

Solutions for Chapter 2: Uncontrolled AC/DC Converters

2.1:

From Equation (2.20) $\phi = \tan^{-1} (\omega L/R) = 78.75^\circ$

Check Figure 2.4 $\beta = 272^\circ$

2.2:

From Equation (2.20) $\phi = \tan^{-1} (\omega L/R) = 72.34^\circ$

Let $\beta_1 = \pi + \phi = 252.34^\circ$

Step	β	$x = \sin (\beta - \phi)$	$y = e^{-\frac{R\beta}{\omega L}} \sin \phi$	Let $x = y$
1	252.34°	0	0.234532	$x = 0.234532$
2 \uparrow	265.907°	+0.234532	0.217508	$x = 0.217508$
3 \downarrow	264.905°	+0.217508	0.218722	$x = 0.218722$
4 \uparrow	264.979°	+0.218722	0.218635	$x = 0.218635$
5 \downarrow	264.977°	+0.218635	0.218641	$x = 0.218641$
6 \downarrow	264.972°	+0.218641	0.218641	$x = 0.218641$
7	264.972°			

So, the extinction angle $\beta = 264.972^\circ$ with high accuracy.

2.3:

For the parameters given

$$Z\sqrt{R^2 + \omega^2 L^2} = 106.9\Omega$$

$$\phi = \tan^{-1}(\omega L/R) = 0.361\text{rad} = 20.7^\circ$$

$$\omega L/R = 0.377$$

(a) From equation (2.25) for current

$$i = 0.936 \sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}$$

β can be found numerically by equating $i = 0$.

β is found to be 3.5 rad.

(b) Average output current

$$I_d = \int_0^{3.5} \frac{1}{2\pi} (0.936 \sin(\omega t - 0.361) + 0.331e^{-\omega t/0.377}) d(\omega t)$$

or

$$I_d = \frac{\sqrt{2V}}{2\pi R} (1 - \cos \beta) = 0.308 A$$

(c) Average output voltage is

$$V_d = \frac{\sqrt{2V}}{2\pi} (1 - \cos \beta) = 30.8 V$$

2.4:

a) From (2.25), the angle α at which D starts to conduct is

$$\alpha = \sin^{-1} m = \sin^{-1} \frac{V_c}{\sqrt{2V}} = \sin^{-1} \frac{100}{110\sqrt{2}} = 40^\circ$$

b) From equation (2.42), γ is

$$\gamma = \pi - 2\alpha = 180 - 80 = 100 \text{ deg}$$

c) From equation (2.43), the average rectified current is

$$I_0 = \frac{1}{2\pi} \int_{40^\circ}^{140^\circ} 110\sqrt{2} \left(\sin 120\pi t - \frac{100}{110\sqrt{2}} \right) d(\omega t) = 10.2 A$$

d) The rms value of the rectified current is

$$I_R = \left[\frac{1}{2\pi} \int_{40^\circ}^{140^\circ} [110\sqrt{2}]^2 (\sin 120\pi t - \frac{100}{100\sqrt{2}})^2 d(\omega t) \right]^{1/2} = 21.2 A$$

e) The power delivered by the ac source is

$$P = RI_R^2 + V_c I_0 = 1 \times 21.2^2 + 100 \times 10.2 = 1469 W$$

f) The power factor is

$$PF = \frac{\text{Power delivered}}{VI_R} = \frac{1469}{110 \times 21.2} = 0.63.$$

2.5:

Calculation of the angle θ . At $\omega t = \theta$, the slopes of the voltage functions are equal to each other as shown in (2.59),

$$\begin{aligned} \sqrt{2V} \cos \theta &= \frac{\sqrt{2V} \sin \theta}{-\omega RC} e^{-(\theta-\theta)/\omega RC} \\ \therefore \frac{1}{\tan \theta} &= \frac{-1}{\omega RC} \\ \text{Thus } \theta &= \pi - \tan^{-1}(\omega RC) \end{aligned}$$

Therefore, $\theta = \pi - \tan^{-1}(100\pi * 100 * 0.0001) = 180 - 72.34 = 107.66^\circ$

Calculation of the angle α using **Iterative method 1**.

