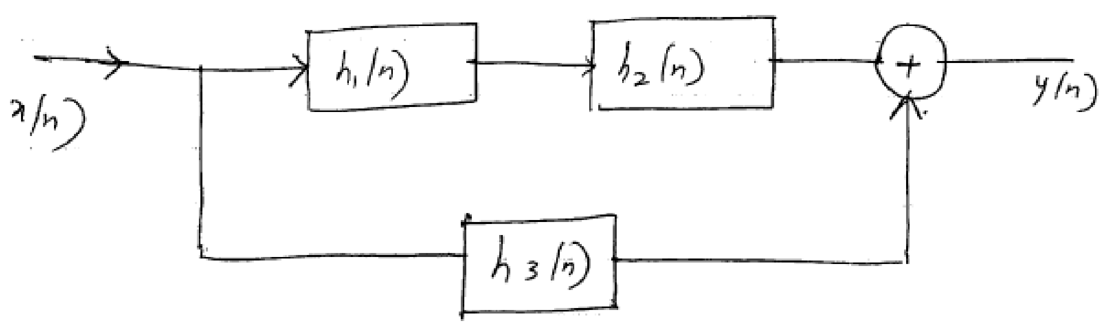
Chapter 2- Solutions

2.5 (a) Determine the overall impulse response of the system shown below in Figure 2.2, where

$$h_1(n) = \delta(n-1) + 3\delta(n)$$
$$h_2(n) = \delta(n-2) + 2\delta(n)$$
$$h_3(n) = 6\delta(n-6) + 7\delta(n-4) - 3\delta(n-1) + \delta(n)$$



Solution

From the system schematic:

$y(n) = x(n)*h_1(n)*h_2(n) + x(n)*h_3(n)$

Taking the z-transform of the equation above:

$Y(z) = X(z)\, H_1(z)H_2(z) + X(z)\, H_3(z)$

Or the system function is:

$H(z) = Y(z)/X(z) = H_1(z)H_2(z) + H_3(z)$

Substituting the values of the system block functions:

$H(z) = (z^{-1} + 3)(z^{-2} + 2) + 6z^{-6} + 7z^{-4} - 3z^{-1} + 1$

$\quad = 6z^{-6} + 7z^{-4} + z^{-3} + 3z^{-2} - z^{-1} + 7$

Taking the inverse z-transform, we get the system impulse response:

$h(n) = 6\delta(n-6) + 7\delta(n-4) + \delta(n-3) + 3\delta(n-2) - \delta(n-1) + 7\delta(n)$

2.5 (b) A 3-point symmetric moving average discrete-time filter is governed by the following difference equation:

$$y(n) = b\{ a\, x(n-1) + x(n) + a\, x(n + 1)\}$$

where a and b are constants.

- Determine the system function $H(z)$.
- Determine the frequency response $H(e^{jw})$ of the filter.
- Determine the scaling factor b such that $H(e^{jw})$ has unity gain at zero frequency.

Solution

- Taking the z-transform of the difference equation:

$$Y(z) = b\, [\, a\, X(z)\, z^{-1} + X(z) + a\, X(z)\, z^{1}]$$

Hence the system function $H(z) = Y(z) / X(z) = b\, [\, a\, z^{-1} + 1 + a\; z^{1}]$

- Putting $z = e^{j\omega}$, we get the frequency response of the system:

$$H(e^{j\omega}) = Y(e^{j\omega}) / X(e^{j\omega}) = b\, [a\, e^{-j\omega} + 1 + a\, e^{j\omega}\,]$$

$$b\, [2a\, \cos(\omega) + 1]$$

- At $\omega = 0$, $H(e^{j0}) = b\, [2a\, \cos(0) + 1] = b[2a+1) = 1$ *(given)*

⇨ $b = 1/(2a+1)$

2.5 (c) A causal, linear, time-invariant discrete-time system has system function:

$$H(z) = \frac{(1-0.5z^{-1})(1+4z^{-2})}{(1-0.64z^{-2})}$$

(i) Find expressions for a minimum phase system $H_1(z)$ and an all pass system $H_{ap}(z)$ such that:

$$H(z) = H_1(z) H_{ap}(z)$$

(ii) Plot the pole-zero plots of $H(z)$, $H_1(z)$ and $H_{ap}(z)$.

Solution

$$H(z) = \frac{(1-0.5z^{-1})(1+4z^{-2})}{(1-0.64z^{-2})}$$
$$= \frac{(1-0.5z^{-1})(1+2jz^{-1})(1-2jz^{-1})}{(1-0.8z^{-1})(1+0.8z^{-1})}$$

