```
Program SOLT RCL.FOR
Completely solve EQ. 40 using Laguerre's Method
      PARAMETER (EPS=1.E-5)
      COMPLEX C(6),ROOTS(5),AD(6),Z1,Z2,Z3
      CHARACTER*3 CH(4)/'m1 ','b1''','b'' ','Y1'''/
      REAL X(4)
      EQUIVALENCE (X(1),FM),(X(2),B1P),(X(3),BP),(X(4),Y1P)
      EPS1=2.*EPS*EPS
5     WRITE(*,*)' Give number of unknown (0=STOP)'
      DO 10 I=1,4
10    WRITE(*,100) I,CH(I)
100   FORMAT(I4,' -',A4)
      READ(*,*) IU
      IF(IU.LT.1 .OR. IU.GT.4) STOP
      WRITE(*,*)' Give values to other variables'
      DO 20 I=1,4
      IF(I.EQ.IU) GO TO 20
      WRITE(*,105) CH(I)
105   FORMAT(A4,' = ',\)
      READ(*,*) X(I)
20    CONTINUE
      IF(IU.EQ.2) THEN
      B1P=.70710678*FM**1.5/((BP*(Y1P+Y1P**2))*SQRT(1.5*FM-Y1P*BP))
      GO TO 50
      ENDIF
      FM3=.5*FM**3
      F15=1.5*FM
      F3=3.*FM
      IF(IU.EQ.4) THEN
      ND=5
      C(6)=CMPLX(1.,0.)
      C(5)=CMPLX(2.-F15/BP,0.)
      C(4)=CMPLX(1.-F3/BP,0.)
      C(3)=CMPLX(-F15/BP,0.)
      C(2)=CMPLX(0.,0.)
      C(1)=CMPLX(FM3/BP**3/B1P**2,0.)
      ELSE IF(IU.EQ.3) THEN
      ND=3
      C(4)=CMPLX(Y1P,0.)
      C(3)=CMPLX(-F15,0.)
      C(2)=CMPLX(0.,0.)
      C(1)=CMPLX(FM3/(B1P*(Y1P+Y1P**2))**2,0.)
      ELSE
      ND=3
      C(4)=CMPLX(.5,0.)
      C(3)=CMPLX(0.,0.)
      BBB=(B1P*BP*(Y1P+Y1P**2))**2
      C(2)=CMPLX(-1.5*BBB,0.)
      C(1)=CMPLX(Y1P*BP*BBB,0.)
      ENDIF
      DO 28 J=1,ND+1
28    AD(J)=C(J)
      DO 30 J=ND,1,-1
      Z1=CMPLX(0.,0.)
      CALL LAGU(AD,J,Z1,EPS)
      IF(ABS(AIMAG(Z1)).LE.EPS1*ABS(REAL(Z1))) Z1=CMPLX(REAL(Z1),0.)
      ROOTS(J)=Z1
      Z2=AD(J+1)
      DO 30 JJ=J,1,-1
```

```
      Z3=AD(JJ)
      AD(JJ)=Z2
30    Z2=Z1*Z2+Z3
      DO 35 J=2,ND
      Z1=ROOTS(J)
      DO 32 I1=J-1,1,-1
      IF(REAL(ROOTS(I1)).LE.REAL(Z1)) GO TO 35
32    ROOTS(I1+1)=ROOTS(I1)
      I1=0
35    ROOTS(I1+1)=Z1
      WRITE(*,106) (ROOTS(J),J=1,ND)
106   FORMAT(' Roots of polynomial',/,5(2F8.4,1X))
      IF(IU.EQ.1) THEN
      X(IU)=REAL(ROOTS(2))
      ELSE
      X(IU)=REAL(ROOTS(ND))
      ENDIF
50    WRITE(*,110) CH(IU),X(IU)
110   FORMAT(' Solution for',A4,' =',F10.4)
      Fr1=SQRT(FM**3*(1.+2.*Y1P)/(B1P**2*(BP*(Y1P+Y1P**2))**3))
      WRITE(*,120) (CH(I),X(I),I=1,4),Fr1
120   FORMAT(' Variables for problem',4(/,A4,' =',F8.3),/' Fr1 =',F8.3,/
     &/' To Solve another problem give 1 (0=STOP')
      READ(*,*) IU
      IF(IU.GT.0) GO TO 5
       END
```

152.

An upstream trapezoidal channel with a bottom slope of $S_{o1} = .0002$, a side slope $m_1 = 1$, and $n = .013$ has a smooth transition to a steep downstream circular channel with D = 10 ft, (a) If the upstream channel has a width $b_1 = 10$ ft, what flowrate will result in uniform flow upstream and what will this uniform depth be, and what will the critical depth be at the beginning of the circular channel? Repeat this solution for larger downstream pipe diameters up to 12 ft, and examine how the flowrates and depths change. (b) Determine how the upstream bottom width must vary for uniform flow to occur in the upstream channel for specified flowrates varying from 400 cfs to 800 cfs in increments of 50 cfs. (Also determine the upstream uniform depth Y_1, and the critical depth Y_{c2} associated with each of these flowrates.) (c) Determine how the upstream bottom slope S_{o1} necessary for uniform flow varies as the flowrates vary from 400 to 800 cfs.

Solution:

The three equations available for this problem are: (1) Critical flow in the downstream circular channel,

$$F_1 = Q^2T_2/(gA_2) - 1 = 0$$

(2) The energy equation,

$$F_2 = Y_1 + (Q/A_1)^2/(2g) - Y_{c2} - (Q/A_{c2})^2/(2g) = 0$$

and (3) The uniform flow equation (Mannings Eq.)

$$F_3 = nQP_1^{2/3} - C_uA_1^{5/3}S_{o1}^{1/2} = 0$$

If Y_{2c}, Y_1 & Q are the unknowns, then Eq. 1 could be used to eliminate Q as an unknown, i.e. $Q^2/g = A_{c2}^3/T_2$ or $Q = (gA_{c2}^3/T_2)^{1/2}$ so

$$F_1 = Y_1 + A_{c2}^3/(2T_2A_1^2) - Y_{c2} - A_{c2}/(2T_2) = 0$$

and

$$F_2 = n(gA_{c2}^3/T_2)^{1/2}P_1^{2/3} - C_uA_1^{5/3}S_{o1}^{1/2} = 0$$

If Q is specified then (1), (2) and (3) can be solved in which another variable is unknown, i.e. b_1, Y_{c2} & Y_1 are unknown, as in the (b) part of this problem. An easier means for solving problems in which Q is specified is to first solve the critical flow equation of Y_{c2} and then solve the energy and Mannings equations simultaneously for the other two variables, i.e. b_1 and Y_1. However, the 3 equations can be solved for three variables even though one of these equations determines the value of one variable independent of the other two unknown.

Program SOLTCIR.FOR is designed to solve both the (a) and (b) parts of this problem plus problems in which other variables may be selected as the unknowns. In using this program one must be cautious to not specify a problem that cannot be solved such as specifying Y_{c2}, D and Q, for example.

Program SOLTCIR.FOR

```
      CHARACTER CH(9)*2/'m1','b1','Y1','Q ','D ','Yc','So','n ','g '/
     &,GUESS*5,FMT*15/'(A3,'' ='',F10.4)'/
      COMMON X(9),Cu,G2
      REAL F(3),FF(3),DJ(3,3)
      INTEGER*2 IU(3),INDX(3)
      EQUIVALENCE (FM,X(1)),(B1,X(2)),(Y1,X(3)),(Q,X(4)),(D,X(5)),(Yc,
     &X(6)),(So,X(7)),(FN,X(8)),(G,X(9))
      WRITE(*,*)' Give output unit number'
      READ(*,*) IOUT
5     WRITE(*,*)' Give 3 numbers for unknowns (0=STOP)'
      DO 10 I=1,7
10    WRITE(*,100) I,CH(I)
100   FORMAT(I4,' -',A4)
      READ(*,*) IU
      DO 2 I=1,3
      IF(IU(I).LT.1 .OR. IU(I).GT.7) STOP
2     CONTINUE
      WRITE(*,*)' Give values to variables'
      DO 20 I=1,9
      GUESS='     '
      IF(I.GT.7) GO TO 19
      DO 18 J=1,3
      IF(I.EQ.IU(J)) GUESS='GUESS'
18    CONTINUE
19    WRITE(*,105) GUESS,CH(I)
105   FORMAT(A6,A3,' = ',\)
20    READ(*,*) X(I)
      G2=2.*G
      Cu=1.486
      IF(G.LT.20.) Cu=1.
29    NCT=0
30    SUM=0.
      CALL FUN(F)
      WRITE(*,*)' F=',F
      II=0
      DO 40 I=1,7
      DO 35 J=1,3
      IF(IU(J).EQ.I) GO TO 36
35    CONTINUE
      GO TO 40
36    XX=X(I)
      II=II+1
      X(I)=1.005*X(I)
      CALL FUN(FF)
      DO 37 J=1,3
37    DJ(J,II)=(FF(J)-F(J))/(X(I)-XX)
      X(I)=XX
40    CONTINUE
      CALL SOLVEQ(3,1,3,DJ,F,1,DD,INDX)
      II=0
      DO 50 I=1,7
      DO 45 J=1,3
      IF(IU(J).EQ.I) GO TO 46
45    CONTINUE
      GO TO 50
46    II=II+1
      X(I)=X(I)-F(II)
      SUM=SUM+ABS(F(II))
50    CONTINUE
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,SUM,(X(IU(J)),J=1,3)
      IF(NCT.LT.30 .AND. SUM.GT.1.E-3) GO TO 30
      WRITE(*,*)' Problem Variables:'
      IF(IOUT.NE.6) WRITE(IOUT,*)' Problem Variables:'
      DO 60 I=1,9
      IF(I.EQ.7) THEN
      FMT(14:14)='7'
      ELSE IF(I.EQ.9) THEN
      FMT(14:14)='2'
      ELSE
      FMT(14:14)='4'
      ENDIF
      IF(IOUT.NE.6) WRITE(IOUT,FMT) CH(I),X(I)
60    WRITE(*,FMT) CH(I),X(I)
      WRITE(*,*)' Give 1 to solve another problem; 2 change v. (0=STOP)'
      READ(*,*) II
      IF(II.EQ.1) GO TO 5
      IF(II.EQ.2) THEN
      READ(*,*) II,X(II)
      GO TO 29
      ENDIF
      END
      SUBROUTINE FUN(F)
```

```
      REAL F(3)
      COMMON X(9),Cu,G2
      EQUIVALENCE (FM,X(1)),(B1,X(2)),(Y1,X(3)),(Q,X(4)),(D,X(5)),(Yc,
     &X(6)),(So,X(7)),(FN,X(8)),(G,X(9))
      COSB=1.-2.*Yc/D
      BETA=ACOS(COSB)
      Ac=.25*D**2*(BETA-COSB*SIN(BETA))
      F(1)=Q*Q*(D*SIN(BETA))/(G*Ac**3)-1.
      A1=(B1+FM*Y1)*Y1
      F(2)=Y1+(Q/A1)**2/G2-Yc-(Q/Ac)**2/G2
      P1=B1+2.*Y1*SQRT(FM*FM+1.)
      F(3)=FN*Q*P1**.66666667-Cu*A1**1.6666667*SQRT(So)
      RETURN
      END
```

Part (a) One might guess a flowrate of 400 cfs, then the critical flow equation gives Y_{c2} = 4.748 ft, E_{c2} = 6.587 ft, with $E_1 = E_{c2}$, Y_1 = 6.357 ft, which when solving Manning's Eq. gives S_{o1} = .000197, which is close to the specified .0002. Thus one selects 3 4 and 6 as the unknown variables, and gets the following solution for part (a).