Since $RC = 0.01$ sec, and $\omega RC = \pi$, we obtain the equation below. At $\omega t = \alpha$, the input voltage is equal to the output voltage,

$$\sin \alpha = (\sin \theta) e^{-(2\pi + \alpha - \theta)/\omega RC}$$

or

$$\sin \alpha = (\sin \theta) e^{-(2\pi + \alpha - \theta)/\omega RC} = 0.953 e^{-(252.34 + \alpha)/\pi}$$

Using the iterative method 1, define:

$$x = \sin \alpha$$

$$y = 0.953 e^{-(252.34 + \alpha)/\pi}$$

Make a table with initial $\alpha = 30^\circ$:

α ($^\circ$)	x	y	x : y
30	0.5	0.1777	>
20	0.34	0.21	>
12	0.2079	0.2194	<
13	0.225	0.21822	>
12.6	0.21814	0.2187	<

12.7	0.21985	0.21858	>
12.63	0.21865	0.21867	≈

We can choose $\alpha = 12.63^\circ$.

2.6:

Calculation of the angle θ . At $\omega t = \theta$, the slopes of the voltage functions are equal to each other.

$$\sqrt{2V} \cos \theta = \frac{\sqrt{2V} \sin \theta}{-\omega RC} e^{-(\theta - \theta)/\omega RC}$$

$$\therefore \frac{1}{\tan \theta} = \frac{-1}{\omega RC}$$

$$\text{Thus } \theta = \pi - \tan^{-1}(\omega RC)$$

$$\theta = \pi - \tan^{-1} / (100\pi * 100 * 0.0001) = 180 - 72.34 = 107.66^\circ$$

With comparison with the discharging angle θ in **Problem 2.5**, it can be seen that both values are same since the R, C and ω have the same value.

Calculation of the angle α using Iterative method 1.

$$RC = 0.01 \text{ sec, and } \omega RC = \pi.$$

We obtain the equation below. At $\omega t = \alpha$, the input voltage is equal to the output voltage,

$$\sin \alpha = (\sin \theta) e^{-(\pi + a - \theta)/\omega RC}$$

$$\sin \alpha = (\sin \theta) e^{-(\pi + a - \theta)/\omega RC} = 0.953 e^{-(72.34 + \alpha)/\pi}$$

Using the iterative method 1, define:

$$x = \sin \alpha$$

$$y = 0.953 e^{-(72.34 + a)/\pi}$$

Make a table with initial $\alpha = 45^\circ$:

$\alpha (^{\circ})$	x	y	$ x >, =, < y ?$
45	0.7071	0.4965	>
35	0.5736	0.5249	>
30	0.5	0.5397	<
33	0.5446	0.5308	>
32.2	0.5328	0.5332	<
32.3	0.5344	0.5329	>
32.22	0.53317	0.53311	\approx

We can choose $\alpha = 32.22^\circ$.

The average output voltage V_d is

$$\begin{aligned}
V_d &= \frac{\sqrt{2}V}{\pi} \left[\int_{\alpha}^{\theta} \sin(\omega t) d\omega t + \int_{\theta}^{\pi+\alpha} 0.953 e^{-(\omega t - \theta)/\pi} d\omega t \right] \\
&= \frac{\sqrt{2}V}{\pi} \left[(\cos \alpha - \cos \theta) + 0.953\pi (1 - e^{-(\pi+\alpha-\theta)/\pi}) \right] \\
&= \frac{240\sqrt{2}}{\pi} [(0.846 + 0.3034) + 0.953\pi (1 - 0.4597)] \\
V_d &= 108.038 [(1.1494) + 0.953\pi (0.5403)] = 298.94 \text{ V}
\end{aligned}$$

The average capacitor current is zero $I_C = 0$

The average resistor and diode current is

$$I_R = I_d = \frac{V_d}{R} = \frac{298.49}{100} = 2.9894 \text{ A}$$