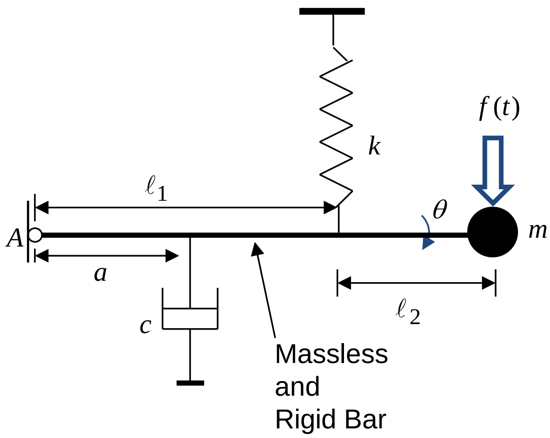
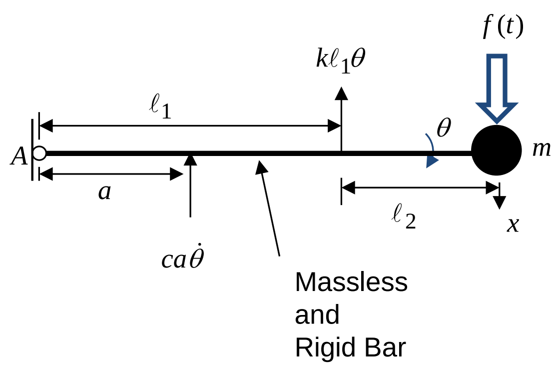
Solution to Chapter 2 Problems

DRAFT

P2.1





Taking moments about A,

$$-ca\dot{\theta}a - k\ell_1\theta\ell_1 + f(t)(\ell_1 + \ell_2) = I_A\ddot{\theta} = m(\ell_1 + \ell_2)^2\ddot{\theta}$$

displacement of mass = $x = (\ell_1 + \ell_2)\theta$

Therefore,

$$-c\left(\frac{a}{\ell_1 + \ell_2}\right)^2 \dot{x} - k\left(\frac{\ell_1}{\ell_1 + \ell_2}\right)^2 x + f(t) = m\ddot{x}$$

or

$$m\ddot{x} + c_{eq}\dot{x} + k_{eq}x = f(t)$$

where

$$c_{eq} = c\left(\frac{a}{\ell_1 + \ell_2}\right)^2 = \frac{25}{256}c$$

$$k_{eq} = k\left(\frac{\ell_1}{\ell_1 + \ell_2}\right)^2 = \frac{25}{64}k$$

$$x_{ss} = \frac{1}{k_{eq}} = 0.01 meter \Rightarrow k_{eq} = 100\, N / meter$$

From (2.1.21),

$$\frac{dx}{dt} = -0.01(-\xi\omega_n)e^{-\xi\omega_n t}(\cos\omega_d t + \chi\sin\omega_d t) - 0.01e^{-\xi\omega_n t}\omega_d(-\sin\omega_d t + \chi\cos\omega_d t) = 0$$

$$(\xi\omega_n)(\cos\omega_d t + \chi\sin\omega_d t) - \omega_d(\sin\omega_d t + \chi\cos\omega_d t) = 0$$

$$\cos\omega_d t(\xi\omega_n - \omega_d\chi) + \sin\omega_d t(\xi\omega_n\chi - \omega_d) = 0$$

$$\xi\omega_n - \omega_d\chi = 0 \Rightarrow \sin\omega_d t = 0 \Rightarrow \omega_d t = \pi \Rightarrow \omega_n t = \frac{\pi}{\sqrt{1-\xi^2}}$$

$$x_p = 0.01[1 - e^{-\xi\omega_n t}(\cos\omega_d t + \chi\sin\omega_d t)] = 0.01[1 + \exp(\frac{-\pi\xi}{\sqrt{1-\xi^2}})]$$

From the plot, $x_p = 0.0175$

$$\exp(\frac{-\pi\xi}{\sqrt{1-\xi^2}}) = \frac{0.0175 - 0.01}{0.01} = 0.75$$

$$\frac{\pi\xi}{\sqrt{1-\xi^2}} = -\ell n(0.75) \Rightarrow \xi^2 = \frac{(\ell n 0.75)^2}{\pi^2 + (\ell n 0.75)^2} \Rightarrow \xi = 0.0912$$

$$Time\ Period = \frac{2\pi}{\omega_d} = 1.3 \Rightarrow \omega_n = \frac{2\pi}{1.3\sqrt{1-\xi^2}} = 4.85 rad./\sec.$$

$$m = \omega_n^2 k_{eq} = 2352\, kg.$$

$$k_{eq} == \frac{25}{64}k = 100 \Rightarrow k = 256\, N/m$$

$$c_{eq} = \frac{25}{256}c = 2m\omega_n\xi \quad \Rightarrow c = \frac{512 m\omega_n\xi}{25} = 21306\, N-s/m$$