Problem Variables:					
m1 =	1.0000	1.0000	1.0000	1.0000	1.0000
b1 =	10.0000	10.0000	10.0000	10.0000	10.0000
Y1 =	5.7124	6.3366	6.7642	7.1233	7.4443
Q =	330.0884	400.8485	453.6035	500.6533	544.8696
D =	10.0000	10.1000	10.2000	10.3000	10.4000
Yc =	4.2943	4.7385	5.0398	5.2912	5.5145
So =	.0002000	.0002000	.0002000	.0002000	.0002000
n =	.0130	.0130	.0130	.0130	.0130
g =	32.20	32.20	32.20	32.20	32.20

Problem Variables:					
m1 =	1.0000	1.0000	1.0000	1.0000	1.0000
b1 =	10.0000	10.0000	10.0000	10.0000	10.0000
Y1 =	7.7402	8.0182	8.2824	8.5357	8.7802
Q =	587.4718	629.1063	670.1646	710.9042	751.5036
D =	10.5000	10.6000	10.7000	10.8000	10.9000
Yc =	5.7193	5.9107	6.0918	6.2648	6.4310
So =	.0002000	.0002000	.0002000	.0002000	.0002000
n =	.0130	.0130	.0130	.0130	.0130
g =	32.20	32.20	32.20	32.20	32.20

Problem Variables:				
m1 =	1.0000	1.0000	1.0000	1.0000
b1 =	10.0000	10.0000	10.0000	10.0000
Y1 =	9.0173	9.5844	10.1235	11.1438
Q =	792.0948	894.1273	997.7149	1211.8520
D =	11.0000	11.2500	11.5000	12.0000
Yc =	6.5917	6.9736	7.3338	8.0086
So =	.0002000	.0002000	.0002000	.0002000
n =	.0130	.0130	.0130	.0130
g =	32.20	32.20	32.20	32.20

For part (b) the unknown variables are selected as 2 3 and 6. The solutions starting with 400 cfs and increasing to 800 cfs in 50 cfs increments are given below. It is interesting that as the flowrate increases the required upstream width b_1 decreases, i.e. is 9.904 ft for Q = 400 cfs, and is 8.552 ft for Q = 800 cfs. The depths Y_1 and Y_{c2} increase with increasing Q.

Problem Variables:		
m1 =	1.0000	1.0000
b1 =	9.9040	9.8011
Y1 =	6.3548	6.7903
Q =	400.0000	450.0000
D =	10.0000	10.0000
Yc =	4.7477	5.0496
So =	.0002000	.0002000
n =	.0130	.0130
g =	32.20	32.20

Problem Variables:

```
m1 =    1.0000
b1 =    9.6749
Y1 =    7.2105
Q  =  500.0000
D  =   10.0000
Yc =    5.3361
So =  .0002000
n  =     .0130
g  =     32.20
```

```
 Problem Variables:
m1 =    1.0000
b1 =    9.5286
Y1 =    7.6182
Q  =  550.0000
D  =   10.0000
Yc =    5.6091
So =  .0002000
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =    9.3643
Y1 =    8.0157
Q  =  600.0000
D  =   10.0000
Yc =    5.8702
So =  .0002000
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =    9.1835
Y1 =    8.4051
Q  =  650.0000
D  =   10.0000
Yc =    6.1205
So =  .0002000
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =    8.9875
Y1 =    8.7877
Q  =  700.0000
D  =   10.0000
Yc =    6.3608
So =  .0002000
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =    8.7769
Y1 =    9.1652
Q  =  750.0000
D  =   10.0000
Yc =    6.5918
So =  .0002000
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =    8.5523
Y1 =    9.5386
Q  =  800.0000
D  =   10.0000
Yc =    6.8139
So =  .0002000
n  =     .0130
g  =     32.20
```

Part (c) To solve this part of the problem the critical depth Y_{c2} could be solved from the critical flow equation and therefrom $E_c = E_1$ can be computed. Next the depth Y_1 could be solved from the energy equation, and final using this as the normal depth the solve S_{o1} could be solve. However the same program SOLTCIR can be used even if the 3 equations could be solved separately for one variable. The unknowns would be identified as 3 6 & 7. The solution to part (c) as produced by SOLTCIR is:

```
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    6.3578
Q  =  400.0000
D  =   10.0000
Yc =    4.7477
So =  .0001967
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    6.7968
Q  =  450.0000
D  =   10.0000
Yc =    5.0496
So =  .0001933
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    7.2214
Q  =  500.0000
D  =   10.0000
Yc =    5.3361
So =  .0001893
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    7.6343
Q  =  550.0000
D  =   10.0000
Yc =    5.6091
So =  .0001849
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    8.0378
Q  =  600.0000
D  =   10.0000
Yc =    5.8702
So =  .0001802
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    8.4338
Q  =  650.0000
D  =   10.0000
Yc =    6.1205
So =  .0001753
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    8.8238
Q  =  700.0000
D  =   10.0000
Yc =    6.3608
So =  .0001702
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    9.2091
Q  =  750.0000
D  =   10.0000
Yc =    6.5918
So =  .0001650
n  =     .0130
g  =     32.20
 Problem Variables:
m1 =    1.0000
b1 =   10.0000
Y1 =    9.5909
Q  =  800.0000
D  =   10.0000
Yc =    6.8139
So =  .0001597
n  =     .0130
g  =     32.20
```

Program SOLTCIR1.FOR is designed to solve part (a) using the above two equations to solve for Y_1 and Y_{c2}, and thereafter solve for the flowrate. Note the solution, which is given after the program listing, agrees with that for part (a) above.

SOLTCIR1.FOR

```
      CHARACTER CH(8)*2/'m1','b1','Y1','Yc','D ','So','n ','g '/
      COMMON FM,B1,X(2),D,So,FN,G,Cu,G2
      REAL F(2),FF(2),DJ(2,2)
      EQUIVALENCE (Y1,X(1)),(Yc,X(2))
      WRITE(*,*)' Give output unit number'
      READ(*,*) IOUT
5     WRITE(*,100) CH
100   FORMAT(' Give values to variables (guesses for Y1 & Yc)'/,8A3)
      READ(*,*) FM,B1,X,D,So,FN,G
      G2=2.*G
      Cu=1.486
      IF(G.LT.20.) Cu=1.
29    NCT=0
30    SUM=0.
      CALL FUN(F)
      DO 40 I=1,2
      XX=X(I)
      X(I)=1.005*X(I)
      CALL FUN(FF)
      DO 37 J=1,2
37    DJ(J,I)=(FF(J)-F(J))/(X(I)-XX)
40    X(I)=XX
      FAC=DJ(2,1)/DJ(1,1)
      DJ(2,2)=DJ(2,2)-FAC*DJ(1,2)
      F(2)=F(2)-FAC*F(1)
      DIF=F(2)/DJ(2,2)
      X(2)=X(2)-DIF
      DIF1=(F(1)-DIF*DJ(1,2))/DJ(1,1)
      X(1)=X(1)-DIF1
      NCT=NCT+1
      WRITE(*,*)' NCT=',NCT,DIF,DIF1,X
      IF(NCT.LT.30 .AND. ABS(DIF)+ABS(DIF1).GT.1.E-3) GO TO 30
```

```
      BETA=ACOS(1.-2.*Yc/D)
      Ac=.25*D*D*(BETA-COS(BETA)*SIN(BETA))
      Q=SQRT(G*Ac**3/(D*SIN(BETA)))
      WRITE(*,*)' Problem Variables:'
      IF(IOUT.NE.6) WRITE(IOUT,*)' Problem Variables:'
      WRITE(*,110) CH(1),FM,CH(2),B1,CH(3),Y1,CH(4),Yc,CH(5),D,CH(6),So,
     &CH(7),FN,CH(8),G,'Q ',Q
      IF(IOUT.NE.6) WRITE(IOUT,110) CH(1),FM,CH(2),B1,CH(3),Y1,CH(4),Yc,
     &CH(5),D,CH(6),So,CH(7),FN,CH(8),G,'Q ',Q
110   FORMAT(6(A3,' =',F10.4,/),A3,' =',F10.6,/,2(A3,' =',F10.4,/))
      WRITE(*,*)' Give 1 to solve another problem; 2 change D (0=STOP)'
      READ(*,*) II
      IF(II.EQ.1) GO TO 5
      IF(II.EQ.2) THEN
      READ(*,*) D
      GO TO 29
      ENDIF
      END
```

```
      SUBROUTINE FUN(F)
      REAL F(2)
      COMMON FM,B1,X(2),D,So,FN,G,Cu,G2
      EQUIVALENCE (Y1,X(1)),(Yc,X(2))
      COSB=1.-2.*Yc/D
      BETA=ACOS(COSB)
      Ac=.25*D**2*(BETA-COSB*SIN(BETA))
      A1=(B1+FM*Y1)*Y1
      T2=D*SIN(BETA)
      F(1)=Y1+Ac**3/(2.*A1*A1*T2)-Yc-.5*Ac/T2
      P1=B1+2.*Y1*SQRT(FM*FM+1.)
      F(2)=FN*SQRT(G*Ac**3/T2)*P1**.66666667-Cu*A1**1.6666667*SQRT(So)
      RETURN
      END
```

Solution:

```
Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    5.7124
 Yc =    4.2943
 D  =   10.0000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  330.0890
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    6.3366
 Yc =    4.7385
 D  =   10.1000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  400.8483
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    6.7642
 Yc =    5.0398
 D  =   10.2000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  453.6039
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    7.1233
 Yc =    5.2912
 D  =   10.3000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  500.6540
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    7.4443
 Yc =    5.5145
 D  =   10.4000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  544.8705
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    7.7403
 Yc =    5.7193
 D  =   10.5000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  587.4724
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    8.0182
 Yc =    5.9107
 D  =   10.6000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  629.1067
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    8.2824
 Yc =    6.0918
 D  =   10.7000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  670.1654
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    8.5358
 Yc =    6.2648
 D  =   10.8000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  710.9053

  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    8.7802
 Yc =    6.4310
 D  =   10.9000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  751.5049

  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    9.0173
 Yc =    6.5917
 D  =   11.0000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  792.0955

  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =    9.5844
 Yc =    6.9736
 D  =   11.2500
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  894.1277
  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =   10.1235
 Yc =    7.3338
 D  =   11.5000
 So =     .0002
 n  =   .013000
 g  =   32.2000
 Q  =  997.7151

  Problem Variables:
 m1 =    1.0000
 b1 =   10.0000
 Y1 =   10.6418
```

```
Yc =    7.6777
D  =   11.7500
So =     .0002
n  =   .013000
g  =   32.2000
Q  = 1103.4860

 Problem Variables:
m1 =    1.0000
```

```
b1 =   10.0000
Y1 =   11.1438
Yc =    8.0086
D  =   12.0000
So =     .0002
n  =   .013000
g  =   32.2000
Q  = 1211.8530
```

153.

