Chapter 2 Problem Solutions

2.1 Show that, for a surface temperature of 5800 K, the sun will deliver 1367 W/m^2 to the Earth. This requires integration of the blackbody radiation formula over all wavelengths to determine the total available energy at the surface of the sun in W/m^2. Numerical integration is recommended. Be careful to note the range of wavelengths that contribute the most to the spectrum. Then note that the energy density decreases as the square of the distance from the source, similar to the behavior of an electric field emanating from a point source. The diameter of the sun is 1.393×10^9 m, and the mean distance from sun to Earth is 1.5×10^{11} m.

Start by plugging the values of the various constants along with T = 5800 K into (2.1). This gives

$$w_\lambda = \frac{3.749\times10^{-16}\lambda^{-5}}{e^{\frac{2.4845\times10^{-6}}{\lambda}}-1}\ \text{W/m}^2\text{/m}.$$

This expression must now be integrated over the interval $0<\lambda<\infty$. Since most of the spectral intensity appears in the wavelength range between 0.1 and 5 μm, values for evaluating the expression must be carefully chosen if numerical integration is chosen. The result of the integration is

$$\int_0^\infty w_\lambda d\lambda = 6.44\times10^7\ \text{W/m}^2.$$

Since this is the power density at the surface of the sun, the total power of the sun can be found by multiplying this power density by the surface area of the sun. The result is

$$P = (6.44\times10^7\ \text{W/m}^2)\times(4\pi)\times(0.5\times1.393\times10^9\ \text{m})^2 = 3.926\times10^{26}\ \text{W}.$$

This power is radiated uniformly in all directions. The power density at the earth is thus found from the total power of the sun divided by the area of the sphere surrounding the sun with radius equal to the distance from the sun to the earth. The result is

$$S = \frac{P}{A} = \frac{3.926\times10^{26}\,\text{W}}{2.827\times10^{23}\,\text{m}^2} = 1389\ \text{W/m}^2,$$

which compares nicely with the value 1367 W/m^2 that results from the not-quite-perfect blackbody spectrum received by the earth.

2.2 If the diameter of the sun is 1.393×10^9 m, and if the average density of the sun is approximately 1.4× the density of water, and if 2×10^{19} kg/yr of hydrogen is consumed by fusion, how long will it take for the sun to consume 25% of its mass in the fusion process?

The density of 1.4 means the density of the sun is 1.4×103 kg/m3 . The mass of the sun thus computes to be 1.981×1030 kg. Hence, 25% of the sun's mass is 4.953×1029 kg.
Hence, (4.953×1029 kg)÷(2×1019 kg/yr) = 24.76 billion years.

2.3 Calculate the zenith angles needed to produce AM 1.5 and AM 2.0 if AM 1.0 occurs at zero degrees.

AM = AM(90^o)cscα . Thus, if AM(90^o) = 1.0, then for AM = 1.5, cscα= 1.5 and α = 41.81^o.
If α = 41.81^o , then $\theta_Z = 90 - \alpha = 48.19^o$.
For AM = 2, sinα = 0.5 so that $\alpha = 30^o$ and $\theta_Z = 90 - \alpha = 60^o$.

2.4 Calculate the zenith angle at solar noon at a latitude of 40^o north on May 1.

At solar noon, $\theta_Z = \phi - \delta = 40^o - 23.45^o \sin\left[\frac{360(121-80)}{365}\right] = 24.79^o$.

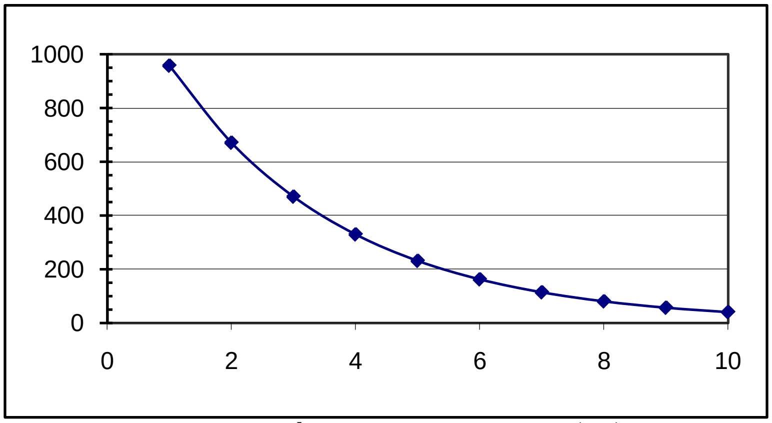
2.5 Calculate the number of hours the sun was above the horizon on your birthday at your birthplace.

