

## CHAPTER 2 GEOMETRICAL OPTICS

**2-1.**  $t = \frac{\sum d_{\text{op}}}{c} = \frac{\sum_i n_i x_i}{c}$

**2-2.** Referring to Figure 2-12 and with lengths in cm,

$$n_0 (x^2 + y^2)^{1/2} + n_i \left( y^2 + (s_o + s_i - x)^2 \right)^{1/2} = n_o s_o + n_i s_i$$

$$(1) (x^2 + y^2)^{1/2} + 1.5 \left( y^2 + (30 - x)^2 \right)^{1/2} = 20 + 1.5 (10) = 35$$

$$2.25 \left[ y^2 + (30 - x)^2 \right] = \left[ 35 - (x^2 + y^2)^{1/2} \right]^2$$

$$1.25 (x^2 + y^2) + 70 (x^2 + y^2)^{1/2} - 135x + 800 = 0$$

Using a calculator to guess and check or using a computer algebra system (like the free program Maxima, for example), one can numerically solve this equation for  $x$  for given  $y$ -values. Doing so results in,

$x$ (cm)	20	20.2	20.4	20.8	21.6	22.4	23.2	24.0	24.8	25.6	26.4	27.2
$y$ (cm)	0	$\pm 1.0$	$\pm 1.40$	$\pm 1.96$	$\pm 2.69$	$\pm 3.20$	$\pm 3.58$	$\pm 3.85$	$\pm 4.04$	$\pm 4.14$	$\pm 4.18$	$\pm 4.13$

**2-3.** Refer to the figure for the relevant parameters.

$$d = d' = \sqrt{30^2 + 2.5^2} = 30.104 \text{ cm}$$

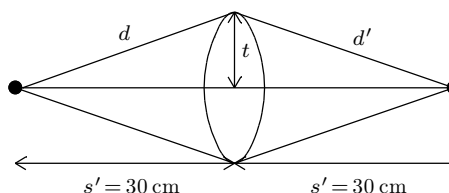
Fermat:  $d + d' = s + s' - t + m t$

$$d + d' = s + s' + t(m - 1)$$

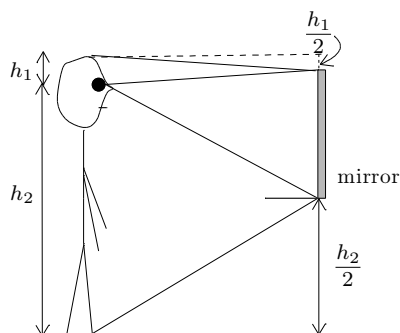
$$2(30.10399) = 60 = t(1.52 - 1)$$

$$t = 4 \text{ mm}$$

$$n = 1.52$$



**2-4.** See the figure below. Let the height of the person be  $h = h_1 + h_2$ .

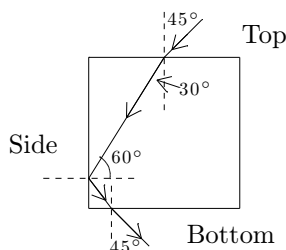


The person must be able to see the top of his head and the bottom of his feet. From the figure, it is evident that the mirror height is:

$$h_{\text{mirror}} = h - h_1/2 - h_2/2 = h/2$$

The mirror must be half the height of the person. So for a person of height six feet, the mirror must be 3 feet high.

**2-5.** Refer to the figure.



At top:  $(1) \sin 45 = \sqrt{2} \sin \theta' \Rightarrow \theta' = 30$

At side:  $\sqrt{2} \sin 60 = (1) \sin \theta', \sin \theta' = \sqrt{1.5} > 1$

Thus total internal reflection occurs.

