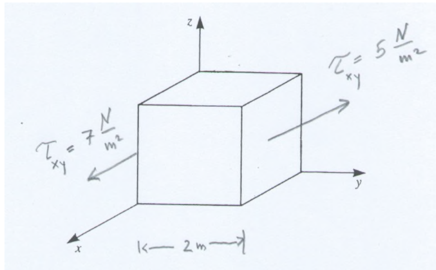
Chapter 2: Problems + Solutions

1. Calculate the net force due to the shear forces acting on two opposite sides of the cube shown in the figure. What is the direction of the resultant (net) force?



solution

$$F_x = \Sigma(\tau \cdot S) = (5-7)(2 \cdot 2) = -8N$$

2.The velocity field in the x-z plane is given by the following expressions

$$u = x$$

$$w = -z$$

Determine the equation describing the streamline passing through the point (2,1), and sketch the flow field described by this formula.

solution

On a streamline the flow is parallel to the stream line:

$$\frac{dx}{u} = \frac{dz}{w} \qquad (2.9)$$

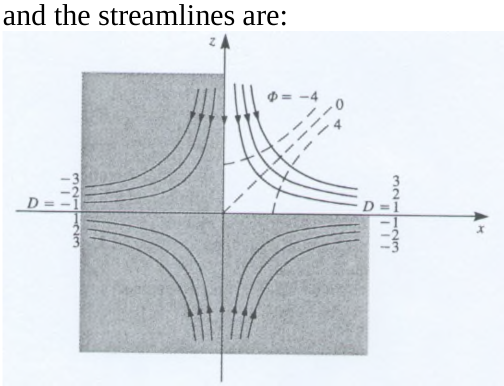
and substituting the velocity components yields

$$\frac{dx}{2x} = \frac{dz}{-2z}$$

Integration by separation of variables results in

$$xz = const = D \qquad (8.28)$$

and the streamlines are:



At a point (2,1) xz=2,therefore the streamline is described by

Z = 2/x

3. The velocity field in the x-z plane is given by the following expressions

$$u = -3z^2$$

$$w = -6x$$

Determine the equation describing the streamline passing through the point (1,1), and sketch the flow field described by this formula.

Solution

On a streamline the flow is parallel to the stream line:

$$\frac{dx}{u} = \frac{dz}{w} \tag{2.9}$$

and substituting the velocity components yields

$$\frac{dx}{-3z^2} = \frac{dz}{-6x}$$

Integration by separation of variables results in

$$-6xdx = -3z^2 dz$$
$$3x^2 = z^3 + c \tag{8.28}$$

The constant c is calculated by using the point (1,1)
Therefore c = 2 and the streamline equation is:

$$z^3 = 3x^2 - 2$$

4. A two-dimensional velocity field is given by the following equation:

$$u = \frac{x}{x^2 + z^2}$$

$$w = \frac{z}{x^2 + z^2}$$

Check if this satisfies the incompressible continuity equation
solution

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = \frac{(x^2+z^2)\cdot 1 - x\cdot 2x + (x^2+z^2)\cdot 1 - z\cdot 2z}{(x^2+z^2)^2} = 0$$

5. The velocity distribution between two parallel horizontal plate is given by the following expression:

$$u(z) = -k\left[\frac{z}{h} - \frac{z^2}{h^2}\right]$$

where k is a constant and h (into the z direction) is the clearance between the two plates. Calculate the shear along a vertical line (between z=0 and z=h). Where the shear force reaches its maximum value?
solution

$$\tau = \mu\frac{\partial u}{\partial z} = -\mu k\left(\frac{1}{h} - \frac{2z}{h^2}\right)$$

The maximum is at the top and bottom $\tau_{\max} = \pm\mu\frac{k}{h}$

6. Based on Eq. 2.37 write down the two-dimensional Euler equation in Cartesian coordinates.
solution

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = f_x - \frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = f_z - \frac{1}{\rho}\frac{\partial p}{\partial z}$$

7. Based on Eq. 2.46 and 2.47 write down the two-dimensional Euler equation in cylindrical coordinates (r - θ directions).
solution
The momentum equation for an incompressible fluid in r direction:

$$\rho\left(\frac{Dq_r}{Dt}-\frac{q_\theta^2}{r}\right)=\rho f_r-\frac{\partial p}{\partial r} \tag{2.46}$$