To find n to find δ, the date is needed. Then the latitude of the birthplace is needed. Finally, solving (2.8) gives

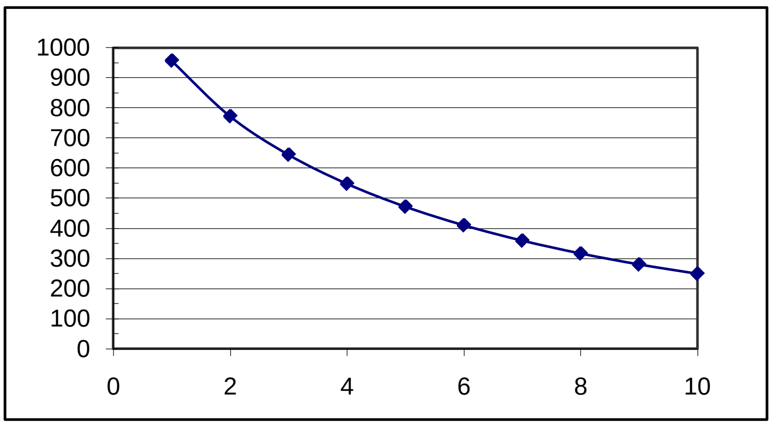
$$DH = \frac{\cos^{-1}(-\tan\phi\tan\delta)}{7.5}\ \text{hr}\ .$$

2.6 Calculate the irradiance of sunlight for AM 1.5 and for AM 2.0, assuming no cloud cover, using (2.2) and (2.3). Then write a computer program that will plot irradiance vs. AM for $1 \le AM \le 10$ for each equation.

Equation (2.2) results in irradiance = 801 W/m^2 for AM1.5 and irradiance = 670 W/m^2 for AM2.0.
Equation (2.3) results in irradiance = 855 W/m^2 for AM1.5 and irradiance = 772.5 W/m^2 for AM2.0.



Irradiance vs. Air Mass using (2.2)



Irradiance vs. Air Mass using (2.3)

2.7 Assume no cloud cover, and, hence, that the solar irradiance is predominantly a beam component. If a nontracking collector is perpendicular to the incident radiation at solar noon, estimate the irradiance on the collector at 1, 2, 3, 4, 5 and 6 hours past solar noon, taking into account air mass and collector orientation. Then estimate the total daily irradiation on the collector in kWh. Assume AM 1.0 at solar noon and a latitude of 20° N.

AM1.0 at solar noon means the sun is directly overhead, or $\alpha = 90^o$ and $\psi = 0^o$. This means the collector is horizontal. Thus, to calculate the direct beam irradiation on the collector at 2, 3 and 4 hours past solar noon, it is necessary to calculate α at these times.

To find α, (2.9) is used: $\sin\alpha = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega$

At solar noon, the hour angle, $\omega = 0$. Also if the sun is directly overhead, $\phi = \delta$ so $\alpha = 90^o$.

Since $-23.45^o < \delta < 23.45^o$, it is necessary to choose a location to determine α. If $\phi = 20^o$ N, it is possible to evaluate α for times when $\omega \neq 0$.

So, at 2 hours past solar noon, $\omega = -30^o$ and $\sin\alpha = \sin^2 20^o + \cos^2 20^o \cos(-30^o) = 0.8817$ so $\alpha = 61.85^o$.

Similarly, at 3 hours past solar noon, $\alpha = 47.85^o$, and at 4 hours past solar noon, $\alpha = 33.95^o$.

Next, for the calculated solar altitudes, the air mass and then the irradiance need to be calculated. Using (2.3), the following table can be generated for solar altitude, air mass, irradiance and the component of irradiance on the horizontal array, given by (Irradiance)×(sinα):

Time	sinα	α, degrees	AM	Irradiance, W/m^2	Direct array component, W/m^2
Solar noon	1.0000	90	1.000	957	957
noon + 1	0.9699	75.9	1.020	950	921
noon + 2	0.8817	61.9	1.134	927	817
noon + 3	0.7414	47.9	1.349	883	655
noon + 4	0.5585	34.0	1.791	805	450
noon + 5	0.3455	20.2	2.894	657	227
noon + 6	0.1170	6.7	8.55	297	35

An estimate of the total daily irradiation on the collector can then be made by summing the direct array components multiplied by appropriate time intervals. Assuming the solar noon component from 11:30 a.m. to 12:30 p.m., the noon + 1 component from 12:30 to 1:30 p.m. and 10:30 to 11:30 a.m., the noon + 2 value from 1:30 to 2:30 p.m. and 9:30 to 10:30 a.m., etc., the approximate total output of the array due to the direct component of the incident sunlight over the 12-hour period becomes

kWh/day = 0.957 + 2×(0.921 + 0.817 + 0.655 + 0.450 + 0.227 + 0.035) = 7.167 .

2.8 Write a computer program using Matlab, Excel or something similar that will plot solar altitude vs. azimuth. Plot sets of curves similar to Figure 2.8 for the months of March, June, September and December for Denver, CO, Mexico City, Mexico, and Fairbanks, AK.

The program needs to plot α and ψ as determined by (2.9) and (2.10), with appropriate values of ϕ and δ for the months and the locations. Note that since δ reaches its maximum and minimum values on or about the 21st of June and December, rather than at the 15th of the months, the curves for May and July, April and August, etc., will be slightly different.

2.9 Using eq. (2.9 and 2.10), show that at solar noon, $\theta_Z = \phi - \delta$ and $\psi = 0$.

Start with equation (2.9) since it does not depend on ψ. Using some trigonometric identities, and ω=0 at solar noon gives:

$$\cos\theta_Z = \sin(90-\theta_Z) = \sin\alpha = \sin\delta\sin\varphi + \cos\delta\cos\varphi = \cos(\varphi-\delta) .$$

This gives $\theta_Z = \pm(\phi-\delta)$. **Explain why it isn't the negative**.