At bottom: reverse of Top:  $\theta' = 45^\circ$

**2-6.** The microscope first focuses on the scratch using direct rays. Then it focuses on the image  $I_2$  formed in a two-step process: (1) reflection from the bottom to produce an intermediate image  $I_1$  and (2) refraction through the top surface to produce an image  $I_2$ . Thus,  $I_1$  is at  $2t$  from top surface, and  $I_2$  is at the apparent depth for  $I_1$ , serving as the object:  $s' = \frac{2t}{n}$  or  $n = \frac{2t}{s'} = \frac{3}{1.87} = 1.60$

**2-7.** Refer to Figure 2-33 in the text. By geometry,  $\tan \theta_c = \frac{7.60/4}{2.25}$  so  $\theta_c = 40.18^\circ$   
Snell's law:  $n \sin \theta_c = (1) \sin 90^\circ \Rightarrow n = \frac{1}{\sin 40.18^\circ} = 1.55$

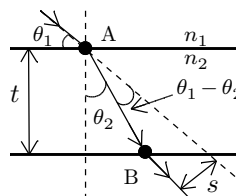
**2-8.** Referring to the figure one can see that

$$s = AB \sin(\theta_1 - \theta_2) \text{ and } AB = \frac{t}{\cos \theta_2} \text{ Therefore,}$$

$$s = \frac{t \sin(\theta_1 - \theta_2)}{\cos \theta_2} \text{ For } t = 3 \text{ cm, } n_2 = 1.50, \theta_1 = 50^\circ,$$

$$\text{Snell's law gives } \sin \theta_2 = \frac{n_1}{n_2} \sin \theta_1 = \frac{1}{1.5} \sin 50^\circ.$$

$$\text{Then, } \theta_2 = 30.71^\circ \text{ and } s = \frac{3 \sin(50^\circ - 30.71^\circ)}{\cos 30.71^\circ} = 1.153 \text{ cm.}$$



**2-9.** Image of near end:  $s = 60 \text{ cm}$ ,  $\frac{1}{60} = \frac{1}{s'} = \frac{1}{-40}$ ,  $s' = -24 \text{ cm}$

Image of far end:  $s = 60 + 100 \text{ cm}$ ,  $\frac{1}{160} + \frac{1}{s'} = \frac{1}{-40}$ ,  $s' = -32 \text{ cm}$ . So,  $L' = \Delta s' = -24 - (-32) = 8 \text{ cm}$

**2-10.** (a) See Figure 2-34 in the text. Image due to rays directly from bubble through plane interface:

$$\frac{n_1}{s} + \frac{n_2}{s'} = 0 \text{ or } \frac{1.5}{s} + \frac{1}{s'} = 0 \Rightarrow s' = -3.33 \text{ cm.}$$

(b) Image due to rays first reflected in spherical mirror, then refracted through plane interface:

$$\text{reflection: } \frac{1}{2} + \frac{1}{s'_1} = -\frac{2}{R} \text{ and } \frac{1}{2.5} + \frac{1}{s'_1} = -\frac{2}{-7.5} \quad s'_1 = -7.5 \text{ cm}$$

$$\text{refraction: } \frac{n_1}{s} + \frac{n_2}{s'_2} = 0 \text{ or } \frac{1.5}{15} + \frac{1}{s'_2} = 0 \quad s'_2 = -10 \text{ cm}$$

Thus the images are at 3.33 cm and 10 cm behind the interface.

**2-11.** There are 5 unknowns:  $s_1$  and  $s'_1$  in position (1),  $s_2$  and  $s'_2$  in position (2), and the focal length  $f$  of the mirror. The five equations that, solved simultaneously, yield the results are:

$$(1) \text{ linear magnification: } s'_1/s_1 = 2 \quad (2) \text{ linear magnification: } s'_2/s_2 = 3$$

$$(3) \text{ focal length from mirror equation: } f = \frac{s_1 s'_1}{s_1 + s'_1} \quad (4) \text{ focal length from mirror equation: } f = \frac{s_2 s'_2}{s_2 + s'_2}$$

$$(5) \text{ image distance relation: } s'_2 = s'_1 + 75$$

One finds  $s_1 = 112.5 \text{ cm}$ ,  $s_2 = 100 \text{ cm}$ ,  $s'_1 = 225 \text{ cm}$ ,  $s'_2 = 300 \text{ cm}$ , and  $f = 75 \text{ cm}$ .

