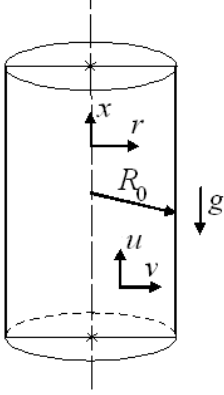


Problem 2.7



Neglecting viscous dissipation, the conservation equations are:

$$\frac{1}{r} \frac{\partial}{\partial r} (rv) + \frac{\partial}{\partial x} (u) = 0$$

$$\rho \left(u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} \right) = -\frac{\partial P}{\partial x} + \mu \left\{ \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial u}{\partial r} \right) + \frac{\partial^2 u}{\partial x^2} \right\} + \rho g_x$$

$$\rho C_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial r} \right) = k \left[\frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial T}{\partial r} \right) + \frac{\partial^2 T}{\partial x^2} \right]$$

$$u^* = \frac{u}{U_m}, v^* = \frac{v}{U_m} \text{Re}_D \text{Pr}, r^* = \frac{r}{R_0}$$

$$x^* = \frac{x/R_0}{\text{Re}_D \text{Pr}}, P^* = \frac{P-P_1}{\rho U_m^2 \text{Pr}}, \theta = \frac{T-T_s}{T_{in}-T_s}$$

where $\text{Re}_D = \rho U_m (2R_0) / \mu$, and U_m represents the mean velocity.

The dimensionless equations will then become:

$$\frac{1}{r^*} \frac{\partial}{\partial r^*} (r^* v^*) + \frac{\partial}{\partial x^*} (u^*) = 0$$

$$\frac{1}{\text{Pr}} \left(u^* \frac{\partial u^*}{\partial x^*} + v^* \frac{\partial u^*}{\partial r^*} \right) = -\frac{\partial P^*}{\partial x^*} + \left\{ \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial u^*}{\partial r^*} \right) + \frac{2}{\text{Re}_D^2 \text{Pr}^2} \frac{\partial^2 u^*}{\partial x^{*2}} \right\} + \frac{D^3}{v^2 \text{Re}_D} g_x$$

$$u^* \frac{\partial \theta}{\partial x^*} + v^* \frac{\partial \theta}{\partial r^*} = \frac{1}{r^*} \frac{\partial}{\partial r^*} \left(r^* \frac{\partial \theta}{\partial r^*} \right) + \frac{2}{\text{Re}_D^2 \text{Pr}^2} \frac{\partial^2 \theta}{\partial x^{*2}}$$

In the last equation, the term $\frac{2}{\text{Re}_D^2 \text{Pr}^2} \frac{\partial^2 \theta}{\partial x^{*2}}$ represents axial conduction in the fluid. In comparison with other terms in the equation, this term is clearly negligibly small when $\text{Re}_D \text{Pr} = \text{Pe} \gg 1$.