P2.2

$$\omega_n = \sqrt{\frac{10000}{100}} = 10\, rad./\sec.$$

$$\xi = \frac{20}{2m_{eq}\omega_n} = 0.01$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$q = \exp(-\frac{\pi\xi}{\sqrt{1-\xi^2}})$$

$$m_1 = \frac{10}{1+q} = 5.0785\, kg$$

$$m_2 = 10 - 5.0785 = 4.9215\, kg$$

$$t_1 = \frac{\pi}{\omega_d} = 0.3142 \text{ sec.}$$

First, place $m_1 = 5.0785kg$. Wait for 0.3142 sec. Then, place $m_2 = 4.9215kg$.

P 2.3

$$\ell = \ell_1 + \ell_2 = 0.8\, meter$$

$$-ca\dot{\theta}a - k\ell_1\theta\ell_1 + f(t)\ell = m\ell^2\ddot{\theta}$$

$$m\ell^2\ddot{\theta} + ca^2\dot{\theta} + k\ell_1^2\theta = f(t)\ell$$

$$\xi = \frac{ca^2}{2\sqrt{k\ell_1^2 m\ell^2}} = \frac{ca^2}{2\ell\ell_1\sqrt{km}}$$

For no overshoot, we must have

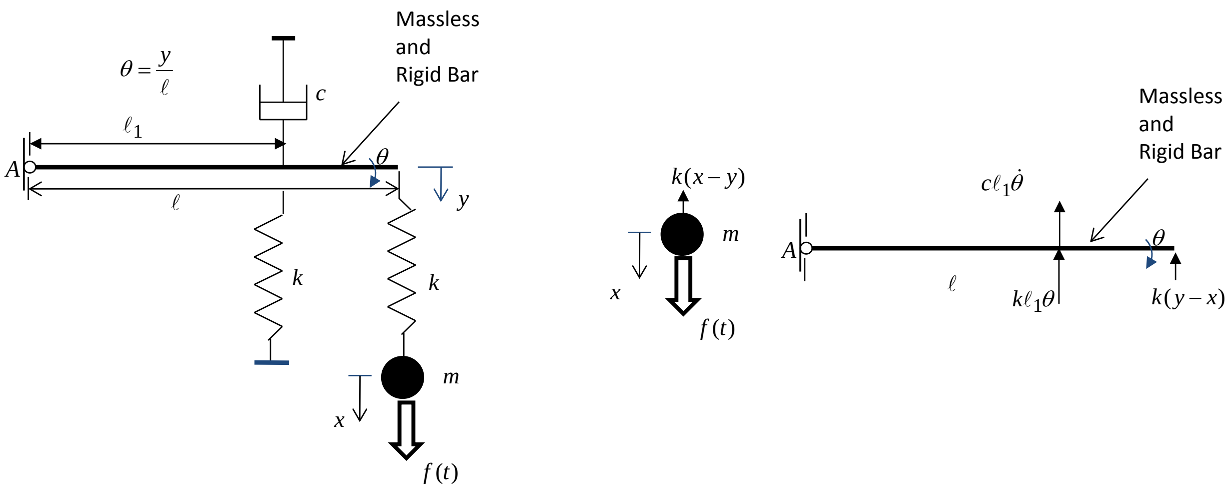
$$\xi \geq 1$$

$$c \geq \frac{2\ell\ell_1\sqrt{km}}{a^2}$$

$$0.5\,kg < m < 2kg$$

$$c = \frac{2\ell\ell_1\sqrt{2k}}{a^2} = 600.37\,N - s/m$$

P2.4



For the massless bar, taking moment about A,

$$-c\ell_1\dot{\theta}\ell_1 - k\ell_1\theta\ell_1 - k(y-x)\ell = 0$$

Because $\theta = \frac{y}{\ell}$,

$$c\frac{\ell_1^2}{\ell^2}\dot{y} + k\frac{\ell_1^2}{\ell^2}y + ky = kx \qquad (1)$$