It has been determined that the side slope of a trapezodial channel must be m = 1.5. What should the bottom width, b, be to have the hydrauically most efficient section, if the channel is to be designed for a flowrate Q = 500 cfs, its Mannings roughness coefficient is n = .015, and it has a bottom slope S_o = 001?

Given: If m=1.5 for side slope of trapezoidal channel what must b be to be the hydraulically most efficient section.

Solution:

$(P-2Y(m^2+1)^{1/2})Y + mY^2 = CP^{2/3}$ Equation 41 and with m=1.5 this equation becomes:

$(P-3.60555Y)Y + mY^2 = CP^{2/3}$

Differentiate with respect to Y and set ∂P/∂Y = 0

(P – 3.60555& +Y(-3.60555) + 3Y = 0

P=4.2111Y ←

P=b+3.0555Y → b = 4.2111Y – 3.60555Y, or b = .60555Y ←

$Q = (C_u/n)A^{5/3}/P^{2/3}S_o^{1/2}$

$500 = (1.486/.015)(.60555Y^2+1.5Y^2)^{5/3}/(4.2111Y)^{2/3}$

$120.3271 = (Y^{10/3})/(Y^{2/3}) = Y^{8/3}$

Y = 6.0275 ft ← and b = 3.65 ft ←

154.

The most efficient trapezoidal sections is to be used to convey 500 cfs of water from a reservoir whose head H = 5 ft above the channel bottom. The channel will have a bottom slope S_o = .0008 and its Mannings n = .013. Find the bottom with of channel to use. The entrance loss coefficient is K_e = .15

Given: H=5 ft, K_e=.15, n=.013 and S_o=.0008

Wanted: Hydraulically most efficient trapezoidal channel to convey Q = 500 cfs

$m=1/\sqrt{3} = .57735$

$Q = (C_u/n)A^{5/3}/P^{2/3}S_o^{1/2}$ (1)

$H = 5 = Y + (1+K_2)(Q/A)^2/(2g)$ (2)

Solve for Y and b

Y = 4.199 ft ← and b = 15.358 ft ←

E=4.896 ft, A=74.675 ft^2, P=25.056 ft, T=20.207 ft, and F_r=0.614

Add additional equation $Y = (\sqrt{3}/2)b = .866025b$

and it is necessary to solve for an additional unknown, i.e. the head of reservoir so that the section is one-half of a hexagon. The TK-Solver Model PB2_154.TK given below solves this problem.

Y = 6.409 ft ← and b = 7.401 ft ← H = 7.291 ft ←

```
==================== VARIABLE SHEET ====================
St Input---- Name--- Output--- Unit----- Comment--------
             Y       6.4091915
             b       7.4007003
             P       22.202092
             A       71.148737
   .57735    m
   500       Q
   .013      n
   .0008     So
             E       7.291088
   .15       Ke
   64.4      g2

==================== RULE SHEET ========================
S Rule---------------------------------------------------
  A=(b+m*Y)*Y
  P=b+2*Y*sqrt(m^2+1)
  Y=.866025*b
  Q=1.486/n*(A/P)^.666667*A*sqrt(So)
  E=Y+(1+Ke)*(Q/A)^2/g2
```

155.

Determine the hydraulically most efficient trapezoidal channel # 2 in the three channel system shown below. This # 2 channel has a Mannings n = .013 and a bottom slope of S_{o2} = .00085. Channel # 1 is trapezoidal also with a bottom width b_1 = 5 m and a side slope m_1 = 1.3, and receives its water from a reservoir with a head H = 3 m. The entrance loss coefficient is K_e = 0.15. Channel # 3 is a pipe with a diameter D = 1.5 m and is steep. Its bottom is 1.2 meter above the level of the other two channels.

Given: A three channel system

Solution:

The 7 unknown variables are: Q_1, Q_2, Q_3, Y_1, Y_2, Y_3, b

Equations:

$$F_1 = Q_1 - Q_2 - Q_3 = 0 \quad (1)$$

$$F_2 = H - Y_1 - (1+K_e)(Q/A)_1^2/(2g) = 0 \quad (2)$$

$$F_3 = Y_1 + (Q/A)_1^2/(2g) - Y_2 + (Q/A)_2^2/(2g) = 0 \quad (3)$$

$$F_4 = Y_1 + (Q/A)_1^2/(2g) - Y_3 + (Q/A)_3^2/(2g) = 0 \quad (4)$$

$$F_5 = Q_3^2 T_3 - gA_3^3 = 0 \quad (5)$$

$$F_6 = n_2 Q_2 P_2^{2/3} - A_3^{5/3} S_{o3}^{1/2} = 0 \quad (6)$$

$$F_7 = Y_2 - (\sqrt{3}/2) b_2 = 0$$

These Equations are solve using the TK-Solver Model PRB2_112.TK

TK-Solver Model PB2_155.TK to solve problem

VARIABLE SHEET

St	Input	Name	Output	Unit	Comment
		Q1	38.798356		
		Q2	33.67562		
		Q3	5.1227359		
		Y1	2.8577567		
		b2	2.7890095		
		Y2	2.4153519		
		beta	2.177458		
		Y3	1.177596		
	3	H			
	.15	Ke			
	5	b1			
	1.3	m1			
	19.62	g2			
	.57735027	m2			
	1.5	D			
		A3	1.4882907		
	1.2	dz			
	9.81	g			
	.013	n2			
	.00085	So2			

```
═════════════════════ RULE SHEET ═════════════════════
S Rule──────────────────────────────────────────────────
* Q1=Q2+Q3
* H=Y1+(1+Ke)*(Q1/((b1+m1*Y1)*Y1))^2/g2
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2=Y2+(Q2/((b2+m2*Y2)*Y2))^2/g2
* cos(beta)=1-2*Y3/D
* A3=.25*D^2*(beta-cos(beta)*sin(beta))
* Y1+(Q1/((b1+m1*Y1)*Y1))^2/g2=Y3+(Q3/A3)^2/g2+dz
* Q3^2*(D*sin(beta))=g*A3^3
* n2*Q2*(b2+2*m2*Y2)^.666667-((b2+m2*Y2)*Y2)^1.666667*sqrt(So2)=0
* Y2=.866025*b2
```

Alternatively the MathCAD model PRB2_120a.MCD solves the eight equations for the eight unknown variables.

Variables

$b1 := 5 \quad m1 := 1.3 \quad H := 3 \quad Ke := .15 \quad n2 := .013 \quad So2 := .00085 \quad z13 := 1.2 \quad D := 1.5 \quad m2 := .5773502$

$Q1 := 33 \quad Q2 := 25 \quad Q3 := 8 \quad b2 := 2.8 \quad Y2 := 2.4 \quad Y1 := 2.9 \quad Y3 := 1.2 \quad beta := 2 \quad g := 9.81 \quad dz := 1.2$

$A1 := 12 \quad A2 := 4 \quad A3 := 1.4$

Given

$A1 = (b1 + m1 \cdot Y1) \cdot Y1 \qquad A2 = (b2 + m2 \cdot Y2) \cdot Y2 \qquad A3 = .25 \cdot D^2 \cdot (beta - \cos(beta) \cdot \sin(beta))$

$\cos(beta) = 1 - 2 \cdot \frac{Y3}{D} \qquad Q1 = Q2 + Q3 \qquad H = Y1 + (1 + Ke) \cdot \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g}$

$Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g} = Y2 + \frac{\left(\frac{Q2}{A2}\right)^2}{2 \cdot g} \qquad Y1 + \frac{\left(\frac{Q1}{A1}\right)^2}{2 \cdot g} = Y3 + \frac{\left(\frac{Q3}{A3}\right)^2}{2 \cdot g} + dz \qquad Q3^2 \cdot (D \cdot \sin(beta)) = g \cdot A3^3$

$Y2 = .8660254 b2 \qquad n2 \cdot Q2 \cdot (b2 + 2 \cdot m2 \cdot Y2)^{.6666667} = ((b2 + m2 \cdot Y2) \cdot Y2)^{1.666667} \cdot \sqrt{So2}$

find(Q1, Q2, Q3, Y1, Y2, Y3, b2, beta, A1, A2, A3) =

	0
0	38.798
1	33.676
2	5.123
3	2.858
4	2.415
5	1.178
6	2.789
7	2.177
8	24.906
9	10.105
10	1.488

156.

If the elevation of the bottom of a channel changes simultaneously with a contraction of its size, critical flow may not occur at the channels throat, or at its end, if it ends in a free overfall. The position where this control (or critical depth) occurs will be where the sum of the critical specific energy E_c, and the elevation of the bottom z is a maximum, or where $H_c = E_c + z$ is maximum. A rectangular channel that is b = 10 ft wide and is carrying a flowrate Q = 250 cfs end in a free overfall. However, before the termination of the channel the width of the channel reduces to 5 ft at its end over a 50 ft length, while simultaneously the bottom of the channel rises and then fall again according to the 2nd degree polynomial $z = 3(2-x/25)(x/25)$ as shown in the sketch below. Find the position x_c where critical depth will occur and this critical depth. To help understand why critical depth occurs where H is maximum, make up a table that gives z, E, Y and F_r^2 as a function of the position x.

Given: The end of a channel reduces in size while it bottom rises and then falls again.
Wanted: Position where critical flow occurs.