**2-12.** The object distance from the front surface is the diameter of the sphere, 5 cm. Then,

$$\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1.5}{5} + \frac{1}{s'} = \frac{1 - 1.5}{-2.5} \Rightarrow s' = -10 \text{ cm} \text{ and } m = -\frac{n_1 s'}{n_2 s} = -\frac{(1.5)(-10)}{(1)(5)} = +3$$

**2-13.** Generally,  $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$

$$(a) \frac{n_1}{f} + \frac{n_2}{\infty} = \frac{n_2 - n_1}{R} \text{ or } f = \frac{n_1 R}{n_2 - n_1} \quad (b) n_2 > n_1: \text{ then } R > 0 \text{ (convex), } n_2 < n_1: \text{ then } R < 0 \text{ (concave)}$$

- 2-14.** (a) In this position the object distance is  $s = 15$  cm. Then, using,  $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R}$  and  $m = -\frac{n_1 s'}{n_2 s}$ ,  

$$\frac{4}{3(15)} + \frac{1}{s'} = \frac{1 - 4/3}{-15} \Rightarrow s' = -15 \text{ cm (center)} \quad \text{and} \quad m = -\frac{(4/3)(-15)}{(1)(15)} \Rightarrow m = 4/3.$$

(b) Similarly, in this position  $s = 7.5$  cm so that

$$\frac{4}{3(15/2)} + \frac{1}{s'} = \frac{1 - 4/3}{-15} \Rightarrow s' = -6.4 \text{ cm}, \quad m = -\frac{n_1 s'}{n_2 s} = -\frac{(4/3)(-45/7)}{(1)(15/2)} = 8/7.$$

- 2-15.** See Figure 2-35 in the text. Rays from the object are (a) refracted through the spherical window, (b) then reflected from the back plane mirror, (c) then refracted out again through the spherical window. Taking these in turn:

(a)  $\frac{n_1}{s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{R} \Rightarrow \frac{1}{30} + \frac{4}{3s'} = \frac{4/3 - 1}{5} \Rightarrow s' = 40$  cm. Then,  $m = \frac{n_1 s'}{n_2 s} = \frac{(1)(40)}{(4/3)(30)} = -1$

(b)  $s = 25 - 40 = -15$  cm (virtual object),  $s' = -s = 15$  cm,  $m = -s'/s = 1$

(c)  $\frac{4/3}{10} + \frac{1}{s'} = \frac{1 - 4/3}{-5} \Rightarrow s' = -15$  cm. Then,  $m = \frac{-(4/3)(-15)}{(1)(10)} = +2$ .

The overall magnification is  $m = (-1)(+1)(+2) = -2$ . Thus a virtual, inverted, double-sized image appears 15 cm behind (right) the spherical window.

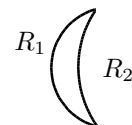
- 2-16.** The plane side of the lens has  $R_1 = \infty$ . The radius of curvature  $R_2$  of the convex side is then found from the lensmaker's equation:

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) \Rightarrow \frac{1}{25} = \frac{1.52 - 1}{1} \left( \frac{1}{\infty} - \frac{1}{R_2} \right) \Rightarrow R_2 = -13 \text{ cm}$$

- 2-17.** In general, the lensmaker's equation gives  $\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

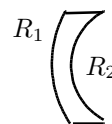
For the positive meniscus lens shown to the right,  $R_1 = 5$  cm and  $R_2 = 10$  cm.

Then,  $\frac{1}{f} = \frac{1.50 - 1}{1} \left( \frac{1}{5} - \frac{1}{10} \right) \Rightarrow f = +20$  cm

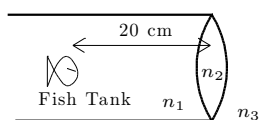


For the negative meniscus lens shown to the right,  $R_1 = 10$  cm and  $R_2 = 5$  cm.

