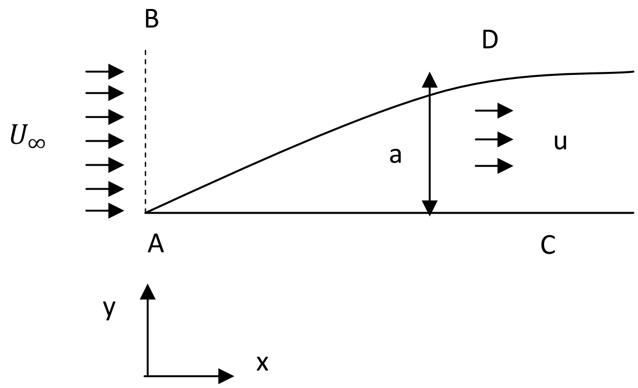
Problem 2.1: Solution



Mass flow rate across AB

$$= \int_0^a \rho_\infty (\dot{V} dA)$$

$$= \int_0^a \rho_\infty U_\infty \, dy$$

Mass flow rate across CD

$$= \int_0^a \rho u dy$$

So the loss of mass flow rate due to BL

$$= \int_0^a \rho_\infty U_\infty \, dy - \int_0^a \rho u dy$$

$$= \int_0^a \rho_\infty U_\infty \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

Let $a \to \infty$ to enclose whole BL, we get

$$= \int_0^\infty \rho_\infty U_\infty \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

$$= \rho_\infty U_\infty \int_0^\infty \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

$$= \rho_\infty U_\infty \delta_1$$

So δ_1(displacement BL) is a measure of loss of mass flow rate due to the BL

also from mass conservation, mass flux across BD would be

$$= \int_0^\infty \rho_\infty U_\infty \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

Momentum flux across AB:

$$= \int_0^\infty \rho_\infty U_\infty \, (U_\infty) dy$$

$$= \int_0^\infty \rho_\infty {U_\infty}^2 dy$$

Momentum flux across CD:

$$= \int_0^\infty \rho u^2 dy$$

Momentum flux across BD = mass flow rate x U_∞ across BD

$$= U_\infty \left\{ \rho_\infty U_\infty \int_0^\infty \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy \right\}$$

So the net momentum flux would be

$$= \int_0^\infty \rho_\infty {U_\infty}^2 dy - \int_0^\infty \rho u^2 dy - \int_0^\infty \rho_\infty {U_\infty}^2 \left(1 - \frac{\rho u}{\rho_\infty U_\infty}\right) dy$$

$$= \int_0^\infty \rho_\infty u U_\infty dy - \int_0^a \rho u^2 dy$$

$$\int_0^\infty \rho u (U_\infty - U) dy$$

$$= \rho_\infty {U_\infty}^2 \left\{ \int_0^\infty \frac{\rho u}{\rho_\infty U_\infty} (1 - \frac{u}{U_\infty}) dy \right\}$$

$$= \rho_\infty {U_\infty}^2 \delta_2$$

So $\delta_2$represents the net loss in momentum flux rate due to the BL.

Energy flux across AB $= \int_0^\infty \rho_\infty {U_\infty}^3 dy$

Energy flux across CD $= \int_0^\infty \rho u^3 dy$

Energy flux across BC $= {U_\infty}^2 \left\{\rho_\infty U_\infty \int_0^\infty (1 - \frac{\rho u}{\rho_\infty U_\infty})\, dy\right\}$

Net loss in energy flux

$$= \int_0^\infty \rho_\infty {U_\infty}^3 dy - \int_0^\infty \rho u^3 dy - \int_0^\infty \rho_\infty {U_\infty}^3 (1 - \frac{\rho u}{\rho_\infty U_\infty})\, dy$$

$$= \int_0^\infty \left(\rho u {U_\infty}^2 - \rho u^3\right) dy$$

$$= \int_0^\infty \rho u {U_\infty}^2 (1 - \frac{u^2}{{U_\infty}^2}) dy$$

$$= \rho_\infty {U_\infty}^3 \int_0^\infty \frac{\rho u}{\rho_\infty U_\infty}\left(1 - \frac{u^2}{{U_\infty}^2}\right) dy$$

$$= \rho_\infty {U_\infty}^3 \delta_3$$

$\rightarrow \delta_3$ represents the loss in energy flux due to BL