Solutions for Chapter 2: Uncontrolled AC/DC Converters

2.1:

From Equation (2.20) $\phi = \tan^{-1}(\omega L / R) = 78.75^O$

Check Figure 2.4 $\beta = 272^O$

2.2:

From Equation (2.20) $\phi = \tan^{-1}(\omega L / R) = 72.34^O$

Let $\beta_1 = \pi + \phi = 252.34^O$

Step	β	$x = \sin(\beta - \phi)$	$y = e^{-\frac{R\beta}{\omega L}} \sin\phi$	Let x = y
1	252.34^O	0	0.234532	x = 0.234532
2 ↑	265.907^O	+0.234532	0.217508	x = 0.217508
3 ↓	264.905^O	+0.217508	0.218722	x = 0.218722
4 ↑	264.979^O	+0.218722	0.218635	x = 0.218635
5 ↓	264.977^O	+0.218635	0.218641	x = 0.218641
6 ↓	264.972^O	+0.218641	0.218641	x = 0.218641
7	264.972^O			

So, the extinction angle β = 264.972^O with high accuracy.

2.3:

For the parameters given

$$Z = \sqrt{R^2 + \omega^2 L^2} = 106.9\Omega$$

$$\phi = \tan^{-1}\left(\omega L / R\right) = 0.361 rad = 20.7^O$$

$$\omega L / R = 0.377$$

(a) From equation (2.25) for current

$$i = 0.936 \sin(\omega t - 0.361) + 0.331 e^{-\omega t / 0.377}$$

β can be found numerically by equating $i = 0$.

β is found to be 3.5 rad.

(b) Average output current

$$I_d = \int_0^{3.5} \frac{1}{2\pi}\left(0.936 \sin(\omega t - 0.361) + 0.331 e^{-\omega t / 0.377}\right) d(\omega t)$$

or

$$I_d = \frac{\sqrt{2}V}{2\pi R}(1 - \cos\beta) = 0.308 A$$

(c) Average output voltage is

$$V_d = \frac{\sqrt{2}V}{2\pi}(1-\cos\beta) = 30.8V$$

2.4:

a) From (2.25), the angle α at which D starts to conduct is

$$\alpha = \sin^{-1} m = \sin^{-1}\frac{V_c}{\sqrt{2}V} = \sin^{-1}\frac{100}{110\sqrt{2}} = 40^O$$

b) From equation (2.42), γ is

$$\gamma = \pi - 2\alpha = 180 - 80 = 100 \text{ deg}$$

c) From equation (2.43), the average rectified current is

$$I_0 = \frac{1}{2\pi}\int_{40^0}^{140^0} 110\sqrt{2}(\sin 120\pi t - \tfrac{100}{110\sqrt{2}})d(\omega t) = 10.2A$$

d) The rms value of the rectified current is

$$I_R = \left[\frac{1}{2\pi}\int_{40^0}^{140^0}\left[110\sqrt{2}\right]^2(\sin 120\pi t - \tfrac{100}{110\sqrt{2}})^2 d(\omega t)\right]^{1/2} = 21.2A$$

e) The power delivered by the ac source is

$$P = RI_R^2 + V_c I_0 = 1 \times 21.2^2 + 100 \times 10.2 = 1469 \text{ W}$$

f) The power factor is

$$PF = \frac{Power\ delivered}{VI_R} = \frac{1469}{110 \times 21.2} = 0.63.$$

2.5:

Calculation of the angle θ. At $\omega t = \theta$, the slopes of the voltage functions are equal to each other as shown in (2.59),

$$\sqrt{2}V\cos\theta = \frac{\sqrt{2}V\sin\theta}{-\omega RC} e^{-(\theta-\theta)/\omega RC}$$

$$\therefore \frac{1}{\tan\theta} = \frac{-1}{\omega RC}$$

$$Thus \qquad \theta = \pi - \tan^{-1}(\omega RC)$$

Therefore, $\theta = \pi - \tan^{-1}(100\pi * 100 * 0.0001) = 180 - 72.34 = 107.66^O$

Calculation of the angle α using **Iterative method 1**.