θ direction:

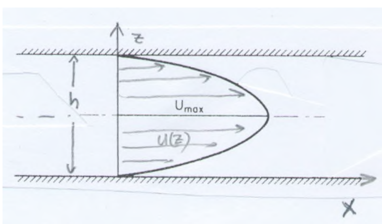
$$\rho\left(\frac{Dq_\theta}{Dt}+\frac{q_r q_\theta}{r}\right)=\rho f_\theta-\frac{1}{r}\frac{\partial p}{\partial \theta} \tag{2.47}$$

8. Based on Eq. 2.37 write down the three-dimensional incompressible Navier-Stokes equation in Cartesian coordinates.
solution

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\frac{-1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=\frac{-1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=\frac{-1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)$$



9. A two dimensional steady state flow between two parallel plates is given by the following function.

$$u=a\left(\frac{z}{h}-\left(\frac{z}{h}\right)^2\right) \qquad 0<z<h$$

$w=0$

Note that the velocity is in the x direction only, but the flow is still two-dimensional.
a) Find the maximum velocity
b) Calculate the flow rate Q at a station x = const, which is given by the integral from the continuity equation

$$Q=\int_{c.s.}\rho(\vec{q}\cdot\vec{n})dS$$

Solution

a) Max velocity is du/dz = 0. This gives $u_{\max}=\frac{a}{4}$

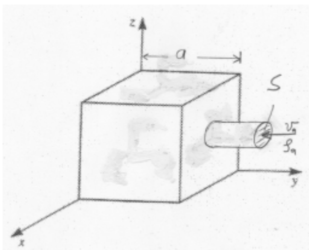
b) The flow per unit width is $Q=\int_{c.s.}\rho(\vec{q}\cdot\vec{n})dS=\rho a\int_0^h\left[\frac{z}{h}-\left(\frac{z}{h}\right)^2\right]dz=\rho a\frac{h}{6}$

10. Calculate the force per unit area (in the previous problem) acting on the surface z =0, and on the surface z = h. Assume fluid viscosity μ.

Solution:

$$F_1 = \tau A = \mu \left.\frac{du}{dz}\right|_{z=0} = \mu a\left(\frac{1}{h} - \frac{2z}{h^2}\right) = \mu\frac{a}{h}$$

$$F_2 = \tau A = \mu \left.\frac{du}{dz}\right|_{z=h} = \mu a\left(\frac{1}{h} - \frac{2h}{h^2}\right) = -\mu\frac{a}{h}$$



11. Fluid is flowing through a circular pipe into a cubical control volume, as shown in the picture. If the cube dimensions are a = 30cm and S = 3cm^2, the velocity vector is (0.-1,0)m/s, and the outside density is ρ_a = 1.0kg/m^3, determine the value of the integral

$$\int_{c.s.} \rho(\vec{q}\cdot\vec{n})dS = 0$$

Solution:

$$\int_{c.s.} \rho(\vec{q}\cdot\vec{n})dS = 1(0,-1,0)\cdot(0,1,0)\cdot 3\cdot 10^{-4} = -3\cdot 10^{-4}\,kg/s$$

12. Evaluate the value of the integral $\frac{\partial}{\partial t}\int_{c.v.}\rho dV$ for the control volume discussed in the previous problem. Assume initial conditions at t = 0, ρ = 0.

Solution:

$$\frac{\partial}{\partial t}\int_{c.v.}\rho dV = \frac{\partial}{\partial t}(\rho a^3) = 3\cdot 10^{-4}\,kg/s$$

$$\frac{d\rho}{dt} = \frac{3\cdot 10^{-4}\,kg/s}{0.3^3 m^3} = 1.11\cdot 10^{-2}\,\frac{kg}{m^3\cdot s}$$

13. Calculate the force and direction of the force acting on the cube. Assume the pressure at the pipe inlet is the same as the pressure surrounding the cube.

Solution:

$$F_y = \rho_a v_2^2 A_2 + (p_a - p_a)S = -1\cdot 1^2\cdot 3\cdot 10^{-4} + 0 = -3\cdot 10^{-4}\,N$$

this means that this external force acts on the control volume in the negative y direction.