For the mass *m*,

$$m\ddot{x} = f(t) - k(x-y)$$

or

$$m\ddot{x} + kx = ky + f(t) \qquad (2)$$

Differentiating (2) wrt time,

$$m\dddot{x} + k\dot{x} = k\dot{y} + \dot{f} \qquad (3)$$

From (1), (2) and (3),

$$(1+\frac{\ell_1^2}{\ell^2})(m\ddot{x}+kx-f)+\frac{c}{k}\frac{\ell_1^2}{\ell^2}(m\dddot{x}+k\dot{x}-\dot{f})=kx$$

$$\frac{c}{k}\frac{\ell_1^2}{\ell^2}m\dddot{x}+(1+\frac{\ell_1^2}{\ell^2})m\ddot{x}+\frac{\ell_1^2}{\ell^2}c\dot{x}+\frac{\ell_1^2}{\ell^2}kx=(1+\frac{\ell_1^2}{\ell^2})f+\frac{c}{k}\frac{\ell_1^2}{\ell^2}\dot{f} \tag{4}$$

When c = 0,

$$(1+\frac{\ell_1^2}{\ell^2})m\ddot{x}+\frac{\ell_1^2}{\ell^2}kx=(1+\frac{\ell_1^2}{\ell^2})f(t)$$

From Prob. 1.12,

$$k=770.486\,N/meter \qquad m=10\,kg.$$

$$\frac{\ell_1^2}{\ell^2}=(0.7)^2=0.49$$

```
%
clear all
close all
%
%Problem 2.4a
coeff=1.49;
f0=coeff*20;
keq=0.49*770.486;
meq=10*coeff;
%
xstat=f0/keq;
x0=0;
v0=0;
omegan=sqrt(keq/meq);
%
A1=x0;
omega1=0.5*omegan;
Amp1=f0/(keq-meq*omega1*omega1);
B1_1=(v0-Amp1*omega1)/omegan;
%
omega2=omegan;
Amp2=-f0/(2*meq*omega2);
B1_2=(v0-Amp2)/omegan;
%
omega3=1.5*omegan;
Amp3=f0/(keq-meq*omega3*omega3);
B1_3=(v0-Amp3*omega3)/omegan;
%
omega4=0.9*omegan;
Amp4=f0/(keq-meq*omega4*omega4);
B1_4=(v0-Amp4*omega4)/omegan;
```

```
%
%
delt=2*pi/(omegan*30);
t=-delt;
for i=1:900
   t=t+delt;
   tv(i)=t*omegan;
   f1(i)=x0*cos(omegan*t)+B1_1*sin(omegan*t)+Amp1*sin(omega1*t);
   f2(i)=x0*cos(omegan*t)+B1_1*sin(omegan*t)+Amp2*t*sin(omega2*t);
   f3(i)=x0*cos(omegan*t)+B1_3*sin(omegan*t)+Amp2*sin(omega3*t);
   f4(i)=x0*cos(omegan*t)+B1_4*sin(omegan*t)+Amp4*sin(omega4*t);
   f5(i)=Amp2*t;
end
figure(1)
plot(tv(1:500),f1(1:500)/xstat)
xlabel('Nondimensional Time, \omega_nt','Fontsize',14)
ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',14)
%legend('0<\xi<1','\xi=1','\xi>1')
grid
figure(2)
plot(tv(1:500),f2(1:500)/xstat,'-',tv(1:500),f5(1:500)/xstat,'--',tv(1:500),-
f5(1:500)/xstat,'--')
xlabel('Nondimensional Time, \omega_nt','Fontsize',14)
ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',14)
%legend('0<\xi<1','\xi=1','\xi>1')
grid
figure(3)
plot(tv(1:500),f3(1:500)/xstat,'-')
xlabel('Nondimensional Time, \omega_nt','Fontsize',14)
ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',14)
%legend('0<\xi<1','\xi=1','\xi>1')
grid
figure(4)
plot(tv,f4/xstat)
xlabel('Nondimensional Time, \omega_nt','Fontsize',14)
ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',14)
%legend('0<\xi<1','\xi=1','\xi>1')
grid
figure(5)
subplot(2,2,1)
plot(tv(1:500),f1(1:500)/xstat)
%xlabel('Nondimensional Time, \omega_nt','Fontsize',14)
ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',12)
title('\omega=0.8\omega_n')
%legend('0<\xi<1','\xi=1','\xi>1')
grid
subplot(2,2,2)
plot(tv(1:500),f2(1:500)/xstat,'-',tv(1:500),f5(1:500)/xstat,'--',tv(1:500),-
f5(1:500)/xstat,'--')
%xlabel('Nondimensional Time, \omega_nt','Fontsize',14)
%ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',14)
%legend('0<\xi<1','\xi=1','\xi>1')
title('\omega=\omega_n')
grid
subplot(2,2,3)
plot(tv(1:500),f3(1:500)/xstat,'-')
xlabel('Nondimensional Time, \omega_nt','Fontsize',12)
```

```
ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',12)
%legend('0<\xi<1','\xi=1','\xi>1')
title('\omega=1.5\omega_n')
grid
subplot(2,2,4)
plot(tv,f4/xstat)
xlabel('Nondimensional Time, \omega_nt','Fontsize',12)
%ylabel('Displacement, x/(f_0/k_e_q)','Fontsize',14)
%legend('0<\xi<1','\xi=1','\xi>1')
title('\omega=0.9\omega_n')
grid
```