Now for equation (2.10). Using the result from above, and a trigonometric identity, we have:

$$\sin\delta = \sin(\varphi-\theta_Z) = \sin(\varphi+\alpha-90) = -\cos(\varphi+\alpha) = \sin\varphi\sin\alpha - \cos\varphi\cos\alpha .$$

Plugging this value into equation 2.10 gives the answer.

2.10 Write a computer program that will generate a plot of solar altitude vs. azimuth for the 12 months of the year for your home town. It would be nice if each curve would have time-of-day indicators.

Appropriate choices need to be made for ϕ and δ for use of (2.9) and (2.10) again.

2.11 Neglecting the analemma effect, calculate the time of day at which solar noon would occur at your longitude. Then compare the north indicated by a compass, corrected for compass declination, with the north indicated by a shadow at calculated solar noon and estimate the error in the shadow direction (i.e., solar noon) due to the analemma effect. Estimate actual solar noon time on the basis of the difference between compass north and the shadow "north."

True north is in the direction of the shadow at solar noon in the northern hemisphere, provided that $\phi > \delta$. If $\phi < \delta$, then the shadow points directly south. As noted in the text, neglecting the analemma effect, solar noon occurs at clock noon at longitudes that are multiples of 15^o from Greenwich, England (with a few noted exceptions). For other longitudes, solar noon is found from interpolation, where 15^o = 60 minutes (i.e., 1 hour), so 1^o = 4 minutes = 1/15 hr. For example, at 80^o W, solar noon occurs at $(80–75)\times60\div15 = 60\div3 = 20$ minutes past noon, clock time.

The difference between true north and compass north is the compass declination, not to be confused with the declination of the sun, δ. This difference is relatively small in eastern United States, but is in the range of 20° in northwestern Canada. Thus, north indicated by a compass must first be corrected to true north from a compass declination table.

Next, the direction of the shadow at calculated solar noon, neglecting the analemma effect, is compared with true north determined from the corrected compass reading. Then, note that if the sun travels 15°/hr, if the shadow "north" is west of true (corrected compass) north, then solar noon has not yet occurred. If the shadow "north" is east of true north, then the sun is west of south and solar noon has already happened. Using the angular spacing between true north and shadow "north," one can now estimate either the time until solar noon or the time elapsed since solar noon occurred.

2.12 Noting the dependence of air mass on θ_z,

a. Generate a table that shows α, ψ and I (using 2.3) vs. time for a latitude of your choice on a day of your choice.
b. Assuming a tracking collector and no cloud cover, estimate the total energy available for collection during your chosen day.

The selection of latitude is to be made by the student. This solution will use 30^o N on March 21. For this case, ϕ = 30^o and δ = 0^o. Hence,

$$\sin\alpha = \sin 0^o \sin 30^o + \cos 0^o \cos 30^o \cos\omega = 0.866\cos\omega \quad \text{and}$$

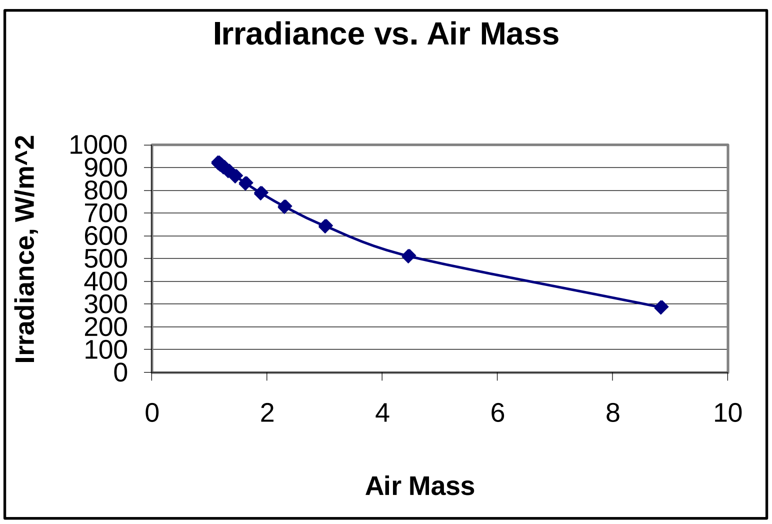
$$\cos\psi = \frac{\sin\alpha\sin\phi - \sin\delta}{\cos\alpha\cos\phi} = 0.5774\tan\alpha\ .$$

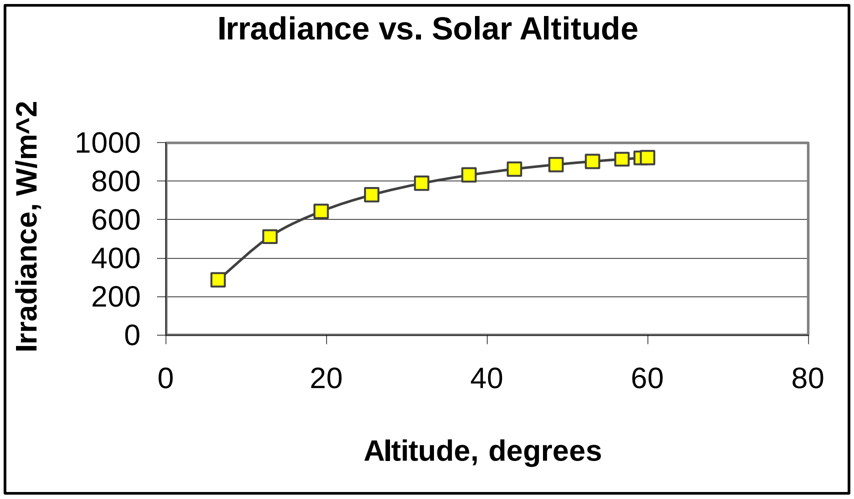
Now, using $\mathrm{AM} = \dfrac{\mathrm{AM}(90^o)}{\sin\alpha} = \dfrac{1}{\sin\alpha}$ and $I = 1367(0.7)^{(\mathrm{AM})^{0.678}}$ with $\omega = 15(12–T)$ for times when the sun is above the horizon enables the construction of a table of I vs. T, which, when summed as $I\Delta T$ over daylight hours, gives the total irradiation per day incident on a tracking collector. The results are tabulated in the following table.