14. A two-dimensional steady-state velocity field is described by the following velocity components: $u = 5x, \quad w = -5z$. Calculate the corresponding acceleration field (using the material derivative). What is the magnitude and direction of the acceleration at point $x = 1, z = 1$?

Solution

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = 0 + 5x \cdot 5 - 5z \cdot 0 = 25x$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = 0 + 5x \cdot 0 + 5z \cdot 5 = 25z$$

at point (1,1), a =(25,25)

15. A two-dimensional steady-state velocity field is described by the following velocity components: $u = \frac{x}{x^2 + z^2} \qquad w = \frac{z}{x^2 + z^2}$. Provide a graphical representation of the velocity distribution along the lines: a) z = 0.,and b) x = 0.

Solution:
See Vortex in Section 8.5.4

16. A two-dimensional steady-state velocity field is described by the following velocity components: $u = \frac{x}{x^2 + z^2} \qquad w = \frac{z}{x^2 + z^2}$. Calculate the corresponding acceleration field (using the material derivative). What is the magnitude and direction of the acceleration at point $x = 0, z = 1$?

Solution

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = 0 + \frac{x}{x^2 + z^2} \cdot \frac{x^2 + z^2 - 2x^2}{(x^2 + z^2)^2} + \frac{z}{x^2 + z^2} \cdot \frac{0 - 2xz}{(x^2 + z^2)^2}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = 0 + \frac{x}{x^2 + z^2} \cdot \frac{0 - 2xz}{(x^2 + z^2)^2} + \frac{z}{x^2 + z^2} \cdot \frac{x^2 + z^2 - 2z^2}{(x^2 + z^2)^2}$$

at point (0,1), a =

17. Try to sketch the streamlines from the previous problem, starting at the origin. What is the shape of this flow?
Solution: Source, see section 8.5.2

18. A two-dimensional steady-state velocity field is described by the following velocity components: $u = \frac{z}{x^2 + z^2} \qquad w = \frac{x}{x^2 + z^2}$. Calculate the corresponding acceleration field (using the material derivative). What is the magnitude and direction of the acceleration at point $x = 0, z = 1$?

Solution

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = 0 + \frac{z}{x^2 + z^2} \cdot \frac{0 - 2xz}{(x^2 + z^2)^2} + \frac{x}{x^2 + z^2} \cdot \frac{x^2 + z^2 - 2z^2}{(x^2 + z^2)^2}$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = 0 + \frac{z}{x^2 + z^2} \cdot \frac{x^2 + z^2 - 2x^2}{(x^2 + z^2)^2} + \frac{x}{x^2 + z^2} \cdot \frac{0 - 2xz}{(x^2 + z^2)^2}$$

at point (0,1), a =

19. Try to sketch the streamlines from the previous problem, starting at the origin. What is the shape of this flow?
Solution: Vortex, see Section 8.5.4

20. A two-dimensional steady-state velocity field is described by the following velocity components: $u = 1 + 2x + z, \quad w = 1 - 2x + 3z$. Calculate the corresponding acceleration field (using the material derivative). What is the magnitude and direction of the acceleration at point $x = 1, z = 1$?

Solution

$$a_x = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + w\frac{\partial u}{\partial z} = 0 + (1 + 2x + z)\cdot 2 + (1 - 2x + 3z)\cdot 1 = 3 + 2x + 5z$$

$$a_z = \frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + w\frac{\partial w}{\partial z} = 0 + (1 + 2x + z)\cdot 2 + (1 - 2x + 3z)\cdot 3 = 5 - 2x + 11z$$

at point (1,1), a =(10,14)

21. An incompressible (one dimensional) steady-state flow in a circular diffuser is flowing from station #1 at x =0 (where the velocity is U, to station #2 at x = x_2. If the inlet radius at station #1 is r_1 and at station #2 the radius is r_2. Assuming $r_2 > r_1$ develop an expression for the fluid acceleration as a function of x.