For this case,  $\frac{1}{f} = \frac{1.50 - 1}{1} \left( \frac{1}{10} - \frac{1}{5} \right) \Rightarrow f = -20$  cm



- 2-18.** The thin lens equation assumes identical, refractive indices on both sides. In this case we can modify the procedure, beginning with Eq. (2-23), to allow for three distinct media as shown.



$$\frac{n_1}{s_1} + \frac{n_2}{s'_1} = \frac{n_2 - n_1}{R_1}, \text{ left lens surface}$$

$$\frac{n_2}{s_2} + \frac{n_3}{s'_2} = \frac{n_3 - n_2}{R_2}, \text{ right lens surface}$$

For a thin lens,  $s_2 \approx -s'_1$ . Adding the equations,  $\frac{n_1}{s_1} + \frac{n_3}{s'_2} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2}$ . Or, simply,

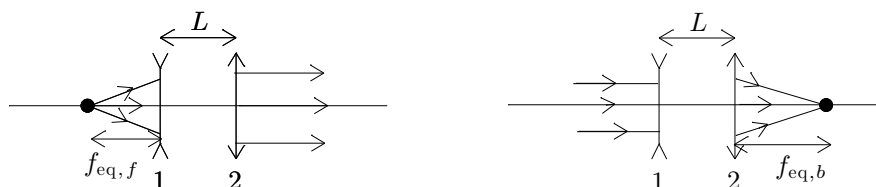
$$\frac{n_1}{s} + \frac{n_3}{s'} = \frac{n_2 - n_1}{R_1} + \frac{n_3 - n_2}{R_2} \Rightarrow \frac{4}{3(20)} + \frac{1}{s'} = \frac{3/2 - 4/3}{30} + \frac{1 - 3/2}{-30}, \text{ so that } s' = -22.5 \text{ cm}.$$

The total magnification is  $m_T = m_1 m_2 = \left( -\frac{n_1 s'_1}{n_2 s} \right) \left( -\frac{n_2 s'_2}{n_3 s_2} \right)$ , where  $s_2 = -s'_1$ . So,

$$m_T = -\frac{n_1 s'}{n_3 s} = -\frac{(4/3)(-22.5)}{(1)(20)} = 1.50.$$

**2-19.** (a) Using  $\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2}$  as in Eq. (2-33),  $\frac{1}{f_{eq}} = \frac{1}{-5} + \frac{1}{20}$  or  $f_{eq} = 6.67$  cm

(b) A pair of separated lenses has a front and a back focal length. The front focal length is the object position from the first lens that leads to an image at infinity. The back focal length is the image position for an object at infinity. These cases are illustrated below. The drawings are generic and not to scale.



Working backwards, for the front focal length: Lens 2:  $\frac{1}{s_2} + \frac{1}{\infty} = \frac{1}{f_2}$  or  $s_2 = f_2$ ,  $s_2 = L - s'_1$  or  $s'_1 = L - f_2$ .

Lens 1:  $\frac{1}{f_{eq,f}} + \frac{1}{L - f_2} = \frac{1}{f_1}$  or  $f_{eq,f} = \frac{f_1(L - f_2)}{L - (f_1 + f_2)} = \frac{(-5)(10 - 20)}{10 - (-5 + 20)} \text{ cm} = -10 \text{ cm}$

For the back focal length: For Lens 1:

$\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f_1} \Rightarrow \frac{1}{\infty} + \frac{1}{s'_1} = \frac{1}{f_1} \Rightarrow s'_1 = f_1$ . Then  $s_2 = L - f_1$ , so that, for Lens 2:

$\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{f_2} \Rightarrow \frac{1}{L - f_1} + \frac{1}{f_{eq,b}} = \frac{1}{f_2} \Rightarrow f_{eq,b} = \frac{f_2(L - f_1)}{(L - f_1) - f_2} = \frac{(20)(10 - (-5))}{(10 - (-5)) - 20} \text{ cm} = -60 \text{ cm}$