Since RC = 0.01 sec, and ωRC = π, we obtain the equation below. At ωt = α, the input voltage is equal to the output voltage,

$$\sin\alpha = (\sin\theta)e^{-(2\pi+\alpha-\theta)/\omega RC}$$

or

$$\sin\alpha = (\sin\theta)e^{-(2\pi+\alpha-\theta)/\omega RC} = 0.953e^{-(252.34+\alpha)/\pi}$$

Using the iterative method 1, define:

$$x = \sin\alpha$$

$$y = 0.953e^{-(252.34+\alpha)/\pi}$$

Make a table with initial α = 30^{O}:

α (^{O})	x	y	|x|: y
30	0.5	0.1777	>
20	0.34	0.21	>
12	0.2079	0.2194	<
13	0.225	0.21822	>
12.6	0.21814	0.2187	<
12.7	0.21985	0.21858	>
12.63	0.21865	0.21867	≈

We can choose α = 12.63^{O}.

2.6:

Calculation of the angle θ. At $\omega t = \theta$, the slopes of the voltage functions are equal to each other.

$$\sqrt{2}V\cos\theta = \frac{\sqrt{2}V\sin\theta}{-\omega RC}e^{-(\theta-\theta)/\omega RC}$$
$$\therefore \frac{1}{\tan\theta} = \frac{-1}{\omega RC}$$
$$Thus \qquad \theta = \pi - \tan^{-1}(\omega RC)$$

$$\theta = \pi - \tan^{-1}(100\pi * 100 * 0.0001) = 180 - 72.34 = 107.66^{O}$$

With comparison with the discharging angle θ in **Problem 2.5**, it can be seen that both values are same since the R, C and ω have the same value.

Calculation of the angle α using Iterative method 1.

RC = 0.01 sec, and ωRC = π.

We obtain the equation below. At ωt = α, the input voltage is equal to the output voltage,

$$\sin\alpha = (\sin\theta)e^{-(\pi+\alpha-\theta)/\omega RC}$$

$$\sin\alpha = (\sin\theta)e^{-(\pi+\alpha-\theta)/\omega RC} = 0.953e^{-(72.34+\alpha)/\pi}$$

Using the iterative method 1, define:

$$x = \sin\alpha$$

$$y = 0.953e^{-(72.34+\alpha)/\pi}$$

Make a table with initial α = 45^{O}:

α (^{O})	x	y	|x| >,=, < y ?
45	0.7071	0.4965	>
35	0.5736	0.5249	>

30	0.5	0.5397	<
33	0.5446	0.5308	>
32.2	0.5328	0.5332	<
32.3	0.5344	0.5329	>
32.22	0.53317	0.53311	≈

We can choose $\alpha = 32.22^{O}$.

The average output voltage V_d is

$$V_d = \frac{\sqrt{2}V}{\pi}\left[\int_{\alpha}^{\theta}\sin(\omega t)d\omega t + \int_{\theta}^{\pi+\alpha} 0.953 e^{-(\omega t-\theta)/\pi} d\omega t\right]$$

$$= \frac{\sqrt{2}V}{\pi}\left[(\cos\alpha - \cos\theta) + 0.953\pi(1-e^{-(\pi+\alpha-\theta)/\pi})\right]$$

$$= \frac{240\sqrt{2}}{\pi}\left[(0.846 + 0.3034) + 0.953\pi(1-0.4597)\right]$$

$$V_d = 108.038\left[(1.1494) + 0.953\pi(0.5403)\right] = 298.94 \text{ V}$$

The average capacitor current is zero $I_C = 0$

The average resistor and diode current is

$$I_R = I_d = \frac{V_d}{R} = \frac{298.94}{100} = 2.9894 \text{ A}$$