Solution

Since this is a steady one-dimensional flow, the acceleration term becomes:

$$a_x = u\frac{\partial u}{\partial x}$$

The velocity u can be calculated, using the one-dimensional continuity equation:

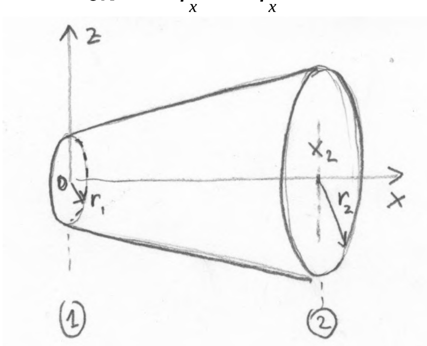
$$u_x = U\frac{A_1}{A_x} = U\frac{r_1^2}{r_x^2}$$

and the radius is

$$r_x = r_1 + \frac{r_2 - r_1}{x_1}x$$

Substituting this into the acceleration term we get:

$$a_x = u\frac{\partial u}{\partial x} = U\frac{r_1^2}{r_x^2}\frac{r_1^2(r_1 - r_2)}{r_x^5}$$



22. An incompressible (one dimensional) steady-state flow in a circular diffuser is flowing from station #1 at x =0 (where the velocity is U=3m/s, to station #2 at x = 10cm. If the inlet radius at station #1 is r_1 = 5cm and at station #2 the radius is r_2=10cm. Calculate the fluid acceleration as a function of x.

Solution

Since this is a steady one-dimensional flow, the acceleration term becomes:

$$a_x = u\frac{\partial u}{\partial x}$$

The velocity u can be calculated, using the one-dimensional continuity equation:

$$u_x = U\frac{A_1}{A_x} = 3\frac{5^2}{(5+0.5x)^2}$$

and the radius is

$$r_x = 5 + \frac{5}{10}x$$

Substituting this into the acceleration term we get:

$$a_x = u\frac{\partial u}{\partial x} = -3\frac{5^2}{(5+0.5x)^2}\cdot\frac{3\cdot 5^2(5+0.5x)}{(5+0.5x)^4} = -\frac{5625}{(5+0.5x)^5}$$

23. An incompressible (one dimensional) steady-state flow in a nozzle with a rectangular cross section is flowing from station #1 at x =0 (where the velocity is U, to station #2 at x = x_2. If the inlet height at station #1 is b_1 and at station #2 is b_2. Assuming $b_2 < b_1$ develop an expression for the fluid acceleration as a function of x.

Solution

Since this is a steady one-dimensional flow, the acceleration term becomes:

$$a_x = u\frac{\partial u}{\partial x}$$

The velocity u can be calculated, using the one-dimensional continuity equation:

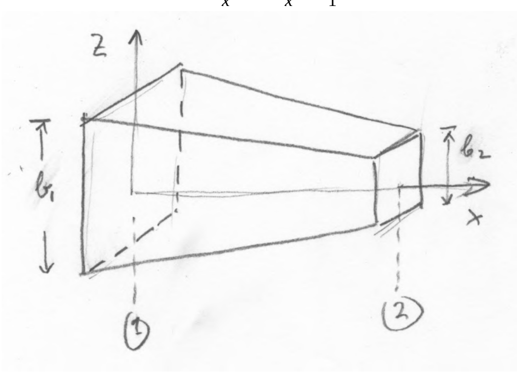
$$u_x = U\frac{A_1}{A_x} = U\frac{b_1^2}{b_x^2}$$

and the height is

$$b_x = b_1 - \frac{b_1 - b_2}{x_1}x$$

Substituting this into the acceleration term we get:

$$a_x = u\frac{\partial u}{\partial x} = -U^2\frac{b_1^2}{b_x^2}\frac{b_1^2(b_1-b_2)}{b_x^4\cdot x_1}$$



24. An incompressible (one dimensional) steady-state flow in a nozzle with a rectangular cross section is flowing from station #1 at x =0 (where the velocity is U=3m/s, to station #2 at x = 10cm. If the inlet height at station #1 is b_1 =10cm and at station #2 is b_2 =5cm, calculate the fluid acceleration as a function of x.

