CHAPTER 2:

PROBLEMS

Problem 2.1

An optical fiber with a step index profile, a core diameter of 62.5 micrometers, a numerical aperture of 0.2 at a wavelength of 1300 nm is used for signal distribution and transmission in a local area network.

(a) What is the V-parameter of this optical fiber?

(b) How many guided modes would it support. Can you comment on this number regarding the velocities of lightwaves.

(c) Select a cladding diameter. Give reasons for your selection.

(d) Find the maximum acceptance angle of this fiber. Hence estimate the coupling loss a laser source with a uniform radiation cone of 30 degrees.

Solution:

(a) and (b)

The V-parameter is given as

$$V = \frac{2\pi}{\lambda} an\sqrt{2\Delta} = \frac{2\pi}{\lambda} a.NA = \frac{2\pi}{1.3x1e-6} 62.5x1e-6x0.2 = 60.4$$

$$N \approx \frac{V^2}{2} = 1824 \,\text{mod}\, es$$

The number of modes is estimated when V>>2.405. Obviously it is a multimode type. There would be several types of modes and the propagation paths of these modes can be (i) straight through the core axis which is the path of the fundamental mode of the spectrum of guided modes. (ii) The zig-zag paths and the spiral path around the core axis. Along the propagation directions of these guided modes the phase velocity of the lightwaves is slowed down by the refractive index of the core region. The propagation constant or phase constant of these guided modes change according to the effective magnitude of the vectors projected to the propagation direction axis z.

(c) *The cladding diameter is chosen such that (i) the modes are well confined in the cladding region, normally about 5 times the core radius and (ii) secondly to offer sufficient strength for drawing the fibers.*
In this case the fiber is a multimode types and the typical cladding diameter is about 125 micrometers which is sufficient to support the guided modes and the length of the fiber.

(d) *Acceptance angle can be estimated from the NA as*

$$NA = (\sin\theta_0)_{\max} = n_1 \cos\theta_c$$

$$0.2 = \frac{1}{2} accep\tan ce...angle \rightarrow accep\tan ce..angle = 2\sin^{-1} 0.2 = 2x11.5^0 = 23^0$$

Thus for a laser with a radiation angle of 30 degrees then we can estimate the loss as a ration between the cones of the 23 degree and 30 degrees as

$$10\text{Log}10\ loss = 10Log_{10}\frac{23}{30} = -1.15dB\ .$$

Problem 2.2A

An optical fiber has the following parameters: index profile = step-like; core diameter = 9.0 μm; numerical aperture NA = 0.11 and a cladding refractive index = 1.48.

(a) Find the normalized frequency of the fiber at 1550 nm wavelength.

(b) Is the fiber operating in the single mode or multimode region at 1550 nm? If it is in the single mode region, estimate its mode field diameter and its spot size. Sketch its field and intensity distribution across the fiber cross section.

(c) Find the cut-off wavelength of the fiber. If lightwaves of wavelength smaller than this cut off wavelength is launched into the fiber is the fiber still operating in single mode region?

SOLUTIONS:

(a) Normalized frequency parameter

$$V = \frac{2\pi}{\lambda} an\sqrt{2\Delta} = \frac{2\pi}{\lambda} a.NA = \frac{2\pi}{1.55x1e-6} 9x1e-6x0.11 = 2.0059$$

(b) The fiber is single mode at the operating wavelength of 1550 nm. Note that the fiber radius is reduced significantly and the numerical aperture is also smaller. The spot size

$$r_0 = \frac{a}{\sqrt{\ln V^2}} = 3.823\mu m$$

can be estimated by:

The mode field diameter = 2x(spot size)

(c) The cut off wavelength can be estimated by setting Vc=2.405

$$V_c = \frac{2\pi}{\lambda_c} an\sqrt{2\Delta} = \frac{2\pi}{\lambda_c} a.NA = 2.405 \rightarrow \lambda_c = 1.55x1e-6\frac{2.0045}{2.405} = 1.291\mu m$$

If a lightwave whose wavelengtgh is shorter than that of the cut off wavelength is launched into this fiber then higher oreder modes are guided and the fiber becomes weakly multimoded.