**2-20.** See Figure 2-36 in the text. Consider the three media as a sequence of three thin lenses. Each has a focal length given by the lensmaker's equation, and the equivalent focal length is given Eq. (2-33) as,

$\frac{1}{f_{eq}} = \frac{1}{f_1} + \frac{1}{f_2} + \frac{1}{f_3}$ . Then,  $\frac{1}{f_1} = (1.5 - 1) \left( \frac{1}{\infty} - \frac{1}{-15} \right) \Rightarrow f_1 = 30 \text{ cm}$ ,

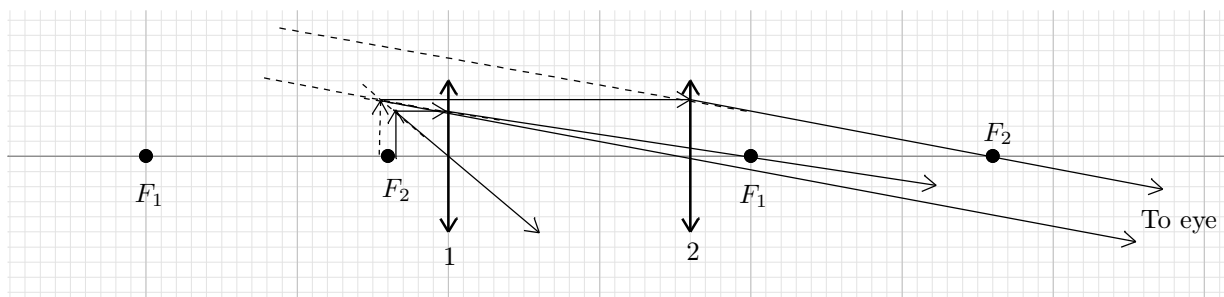
$\frac{1}{f_2} = (1.65 - 1) \left( \frac{1}{-15} - \frac{1}{15} \right)$  or  $f_2 = -\frac{150}{13} \text{ cm}$ , and  $\frac{1}{f_3} = \text{same as for } f_1 : f_3 = 30 \text{ cm}$ . Then,

$\frac{1}{f_{eq}} = \frac{1}{30} + \frac{-13}{150} + \frac{1}{30}$  and so  $f_{eq} = -50 \text{ cm}$ .

**2-21.** (a) One can use the formula derived in problem 2-19b, or do the calculation at first hand:

Second lens:  $\frac{1}{s_2} + \frac{1}{\infty} = \frac{1}{20}$  or  $s_2 = 20 \text{ cm}$ , First lens:  $\frac{1}{s_1} + \frac{1}{-4} = \frac{1}{20}$  or  $s_1 = 3.33 \text{ cm}$ . The object should be placed 3.33 cm before the first lens.

(b) In the figure below, the dashed arrow is the intermediate image that acts as the object for the second lens. Since the image is "at infinity", it is described by an angular magnification. The image appears erect and magnified.



**2-22.** Refer to Figure 2-37 in the text.

(b) Lens heading toward mirror:  $\frac{1}{3f/2} + \frac{1}{s'} = \frac{1}{-f}$  or  $s' = -3f/5$ .  $m_1 = -\frac{s'}{s} = -\frac{-3f/5}{3f/2} = 2/5$

Mirror:

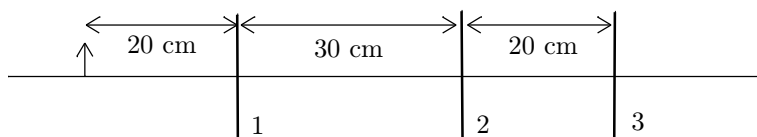
$$s = 3f + 3f/5 = 18f/5 \Rightarrow \frac{5}{18f} + \frac{1}{s'} = \frac{1}{f} \Rightarrow s' = 18f/13, m_2 = -\frac{s'}{s} = -(18f/13)/(18f/5) = -5/13$$

Lens after reflection:

$$s = 3f - 18f/13 = 21f/13 \Rightarrow \frac{13}{21f} + \frac{1}{s'} = \frac{1}{-f} \text{ or } s' = 21f/34, m_3 = -s'/s = -(21f/34)/(21f/13) = \frac{13}{34}$$

$m_T = \left(\frac{2}{5}\right)\left(-\frac{5}{13}\right)\left(\frac{13}{34}\right) = -17$ . The image is inverted,  $(21/34)f$  behind (right of) lens, inverted, and  $1/17$  original size.