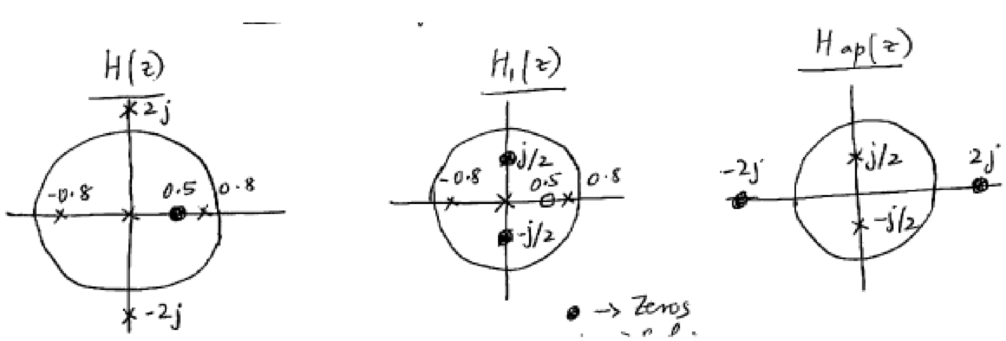
(i) The minimum phase system is:

$$H_1(z) = \frac{(1-0.5z^{-1})(1-\frac{j}{2}z^{-1})(1+\frac{j}{2}z^{-1})}{(1-0.8z^{-1})(1+0.8z^{-1})}$$
$$= \frac{(1-0.5z^{-1})(1+\frac{1}{4}z^{-2})}{(1-0.64z^{-2})}$$

The all-pass system is:

$$H_{ap}(z) = \frac{(1-2jz^{-1})(1+2jz^{-1})}{(1-\frac{j}{2}z^{-1})(1+\frac{j}{2}z^{-1})}$$
$$= \frac{(1+4z^{-2})}{(1+\frac{1}{4}z^{-2})}$$

The pole-zero plots are given below:

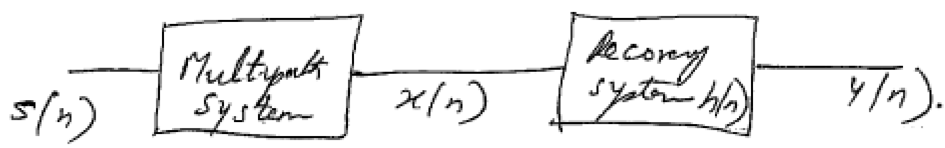


2.5 (d) A simple model for multipath channel is described by the difference equation:

$$x(n) = s(n) - e^{-8\alpha} s(n-8)$$

We wish to recover s(n) from *x(n)* with a linear time-invariant system. Find the causal and stable system function *H(z) = Y(z)/X(z)* such that its output *y(n) = s(n)*

Solution



Taking the z-transform of the channel system difference equation, we get:

$$X(z) = S(z)\ (1 - e^{-8\alpha} z^{-8})$$

And the output of the recovery system is:

$$Y(z) = H(z)\ X(z)$$

$$= H(z)\ S(z)\ (1 - e^{-8\alpha} z^{-8}) = S(z) \text{ (desired output for complete recovery)}$$

Hence the recovery system function is given by:

$$H(z) = 1/(1 - e^{-8\alpha} z^{-8})$$

$$= z^{8}/(z^{8} - e^{-8\alpha})$$

For a stable system, the poles of the recovery system should lie inside the unit circle, which gives the condition:

$$z^{8} = e^{-8\alpha} < 1$$

$$\Rightarrow\ z = e^{-\alpha} < 1$$

$$\Rightarrow\ \alpha > 0$$

2.5 (e) Consider a causal LTI system described by the difference equation:

$$y(n) = p_0\ x(n) + p_1\ x(n-1) - d_1\ y(n-1)$$

where *x(n)* and *y(n)* denote, respectively, its input and output. Determine the difference equation of its *inverse* system.

Solution

Taking the z-transform of the difference equation, we get:

$$Y(z) = p_0 X(z) + p_1 X(z) z^{-1} - d_1 Y(z) z^{-1}$$

The system function is:

$$H(z) = \frac{Y(z)}{X(z)} = \frac{p_0 + p_1 z^{-1}}{1 + d_1 z^{-1}}$$

The system function of the inverse function is:

$$H_i(z) = \frac{1 + d_1 z^{-1}}{p_0 + p_1 z^{-1}}$$

Expanding the system function:

$$Y(z)(p_0 + p_1 z^{-1}) = X(z)(1 + d_1 z^{-1})$$

And taking the inverse z-transform, we get the difference equation of the inverse system:

$$p_0 y(n) + p_1 y(n-1)) = x(n) + d_1 x(n-1)$$

Chapter 3- Solutions

3.6 (a) Suppose that we are given an ideal low pass discrete-time filter with frequency response

$$H(e^{j\omega}) = 1, \qquad 0 \leq |\omega| < \pi/4$$
$$H(e^{j\omega}) = 0, \qquad \pi/4 < |\omega| \leq \pi$$

We wish to derive new filters from this prototype by manipulations of the impulse response *h(n)*.

i. Plot the frequency response $H_1(e^{j\omega})$ for the system whose impulse response is $h_1(n) = h(2n)$.

ii. Plot the frequency response $H_2(e^{j\omega})$ for the system whose impulse response is as follows:

$$h_2(n) = h(n/2),\ n = 0,\ \pm 2,\ \pm 4, ...$$
$$h_2(n) = 0, \qquad otherwise$$

iii. Plot the frequency response $H_3(e^{j\omega})$ for the system whose impulse response is $h_3(n) = e^{j\pi n}h(n)$.

Solution

i. h(2n) is a downsampled version of the filter. Therefore, the frequency response will be expanded by a factor of two with a gain of ½. This will still be a low pass filter.