2. How (Most) Physicists Approach Biophysics

Problem 2.1 Calculating elementary physical distances related to solid matter.

Solutions:

(1)

$$\text{Density} = 2.33\text{g/cm}^3;\ \text{MW} = 28.1\text{g/mole}$$

$$\frac{6.02\times10^{23}\text{molec}}{\text{mole}}\cdot\frac{1\text{mole}}{28.1\text{g}}\cdot\frac{2.33\text{g}}{\text{cm}^3} = 4.99\times10^{22}\frac{\text{molec}}{\text{cm}^3}$$

$$\left(4.99\times10^{22}\frac{\text{molec}}{\text{cm}^3}\right)^{1/3} = 3.68\times10^{9}\,\text{molec/m} \Rightarrow 0.272\text{nm/molec}$$

For better understanding, consider eight 1-cm diameter spheres on the corners of a 1-cm cube. Only 1/8th of each sphere is "in" the cube.

(2) Carbon in graphite and in diamond:

$$\text{Graphite density} = 2.27\text{g/cm}^3;\ \text{MW} = 12.0\text{g/mole}$$

$$\frac{6.02\times10^{23}\text{molec}}{\text{mole}}\cdot\frac{1\text{mole}}{12.0\text{g}}\cdot\frac{2.27\text{g}}{\text{cm}^3} = 1.14\times10^{23}\frac{\text{molec}}{\text{cm}^3}$$

$$\left(1.14\times10^{23}\frac{\text{molec}}{\text{cm}^3}\right)^{1/3} = 4.85\times10^{9}\,\text{molec/m} \Rightarrow 0.206\text{nm/molec}$$

$$\text{Diamond density} = 3.52\text{g/cm}^3;\ \text{MW} = 12.0\text{g/mole}$$

$$\frac{6.02\times10^{23}\text{molec}}{\text{mole}}\cdot\frac{1\text{mole}}{12.0\text{g}}\cdot\frac{3.52\text{g}}{\text{cm}^3} = 1.77\times10^{23}\frac{\text{molec}}{\text{cm}^3}$$

$$\left(1.77\times10^{23}\frac{\text{molec}}{\text{cm}^3}\right)^{1/3} = 5.61\times10^{9}\,\text{molec/m} \Rightarrow 0.178\text{nm/molec}$$

(3) Electrical properties like conductivity, band gap and, of course, density. Chemical and elastic properties (hardness, compressibility) depend only slightly on atom-atom distances, more strongly on the strength of bonds.

Problem 2.2 Calculating elementary physical distances related to liquid matter.

Solutions:

(1) Average distance between water molecules in liquid water.

$$\left(\frac{1\text{m}^3}{1000\text{kg}}\,\frac{0.018\text{kg}}{6.02\times10^{23}\text{molec}}\right)^{1/3} = 0.310\ \text{nm}$$

(2) Average distance between sodium ions in a 1 molar (1 M), aqueous solution of NaCl. The concentration of both [Na^+] and [Cl^-] are 1 M, so

$$\text{Distance} = \left(1\text{M}\,\frac{6.02\times10^{26}\,\text{molec}/\text{m}^3}{\text{M}}\right)^{-1/3} = 1.18\ \text{nm}$$

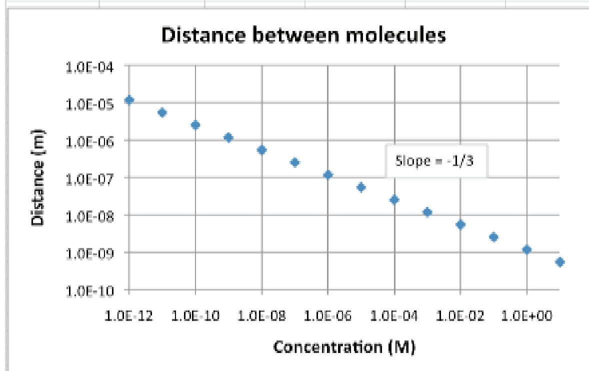
(3) Solution: see below.

(4)

$$\text{Distance} = \left(\text{Molarity}\cdot 6\text{x}10^{26}\,\frac{\text{molec}/\text{m}^3}{\text{M}}\right)^{-1/3}$$

Slope = -1/3. Does not change if units are changed.

Molarity	Distance between molecules
1.0E-12	1.2E-05
1.0E-11	5.5E-06
1.0E-10	2.6E-06
1.0E-09	1.2E-06
1.0E-08	5.5E-07
1.0E-07	2.6E-07
1.0E-06	1.2E-07
1.0E-05	5.5E-08
1.0E-04	2.6E-08
1.0E-03	1.2E-08
1.0E-02	5.5E-09
1.0E-01	2.6E-09
1.0E+00	1.2E-09
1.0E+01	5.5E-10

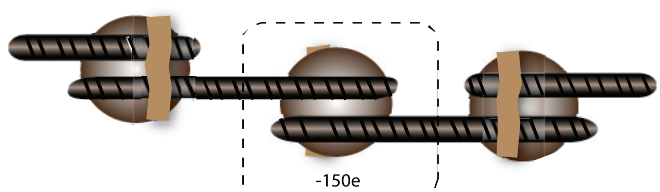


Problem 2.3 Biological importance of Problem 2.2.

