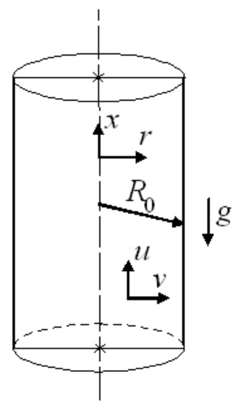
Problem 2.6



Neglecting viscous dissipation, the conservation equations are:

$$\frac{1}{r}\frac{\partial}{\partial r}(rv)+\frac{\partial}{\partial x}(u)=0$$

$$\rho\left(u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}\right)=-\frac{\partial P}{\partial x}+\mu\left\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right)+\frac{\partial^2 u}{\partial x^2}\right\}+\rho g_x$$

$$\rho C_P\left(u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}\right)=k\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\partial^2 T}{\partial x^2}\right]$$

$u^* = \frac{u}{U_m}$, $v^* = \frac{v}{U_m}\mathrm{Re_D Pr}$, $r^* = \frac{r}{R_0}$

$x^* = \frac{x/R_0}{Re_D Pr}$, $P^* = \frac{P-P_1}{\rho U_m^2 Pr}$, $\theta = \frac{T-T_s}{T_{in}-T_s}$

where $Re_D = \rho U_m(2R_0)/\mu$, and U_m represents the mean velocity.

The dimensionless equations will then become:

$$\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^* v^*\right)+\frac{\partial}{\partial x^*}\left(u^*\right)=0$$

$$\frac{1}{\mathrm{Pr}}\left(u^*\frac{\partial u^*}{\partial x^*}+v^*\frac{\partial u^*}{\partial r^*}\right)=-\frac{\partial P^*}{\partial x^*}+\left\{\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial u^*}{\partial r^*}\right)+\frac{2}{\mathrm{Re}_D^2\,\mathrm{Pr}^2}\frac{\partial^2 u^*}{\partial x^{*2}}\right\}+\frac{D^3}{\nu^2\,\mathrm{Re}_D}g_x$$

$$u^*\frac{\partial\theta}{\partial x^*}+v^*\frac{\partial\theta}{\partial r^*}=\frac{1}{r^*}\frac{\partial}{\partial r^*}\left(r^*\frac{\partial\theta}{\partial r^*}\right)+\frac{2}{\mathrm{Re}_D^2\,\mathrm{Pr}^2}\frac{\partial^2\theta}{\partial x^{*2}}$$