T	ω	sinα	α	ψ	AM	I, kW/m^2	cum I, kWh/m^2
6	90	0.0000	0.00	90.01	870319	0	0
6.5	82.5	0.1130	6.49	86.24	8.847	286	0.07
7	75	0.2241	12.95	82.37	4.462	511	0.27
7.5	67.5	0.3314	19.36	78.30	3.017	643	0.56
8	60	0.4330	25.66	73.90	2.309	729	0.90
8.5	52.5	0.5272	31.82	69.01	1.897	788	1.28
9	45	0.6124	37.76	63.44	1.633	831	1.69
9.5	37.5	0.6870	43.40	56.91	1.456	863	2.11
10	30	0.7500	48.59	49.10	1.333	886	2.55
10.5	22.5	0.8001	53.14	39.63	1.250	903	2.99
11	15	0.8365	56.78	28.17	1.195	914	3.45
11.5	7.5	0.8586	59.16	14.72	1.165	920	3.91
12	0	0.8660	60.00	0.00	1.155	923	4.37
12.5	-7.5	0.8586	59.16	14.72	1.165	920	4.83
13	-15	0.8365	56.78	28.17	1.195	914	5.29
13.5	-22.5	0.8001	53.14	39.63	1.250	903	5.74
14	-30	0.7500	48.59	49.10	1.333	886	6.19
14.5	-37.5	0.6870	43.40	56.91	1.456	863	6.63
15	-45	0.6124	37.76	63.44	1.633	831	7.05
15.5	-52.5	0.5272	31.82	69.01	1.897	788	7.45
16	-60	0.4330	25.66	73.90	2.309	729	7.83
16.5	-67.5	0.3314	19.36	78.30	3.017	643	8.18
17	-75	0.2241	12.95	82.37	4.462	511	8.47
17.5	-82.5	0.1130	6.49	86.24	8.847	286	8.66
18	-90	0.0000	0.00	90.01	870318.7	0	8.74

2.13 Plot I vs. AM and then plot I vs. α.

The plots can be generated from the data from Problem 2.11 with the following results:





2.14 Calculate the collector orientation that will produce maximum summer output between 2 p.m. and 5 p.m. in Tucson, AZ. Use July 21 as the assumed midpoint of summer and correct for the longitude of the site.

The collector should face the sun at 3:30 p.m. (assume MDT) on July 21. Hence, the sun position in Tucson, AZ is needed at 3:30 p.m. on July 21. Note that on July 21, 3:30 p.m. is daylight savings time, or 2:30 p.m. standard time. The longitude of Tucson is 110.9°W, so solar noon occurs at 12:24, sun time. Thus, 2:30 p.m. standard time is equivalent to 2:06 p.m. sun time. For Tucson, $\phi = 32.2^{o}$ N and for July 21,

$\delta = 23.45 \sin \frac{360(202-80)}{365} = 20.24^{o}$ and $\omega = 15(12–T) = 15(12–14.1) = –31.5^{o}$. Hence,

$\sin\alpha = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega = 0.861$ and thus $\alpha = 59.5^{o}$.

Next, $\cos\psi = \frac{\sin\alpha\sin\phi - \sin\delta}{\cos\alpha\cos\phi} = .188$ so that $\psi = -79.2^{o}$. (Note negative since west of south.) Hence, the collector is pointed close to west at an altitude of 59.5°. As an interesting follow-up exercise, one might want to calculate the position without the longitude correction. Another interesting follow-up is to note that Arizona is on pacific time instead of mountain time. Thus, the problem could also be worked using 3:30 p.m. PDT, which is based on longitude of 120°, rather than 105° for mountain time solar noon. This means solar noon is at 11:24 a.m. rather than 12:24 p.m. PST. Obviously the same procedures will be followed.

2.15 Calculate the collector orientation that will produce maximum summer output at 9 a.m. Daylight Savings Time in Minneapolis, MN, using July 21 as the assumed midpoint of summer and using a longitude correction.

Minneapolis, MN has $\phi = 45^{o}$ N and longitude = 93.3° W. For July 21, $\delta = 20.24^{o}$ (See Problem 2.13). To find how 9 a.m. CDT relates to solar noon, note that at 90° W, solar noon is at 1 p.m. CDT. Hence, at 93.3° W, solar noon is 3.3×60÷15 = 13.2 minutes past the hour, or at 1:13 p.m. This means that 9 a.m. CDT corresponds to 4 hr 13 min before solar noon. (4 hr 13 min = 4.2167 hr) Hence T–12 = –4.2167 hr and ω = 15(12–T) = 63.25°. Solving for α and ψ gives

$\sin\alpha = \sin\delta\sin\phi + \cos\delta\cos\phi\cos\omega = 0.5432$ and thus $\alpha = 32.9^{o}$ and

$\cos\psi = \frac{\sin\alpha\sin\phi - \sin\delta}{\cos\alpha\cos\phi} = .0643$ so that $\psi = 86.316^{o}$ East of South.