Solution

Since this is a steady one-dimensional flow, the acceleration term becomes:

$$a_x = u\frac{\partial u}{\partial x}$$

The velocity u can be calculated, using the one-dimensional continuity equation:

$$u_x = U\frac{A_1}{A_x} = 3\frac{10^2}{(10-0.5x)^2}$$

and the height is

$$b_x = 10 - \frac{5}{10}x$$

Substituting this into the acceleration term we get:

$$a_x = u\frac{\partial u}{\partial x} = 3\frac{10^2}{(10-0.5x)^2}\cdot\frac{3\cdot 10^2(10-0.5x)}{(10-0.5x)^4} = \frac{300^2}{(10-0.5x)^5}$$

25. A two dimensional steady state flow is given by the following function.

$$u = a_1 z + a_2 z^2 \qquad\qquad 0 < z < h$$

$$w = 0$$

Note that the velocity is in the x direction only, but the flow is still two-dimensional.
a) Sketch the velocity distribution along a vertical line (between z = 0, and z = h)
b) Calculate the shear stress τ_{xz}, and its value at z = 0.

Solution

$$\tau_{xz} = \mu\frac{du}{dz} = \mu(a_1 + 2a_2 z)$$

at z = 0 $\tau_{xz} = \mu a_1$

26. The velocity profile above a horizontal flat plate (within the range of 0 < z < δ) is described by the following function

$$\frac{u}{U_e} = \sin(\frac{\pi z}{2\delta}) \qquad\qquad z \le \delta$$

Calculate the force per unit area acting on the surface z =0. Assume fluid viscosity μ.

Solution:

$$F_1 = \tau A = \mu\left.\frac{du}{dz}\right|_{z=0}\cdot A = \mu\frac{\pi U_e}{2\delta}\cos\left(\frac{\pi z}{2\delta}\right)_{z=0}\cdot A = \mu\frac{\pi U_e}{2\delta}A$$

27. The velocity profile above a horizontal flat plate (within the range of 0 < z < δ) is described by the following function

$$\frac{u}{U_e} = \left(\frac{z}{\delta}\right)^{\frac{1}{9}} \qquad\qquad z \le \delta$$

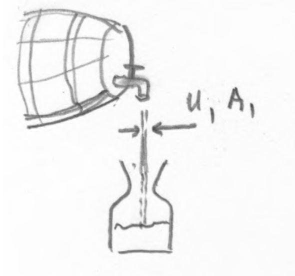
a) Calculate the force per unit area acting on the surface z =0. Assume fluid viscosity μ.

b) Compare with the force calculated in the previous problem. Which one is higher? (Note that these two formulas relate to the shape of a boundary layer above a flat plate, the latter is for turbulent flows).

Solution:

$$F_1 = \tau A = \mu \frac{du}{dz}\bigg|_{z=0} \cdot A = \mu \frac{U_e}{9\delta}\left(\frac{z}{\delta}\right)^{\frac{-8}{9}}_{z=0} \cdot A$$

This force is larger since at z = 0 F --→ ∞



28. Wine in poured froma barrel into a liter bottle, as shown. At a certain point, the fluid velocity is 1m/s, and the cross section area is 0.5 cm^2. How long it takes to fill the 1 liter bottle?

Solution:
The flow rate is

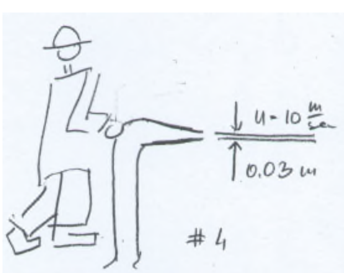
$$Q = uA = 100\frac{cm}{s} \cdot 0.5cm^2 = 50\frac{cm^3}{s}$$

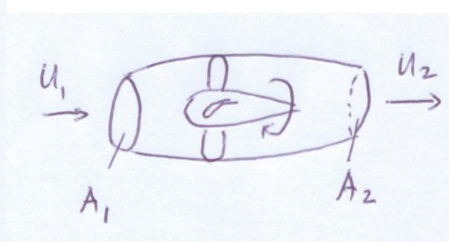
And the time to fill the bottle is

$$t = \frac{V}{Q} = \frac{1000}{50} = 20s$$

29. A fire fighter holds a water hose as shown. The water leaves the nozzle at a velocity of 10 m/sec, and the diameter of the circular jet is 0.03 m. Assuming the pressure at the exit of the nozzle is atmospheric, calculate the force pushing the fire fighter backward. (ρ_{water} = 1000 kg/m^3).
solution

$$F = \dot{m}u = \rho u^2 S = 1000 \cdot 10^2 \cdot \pi \frac{0.03^2}{4} = 70.68N$$