Solution:

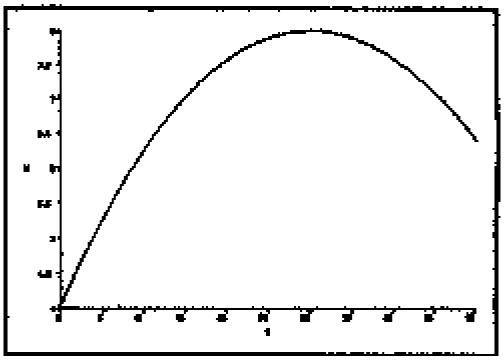
Since this is a rectangular channel $E_c = 1.5Y_c$ and $Y_c = (q^2/g)^{1/3}$, and therefore the total critical head H_c is defined by,

$$H_c=1.5\{(Q/b)^2/g\}^{1/3} + 3(2-x/25)(x/25) \text{ in wich } b = 10 - 0.1 x.$$

A TK-Solver model that list solves this equation is given below, along with a graph of H_c versus position x.

```
==================== VARIABLE SHEET ====================
St Input----- Name---- Output---- Unit------ Comment----------
LG 4          H
L  0          x

==================== RULE SHEET ========================
S Rule-------------------------------------------------------
  H=1.5*((250/(10-.1*x))^2/32.2)^.3333333+3*(x/25)*(2-x/25)

==================== TABLE: Prb2_131 ===================
Title:
Element x---------- H---------- ------- ------- ------- ---
1       0           4.03118378
2       1           4.2934843
3       2           4.54664493
4       3           4.79067831
5       4           5.02559754
6       5           5.25141623
7       6           5.46814853
8       7           5.67580914
9       8           5.87441335
10      9           6.06397705
11      10          6.2445168
12      11          6.41604982
13      12          6.57859406
14      13          6.73216821
15      14          6.87679177
16      15          7.01248507
17      16          7.13926934
18      17          7.25716673
19      18          7.36620038
```

20	19	7.46639448
21	20	7.55777434
22	21	7.64036641
23	22	7.7141984
24	23	7.77929935
25	24	7.83569967
26	25	7.88343128
27	26	7.92252766
28	27	7.95302396
29	28	7.97495715
30	29	7.98836607
31	30	7.99329161
32	31	7.98977682
33	32	7.97786706
34	33	7.95761018
35	34	7.92905668
36	35	7.89225989



The following TK-Solver Model is used to make up the table that eventually gives the Froude Number F_r^2 as a function of the position x. The table gives the solution twice. The first is correct root that gives Froude Number larger than unity after the control section, i.e., the flow will be supercritical as it goes off the end of the channel. The second gives Froude Numbers smaller than unity after the control section. The alternative subcritical depths were computed because the "guesses" for Y resulted in the incorrect root being solved.

TK-Solver Model PB2_156B.TK

```
===================== VARIABLE SHEET ======================
St Input----- Name---- Output---- Unit------ Comment---------
L             z
L  0          x
L             b
L             q
   250        Q
L             Yc
   32.2       g
L             Ec
L             E
   7.99329    H
L             Fr2
LG 0          Y

===================== RULE SHEET ========================
S Rule-------------------------------------------------
  z=3*(2-x/25)*x/25
  b=10-.1*x
  q=Q/b
  Yc=(q^2/g)^.3333333
  Ec=1.5*Yc
  E=H-z
  E=Y+(q/Y)^2/(2.*g)
  Fr2=q^2/(g*Y^3)
```

Table giving the correct root, i.e. the supercritical depth, after the control section at element 31. (Model PB2_156B.TK)

===================== TABLE: ========================

Element	x	b	Yc	Ec	E	Y	Fr2
1	0	10	2.687456	4.031184	7.99329	7.835204	.0403528
2	1	9.9	2.705523	4.058284	7.75809	7.586024	.045364
3	2	9.8	2.723897	4.085845	7.53249	7.345191	.0509991
4	3	9.7	2.742586	4.113878	7.31649	7.112601	.0573318
5	4	9.6	2.761598	4.142398	7.11009	6.888144	.0644429
6	5	9.5	2.780944	4.171416	6.91329	6.671703	.0724214
7	6	9.4	2.800632	4.200949	6.72609	6.463154	.0813645
8	7	9.3	2.820673	4.231009	6.54849	6.262368	.0913782
9	8	9.2	2.841076	4.261613	6.38049	6.069208	.1025776
10	9	9.1	2.861851	4.292777	6.22209	5.88353	.1150873
11	10	9	2.883011	4.324517	6.07329	5.705187	.1290415
12	11	8.9	2.904567	4.35685	5.93409	5.534024	.1445843

13	12	8.8	2.926529	4.389794	5.80449	5.36988	.1618695
14	13	8.7	2.948912	4.423368	5.68449	5.212592	.1810608
15	14	8.6	2.971728	4.457592	5.57409	5.06199	.2023314
16	15	8.5	2.99499	4.492485	5.47329	4.917902	.2258638
17	16	8.4	3.018713	4.528069	5.38209	4.780151	.2518492
18	17	8.3	3.042911	4.564367	5.30049	4.64856	.280487
19	18	8.2	3.0676	4.6014	5.22849	4.522947	.3119837
20	19	8.1	3.092796	4.639194	5.16609	4.403132	.3465525
21	20	8	3.118516	4.677774	5.11329	4.288932	.3844118
22	21	7.9	3.144778	4.717166	5.07009	4.180166	.425784
23	22	7.8	3.171599	4.757398	5.03649	4.076653	.4708945
24	23	7.7	3.199	4.798499	5.01249	3.978214	.5199699
25	24	7.6	3.227	4.8405	4.99809	3.884672	.5732366
26	25	7.5	3.255621	4.883431	4.99329	3.795852	.6309195
27	26	7.4	3.284885	4.927328	4.99809	3.711581	.6932404
28	27	7.3	3.314816	4.972224	5.01249	3.631689	.7604184
29	28	7.2	3.345438	5.018157	5.03649	3.555996	.8326744
30	29	7.1	3.376777	5.065166	5.07009	3.484272	.9102725
31	30	7	3.408861	5.113292	5.11329	3.410047	.9989574
32	31	6.9	3.441718	5.162577	5.16609	3.353488	1.081025
33	32	6.8	3.475378	5.213067	5.22849	3.293049	1.17547
34	33	6.7	3.509873	5.26481	5.30049	3.236254	1.275695
35	34	6.6	3.545238	5.317857	5.38209	3.182867	1.381913
36	35	6.5	3.581507	5.37226	5.47329	3.132749	1.494241
37	36	6.4	3.618717	5.428076	5.57409	3.085778	1.612761
38	37	6.3	3.65691	5.485365	5.68449	3.041846	1.737527
39	38	6.2	3.696127	5.54419	5.80449	3.00085	1.868564
40	39	6.1	3.736412	5.604618	5.93409	2.962696	2.005871
41	40	6	3.777813	5.66672	6.07329	2.927299	2.149416
42	41	5.9	3.820381	5.730571	6.22209	2.894578	2.299135
43	42	5.8	3.864168	5.796252	6.38049	2.86446	2.454933
44	43	5.7	3.909232	5.863848	6.54849	2.836881	2.616683
45	44	5.6	3.955633	5.933449	6.72609	2.811781	2.78422
46	45	5.5	4.003436	6.005154	6.91329	2.789109	2.957346
47	46	5.4	4.05271	6.079064	7.11009	2.768822	3.135823
48	47	5.3	4.103528	6.155292	7.31649	2.750881	3.319379
49	48	5.2	4.15597	6.233955	7.53249	2.735259	3.507699
50	49	5.1	4.210121	6.315181	7.75809	2.721932	3.70043
51	50	5	4.26607	6.399105	7.99329	2.710886	3.897179

Table giving the incorrect root, i.e. the supercritical depth, after the control section at element 31. (Model PB2_156C.TK) Notice in this table that the Froude Number squared increases to element 31, where it is unity, and then decreases thereafter again.

═══════════════ TABLE: ══════════════════════

Title:

Element	x	b	Yc	Ec	E	Y	Fr2
1	0	10	2.687456	4.031184	7.99329	7.835204	.0403528
2	1	9.9	2.705523	4.058284	7.75809	7.586024	.045364
3	2	9.8	2.723897	4.085845	7.53249	7.345191	.0509991
4	3	9.7	2.742586	4.113878	7.31649	7.112601	.0573318
5	4	9.6	2.761598	4.142398	7.11009	6.888144	.0644429
6	5	9.5	2.780944	4.171416	6.91329	6.671703	.0724214
7	6	9.4	2.800632	4.200949	6.72609	6.463154	.0813645
8	7	9.3	2.820673	4.231009	6.54849	6.262368	.0913782
9	8	9.2	2.841076	4.261613	6.38049	6.069208	.1025776
10	9	9.1	2.861851	4.292777	6.22209	5.88353	.1150873
11	10	9	2.883011	4.324517	6.07329	5.705187	.1290415
12	11	8.9	2.904567	4.35685	5.93409	5.534024	.1445843
13	12	8.8	2.926529	4.389794	5.80449	5.36988	.1618695
14	13	8.7	2.948912	4.423368	5.68449	5.212592	.1810608
15	14	8.6	2.971728	4.457592	5.57409	5.06199	.2023314
16	15	8.5	2.99499	4.492485	5.47329	4.917902	.2258638
17	16	8.4	3.018713	4.528069	5.38209	4.780151	.2518492
18	17	8.3	3.042911	4.564367	5.30049	4.64856	.280487
19	18	8.2	3.0676	4.6014	5.22849	4.522947	.3119837
20	19	8.1	3.092796	4.639194	5.16609	4.403132	.3465525
21	20	8	3.118516	4.677774	5.11329	4.288932	.3844118
22	21	7.9	3.144778	4.717166	5.07009	4.180166	.425784
23	22	7.8	3.171599	4.757398	5.03649	4.076653	.4708945
24	23	7.7	3.199	4.798499	5.01249	3.978214	.5199699
25	24	7.6	3.227	4.8405	4.99809	3.884672	.5732366

26	25	7.5	3.255621	4.883431	4.99329	3.795852	.6309195
27	26	7.4	3.284885	4.927328	4.99809	3.711581	.6932404
28	27	7.3	3.314816	4.972224	5.01249	3.631689	.7604184
29	28	7.2	3.345438	5.018157	5.03649	3.555996	.8326744
30	29	7.1	3.376777	5.065166	5.07009	3.484272	.9102725
31	30	7	3.408861	5.113292	5.11329	3.410047	.9989574
32	31	6.9	3.441718	5.162577	5.16609	3.533071	.924419
33	32	6.8	3.475378	5.213067	5.22849	3.671419	.8482122
34	33	6.7	3.509873	5.26481	5.30049	3.815221	.7786021
35	34	6.6	3.545238	5.317857	5.38209	3.964743	.7149755
36	35	6.5	3.581507	5.37226	5.47329	4.120162	.6568323
37	36	6.4	3.618717	5.428076	5.57409	4.281639	.603718
38	37	6.3	3.65691	5.485365	5.68449	4.449326	.5552139
39	38	6.2	3.696127	5.54419	5.80449	4.623373	.510933
40	39	6.1	3.736412	5.604618	5.93409	4.803924	.4705178
41	40	6	3.777813	5.66672	6.07329	4.991119	.4336384
42	41	5.9	3.820381	5.730571	6.22209	5.185095	.3999908
43	42	5.8	3.864168	5.796252	6.38049	5.385982	.3692951
44	43	5.7	3.909232	5.863848	6.54849	5.593906	.3412942
45	44	5.6	3.955633	5.933449	6.72609	5.80899	.3157521
46	45	5.5	4.003436	6.005154	6.91329	6.031348	.2924527
47	46	5.4	4.05271	6.079064	7.11009	6.261093	.2711979
48	47	5.3	4.103528	6.155292	7.31649	6.498328	.2518068
49	48	5.2	4.15597	6.233955	7.53249	6.743155	.2341143
50	49	5.1	4.210121	6.315181	7.75809	6.995668	.2179697
51	50	5	4.26607	6.399105	7.99329	7.255955	.2032359

157.

For the channel of the previous problem investigate how the position, and magnitude of the critical depth varies with the flowrate, using values of Q from 50 cfs to 400 cfs.