2.16 A collector in Boca Raton, FL (ϕ = 26.4^o N, longitude = 80.1° W) is mounted on a roof with a 5:12 pitch, facing 30^o S of W. Determine the time of day and days of the year that the direct beam radiation component is normal to the array.

A roof with a 5:12 pitch has $\theta = \tan^{-1}\dfrac{5}{12} = 22.62^o$. Thus, $\alpha = 90 - \theta = 67.38^o$ and 30^o S of W corresponds to $\psi = -60^o$. For Boca Raton, $\phi = 26.36^o$ N with longitude of 80.08^o W. From this information it is necessary to determine δ and ω. Equation (2.12) can be solved for sinδ. The result is

$\sin\delta = \sin\alpha\sin\phi - \cos\alpha\cos\phi\cos\psi = 0.2375$ and thus $\delta = 13.74^o$.

Next, (2.11) can be used to determine ω, with the result

$\cos\omega = \dfrac{\sin\alpha - \sin\delta\sin\phi}{\cos\delta\cos\phi} = 0.9394$ and thus $\omega = -20.05^o$ (i.e., W of S).

To find the days of the year that the sun shines directly onto the array, solve for the days for which $\delta = 13.74^o$. This is done using (2.5), which gives the result

$\dfrac{360(n-80)}{365} = \sin^{-1}\dfrac{13.74}{23.45} = 35.87^o$ or 144.13^o.

For 35.87^o, n = 116, corresponding to April 26. For 144.13^o, n = 226, corresponding to August 14.

The time of day relative to solar noon for direct sun is found from (2.8), resulting in $T = 12 + \dfrac{20.05}{15} = 13.3367$, which corresponds to 13 hr 20 min solar time. To find when solar noon occurs at 80.08^o W, note that solar noon occurs at $\dfrac{80.08-75}{15}\times 60 = 20$ minutes past the hour, i.e., at 12:20 p.m. EST, or at 1:20 p.m. EDT. Since direct sun occurs 1 hr 20 min later, it occurs at 2:40 p.m. on April 26 and August 14.

2.17 Determine the location (latitude and longitude) where, on May 29, sunrise is at 6:30 a.m. and sunset is at 8:08 p.m. EDT. At what time does solar noon occur at this location?

Solar noon is halfway between sunrise and sunset. Thus, solar noon is at $\dfrac{1}{2}\left(6.5 + 20 + \dfrac{8}{60}\right) = 13.317$ hr, which corresponds to 1:19 p.m. EDT. From this information, the longitude can be determined to be 75^o + $\dfrac{19}{60}\times 15^o = 79.75^o$ W.

The declination on May 29 is found to be $\delta = 23.45\sin\dfrac{360(149-80)}{365} = 21.75^o$.

From sunrise to sunset, DH = 20.1333–6.500 = 13.633 hours on May 29.

Finally, since DH is known to be 13.633, and δ is known, (2.8) can be used to solve for the latitude.

$DH = \dfrac{\cos^{-1}(-\tan\phi\tan\delta)}{7.5} = 13.633$, so that $\tan\phi = \dfrac{0.212}{0.399} = 0.531$ and thus $\phi = 27.985^o$ N.

2.18 Using Figure 2.14, make a table of unshaded collector times for each month of the year.

The line is drawn on the face of the Solar Pathfinder such that above the line represents shaded and below the line is unshaded. Shaded or unshaded is determined by the reflection of shading objects from the face of the instrument. Hence, the following table can be generated based on the position of the chalk/crayon line on the instrument face.

Month	Unshaded times		Month	Unshaded times
Jan	12:45 – 1:30 p.m.		Jul	5:15 a.m. – 6:45 p.m.
Feb	noon – 2:00 p.m.		Aug	5:30 a.m. – 6:30 p.m.
Mar	9:30 a.m. – 2:45 p.m.		Sep	9:15 a.m. – 3:15 p.m.
Apr	5:30 – 6:30 a.m. & 8:30 a.m. – 6:30 p.m.		Oct	10:00 a.m. – 2:15 p.m.
May	5:15 a.m. – 6:45 p.m.		Nov	12:30 – 1:30 p.m.
Jun	5:00 a.m. – 7:00 p.m.		Dec	12:45 – 1:15 p.m.

2.19 An array of collectors consists of three rows of south-facing collectors as shown in Figure P2.1. If the array is located at 40° N latitude, determine the spacing, d, between the rows needed to prevent shading of one row by another row.

The smallest value of α over the year is needed for this problem. If sunrise to sunset is considered, then $\alpha = 0$ and the spacing must be infinite. Normally it is considered to be adequate if the collector is unshaded between 9 a.m. and 3 p.m. So this means that $\omega=\pm45°$. For either value of ω, α will be the same. The smallest value of α will occur when $\delta = -23.45°$. So for $\phi = 40°$,

$\sin\alpha = \sin(-23.45°)\sin40° + \cos(-23.45°)\cos40°\cos45° = 0.2411$, so $\alpha = 13.95°$.

