1 Chapter 2 Dynamic Simulation

1. A voltage source is serving an RLC series connected circuit. Let $R = 0.01\Omega$, $L = 0.01$ H, $C = 0.001$ F. The compensation degree of the system is X_c/X_L, approximately 70.36%. Find its current response for a step response of the voltage source, and a sinusoidal 60 Hz input (amplitude 1 V)of the voltage source. Use Laplace transformation to find the current in Laplace domain and current in time domain.

Solution: We will first find the transfer function from the source voltage to the current by implementing the impedance model. The impedance model of the circuit is $R + Ls + \frac{1}{Cs}$. Therefore, the transfer function is

$$\frac{I(s)}{V_s(s)} = \frac{1}{R + Ls + \frac{1}{Cs}}. \tag{1}$$

Next, we consider two cases. Case 1, the source voltage is a step response and its Laplace transform is $\frac{1}{s}$. Case 2, the source voltage is sinusoidal.

Case 1: $V_s(s) = \frac{1}{s}$ for a step response. Therefore, we may find the current's Laplace transform as

$$I(s) = \frac{1}{s}\frac{Cs}{LCs^2 + RCs + 1} = \frac{0.001}{10^{-5}s^2 + 10^{-5}s + 1} = \frac{100}{s^2 + s + 10^5} \tag{2}$$

A few algebraic manipulations will lead to

$$\boxed{I(s) = \frac{100}{(s + 0.5)^2 + 316^2}}. \tag{3}$$

Since the inverse Laplace transform for $\frac{b}{(s-a)^2+b^2}$ is $e^{at}\sin(bt)$, we then find $i(t)$ in the following expression.

$$\boxed{i(t) = 0.3162e^{-0.5t}\sin(316t) \text{ A}} \tag{4}$$

Case 2: $v_s(t) = \sin(377t)$. Its Laplace transform is $V_s(s) = \frac{377}{s^2+377^2}$. We may find the current's Laplace transform as

$$\boxed{I(s) = \frac{377}{s^2 + 377^2}\frac{0.001s}{10^{-5}s^2 + 10^{-5}s + 1} = \frac{37700s}{(s^2 + 377^2)(s^2 + s + 10^5)}} \tag{5}$$

The denominator has four roots: $\pm j377$ and $-0.5 \pm j316$. In order to find $i(t)$, we need to carry out inverse Laplace transform. The technique used is Partial Fraction Expansion to split up a complicated fraction into forms that are in the Laplace transform table. For the above $I(s)$, its fraction expansion expression is

$$I(s) = \frac{k_1 s + k_2}{s^2 + 377^2} + \frac{k_3 s + k_4}{s^2 + s + 10^5} \tag{6}$$

where k_is are real numbers.

To find k_1 and k_2, let (5) and (6) both be multiplied by $(s^2 + 377^2)$. Then we evaluated the right side terms of the two equations at $\pm j377$. Note that $\frac{k_3 s+k_4}{s^2+s+10^5}(s^2 + 377)$ will be 0 if it is evaluated at $s = \pm j377$. Therefore the following equations will be found.

$$k_1 s + k_2 \Big|_{-j377} = \frac{37700s}{s^2 + s + 10^5}\Big|_{-j377} \tag{7a}$$

$$k_1 s + k_2 \Big|_{j377} = \frac{37700s}{s^2 + s + 10^5}\Big|_{j377} \tag{7b}$$

$$\Longrightarrow k_1 = -0.8948, k_2 = 3.0187$$

Similarly, we can find $k_3 = 0.8948$ and $k_4 = -2.1239$.

The inverse Laplace transform is

$$\begin{aligned} i(t) &= k_1 \cos(377t) + \frac{k_2}{377}\sin(377t) + k_3 e^{-0.5t}\sin(316t) + \frac{k_4}{316}e^{-0.5t}\cos(316t) \\ &= -0.8948\cos(377t) + 0.008\sin(377t) + 0.8948e^{-0.5t}\sin(316t) - 0.0067e^{-0.5t}\cos(316t) \end{aligned} \tag{8}$$

The above expression can be further arranged as follows.

$$\boxed{i(t) = 0.8948\sin(377t - 1.5618) + 0.8948e^{-0.5t}\sin(316t - 0.0075)} \tag{9}$$

Remarks: We may examine the above expression and know that the steady-state response of the current is $0.8948\sin(377t - 1.5618)$. This time-domain function can also be obtained by phasor-based calculation.

The impedance at 60 Hz or 377 rad/s is $Z = R + j377L - j\frac{1}{377C} = 1.1175e^{j1.5618}$.

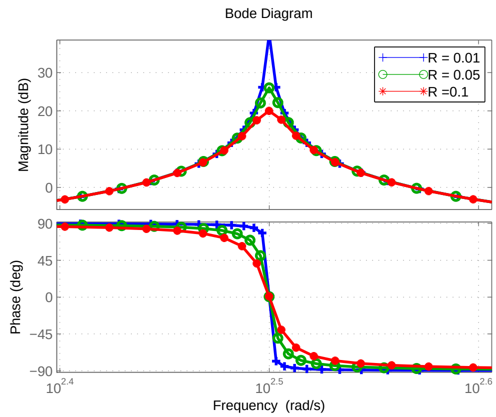
Therefore the current phasor magnitude should be the voltage source magnitude divided by $|Z| = 1.1175$ and the phase shift should be -1.5618 radian. The steady-state current should be