30. A small fan engine (as shown in the figure) was tested and the incoming speed u_1 was 100m/s. The inlet area is A_1 = 0.1m^2, and the exhaust speed is u_2 = 200m/s. Assuming that the flow is accelerated by the fan and that the pressure and density are

the same at the inlet and exit (and $\rho = 1.2$ kg/m^3), calculate the exit area A_2. Also calculate the thrust generated by this unit.

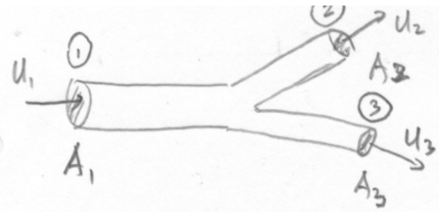
solution

$$A_2 = A_1 \frac{u_2}{u_1} = 0.05 m^2$$

$$F = \dot{m}(u_2 - u_1) = 1.2 \cdot 100 \cdot 0.1(200 = 100) = 1200 N (122.5 kgf)$$

31. Water enters a 0.05m diameter tube at section #1 at an average speed of 0.5 m/s and exits through station #2 and #3, as shown. The diameter of the exit at station %2 is 0.03m and the average velocity there is 0.5 m/s, as well. Calculate the average exit velocity at station #3 if the tube diameter there is 0.02m.

solution



$$Q = \rho u A = 1000 \times 0{,}5 \times \pi \frac{0{,}05^2}{4} = 0{,}981 \frac{kg}{sec}$$

$$u_2 = 0{,}5$$

$$A_2 \Rightarrow D_2 = 0{,}03$$

$$A_3 \Rightarrow D_2 = 0{,}02$$

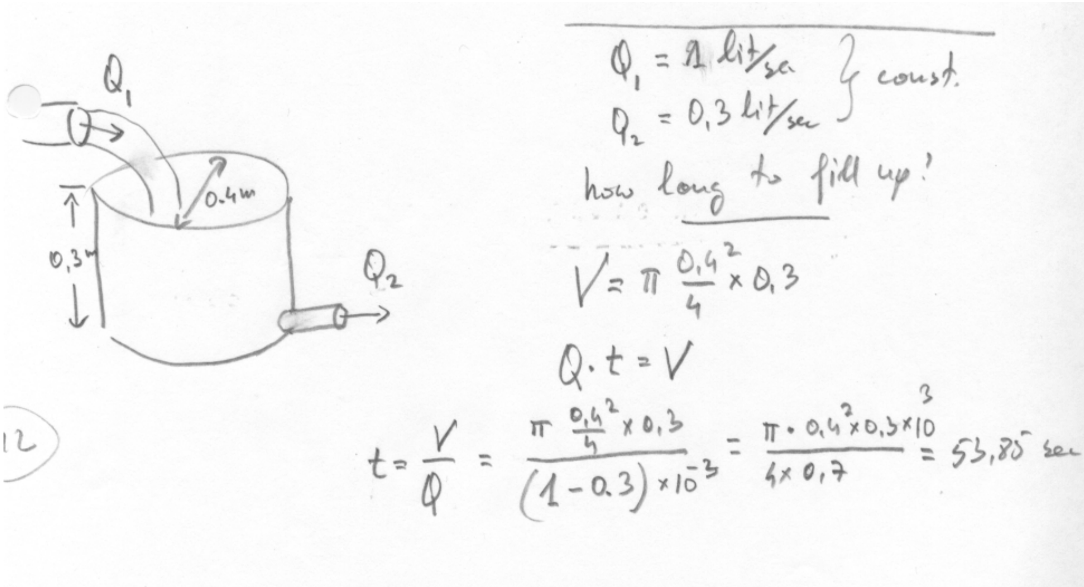
$$u_3 = ?$$

$$Q = \rho \cdot 0{,}5 \times \pi \frac{0{,}05^2}{4} = \rho\left(0{,}5 \cdot \pi \frac{0{,}03^2}{4} + u_3 \cdot \pi \frac{0{,}02^2}{4}\right)$$