Then, since $\tan\alpha = 3\div d'$, where d′ is the shadow length, $d' = 3\div\tan\alpha = 12.07$ ft. But this shadow distance is measured from the projection of the high side of the module onto the roof toward the northwest, since the azimuth of the sun in the morning is toward the southeast. So the value of ψ is also needed. Using 2.10, $\psi = -42°$ (i.e., east of south).

The distance between rows, d, is thus found from $d = d'\cos\psi = 8.97$ ft.

2.20 The top of a tree is found to be at an angle of 15° between the horizontal and the corner of a collector and at an azimuth angle of – 30°. If the site is at a latitude of 30° N, determine the months (if any) when the tree will shade the array at the point from which the measurements are taken.

This problem would most easily be solved with an instrument to calculate sun path. But, of course, for an engineer, this would be too simple. So the mathematical approach is used instead. The problem is to determine whether the sun altitude dips below 15° when the azimuth is –30° during any month of the year. So this means calculating the declination when the altitude, azimuth and latitude are known. Solving (2.10) for the declination gives $\sin\delta = \sin\alpha\sin\phi - \cos\psi\cos\alpha\cos\phi$. Plugging in $\alpha = 15°$, $\psi = -30°$ and $\phi = 30°$ gives $\sin\delta = -0.60$ and thus $\delta = -36.52°$. Since $-23.45° < \delta < 23.45°$, this means that the tree will never be a problem unless it grows taller. For the curious person, it is interesting to calculate how tall the tree can grow before it becomes a problem, i.e., for what value of α is $\delta = -23.45°$.

2.21 A residence has a 5:12 roof pitch and is located at latitude 27°N and longitude 83°W, facing 15° west of south. If two rows of 66 cm x 142 cm collectors are to be mounted on the back roof so they will have a tilt of latitude, as shown in Figure P2.2,

a. How far apart will the rows need to be if collectors are to remain unshaded for 6 hours on December 21?
b. Assuming Eastern Standard Time, over what time period on December 21 will the collectors be unshaded, assuming the collectors are facing 15° west of south.
c. Assuming no other shading objects, over what time period will the collectors be unshaded on March 21?

a. On December 21, $\delta = -23.45°$. For 6 hours of unshaded operation, the collector will remain unshaded from 9 a.m. to 3 p.m. sun time. Thus, the altitude must be computed when $\omega = 45°$ at the latitude of 27°. The fact that the collector is facing slightly west of south means that if the collector is not shaded at 9 a.m., then it will not be shaded at 3 p.m. so, in fact, the collector will receive a bit more than 6 hours of sun. Using (2.9) to find α gives $\alpha = 23.4°$.

Referring to Figure P2.2, note that $\alpha = \beta + \gamma$. The 5:12 roof slope determines $\beta = \tan^{-1}(5/12) = 22.62°$. This means that $\gamma = 0.78°$.

Next, note that $\theta = \phi + \beta = 49.62°$. If the modules are mounted in landscape position, then $\sin\theta = y/66$ and $\cos\theta = x/66$ (x and y in cm). So x = 42.8 cm and y = 50.3 cm.

Finally, $\tan\gamma = y/(d-x)$, so d = 3737 cm = 37.37 m. Hence, it is unlikely that the roof will be large enough to meet the spacing constraint.

b. As a first approximation, the collectors will be unshaded from 9 a.m. to 3 p.m. sun time if the spacing of part a can be achieved. At a longitude of 83°W, solar noon will occur at 12:32 p.m. EST, so the collectors will be unshaded from 9:32 a.m. to 3:32 p.m.

c. On March 21, $\delta = 0$, so if $\alpha = 23.4°$ and $\phi = 27°$, then, using (2.11), $\omega = \pm 63.5°$. This corresponds to unshaded hours 4.24 hr either side of solar noon, or from 7:46 a.m. to 4:14 p.m. sun time. Correcting time for longitude, this results in unshaded hours from 8:18 a.m. to 4:46 p.m.

2.22 A commercial building has a flat roof as shown in Figure p2.3. The roof has a parapet around it that is 3 ft high, and an air conditioner located as shown that is 4 ft above the roof. The building is located at 40 degrees north. A PV system is to be installed using modules that measure 3 ft by 5 ft, in rows that will remain unshaded for at least 6 hours every day of the year. The modules are to be mounted at a tilt of 25 degrees, in portrait mode, facing south. The bottom edges of the modules will be 6 inches above the roof. Determine the maximum number of modules that can be installed on the roof so that no module will be shaded during the 6-hour period. Pay attention to module rows shading other rows, shading from the air conditioner and shading from the parapet. Draw a diagram that shows your result.

Since the modules are raised 6 inches above the roof, this problem can be solved by treating the modules as though their bottom edges are at ground (roof) level, the parapet is 2.5 ft tall, and the AC is 3.5 ft tall.

It may be wise to first check that at least 6 hours of sunlight are available every day of the year. The least amount of daylight hours occur when $\delta = -23.45°$. Using equation (2.8) it is found that the shortest day has about 9.15 hours of sunlight. Hence the problem is reasonable. It should also be clear that the desirable unshaded hours are from 9am – 3 pm solar time.