Problem 2.2B

A single mode step index optical fiber has the following parameters: core diameter = 8.0 μm, cladding diameter = 0.125 mm, core refractive index = 1.460, relative index difference = 0.2 % at 1550 nm.

(a) Confirm that the fiber can be operating in the single mode regime at 1550 nm wavelength.

(b) Find the fiber cut-off wavelength.

(c) What is the fiber mode field diameter if it is operating at 1550 nm wavelength.

SOLUTIONS:

(a)

The normalized frequency parameter is given as

$$V = \frac{2\pi}{\lambda} an\sqrt{2\Delta} = \frac{2\pi}{1.55x1e-6} 4x1e-6.1.46\sqrt{2.0.002} = 1.497$$

So the fiber is operating in the single mode region at this wavelength 1.550 nm.

(b)

The cut off wavelength is given as

$$V_c = \frac{2\pi}{\lambda_c} an\sqrt{2\Delta} = 2.405 \rightarrow \lambda_c = 1550nm\frac{1.497}{2.405} = 964nm$$

(c) Mode field diameter

$$MFD = 2r_0 = \frac{a}{\sqrt{\ln V^2}} = \frac{4}{\sqrt{\ln(1.497)^2}} = 2x4.452\mu m$$

We note that the mode field diameter (MFD) is greater than that of the core diameter, thus more power of the lightwave is distriubuted in the cladding region as compared to the Problem 2.2A.

Problem 2.3

For the optical fiber in Problem 2.2B, if the refractive index profile is parabolic (α =2) or triangular (α=1) with the above numerical aperture at the central axis, repeat (a), (b) and (c).

SOLUTIONS:

In the case that the index profile of the fiber given in problem 2.2B is graded with parabolic (α=2) or triangular profile (α=1) we can use the equivalent index method, thus the effective fiber parameters are related to the parameters of a step index profile with the maximum values as the values at the center of the core, given as:

$$\frac{V_e}{V} = \left(\frac{\alpha}{\alpha+2}\right)^{1/2} ; \lambda_{ec} = \frac{V}{2.405}\left(\frac{\alpha}{\alpha+2}\right)^{1/2}$$

Now assume that the parameters given in Problem 2.2 are given for the values at the center of the core we then can estimate the effective values as:

$$\frac{V_e}{V} = \left(\frac{\alpha}{\alpha+2}\right)^{1/2} \longrightarrow V_e = 1.497\left(\frac{2}{2+2}\right)^{1/2} = \frac{1.497}{\sqrt{2}} = 1.0586$$

for parabolic profile. This value is close to unity and thus the mode spot size becomes very large.\

For triangular profile we have

$$\frac{V_e}{V} = \left(\frac{\alpha}{\alpha+2}\right)^{1/2} \longrightarrow V_e = 1.497\left(\frac{1}{1+2}\right)^{1/2} = \frac{1.497}{\sqrt{3}} = 0.864$$

This value of V is less than one and the mode spot size becomes impossible value, thus this waveguide may not exist.

Problem 2.4

Single mode optical fibers produced by Corning (and Optical Waveguides Pty. Ltd. Noble Park Vic Australia) have typical characteristics as per technical data sheet attached at the end of this Chapter.

(a) Using the fiber physical characteristics and technical data on its numerical aperture confirm the fiber functional characteristics such as the cut-off wavelength range etc.

(b) If this fiber is launched with a 850 nm laser, how many modes would it support. Sketch the mode fields for LP_{01} and LP_{11} modes.

(c) If lightwaves at 1300 nm travels over 10 Km of this fiber, calculate the travel time of this waves.

(d) Estimate the fiber mode field diameter at 1300 nm wavelength.

(e) If the same spot size of (d) is required for the fiber to operate at 1550 nm, can you advise the manufacturer on any change of the fiber physical parameters.