$$\frac{0{,}5 \cdot (0{,}05^2 - 0{,}03^2)}{0{,}02^2} = u_3 = 2 \frac{m}{sec}$$

32. At t = 0, the pipe above an empty cylindrical container is opened pouring water at a rate of Q_1 = 1 liter/s. At the bottom of the container, an open pipe can drain the container at a rate of Q_2 = 0.3 liter/s. How long it will take to fill up the container (dimensions shown in the figure)?

solution

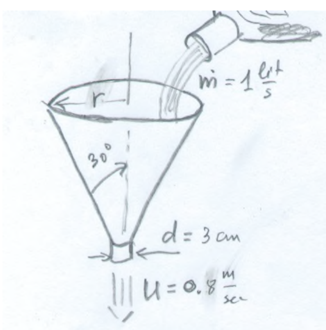


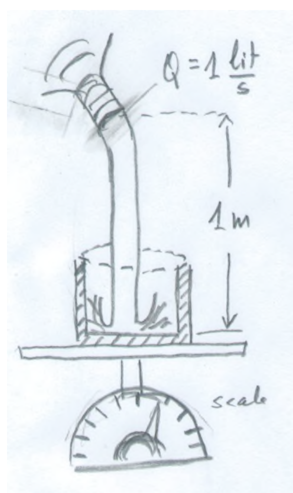
33.A water stream is flowing from a pipe at a rate of Q = 1 lit/s into a container placed on a scale, as shown. Assuming the effective height of the water column is 1 m (e.g., the vertical velocity is zero at that height), then calculate the force measured by the scale?

solution

$u = \sqrt{2gh}$

$F = \dot{m}u = \rho Q\sqrt{2gh} = 1000 \cdot 1\sqrt{2 \cdot 9.8 \cdot 1} = 4.42N$





34. Oil is poured at a rate of Q = 1 lit/s into a conical funnel of r = 0.3m as shown. Assuming it exits at the bottom of the funnel at a constant average speed of U = 0.8 m/s through a 0.03m diameter tube, calculate how long it will take to fill up the conical section. (the volume of a cone = 1/3 x base area x height).

solution

$$V = \frac{1}{3}\pi r^2 h \qquad r = 0{,}3\,m$$

$$h = \frac{r}{\tan 30}$$

$$Q_{out} = \pi \frac{d^2}{4}\cdot u = \pi \frac{0.03^2}{4}\times 0{,}8 = 5{,}65\times10^{-4}\ \frac{m^3}{s}$$

$$\left[0{,}565\ \frac{lit}{s}\right]$$

$$V = \frac{1}{3}\pi r^2 h = (\dot m_{in} - \dot m_{out})\cdot t$$

$$= \left(1\ \frac{lit}{sec} - 0{,}565\right)\cdot t$$

$$t = \frac{V}{Q_{in} - Q_{out}} = \frac{\pi\, 0{,}3^3}{3\cdot\tan 30\,(1 - 0{,}565)\,10^{-3}} = 0.112\times10^{3}$$

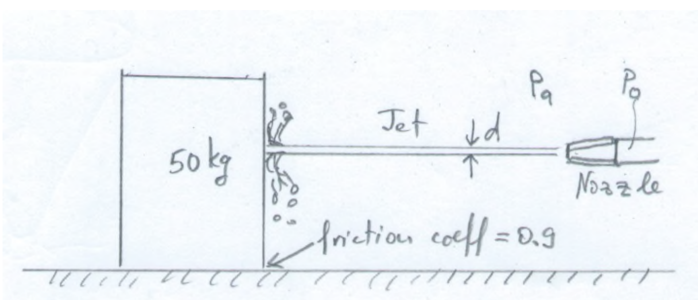
$$= 112\ sec = 1\ min\ 52\ sec$$

35. A water jet hits horizontally a 50kg block. The friction coefficient between the block and the ground is 0.9. What is the minimum diameter *d* of the water jet in order for the block to slide to the right Assume the jet speed, as it hits the block, is 20 m/s.

solution

$$d^2 = \frac{4F}{\rho u^2 \pi} = \frac{4\times 0{,}9\times 50\times 9{,}8}{1000\times 20^2\times\pi} = 1{,}4\times10^{-3}$$

$$d = 0{,}037\ m$$



36. Consider the flow in a circular pipe of radius R. Assuming the axis symmetric velocity distribution is given by the expression $q_x = A\left[1-\left(\frac{r}{R}\right)^2\right]$, where A is a constant. Calculate the volumetric flow rate and the average velocity.