To calculate the shadow length (and direction) for a given day and time it is necessary to find α and ψ. In the following table, using equations (2.6), (2.9), and (2.10) the required angles for the first day of winter, spring/fall, and summer can be found. Also, since cosine is an even function, one must be careful to make sure the sign of the azimuth angles calculated are correct.

Rather than calculate the shadow length of each side of the parapet, the modules, and the AC for each of the 3 declinations, the shadow length of a pole 1 foot high will be found. Then it is only necessary to multiply any of these shadow lengths by the actual height of our object to get the actual shadow length. In the table, Length corresponds to the shadow length, N Length corresponds to the length of the shadow in the northern direction, and similarly for W Length ($(\text{N Length})^2 + (\text{W Length})^2 = (\text{Length})^2$). Length was found using $(1\text{ft})/\tan\alpha$, and for N Length and W Length, Length is multiplied by $\cos\psi$ and $\sin\psi$ respectively.

		|------ Winter------|		**|----Fall/Spring----|**		**|----- Summer------|**	
TIME:	**ω**	**α**	**ψ**	**α**	**ψ**	**α**	**ψ**
9:00	45.00	13.95	41.95	32.80	57.27	48.83	80.19
9:30	37.50	17.56	35.86	37.43	50.05	54.42	73.73
10:00	30.00	20.66	29.36	41.56	41.93	59.82	65.83
10:30	22.50	23.17	22.45	45.05	32.80	64.83	55.65
11:00	15.00	25.03	15.19	47.73	22.63	69.17	41.89
11:30	7.50	26.17	7.67	49.42	11.57	72.28	23.17
12:00	0.00	26.55	0.00	50.00	0.00	73.45	0.00
12:30	-7.50	26.17	-7.67	49.42	-11.57	72.28	-23.17
1:00	-15.00	25.03	-15.19	47.73	-22.63	69.17	-41.89
1:30	-22.50	23.17	-22.45	45.05	-32.80	64.83	-55.65
2:00	-30.00	20.66	-29.36	41.56	-41.93	59.82	-65.83
2:30	-37.50	17.56	-35.86	37.43	-50.05	54.42	-73.73
3:00	-45.00	13.95	-41.95	32.80	-57.27	48.83	-80.19

Shadow lengths for a one foot pole

	WINTER			FALL/SPR			SUMMER		
TIME	**Length**	**N Length**	**W Length**	**Length**	**N Length**	**W Length**	**Length**	**N Length**	**W Length**
9:00	4.02	2.99	2.69	1.55	0.84	1.31	0.87	0.15	0.86
9:30	3.16	2.56	1.85	1.31	0.84	1.00	0.72	0.20	0.69
10:00	2.65	2.31	1.30	1.13	0.84	0.75	0.58	0.24	0.53
10:30	2.34	2.16	0.89	1.00	0.84	0.54	0.47	0.27	0.39
11:00	2.14	2.07	0.56	0.91	0.84	0.35	0.38	0.28	0.25
11:30	2.04	2.02	0.27	0.86	0.84	0.17	0.32	0.29	0.13
12:00	2.00	2.00	0.00	0.84	0.84	0.00	0.30	0.30	0.00
12:30	2.04	2.02	-0.27	0.86	0.84	-0.17	0.32	0.29	-0.13
1:00	2.14	2.07	-0.56	0.91	0.84	-0.35	0.38	0.28	-0.25
1:30	2.34	2.16	-0.89	1.00	0.84	-0.54	0.47	0.27	-0.39
2:00	2.65	2.31	-1.30	1.13	0.84	-0.75	0.58	0.24	-0.53
2:30	3.16	2.56	-1.85	1.31	0.84	-1.00	0.72	0.20	-0.69
3:00	4.02	2.99	-2.69	1.55	0.84	-1.31	0.87	0.15	-0.86

Note that the largest shadow is given in the winter at 9am/3pm. Thus, using 2.99 ≈ 3.00 and 2.69 ≈ 2.70 for N and W Lengths, the shadow caused by the parapet on the southern edge extends 3.00×2.5 ft = 7.5 ft in the northern direction. Similarly, the east and west edges of the parapet cast a shadow extending 2.70×2.5 ft = 6.75 ft in the west and east directions, respectively. Noting that there are no negative N Lengths in the table, and thus no shadow directed southward, and hence the north side of the parapet can be ignored. (Note, that even in cases where the latitude is greater than 23.45° it is possible to have a shadow extending in the southern direction. In your spreadsheet replace the latitude of 40° to 25° to see this. In fact, earlier and later in the day, shadows will have a south-pointing component as the sun is north of east and north of west).

