Answers to chapter 2 problems

1. Linearity: $aX\left[z\right]=a\sum_{n=0}^{\infty}x\left[n\right]z^{-n}=Z\left\{ax[n]\right\}$, $aY\left(s\right)=a\int_{0}^{\infty}y\left(t\right)e^{-st}dt=L\left\{ay\left(t\right)\right\}$

Also see eqn (2.1.4).

Shift invariance: $\sum_{n=0}^{\infty}x[n-k]z^{-n}=\sum_{m=0}^{\infty}x[m]z^{-(m+k)}=\sum_{n=0}^{\infty}\left\{x[m]z^{-k}\right\}z^{-m}=z^{-k}X\left[z\right]$

Note that for $k > 0$, $x[-k] = 0$ allowing a simple substitution of $m=n-k$.

$\int_{0}^{\infty}y\left(t-\tau\right)e^{-st}dt=\int_{0}^{\infty}y\left(t'\right)e^{-s(t'+\tau)}dt=e^{-s\tau}Y\left(s\right)$ Also, see eqn (2.1.5).

2. Let the digital signal be modeled as $x\left[n\right]=A^{n}z^{n}$ where “z” is some complex number, “n” is sample number, and “A” magnitude equal to unity. If the imaginary part of “z” is greater than zero, one can imagine a complex digital sinusoid where each sample is on the unit circle at an angle of “nθ” where “θ” is the angle of “z”. This signal is stable because as “n” approaches infinity x[n] remains finite. But if “A” > 1, the signal is unstable because it eventually becomes infinite as “n” approaches infinity. If “A” < 1 the signal approaches zeros as “n” approaches infinity. Therefore, the “stable” region on the z-plane is the region on or inside the unit circle.

For an analog signal, let $y\left(t\right)=Ae^{st}=Ae^{(\sigma+j\omega)t}$ where “ω” is radian frequency and “t” is time. If “σ” = 0 we have a complex sinusoid which is stable. But if “σ” > 0 the magnitude of this signal grows with increasing time. Therefore the region of stability on the s-plane is the region where “σ” is less than or equal to zero.

3. Since the magnitude of “α” is unity, there is no stretching or compressing of the frequency axis, but the phase of “∝” has the effect of rotating the z-domain spectrum counter-clockwise by the angle of “α” which is assumed complex in equation (2.1.13). When “a” is imaginary in equation (2.1.12), the spectrum slides up the “jω” by “a”. Both of these effects are called “modulation” where the time domain signal is multiplied by a complex sinusoid and the resulting spectrum is shifted up on the s-plane or rotated counter-clockwise on the z-plane.

4. The shift in phase (overall negative slope to the phase) of the digital system in Figs 2.6 and 2.7 is due to the “z^{-1}” in the numerator of equation (2.3.3-4), which is the result of our choosing a causal system representation precisely. Carrying this delay is really just a matter of choice.

5. When you have conjugate poles or zeros, each conjugate pair can be represented as a second order system with real coefficients. So an even-order conjugate pair system can be factored into a number of 2nd order systems with real coefficients, which can be cascaded resulting in a real signal output response for a real signal input.

6. Equation (2.1.22) has a zero at z = 1, which corresponds to 0 Hz, same as a zero at “s”= 0 in equation (2.1.15).

7. Using eqn (2.3.1), the poles on the z-plane have magnitude $e^{-\frac{40}{1000}} \approx 0.912$ and a frequency angle of 0.175π radians, or about $\pm$31.5 degrees.