**2-23.** The arrangement of the object and lenses is shown below.



(a)  $f_1 = +10$  cm,  $f_2 = +15$  cm,  $f_3 = +20$  cm

1st lens:  $\frac{1}{20} + \frac{1}{s'} = \frac{1}{10}$   $s' = 20$   $m_1 = -20/20 = -1$

2nd lens:  $\frac{1}{10} + \frac{1}{s'} = \frac{1}{15}$   $s' = -30$   $m_2 = -(-30)/10 = +3$

3rd lens:  $\frac{1}{50} + \frac{1}{s'} = \frac{1}{20}$   $s' = 100/3$   $m_3 = -100/3(50) = -2/3$

$$m_T = m_1 m_2 m_3 = +2$$

(b)  $f_1 = +10$  cm,  $f_2 = -15$  cm,  $f_3 = +20$  cm

1st lens:  $\frac{1}{20} + \frac{1}{s'} = \frac{1}{10}$   $s' = 20$   $m_1 = -20/20 = -1$

2nd lens:  $\frac{1}{10} + \frac{1}{s'} = \frac{1}{-15}$   $s' = -6$   $m_2 = -(-6)/10 = +0.6$

3rd lens:  $\frac{1}{26} + \frac{1}{s'} = \frac{1}{20}$   $s' = 520/6$   $m_3 = -520/(6 \times 26) = -\frac{10}{3}$

$$m_T = m_1 m_2 m_3 = +2$$

(c)  $f_1 = -10$  cm,  $f_2 = +15$  cm,  $f_3 = -20$  cm

1st lens:  $\frac{1}{20} + \frac{1}{s'} = \frac{1}{-10}$   $s' = 20/3$   $m_1 = -(-20)/3(20) = \frac{1}{3}$

2nd lens:  $\frac{3}{110} + \frac{1}{s'} = \frac{1}{15}$   $s' = 330/13$   $m_2 = -\frac{(330)(3)}{(13)(110)} = -\frac{9}{13}$

3rd lens:  $\frac{-13}{70} + \frac{1}{s'} = \frac{1}{-20}$   $s' = 140/19$   $m_3 = -\frac{(140)(13)}{(19)(-70)} = \frac{26}{19}$

$$m_T = m_1 m_2 m_3 = -6/19$$

**2-24.** Using the lensmaker's formula,  $\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$  gives

in air:  $\frac{1}{30} = \frac{1.50 - 1}{1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$

and in the liquid:  $-\frac{1}{188} = \frac{1.50 - n_L}{n_L} \left( \frac{1}{R_1} - \frac{1}{R_2} \right)$ .

Dividing the two equations gives  $\frac{-188}{30} = \frac{0.5n_L}{1.5 - n_L}$  or  $n_L = 1.63$ .

**2-25.** Use the lensmaker's formula to find the focal length of the lens,

$$\frac{1}{f} = \frac{n_2 - n_1}{n_1} \left( \frac{1}{R_1} - \frac{1}{R_2} \right) = \frac{1.5 - 1}{1} \left( 0 + \frac{1}{60} \right) \Rightarrow f = 120 \text{ cm}$$

The Newtonian equations are,  $m = -\frac{f}{x} = -\frac{x'}{f}$ . For  $m = -4$ :

$$\begin{aligned} -4 &= -\frac{f}{x} = -\frac{120}{x} \text{ or } x = 30 \text{ cm} \\ -4 &= -\frac{x'}{f} = -\frac{x'}{120} \text{ or } x' = 480 \text{ cm} \end{aligned}$$

Thus,  $s = x + f = 30 + 120 = 150 \text{ cm}$  and  $s' = x' + f = 480 + 120 = 600 \text{ cm}$ .