P2.5

Total bearing stiffness = k_{eq}

m = $10\,kg$, e = $0.5\,cm$

$$\omega = \frac{4200 \times 2\pi}{60} = 140\pi\, rad./\sec.$$

Differential equation of motion:

$$m\ddot{x} + k_{eq}x = me\omega^2 \sin\omega t$$

Steady State Response

$$x(t) = A\sin\omega t$$

$$A = \frac{me\omega^2}{k_{eq} - m\omega^2} \le 0.05 \times 10^{-2}$$

$$10m\omega^2 \le k_{eq} - m\omega^2 \quad \Rightarrow k_{eq} \ge 11m\omega^2 = 2.13 \times 10^7 N/m$$

stiffness of each bearing = $\dfrac{k_{eq}}{2}$

P2.6

shaft stiffness $k_{eq} = \dfrac{48EI}{\ell^3}$

mass $m = 10\,kg$,eccentricity e = $0.5\,cm$, length $\ell = 0.5\,meter$, diameter $d = 5\,cm$

modulus of elasticity (steel), $E = 2\times10^{11}\ N/m^2$

Area Moment of Inertia $I = \dfrac{\pi d^4}{64}$

shaft stiffness $k_{eq} = \dfrac{48EI}{\ell^3} = 2.36\times10^7\ N/m^2$

a. Critical Speed of Rotor

$$\omega_n = \sqrt{\frac{k_{eq}}{m}} = 1536.2\ rad./\sec.$$

b. Yield stress (steel), $\sigma_y = 2.5\times10^8\ N/m^2$

Displacement at mid-span, $= x$

Force at mid-span, $F = k_{eq}x$

Reaction force at each support $= \dfrac{F}{2}$

Bending moment at mid-span, $M = \dfrac{F}{2}\dfrac{\ell}{2}$

Maximum bending stress at mid-span, $\sigma = \dfrac{M(d/2)}{I} = \dfrac{F\ell d}{8I} = \dfrac{k_{eq}x\ell d}{8I}$

$$\sigma < 0.7\sigma_y \Rightarrow x < 0.7\frac{8\sigma_y I}{k_{eq}\ell d} = 0.7\frac{\sigma_y \ell^2}{6Ed} = 0.0007 meter$$

Differential equation of motion:

$$m\ddot{x} + k_{eq}x = me\omega^2 \sin\omega t$$

For $\omega = \omega_n$, equation (2.2.32) yields

$$x(t) \approx A_r t\cos\omega t$$

Where

$$A_r = -\frac{me\omega^2}{2}\frac{\omega_n}{2k_{eq}} = -\frac{e\omega_n}{4}$$

Time required to reach $|x| = 0.0007\, meter$

$$t = \frac{0.0007}{|A_r|} = \frac{0.0028}{e\omega_n} = 3.65 \times 10^{-4}\ \text{sec}.$$

P2.7

Transmissibility = $\frac{\sqrt{1+(2\xi r)^2}}{\sqrt{(1-r^2)^2+(2\xi r)^2}} \le 0.65$

With equality sign,

$$\frac{1+(2\xi r)^2}{(1-r^2)^2+(2\xi r)^2} = (0.65)^2$$

Or

$$\frac{1+(2\xi r)^2}{(0.65)^2} = (1-r^2)^2 + (2\xi r)^2$$

With $\xi = 0.045$, we get

$$(r^2)^2 - 2.0111 r^2 - 1.3669 = 0$$

Or

$$r^2 = \frac{2.0111 \pm \sqrt{(2.0111)^2 + 4 \times 1.3669}}{2}$$

With ‘+’ sign,

$$r^2 = 2.5476 \Rightarrow r = 1.5961$$

Therefore, for transmissibility to be less than 0.65,

$$r = \frac{\omega}{\omega_n} \ge 1.5961 \ \Rightarrow \ \omega_n \le \frac{\omega}{1.5961}$$

Given,

$$\frac{1800 \times 2\pi}{60} \le \omega \le \frac{2300 \times 2\pi}{60}$$

Therefore,

$$\omega_n = \frac{1800 \times 2\pi}{60 \times 1.5961} = 118.0976 \; rad./\sec.$$

Lastly,

$$k_{eq} = m_{eq}\omega_n^2 = 1.813 \times 10^5 \; N/met.$$