$$i(t) = \frac{1}{1.1175}\sin(377t - 1.5618) = 0.8948\sin(377t - 1.5618).$$

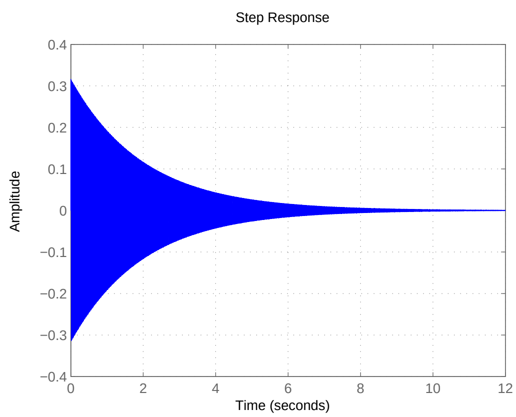
2. Use MATLAB linear system analysis tools to define a linear system for the above RLC circuit. Treat the voltage source as the input while the current as the output. Give a set of Bode plots of the system by varying R. Notate the plot properly. Use MATLAB function *step* to examine the dynamic response of the current with a step response of the voltage source. Use Matlab function *lsim* to examine the dynamic response of the current with a sinusoidal input.

Solution: The MATLAB codes are as follows.

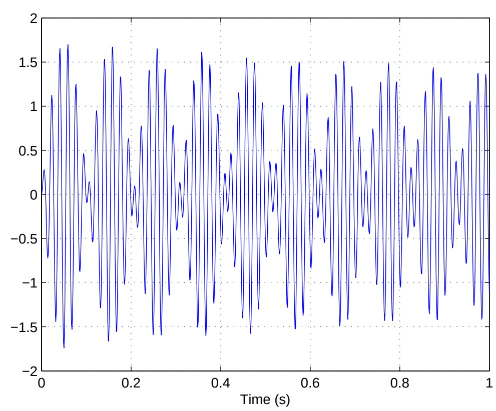
```
R = [0.01, 0.05, 0.1] ; L = 0.01; C = 0.001;
s = tf('s');
for i=1: length(R)
    plant(i) = 1/(R(i) + L*s + 1/(C*s));
    bode(plant(i)); hold on;
end

figure
step(plant(1)); grid on;

t=0:0.001:1; u = sin(377*t);
y = lsim(plant(1), u, t, 0);
figure
plot(t,y);
xlabel('Time (s)'); grid on;
```



(a) Bode plots for three different Rs.



(b) Problem 2: *step* output for a step input.



(c) Problem 2: *lsim* output for a sinusoidal input.

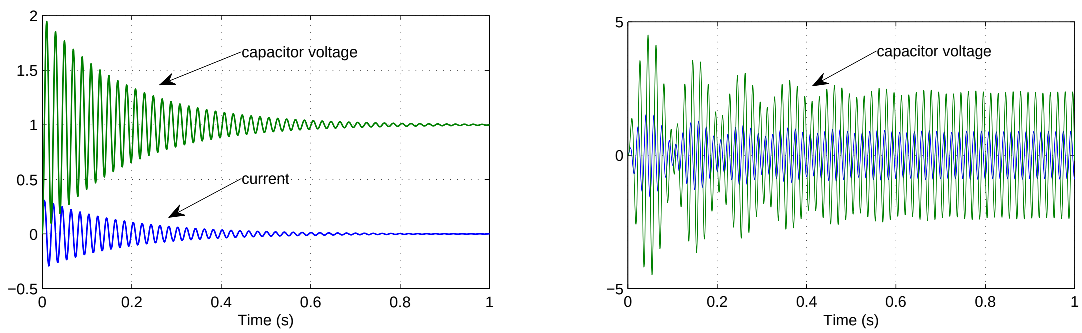
Figure 1: Problem 2.

The generated figures are shown in Figs. 1a, 1b, and 1c.

3. For the above RLC circuit, build a two-order state-space model. The state variables are the current and the voltage across the capacitor. Use MATLAB function *ode* to simulate the dynamic response of the current for a step input and a sinusoidal input.

Solution: The state-space model has been built in (2.19). The MATLAB codes to conduct the two simulation case studies are as follows.

```
R = 0.1 ; L = 0.01; C = 0.001;
dxdt_step = @(t, x, R, L, C)[(1-R*x(1)-x(2))/L; x(1)/C] ;
dxdt_sin  = @(t, x, R, L, C)[(sin(377*t)-R*x(1)-x(2))/L; x(1)/C] ;
[T1,y1]= ode23(@(t,x)dxdt_step(t, x, R, L, C), [0 1],[0; 0]);
[T2,y2]=  ode23(@(t,x)dxdt_sin(t, x, R, L, C), [0 1],[0; 0]);

figure
plot(T1,y1); grid on; xlabel('Time (s)');
figure
plot(T2,y2); grid on; xlabel('Time (s)');
```



(a) (b)

Figure 2: Problem 3: (a) Dynamic response for a step input; (b) Dynamic response for a sinusoidal input.

2 Chapter 3 Frequency Control

1. Use parameters in Example 1 Fig. 3.29. For a single generator load serving system, derive the linear system model and build the model in MATLAB/Simulink. Find the droop to make $\Delta\omega = -0.2$ for $\Delta P_L = 0.1$.

- Find the bandwidth of the system with only primary frequency control.
- Provide the dynamic simulation of the system frequency due to a step response of load increase 0.1.
- Specify ΔP_c to bring $\Delta\omega$ back to zero.