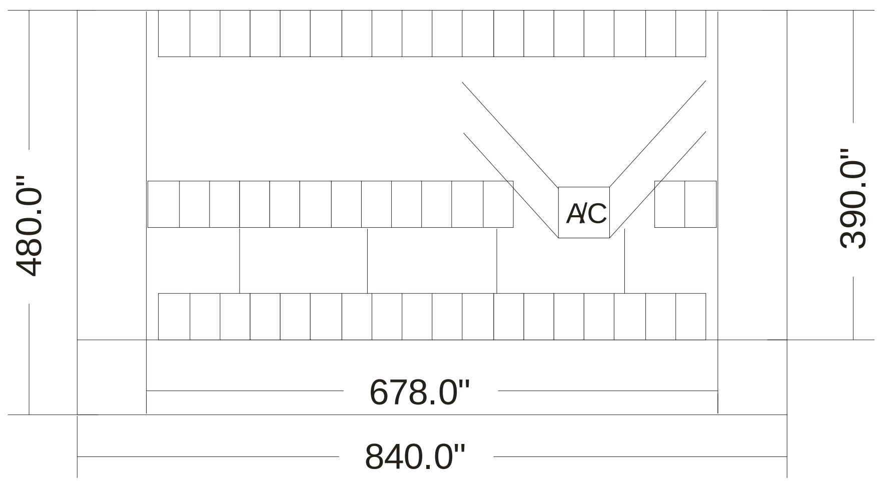
Subtracting these shadows (and for the moment ignoring the AC) gives an available roof space of 32.5 ft in the N/S direction and 56.55 ft in the W/E direction. For this problem, assume that it is unnecessary to leave space between adjacent modules in the row. Since 56.55ft/3ft = 18.85, note that it is possible to fit at most 18 modules in a row. Also, the height of the module is given by 5ft×sin25°=2.11 ft, and the length (in the N/S direction) is given by 5ft×cos25°= 4.53ft. From the height we get that the N/S shadow is 2.11ft×3 = 6.33 ft at the most. Three rows would stretch, including shadows, 3×(4.53ft + 6.33ft) = 32.58ft. It is thus possible to have at most 3 rows. So, still ignoring the AC, a maximum of 3×18 = 54 modules will fit. (Note that in portrait mode, there could be 4 rows of 11, which is less).

One can quickly verify that the shadow extending northward from the AC is at most 10.5 ft. This means the 3rd row of modules can be placed to the very north of the roof without being affected by the AC. Also, as is reasonable, if the 1st row is placed directly behind the parapet shadow, this means the 2nd row needs to be placed between (4.53ft + 6.33ft) = 10.86 ft and (40ft – 7.5ft – 10.86ft – 4.53ft) = 17.11 ft north of the first row. However, considering the AC shadow spreads out more as one moves more north of the AC, it should be clear that this 2nd row should be closer to the 1st row than to the 3rd. This means the 2nd row will be 18.36 ft from the south side of the roof (7.5 ft due to parapet + 10.86 ft due to first row), or put differently, 0.86 ft north of the southern edge of the AC.

Now, it may be possible to place a panel in a spot that shows shadow coverage. This is because the shadow could pass underneath the raised module. It is interesting to calculate exactly how close the module can be placed to the AC. This can be done using 3d vectors. Since the height of the AC is 3.5 ft, the shadow extends from any top corner of the AC at most 10.5 ft north, and 9.45 ft east or west. In the 3d coordinate frame (x,y,z), let the SE top corner of the AC be represented by (0, 0, 3.5). Then, considering x = east, and y = north, at 3pm on the first of winter, the shadow from this corner touches "ground" at (9.45, 10.5, 0). This can be described as a shadow vector going from (0, 0, 3.5) to (9.45, 10.5, 0). Likewise, the west side of the module touches ground x (where x is to be determined) feet away from the AC, and 0.86 feet north of the AC base to gives a coordinate of (x, 0.86, 0). The module ends at (x, 5.392, 2.11). Hence, the shadow vector is given by (9.45t, 10.5t, 3.5 – 3.5t) and the module vector can be given by (x, 0.86 + 4.53s, 2.11s). Now, setting these equal to each other the value of x can be determined, which is how close the module must be to the AC for the shadow to just touch it at 3pm. This value is given by x $\approx$ 4.4 ft.

Hence if the 2nd row is placed as far south as possible, modules can be placed no closer than 4.4 ft W or E of the AC unit. This means to the east of the AC unit the space available is (17.5 ft – 6.75 ft – 4.4 ft) = 6.35 ft. This is enough room for 2 modules. To the west of the AC unit, (47.5 ft – 6.75 ft – 4.4 ft) = 36.35 ft is available. This is enough room for 12 modules. Hence, 14 modules can be squeezed into the 2nd row. This gives a total of 50 modules. So, the AC unit only cost 4 modules.

Note that a little bit of slack is available on the 2nd row. Hence the modules could be placed slightly further from the AC than the minimum of 4.4 ft, and also slightly more north than 0.86ft from the base of the AC unit. Doing this would give a little bit more than the 6 hours of sunlight daily.



2.23 Refer to the tables in Appendix A for Miami, FL; Sacramento, CA; Boston, MA and Fairbanks, AK. Create a table that shows the annual collection losses, in percent, for fixed-tilt collectors, compared to maximum collection when the collector is tilted at latitude. Include all of the tilt angles listed in the tables in your table. If all that counts is total annual peak sun hours, what generalizations can you offer with regard to array tilts?

	Annual psh for array fixed at latitude	% loss for array fixed at latitude -15º	% loss for array fixed at latitude +15º
Miami	5.2	1.92%	1.92%
Sacramento	5.5	0.00%	5.45%
Boston	4.6	2.17%	4.35%
Fairbanks	3.3	-3.03%	6.06%

The differences between the 3 fixed array tilts are not that large. In one case there is actually an improvement by tilting at –15º from latitude. Since this 30º span doesn’t result in a lot of difference, it might be worth tilting at a different angle if it is convenient otherwise. In particular, placing modules flat on a roof that has a slope within 15º of latitude, rather than trying to tilt them to match latitude, which could be more costly due to material and/or labor costs, may be a good idea.