Wanted: Solve the previous problem for different flow rates.

Solution:

The following table is made up using the TK-Solver model in the previous problem, PB2_156A.TK, by changing the 250 for Q in the rule sheet to the proper flow rate. Note the position of critical flow is relatively stable.

Q(cfs)	x(ft)	H_c(ft)	Y_c(ft)
50	27	4.651	3/31
100	28	5.681	3.35
150	28	6.527	3.35
200	29	7.288	3.38
250	30	7.993	3.411
300	31	8.657	3.44
350	32	9.289	3.48
400	32	9.896	3.48

158.

Rather than have the bottom of the channel in the previous two problem rise according to a 2nd degree polynomial the bottom rise 3 ft linearly over the first 10 ft of the transition, and then falls 3 ft over the last 40 ft of the transition. Now where does critical depth occur?

Given: Rather than have the bottom of the channel in the previous two problems rise according to a 2^{nd} degree polynomial the bottom rises 3 ft linearly over the first 10 ft of the transition, and then falls 3 ft over the last 40 ft of the transition. Now where does critical depth occur?

Solution:

Now the control is at the top of the linearly varying channel bottom, i.e. at x = 10 ft. Two TK-Solver models, one for the first 10 ft, and the second for the last 40 ft show that H_c occurs at x = 10 ft.

Now the control is at the top of the linearly varying channel bottom, i.e. at x = 10 ft. Two TK-Solver models, one for the first 10 ft and the second for the last 40 ft show that H_c occurs at x = 10 ft.

TK-Solver Model PB2 158A.TK (for first 10 ft, i.e. position 0 to 10ft)

```
==================== VARIABLE SHEET ====================
St Input---- Name--- Output--- Unit----- Comment--------
LG 7.7811838 H
L  0         x

==================== RULE SHEET ========================
S Rule--------------------------------------------------
  H=1.5*((250/(10-.1*x))^2/32.2)^.3333333+x/3.3333333

==================== TABLE: Prb2_158a ==================
Title:
Element x--------  H--------
1       0          4.03118378
2       .5         4.1946773
3       1          4.3582843
4       1.5        4.52200633
5       2          4.68584494
6       2.5        4.84980173
7       3          5.01387832
8       3.5        5.17807636
9       4          5.34239755
10      4.5        5.50684359
11      5          5.67141624
12      5.5        5.83611729
13      6          6.00094855
14      6.5        6.16591187
15      7          6.33100916
16      7.5        6.49624234
17      8          6.66161337
18      8.5        6.82712427
19      9          6.99277708
20      9.5        7.15857389
21      10         7.32451683
```

TK-Solver Model PB2_158.TK (for last 40 ft, i.e. position 10 to 50ft)

```
==================== VARIABLE SHEET ====================
St Input---- Name--- Output--- Unit----- Comment--------
L            H       7.7811838
L  0         x
==================== RULE SHEET ========================
S Rule--------------------------------------------------
  H=1.5*((250/(10-.1*x))^2/32.2)^.3333333+3-(x-10)/13.3333333
```

═══════════════════ TABLE: Prb2_158 ═══════════════════

Title:

Element	x	H
1	10	7.3245168
2	11	7.28184982
3	12	7.23979406
4	13	7.19836821
5	14	7.15759177
6	15	7.11748507
7	16	7.07806934
8	17	7.03936673
9	18	7.00140038
10	19	6.96419448
11	20	6.92777433
12	21	6.8921664
13	22	6.8573984
14	23	6.82349935
15	24	6.79049967
16	25	6.75843128
17	26	6.72732765
18	27	6.69722396
19	28	6.66815715
20	29	6.64016607
21	30	6.6132916
22	31	6.58757681
23	32	6.56306706
24	33	6.53981018
25	34	6.51785668
26	35	6.49725989
27	36	6.4780762
28	37	6.46036525
29	38	6.44419024
30	39	6.42961812
31	40	6.41671994
32	41	6.40557113
33	42	6.39625192
34	43	6.38884764
35	44	6.38344921
36	45	6.3801536
37	46	6.37906434
38	47	6.38029208
39	48	6.38395528
40	49	6.39018088
41	50	6.39910507

159.

A trapezoidal channel which is b = 5 m wide with a side slope m = 1, except near its end carries a flowrate Q = 25 m^3/s. Before ending in a free overfall at its end the channel has a 25 m long transition in which the bottom width reduces to 2.5 m, and the side slope m to 0, both linearly. Simultaneously the bottom of the channel rises according to the 2nd degree polynomial, z = (x/12.5)(2 - x/12.5). Find the position where critical depth occurs, and then make up a table as in Problem 156 that gives the Froude number as a function of x.

Given: Trapezoidal channel with b = 5 m, m=1 and Q=25 m^3/s has bottom width reduce and bottom rise.
Find: Position where critical flow occurs and then make up a tabale that gives the Froude Numbers as a function of x.

Solution:

Since simplified equations do not occur as for a rectangular channel it is necessary that first the critical flow equation be solved for Y_c at each position x along the transition over the last 25 m of channel length. From this Y_c, E_c is solved and there from $H_c=E_c+z$. The TK-Solver model to accomplish this "list" solution is given below. Note that the critical depth occurs at x = 18 m when H_c = 3.3203 m (because this is where E=2.5 m, the head available at the reservoir.)

```
TK-Solver Model PB2 159.TK
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment----------
L             E
LG 2          Y
   25         Q
L             A
   9.81       g
L             H
L             z
L             b
L             m
L             T
L  0          x

==================== RULE SHEET ==========================
S Rule-------------------------------------------------------
* E=Y+(Q/A)^2/(2*g)
* H=E+z
* A=(b+m*Y)*Y
* T=b+2*m*Y
* m=1-x/25
* b=5-.1*x
* z=(x/12.5)*(2-x/12.5)
* Q^2*T/(g*A^3)=1

==================== TABLE: prb2_159 =====================
Title:
```

Element	x	z	m	b	T	A	H	Y
1	0	0	1	5	7.50159	7.818464	1.771916	1.250795
2	.5	.0784	.98	4.95	7.418735	7.789572	1.862952	1.259558
3	1	.1536	.96	4.9	7.335508	7.760334	1.951051	1.268494
4	1.5	.2256	.94	4.85	7.251902	7.730738	2.036222	1.277607
5	2	.2944	.92	4.8	7.167904	7.700774	2.118475	1.286904
6	2.5	.36	.9	4.75	7.083506	7.670431	2.197821	1.296392
7	3	.4224	.88	4.7	6.998695	7.639695	2.274271	1.306076
8	3.5	.4816	.86	4.65	6.91346	7.608554	2.347836	1.315965
9	4	.5376	.84	4.6	6.827789	7.576996	2.418529	1.326065
10	4.5	.5904	.82	4.55	6.74167	7.545004	2.486364	1.336384
11	5	.64	.8	4.5	6.655089	7.512566	2.551353	1.346931
12	5.5	.6864	.78	4.45	6.568033	7.479664	2.613512	1.357713

13	6	.7296	.76	4.4	6.480486	7.446283	2.672857	1.368741
14	6.5	.7696	.74	4.35	6.392434	7.412404	2.729403	1.380023
15	7	.8064	.72	4.3	6.303861	7.37801	2.783168	1.39157
16	7.5	.84	.7	4.25	6.21475	7.34308	2.834171	1.403393
17	8	.8704	.68	4.2	6.125084	7.307593	2.882433	1.415503
18	8.5	.8976	.66	4.15	6.034843	7.271528	2.927974	1.427912
19	9	.9216	.64	4.1	5.944009	7.23486	2.970816	1.440632
20	9.5	.9424	.62	4.05	5.85256	7.197566	3.010985	1.453678
21	10	.96	.6	4	5.760476	7.159617	3.048506	1.467063
22	10.5	.9744	.58	3.95	5.667732	7.120986	3.083408	1.480803
23	11	.9856	.56	3.9	5.574305	7.081641	3.115719	1.494915
24	11.5	.9936	.54	3.85	5.480169	7.041551	3.145473	1.509416
25	12	.9984	.52	3.8	5.385296	7.00068	3.172704	1.524323
26	12.5	1	.5	3.75	5.289658	6.95899	3.19745	1.539658
27	13	.9984	.48	3.7	5.193223	6.916441	3.219751	1.555441
28	13.5	.9936	.46	3.65	5.095959	6.872989	3.239652	1.571695
29	14	.9856	.44	3.6	4.997831	6.828587	3.257199	1.588444
30	14.5	.9744	.42	3.55	4.898801	6.783184	3.272446	1.605715
31	15	.96	.4	3.5	4.798829	6.736724	3.285449	1.623536
32	15.5	.9424	.38	3.45	4.697872	6.689147	3.296271	1.641937
33	16	.9216	.36	3.4	4.595885	6.640387	3.304979	1.660951
34	16.5	.8976	.34	3.35	4.492817	6.590372	3.311648	1.680613
35	17	.8704	.32	3.3	4.388616	6.539023	3.316361	1.700962
36	17.5	.84	.3	3.25	4.283224	6.486254	3.319209	1.722039
37	18	.8064	.28	3.2	4.176578	6.431968	3.320294	1.743889
38	18.5	.7696	.26	3.15	4.068612	6.37606	3.319728	1.766561
39	19	.7296	.24	3.1	3.959252	6.318414	3.317639	1.790109
40	19.5	.6864	.22	3.05	3.848419	6.258897	3.314166	1.814589
41	20	.64	.2	3	3.736026	6.197363	3.309471	1.840065
42	20.5	.5904	.18	2.95	3.621978	6.13365	3.303733	1.866607
43	21	.5376	.16	2.9	3.506172	6.067569	3.297158	1.894289
44	21.5	.4816	.14	2.85	3.388494	5.998913	3.289981	1.923193
45	22	.4224	.12	2.8	3.268818	5.927441	3.282473	1.953409
46	22.5	.36	.1	2.75	3.147007	5.852879	3.274946	1.985034
47	23	.2944	.08	2.7	3.022908	5.774911	3.267765	2.018174
48	23.5	.2256	.06	2.65	2.896353	5.69317	3.261359	2.052942
49	24	.1536	.04	2.6	2.767157	5.607229	3.256235	2.08946
50	24.5	.0784	.02	2.55	2.635114	5.516583	3.252999	2.127854
51	25	0	0	2.5	2.5	5.420637	3.252382	2.168255

The TK-Solver model to make a table that eventually gives the Froude Number as a function of the position along the channel can be as follows: (The first tabale shows that the Froude Numbers after the critical section are larger than unity. This is what will occur, i.e., the flow will be supercritical when it goes off the end of the channel. The second tabale gives a solution in which the Froude Numbers increase to unity, and then decrease again after the critical section. This is not the correct solution, but can be obtained depending upon the guesses used for Y. It represents the alternative depths after the critical section.

