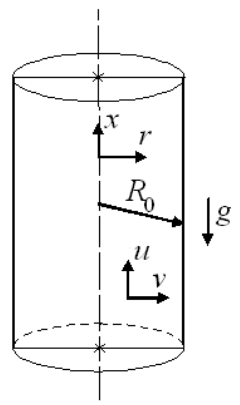
Problem 2.10



Assume adiabatic flow (i.e., no heat transfer) and constant properties. The conservation equations are:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv)+\frac{\partial}{\partial x}(u)=0$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial P}{\partial x}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial x^2}\right\}+\rho g_x$$

$$\rho\left(u\frac{\partial m_1}{\partial x}+v\frac{\partial m_1}{\partial r}\right)=\mathcal{D}_{12}\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial x^2}\right]$$

Now, define dimensionless parameters as:

$$u^* = \frac{u}{U_m},\ v^* = \frac{v}{U_m}\mathrm{Re_D Sc},\ r^* = \frac{r}{R_0}$$

$$x^* = \frac{x/R_0}{Re_D Sc}\ ,\ P^* = \frac{P-P_1}{\rho U_m^2 Sc},\ \phi = \frac{m_1-m_{1,s}}{m_{1,in}-m_{1,s}}$$

where $Re_D = \rho U_m(2R_0)/\mu$, and U_m represents the mean velocity.

The dimensionless equations will then become:

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* v^*\right)+\frac{\partial}{\partial x^*}\left(u^*\right)=0$$

$$\frac{1}{\mathrm{Pr}}\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial r^*}\right)=-\frac{\partial P^*}{\partial x^*}+\left\{\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial u^*}{\partial r^*}\right)+\frac{2}{\mathrm{Re}_D^2\,\mathrm{Pr}^2}\frac{\partial^2 u^*}{\partial x^{*2}}\right\}+\frac{D^3}{\nu^2\,\mathrm{Re}_D}g_x$$

$$u^*\frac{\partial \phi}{\partial x^*}+v^*\frac{\partial \phi}{\partial r^*}=\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial \phi}{\partial r^*}\right)+\frac{2}{\mathrm{Re}_D^2\,Sc^2}\frac{\partial^2 \phi}{\partial x^{*2}}$$

In the last equation, the term $\frac{2}{\mathrm{Re}_D^2 \, Sc^2} \frac{\partial^2 \theta}{\partial x^{*2}}$ represents axial conduction in the fluid. In comparison with other terms in the equation, this term is clearly negligibly small when $\mathrm{Re}_D \, Sc = \mathrm{Pe}_{ma} >> 1$.