SOLUTIONS:

(a) We assume the fiber is the standard single mode fiber – Corning SMF-28. The physical characteristics of this fiber are given as:

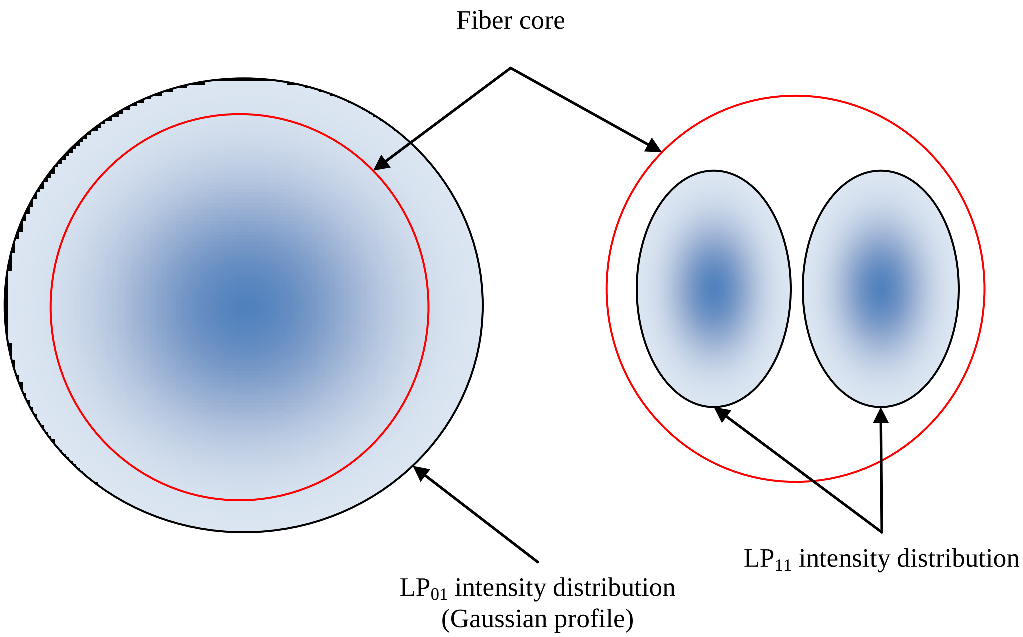
Core diameter 8.2 μm; Cladding parameter: 125 μm±1; Relative refractive index difference = 0.36%; step index profile (effective), NA = 0.14; Effective group index of refraction = 1.4682 at 1550 nm wavelength.
Using these values one can confirm the cut-off wavelength of the SMF-28 fibers.
Cut-off wavelength ≤ 1260 nm
Mode field diameter = 10.4μm

(b) At 850 nm the V parameter becomes

$$V = \frac{2\pi}{\lambda} aNA = \frac{2\pi}{\lambda} aNA = 2.32 < 2.405$$

at wavelength of 1550 nm.

So at 850 nm the V parameter is 4.23. Therefore at this wavelength the fiber may support about 16 modes.



(c) At 1300 nm – rather we should say 1550 nm the effective index of the group is 1.468 so over 10 km of fiber the travel time is: (10,000/3x1e8)x1.468 = 2.041e-5 sec. = 20.14 micro-seconds.

(d) Mode field diameter is strongly dependent on the V parameter – so at 1300 nm the V parameter is given as: V = 2.32 x (1.55/1.3)=2.766. At this value of V, in practice it supports the fundamental mode a very week second order mode LP_{11}. So the estimation of the mode field diameter is still valid and given by

$$MFD = 2r_0 = \frac{a}{\sqrt{\ln V^2}} = \frac{4.1}{\sqrt{\ln(2.767)^2}} = 2x5.84 = 11.69\mu m$$

Note that there would be two LP_{01} and four LP_{11} modes according to the directions of the polarizations of the field distribution.

(e) If this mode field diameter is required for 1550 nm wavelength, then the V parameter must reach the value of 2.767 – in order to achieve this we can increase the refractive index difference keeping the radius the same or increase the core radius but keeping the relative index difference the same. It seems that the easiest thing to do is to increase the core radius.