TK-Solver Model PB2 159A.TK

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
L             E
LG 2          Y
   25         Q
L             A
   9.81       g
   3.3203     H
L             z
L             b
L             m
L             T
L  0          x
L             Fr2
```

```
═══════════════════ RULE SHEET ════════════════════════
S Rule─────────────────────────────────────────────────
  E=Y+(Q/A)^2/(2*g)
  H=E+z
  A=(b+m*Y)*Y
  T=b+2*m*Y
  m=1-x/25
  b=5-.1*x
  z=(x/12.5)*(2-x/12.5)
  Fr2=Q^2*T/(g*A^3)
```

Correct Supercritical depths after the control section at element 39 and beyond

═══════════════════ TABLE: prb2_159 ═══════════════════

Title:

Element	x	z	m	b	T	A	E	Y	Fr2
1	0	0	1	5	11.554	27.12374	3.3203	3.277001	.0368888
2	.5	.0784	.98	4.95	11.21039	25.80875	3.2419	3.194076	.0415461
3	1	.1536	.96	4.9	10.87872	24.56683	3.1667	3.113918	.0467457
4	1.5	.2256	.94	4.85	10.55861	23.39407	3.0947	3.036494	.0525412
5	2	.2944	.92	4.8	10.24965	22.28677	3.0259	2.961766	.05899
6	2.5	.36	.9	4.75	9.951457	21.24139	2.9603	2.889698	.0661529
7	3	.4224	.88	4.7	9.663642	20.25454	2.8979	2.820251	.0740943
8	3.5	.4816	.86	4.65	9.38582	19.323	2.8387	2.753384	.0828819
9	4	.5376	.84	4.6	9.117612	18.44371	2.7827	2.689055	.0925865
10	4.5	.5904	.82	4.55	8.858644	17.61374	2.7299	2.627222	.1032817
11	5	.64	.8	4.5	8.608545	16.83033	2.6803	2.567841	.1150438
12	5.5	.6864	.78	4.45	8.366951	16.09082	2.6339	2.510866	.1279509
13	6	.7296	.76	4.4	8.133505	15.39273	2.5907	2.456253	.1420828
14	6.5	.7696	.74	4.35	7.907856	14.73368	2.5507	2.403957	.1575202
15	7	.8064	.72	4.3	7.689658	14.1114	2.5139	2.353929	.1743442
16	7.5	.84	.7	4.25	7.478576	13.52378	2.4803	2.306125	.192635
17	8	.8704	.68	4.2	7.274279	12.9688	2.4499	2.260499	.2124719
18	8.5	.8976	.66	4.15	7.076447	12.44455	2.4227	2.217005	.2339316
19	9	.9216	.64	4.1	6.884767	11.94922	2.3987	2.175599	.2570876
20	9.5	.9424	.62	4.05	6.698933	11.48113	2.3779	2.136236	.2820089
21	10	.96	.6	4	6.518649	11.03866	2.3603	2.098874	.3087588
22	10.5	.9744	.58	3.95	6.343628	10.62031	2.3459	2.063473	.3373937
23	11	.9856	.56	3.9	6.173591	10.22466	2.3347	2.029992	.3679619
24	11.5	.9936	.54	3.85	6.008267	9.850357	2.3267	1.998395	.4005016
25	12	.9984	.52	3.8	5.847394	9.49616	2.3219	1.968648	.4350401
26	12.5	1	.5	3.75	5.690717	9.160881	2.3203	1.940717	.4715918
27	13	.9984	.48	3.7	5.537992	8.843414	2.3219	1.914575	.5101565
28	13.5	.9936	.46	3.65	5.38898	8.54272	2.3267	1.890196	.5507178
29	14	.9856	.44	3.6	5.24345	8.257824	2.3347	1.867557	.5932412
30	14.5	.9744	.42	3.55	5.101179	7.987817	2.3459	1.846642	.637672
31	15	.96	.4	3.5	4.96195	7.731845	2.3603	1.827438	.6839338
32	15.5	.9424	.38	3.45	4.825552	7.489112	2.3779	1.809937	.7319258
33	16	.9216	.36	3.4	4.691777	7.258872	2.3987	1.794135	.7815215
34	16.5	.8976	.34	3.35	4.560425	7.040425	2.4227	1.780037	.8325681
35	17	.8704	.32	3.3	4.431293	6.833094	2.4499	1.767646	.88489
36	17.5	.84	.3	3.25	4.304167	6.636125	2.4803	1.756944	.9383316
37	18	.8064	.28	3.2	4.178036	6.442846	2.5139	1.746493	.9952908
38	18.5	.7696	.26	3.15	4.081896	6.480165	2.5507	1.792107	.9556848
39	19	.7296	.24	3.1	3.933393	6.105809	2.5907	1.736234	1.100902
40	19.5	.6864	.22	3.05	3.81252	5.946376	2.6339	1.733001	1.155224
41	20	.64	.2	3	3.692732	5.795338	2.6803	1.73183	1.208711
42	20.5	.5904	.18	2.95	3.573818	5.652324	2.7299	1.732827	1.260847
43	21	.5376	.16	2.9	3.455563	5.517051	2.7827	1.736133	1.311019
44	21.5	.4816	.14	2.85	3.33774	5.389302	2.8387	1.741929	1.35852
45	22	.4224	.12	2.8	3.220107	5.268933	2.8979	1.750445	1.402533
46	22.5	.36	.1	2.75	3.102395	5.15588	2.9603	1.761973	1.442114
47	23	.2944	.08	2.7	2.984302	5.050184	3.0259	1.776888	1.476156
48	23.5	.2256	.06	2.65	2.865482	4.952027	3.0947	1.795682	1.503349
49	24	.1536	.04	2.6	2.745521	4.861791	3.1667	1.819015	1.522114
50	24.5	.0784	.02	2.55	2.623912	4.780163	3.2419	1.847794	1.530496
51	25	0	0	2.5	2.5	4.708332	3.3203	1.883333	1.525983

Incorrect alternative depths after the control section

═══════════════════ TABLE: prb2_159 ═══════════════════

Title:

Element	x	z	m	b	T	A	E	Y	Fr2

1	0	0	1	5	11.554	27.12374	3.3203	3.277001	.0368888
2	.5	.0784	.98	4.95	11.21039	25.80875	3.2419	3.194076	.0415461
3	1	.1536	.96	4.9	10.87872	24.56683	3.1667	3.113918	.0467457
4	1.5	.2256	.94	4.85	10.55861	23.39407	3.0947	3.036494	.0525412
5	2	.2944	.92	4.8	10.24965	22.28677	3.0259	2.961766	.05899
6	2.5	.36	.9	4.75	9.951457	21.24139	2.9603	2.889698	.0661529
7	3	.4224	.88	4.7	9.663642	20.25454	2.8979	2.820251	.0740943
8	3.5	.4816	.86	4.65	9.38582	19.323	2.8387	2.753384	.0828819
9	4	.5376	.84	4.6	9.117612	18.44371	2.7827	2.689055	.0925865
10	4.5	.5904	.82	4.55	8.858644	17.61374	2.7299	2.627222	.1032817
11	5	.64	.8	4.5	8.608545	16.83033	2.6803	2.567841	.1150438
12	5.5	.6864	.78	4.45	8.366951	16.09082	2.6339	2.510866	.1279509
13	6	.7296	.76	4.4	8.133505	15.39273	2.5907	2.456253	.1420828
14	6.5	.7696	.74	4.35	7.907856	14.73368	2.5507	2.403957	.1575202
15	7	.8064	.72	4.3	7.689658	14.1114	2.5139	2.353929	.1743442
16	7.5	.84	.7	4.25	7.478576	13.52378	2.4803	2.306125	.192635
17	8	.8704	.68	4.2	7.274279	12.9688	2.4499	2.260499	.2124719
18	8.5	.8976	.66	4.15	7.076447	12.44455	2.4227	2.217005	.2339316
19	9	.9216	.64	4.1	6.884767	11.94922	2.3987	2.175599	.2570876
20	9.5	.9424	.62	4.05	6.698933	11.48113	2.3779	2.136236	.2820089
21	10	.96	.6	4	6.518649	11.03866	2.3603	2.098874	.3087588
22	10.5	.9744	.58	3.95	6.343628	10.62031	2.3459	2.063473	.3373937
23	11	.9856	.56	3.9	6.173591	10.22466	2.3347	2.029992	.3679619
24	11.5	.9936	.54	3.85	6.008267	9.850357	2.3267	1.998395	.4005016
25	12	.9984	.52	3.8	5.847394	9.49616	2.3219	1.968648	.4350401
26	12.5	1	.5	3.75	5.690717	9.160881	2.3203	1.940717	.4715918
27	13	.9984	.48	3.7	5.537992	8.843414	2.3219	1.914575	.5101565
28	13.5	.9936	.46	3.65	5.38898	8.54272	2.3267	1.890196	.5507178
29	14	.9856	.44	3.6	5.24345	8.257824	2.3347	1.867557	.5932412
30	14.5	.9744	.42	3.55	5.101179	7.987817	2.3459	1.846642	.637672
31	15	.96	.4	3.5	4.96195	7.731845	2.3603	1.827438	.6839338
32	15.5	.9424	.38	3.45	4.825552	7.489112	2.3779	1.809937	.7319258
33	16	.9216	.36	3.4	4.691777	7.258872	2.3987	1.794135	.7815215
34	16.5	.8976	.34	3.35	4.560425	7.040425	2.4227	1.780037	.8325681
35	17	.8704	.32	3.3	4.431293	6.833094	2.4499	1.767646	.88489
36	17.5	.84	.3	3.25	4.304167	6.636125	2.4803	1.756944	.9383316
37	18	.8064	.28	3.2	4.178036	6.442846	2.5139	1.746493	.9952908
38	18.5	.7696	.26	3.15	4.081896	6.480165	2.5507	1.792107	.9556848
39	19	.7296	.24	3.1	3.986221	6.541622	2.5907	1.846294	.9072271
40	19.5	.6864	.22	3.05	3.886664	6.595068	2.6339	1.90151	.8632373
41	20	.64	.2	3	3.783092	6.639729	2.6803	1.957729	.8233924
42	20.5	.5904	.18	2.95	3.67534	6.674481	2.7299	2.014834	.7875098
43	21	.5376	.16	2.9	3.563251	6.698061	2.7827	2.07266	.7554576
44	21.5	.4816	.14	2.85	3.446677	6.709078	2.8387	2.13099	.7271484
45	22	.4224	.12	2.8	3.32549	6.70601	2.8979	2.189542	.702545
46	22.5	.36	.1	2.75	3.19959	6.687187	2.9603	2.247949	.6816711
47	23	.2944	.08	2.7	3.068916	6.650774	3.0259	2.305727	.6646292
48	23.5	.2256	.06	2.65	2.933468	6.594734	3.0947	2.362236	.6516291
49	24	.1536	.04	2.6	2.793329	6.516777	3.1667	2.416607	.6430348
50	24.5	.0784	.02	2.55	2.648706	6.414265	3.2419	2.467639	.6394461
51	25	0	0	2.5	2.5	6.284044	3.3203	2.513618	.6418497

The problem statement specified a flow rate Q = 25^3/s, and ask that the reservoir head H be found that will result in this flow rate. If we recognize that the linear reduction of b and m with x could be used in place of these variable where ever they occur, and that A and T are geometry equations that could be included in the other equations also, and H=E+z just introduces the specific energy E, then there are basically only three equations, the energy equation, the critical flow equation, and the given equation giving z as a function of x. Therefore three variables may be solved. Rather than specifying the flow rate Q let's assume that the reservoir head H is specified, and the three variables solved are Y, E and Q. If these are the unknowns, then the position x where critical flow is to occur must be specified. Below solutions are given for which the head H of the reservoir has been specified at H = 3.5 m and H = 4.0 m, and the position x where critical flow occurs as been given from 5 m to 25 m (the end of the converging channel.) TK-Solver Model PB2_159C.TK has been used to obtain these solutions.

H=3.5 m and x = 5 m

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output--- Unit----- Comment---------
              Y        2.0508
              E        2.86
              Q        50.17812
   5          x
              A        12.593224
   9.81       g
   3.5        H
              z        .64
              b        4.5
              m        .8
              T        7.7812799

==================== RULE SHEET =========================
S Rule----------------------------------------------------
  E=Y+(Q/A)^2/(2*g)
  H=E+z
  A=(b+m*Y)*Y
  T=b+2*m*Y
  m=1-x/25
  b=5-.1*x
  z=(x/12.5)*(2-x/12.5)
  Q^2*T/(g*A^3)=1
```