Solution: Distances between water molecules are about 0.3 nm, between 1 M Na^+ ions, about 1 nm. (If the NaCl concentration were closer to normal physiological levels, ~0.15 M, the distance between Na^+s would be larger, several nm.) If a cell suddenly encountered a different NaCl concentration, the time for the new Na^+ concentration to completely surround the cell would be important. This time would either be the time for Na^+ to diffuse in water or flow a distance of about half a cell circumference. The characteristic distance a Na^+ ion or water molecule might move after a collision with a water molecule should be about 0.3 nm or less. Note the Na^+ ion is more likely to collide with water than with another Na^+ or Cl^-. In order to affect interior cell structures or processes, a Na^+ or Cl^- ion needs to be transported across the cell membrane. Since the average distance between ions is a few nm or less and the membrane thickness is 5-10 nm, and there will be many ions near the $\sim 4\pi R^2 \sim 10^7$ nm^2 surface of the cell, there will be an almost constant, large supply of ions ready to cross the cell membrane: a continuum model should provide an adequate description of the diffusion and other motion of ions and water around and into the cell. Other physical effects: typical distances for electric force and energy calculations involving Na^+ or Cl^- should be less than about 10 nm.

Problem 2.4 DNA

Solution:



Number of charges on 2.5 turns of DNA:

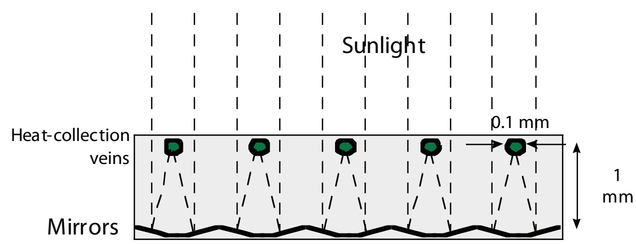
$$\text{Avg radius} \cong 4 \text{ nm}$$
$$\text{Avg Circum} \cong 2\pi \cdot 4\text{nm} = 25 \text{ nm}$$
$$\#\text{phosphates} = 25\text{nm}\left(\frac{1 \text{ basepair}}{0.34\text{nm}}\right)\left(\frac{2 \text{ phos}}{\text{basepair}}\right)(2.5 \text{ wraps}) \cong 370$$

So there is a charge of about -370e from the DNA phosphate charges. This high charge will probably be partly neutralized by ions in the solvent and electric fields reduced by the high dielectric constant of water. The sphere should have compensating positive charge on it to encourage DNA wrapping. In fact, the net charge on a nucleosome unit has been estimated at -150e, so half the DNA charge is compensated for in histone positive charge.

The bending rigidity of DNA cannot be too large: it must be able to bend in a ~4 nm radius circle without increasing the energy too much (should not be too many k_BT's of energy.)
Doing some research (see Chapter 16, also), the energy of a length, L, of a rod of "persistence length" A and radius of curvature is $E = \frac{k_B TAL}{2R^2} \cong \frac{k_B TA(2.5 \cdot 2\pi R)}{2R^2}$. Assuming the storage is as compact as possible, the persistence length, the distance over which the direction of the rod tends to persist, should be about half a circumference, so $E = \frac{k_B T(\pi R)(2.5 \cdot 2\pi R)}{2R^2} \cong 25 k_B T$, which isn't too large. (The measured persistence length in normal solution is actually ~50 nm, so E is closer to $100 k_B T$.

Problem 2.5

Solution:
(1) See diagram.
(2) The focal length of the mirrors should be 1 mm, which means the mirror radius of curvature should be 2 mm.



If the veins are also separated by 1 mm, each mirror should be 1 mm across and there should be about 200 such mirrors across the 20-cm leaf. This all is physically possible.

(3) The amount of power falling on each mirror would be $P_{mirror} = 1000 \frac{W}{m^2}\left(10^{-3} m\right)^2 = 10^{-3} W$. A vein spanning half the leaf would have 100 mirrors feeding power to it, which means it must transport up to 100 mW. The 10^{-3} W of power would be focused onto a tube of radius 0.05 mm and length 1 mm. The volume of this tube over the mirror is
$\left(\pi\left(5 \times 10^{-5} m\right)^2\right)\left(10^{-3} m\right) = 7.8 \times 10^{-12} m^3$. If this were water-filled, the amount of heat deposited in 1 s would be 10^{-3} J, which would heat the water up

$Q = 10^{-3}\,\text{J} = mc\Delta T = \left(7.8 \times 10^{-9}\,\text{kg}\right)\left(4186\,\frac{\text{J}}{\text{kg}\cdot{}^{\circ}\text{C}}\right)\Delta T \Rightarrow \Delta T \cong 31^{\circ}\text{C}$. Since the veins would pass over perhaps 100 other mirrors on the way out of the leaf, this is probably a 100x too-large a temperature rise, so the vein probably needs to transport the heated water out of the space above a mirror in about 0.01 s. The water velocity would then be about $\frac{10^{-3}\,\text{m}}{0.01\,\text{s}} = 0.1\,\text{m/s} = 100\,\text{mm/s}$. This seems physically possible, but we should check on the fluid friction in this small tube. We'll leave this for later.