International Relations Theory:

The Game Theoretic Approach

Exercise Answer Key

Chapter 2: What States Want*

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Exercise 1. In general, when territory is linear in value, states will care only about how much of it they have, not about how it is arranged. When territory has increasing returns to size of contiguous blocs, then having large blocks of territory is preferable.

1. (a) No. If the player's utility is equivalent to the length of the interval they own, changing the ownership vector so a province changes hands will benefit the gainer at the expense of the loser. For instance, if p_i is transferred from player 1 to player 2, player 1's utility declines by s_i and player 2's utility increases by s_i.

   (b) Again, no. Increasing p_i must come at the expense of p_{i-1} and or p_{i+1}. If the losing provinces are both owned by the same player as the gaining province, there is no change. If one of the losing provinces is owned by the other player, there will be a loss for that player.

*Please send comments and corrections to kydd@wisc.edu. I will update this document as I discover or receive information about errors.

(c) Again, no. Imagine player 1 owns both p_k and p_{k+1}. The payoff from these two provinces is the sum of their length, $s_k + s_{k+1}$. If the provinces are combined, the new province's length is $s_k + s_{k+1}$, so the payoff is the same.

2. (a) No. Exchanging existing provinces is a zero sum transfer, the previous owner loses s_k^2 and the new one gains s_k^2.

   (b) Yes. If player 1 owns provinces p_k and p_{k+1}, the payoff is $s_k^2 + s_{k+1}^2$. If these provinces are combined, the payoff is $(s_k + s_{k+1})^2$ which equals $s_k^2 + 2s_k s_{k+1} + s_{k+1}^2$, which is larger than for the unconsolidated case.

   (c) The only divisions that are efficient are those in which a single boundary divides player 1's territory from player 2's territory. If you take any given ownership structure and rearrange the provinces so they are contiguous, with player 1's provinces to the left and player 2's to the right, the payoffs are the same as before. If you then combine the provinces for each player, you maximize the payoff for each player.

Exercise 2. The answers all refer to Figure 1.

1. The ideal points are located on the axes, as shown in Figure 1.

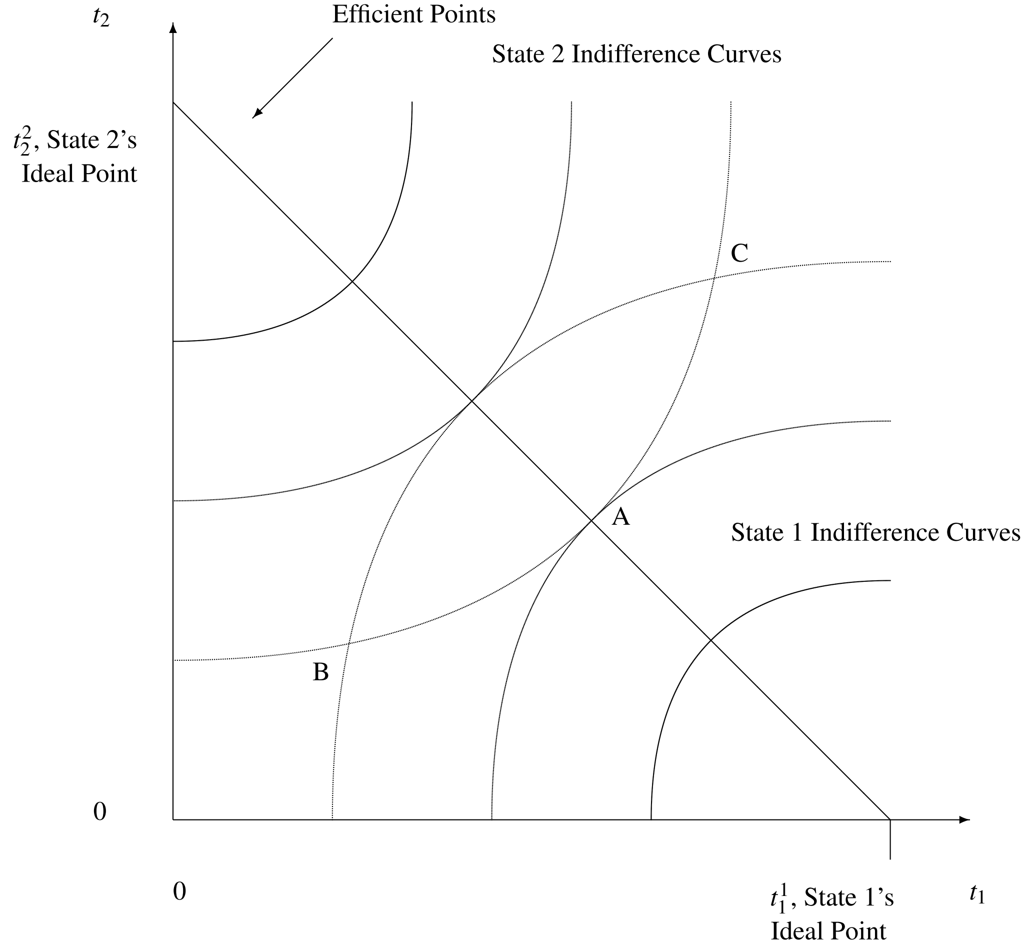
2. The indifference curves are circles. If we choose some reference level of utility $-k$ and set it equal to the utility function we get

$$\begin{aligned} -k &= -\sqrt{(t_1 - t_1^i)^2 + (t_2 - t_2^i)^2} \\ k^2 &= (t_1 - t_1^i)^2 + (t_2 - t_2^i)^2 \end{aligned}$$

   which is the equation of a circle with center at (t_1^i, t_2^i) and radius k^2.

3. The contract curve is the set of tangent points between the utility curves, such as point A. In this case it is a straight line described by the equation $t_2 = t_2^2 - \frac{t_2^2}{t_1^1} t_1$. If we pick any ray out of a point, circles around that point will be perpendicular to it. To see that, imagine the ray is the y axis and take the derivative of the equation for the circle at x=0, the result will be zero. Since the line between the two ideal points is a ray from both points,

Figure 1: Utility Over Tariff Levels



the circles from each side will be perpendicular to it, and hence parallel to each other, therefore the ones that meet will be tangents.

4. Below the contract curve, at a point such as B, both sides can be made better off by increasing their tariffs, so that they end up inside the lens between B and C. Above the contract curve, at a point such as C, both parties can be made better off by reducing their tariffs to go inside the lens.

Exercise 3. The derivative of x^{ρ} with respect to x is $\rho x^{\rho-1}$, and the second derivative is $\rho(\rho-1)x^{\rho-2}$. We know that $x^{\rho-2}$ is always positive, so the sign depends on $\rho(\rho-1)$, which in turn depends on $\rho-1$. If $\rho > 1$, this is positive, so the second derivative is increasing, so the function is concave, or risk acceptant. If $\rho < 1$, it is negative, so the second derivative is negative, returns are diminishing, and the function is risk averse. When $\rho = 1$ then the function is linear and risk neutral.