H=3.5 m and x = 10 m

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output--- Unit----- Comment---------
              Y        1.7983712
              E        2.54
              Q        34.841937
   10         x
              A        9.133968
   9.81       g
   3.5        H
              z        .96
              b        4
              m        .6
              T        6.1580454
```

H=3.5 m and x = 15 m

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output--- Unit----- Comment---------
              Y        1.7790497
              E        2.54
              Q        28.951088
   15         x
              A        7.4926813
   9.81       g
   3.5        H
              z        .96
              b        3.5
              m        .4
              T        4.9232398
```

H=3.5 m and x = 20 m

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output--- Unit----- Comment---------
              Y        1.9752978
              E        2.86
              Q        27.940105
   20         x
              A        6.7062536
   9.81       g
   3.5        H
              z        .64
```

```
             b       3
             m       .2
             T       3.7901191

H=3.5 m and x = 25 m
==================== VARIABLE SHEET =====================
St Input──── Name─── Output─── Unit───── Comment─────────
             Y       2.3333333
             E       3.5
             Q       27.908705
   25        x
             A       5.8333333
   9.81      g
   3.5       H
             z       0
             b       2.5
             m       0
             T       2.5

H=4.0 m and x = 5 m
==================== VARIABLE SHEET =====================
St Input──── Name─── Output─── Unit───── Comment────────
             Y       2.427419
             E       3.36
             Q       66.888874
   5         x
             A       15.637276
   9.81      g
   4         H
             z       .64
             b       4.5
             m       .8
             T       8.3838704

H=4.0 m and x = 10 m
==================== VARIABLE SHEET =====================
St Input──── Name─── Output─── Unit───── Comment────────
             Y       2.1691886
             E       3.04
             Q       47.534476
   10        x
             A       11.499982
   9.81      g
   4         H
             z       .96
             b       4
             m       .6
             T       6.6030263

H=4.0 m and x = 15 m
==================== VARIABLE SHEET =====================
St Input──── Name─── Output─── Unit───── Comment────────
             Y       2.1442071
             E       3.04
             Q       39.171992
   15        x
             A       9.3437747
   9.81      g
   4         H
             z       .96
             b       3.5
             m       .4
             T       5.2153657

H=4.0 m and x = 20 m
==================== VARIABLE SHEET =====================
St Input──── Name─── Output─── Unit───── Comment────────
             Y       2.3321997
             E       3.36
             Q       36.3039
   20        x
```

```
             A       8.0844302
   9.81      g
   4         H
             z       .64
             b       3
             m       .2
             T       3.9328799
```

H=4.0 m and x = 25 m

```
==================== VARIABLE SHEET =====================
St  Input----- Name---- Output---- Unit------ Comment----------
             Y       2.6666667
             E       4
             Q       34.097898
   25        x
             A       6.6666667
   9.81      g
   4         H
             z       0
             b       2.5
             m       0
             T       2.5
```

In the problem statement the flow rate was specified to be 25 m^3/s and the reservoir head was not given, i.e. it became part of the solution such that somewhere within the reducing channel near its end critical flow occurs and the flow became supercritical thereafter. Still another way one could pose the problem differently is to ask for a given reservoir head H and flow rate Q where would the flow become critical, then the problem can be solved with the model below in which Y and x are the two variables being solves with H and given. In this model the variable E has been eliminated. The first variable sheet show the solution in which H=3.3203 m as found above. In this model a G is place in the status column for the first two variables Y and x. After this solution a series of solution are given in which the reservoir head H has been modestly decreased to 2.5 m in a few steps. Because of the nature of the equations being solved close guesses are needed. Also another root is possible that gives the position x larger than the 18.13 m as H is modestly decreased. It is not possible to solve the problem for H's larger than 3.3202 m. The position x gets larger than the end of the channel, i.e. larger than 25 m.

TK-Solver Model PB2 159B.TK

```
==================== VARIABLE SHEET =====================
St  Input----- Name---- Output---- Unit------ Comment---------
             Y       1.7497944
             x       18.132035
   25        Q
             A       6.4173667
   9.81      g
   3.3203    H                            <--
             z       .7969932
             b       3.1867965
             m       .27471862
             T       4.1481987

==================== RULE SHEET =========================
S Rule------------------------------------------------------
  H=Y+(Q/A)^2/(2*g)+z
  A=(b+m*Y)*Y
  T=b+2*m*Y
  m=1-x/25
  b=5-.1*x
```

```
  z=(x/12.5)*(2-x/12.5)
  Q^2*T/(g*A^3)=1
```

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
                Y       1.7335914
                x       17.76662
   25           Q
                A       6.4575014
   9.81         g
   3.32         H                                <--
                z       .82248134
                b       3.223338
                m       .28933518
                T       4.2265159
```

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
                Y       1.6751003
                x       16.361511
   25           Q
                A       6.6043549
   9.81         g
   3.31         H                                <--
                z       .90456789
                b       3.3638489
                m       .34553956
                T       4.5214757
```

This VARIBLE SHEET shows another possible solution in which x has increased

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
                Y       1.8374235
                x       19.949086
   25           Q
                A       6.2037261
   9.81         g
   3.31         H                                <--
                z       .64487114
                b       3.0050914
                m       .20203655
                T       3.7475448
```

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
                Y       1.6494598
                x       15.699803
   25           Q
                A       6.6698079
   9.81         g
   3.3          H                                <--
                z       .93447208
                b       3.4300197
                m       .37200789
                T       4.6572438
```

```
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
                Y       1.487862
                x       10.751778
   25           Q
                A       7.1012646
   9.81         g
   3.1          H                                <--
                z       .9804398
                b       3.9248222
                m       .5699289
                T       5.6207734
```

```
════════════════════ VARIABLE SHEET ═════════════════════
St Input──── Name─── Output─── Unit───── Comment────────
             Y       1.4499894
             x       9.3599255
   25        Q
             A       7.2080785
   9.81      g
   3         H                               <--
             z       .93689556
             b       4.0640075
             m       .62560298
             T       5.8782428

════════════════════ VARIABLE SHEET ═════════════════════
St Input──── Name─── Output─── Unit───── Comment────────
             Y       1.4201667
             x       8.1893712
   25        Q
             A       7.2940029
   9.81      g
   2.9       H                               <--
             z       .88107827
             b       4.1810629
             m       .67242515
             T       6.0909745

════════════════════ VARIABLE SHEET ═════════════════════
St Input──── Name─── Output─── Unit───── Comment────────
             Y       1.3953715
             x       7.1620517
   25        Q
             A       7.3667484
   9.81      g
   2.8       H                               <--
             z       .81764037
             b       4.2837948
             m       .71351793
             T       6.2750399

════════════════════ VARIABLE SHEET ═════════════════════
St Input──── Name─── Output─── Unit───── Comment────────
             Y       1.3740549
             x       6.2369363
   25        Q
             A       7.4302918
   9.81      g
   2.7       H                               <--
             z       .74895381
             b       4.3763064
             m       .75052255
             T       6.4388247

════════════════════ VARIABLE SHEET ═════════════════════
St Input──── Name─── Output─── Unit───── Comment────────
             Y       1.3553063
             x       5.3893528
   25        Q
             A       7.4869859
   9.81      g
   2.6       H                               <--
             z       .67640766
             b       4.4610647
             m       .78442589
             T       6.5873394