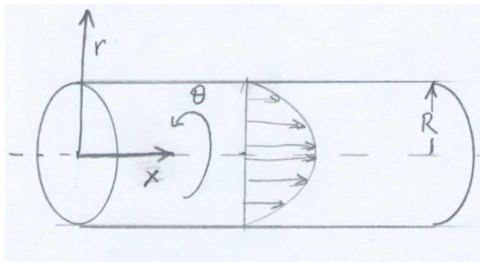
solution

Solution:

The volumetric flow rate Q is

$$Q = \int_0^R q_x dS = A\left[1-\left(\frac{r}{R}\right)^2\right]2\pi r dr = \frac{A}{2}\pi R^2$$

$$U_{av} = \frac{Q}{S} = \frac{A}{2}$$



37. Calculate the shear stress on the inner wall of the pipe shown in the previous problem and estimate the shear force per unit length of the pipe, Find where in the fluid the shear stress is minimum.

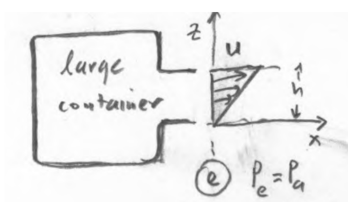
solution

$$\tau = \mu \frac{\partial q}{\partial r} = -\mu A \frac{2r}{R^2}$$

$$\tau_{max} = -\mu \frac{2A}{R}$$

$$F = -2\pi R \cdot \mu \frac{2A}{R} = -4\pi\mu A$$

$\tau_{min} \rightarrow$ at $r = 0$



38. A fluid jet exits a large container and the two-dimensional velocity distribution can be described as $u(z) = U\left(\frac{z}{h}\right)$ where U is the maximum velocity at the top. Assuming the the exit pressure is the same as the ambient, calculate the flow rate per unit width, the average velocity, and the force on the container.

solution

$$u(z) = U\left(\frac{z}{h}\right) \qquad 0 \le z \le h$$

$$Q = \int_0^h u(z)\,dz = \frac{U}{h}\frac{z^2}{2}\Big|_0^h = U\frac{h}{2}$$

$$u_{av} = \frac{Q}{h} = \frac{U}{2}$$

$$F_x = \rho\int_0^h u^2(z)\,dz = \rho\frac{U^2}{h^2}\frac{z^3}{3}\Big|_0^h = \rho U^2\frac{h}{3}$$

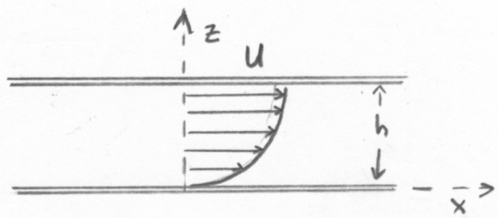
39. The two-dimensional velocity profile between a plate moving at a velocity U and a stationary plate is given by

$$u(z) = U \cdot \sin\left(\frac{\pi z}{2h}\right) \qquad 0 \le z \le h$$

Calculate the two-dimensional flow rate (per unit width) crossing the x = 0 plane

solution

(19)



$$Q = \int_0^h u(z)\,dz = U\int_0^h \sin\frac{\pi z}{2h}\,dz = -U_\infty\frac{2h}{\pi}\cos\frac{\pi z}{2h}\Big|_0^h = \frac{2Uh}{\pi}$$

40. Calculate the shear stress on the upper and lower walls in the previous problem.

solution

$$\tau = \mu\frac{du}{dz} = \mu U\cos\left(\frac{\pi z}{2h}\right)$$

$$\tau_h = 0$$

$$\tau_0 = \mu U$$