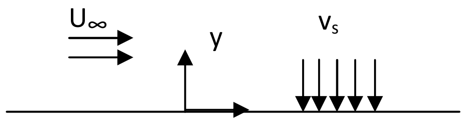
Problem 2.4: Solution



Assume steady, incompressible, constant-property, laminar flow. Also, assume negligible gravity effect. For an infinitely large plane no dependence on x is possible, therefore

$$\rho\left(\cancelto{0}{\frac{du}{dt}} + \cancelto{0}{u\frac{du}{dx}} + v\frac{du}{dy}\right) = -\cancelto{0}{\frac{dP}{dx}} + \mu\left(\cancelto{0}{\frac{d^2u}{dx^2}} + \frac{d^2u}{dy^2}\right)$$

$$v\frac{du}{dy} - \nu\frac{d^2u}{dy^2} = 0 \qquad (1)$$

The y-momentum gives

$$\rho\left(\cancelto{0}{\frac{dv}{dt}} + \cancelto{0}{u\frac{dv}{dx}} + v\frac{dv}{dy}\right) = -\cancelto{0}{\frac{dP}{dy}} + \mu\left(\cancelto{0}{\frac{d^2v}{dx^2}} + \frac{d^2v}{dy^2}\right)$$

$$v\frac{dv}{dy} - \nu\frac{d^2v}{dy^2} = 0 \qquad (2)$$

However, mass conservation says,

$$\cancelto{0}{\frac{du}{dx}} + \frac{dv}{dy} = 0 \;\rightarrow\; \frac{dv}{dy} = 0$$

$$\rightarrow v = constant = -v_s \qquad (3)$$

Evidently, $v = v_s$ everywhere, and that satisfies Eqn. (2).

Eqn. (1) can now be written as

$$\frac{d^2u}{dy^2} + \frac{v_s}{\nu}\frac{du}{dy} = 0 \qquad (4)$$

At $y = 0, u = 0$

At $y \rightarrow \infty, u = U_\infty$

The solution will be

$$u = U_\infty\left[1 - \exp\left(-\frac{v_s}{\nu}y\right)\right] \qquad (5)$$

(Note that, by our definition, $v_s > 0$ for suction at the wall.)

If we define the edge of the boundary layer as the location where $\frac{u}{U_\infty} = \zeta^*$, then

$$\zeta^* = \left[1 - \exp\left(-\frac{v_s}{\nu}\delta\right)\right] \tag{6}$$

For $v_s = 0.1\, {}^{m}/_{s}$, and assuming $\nu = 8.6\, X\, 10^{-7}\, {}^{m^2}/_{s}$ for water, and $\zeta^* = 0.999$, then

$$\delta \approx 6\, X\, 10^{-5}\text{m} \tag{7}$$

For blowing, $v_s < 0$, and since $\zeta^* < 1$, will set,

$$\delta = -\frac{\nu}{v_s}\ln(1-\zeta^*) < 0 \tag{8}$$

(No acceptable solution)

a) In steady-state, neglecting the viscous dissipation, the energy equation becomes

$$\underbrace{u\cancel{\frac{dT}{dx}}}_{0} - v_s\frac{dT}{dy} = \alpha\frac{d^2T}{dy^2}$$

$$\rightarrow \frac{dT}{dy} + \frac{\alpha}{v_s}\frac{d^2T}{dy^2} = 0 \tag{9}$$

$$\text{At } y = 0, T = T_s \tag{10}$$

$$\text{At } y \rightarrow \infty, T \rightarrow \infty \tag{11}$$

The solution will be

$$\frac{T-T_\infty}{T_s-T_\infty} = 1 - \exp\left(-\frac{v_s}{\alpha}y\right) \tag{12}$$

For air, with $\nu = .158\, X\, 10^{-4}\, {}^{m^2}/_{s}$, will have for the suction case,

$$\delta = 0.00109\, m$$

$$\delta_3 = \int_0^\infty \frac{\rho u}{\rho_\infty U_\infty}\left(1 - \frac{u^2}{U_\infty^2}\right)dy$$

$$= \left(\frac{U_\infty}{\nu x}\right)^{-1/2}\int_0^5 \left(sin\frac{\pi}{10}\eta - sin^3\frac{\pi}{10}\eta\right)d\eta$$

$$= \left(\frac{U_\infty}{\nu x}\right)^{-1/2}\left(\frac{10}{3\pi}\right)$$