════════════════════ VARIABLE SHEET ═════════════════════
St Input──── Name─── Output─── Unit───── Comment────────
             Y       1.3385404
             x       4.6031219
   25        Q
```

	A	7.5383513	
9.81	g		
2.5	H		<--
	z	.60089162	
	b	4.5396878	
	m	.81587512	
	T	6.7238515	

160.

Figure A-2 (in Appendix A) provides a plot of several dimensionless variables as functions of the dimensionless dpeth Y= = Y/D for circular channels. Prove that the maximum flow rate Q will occur in a given circular channel (D, S_o and n fixed) according to Manning=s Equation when β = 2.6391 rad (a corresponding dimensionless depth Y= = Y/D = .93818. Also prove that the dimensionless hydraulic radius R_h > = R_h/D^2 occurs when β = 2.2468 rad, and the maximum dimensionless conveyance K= = A=(A=/P=) = .5015, i.e. verify some of the values given on Fig. A-2. What value of A= is associated with the maximum Q? Give a physical explanation why the maximum Q is associated with a larger β than the maximum hydraulic radius.

Wanted: Verify results in Figure A-2 of Appendix A.

Maximum Q in Circular Channels using Mannings Equation

$Q=C_u/nA^{5/3}/P^{2/3}S_o^{1/2} = C(A^{5/3})/(P^{2/3})$

$$\frac{dQ}{d\beta} = C[A^{5/3}(-2/3)P^{-5/3}\frac{dP}{d\beta} + \frac{5}{3}\frac{A^{2/3}}{P^{2/3}}\frac{dA}{d\beta}] = 0$$

divide by $(2/3)(A/P)^{2/3}C$, with $P=D\beta$, $dP/d\beta=D$

$A=D^2/4(\beta-\cos\beta\sin\beta)$, $dA/d\beta=D^2/4(1-\cos^2\beta+\sin^2\beta)=(D^2/2)\sin^2\beta$

$(5/4)D^2\sin^2\beta-AD/P = 0$

$(5/4)D^2\sin^2\beta-D^2/4(\beta-\cos\beta\sin\beta)D/(D\beta) = 0$

$5\sin^2\beta - 1 + \sin\beta\cos\beta/\beta = 0$ ← or $5\sin^2\beta + \sin\beta\cos\beta/\beta = 1$ ←

Solution: β = 2.6390536 rad ←

$\cos\beta = 1-2Y/D$, or $Y'=Y/D = .5[1-\cos\beta]= .93818122$ ←

$A_d =(\beta-\cos\beta\sin\beta)/4 = 3.0611555/4 = .7652888$ ←

$P' = \beta$ $A' = 4A_d/\Pi = .97439606$ ←

$R_h' = A'/P' = .97439606/2.6390536 = .3692217$ ←

$K' = A'(A'/\beta)^{2/3} = .5014878$ ←

Maximum Hydraulic Radius, R_h $R_h = A/P = AP^{-1}$

$$\frac{dR_h}{d\beta} = -AP^{-2}\frac{dP}{d\beta} + P^{-1}\frac{dA}{d\beta} = 0 \quad \text{or} \quad -\frac{AD}{P^2} + \frac{D^2\sin^2\beta}{2P} = 0$$

$$\frac{D\sin^2\beta}{2} - \frac{D^2/4(\beta-\cos\beta\sin\beta)}{D\beta} = 0$$

$2\beta\sin^2\beta - \beta + \cos\beta\sin\beta = 0$ ← Solution $\beta = 2.246707$ ←

$A_d = 2.7347645/4$, $A' = .87050259$ ←

$R_h' = A'/P' = A'/\beta = .3874575$ ←

The reason why β associated with R_h-max. is that in Mannings Equation Q is linearly dependent upon $AR_h^{2/3}$ and A continues to increase with β so $AR_h^{2/3}$ – Maximum is associated with a larger β than R_h-max.

The question might also be ask, "What portion of full flow in a circular channel produces the maximum velocity of flow according to Mannings Equation?"

$V=(C_u/n)A^{2/2}/P^{2/3}S_o^{1/2} = CA^{2/3}P^{-2/3}$

$dV/d\beta = C[A^{2/3}(-2/3)P^{-5/3}(dP/d\beta) +(2/3)P^{-2/3}A^{-1/3}(dA/d\beta) = 0$

with $dP/d\beta = D$ and $dA/d\beta = D^2\sin^2\beta/2$

$$\frac{D^2\sin^2\beta}{2P^{2/3}A^{1/3}} = \frac{A^{2/3}D}{P^{5/3}} = 0$$

$D\sin^2\beta - 2(A/P) = 0$

$D\sin^2\beta - \beta + \cos\beta\sin\beta = 0$

Solution: $\beta = 2.2467047$ rad ←

$Y'=Y/D = .5(1-\cos\beta) = .81280312$ ←

$A_d= (\beta - \cos\beta\sin\beta)/4 = 2.7347645/4 = .68369113$ ←

$A' = 4A_d/\pi = .8705026$ ←

$R_h' = A'/\beta = .3874575$

$K' = A'R_h'^{2/3} = .46265023$ ←

161.

As in the previous problem determine the angle β associated with the maximum flow rate Q that will occur in a circular channel based on Chezy=s Equation, rather than Manning=s Equation, assuming that Chezy=s C is constant. Compute A=, A_d, R_h= and K= associated with this condition. Give a physical explanation why this β is larger than the β of the previous problem.

Wanted: Same as previous problem except based on Chezy's Equation

Chezy's Equation with C = Constant

$Q = CA(R_hS_o)^{1/2} = C\sqrt{S_o}A^{1/5}P^{-1/2}$

$dQ/d\beta = C\sqrt{S_o}[A^{1.5}P^{-1.5}(dP/d\beta(-.5) + P^{-5}(1.5A^{.5})(dA/d\beta] = 0$

divide by $.5(A/P)^{.5}C\sqrt{S_o}$

$$-\frac{A}{P}\frac{dP}{d\beta} + 3\frac{dA}{d\beta} = \frac{3D^2}{2}\sin^2\beta - \frac{D^2/4(\beta-\cos\beta\sin\beta)D}{D\beta} = 0$$

Or

$6\sin^2\beta - (\beta-\cos\beta\sin\beta)/\beta = 0$

Solution:

$\beta = 2.689255$ ←

$A_d = (\beta-\cos\beta\sin\beta)/4 = 3.082367/4 = .7706418$ ←

$A' = 4A_d/\pi = .981148$ ←

$R_h' = A'/P' = A'/\beta = .36484$ ←

$K' = a'R_H'^{2/3} = .5009598$ ←

The reason why β is larger than in previous problem based on Mannings Equation is that now the flow rate Q is proportional to $A^{1.5}$ rather than $A^{1.66667}$.

The question might also be ask, "What portion of the full flow in a circular channel produces the maximum velocity of flow according to Chezy's Equation if C = Constant?"

$V = C(R_hS_o)^{1/2} = KA^{.5}P^{-.5}$

$dV/d\beta = K[A^{.5}(-.5)P^{-3/2}(dP/d\beta) + P^{.5}(.5)A^{-.5}(dA/d\beta) = 0$

multiply by $.5(A/P)^{1/2}$

$dA/d\beta - (A/P)(dP/d\beta) = 0$ $\quad .5D^2\sin^2\beta - [.25D^2(\beta-\cos\beta\sin\beta)D]/(D\beta) = 0$

$2\sin^2\beta - 1 + \cos\beta\sin\beta/\beta = 0$ ← or $2\beta\sin^2\beta - \beta + \cos\beta\sin\beta = 0$ ←

which is identical to the results obtained in the previous problem based on Mannings Equation, or,

$\beta = 2.2467047$ rad ←, $Y' = Y/D = .5(1-\cos\beta) = .81280312$ ←

$A_d = (\beta - \cos\beta\sin\beta)/4 = .68369113$ ←, $A' = .8705026$ ←

$R_h' = A'/\beta = .3874575$, $K' = A'R_h'^{2/3} = .46265023$ ←

One would expect this result since V is proportional to $R_h^{2/3}$ or V is proportional $R_h^{1/2}$ will have the maximum at the same dimensionless depth.

162.

In the previous problem you were to determine angle β associated with the maximum Q in a circular channel based on Chezy=s C being constant. Determine this angle for several values of the relative roughness e/R_h if the flow is wholly rough.

Wanted: Determine angle β for several values of the relative roughness e/R_h from the previous problem if the flow is wholly rough.

Solution:

Chezy's C Equation for wholly rough flow $C = (32.2g)^{1/2}\text{Log}_{10}(12R_h/e)$

$$\frac{dC}{d\beta} = (32.2g)^{1/2}\text{Log}_{10}(e)\left[\frac{e}{12R_h}\ \frac{dR_h}{d\beta}\right] = K\left[\frac{e}{12R_h}\left(\frac{D^2\sin^2\beta}{2P} - \frac{AD}{P^2}\right)\right]$$

in which $K = (32.2g)^{1/2}\text{Log}_{10}(e)$

Chezy's Equation $Q = CA(R_hS_o)^{1/2} = C\sqrt{S_o}(AR_h^{1/2})$

$$\frac{1}{\sqrt{S_o}}\ \frac{dQ}{d\beta} = C\left[\frac{A}{R_h^{1/2}}(1/2)\frac{dR_h}{d\beta} + R_h^{1/2}\frac{dA}{d\beta}\right] + AR_h^{1/2}\frac{dC}{d\beta} = 0$$

divide by $AR_h^{1/2}$

$$C\left\{\frac{1}{2}\ \frac{1}{R_h^{1/2}}\left[\frac{D^2\sin^2\beta}{2P} - \frac{AD}{P^2}\right] + \frac{D^2}{2A}\sin^2\beta\right\} + \frac{dC}{d\beta} = 0$$

replacing A and P by their definitions, and the derivative of C

$$C\left\{\frac{1}{2R_h^{1/2}}\left[\frac{D^2\sin^2\beta}{2D\beta} - \frac{D^3/4(\beta-\cos\beta\sin\beta)}{D^2\beta^2}\right] + \frac{D^2\sin^2\beta}{D^2/2(\beta-\cos\beta\sin\beta)}\right\} +$$

$$\frac{Ke}{12R_h}\ \frac{D^2\sin^2\beta}{2D\beta} - \frac{D^3/4(\beta-\cos\beta\sin\beta)}{D^2\beta^2}\ \} = 0$$

$R_h/D = A'/P' = (\beta-\cos\beta\sin\beta)/\beta = R_h'$

$$C\left\{\frac{\beta}{(\beta-\cos\beta\sin\beta)}\left[\frac{\sin^2\beta}{2\beta} - \frac{\beta-\cos\beta\sin\beta)}{4\beta^2}\right] + \frac{2\sin^2\beta}{(\beta-\cos\beta\sin\beta)}\right\} +$$

$$\frac{Ke}{\quad}\ \frac{\sin^2\beta}{\quad} - \frac{(\beta-\cos\beta\sin\beta)}{\quad}\ \} = 0$$

$$12R_h \qquad \beta \qquad 4\beta^2$$

divide by $(32g)^{1/2}$

$$\mathrm{Log}_{10}\left\{\frac{12R_h}{e}\right\}\left\{\frac{\beta}{(\beta-\cos\beta\sin\beta)}\left[\frac{\sin^2\beta}{2\beta} - \frac{\beta-\cos\beta\sin\beta)}{4\beta^2}\right] + \frac{2\sin^2\beta}{(\beta-\cos\beta\sin\beta)}\right\} +$$

$$\frac{\mathrm{Log}_{10}(e)e}{12R_h}\left(\frac{\sin^2\beta}{\beta} - \frac{(\beta-\cos\beta\sin\beta)}{4\beta^2}\right\} = 0$$

let $R_h' = (\beta-\cos\beta\sin\beta)/\beta$ and $\mathrm{Log}_{10}(e) = .43429448$

$$\mathrm{Log}_{10}\left\{\frac{12R_h}{e}\right\}\left\{\frac{1}{R_h'}\left[\frac{\sin^2\beta}{2\beta} - \frac{R_h'}{4}\right] + \frac{2\sin^2\beta}{\beta R_h'}\right\} + .43429448(e/R_h)(\sin^2\beta+R_h'/(4\beta)=0$$

Solution results:

e/R_h	β	A_d	A'	R_h'	K'
.001	2.55992	.754746	.960973	.375392	.500074
.0001	2.55989	.754742	.960967	.375394	.500073
.00001	2.559886	.7547416	.960967	.375394	.500073
.005	2.56007	.754769	.9610024	.375381	.500073
.01	2.56024	.754803	.9610446	.375365	.500088
.05	2.56250	.755135	.9614674	.375207	.500167

```
TK-Solver Model PB2_162.TK
==================== VARIABLE SHEET =====================
St Input----- Name---- Output---- Unit------ Comment---------
              Rhp      1.1787465
              B        2.5625008
   .05        re
              Ad       .75513473
              Ap       .9614674
              Rprime   .37520668
              Kp       .50016687

==================== RULE SHEET =========================
S Rule---------------------------------------------------
* Rhp=(B-COS(B)*SIN(B))/B
* LOG(12/re)*((SIN(B)^2/(2*B)-Rhp/4)/Rhp+2*SIN(B)^2/(B*Rhp))+.43429448*re*(SIN(
* Ad=(B-COS(B)*SIN(B))/4
* Ap=4*Ad/PI()
* Rprime=Ap/B
* Kp=Ap*Rprime^.6666667
```