Check:  $\frac{1}{s} + \frac{1}{s'} = \frac{1}{150} + \frac{1}{600} = \frac{1}{120} = \frac{1}{f}$

**2-26.** (a)  $f_1 = 10 \text{ cm} \Rightarrow P_1 = \frac{1}{0.01} = +10 D$ ,  $f_2 = 20 \text{ cm} \Rightarrow P_2 = \frac{1}{0.2} = +5 D$ ,  $f_3 = -40 \text{ cm} \Rightarrow P_3 = \frac{1}{-0.4} = -2.5 D$

Then,  $P = P_1 + P_2 + P_3 = 10 + 5 - 2.5 = +12.5 D$

(b)  $\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$ ,  $V + V' = P$ , where  $V = \frac{1}{s} = \frac{1}{0.12} = +8.33 D$ ,

$$V' = 4.167 D \text{ or } s' = \frac{1}{V'} = \frac{1}{4.167} = 0.24 \text{ m} = 24 \text{ cm}$$

**2-27.** See Figure 2-38 in the text. The applicable relations are

Lens equations:  $\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f}$  and  $\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{f}$ ,

Geometrical:  $L = s_1 + s'_1 = s_2 + s'_2$ ,  $D = s_2 - s_1 = s'_1 - s'_2$

Thus,

$$f = \frac{s_1 s'_1}{s_1 + s'_1} = \frac{s_1 s'_1}{L} = \frac{s_2 s'_2}{s_2 + s'_2} = \frac{s_2 s'_2}{L} \quad (1)$$

Because the lens equation can be satisfied the second time by simply interchanging object and image distances,

$$s_2 = s'_1 \text{ and } s'_2 = s_1 \quad (2)$$

Adding and subtracting the equations  $L = s_1 + s_2$  and  $D = -s_1 + s_2$ , we get

$L - D = 2s_1$  and  $L + D = 2s_2$ . Their product is by Eq. (1),  $L^2 - D^2 = 4s_1 s_2$ , or by Eq. (2),  $L^2 - D^2 = 4fL$ .

Thus,  $f = \frac{L^2 - D^2}{4L}$ .

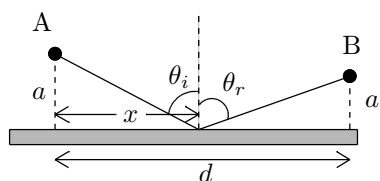
**2-28.** Lens equations:  $\frac{1}{s_1} + \frac{1}{s'_1} = \frac{1}{f}$  and  $\frac{1}{s_2} + \frac{1}{s'_2} = \frac{1}{f}$ . Then calculate,

$$\frac{1}{M_1} - \frac{1}{M_2} = -\frac{s_1}{s'_1} + \frac{s_2}{s'_2} = -\frac{s_1}{s_1 f / (s_1 - f)} + \frac{s_2}{s_2 f / (s_2 - f)} = \frac{s_2 - f}{f} - \frac{s_1 - f}{f} = \frac{s_2 - s_1}{f}$$

Thus,

$$f = \frac{s_2 - s_1}{1/M_1 - 1/M_2}$$

**2-29.** Consider an arbitrary path from point A to point B by reflection from a mirror surface,



The path distance  $D$  from A to B is  $D = \sqrt{a^2 + x^2} + \sqrt{b^2 + (d - x)^2}$

$$\frac{dD}{dx} = \frac{x}{\sqrt{a^2 + x^2}} + \frac{-(d - x)}{\sqrt{b^2 + (d - x)^2}} = 0$$

$$\sin \theta_i - \sin \theta_r = 0 \Rightarrow \theta_i = \theta_r$$