Problem 2.5

(a) The optical fiber in Problem 2.3 is used in an optical fiber transmission system with a laser source operating at 1310 nm having an output power of 1.0 mW. The fiber length is 50 Km. An optical receiver which can detect an average optical power of 0.1 μW. Is it possible to detect the optical power at the end of the fiber length.

(b) Referring to the technical data of the standard optical fiber estimate the spreading of optical pulse after transmitting through the 50 Km length fiber if the source has an optical line width of 2.0 nm.

SOLUTIONS:

(a) 1 mW = 0 dBm; 0.1μW= -40dBm in dB scale. The fiber loss at 1300 nm is about 0.35 dB/km so over 50 km the loss is 50x0.35 = 17.5 dB.

With 0dBm power at the input the output power is -17.5 dBm. The receiver can detect signal at -40dBm and higher thus with -17.5dBm there would be no problem.

(b) If a source of a linewidrth of 2 nm and at central wavelength of 1550 nm then the fiber dispersion is +17 ps/nm/km then after 50 km we have the broadening o f the pulse is: 17x50x2 =

Problem 2.6

A step index optical fiber is used for an optical communication systems operating at 1310 nm has a core radius of 25 μm, refractive indices in the core and cladding regions of 1.460 and 1.4550 respectively.

(a) what is the numerical aperture of the fiber

(b) estimate the number of guided modes.

SOLUTIONS:

This problem is similar to problems 2.1 and 2.2.

Problem 2.7

(a) Show that for a graded index fiber having a core refractive index $n^2(r)=n_2^2\left[1+2\Delta s\left(\frac{r}{a}\right)\right]$ with s(r/a) =1-(r/a)$^{\alpha}$the acceptance angle α(r) is given by

$$\sin\alpha(r)=\left[n^2(r)-n_2^2\right]^{1/2}$$

(b) If the optical fiber has a parabolic profile shape show that $\sin\alpha(r)=NA\sqrt{1-\left(\frac{r}{a}\right)^{1/2}}$ where NA = $n_2(2\Delta)^{1/2}(1+\Delta)$.

(c) A parabolic graded index silica optical fiber has a cladding refractive index of 1.460 and a relative index difference at the core axis of 1%. Find the maximum acceptance angle at the core axis of the fiber. Plot sinα(r) as a function of r. What is the acceptance angle of the fiber at the core and cladding interface. Comment on the launch of a laser source into this fiber.

SOLUTIONS:

(a) This problem is related to the definition and derivation of the numerical aperture of optical fiber. The steps for solving this problem are: (i) Find the critical angle of the core of the fiber using the Snell's lay of refraction and then set this critical angle at pi/2, (ii) find the angle of incidence with respect to the core axis. The sinus of this angle is the numerical aperture.

(b) For parabolic profile a = 2 and substitute this into the expression given in (a) leading to the expression for the NA in (b).

(c) Similar to the finding of the acceptance angle at the core center by finding the numerical aperture and thence the arcsinus leads to the angle.

Problem 2.8

(a) For a single mode optical fiber having a graded index central dip, that is s(r/a) = 1-(1-r/a)$^{\alpha}$ the ESI parameters of V and radius are given by:

$$\frac{V_e}{V} = \left[1 - \frac{2\gamma}{(\alpha+1)(\alpha+2)}\right]^{1/2} ....and.... \frac{a_e}{a} = \frac{(\alpha+1)(\alpha+2)(\alpha+3)-6\gamma}{(\alpha+1)(\alpha+2)(\alpha+3)-2\gamma}$$

where $V=ka(2\Delta)^{1/2}$.

(b) The fiber has a physical core radius of 8.0 μm, a maximum relative index difference of 0.3 % and a cladding refractive index of 1.460. Find its ESI parameters for the normalized frequency and radius at 1550 nm wavelength. Find also its ESI cut off wavelength and its mode field diameter at this wavelength.

SOLUTIONS:

(a) This part is given for information to estimate the ESI parameters of the single mode graded index profile fiber- that is the graded index fiber can be converted or stated as an equivalent of the single mode step-index fiber.

(b) Very straight forward to substitute the given parameters into expressions given in (a).